Robust optimization based EV charging

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***Abstract*—With the introduction of new technologies like electric vehicles and smart grids the operation and planning of power systems are subject to major changes. These technologies can bring various flexibilities to different entities involved in decision making. This paper proposes a robust optimization based method to optimal charging/discharging of electric vehicles considering the electricity price uncertainties. The objective function is defined as the total operating costs of energy procurement in distribution networks which is tried to be minimized while considering the technical constraints of the problem.**

I. Introduction

The new paradigm of holistic transportation and power system management has attracted a great deal of attention specially with the advent of smart grids. The smart grid is defined as an electric grid in which there exists bidirectional data/command exchange between market players [1]. This is done in order to ensure the reliability and security of energy transition from supply section to end users. The innovative technologies like electric vehicles have the potential to facilitate this task. The key point is that how to capture these potential benefits and promote their role in future. The electric vehicle (EV) can play different roles such as load, generation and energy storage unit [2]. Different energy management and control strategies have been proposed in the literature to capture the potential flexibilities that electric vehicle (EV) can bring to different power system entities as shown in Fig.1.

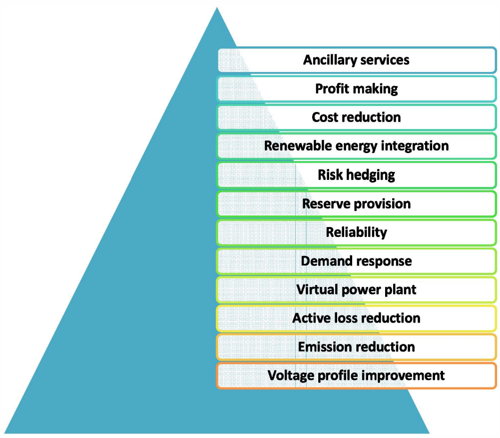

Fig. 1. The potential flexibilities of electric vehicles

These flexibilities range from end users electricity bill reduction to distribution and transmission impact assessment by operators and utilities. This paper, would provide a model to optimally schedule the EVs in an uncertain electricity market in order to minimize the total energy payments.

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II. Literature review

Different approaches have been proposed in the literature to facilitate the flexibility provision by EV to the electric grids. The physical nature of power system requires the instantaneous balance between the demand and generation. With the increasing share of renewable energy in energy procurement procedure, the importance of flexibility services has become more vital. These services play a key role in maintaining the balance between demand and supply side. The EVs have been identified as an interesting flexibility service provider in different circumstances such as :

- Spinning reserve supply
- Provision of system regulation
- Valley filling and target load following
- Household energy management [3]
- Shaving the peak of the load curve [4]
- Congestion Prevention in the Distribution networks
- Offering secondary reserve by a pool of EVs
- Releasing spinning reserve to electric systems
- Facilitating the load shifting and shaving
- Allowing more flexibility in setting the tradeoff between cost and user comfort
- Load shifting potential of electric vehicles
- Minimizing losses and improve voltage profile [5], [6]
- Possible positive impact on life duration of a transformer supplying a residential area

III. Problem Formulation

The optimal decisions on energy procurement strategy should be taken at each stage to not only reduce the total payments but also not violating the technical constraints. The following optimization problem is solved:

$$\min_{\mathfrak{D}} z = \sum_{t\in\Psi_T} \mathfrak{E}_t\tilde{\Lambda}_t \quad (1)$$

$$\mathbf{F}(\mathfrak{D},\Pi) \leq \mathbf{0} \quad (2)$$

$$\mathbf{G}(\mathfrak{D},\Pi) = \mathbf{0} \quad (3)$$

$\mathfrak{E}_t$ in (1) is the hourly energy bought from the upstream network. $\mathfrak{D}$ and Π represent the decision variables and input parameters (price values and technical data), respectively. T is the operating horizon. $\tilde{\Lambda}_t$ is the uncertain electricity price at time t in day ahead market. **F** and **G** represent the inequality and equality constraints of the optimization framework as described in (6) to (19), respectively. In this paper, a new strategy is proposed that tries to minimize total payments related to the active power procurement. Obviously, the optimal actions $\mathfrak{D}$ directly depend on the input parameters (Π) including price values for the day ahead market. The issue is that usually there is limited information about the electricity prices of the next day. The optimization problem can therefore be formulated as follows:

$$\min_{\mathfrak{D}} z = \sum_{t\in\Psi_T} \mathfrak{E}_t\tilde{\Lambda}_t \tag{4}$$

