

Real-Time Dispatchability of Bulk Power Systems with Volatile Renewable Generations

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Abstract—The limited predictability and high variability of renewable generations has brought significant challenges on the real-time operation of bulk power systems. This paper proposes the concept of real-time dispatchability (RTDA) of power systems with variable energy resources, which focuses on investigating the impact of operating constraints and the cost of corrective actions on the flexibility of real-time dispatch. RTDA is the largest region in the uncertainty space, such that all the elements in it will not cause infeasibility while deploying a corrective action. This paper proposes a closed polyhedral form of RTDA. Moreover, an adaptive constraint generation algorithm is proposed to compute the boundaries of RTDA. Three potential applications are suggested. Case studies on the IEEE 118-bus system illustrate the RTDA concept and demonstrate the validity and efficiency of the proposed method in practical applications.

Index Terms—Bulk power system, real-time dispatch, renewable power generation, uncertainty.

NOMENCLATURE

Major symbols and notations used in this paper are defined below for quick reference. Others are defined following their first appearance as required.

A. Parameters

α_i, b_i	Production cost coefficients of generator i .
b_i^+	Up regulation cost coefficient of generator i .
b_i^-	Down regulation cost coefficient of generator i .
C^R	Available cost of real-time dispatch.
C_j^W	Capacity of variable energy resource (VER) plant j .
F_l	Transmission capacity of line l .
N_W	The number of VER plants.
P_i^l	Minimal output of generator i .
P_i^u	Maximal output of generator i .
p_i	Current output of generator i .
p_q	Power demand of load q .
R_i^+	Ramp-up limit of generator i .
R_i^-	Ramp-down limit of generator i .
Δt	Time duration of the current dispatch interval.
w_j^e	Current output of VER plant j .
π_{il}	Power flow distribution factor from unit i to line l .
π_{jl}	Power flow distribution factor from VER j to line l .
π_{ql}	Power flow distribution factor from load q to line l .

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B. Decision Variables

p_i^+	Up-regulation power of generator i .
p_i^-	Down-regulation power of generator i .
Δw_j	Uncertain output of VER plant j .

C. Abbreviations

Ad-CG	Adaptive Constraint Generation
BLP	Bilinear Program
DNE	Do-not-exceed
ITLP	Iterative Linear Programming
LP	Linear Program
LPCC	Linear Program with Complementarity Constraints
MILP	Mixed Integer Linear Program
OPF	Optimal Power Flow
RO	Robust Optimization
RTD	Real-time Dispatch
RTDA	Real-time Dispatchability
SO	Stochastic Optimization
TED	Traditional Economic Dispatch
VER	Variable Energy Resource

D. Vector and Matrix Notations

Vectors and matrices used in the compact formulations are defined as follows. Vector $p = \{p_i\}, \forall i$, $p^+ = \{p_i^+\}, \forall i$, $p^- = \{p_i^-\}, \forall i$, $w^e = \{w_j^e\}, \forall j$, $\Delta w = \{\Delta w_j\}, \forall j$. Stacked vector $y = \{p^+, p^-\}$ denotes the corrective action in real-time dispatch (RTD) after Δw is realized. Matrices A , B , C , and vector b are used in the compact form of the operating constraints. H and h represent the matrix and vector in the polyhedral formulation of RTDA.

I. INTRODUCTION

THE increasing penetration of variable energy resources (VERs), such as the wind and solar generation, has been increased dramatically in modern bulk power systems during the past decade [1]–[3]. As a result, maintaining power balance and managing transmission congestion is becoming more challenging [4]–[6] in power system operations.

From the perspective of the short-term generation scheduling, mitigating the increasing level of uncertainties requires more reserve, storage and ramping capacities. The former two requirements mainly aim at preserving the generation adequacy in the time-scale from half an hour to several hours, while the last one usually focuses on enhancing the load following capability in the time-scale within half an hour. Ref. [7] proposes a method for calculating reserve

capacities in the time-scale of 1-48 hours. Ref. [8] discusses the energy balancing issues in the time-scale from several minutes to several days, covering the areas of automatic generation control, economic dispatch and unit commitment. Ref. [9] addresses the ramping scheduling problem in economic dispatch. Storage devices provide various kinds of reserves. The pumped-storage unit is shown to be effective in mitigating wind power uncertainty [10], [11]. It is also noticed that the plug-in electric vehicles could play a very important role of storage devices by V2G interactions [12], [13].

To operate power systems with VERs in a reliable and economical way, some advanced optimization methods have been proposed. For instance, the stochastic optimization (SO) method in [14]–[18] uses probability distributions to describe uncertainties, while the robust optimization (RO) method in [19]–[23] uses a pre-specified set to model uncertainties. Both methods have been applied to unit commitment and economic dispatch problems. Other methods [24]–[26] motivated from operating experiences also appear to be effective. These studies focus on producing a reliable short-term (from hourly-ahead to day-ahead) generation scheduling plan that admits a feasible corrective action to restore the system, despite the anticipatory variations of VERs. Their theoretical soundness and compatibility with power system operating patterns make them promising to be used in the near future.

