A mixed integer programming approach to reduce fuel load accumulation for prescribed burn planning

Ramya Rachmawati[a,*], *Melih Ozlen*[a], *John W Hearne*[a], *Karin Reinke*[a]

[a]School of Mathematical and Geospatial Sciences RMIT University, GPO Box 2476, Melbourne, Victoria, 3001, Australia

*Corresponding author. Phone number +61 39925 3767.

Email addresses: ramya.rachmawati@rmit.edu.au; melih.ozlen@rmit.edu.au; john.hearne@rmit.edu.au; karin.reinke@rmit.edu.au

Abstract

The increasing frequency of destructive wild land fires, with a consequent loss of life and property, has led to fire and land management agencies initiating extensive fuel management programs. This involves long-term scheduling of the location of fuel reduction activities such as prescribed burning or mechanical clearing. In this paper a Mixed Integer Programming (MIP) model is formulated which includes multiple vegetation types in the landscape. The model keeps track of the age of each vegetation class in each treatment unit. Each vegetation type is subject to a minimum and maximum Tolerable Fire Interval (TFI). The objective is to minimise fuel load over the planning horizon. The efficacy of the model was tested using randomised data from 711 treatment units in the Barwon-Otway district of Victoria. For a landscape comprising this many treatment units, solutions could be obtained for up to 10-year prescribed burn planning using a standard implementation utilising CPLEX.

Keywords: MIP, prescribed burns, fuel reduction planning, optimisation, wildfires, fuel management

1 Introduction

Fire is a natural ecosystem process. However, uncontrolled wildfires can cause significant damage. Loss of human life, destruction of properties and natural resources are amongst the problems caused by wildfires (King et al., 2008). An increase in wildfire severity and extent has been noted in many countries such as the USA, Canada, Australia and also in southern Europe (Boer et al., 2009). This is, to some extent, due to fire suppression-focused twentieth century fire management practices, which according to Loehle (2004) and Reinhardt et al. (2008) results in uncharacteristically high fuel loads. There are three key factors affecting fire behaviour: fuel, weather, and topography. Among these factors, only fuel can be actively controlled (Schmidt et al., 2008).

Kim et al. (2009) identify two ways to prepare for wildfires: firstly, by providing fire suppression forces such as air tankers or fire crews adjacent to communities to prevent fires from escaping; and secondly, by altering fuel loads in the surrounding landscape to lessen the potential for severe fires. Finney (2007) and Reinhardt et al. (2008) recommend reducing fuel load as the best possible method to slow fire growth. This is because suppression efforts usually fail in tackling large destructive fires due to hot, dry and windy weather conditions. Fuel management is the process of altering the amount and structure of fuels through methods including prescribed burning and mechanical clearing (King et al., 2008). Fuel management is undertaken for both wildfire hazard reduction and ecological restoration (Reinhardt et al., 2008; Penman et al., 2011). In this paper, fuel treatment will be considered as a means of reducing the accumulated fuel, and therefore, the risk posed by wildfires.

Fuel management is a complex activity that involves both spatial and temporal decisions (Belval et al., 2014). The development of decision support tools for fuel management programs is an ongoing and active research area Martell (2011). Operations Research (OR) has been successfully applied to a wide range of problems related to fire management, forestry management, and ecological management (Martelll, 2007) and has great value in providing land management agencies with a framework for optimising fuel reduction planning over the landscape. A discussion of OR techniques used for solving fuel management problems can be found in the review paper by Minas et al. (2012). As an example, Wei et al. (2008) formulated an integer programming approach to reduce expected loss incurred on a landscape. Wei (2012) later proposed a Mixed Integer Programming (MIP) method to locate fuel reduction treatments to set up potential control locations for future fires. Minas et al. (2014) developed a model that deals with fuel treatment scheduling to break the connectivity of high risk treatment units applied in a landscape. However, a limitation of the models proposed, such as that by Minas et al. (2014) is that it only handles a single vegetation type, and fuel accumulation is a linear function of time. In reality, the fire landscape is made up of multiple vegetation types, of mixed ages, with fuel accumulation taking on non-linear functions depending on vegetation type.

In this paper, we propose a MIP approach to solve a multi-period prescribed burn planning problem.

The objective function of the model is to reduce fuel load accumulation in a landscape with multiple vegetation types of mixed ages, with non-linear fuel accumulation functions. It is the first multi-period and multi-vegetation landscape fuel treatment model. We illustrate the method with a case study in the Barwon-Otway district of Victoria.

2 Problem formulation

In this paper, the candidate locations for fuel reduction treatments are represented by 'treatment units'. A treatment unit is a spatial feature consisting of key attributes relating to the size of the area (treatment unit), the land ownership (public or private), vegetation type and vegetation age, each attributes as critical information to the problem.

The question addressed by the model in this paper is where and when to conduct fuel reduction treatments in order to minimise the risk of wildfire, by minimising the total fuel load accumulation, while considering the ecological requirements of the vegetation present. The ecological requirements can be described as the minimum and maximum Tolerable Fire Intervals (TFI). The minimum TFI is defined as the minimum time required between two consecutive fire events at a location and is normally based on the time to reach maturity of the sensitive species in the vegetation class, while the maximum TFI refers to the maximum time needed between fire events at a location that considers the fire interval required for fire-adapted species rejuvenation (Cheal, 2010). A treatment unit should not be treated if the age of vegetation growing in that location is under minimum TFI. In contrast, treatment units with vegetation over the maximum TFI must be treated. In the following section, the mathematical model used in this study is presented.

3 Methods

We consider the landscape divided into treatment units. It is assumed that all the vegetation of each kind is of the same age within each treatment unit. With the decision to determine when and where to treat every year to minimise total fuel load of certain regions, the following mixed integer programming model is formulated.

