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An Overview on Mathematical Programming Approaches for the Deterministic Unit Commitment Problem in Hydro Valleys

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Abstract With the fast-growing demand in the electricity market of the last decades, attention has been focused on alternative and flexible sources of energy such as hydro valleys. Managing the hydroelectricity produced by the plants in hydro valleys is called the hydro unit commitment problem. This problem consists in finding the optimal power production schedule of a set of hydro units while meeting several technical, physical, and strategic constraints. The hydro unit commitment has always been a crucial and challenging optimization problem, not only because of its strong nonlinear and combinatorial aspects, but also because it is a large-scale problem that has to be solved to (near) optimality in a reasonable amount of time. This paper presents a review on mathematical programming approaches for the deterministic hydro unit commitment problem. We first provide a survey of the different variants of the problem by exposing a variety of the assumptions, objectives, and constraints considered in the literature. Then, we review the main contributions on resolution approaches with a particular focus on methods based on mathematical programming techniques.

Keywords Hydro unit commitment · Hydro valleys · Mathematical programming · MI(N)LP

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1 Introduction

The continuously increasing demand of electricity motivates the study of different and alternative sources of energy. One of the most flexible, then crucial, sources are hydro plants.

For example, at Électricité de France (EdF), the largest French electric utility company, hydro energy is the most widely used renewable energy, i.e., it represents 7.6 % of the total energy production, the second source for amount of energy produced right after nuclear energy (see [41]). More generally speaking, hydroelectricity is produced in 150 countries all over the world and it represents the second source of energy with 16.3 % of the world-wide production and the first renewable energy with 78 % of the world-wide renewable energy production (source: EdF website `http://jeunes.edf.com/article/l-hydraulique-dans-le-monde,89`). For all these reasons, optimizing the generation schedules for large-scale hydro power systems can substantially reduce the operating costs.

We focus on the short term Hydro Unit Commitment (HUC) problem, also known as the Short Term Hydro Scheduling (STHS) problem, that aims at finding an optimal scheduling and operational level of plants' units for hydro valleys for a short term period ranging from several hours to a few days. For mid and long term, the reader can, for instance, refer to [10,31,74,84,85,88,124,131,138].

The STHS represents just a subproblem of the entire unit commitment problem solved daily by electric utility companies and it is very challenging, see, for example, [67]. The causes of the difficulties can be identified as the intrinsic nonlinear and combinatorial aspects that will be listed in the next section. Clearly, an additional challenge resides in the fact that utility companies are typically interested in solving large-scale instances.

From a mathematical point of view, the problem has a mixed integer nonlinear and nonconvex structure. The nonlinearity (nonconvexity) is due to the turbines output characteristics, i.e., the produced power depends nonlinearly on the water flow and the head effect. The combinatorial aspect is caused, for example, by the discreteness of some elements of the problem such as the startup/shutdown costs and on/off status of the hydro units. STHS poses also discontinuous optimization problem mainly because of forbidden operational zones.

This makes the HUC problem hard to handle and solve to optimality. Different mathematical programming techniques have hence been used such as Mixed Integer Linear (or Nonlinear) Programming (MI(N)LP), dynamic programming, decomposition methods, Lagrangian relaxation, and (meta)heuristics, which will be detailed in the sequel. However, the focus of the paper will be on exact methods, while we will discuss (quickly) some (meta)heuristic approaches without being exhaustive.

A few decades ago, realistic modeling was not possible because of intractability of the problem to the "at that time" state of the art solvers. Thus, the typical approach was to consider only a subset of the real constraints and to simplify the functions involved. For example, highly nonlinear function were assumed to be univariate, linear or concave (as they appeared in a maximization objective function), no head effect was considered, and, at the very beginning, no discontinuities or fixed costs were taken into account so as to stick with continuous variables. Over the past years, efforts were focused on providing more accurate and computationally efficient models for real-world problems. As the computers become more and more powerful, the approaches to solve MI(N)LP problems improve and the size of the instances that are considered tractable grows.

In the scope of this paper, we are situated in the deterministic context. That is, we suppose that the day-ahead price, demand, and inflows are known in advance. In the literature, several stochastic HUC problems have been studied, see, for example, [1,7,37,47,48,57,68,107,108]. For a general overview, the reader is referred to the recent survey by Tahanan et al. [126].

For extensive literature surveys on the general UC problem, the reader can refer to [5,14,101,120,129]. To the best of our knowledge, the only surveys on the hydro UC problem are [75] and the most recent one [99].

As several abbreviations will be used in the rest of the paper, we end the Introduction by summaryzing them in Table 1.

DP	Dynamic programming
EdF	Électricité de France
HUC	Hydro unit commitment
GS	Generation schedule
LR	Lagrangian relaxation
MILP	Mixed integer linear programming
MINLP	Mixed integer nonlinear programming
NLP	Nonlinear programming
PSO	Particle swarm optimization
STHS	Short term hydro scheduling
TGP	Three Gorges Project

Table 1 Table of abbreviations.