This paper investigates the feasibility issue of RTD from the perspective of uncertainties: starting from the current operating point, how much uncertainty of nodal injection the power system can accommodate in a certain dispatch interval, which is called the real-time dispatchability (RTDA). The terminology dispatchability is used in [27] to describe a coordinate operation of wind generations and pumped storage units. The formal definition of RTDA discussed in this paper will be given in Section II. Now we just mention RTDA is a deterministic set in the subspace of uncertain variables, rather than a scalar index or a strategic solution of an optimal decision-making problem. Similar concepts include the do-not-exceed (DNE) limit proposed in [28] and the flexibility measure defined in [29]. The DNE limit is a box set of uncertainty scenarios those will not cause infeasibility. Ref. [28] suggests an economic dispatch model that exhibits the largest DNE limit, which is an attractive feature. RTDA does not outcome any generation plan. It is an analogue of the box set in DNE method, however, RTDA is a more complicated set that describes exactly how much uncertainty the corrective action can handle. In this regard, the DNE set is an inner-box approximation of RTDA. The concept of flexibility defined in [29] generalizes the DNE limit into a multi-period setting. To retain computational tractability, a box description is still adopted. Since the RTDA depends on the current operating point, it is restricted in a single-period setting, but gives larger estimation of dispatchability. There are some other definitions of flexibility, such as those in [30]–[33], but they have different modeling paradigms and quantifications compared with RTDA in this paper.

A recent study [34] proposes a method to compute the dispatchable region of wind power generation, which is the original idea of RTDA. This paper extends the definition of

dispatchable region, proposes a more efficient algorithm to compute RTDA, and provides three potential applications of the proposed concept and method. Details of our contributions are summarized as follows:

1) The concept of real-time dispatchability of power systems with VER generations. The physical interpretation of RTDA is the maximal ability of the power system to accommodate uncertain VER fluctuations in RTD, or a security region in the uncertainty space, which is similar to the dispatchable region proposed in [34]. We describe the differences between RTDA and the dispatchable region below. In the basic setting of the dispatchable region, the current generation and reserve portfolio are provided by a joint energy and reserve dispatch problem. If a generator does not offer reserve capacity, its output is a constant in RTD. Note that the optimal reserve capacity offered by each generator depends on the anticipated uncertainty in the energy and reserve dispatch problem, say the uncertainty set or some sampled scenarios, which are somehow subjective. Moreover, in the dispatchable region problem, the corrective actions are assumed to be free of charge. In this paper, the RTDA extends the dispatchable region in two ways. On the one hand, all generators are assumed to be able to adjust their output in RTD subjecting to their ramping limits and generating capacities. Moreover, RTDA explicitly consider the cost of RTD, which is ignored in [34]. The first extension makes RTDA do not rely on subjective assumptions on the underlying uncertainties of VER generations. The second extension incorporates economic considerations, which is important in evaluating flexibilities. They make RTDA more close to the actual situation of RTD. However, the former extension will introduce more decision variables in RTD; the latter extension will in general introduce more boundaries in RTDA than the dispatchable region in [34]. Thus the requirement on the computational efficiency is more demanding.

2) An efficient algorithm to compute RTDA. RTDA is a set of uncertain nodal injections that will not cause infeasibility in RTD. Computing such a region is different from solving an optimization problem. We give an explicit polyhedral form of RTDA. Ref. [34] only claims the dispatchable region is a polytope, but does not reveal its closed form. To fulfill the requirement on the computational efficiency, we propose an adaptive constraint generation (Ad-CG) algorithm to retrieve the boundaries of RTDA. The Ad-CG algorithm in this paper has different mathematical background compared with the algorithm in [34]. The advantage of the Ad-CG algorithm is that it no longer requires finding the boundary point in each iteration, thus the computational efficiency can be enhanced remarkably. A mixed integer linear program (MILP) based oracle as well as an iterative linear program (ITLP) based oracle is suggested to implement the Ad-CG procedure.

3) Several potential applications of RTDA are summarized. In addition to those mentioned in [34], this paper reveals that RTDA can imply necessary information on the vulnerable elements of the power system which prevent the further accommodation of VER generations.

The remaining parts of this paper are organized as follows. The mathematical formulations of RTDA as well as its closed polyhedral formulation are presented in Section II. The Ad-CG

algorithms for computing RTDA and its practical implications are described in section III. Numerical experiments on the IEEE 118-bus system are reported in Section IV. Conclusions are given in Section V.

II. FORMULATION, DEFINITION AND GEOMETRIC PROPERTY OF RTDA

A. Mathematic Formulation

Symbols used in this section are defined in Nomenclature. RTD is based on the current operating point, in other words, the output p of generating units and the output w^e from VER plants are known. In the current dispatch interval, VERs' output may deviate from w^e , their actual output is $w = w^e + \Delta w$, where Δw is the deviation, or the forecast error. Once w is observed, corrective actions $\{p^+, p^-\}$ are deployed to recover operating constraints, after which the output of generators is changed to $p + p^+ - p^-$. Due to the limited ramping capability and the cost of RTD, the system cannot accommodate arbitrary large fluctuations of VERs' output. The RTDA problem is to identify the largest set W , such that the corrective actions $\{p^+, p^-\}$ always exist for all $w = w^e + \Delta w, \forall \Delta w \in W$.

First, the RTD problem is stated as follows. For a given Δw and the available re-dispatch cost C^R , the RTD problem renders finding a feasible solution $\{p^+, p^-\}$ in the following constraint set

$$\sum_i (p_i + p_i^+ - p_i^-) + \sum_j (w_j^e + \Delta w_j) = \sum_q p_q \quad (1.1)$$

$$-F_l \leq \sum_i \pi_{il}(p_i + p_i^+ - p_i^-) +$$

$$\sum_j \pi_{jl}(w_j^e + \Delta w_j) - \sum_q \pi_{ql} p_q \leq F_l, \forall l \quad (1.2)$$

$$P_i^l \leq p_i + p_i^+ - p_i^- \leq P_i^u, \forall i \quad (1.3)$$

$$0 \leq p_i^+ \leq R_i^+ \Delta t, 0 \leq p_i^- \leq R_i^- \Delta t, \forall i \quad (1.4)$$

$$\sum_i (b_i^+ p_i^+ + b_i^- p_i^-) \leq C^R \quad (1.5)$$

where constraints (1.1) – (1.3) are the power balancing condition, security limitations on the active power flows of transmission lines, and the capacity restrictions on generating output, respectively; constraint (1.4) stipulates that the regulation power is subjected to the ramping capacity; budget constraint (1.5) restricts the total cost of RTD, imposing a coupled constraint on the regulation power of all units. Some additional remarks are given.