$$\Lambda_t^{min} \leq \tilde{\Lambda}_t \leq \Lambda_t^{max} \tag{5}$$

$$(6) to (19)$$

$\Lambda_t^{min}, \Lambda_t^{max}$ are the lower and upper bounds of $\tilde{\Lambda}_t$, respectively. It is assumed that no information is available from day ahead market prices other than these bounds. The power flow equations to be satisfied $\forall i \in \Psi_n, \forall t \in \Psi_T, \forall \ell \in \Psi_L$ are:

$$\mathfrak{E}_t = \sum_{i\in\Psi_n} P_{i,t}^G \tag{6}$$

$$P_{i,t}^{net} = P_{i,t}^G - P_{i,t}^D - \kappa_{i,v}(P_{t,v}^{ch} - P_{t,v}^{dch}) \tag{7}$$

$$Q_{i,t}^{net} = Q_{i,t}^G - Q_{i,t}^D \tag{8}$$

$$P_{i,t}^{net} = V_{i,t}\sum_{j\in\Psi_n} Y_{ij}V_{j,t}cos(\delta_{i,t} - \delta_{j,t} - \theta_{ij}) \tag{9}$$

$$Q_{i,t}^{net} = V_{i,t}\sum_{j\in\Psi_n} Y_{ij}V_{j,t}sin(\delta_{i,t} - \delta_{j,t} - \theta_{ij}) \tag{10}$$

$$V_{min} \leq V_{i,t} \leq V_{max} \tag{11}$$

$$I_{\ell,t} = Y_{\ell=ij}(V_{i,t} - V_{j,t}) \leq \overline{I}_\ell \tag{12}$$

where $P_{i,t}^{net}, Q_{i,t}^{net}$ in (7) and (8) are the net injected active and reactive power to bus i, respectively. Y_{ij}, θ_{ij} are the magnitude and angle of the admittance connecting bus i to j, respectively. $V_{i,t}, V_{min}, V_{max}$ in (11) are the voltage magnitude, min/max operating limits of each bus, respectively. I_ℓ in (12) is the current passing through feeder ℓ and $\overline{I}_\ell$ in (12) is the maximum allowable current in feeder ℓ. $P_{i,t}^G, Q_{i,t}^G$ in (7) and (8) are the active and reactive power injected to the network by the DG units or grid connection. $\kappa_{i,v}$ is a binary parameter indication if vehicle v is connected to bus i ($\kappa_{i,v} = 1$) or not ($\kappa_{i,v} = 0$). Ψ_n, Ψ_T, Ψ_L are the set of system nodes, operating hours, feeders, respectively. $P_{t,v}^{ch/dch}$ is the charged/discharged power of EVs in (7).

The operation modeling of EV is described in (13) to (19).

$$Soc_{t,v} = Soc_{t-1,v} + \eta_v^c P_{t,v}^{ch} - \frac{P_{t,v}^{dch}}{\eta_v^d} - P_{t,v}^{tr} \tag{13}$$

$$Soc_{t,v} = E_v^{ini} + \eta_v^c P_{t,v}^{ch} - \frac{P_{t,v}^{dch}}{\eta_v^d} - P_{t,v}^{tr} \tag{14}$$

$$Soc_{min}^v \leq Soc_{t,v} \leq Soc_{max}^v \tag{15}$$

$$P_{t,v}^{tr} = \Delta D_{t,v}\epsilon_v \tag{16}$$

$$P_{min}^{ch,v}U_{t,v}^{ch} \leq P_{t,v}^{ch} \leq P_{max}^{ch,v}U_{t,v}^{ch} \tag{17}$$

$$P_{min}^{dch,v}U_{t,v}^{dch} \leq P_{t,v}^{dch} \leq P_{max}^{dch,v}U_{t,v}^{dch} \tag{18}$$

$$U_{t,v}^{ch} + U_{t,v}^{dch} \leq 1 \tag{19}$$

The state of charge in v^{th} EV at time t ($Soc_{t,v}$) depends on the state of charge at time $t-1$ ($Soc_{t-1,v}$) as well as the charging/discharging or traveling state of the EV as modeled in (13) and (14). The relation between the required energy for traveling of v^{th} EV ($P_{t,v}^{tr}$) depends on the traveling distance ($\Delta D_{t,v}$) and also the efficiency of the vehicle (ϵ_v) as described in (16). The state of charge should be kept between operating limits as (15). The charging and discharging rate of each EV are limited by technical characteristics as well as the operating state as enforced by (17) and (18). It is assumed that each EV is either in charging $U_{t,v}^{ch} = 1$ / discharging state $U_{t,v}^{dch} = 1$ or traveling state $U_{t,v}^{ch} + U_{t,v}^{dch} = 0$ as described in (19).