2 The Hydro Unit Commitment problem

The hydro system can be divided in a set of hydro chains or valleys that consist of several reservoirs. Each reservoir is associated to a hydro plant composed of different or similar hydro units. Each unit can work as turbine and/or pump. The plants of a given valley can be connected either in series or in parallel. The release of an upstream plant contributes to the inflow of downstream plants together with "natural" inflows given by rivers, rain, etc.

The HUC problem aims at determining the startup and shutdown slots for turbines and pumping units together with the operational level of each unit.

It also involves the scheduling of activated units over a specific short term time horizon, in order to minimize the operational cost or to maximize the profit of a generating company. We assume that the "natural" water inflows are forecast. As for the electricity prices, we discuss it in the next section.

The problem is difficult even in the case of one reservoir. As mentioned in [137], hydro scheduling has complicated discrete and dynamic operating constraints and multiple operating zones. The authors also discuss the feasibility problem, i.e., finding a feasible solution or prove that the problem is infeasible, that might be challenging in practice for the HUC.

In the following, we report a very simplified mathematical model to show the main characteristics of the HUC problem. Note that, for simplicity, we assume that a time period is needed to the water to go from a reservoir to its upstream/downstream reservoirs. Let us now define sets

- I: plants/reservoirs set.
- T: set of time periods.
- I_i^+: upstream reservoirs set for reservoir i ($i \in I$).
- I_i^-: upstream reservoirs set for reservoir i ($i \in I$).

and variables

- q_{it}: water flow passing through plant i in time period t.
- p_{it}: generated/consumed power by plant i in time period t.
- v_{it}: water volume in reservoir i at time period t.

A simple model can then be written as follows

$$\max \sum_{i \in I} \sum_{t \in T} \lambda_{it} p_{it} \tag{1}$$

$$p_{it} = \Phi(q_{it}, v_{it}) \qquad \forall t, \forall i \tag{2}$$

$$v_{it} = v_{i(t-1)} + I_{it} + \Delta T(-q_{it} + \sum_{r \in I_i^+} q_{r(t-1)} - \sum_{r \in I_i^-} q_{r(t-1)}) \qquad \forall t, \forall i \tag{3}$$

$$q_{it} \in \{Q_i^-\} \cup \{0\} \cup [\underline{Q}_i, \overline{Q}_i] \qquad \forall t, \forall i \tag{4}$$

$$\underline{V}_i \leq v_{it} \leq \overline{V}_i \qquad \forall t, \forall i \tag{5}$$

where λ_{ij} are the marginal costs, Φ is a nonconcave function $\mathbb{R}^2 \to \mathbb{R}$, I_{it} are the external inflows, ΔT is the duration of a time period in seconds. The objective (1) is to maximize the profit while satisfying constraints (2)-(5). Constraints (2) give the produced power with respect to the water flow and the head-dependency. Constraints (3) ensure the water balance at the reservoirs at each time period. Constraints (4) and (5) represent simple bounds on the water flow passing through a plant and on the reservoirs water volume, respectively.

In the next two sections we discuss in details the objective functions and constraints that have been considered in the literature.

2.1 HUC bjective functions

Utility companies can be divided in two categories: the price takers and price makers.

A *price taker* is a utility company acting in the day-ahead electricity market and small enough to have a negligible influence in the market. In this case, we can consider the electricity price to be exogenous, thus they can be considered as an input parameter and, in our case, they are forecast.

A *price maker* is a utility company influential enough to affect the energy market where it acts and, in particular, the electricity price (see, for example, [109]). In this case, bidding strategies considering the effect of the plant selling bids on the market clearing procedure is crucial. The literature is rich of papers on bidding strategies. However, they are not peculiar of the hydro unit commitment, therefore we will focus on the price taker context. Moreover, as mentioned in Section 1, the HUC is a subproblem of the whole UC problem that price maker utility companies are interested in solving. Typically, UC is decomposed through Lagrangian decomposition and the objective is to maximize the profit given by the price signals, see, for example, [67]. Thus, we can assume the price taker context being of interest also for price maker utility companies.

Profit maximization is a classic objective function [37,69]. However, other conflicting objectives can be identified. Examples of such objectives follow:

- Keep the storage volume at each reservoir as close as possible to the target volume [39,67]. If we just consider to maximize the profit, the optimization process will guide us so as to consume all the water in the reservoir because power production means water consumption and the search for optimality would guide us to empty the reservoir. This is caused by the fact that we do not take into account the future, i.e., that we will need water tomorrow to produce power. The target volume has the aim of avoiding to consume too much water and to take into account the future need of water. An alternative could be giving a value to the water left in the reservoir at the end of the time horizon and to maximize it or to add a constraint on target volumes, see next section.
- Minimize the costs of power generation loss [6].
- Minimize the number of generating unit startup/shutdowns [6,34].
- Maximize the daily plants global efficiency [34].
- Minimize the hydro-logic alteration, i.e., modification of the natural flow regime [39,114].
- Minimize the damage by inundation, the risk due to overtopping, and the sum of the reservoir releases through the bottom outlet [39].