1. To acquire the optimal corrective actions, one should minimize the cost $C^R = \sum_i (b_i^+ p_i^+ + b_i^- p_i^-)$ subjecting to constraints (1.1) – (1.4), which boils down to a LP. In the RTDA problem raised in the next sub-section, we study the existence of corrective actions rather than acquiring a strategic solution. So the RTD problem is formulated as a constraint set in this paper.

2. The current generation strategies $\{p, w^e\}$ are known in RTD constraint set (1), thus they are regarded as parameters. In practical engineering, the current output of VER plants w^e has already been observed. The current output p of generators is also available from the energy management system. If one wish to optimize the current dispatch strategy p , the traditional economic dispatch (TED), the SO and RO based approaches

in [18] and [22] as well as others in [24]–[26] can be applied. However, this is not the main concern of this paper. The current dispatch strategy p acts as the input of RTD, whose corresponding RTDA is under investigation. For the purpose of simplification, we assume p is the optimal solution of the following TED problem, although it may not be the best choice from a system operation point of view

$$\min \sum_i (a_i p_i^2 + b_i p_i) \quad (2.1)$$

$$s.t. P_i^l \leq p_i \leq P_i^u, \forall i \quad (2.2)$$

$$\sum_i p_i + \sum_j w_j^e = \sum_q p_q \quad (2.3)$$

$$-F_l \leq \sum_i \pi_{il} p_i + \sum_j \pi_{jl} w_j^e - \sum_q \pi_{ql} p_q \leq F_l, \forall l \quad (2.4)$$

where objective (2.1) is the generation cost; constraint (2.2) – (2.4) is the generation capacity restrictions, the power balancing condition and active power flow limits on transmission lines associating with w^e , respectively. In both RTD constraint set (1) and TED (2), we adopt the DC power flow model. We are aware that AC power flow equations can provide more accurate operating conditions of the entire network in real-time dispatch. Since we will develop our method resting on linear optimization theories, it is difficult to incorporate AC power flow equations in our model because they are nonlinear and non-convex. Nevertheless, because we restrict our method to bulk power systems with centralized renewable integrations, we believe the DC power flow model can provide satisfactory approximations for active power flows in such high-voltage transmission networks.

B. Definition of RTDA and its Polyhedral Formulation

To simplify notations and derivations, constraint set (1) can be arranged into a compact form as

$$Ap + By + C(w^e + \Delta w) \leq b^0$$

where matrices A, B, C and vector b^0 are the coefficients associated in constraints (1.1) – (1.5). Define the feasible set of RTD under given p and Δw as

$$Y(p, \Delta w) = \{y \mid By \leq b - Ap - C\Delta w\} \quad (3)$$

where the vector $b = b^0 - Cw^e$. The compact formulation (3) brings up the definition of RTDA as follows

Definition 1 (Real-time Dispatchability): The RTDA is a set W^{RTD} in the uncertainty space that satisfies

$$W^{RTD}(p) = \{\Delta w \mid \exists y : By + C\Delta w \leq b - Ap\}$$

Geometrically, W^{RTD} is the largest region in the uncertainty space that guarantees that the variations of VERs will not cause infeasibility in constraint set (1). Certainly, it depends on the current dispatch strategy p . In the following context, we will omit the dependence of W^{RTD} on parameter p without causing confusions. The following theorem reveals that W^{RTD} has a closed polyhedral formulation.

Theorem 1: RTDA has the following polyhedral form

$$W^{RTD} = \{\Delta w \mid u^T C \Delta w \geq u^T (b - Ap), \forall u \in \text{vert}(U)\}$$

where $U = \{u \mid B^T u = 0, -1 \leq u \leq 0\}$, the set $\text{vert}(U)$ represents for all the vertices of the polytope U .

Proof: For fixed p and Δw , consider the following LP

$$\begin{aligned} \min \quad & 1^T s^+ + 1^T s^- \\ \text{s.t.} \quad & B y + I s^+ - I s^- \leq b - Ap - C \Delta w \\ & s^+ \geq 0, s^- \geq 0 \end{aligned} \quad (4)$$

where s^+ and s^- are non-negative slack variables, 1^T and I are the unit vector and identity matrix with compatible dimensions, respectively. The dual of LP (4) is

$$\begin{aligned} \max \quad & u^T (b - Ap - C \Delta w) \\ \text{s.t.} \quad & B^T u = 0, -1 \leq u \leq 0 \end{aligned} \quad (5)$$

where u is the dual variable. The set $Y(p, \Delta w) \neq \emptyset$ if and only if the optimal value of LP (4) is 0. According to the strong duality of LP, the optimal value of the dual LP (5) is also 0, implying

$$u^T (b - Ap - C \Delta w) \leq 0, \forall u \in U$$

Regarding Δw as a parameter in the above condition, we can claim that W^{RTD} has the following polyhedral representation by noting that the optimal solution of LPs can always be found at one of its vertices

$$\begin{aligned} W^{RTD} &= \{\Delta w \mid u^T C \Delta w \geq u^T (b - Ap), \forall u \in U\} \\ &= \{\Delta w \mid u^T C \Delta w \geq u^T (b - Ap), \forall u \in \text{vert}(U)\} \end{aligned}$$

This completes the proof. ■

Theorem 1 gives an explicit polyhedral form of W^{RTD} based on vertex enumeration. Some further discussions are provided.