IV. PROPOSED STRATEGY

One of the most important issues of energy management systems is risk management. The term "risk" has various definition. In financial analysis, it may be defined as a loss in profit or excess in a cost function. In technical analysis, it may be defined as a contingency, voltage instability, overloading a line, over voltage or under voltage phenomena. Generally speaking, risk is defined as any undesired event due to the occurrence of unpredicted issue. Different uncertain parameters may lead to risk. These uncertain parameters are usually the output power of renewable energy resources such as wind turbines or photo voltaic cells, demand uncertainty, equipment failures and competitive behavior of electricity market participants [7]. The risk hedging tools are used in order to mitigate the technical or financial risks [8]. Different models and techniques can be found in the literature which propose risk hedging approaches. Some of these models are as follows: stochastic techniques [9], [10], fuzzy methods [11], Information Gap Decision Theory (IGDT) [12] and robust optimization [13], [14]. This paper uses the robust optimization technique to model the uncertainties associated with electricity prices. The robust optimization concept can be used to minimize z in eq. (4) without knowing the exact values of Λ_t. Additionally, the optimal decision making should be done in a way that these actions still remain optimal even though the actual values (Λ_t^a) of uncertain parameters deviate (to some degree Γ) from the forecasted values Λ_t^f. Two cases may happen: first, the actual price Λ_t^a is more than the forecasted price Λ_t^f. The constraint for uncertainty modelling of the price can be expressed as:

$$\Lambda_t^a = \Lambda_t^f + \Delta_t^+\xi_t \tag{20a}$$

$$\Delta_t^+ = \Lambda_t^{max} - \Lambda_t^f \tag{20b}$$

$$0 \leq \xi_t \leq 1 \tag{20c}$$

where, ξ_t is the prediction error.
Actually the main concern of the decision maker is on (20a) , (20b) where the actual prices may be more than the forecasted values. Thus, the formulation expressed in (4) , (5) can be replaced by the following one:

$$\min_{\mathfrak{D}} z = \sum_{t\in T} \mathfrak{E}_t\Lambda_t^f + \mathfrak{E}_t\Delta_t^+\xi_t \tag{21a}$$

$$0 \leq \xi_t \leq 1 \tag{21b}$$

$$\sum_{t\in T} \xi_t \leq \Gamma \tag{21c}$$

$$(6) to (19)$$

Γ in (21c) is the maximum total deviation that can be tolerated. It varies from 0 (no prediction error) to 24 (100% prediction error) (increases with the conservativeness of the decision maker). For example, if $\Gamma = 4$ this means that the algorithm will remain robust even though the maximum total prediction error is 100% in 4 hours or 50% in 8 hours of the day ahead market. The robust counterpart of (21) is represented as [15]:

$$\min_{\mathfrak{D}} z = \sum_{t \in T} \mathfrak{E}_t \Lambda_t^f + \begin{Bmatrix} \max_{\xi_t} \mathfrak{E}_t \Delta_t^+ \xi_t \\ (21b), (21c) \end{Bmatrix} \tag{22}$$
$$(6) to (19)$$

The formulation described in (22), requires to solve a bi-level optimization since the inner maximization tries to simulate the worst case realization of uncertain price (by changing ξ_t) while the outer minimization attempts to decrease the undesired impacts of uncertain prices by controlling $\mathfrak{D}$. According to the duality theory [14], (22) is equivalent to (23) as follows:

$$\min_{\mathfrak{D}} z = \Lambda_t^f \mathfrak{E}_t + \sum_{t \in T} \zeta_t + \Upsilon \Gamma \tag{23a}$$
$$\Upsilon + \zeta_t \geq (\Lambda_t^{max} - \Lambda_t^f) \mathfrak{E}_t \tag{23b}$$
$$\Upsilon, \zeta_t \geq 0 \tag{23c}$$
$$(6) to (19)$$

where ζ_i, Υ are dual variables.
It is obvious that the resulted single level optimization is easier to solve than the original bi-level optimization structure. The decision variables ($\mathfrak{D}$), parameters (Π) and the sets are as follows:

$$\mathfrak{D} = \begin{Bmatrix} \mathfrak{E}_t, (P/Q)_{i,t}^G, I_{\ell,t}, \xi_t, \zeta_t, \Upsilon \\ (P/Q)_{i,t}^{net}, P_{t,v}^{ch/dch}, V_{i,t}, \delta_{j,t}, Soc_{t,v} \end{Bmatrix} \tag{24}$$