The reader is referred to [34,39] for a discussion of the objective functions mentioned above. It is clear that sophisticated multiobjective methods cannot be applied to such difficult and large-scale problem. Thus, the typical approach is to combine several objective functions in a single one (say, weighting method) and/or to trasform all the objective functions but one in constraints (say, epsilon constraint method). In [75], the authors cite Ko et al. 1992 as an example of multiobjective optimization. They compare the standard multiobjective methods, i.e., weighting and epsilon constraint method, on the instance of the Han River Reservoir system in Korea. Four objectives were evaluated: i. maximizing total energy production; ii. maximizing firm energy; iii. maximizing minimum downstream discharges for water supply and water quality maintenance purposes; and iv. maximizing the reliability of satisfying downstream water supply requirements. To the best of our knowledge, it is one of the few papers dealing with the multiobjective issue in the hydro UC context together with [6,81] where a dynamic programming approach is presented and [34] where the authors propose a genetic algorithm.

However, the multiobjective aspect has been more studied in the case of general hydro-thermal UC problem, see, for example, [2,26,29,60,112,116].

2.2 HUC constraints

In the HUC problem, the constraints can be divided into physical and strategic constraints. The former are derived from physical laws that model the system's behaviour. The latter are utility company's choice, laws to be respected, etc. meaning that typically they are country/company/environment specific. It is important to emphasize this distinction because the former are hard constraints, while the latter are soft ones and can typically be modeled in different ways.

The main physical constraints are listed in the following:

- Initialization: the system status at the beginning of the time horizon is given (continuity and coherence with respect to the past).
- Water flow balance equations: the volume in a reservoir at the current time step is equal to the volume in the reservoir at the previous time step plus the inflows (external, due to turbining of uphill plants, or to pumping of downhill plants in the previous time steps) minus the outflows (external, due to turbining, or to pumping), see (3).
- Each reservoir has an upper and lower bound for the stored water volume (water volume bounds), see (5).
- It is forbidden to simultaneously pump and turbine.
- The units have to respect the allowed operational points. These can be defined as continuous or discrete.
- Operational forbidden zones: in complex hydro units, mechanical vibrations and cavitation strongly discourage to use certain intervals of turbined water (because of low efficiency or high output variation). Thus, we can impose through constraints that the turbined water lies outside of these forbidden zones, see, for example, [126]. It is strictly linked to the previous point.
- The power production function depends on water flow and head effect, i.e., a dependent variable with respect to the reservoir water volume. In particular, the head effect models the height from which the water falls and, consequently, the pressure applied to the turbine that influence the

quantity of produced electricity, see, for example, (2) in the simple model of Section 2 and paper [15].
- Spinning reserves: spinning reserve requirements are necessary to compensate the load in case of disruption or fault so as to minimize the possibility of load interruption.
- Modeling of the spillage due to startup needs or to strategic constraints, for example (see below).
- For each unit, we possibly have a minimum starting up/down time periods, i.e., a minumum number of time periods to spend on or off.
- Additional constraints might be needed to model startup/shutdown costs.

The strategic constraints are:

- Load balance equations constraint or demand satisfaction. This constraint is present when we assume to be in the context of minimization of cost subject to electricity demand satisfaction, while it is not present when we assume that all the produced electricity is sold aiming, thus, at maximizing the profit or when the HUC is just a subproblem of the general UC problem. In the latter case, the demand satisfaction represents the global constraint that links all the subproblems and, as already mentioned, it is typically relaxed through Lagrangian Relaxation (LR), see [67].
- Ramp up/down bound constraints: the absolute value of the water flows difference from one time step to the next has a bound. It is typically due for safety reasons whenever the water flow involves the level of water in rivers used for leisure activities as, for example, fishing, kayaking, etc.
- Final target reservoir level (or water consummation constraint). As already mentioned in Section 2.1, it can be treated, alternatively as an objective function.
- Water rights/irrigation requirements (and ecological/environmental flows and mandatory run-of-river operation, see, for example, [57, 83, 103]): whenever a minimum amount of water release has to be guarantee (and bound of the maximum amount of water release to be respected), see spillage.

It is clear that, while the physical constraints are typically present in the mathematical models we find in the literature, the strategic ones are more specific and might be different in diverse papers. For this reason, a resolution approach might be effective in one case, while not really in others. We will summarize the mathematical programming based resolution approaches in the next section.

3 Approaches of resolution

In this work, we choose to focus on mathematical programming based approaches which, in many cases, proved to be efficient for the resolution of the HUC problem. For a review on further practical modelling and optimization techniques used by industrial utility companies the reader can refer to [119].