1. The polytope U only depends on the matrix B , which depends on the parameters of generators and the transmission network, and is independent of the current operating condition, so is fixed for a given system. For some small-scale power systems, the vertices of polytope U can be computed off-line, then W^{RTD} can be directly constructed according to Theorem 1 after receiving the current generation dispatch p and VERs' output w^e (contained in the vector $b = b^0 - C w^e$).

2. In fact, in the vertex enumeration based formulation in Theorem 1, most constraints in W^{RTD} will be redundant. The method proposed in [35] can be applied to remove redundant constraints through solving a LP.

3. Due to the difficulty and complexity of vertex enumeration [36], it is usually impossible to enumerate all vertices of set U even for medium scale power systems. In the next subsection, an algorithm will be proposed to identify binding vertices in U and generate the boundaries of W^{RTD} adaptively without seeking any boundary point, which is different from that in [34].

III. SOLUTION APPROACH

Theorem 1 indicates that W^{RTD} can be formulated as a polytope in Δw -subspace by enumerating the vertices in set U . It is also mentioned that most constraints will be redundant, so an efficient strategy is desired to identify a set of critical vertices in U that will create non-redundant constraints in W^{RTD} . We start this section with two separation oracles that can identify one critical vertex in the set U , and then present the Ad-CG algorithm to compute W^{RTD} , finally summarize three possible applications of RTDA.

A. The Separation Oracles

According to Theorem 1, the following inequality holds

$$u^T C \Delta w \geq u^T (b - Ap), \forall u \in U, \forall \Delta w \in W^{RTD} \quad (6.1)$$

Recall Definition 1, W^{RTD} is the largest set that makes (6.1) holds true, indicating that if $\Delta w^* \notin W^{RTD}$, there must be some vertex $u^* \in U$ that will make

$$(u^*)^T (b - Ap - C \Delta w^*) > 0 \quad (6.2)$$

Definition 2 (Critical Vertex): An element $u^* \in U$ is called a critical vertex corresponding to $\Delta w^* \notin W^{RTD}$ if equation (6.2) holds.

Given some $w^* \notin W^{RTD}$, the following hyperplane strictly separates Δw^* from W^{RTD}

$$(u^*)^T C \Delta w = (u^*)^T (b - Ap) \quad (7)$$

It should be emphasised that hyperplane (7) will not remove any point in W^{RTD} . More precisely,

$$\{\Delta w \mid (u^*)^T C \Delta w < (u^*)^T (b - Ap)\} \cap W^{RTD} = \emptyset$$

because of constraint (6.1). Our strategy for computing W^{RTD} is to create a large enough initial box set W^B such that $W^{RTD} \subseteq W^B$. Then separate every $\Delta w^* \notin W^{RTD}$ by identifying corresponding critical vertex and creating the hyperplane in equation (7), until $W^B \subseteq W^{RTD}$, thus $W^B = W^{RTD}$. This procedure involves the following important problem:

Seperation problem: Given a polytope W , check if equation (6.1) is satisfied for sets U and W . If not, find $w^* \in W$ and $u^* \in U$ such that equation (6.2) holds.

The seperation problem can boil down to a bilinear program

$$\begin{aligned} R(p) &= \max u^T (b - Ap - C \Delta w) \\ \text{s.t.} \quad & u \in U, \Delta w \in W \end{aligned} \quad (8)$$

Since $u = 0$ is always feasible, the optimal value $R(p)$ must be non-negative. If $R(p) = 0$, then $\forall \Delta w \in W, Y(p, \Delta w) \neq \emptyset$, or equivalently, equation (6.1) holds true for sets U and W , and $W \subseteq W^{RTD}$. Otherwise, if $R(p) > 0$, the optimal solution $\Delta w^* \notin W^{RTD}$ and solution u^* is a critical vertex corresponding to Δw^* . Equation (6.2) holds true at the pair (w^*, u^*) . In the remaining part of this subsection, two methods are introduced to solve BLP (8).

a) A MILP based oracle

This oracle is motivated from [37] and also used in [34]. It is confirmed in Theorem 1 that RTDA can be written as

linear inequalities, suppose its current outer approximation is $W = \{\Delta w \mid H\Delta w \geq h\}$. It is shown in [37] that if W and U are separated polytopes, a BLP can be transformed into a MILP following three steps. First consider the problem

$$R(x) = \max_{u \in U} \left(u^T (b - Ap) + \max_{\Delta w} (-u^T C \Delta w) \right) \quad (9.1)$$

$$s.t. H\Delta w \geq h : \xi \quad (9.2)$$

where ξ is the dual variable of the constraints in set W . The KKT condition of the inner LP (parameterized in u) is

$$C^T u + H^T \xi = 0 \quad (10.1)$$

$$0 \geq \xi \perp h - H\Delta w \leq 0 \quad (10.2)$$

Subjecting to constraints (10.1) and (10.2), the following equation holds because strong duality holds for LPs

$$-u^T C \Delta w = h^T \xi \quad (10.3)$$

Equation (10.3) allows to replace the bilinear term $-u^T C \Delta w$ in the objective of BLP (8) with a linear term $h^T \xi$. The non-linearity is moved into the complementary constraint (10.2). Moreover, the complementary constraint (10.2) can be linearized by using the disjunctive method in [38] as follows

$$\begin{aligned} -M\theta &\leq h - H\Delta w \leq 0 \\ -M(1 - \theta) &\leq \xi \leq 0 \\ \theta &\in \{0, 1\}^{N_C} \end{aligned} \quad (11)$$

where θ is a vector consisting of N_C binary variables; N_C is the number of linear inequalities in W ; M is a large enough constant. Because W is a bounded polytope, the inner LP of problem (9) must have a bounded optimal solution, so does the dual problem. Consequently, the dual variable ξ must be bounded at the optimal solution, thus M is also finite.