Sets:

V_i is the set of vegetation types growing in treatment unit i

T is the planning horizon

C is the set of treatment units of which total fuel load is to be minimised

Indices:

i = treatment unit

j= vegetation type

k = vegetation age

t = period, $t = 0, 1, 2, \ldots$

Parameters:

w_i = relative importance (weight) of treatment unit i

$m_{i,j}$ = initial age of vegetation type j at treatment unit i

$A_{i,j}$ = area of treatment unit i with vegetation type j

ρ = treatment level (in percentage), i.e. the maximum proportion of the total treatable area in a landscape selected for treatment

R = the total treatable area in a landscape

c_i= area of treatment unit i

$L_{j,k}$ = fuel load (ton/hectare) of vegetation j, at age k

$maxTFI_j$= maximum TFI of vegetation type j

$minTFI_j$ = minimum TFI of vegetation type j

Decision variables:

$$x_{i,t} = \begin{cases} 1 & \text{if treatment unit } i \text{ is treated in time period } t \\ 0 & \text{otherwise} \end{cases}$$

$$y_{i,j,k,t} = \begin{cases} 1 & \text{if in treatment unit } i\text{, there is vegetation type } j\text{, at age } k\text{, in time } t \\ 0 & \text{otherwise} \end{cases}$$

minimise total weighted fuel load

$$z = \sum_{t=1}^{T}\sum_{k}\sum_{j\in V_i}\sum_{i\in C} w_i L_{j,k} A_{i,j} y_{i,j,k,t} \tag{1}$$

subject to

$$y_{i,j,k,0} = 1,\ \forall i, j \in V_i,\ k\ = m_{i,j} \tag{2}$$

$$y_{i,j,k+1,t+1} \geq y_{i,j,k,t} - x_{i,t},\ \forall i, j \in V_i,\ k\ = 1, 2, \ldots, maxTFI_j - 1, \forall t \tag{3}$$

$$y_{i,j,k,t} \leq x_{i,t},\ \forall i, j \in V_i, \forall t, \text{ for } k = maxTFI_j \tag{4}$$

$$y_{i,j,1,t+1} \geq x_{i,t},\ \forall i, j \in V_i, \forall t \tag{5}$$

$$\sum_{k} y_{i,j,k,t} \leq 1,\ \forall i, j \in V_i, \forall t \tag{6}$$

$$\sum_{j\in V_i}\sum_{k<minTFI_j} y_{i,j,k,t} - \mid V_i \mid \sum_{j\in V_i}\sum_{k=maxTFI_j} y_{i,j,k,t} \leq \mid V_i \mid (1 - x_{i,t}),\ \forall i, j \in V_i, \forall t \tag{7}$$

$$\sum_{i} c_i x_{i,t} \leq \rho R,\ \forall t \tag{8}$$

$$y_{i,j,k,t} \in \{0, 1\} \tag{9}$$

$$x_{i,t} \in \{0, 1\} \tag{10}$$

The objective function (1) minimises the weighted total fuel load of all vegetation at all regions throughout a planning horizon.

Constraint (2) sets the initial conditions. Constraint (3) indicates that when $x_{i,t} = 0$, which means fuel treatment is not conducted, the vegetation in that area will continue growing until the following period, and the age will be incremented by one.

Constraint (4) ensures that vegetation will be treated once it has reached maximum TFI. The vegetation with age 1 in the next period comes from the areas that are treated in the current period, as denoted in constraint (5). Constraint (6) ensures a unique age for each vegetation type within each treatment unit for each time period. Constraint (7) enforces that the vegetation under minimum TFI cannot be treated unless there is another vegetation type in the same treatment unit which is over the maximum TFI to avoid a deadlock. Here $\mid V_i \mid$ represents the number of different vegetation types in treatment unit i.

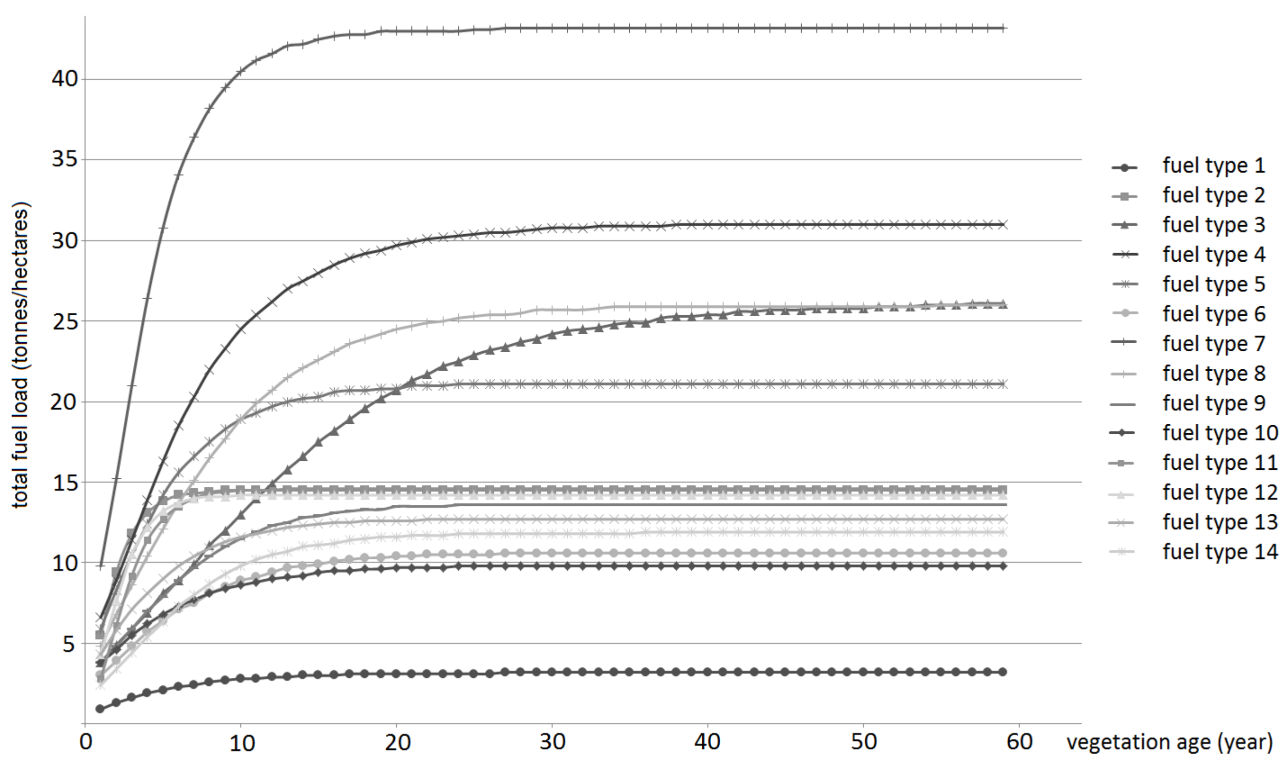
Constraint (8) specifies that the total area selected for fuel treatment each year is not more than the annual area allotted (target) for fuel treatment (in hectares). Here, the target is obtained by multiplying the treatment level and the total treatable area in a landscape. Constraint (9) and (10) ensure that the decision variables $y_{i,j,k,t}$ and $x_{i,t}$ take binary values.