3.1 Mixed Integer Programming Approaches

In order to solve large-scale discrete HUC problems with high accuracy, mixed integer programs turn out to be one of the most adapted approaches. In fact, conventional approximation algorithms do not generally converge to an optimal or near optimal solution. Moreover, when continuous, the HUC problem is, in general, efficiently solved to optimality by any (Non) Linear Programming solver.
In what follows, we will detail (mixed integer) nonlinear and linear programming approaches respectively.

3.1.1 (Mixed Integer) Nonlinear Programming

One of the goals to achieve when studying the HUC problem is to obtain optimal feasible solutions for realistic hydro valleys. For this purpose, one has to be the nearest to reality and to express accurately the plants characteristics, mainly by considering the head variation effects on the output power. This generally implies a nonlinear objective function, in particular when the objective is to maximize the profit. In this case, the problem has to be handled through (mixed integer) nonlinear programming methods which have been also used to solve the general UC problem [4].

In this framework, Catalão and co-authors have brought many contributions to solve the (MI)NLP HUC problem. Catalão and co-authors have generally considered a competitive electricity market where the goal is to maximize the total hydroelectric generation throughout the considered time horizon, and such as to maximize the profit of the hydroelectric company [86,87]. In particular, in their papers, mainly those dealing with the continuous HUC problem, they focus on the head-dependency or head-sensitivity in hydroelectric power generation, which is usually neglected for large hydro plants (see for instance [17,20]). The authors justify this choice by the fact that considering the head-dependency on the short term hydro scheduling allows to obtain more realistic and feasible results. In this optic, the authors in [18] study the impact of the physical data defining hydro chains parameters used in the nonlinear model on the HUC problem resolution. In [19], Catalão et al. propose a new nonlinear programming formulation to solve the HUC problem taking into account complicated constraints caused by a head-sensitive cascaded hydro system. In further works, Catalão et al. consider not only head-dependency, but also startup/shutdowns of units, discontinuous operating regions, and discharge ramping constraints, including hence the discrete dimension solved using MINLP techniques [21–24]. Always in the discrete context, and still confirming the idea that "nonlinearity is closer to reality", Diaz et al. [38] consider the hill diagrams provided by turbine manufacturers. In the framework of a MINLP, the authors include an estimation as a quadratic function of net head and water discharge of the multiple nonlinear regression analysis of the unit's technical efficiency. They prove that the MINLP model provides better results compared to the MILP model.

Many other researchers have also provided nonlinear models for the HUC problem. In [102], Pérez and Wilhelmi present a nonlinear programming technique in order to solve the self-scheduling problem of a single-unit small hydro plant. The proposed model considers not only head-dependency, but also unit startups and shutdowns within a continuous context. The authors shows that their model performs very good results on the Spanish hydro system data compared to previous methods. Shu et al. [121] also studied the self scheduling problem. The authors propose a multistage nonlinear model containing complex constraints where the discrete aspect of the problem is relaxed. Using the nonlinear complementarity method, the problem is transformed into nonlinear equations. Then, in order to solve the Karush-Kuhn-Tucker conditions, the system of the model equations is transformed into a sequence of nonlinear algebraic equations based on the Fischer-Burmeister function. This method helped solving efficiently the problem compared to an improved genetic algorithm, but was very limited especially regarding the size of the instances.

As it has been shown in the above-mentioned works, modeling the hydropower plant using nonlinear methods and taking the head-dependency into account, is very efficient, and allows to have more realistic solutions. However, dealing with nonlinearities is a quite hard task that has its limits mainly considering the size of the instances and the tools of resolution. It is in fact known that (mixed integer) linear programming solvers are generally more efficient that nonlinear ones. Consequently, several researchers were motivated to deal with the problem thanks to linear programming techniques.

3.1.2 (Mixed Integer) Linear Programming

The availability of highly efficient software environment often encourages dealing with large-scale problems using (MI)LP. In fact, MILP proves to be a method of choice to solve large-scale real-world instances. In [80], for instance, Li et al. develop a MILP for solving the HUC problem for the Three Gorges Project in China, the largest and most complex hydropower system in the world.

A further advantage of this technique is that it ensures a tradeoff between the solution accuracy and the execution time. Moreover, in some cases, linearization has an extreme influence on the solution feasibility and the coherence of obtained solutions with the real world application. This issue was the topic of a study lead by Tong et al. [127] who emphasize on the impact of linearization on solution feasibility.

Linearization features

As pointed in [56], one of the main difficulties of the hydro scheduling problem is the nonlinear relationship between the hydroelectric generation, the water discharge flow, and the net hydraulic head. Hence, in order to reduce the HUC problem to a (MI)LP, one should opt for linearization techniques. The linearization approach calls for the nonlinear functions such as forebay level, tailrace level, penstock loss, and hydropower production to be replaced by their linear approximations [127].