Finally, BLP (8) is equivalent to the following MILP

$$\begin{aligned} R(p) &= \max u^T (b - Ap) + \xi^T h \\ s.t. \quad u &\in U, \quad \theta \in \{0, 1\}^{N_C} \\ C^T u + H^T \xi &= 0 \\ -M\theta &\leq h - H\Delta w \leq 0 \\ -M(1 - \theta) &\leq \xi \leq 0 \end{aligned} \quad (12)$$

For a given set W , the number of binary variables in MILP (12) depends on the number of constraints in W , and is independent of the power system model, implying MILP (12) can be efficiently solved for large-scale power systems providing N_C is small. However, with the computation going on, the number of constraints in set W increases. Thus MILP (12) will gradually involve more binary variables. Some weak points are analyzed and summarized at the end of next subsection.

b) An iterative LP based oracle

By noting that W and U are separated polytopes, an iterative LP (ITLP) based method proposed in [39] can be applied to compute a local optimal solution of BLP (8). By properly choosing a set of initial values of Δw , this approach can produce a high quality solution. The issue of selecting initial values will be discussed in the next subsection. The

algorithm is described as follows

Algorithm 1: ITLP oracle

Step 1: Choose a tolerance $\varepsilon > 0$, and an initial $w^* \in W$;
Step 2: Solve LP (13) with current w^* , the optimal solution is u^* and the optimal value is R_1 ;

$$R_1 = \max_{u \in U} u^T (b - Ap - C \Delta w^*) \quad (13)$$

Step 3: Solve LP (14) with current u^* , the optimal solution is w^* and the optimal value is R_2 ;

$$R_2 = \max_{\Delta w \in W} (u^*)^T (b - Ap - C \Delta w) \quad (14)$$

Step 4: If $R_2 - R_1 \leq \varepsilon$, report the optimal value $R(p) = R_2$ and the optimal solution $\Delta w^*, u^*$, terminate; otherwise, go step 2.

The optimal solution of LP (13) and LP (14) can always be found at one of the vertices of polytopes U and W , respectively, i.e. $\Delta w^* \in \text{vert}(W)$ and $u^* \in \text{vert}(U)$ holds. Algorithm 1 will terminate in finite number of iterations [39] because both U and W have finite number of vertices. Due to the non-convexity of BLP (8), Algorithm 1 does not have a theoretical guarantee on the global optimality. Nevertheless, its performance is confirmed by the robust unit commitment and multi-period economic dispatch applications in refs. [40]–[42]. This oracle is especially suited for the instances with a large number of uncertain resources.

B. Adaptive Constraint Generation Algorithm

Based on Theorem 1 as well as both separation oracles in the previous sub-section, the Ad-CG algorithm is formally provided below

Algorithm 2 Ad-CG-MILP

Step 1: Choose a sufficiently large set $W^B = \{\Delta w \mid H\Delta w \geq h\}$ and tolerance $\delta > 0$;

Step 2: Solve MILP (12) with the current W^B , the optimal solution is u^* and Δw^* , and the optimal value is $R(p)$.

Step 3: If $R(p) \leq \delta$, terminate, report $W^{RTD} = W^B$; else add the following constraint to W^B , update matrix H and vector h in W^B , and go step 2;

$$(u^*)^T C \Delta w \geq (u^*)^T (b - Ap)$$

Algorithm 3 Ad-CG-ITLP

Step 1: Choose a sufficiently large set $W^B = \{\Delta w \mid H\Delta w \geq h\}$ and a tolerance $\delta > 0$.

Step 2: Create the set of initial points W^I (see discussion).

Step 3: Pick up some $\Delta w^0 \in W^I$, solve BLP (8) with Δw^0 and the current W^B using Algorithm 1, the optimal solution is u^* and Δw^* , and the optimal value is $R(p)$, $W^I = W^I - \{\Delta w^0\}$.

Step 4: If $R(p) \geq \delta$, add the following constraint to W^B , and go step 2; else if $R(p) < \delta$ and $W^I \neq \emptyset$, go step 3; else go step 5;

$$(u^*)^T C \Delta w \geq (u^*)^T (b - Ap)$$

Step 5: Terminate, report $W^{RTD} = W^B$.

Some details are discussed below.

1. There are several ways to choose the initial points set W^I . If the number of VER plants N_W is small, we can use the extreme points of the initial hypercube W^B , otherwise, we can also use the projection of the origin on the boundaries of current W^B . The key point is, these initial points should cover most directions of the Δw -subspace. This heuristic helps Algorithm 1 be able to find a high quality solution of BLP (8).

2. The following fact is used in Algorithm 3 to accelerate convergence: any feasible solution of BLP (8) that makes $R(p) > 0$ indicates a critical vertex. Therefore, a boundary is immediately created in step 4 when $R(p) \geq \delta$ is found, without further exploring the remaining initial values. Such technique can be adopted in Algorithm 2 as well. In such circumstance, instead of solving MILP (12) to optimality, a feasible solution $u \in U$ in the following constraint set is desired in step 2 of Algorithm 2

$$\begin{aligned} u^T(b - Ap) + \xi^T h &\geq \delta, \quad u \in U \\ C^T u + H^T \xi &= 0, \quad \theta \in \{0, 1\}^{N_C}, \forall i \\ -M\theta &\leq h - H\Delta w \leq 0 \\ -M(1 - \theta) &\leq \xi \leq 0 \end{aligned}$$

This can reduce the burden of branch and bound computation. Both Algorithm 2 and Algorithm 3 will terminate in finite number of iterations because polytope U has finite number of vertices. These two algorithms are different from the method proposed in [34]. Several advantages of theirs are given below

1. In Algorithm 2 and Algorithm 3, a boundary of W^{RTD} is immediately generated using the critical vertex u^* after solving BLP (8), while the method of [34] requires finding an approximated boundary point on the line segment connecting the origin and w^* in order to create a boundary hyperplane of W^{RTD} . The boundary point searching procedure contributes most of the computational efforts in each iteration. If all the generators are allowed to adjust their output, there will be more constraints in the primal constraint set (1) and thus more dual variables in vector u as well, and the boundary point searching will consume even more time. Therefore, the algorithms in this paper are more efficient than that in [34].