The model is capable of handling multiple vegetation types and ages. Each vegetation type has different minimum and maximum TFI, and at any period each vegetation type may have a different age even within a single treatment unit. The fuel curve representing each age of certain vegetation can also be a nonlinear function (Figure 1).

3.1 Model improvement

The solution time of a mixed integer programming problem can generally be improved by reducing the number of variables, or restricting the values that they can take. Age index k should be based on the set of possible ages that vegetation type j can take in treatment unit i at time t. The maximum possible periods between two consecutive treatments for any treatment unit can be derived by finding

Fig. 1: Fuel load accumulation curves over time for different fuel types listed for the Barwon-Otway



the minimum of the maximum TFI values of all vegetation types available within that unit. This sets an upper limit on the values k can take within that treatment unit.

We can also tighten the mixed integer programming formulation by introducing valid inequalities on the frequency of treatment event in each unit as follows

$a_{i,j}$ = initial age of vegetation type j at treatment unit i

$q = \min(maxTFI_j - a_{i,j})$

$$\sum_{t=0}^{q-1} x_{i,t} \geq 1,\ \forall i \tag{11}$$

$p = \min(maxTFI_j)$

$$\sum_{t}^{t+p-1} x_{i,t} \geq 1,\ \forall i\ for\ t = 0, 1, ..., T - p \tag{12}$$

$$x_{i,t} = 0,\ \forall i, \forall t < \min(\min(minTFI_j - a_{i,j}), \min(maxTFI_j - a_{i,j})),\ j \in V_i \tag{13}$$

Constraint (11) ensures that a treatment unit will be treated when the most critical vegetation type (i.e., the vegetation type which sets the minimum of the maximum TFI value among all vegetation types available within a treatment unit) reaches its maximum TFI. In other words, we have to treat the treatment unit at some time in the first q periods of the planning horizon. Constraint (12) generalises this idea to the rest of the planning horizon by setting a frequency to treat. It ensures that treatment unit i must be treated at least once every p years. It is assumed that each treatment unit has a critical vegetation type (i.e. the vegetation in the treatment unit which has the least maximum TFI) that determines the treatment cycle. However, constraint (12) can only help to speed up the computation time when the planning horizon is longer than the burning frequency in the treatment units. Constraint (13) reduces the number of binary variables by setting the burn variables to 0 for burns that are not allowed based on the TFI values. We can also improve solution time by treating variable $y_{i,j,k,t}$ as a continuous variable instead of a binary variable. In other words, we replace constraint (9) with constraint (14) as follows.

$$0 \leq y_{i,j,k,t} \leq 1 \tag{14}$$

3.2 Implementation approach

In this paper, the model is run in ten-yearly planning horizons, in part dictated by computational efficiency. As an illustration, at year = 0, the model is run for prescribed burn planning for the first ten years. The solution of the model suggests which treatment units should be treated every year. Then, the initial vegetation age data is updated based on the solution and the unplanned wildfires in that area. At year = 10, the model is re-run based on the updated data. This process continues for subsequent ten-year planning horizons. At the ρ treatment level, the solution will be infeasible if at the end of the first ten years, there is more than ρ of the landscape that is over the maximum Tolerable Fire Interval (TFI). In this study, we define an 'old treatment unit' as a treatment unit that that comprises some vegetation type whose age is over the maximum TFI. Constraints (15) and (16) are imposed on the model to avoid infeasibility.

$$\theta_i = \begin{cases} 1 & \text{if treatment unit } i \text{ is classified as an 'old treatment unit' at the end of a planning horizon } T \\ 0 & \text{otherwise} \end{cases}$$

α_i is the area of treatment unit i;

ρ is the treatment level (in percentage);

R is the total treatable area in a landscape;

V_i is the set of vegetation type growing in treatment unit i

$$\sum_i \alpha_i \theta_i \leq \rho R \tag{15}$$

$$\sum_{j \in V_i} \sum_{k = maxTFI_j} y_{i,j,k,t} \leq \mid V_i \mid \theta, \qquad \forall i, t = T \tag{16}$$

Constraint (15) potentially ensures that at the end of each planning horizon, the 'old treatment units' are less than ρ of the total area of the landscape. Constraint (16) enforces that θ_i is equal to

Fig. 2: Location of the case study in the Barwon-Otway district of Victoria, Australia