Obviously, nonlinearities of the problem can be accurately incorporated using several approximation techniques such as piecewise linear approximation [28]. In [56], the authors summarized several approaches to cope with nonlinearity caused by the head variation effect. One can for instance consider a constant average of the net head [28], or a discrete family of curves predefined for the expected range of variations [35] or also built an approximation of the input-output surface by meshing and triangulation [71,115]. In [35], the nonlinear features in the hydroelectric power, the discharged water and the head of the associated reservoir were represented through a specialized approximation methodology based on a three univariate functions between power, volume and flow. This has been generalized later using a parametric number of fixed volume in [15] and [36]. The major contribution of [15] is its ability to further enhance the solution power output accuracy of the approach proposed in [35] through two-dimensional considerations of both water storages and power releases. A three-dimensional interpolation technique has been further adopted to accurately represent the nonlinear power generation function of each individual unit, taking into account the time-varying head as well as the non-smooth limitations for power output and power release [80].

Several improvements have also been combined with the classical piecewise linear approximation. In [57], for instance, the authors treated the nonlinear aspect of the power output by means of an iterative procedure. The first iteration of the procedure begins by an initialization of the net head values as a constant so as to obtain a simplified MILP. Then, at each iteration the net head value is updated until the convergence is reached.

In [118], Santos and Diniz study the impact of increasing breakpoints on the approximation of the hydro power generation. They compare two approaches of hydro generation representation, namely static and dynamic piecewise linearization. In the static approach, all hyperplanes defining the power function are included at once in the linear programming to be solved. However, in the dynamic approach, hyperplanes are included dynamically as the solving procedure evolves. Authors prove that the dynamic approach enables to have the same results as the static approach, but with better time of execution. They also conclude that in order to calibrate the piecewise linearization quality, one should not only compare the deviations between the piecewise linear model and the true nonlinear function, but also perform a sensitivity analysis of this deviation.

Advances in MILP

The first interests in solving the HUC problem using MILP appeared in the mid 1990's with the work of Christoforidis et al. [32]. Several mixed integer linear programming formulations have then been proposed to different variants of the HUC problem. In [27], Chang and Waight are interested to the HUC problem that aims at determining optimal half-hourly schedules for some hydro units while respecting system and hydraulic constraints. The nonlinear hydro unit characteristic is approximated by a two-segment linear curve for a given head level. The problem is then modeled as a MILP and tested on real hydraulic system data. Later, Chang et al. [28] propose a more complete MILP considering further discrete and dynamic constraints such as unit startup/shutdown and minimum-up/minimum-down time limits. Using AMPL/CPLEX, the MILP is tested over two test systems, the first is of the Southern Generation Group of the Electricity Corporation of New Zealand Limited and the second is taken from the Swiss Rail hydro resources data. In the same period, García-González and Castro [55] studied the STHS problem considering explicitly the relation among the electrical power, the net head and the turbine water discharge. They present a new approach where the nonlinear input-output surfaces are linearized using binary variables. A numerical test of a three cascaded and head dependent reservoirs for a 24 hours planning horizon is also presented. In recent research, a new operational constraint has captivated attention namely the water time delay. In a cascade, the water time delay can be defined as the time required for discharging water from an upstream reservoir to its downstream reservoir [58]. In [127], authors show that the real number water delay can be handled in a MILP without destroying the linear structure of the water balance constraint. More recently, Ge et al. [58] studied the short term hydropower optimal scheduling considering the optimization of water time delay. The water time delay is formulated as a nonlinear function the outflow from upstream reservoir. The authors studied the linearization of this function as well as the water-to-power conversion function and the power limits.

MILP techniques have been also combined with graph theory. In [16], Bregar presents the Soča hydro system composed of four hydro power plants as a directed graph. Vertices correspond to regulating basins and links to the hydro power plants. To each vertex a volume variable v is assigned and, similarly, to each link a turbined flow variable q is assigned. The author used then a MILP to solve the corresponding hydro scheduling problem.

As a conclusion, both (MI)LP and (MI)NLP approaches have advantages and drawbacks. The choice of one over the other is generally motivated by the main objectives of solving the HUC problem. One can, for instance, aim at solving accurately the problem keeping the nonlinear difficulty, or linearize the model and have quickly a good approximation of the solution. In [10], Barros et al. propose nonlinear and linear models to solve the HUC problem for the Brazilian hydropower system, and lead a comparative analysis on the results obtained from the nonlinear and the linear programs respectively.

3.1.3 Reformulation techniques and polyhedra: towards tighter relaxations

In this section, we survey works that have been achieved for the general UC problem, and that can be interesting to apply in the future for the hydro context.

In order to achieve a small gap for MI(N)LP, one can think about a tight relaxation that help well approximate the problem. Better lower bounds can this way be obtained and optimal integral solutions can fast be reached. In [64, 65], Günlük and Linderoth survey recent works on the prespective reformulation approach that generates tight, tractable relaxations for convex MINLP. Using this technique, Frangioni and Gentile [53] propose a perspective reformulation that tightens the mixed integer programming formulation of the general UC problem with quadratic cost curves, making use of a crucial class of valid inequalities called the perspective cuts [50]. In the same scope, Jabr [70] and Quan [111] propose tighter relaxation method based on second-order cone programming and valid inequalities previously studied in [13, 51].