2. The range of W^{RTD} is not known in advance. It's prudent to choose large bounds in the initial set W^B in order to guarantee $W^{RTD} \subseteq W^B$, so does the method of [34]. However, to reduce the computational burden in searching the boundary points, set W^B should be as small as possible. Such trade-off is not required in Algorithm 2 and Algorithm 3, whose efficiency is almost independent of the choice of W^B .

Some possible weak points of Algorithm 2 and Algorithm 3 are also provided below.

1. Because Algorithm 1 may not report the global optimal solution of BLP (8), consequently, Algorithm 3 would possibly miss some critical vertices of U . However, it is still able to discover the majority of valid boundaries of W^{RTD} , because the choice of initial points of Δw covers most directions of the Δw -subspace. An alternative choice may be using the result of Algorithm 3 as the initial set W^B in Algorithm 2.

2. With the computation going on, the number of constraints N_C in set W^B increases. Thus MILP (12) involves more binary variables. The total number of constraints in the final result of W^{RTD} depends on the parameter of generators and the transmission network, the current operating point (p, w^e) , and the cost of RTD C^R . However, one does not have priori knowledge on how many constraints W^{RTD} has. In view that solving MILP is NP-hard, computing W^{RTD} is also an NP-hard problem.

3. Despite the NP-hardness, when there are only a few VER plants in the system, Algorithm 2 can be quite efficient. When the number of VER plants grows larger, Algorithm 3 can be adopted. Although this paper treats VER as the only source of uncertainty, the proposed model and algorithm is able to incorporate load variations as well. Because the dimension of load uncertainty is usually much higher than that of VERs, online-use may not be applicable even Algorithm 3 is used. One way to alleviate the computation burden is to merge several adjacent small uncertainty sources into a larger aggregated one, and simplify the related part in the network.

Finally, because we use a larger set W^B to approximate W^{RTD} in the first step, redundant constraints may occur in the resulting W^{RTD} , and can be removed by using the method in [35]. According to our experiences, the redundant constraints are rare, even there is any.

C. Applications

The polytope W^{RTD} provides a geometrical measurement on the dispatchability of the power system, characterizing exactly how much uncertainty the system can tackle. It is useful in some power system assessment issues. In this subsection, several potential applications are suggested.

a. Vulnerability assessment

Given a VER output variation $\Delta w \notin W^{RTD}$, RTD cannot restore the power system, there must be one or more operating constraints violated, due to insufficient ramping limits, or the lack of transmission capacities, or the shortage of cost, or the combination of them. The terminology "vulnerability" here means the factors which prevent the system from being restored. To identify such factors, it is necessary to pick up the violated inequalities in W^{RTD} such that

$$(u^i)^T C \Delta w < h_i, \quad i \in I_V$$

where I_V corresponds to the indices of the constraints in W^{RTD} violated due to Δw . Each non-zero element in u^i , $i \in I_V$ suggests a binding constraint in (1), which indicates that the associated resource will be used up in RTD, thus the corresponding element is vulnerable to deviation mode Δw . The magnitudes of these elements may naturally give a ranking of vulnerable components.

b. Security assessment

Given a VER output variation $\Delta w \in W^{RTD}$, the corrective action $y \in Y(p, \Delta w)$ can restore the power system in RTD. From a practical point of view, the minimal distance d from Δw to the boundaries of W^{RTD} is desired, reflecting the

security margin to infeasibility. The distance can be explicitly calculated from the following equation [34]

$$d = \min \left\{ \frac{|(\Delta w - \Delta w^{bi})^T H^i|}{\sqrt{(H^i)^T H^i}}, \forall i \right\} \quad (15)$$

where H^i is the i -th row of matrix H , h_i is the i -th element of vector h , Δw^{bi} is an arbitrary point on the i -th hyperplane, satisfying $(H^i)^T \Delta w^{bi} = h_i$. Because the expression of W^{RTD} is known at this stage, it's quite easy to acquire such point. It should be mentioned that the actual output of a practical VER plant will not exceed its capacity C_i^w nor will it be negative. If RTDA could cover the variability of VERs in such ranges, the system dispatch would be quite robust.

c. Reliability assessment

In addition, if the probability distribution of VERs output is available, one can test the system reliability by verifying the probability that Δw falls in the set W^{RTD} from Mont Carlo simulation without solving extensive power flow equations, thus can be implemented online. Moreover, the system reliability level under different cost can be easily examined, providing a reference for operators with different risk preferences and economic interests. Because we are focusing on the RTD, whose time scale is typically from 15 minutes to 1 hour, we do not consider generator outage and transmission line tripping as the long term reliability issue does.

Besides, RTDA can be used to compare the dispatchability of power systems with different dispatch strategies offered by different optimization methods, and investigate the impact of various generation facilities on the operating flexibility, such as the carbon capture plants and storage devices.