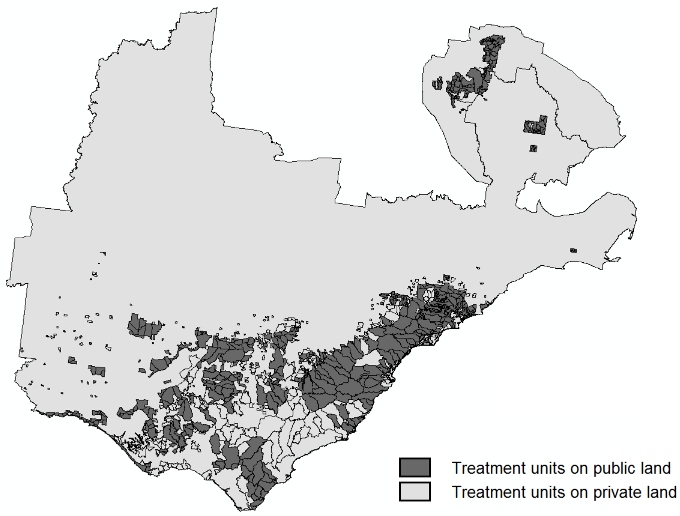
one if there are parts of a treatment unit in which the vegetation age is over the maximum TFI. We hope to achieve feasibility with these constraints. However, if the constraints cannot be met, we can use the Phase 1 approach that is discussed in the following section to guarantee feasibility.

4 An Australian case study

An Australian case study is presented to demonstrate the model. The study location is situated in the Barwon-Otway district of Victoria, Australia, and covers approximately 1,150,000 hectares (Figure 2). Data used in this case study considers land ownership, vegetation type and age in each treatment unit, minimum and maximum TFI, and fuel load for the specific age of vegetation. In this case study, we categorise the treatment units according to land ownership (i.e. public or private). It is assumed that treatments can only occur on public land, so the candidate locations for prescribed burn planning are represented in these treatment units only. A total of 711 of treatment units exist over 73,535 hectares. Figure 3 shows the public land treatment units.

Each vegetation type in this case study has its own fuel type and fuel accumulation loads over time as described in Figure 1. The curves show that each vegetation type has a different level of fuel load

Fig. 3: Map showing the distribution of treatment units within the Barwon-Otway case study area case study area



depending on age. In addition, there are some aquatic vegetation types or communities that have zero fuel loads and as such require no treatment. In this paper those vegetation types are excluded. Table 1 lists the Ecological Vegetation Class (EVC) name and its fuel type used in this case study.

The case study is conducted in two Phases. Phase 1 is a preliminary stage before the Phase 2 approach (described in the methods section) is executed. In Phase 1, the 'old treatment units' in the landscape are identified. The purpose of Phase 1 is to handle any infeasibility that might arise based on the initial data. This is necessary for ensuring feasibility of the Phase 2 approach. Infeasibility may arise due to conflicting constraints, especially constraints (4), (7) and (8). The Phase 2 approach requires that all 'old treatment units' must be treated. However, treating all of these old treatment units (in this case, 35 percent of the total treatable area in the landscape, as can be seen in Figure 4a) would be costly and impractical in a single year. Moreover, The 2009 Victorian Bushfires Royal Commission nominates a target of five percent of the public land to be treated each year across the state in order to reduce the threat of fire for the coming fire season (Teague et al., 2010). Using a five percent treatment level across the case study area means that imposing the maximum TFI leads to infeasibility of the Phase 2 approach. Therefore, to reduce the number of 'old treatment units' and achieve feasibility first, in Phase 1 the treatment level must be increased. For Phase 1 of the case study, a treatment level of seven percent of the total area of the landscape each year is imposed. Interestingly, (Penman et al., 2011) note that when more than seven percent of the total area has been burnt by

Tab. 1: Ecological Vegetation Class (EVC) and associated fuel types

EVC name	min TFI	max TFI	fuel type	area (hectare)	area (percentage)	initial fuel load (ton)
Creekline Grassy Woodland	20	150	7	6.14	0.008	65.08
Hills Herb-rich Woodland	15	150	7	641.42	0.872	6545.51
Creekline Herb-rich Woodland	15	150	7	281.36	0.383	2409.04
Grassy Woodland	5	45	7	141.44	0.192	1285.21
Valley Slopes Dry Forest	10	100	7	12.40	0.017	131.44
Sedgy Riparian Woodland	20	85	7	532.54	0.724	4946.06
Scoria Cone Woodland	4	15	7	20.74	0.028	219.84
Wet Forest	45	300	9	218.10	0.297	9396.53
Shrubby Wet Forest	25	150	9	825.47	1.123	34644.30
Riparian Forest	10	80	10	3.56	0.005	92.29
Swampy Riparian Woodland	15	125	10	1.89	0.003	43.65
Riparian Scrub or Swampy Riparian Woodland Complex	10	80	11	2561.76	3.484	30299.40
Wet Sands Thicket	15	90	11	27.27	0.037	370.87
Stream Bank Shrubland	15	90	11	38.32	0.052	521.15
Cool Temperate Rainforest	45	999	1	0.60	0.001	5.88
Wet Heathland	12	45	13	1416.63	1.926	18692.73
Damp Heath Scrub	10	90	13	1142.88	1.554	15908.60
Damp Heath Scrub/Heathy Woodland Complex	10	90	13	16.05	0.022	234.33
Sand Heathland	8	45	14	132.81	0.181	1684.73
Clay Heathland	10	45	14	30.58	0.042	405.60
Coastal Dune Scrubor Coastal Dune Grassland Mosaic	10	90	1	253.84	0.345	3016.53
Coastal Headland Scrub	8	90	1	1077.69	1.466	12587.77
Coastal Headland Scrub/Coastal Tussock Grassland Mosaic	8	90	1	98.98	0.135	1177.86
Coast Gully Thicket	10	90	1	1.67	0.002	15.52
Coastal Alkaline Scrub	10	70	1	11.82	0.016	140.65
Coastal Saltmarsh/Mangrove Shrubland Mosaic	8	90	2	4.52	0.006	14.46
Coastal Tussock Grassland	5	40	3	260.27	0.354	3773.91
Heathy Woodland	5	45	4	15985.16	21.738	313589.23
Shrubby Woodland	10	45	4	220.56	0.300	3465.91
Lowland Forest	8	80	5	21454.24	29.175	574823.49
Heathy Dry Forest	10	45	5	3958.52	5.383	95741.43
Shrubby Dry Forest	5	45	5	2299.87	3.128	64937.21
Grassy Dry Forest	5	45	6	2006.33	2.728	38475.14
Herb rich Foothill Forest	8	90	6	1670.13	2.271	34302.81
Shrubby Foothill Forest	8	90	6	12945.85	17.605	258807.84
Herb-rich Foothill Forest/Shrubby Foothill Forest Complex	8	90	6	2027.99	2.758	39253.237
Damp Sands Herb Rich Woodland	10	90	7	270.13	0.367	2776.23
Valley Grassy Forest	10	100	7	397.99	0.541	4054.89
Plains Grassy Woodland	4	15	7	482.38	0.656	4589.66
Alluvial Terraces Herb-Rich Woodland	4	15	7	56.07	0.076	594.34