In another side, some restrictions of the UC problem can be very interesting to study from a theoretical point of view and hide nice polyhedral properties. One of the first works that have been interested to these kind of study is the one of Lee et al. [76]. In [76], the authors study the min-up/min-down polytope. They describe new family of valid inequalities called the alternating-up/alternating-down inequalities and characterize the polyhedral structure of the associated model. In a further work, Rajan and Takriti [113] improve these inequalities and define the so-called turn-on/turn-off constraints. They give a complete description of the restriction of the UC polytope on these inequalities, discuss their separation and devise branch-and-bound and branch-and-cut algorithms.

More recently, Morales-España et al. [91] provide a description of the convex hull of the basic constraints for the power-based unit commitment problem including both slow- and quick-start generating units. These constraints are: generation limits, and minimum up and down times, startup and shutdown power trajectories for slow-start units, and startup and shutdown capabilities for quick-start units. These constraints are also considered for the thermal UC problem in [92]. Other tighter formulations and polyhedral description of the UC problem can be found in [59, 100].

Along with (mixed integer) linear programming, dynamic programming has likely been one of the most used optimization technique to solve hydro scheduling problems. More details are given in the next section.

3.2 Dynamic Programming

Even though Dynamic Programming (DP) it is not typically a mathematical programming method, we choose to talk about it, mainly because it is a crucial brick of several methods and it has been widely used in the literature. Moreover, dynamic programming has probably been one of the first methods to tackle the hydropower scheduling problem. This approach has first been used in 1986 by Allen and Bridgeman [3] who apply it to solve three case studies involving hydropower scheduling. Its major interest lies in the fact that it helps overcome the nonlinear and nonconcave aspect of the hydro scheduling problem. It has, however, some limits to manage the unit discrete startups and shutdowns constraints.

In [6], Arce et al. present a dynamic programming model in order to determine the number of hydro generating units in operation on an hourly basis at the Itaipú hydro plant. The objective is to satisfy the total generation scheduling of the plant while minimizing the tradeoff between the costs of power generation loss and generating unit startup/shutdowns. Variations in tailrace elevation, penstock head losses and turbine-generator efficiencies are also taken into account. The model is solved efficiently by a DP technique where the stage is an hour, the state variable is the number of generating units in operation for each stage, and the control variable is the number of startups or shutdowns of generating units for each stage [6].

In [104], Pérez-Díaz et al. propose a DP model to solve the short term hydro scheduling problem. The model is designed to determine in each hour of the planning horizon, the optimal number of units in operation as well as the power to be generated by the committed units. The power generated by each hydro unit is considered as a nonlinear function of the water discharge and the volume of the associated reservoir. The dependence of the units operating maximum and minimum water flows on the actual gross head has also been also taken into account. The resolution approach is based on two major steps. First, a dynamic programming procedure is applied to obtain a cloud of plant operating points, each representing the optimal instantaneous unit commitment and dispatch for a given reservoir volume and water discharge. Then, in a second step, these operating points are fitted by a family of piecewise linear power-discharge curves, and the resulting short term scheduling problem is solved using dynamic programming also. A similar model and procedure is proposed in [105].

Generally, the dynamic programming models in [104,105] are based on nonlinear programming formulations of the HUC problem that help defining the structure of the problem. The resolution of NLP based models are in practice well handled by dynamic programming. However, when a discrete aspect comes into play, this approach attain fast its limits and give place to other techniques that turn out to be more suitable, like the MI(N)LP based methods described above. Moreover, solving the HUC problem for large-scale cascaded hydro-plants by DP approach can be numerically impossible because of the so-called curse of dimensionality. Other methods are here more efficient, such as decomposition approaches and mainly Lagrangian decomposition that will be detailed in Section 3.3.

Furthermore, as it has been shown above, dynamic programming is a classic choice and a sophisticated optimization method that demonstrates its capabilities for real-time optimal unit scheduling [130], mainly for the small-scale problems. However, as pointed in [30] the DP algorithm becomes limited and can not even be implemented in practice for large-scale problems. This is mainly because its execution time exponentially increases time with the increase of units number. In this case, the use of approximation methods and heuristics can be more efficient. In [30], for instance, Cheng et al. show that the Particle Swarm Optimization (PSO) metaheuristic outperforms dynamic programming procedure for large HUC problems. More details about the heuristic-based methods of resolution will be developed in Section 3.4.

3.3 Decomposition approaches and Lagrangian relaxation

Decomposition techniques are considered as attractive methods to solve large-scale Unit Commitment problems. Benders decomposition, for example, proved to be efficient for the UC problem in some specific contexts [122,125]. However, as indicated in [122], the most successful optimization technique so far applied to solve the UC problem has been Lagrangian relaxation. Frangioni et al. [52] proved, indeed, that LR is faster and more competitive than MILP for the hydrothermal unit commitment, especially for large-scale instances. For further papers dealing with LR for the general UC problem, the reader is referred to [9,40,61–63,79,89].