IV. CASE STUDIES

In this section, the proposed method is applied to the IEEE 118-bus system to illustrate the concept of RTDA, and demonstrate the efficiency of the Ad-CG algorithm. The data of the IEEE 118-bus system are provided online at: http://motor.ece.iit.edu/data/JEAS_IEEE118.doc. All numeric experiments are conducted on a laptop computer with Intel i5-3210M CPU and 4 GB memory. MILPs and LPs are solved by CPLEX 12.5.

This test system consists of 54 conventional generators and 186 transmission lines. In the considered dispatch interval, the total demand is 5500MW. The up/down regulation cost coefficient b_i^+/b_i^- of each generator is assumed to be 10% of its production cost coefficient b_i according to the setting in [18]. The up/down ramping limit R_i^+/R_i^- of each generator is assumed to be 25% of its capacity. Virtual wind farms are connected. The methodology proposed in previous sections is applied to compute the RTDA of this system for two purposes: simple cases with two or three wind farms are considered first, such that the set W^{RTD} can be easily visualized; after that, the number of wind farms are increased to test the efficiency of the Ad-CG algorithm when the dimension of uncertainty grows higher.

Case 1: Two wind farms are connected at bus #70 in Zone 1 and bus #49 in Zone 2. Their respective current output is equal

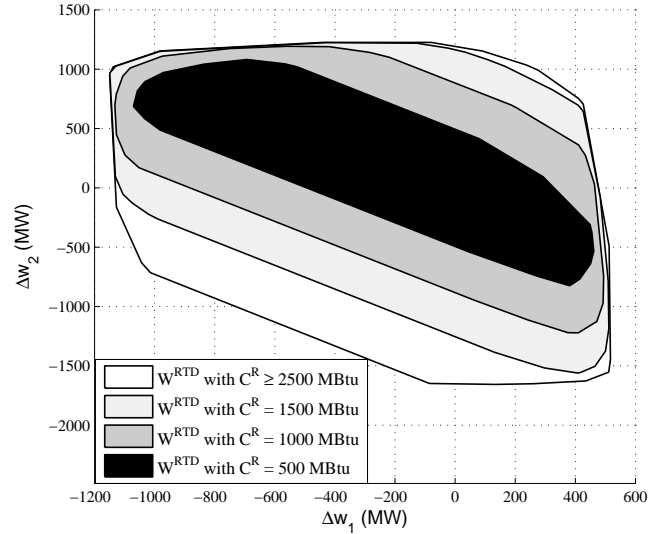


Fig. 1. W^{RTD} of two wind farms under different C^R . With C^R increasing in a certain range, W^{RTD} grows larger. When C^R is big enough, the boundaries of W^{RTD} will mainly depend on other factors, such as ramping capacities of generators and power flow limits of transmission lines.

to 350MW. The TED problem (2) is firstly solved to retrieve the current dispatch strategy, and then Algorithm 2 is applied to compute the W^{RTD} under different RTD cost C^R . The pure computation time (which does not include preparing system data and formulating constraints) is less than 5 seconds in all these tests. Results are shown in Fig. 1. The algorithm proposed in [34] can also be applied to compute the RTDA. The computation time varies from 20-25 seconds in these tests.

For each case, 1000 uniformly distributed samples Δw^s are generated and the feasibility of system (1) is tested. It is found that no constraints are violated in system (1) if and only if $\Delta w^s \in W^{RTD}$. This certifies that W^{RTD} satisfies Definition 1 and the Ad-CG algorithm is correct. The impact of available RTD cost C^R on RTDA can be observed from Fig. 1, showing that W^{RTD} grows larger with C^R increasing. However, when C^R is sufficiently large, the cost constraint (1.5) never becomes binding, and the feasibility of RTD will mainly depend on the ramping limits of generators and power flow capacities of transmission lines, thus C^R becomes less important.

One more wind farm is connected at bus #100 in Zone 3. The output of the three wind farms is equal to 250MW. The TED problem (2) is solved with current wind generation and load, after that Algorithm 2 is applied to compute the corresponding W^{RTD} with the associating RTD cost C^R varying from 200MBtu to 2500 MBtu. Results are shown through Fig. 2 to Fig. 4, from which we can see clearly W^{RTD} consists of hyperplanes and is polytope. Similar to Fig. 1, W^{RTD} grows larger with C^R increasing.

This case demonstrates that RTDA provides both analytical tools as well as visualized synthesis to study the ability of power systems to accommodate uncertain wind generations.

Case 2: The efficiency of Algorithm 2, the combination of Algorithm 3 and Algorithm 2 (denote by Algorithm 3-2),

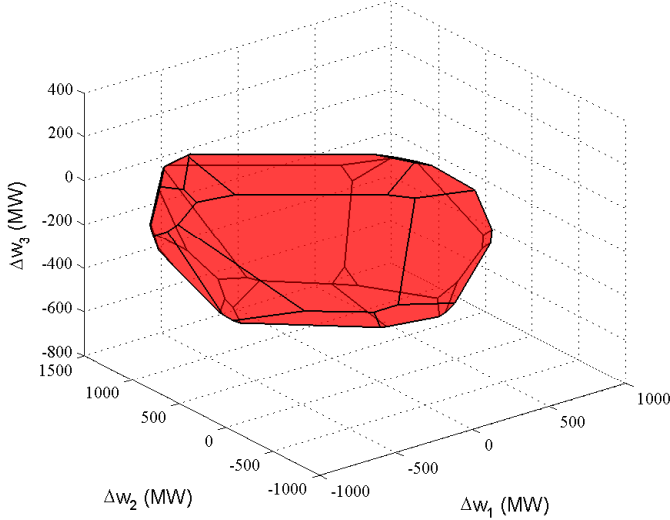


Fig. 2. W^{RTD} with three wind farms and $C^R=200\text{MBtu}$, consisting of 28 boundary-hyperplanes.