prescribed fire, the total area burnt by unplanned fire will be close to zero. This Phase corresponds to the following MIP:

L_i is the total fuel load of 'old treatment unit' i

$$x_{i,t} = \begin{cases} 1 & \text{if 'old treatment unit' } i \text{ is treated in time period } t \\ 0 & \text{otherwise} \end{cases}$$

A_i is area of 'old treatment unit' i

R is the total treatable area of the landscape

maximise total fuel load:

$$z = \sum_{t=0} L_i x_{i,t} \tag{17}$$

subject to

$$\sum_i A_i x_{i,t} \leq 0.07R, \qquad t = 0 \tag{18}$$

The objective function (17) is to maximise total fuel load of all 'old treatment units', subject to the single constraint (18). This constraint limits the area that can be treated per year to be seven percent of the total treatable area in the landscape. The model will choose seven percent of the treatment units containing the highest fuel load to be burned every year. The model, which is a basic knapsack problem, is solved for consecutive years using the solution of the previous year as an input until the problem is reconciled, containing less than five percent of 'old treatment units' in the landscape, as can be seen in Figure 4. Based on the initial data, it would take six years to achieve that for our case study. The model data, now feasible, enables us to move to Phase 2.

In Phase 2, the model described in the methods section is applied in ten-yearly planning horizons. The objective function is to minimise the total fuel load whilst meeting the constraints that have been