Concerning large-scale HUC problem, one can also affirm that LR is one of the most powerful methods of resolution. An important feature of the LR approach is that it allows neglecting the complicating linking constraints of the problem. The original problem is hence split into a sequence of smaller subproblems coordinated by a dual master program.

LR decomposition technique is based on three majors phases: dualization, dual problem solving, and primary recovery phase. The dualization phase is a crucial step and can generally be ensured using two methods. The first and most common method is to dualize the linking constraints, which, for the HUC problem, are hard constraints expressing the physical relationship between upstream and downstream reservoirs. The second approach of dualization is the use of variables duplication, a strategy basically used for mixed integer nonlinear programs [78]. This is based on the introduction of artificial variables and constraints, and then the relaxation of the artificial constraints (see for instance [42–44,46]). In [45], Finardi et al. detailed these two dualization methods and lead a comparative analysis of the corresponding dual problems for solving the HUC problem.

After dualization, an important phase is to solve the dual subproblems and update the Lagrange multipliers. This can generally be handled by the use of nonsmooth algorithms like subgradient method [96]. In case of nonlinear subproblems, Finardi et al. apply the so-called sequential quadratic programming method [43,44]. Another interesting method to solve large-scale dual problems with high precisions is the bundle method [42], which helps giving good starting points for recovering primal solutions. Bundle methods have been before efficiently applied to the general unit commitment problem, see for example [8, 11,77].

The last step, after solving the dual problems, is to find a primal feasible solution. One of the most used methods here is augmented Lagrangian [25] or inexact augmented Lagrangian [43,44].

In practice, it would be interesting to study hybrid methods that combines MILP with the LR solutions as a starting point. LR can also be combined with a heuristic procedure. In [123] for instance, Soares et al. proposed a heuristic based on a LR relaxing the daily generation targets for all the plants and the load attainment constraints. Another option which is worth to be explored is to perform a branch-and-bound procedure entirely based on LR. A related work dealing with a sequential Lagrangian-MILP solution is proposed in [54], where a Lagrangian lower bound is provided to the MILP solver that uses it in the context of a branch-and-bound algorithm.

3.4 Heuristics and Metaheuristics

Even though heuristic-based methods cannot guarantee optimality, they can provide a good enough approximation of the solution for large-scale instances in a reasonable execution time. In this section we quickly discuss some interesting heuristic approaches without the aim of being exhaustive. In fact, we focus on (meta)heuristic methods hybridized with methods presented in the previous sections.

These methods can be either applied alone or combined with other methods, resulting hence into efficient hybrid methods. In [117], Santos and Ohishi describe a hybrid approach that determine the daily hydro unit startup/shutdown schedule and the corresponding power output schedule. The method is based on a genetic algorithm combined with nonlinear optimization techniques. The HUC problem is divided into two subproblems. The first one determines the startup/shutdown schedule using a genetic algorithm method, and the second subproblem calculates by NLP techniques the power output of all committed hydro units selected by the genetic algorithm. The approach is shown efficient by application to three examples from the Brazilian Shouteast Hydroelectric System. Later, in [98], the authors present a comparison of the results of two heuristics approaches for the HUC problem. The objective is to optimize the performance criterion which depends on variations in tailrace elevation, penstock head losses and turbine-generator efficiency, as well as the cost of startup/shutdown of the hydro generating units. The first heuristic consists in decomposing the HUC problem into Generation Schedule (GS) and Unit Scheduling subproblems. The two subprolems are then solved by LR and DP, respectively. The second heuristic is based on a genetic algorithm combined with Lagrangian relaxation to solve the whole HUC problem. The two heuristics are tested over a system composed of sixteen hydro plants taken from the Brazilian power system. Genetic algorithm has been also efficiently combined with other techniques. Yuan et al [133], for instance, propose a novel hybrid chaotic genetic algorithm to solve the HUC problem. This is based on introducing the chaotic sequence in the evolutionary process of the genetic algorithm. Similar hybrid methods are described in [33,132,134].

Apart from the genetic algorithm based resolution approaches, we find works based on the PSO. In [30], this method has been adopted, and shown to be more efficient than the dynamic programming for large-scale HUC problems. In addition, in [83], Mahor and Rangnekar propose a novel self adaptive inertia weight PSO approach in order to determine the optimal generation schedule of real operated cascaded hydroelectric system located at Narmada river in Madhya Pradesh, India. The GS problem is formulated considering two situations that take into account not only the generation power purpose but also the fulfillment of irrigation requirements.