$$\Pi = \begin{Bmatrix} (P/Q)_{i,t}^D, Soc_{min/max}^v, P_{min/max}^{ch/dch,v} \\ \Lambda_t^{max/min}, \Lambda_t^{f/a}, \theta_{ij}, Y_{ij} \\ \Delta_t^{\pm}, \Gamma, V_{min/max}, \overline{I}_\ell, \eta_{ch/dch} \end{Bmatrix} \tag{25}$$

$$Sets = \{\Psi_T, \Psi_n, \Psi_L\} \tag{26}$$

V. SIMULATION RESULTS

The proposed robust optimization based model is examined on a 33-bus distribution system. The data of this system including the data of loads, and feeders are given in [16]. In order to better explore the benefits of electric vehicles on distribution networks, the load level is decreased by 50%. The proposed algorithm is implemented in GAMS environment solved by SNOPT solver running on an Intel®Xeon ™CPU E5-1620 3.6 GHz PC with 8 GB RAM. It is assumed that there are only 10 EVs in the system under study. The technical data regarding the EVs is given in Table I.

TABLE I. ELECTRIC VEHICLE CHARACTERISTICS DATA

Parameter	Value	Unit
η_v^d	93	%
η_v^c	90	%
E_v^{ini}	3	kWh
Soc_{max}^v	40	kWh
Soc_{min}^v	1	kWh
$P_{min}^{ch,v}$	0	kW
$P_{max}^{ch,v}$	20	kW
$P_{min}^{dch,v}$	0	kW
$P_{max}^{dch,v}$	20	kW
ϵ_v	$\frac{1}{6}$	kW/km

The traveling patterns of EVs are given in Table II. It is assumed that the required energy for demand supply is the pool market. The purchased power from pool market is injected to the network through slack bus which is bus $i = 1$. The electric demand and electricity price patterns are shown in Fig. 2.

TABLE II. ELECTRIC VEHICLE TRAVELING PATTERN (KM) AND CONNECTED NODES

Time	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9	v_{10}
t_1	0	0	0	4.6	0	0	0	4	0	0
t_2	0	0	0	1.8	0	2	0	0	0	0
t_3	0	5	0	0	0	2	0	0	0	2
t_4	0	0	0	0	0	0	0	1	0	0
t_5	2.4	0	0	0	0	4	2.8	3	0	4
t_6	0	4.8	0	0	0	1	0	0	0	0
t_7	0	0	0	0	0	0	0	0	0	0
t_8	4.8	0	1	2	0	0	0	0	0	2
t_9	0	0	0	0	1	0	0	0	0	1
t_{10}	0	2.4	0	0	0	0	0	0	0	1
t_{11}	0	0	0	4.6	2	0	0	4.4	0	0
t_{12}	4	0	0	0	4	3	0	0	0	0
t_{13}	0	0	0	0	0	0	0	0	4.2	0
t_{14}	0	0	0	0	0	0	0	4	0	0
t_{15}	0	0	0	0	0	0	0	0	3.8	0
t_{16}	3.6	0	0	4.6	0	0	0	0	0	4
t_{17}	0	0	3.6	0	0	0	0	3	0	4
t_{18}	0	0	0	4	0	0	0	1.8	0	0
t_{19}	0	0	4	0	0	2.6	0	0	2	4
t_{20}	0	0	0	0	0	0	4.2	3.2	2.2	0
t_{21}	0	0	4.8	3.8	0	0	0	2.6	1	1
t_{22}	0	0	0	0	0	0	0	0	0	0
t_{23}	0	4.8	0	0	0	0	0	0	0	2.2
t_{24}	0	0	0	0	0	0	3.2	0	0	0
$i : \kappa_{i,v} = 1$	12	20	23	21	32	15	28	17	22	33

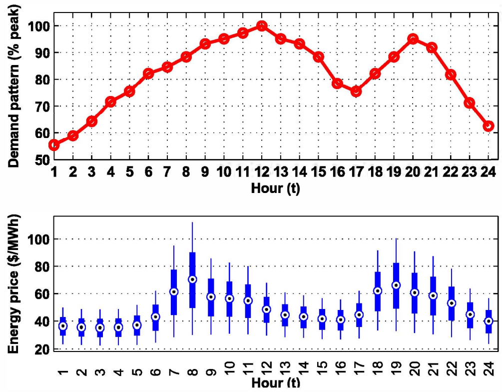

Fig. 2. The electric demand and electricity price patterns

Two case studies have been performed to demonstrate the effectiveness of the proposed methodology namely, random case and centrally controlled charging.