Fig. 4: Solution of Phase 1

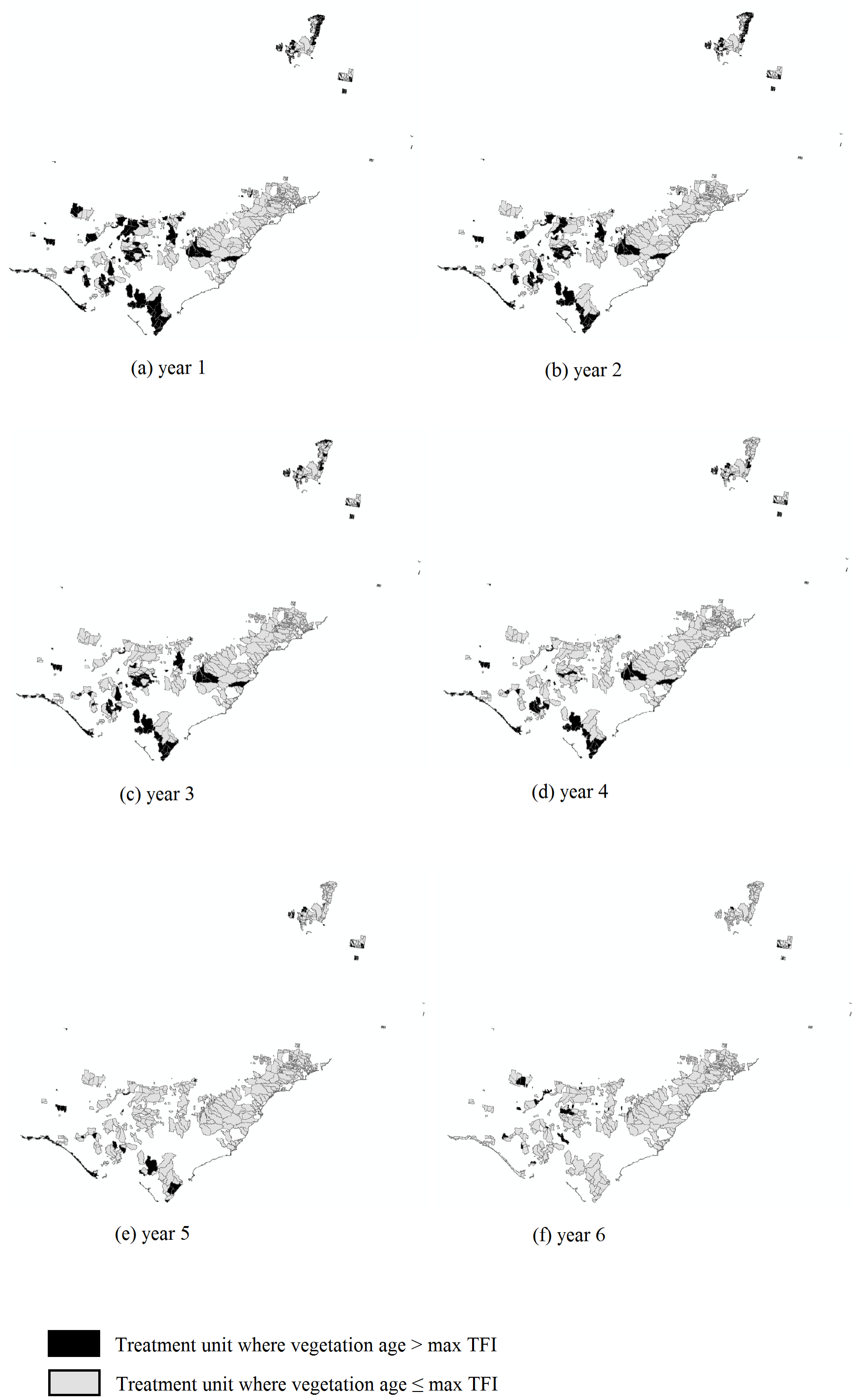
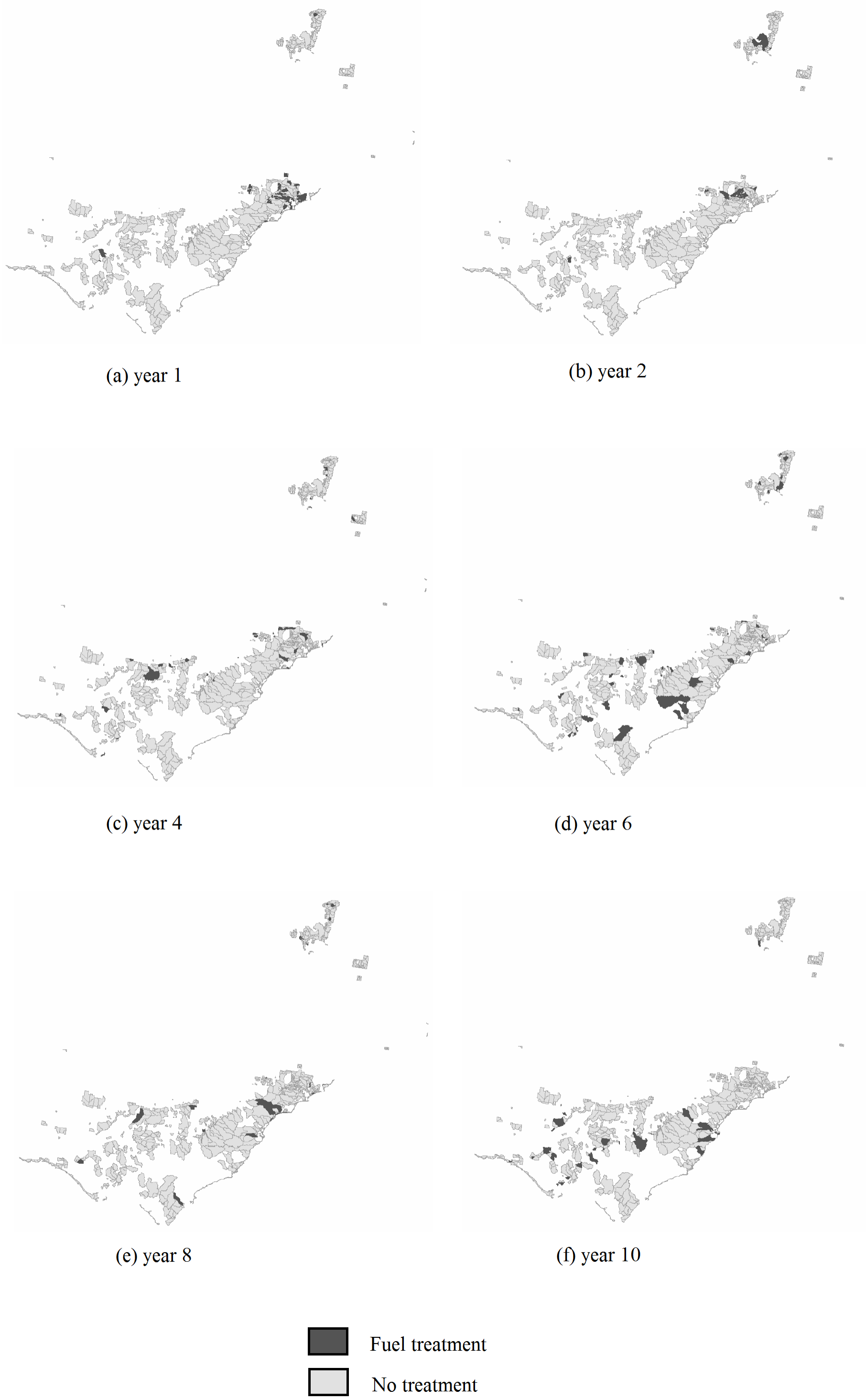


Fig. 5: Solution of Phase 2



Tab. 2: Computational comparison between the three model configurations using a 5% treatment level

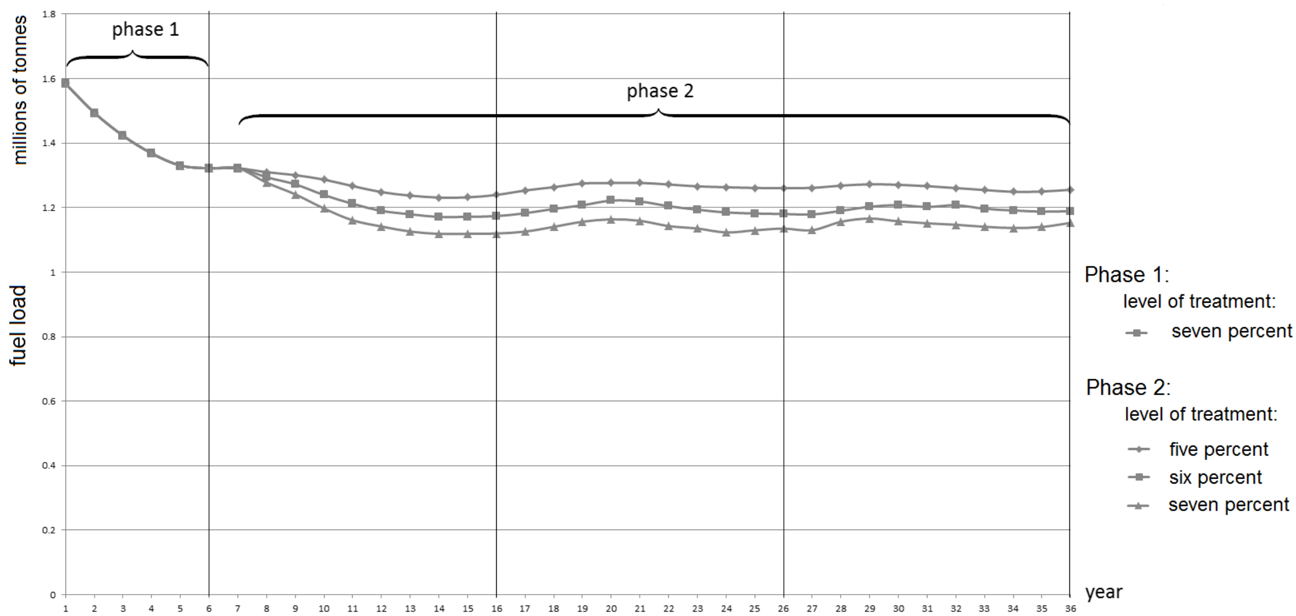
Length of planning horizon	Solution time (seconds) or optimality gap (%) at 10800 seconds		
	'total'	'subset'	'random'
5 years	6.35	3.55	13.24
10 years	901.21	33.45	794.61
15 years	(1.88%)	19764.57	(1.4%)
20 years	(6.58%)	(1.16%)	(4.91%)
25 years	(11.73%)	(3.17%)	(8.81%)