3.5 Other interesting approaches

In the framework of this paper, we focused on mathematical programming and hybrid methods of resolution. In the literature, the HUC problem has been tackled with several other resolution approaches that we do not detail in this review. One of these work is based on simulation techniques. Recall that, as stated in Sections 3.3 and 3.4, some papers use decomposition of the HUC problem into subproblems that can be then solved using heuristic-based methods. In the same scope, in [73], the authors solve the HUC problem using a decomposition approach based on optimization-simulation techniques. The approach begins first by solving a relaxed version of the original problem. Then, a simulation phase is computed to overcome eventual hydraulic infeasabilities. The approach is tested over the Brazilian power system data. Some other methods based on neural networks and model predictive control theories have also been studied and proved efficient on real-data instances (see [93,94] and [82,97,106,110,135,136], respectively).

4 Solving real-world hydro valley problems

As mentioned in the Introduction, hydroelectricity is produced in 150 countries all over the world. In 2012, the Asia-Pacific region is ranked first with 33.7% of the world-wide hydro energy production. China is the largest hydroelectricity producer, with 823.3 terawatt-hours of production, representing around 22.5% percent of the world-wide production (source: EdF website http://jeunes.edf.com/article/l-hydraulique-dans-le-monde,89).

The most famous Chinese hydroelectric station is the Three Gorges Dam, which is also the world's largest and most complex hydropower system in operation, with an installed capacity of 22.5 GW [80]. The Three Gorges is a large-scale, real-world, and highly complex multi-unit hydropower system, supplying the electricity demand for nine provinces and two cities in China. In [80], Li et al. develop a MILP model for solving the HUC problem of the Three Gorges Project (TGP). The authors underline that although there is only one reservoir in the TGP that feeds water to all 32 units, the corresponding HUC problem is difficult to solve because the multiple units are considered individually, introducing hence high-dimensionality. There is in addition a complication in modeling the tailrace water level, as this level is affected by the forebay water level of the Gezhouba, a reservoir immediately downstream from the TGP. The same dam has interested authors in [90]. Mo et al. [90]

present a hybrid algorithm based on multi ant colony system and differential evolution for solving the HUC problem for Three Gorges-Gezhouba cascaded hydropower plants. Later, in [58], Ge et al. propose to develop a mixed integer model to solve the short term hydropower optimal scheduling problem. The proposed formulation has been applied to solve a real-world case in China based on 13 reservoirs and 44 hydropower units.

The second largest producer of hydropower in the world is Brazil, with 416.8 terawatt-hours of production in 2012, representing around 11.4% percent of the world-wide production. Producing hydroelecticity has been a major interest for Brazil last years, mainly because almost 80% of the energy generated and consumed domestically originates from hydro plants (source: http://thebrazilbusiness.com/article/hydro-electricity-in-brazil, 2013). Many academic researchers and practitioners have then been motivated to study and solve the HUC problem for the Brazilian dams. A special interest is granted to the Itaipú hydropower system. Itaipú is a 12.6 GW hydro plant, located on the Paraná river, in South America, composed of 18 identical 700 MW generating units. It is the second largest hydroelectric plant in operation in the world shared by Brazil and Paraguay, contributing to 20% of the Brazilian demand and 95% of the Paraguayan demand for electric energy. In [6] and [123], the authors have been respectively concerned with the optimal and dynamic dispatch of generating units of Itaipú. In [6], Arce et al. develop a dynamic programming model to optimize the number of generating units in operation at each hour of the day in order to attain the total generation scheduling of the plant in the most economic way. However, a heuristic procedure based on Lagrangian Relaxation is applied in [123] in order to solve the dynamic dispatch problem of scheduling generation on an hourly basis during a day. Further works have been based on the use of the real Brazilian hydroelectric system data. The reader can for instance refer to [42–46, 73, 98, 118, 125].

In the third place of the largest hydropower producers in the world, comes Canada with 380.1 terawatt-hours of production in 2012, which represents 10.4% percent of the world-wide production. The HUC problem for hydropower systems in Quebec, Ontario, and British-Columbia have been considered in [128], [66], and [90], respectively.

In addition to these three largest producers, hydropower is one of the main produced and consumed energy in many other countries. In Norway, for example, more than 95% of the Norwegian production comes from hydro plants [12, 49, 72]. It is also a very promising and emerging renewable source of energy for other countries such as Sweden [95], France [25], Mexico [4], and Portugal [17–19, 21–24, 86].

5 Concluding remarks

Although in practice it represents only a subproblem of a more general one, the hydro unit commitment problem has received a great deal of attention over the past years. Based on various assumptions, different variants of the problem have been studied, and solved using several methods of resolution. Based on the most recent articles dealing with the HUC problem, this paper gives an overview on the various aspects and assumptions of the problem. It provides a detailed description of the different hypothesis, the possible objective functions as well as the constraints that can be considered. It also presents a review on effective approaches used to solve the problem, with a specific focus granted for the mathematical programming based methods. The work is presented in a recital way whose main objective is to list the recent achievements in solving the HUC problem without any evaluation or comparison of the resolution methods or results. It is clear from this survey that each resolution method has its advantages and disadvantages, and is generally efficient in a specific context. As a consequence, an obvious consensus points toward the hybrid methods which can be able to simultaneously deal with various challenges and solve more efficiently large-scale problem.

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