A. Random charging case

In this case, it is assumed that the EV owners perform the charging and do not inject any energy to the grid (e.g. $P_{t,v}^{dch} = 0$). The charging pattern is completely random and do not consider the price (as well as its uncertainties). The total energy charged (P_t^{ch}) in random case is depicted in Fig. 3.

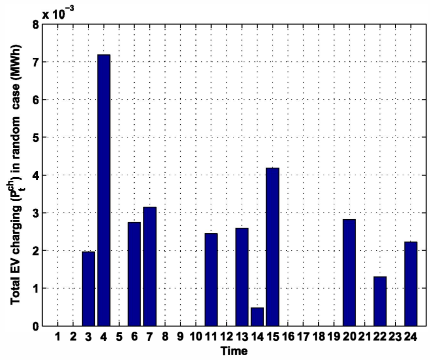

Fig. 3. The total energy charged (P_t^{ch}) in random case

The total Eoc_t in random charge case is shown in Fig. 4.

controlled case is shown in Fig. 7. The total state of charge vs Γ in centrally controlled case is shown in Fig. 8.

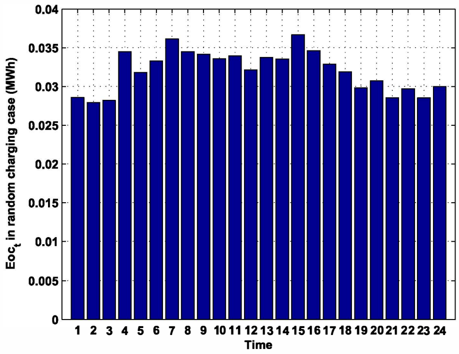

Fig. 4. The total Eoc_t in random charge case

The hourly energy losses vs Γ in random case are shown in Fig. 5. The total energy losses are 0.51625 MWh.

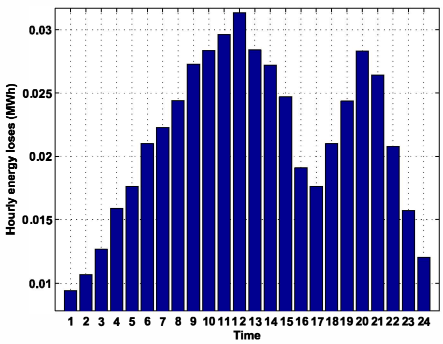

Fig. 5. The hourly energy losses vs Γ in random case

The total payments depend on price uncertainty. As the price uncertainty increases, the total payments would increase. The total energy payments compared are shown in Fig. 11.

B. Centrally controlled charging/discharging case

In this case, it is assumed that the charging/discharging of EVs is performed centrally. The charging/discharging pattern is optimally determined with considering the electricity price uncertainties.

The total energy charged (P_t^{ch}) in centrally controlled case is depicted in Fig. 6. The total energy discharged (P_t^{dch}) in