described in methods section. Figure 5 represents the result of Phase 2 and identifies the location of treatments for each year to minimise the total fuel load while satisfying the minimum and maximum TFI constraints. The length of the planning horizon is 10 years and the treatment level of each year is less than or equal to five percent.

The model is solved using ILOG CPLEX 12.6 with the Python 2.7 programming language. Computational experiments are performed on Trifid, a V3 Alliance high performance computer cluster. Computation time against different model configurations is tested and the results are represented in Table 2. The CPU time or the gap between the best solution identified and the current linear programming relaxation is presented. The solution may actually be optimal but CPLEX may need a long time to prove it. The three model configurations are: 'total' (total fuel load where all treatment units are considered equal), 'subset' (total fuel load where a subset of treatment units are prioritised) and 'random' (total fuel load where random weights are assigned to treatment units). In 'total', all w_i's $= 1$. It means that the model minimises the total fuel load in all treatment units in the landscape, without prioritising certain regions. In 'subset', the value of $w_i = 1$ for some priority regions, and $w_i = 0$ for the other region. This priority may be due to proximity to towns. In 'random', $0 < w_i < 1$ assigned a relative importance weight to treatment unit which may be based on the population at risk or any other measure of defining relative importance.

The optimal solutions of Phase 1 and Phase 2 are represented in Figure 4 and Figure 5, respectively. The solutions suggest where and when to conduct fuel treatments so as to minimise total fuel load, and in turn reduce the risk posed by wildfires. Figure 6 summarises the total fuel load over time for Phase 1 and Phase 2 for five, six and seven percent annual treatment levels. From the graph it

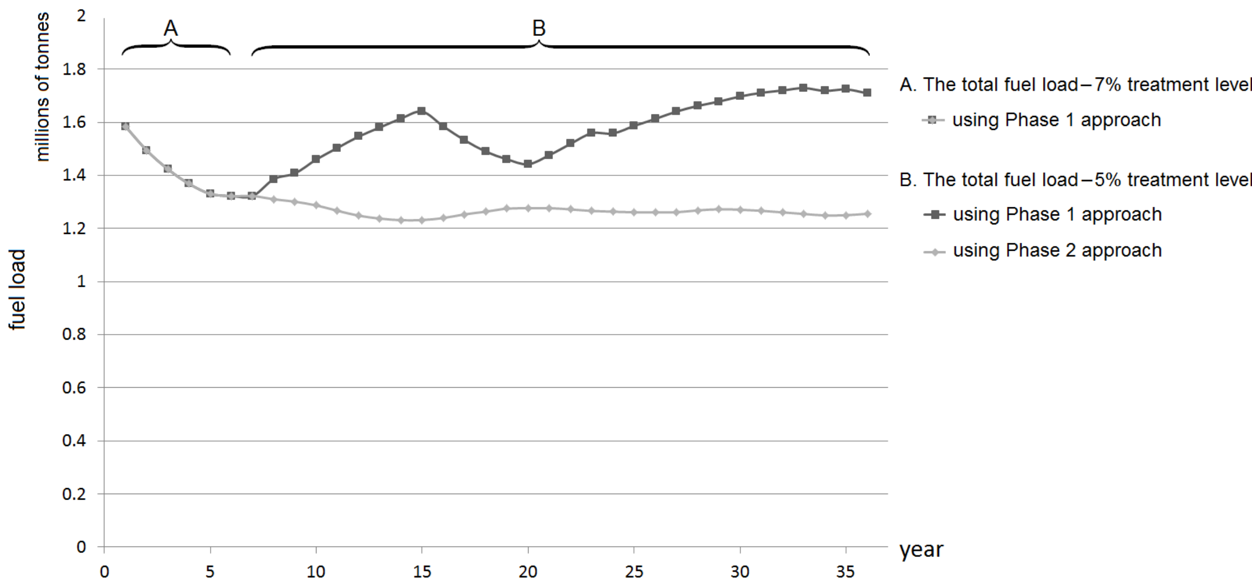
Fig. 6: Total fuel load over time



is clear that the seven percent treatment level has the least total fuel load at every point in time, which is to be expected. However, a five percent treatment level has the most stable total fuel load in the long term. In other words, less variation is seen between years. For Phase 2 (i.e. from year 7 to 36), the approximate mean total fuel load and standard deviations in the landscape for five, six and seven percent treatment level are 1.265 million tonnes (standard deviation 21,000 tonnes), 1.205 million tonnes (standard deviation 35,000 tonnes) and 1.157 million tonnes (standard deviation 46,000), respectively.