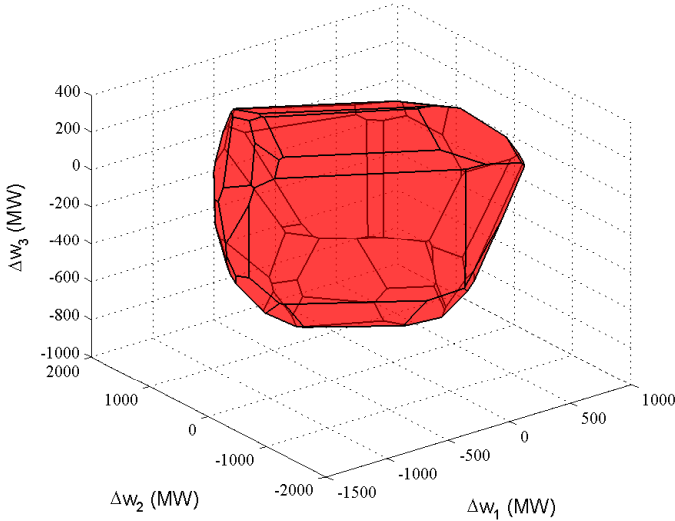


Fig. 3. W^{RTD} with three wind farms and $C^R=600\text{MBtu}$, consisting of 36 boundary-hyperplanes.

in which the former runs first, the latter uses the result of the former as the initial set W^B , as well as the algorithm proposed in [34] are tested by increasing the number of wind farms. When adding wind farms to the grid, the total current wind generation is maintained at 700MW, the output of each wind farm is equal to $700/N_W$, N_W is the number of wind farms. Results shown in Table I demonstrate that both algorithms proposed in this paper outperform that proposed in [34], because the boundary point searching is no longer required. We also found that the ITLP oracle sometimes can fail to find the global optimal solution of BLP (8). To guarantee a reliable result, we incorporate a checking step in Algorithm 3 by using the MILP based oracle, rendering Algorithm 3-2.

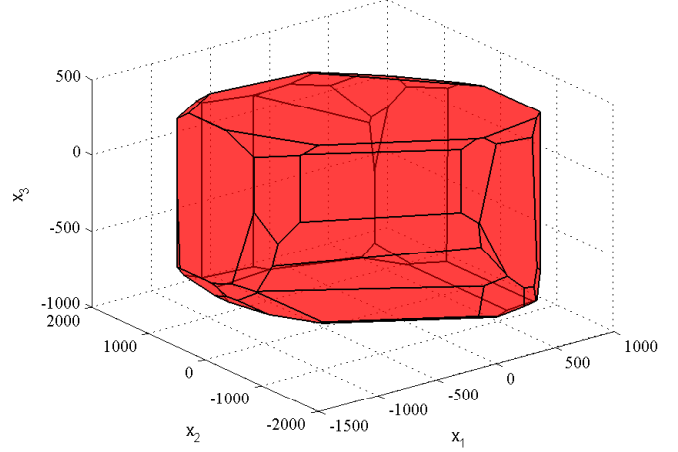


Fig. 4. W^{RTD} with three wind farms and $C^R=2500\text{MBtu}$, consisting of 47 boundary-hyperplanes.

TABLE I
COMPUTATION TIMES WITH DIFFERENT N_W

N_W	Computational time (seconds)		
	Algorithm 2	Algorithm 3-2	Algorithm in [34]
4	8	9	43
8	50	46	218
12	187	159	536
15	332	273	902

Table I suggests Algorithm 3-2 outperforms Algorithm 2 only when N_W becomes large, otherwise the MILP solver will quickly find a solution of MILP (12) with moderate branch-and-bound computation.

Finally, we would like to point out that in bulk power systems, the large-scale wind and solar generation is usually centrally integrated, especially in China. So there are often a few huge VER generation centers in such systems. The MILP based Algorithm 2 is especially suitable for these instances. Algorithm 3-2 is valuable when a lot of wind farms or the load variations should be taken into account. In the distribution networks, there are usually a mass of distributed VER generations. However, there are plenty of storage devices whose regulating speed is fast compared to their capacities, so VER uncertainty has less impact on such systems compared with the high-voltage transmission networks.

V. CONCLUSION

This paper proposes the concept of real-time dispatchability of power systems with volatile renewable power generations, which indicates exactly how much nodal injection uncertainty the system can tackle. An explicit polyhedral representation of RTDA is given. An efficient Ad-CG algorithm is proposed to generate the boundaries of RTDA provided the current operating status. Technically, our method is easy to implement because it only requires linear (MILP and LP) solvers.

Numerical experiments on the IEEE 118-bus system with wind generations provide a visualized description of RTDA.

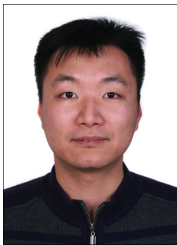
The computation times under different conditions demonstrate that the proposed method meets the requirement of online applications, and is especially suitable for bulk power systems with centralized VER integrations. RTDA provides the system operator the security boundary in the uncertainty space. Such outcomes are desired by operators and engineers to learn the impact of uncertain generations and demands on the feasibility of the real-time dispatch.

Some interesting research directions are open. For example, using the outcome of RTDA, it would be convenient to study the security and reliability of the power system operation in a quantitative way. It also provides an alternative approach to compare the flexibility of power systems under different dispatch strategies. It will be interesting to study how the system flexibility is influenced by various kinds of generating facilities, such as the carbon capture plants, storage devices, etc. Another very interesting topic is to evaluate the probability of infeasible real-time dispatch even in the absence of the probability distribution function of renewable generations, using the generalized probability inequality approach developed in refs. [43]–[45], which is one of our ongoing research.

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