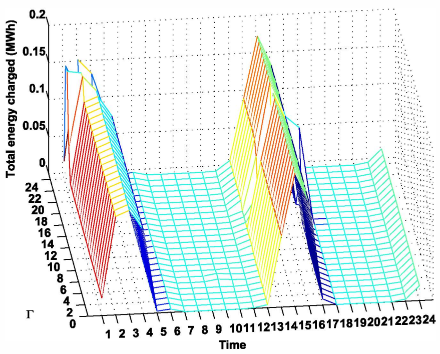

Fig. 6. The total energy charged (P_t^{ch}) in controlled case

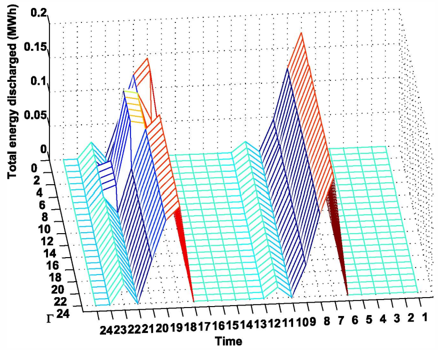

Fig. 7. The total energy discharged (P_t^{dch}) in controlled case

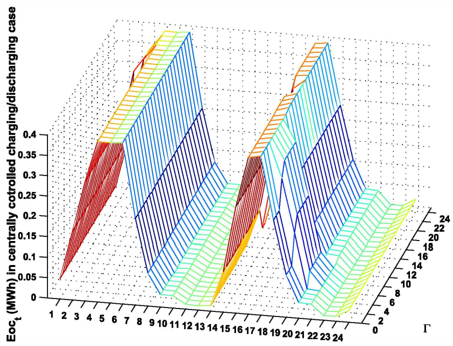

Fig. 8. The state of charge vs Γ in centrally controlled case

The charging and discharging pattern of EVs directly affects the active losses in distribution network. This impact is investigated in this case. The hourly energy losses vs Γ in controlled case are given in Fig. 9. The total energy losses vs Γ in controlled case is shown in Fig. 10. The total energy payments in both cases are compared in Fig. 11.

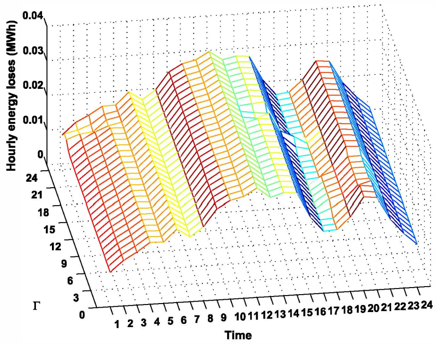

Fig. 9. The hourly energy losses vs Γ in controlled case

Comparing the payments in both cases we observe that, the centrally controlled case results in lower total operating costs in every budget of uncertainty (Γ).

If the percentage of EV loads of the the total load in distribution network is increased, then the energy payments

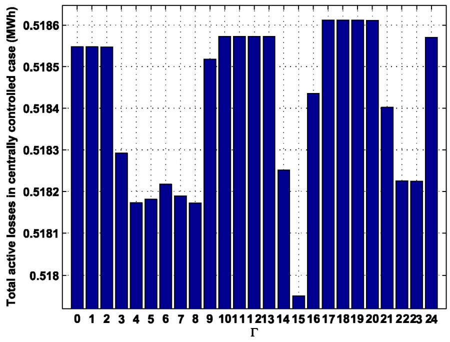

Fig. 10. The total energy losses vs Γ in controlled case

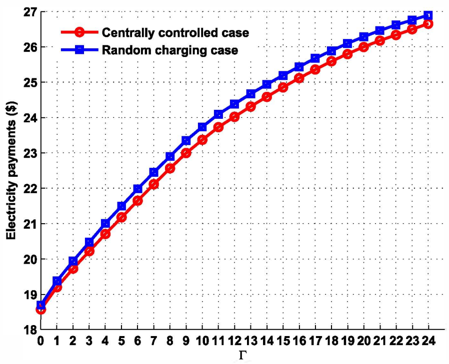

Fig. 11. The total energy payments compared in both cases

reduction would be more significant. Uncertainty modeling is important because it would enable the DSO to avoid the risk of high values of electricity prices. The level of conservatism can be adjusted by choosing the appropriate (Γ) level. The proposed formulation is computationally attractive. The uncertainty modeling of electricity prices has a linear structure and does not add more complexity to the existing model.

VI. Discussions

Considering the operating strategy carried out, the general conclusions below are in order:

1) The proposed framework can be extended to consider the impacts of any renewable distributed generation (DG) units existing in the network.
2) The existing model can be enhanced if it is integrated with (Demand Response) DR programs.
3) Other uncertainties can be considered in the model to get closer to the reality. These uncertainties may be related to the demand, generation of renewable power generations or conventional technologies.
4) Other uncertainty modeling techniques can be used to deal with uncertainties such as Information Gap Decision Theory (IGDT). The IGDT technique can be used to model opportunistic behavior of decision maker (DSO).
5) Using the information provided by smart grid facilities, the price intervals can be updated. This means it is not necessary to run the model on a day ahead basis. The rolling window can move from t_1 and updates the available information about the uncertain parameters.
6) The distribution network reconfiguration can also be regarded as an option for reducing the operating costs.
7) The capacitor switching can also be used to reduce the operating costs.

VII. Conclusion

An optimization based technique was proposed to obtain the optimal scheduling of the EVs. The robust optimization approach is used to deal with the uncertainty of electricity prices. The electric vehicles have been used as the energy storage device in order to maximize the flexibility of distribution network operator. The obtained results from the proposed risk-averse strategy ensure the decision maker that although the predicted values of the uncertain input parameters are not exact but the outcome of the proposed model (payments) would be immune against the prediction error to some controlled extent (Γ).

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