By using the Phase 1 approach solely, only the 'old treatment units' can be treated. By minimising the total fuel load, Phase 2 approach provides a better optimal solution (less total fuel load) than that of the Phase 1 approach only, as can be seen in Figure 7. This is because in the Phase 1 approach, the treatment units can only be treated if there is a vegetation type in the treatment unit that hits the maximum TFI. In the MIP approach, we can treat a treatment unit even if the vegetation age that grows in the treatment unit is above the minimum TFI and below the maximum TFI. In summary, the Phase 2 approach gives us more options to optimise the total fuel load than that of the Phase 1 approach.

Fig. 7: Comparison of the total fuel load using Phase 1 approach and Phase 2 approach



5 Conclusion

The model developed in this paper, and embedded in a real-world case study, demonstrates the utility offered by using a mixed integer programming approach for finding optimal prescribed burning schedules that include multiple vegetation types of mixed ages in the landscape. The model takes into account both the spatial and temporal changes in the landscape. Hence it provides land managers with a more realistic planning approach to prescribed burning scheduling.

The model determines when and where to conduct fuel treatment in order to reduce the total fuel load in a landscape, while also considering the ecological constraints relating to the Tolerable Fire Interval of each vegetation community. The model keeps track of the vegetation age in each treatment unit in each planning cycle. It was applied to a 10-yearly prescribed burn planning schedule using real data belonging to the Barwon-Otway district of Victoria. The data consisted of 711 treatment units with each treatment unit consisting of sub-units to further describe the landscape in terms of land tenure, vegetation type and age, amongst others. The solution was obtained in a reasonable computational time ($<$ 15 minutes) for such a long-term planning exercise. The case study shows that the Phase 2 approach provides a better optimal solution than that of the Phase 1 approach alone.

Future research is planned to extend this study by incorporating other ecological requirements such as habitat connectivity in the landscape. However, this added complexity will increase computational effort and there may be a need to develop heuristic approaches to these problems as we increase the number of constraints and objectives built into our models. This paper shows how a mixed integer programming approach can be used to efficiently address multi-objective, spatio-temporal planning schedules for prescribed burning, and may serve as a benchmark for future studies.

Acknowledgement

The second author is supported by the Australian Research Council under the Discovery Projects funding scheme (project DP140104246).

References

Belval, E. J., Y. Wei, and M. Bevers (2014). A mixed integer program to model spatial wildfire behavior and suppression placement decisions. *Canadian Journal of Forest Research* (ja).

Boer, M., R. Sadler, R. Wittkuhn, L. McCaw, and P. Grierson (2009). Long-term impacts of prescribed burning on regional extent and incidence of wildfires-evidence from 50 years of active fire management in SW Australian forests. *Forest Ecology and Management 259*(1), 132–142.

Cheal, D. (2010). Growth stages and tolerable fire intervals for Victoria's native vegetation data sets. In *Fire and adaptive management report no. 84*. Department of Sustainability and Environment, East Melbourne, Victoria, Australia.

Finney, M. (2007). A computational method for optimising fuel treatment locations. *International Journal of Wildland Fire 16*(6), 702–711.

Kim, Y.-H., P. Bettinger, and M. Finney (2009). Spatial optimization of the pattern of fuel management activities and subsequent effects on simulated wildfires. *European Journal of Operational Research 197*(1), 253–265.

King, K., R. Bradstock, G. Cary, J. Chapman, and J. Marsden-Smedley (2008). The relative importance of fine-scale fuel mosaics on reducing fire risk in South-West Tasmania, Australia. *International Journal of Wildland Fire 17*(3), 421–430.

Loehle, C. (2004). Applying landscape principles to fire hazard reduction. *Forest Ecology and Management 198*(1-3), 261–267.

Martell, D. (2011). The development and implementation of forest and wildland fire management decision support systems: reflections on past practices and emerging needs and challenges. *Mathematical and Computational Forestry & Natural-Resource Sciences (MCFNS) 3*(1), Pages–18.

Martelll, D. (2007). Forest fire management. In A. Weintraub, C. C. Romero, T. Bjorndal, R. R. Epstein, and J. Miranda (Eds.), *Handbook of operations research in natural resources*, pp. 489–509. New York Springer.

Minas, J., J. Hearne, and J. Handmer (2012). A review of operations research methods applicable to wildfire management. *International Journal of Wildland Fire 21*(3), 189–196.

Minas, J., J. Hearne, and D. Martell (2014). A spatial optimisation model for multi-period landscape level fuel management to mitigate wildfire impacts. *European Journal of Operational Research 232*(2), 412–422.

Penman, T., F. Christie, A. Andersen, R. Bradstock, G. Cary, M. Henderson, O. Price, C. Tran, G. Wardle, R. Williams, et al. (2011). Prescribed burning: how can it work to conserve the things we value? *International Journal of Wildland Fire 20*(6), 721–733.

Reinhardt, E. D., R. E. Keane, D. E. Calkin, and J. D. Cohen (2008). Objectives and considerations for wildland fuel treatment in forested ecosystems of the interior western United States. *Forest Ecology and Management 256*(12), 1997 – 2006.

Schmidt, D., A. Taylor, and C. Skinner (2008). The influence of fuels treatment and landscape arrangement on simulated fire behavior, Southern Cascade range, California. *Forest Ecology and Management 255*(8-9), 3170–3184.

Teague, B., R. McLeod, and S. Pascoe (2010). Final report, 2009 victorian bushfires royal commission. *Parliament of Victoria, Melbourne Victoria, Australia*.

Wei, Y. (2012). Optimize landscape fuel treatment locations to create control opportunities for future fires. *Canadian Journal of Forest Research 42*(6), 1002–1014.

Wei, Y., D. Rideout, and A. Kirsch (2008). An optimization model for locating fuel treatments across a landscape to reduce expected fire losses. *Canadian Journal of Forest Research 38*(4), 868–877.