Solving MIPs via Scaling-based Augmentation

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Abstract

Augmentation methods for mixed-integer (linear) programs are a class of primal solution approaches in which a current iterate is augmented to a better solution or proved optimal. It is well known that the performance of these methods, i.e., number of iterations needed, can theoretically be improved by scaling methods. We extend these results by an improved and extended convergence analysis, which shows that bit scaling and geometric scaling theoretically perform similarly well in the worst case for $0/1$ polytopes as well as show that in some cases geometric scaling can outperform bit scaling arbitrarily.

We also investigate the performance of implementations of these methods, where the augmentation directions are computed by a MIP solver. It turns out that the number of iterations is usually low. While scaling methods usually do not improve the performance for easier problems, in the case of hard mixed-integer optimization problems they allow to compute solutions of very good quality and are often superior.

1 Introduction

A standard approach to solving mixed integer linear programs (MIPs) is via a combination of branch-and-bound and cutting planes. This approach can be considered largely as *dual*, since in practice the nodes of the branch-and-bound tree are, to a large extent, pruned by objective bounds. An alternative view to solving MIPs is via *primal augmentation approaches*. Here the idea is to start from a feasible solution to the considered MIP and then move to a new solution with improved objective function value by means of an *augmentation step*.

In this work, we will consider a specific class of primal augmentation approaches, namely those arising from *scaling*. In a nutshell, here the objective function is adjusted to include a *potential* that guides the search away from the boundary, deep into the feasible regions similar to interior point methods in convex optimization. In the same vein, a scaling parameter μ controls the tradeoff between depth in the feasible region and optimizing the objective function. A key insight is that via appropriate scaling only a polynomial number of augmentation steps is needed. If now the computation of an augmenting direction can be performed fast, one can obtain, *in theory*, fast algorithms for solving MIPs. For example, this augmentation can be performed very fast for network flows, which, in fact, motivated several scaling approaches for MIPs in the first place (see e.g., Wallacher and Zimmermann [1992], Orlin and Ahuja [1992]). Traditional scaling approaches in the context of network flows include the well-known capacity scaling (scaling in the dual) and cost scaling (scaling in the primal).

While our focus is on the *computational feasibility and performance* of these scaling approaches for MIPs, we also provide new theoretical insights in terms of worst-case examples for bit scaling, and we slightly improve the analysis of geometric scaling.

1.1 Related work

Primal augmentation approaches in the context of MIPs have been well-studied, both from an algebraic point of view using test sets, but also in the context of solving linear programs and mixed-integer (nonlinear) programs exactly and approximately. Graver [1975] studied test sets (or Graver bases), i.e., the sets of feasible (integer) directions, which give rise to a natural converging augmentation algorithm; see also Scarf [1997]. Algebraic approaches (see e.g., De Loera et al. [2013, 2014] and the references contained therein) are usually based on an algebraic characterization of test sets; then an improving direction is used for augmentation.

Augmentation methods have recently become important to investigate mixed-integer nonlinear problems (MINLPs), see, e.g., Hemmecke et al. [2010] and Onn [2010] for an overview. Here, test sets are used to solve or approximate MINLPs; some selected references are Hemmecke et al. [2014, 2011], Lee et al. [2012, 2008], De Loera et al. [2008]. However, in this paper we concentrate on mixed-integer linear programs.

In Bienstock [1999, 2002], among other approaches, an exponential penalty function framework is considered for (approximately) solving linear programs. Interestingly, this approach can be considered somewhat dual to the approximate LP solving framework via multiplicative weight updates in Plotkin et al. [1995] for fractional packing and covering problems (see also Arora et al. [2012]). In Letchford and Lodi [2003], the authors consider an integrated augment-and-branch-and-cut framework for mixed 0/1 programs. A proximity search heuristic is considered in Fischetti and Monaci [2014], where the objective function is replaced by a proximity function to explore the neighborhood around a feasible solution.

Our approach here is mostly based on *geometric scaling* introduced in Schulz and Weismantel [2002], which in turn is inspired by classical scaling algorithms for flow problems and certain linear programs (see e.g., Wallacher and Zimmermann [1992], Orlin and Ahuja [1992], McCormick and Shioura [2000]), as well as *bit scaling* introduced in Schulz et al. [1995], which is based on an article by Edmonds and Karp [1972]. Other approaches that use scaling implicitly are the *multiplicative weights update method* (see e.g., Arora et al. [2012]), which is also at the core of the algorithm in Garg and Koenemann [2007] for multicommodity flows.

On a high level, the augmentation methods considered here are similar to proximal methods for nonlinear programs (see, e.g., Rockafellar [1976]) in the sense that the deviation from the current iterate is penalized in the objective function; this is also the viewpoint of Fischetti and Monaci [2014], mentioned above. On the other hand, local branching, see Fischetti and Lodi [2003], would be the analogue of trust region methods, see, e.g., Conn et al. [2000].

1.2 Contributions

Our contributions fall into two main categories:

1. *Theoretical analysis of primal scaling approaches.* In the first part we revisit bit scaling and geometric scaling. We establish a new upper bound on the number of required augmentations for geometric scaling (Theorem 3.13), which improves over the bound of Schulz and Weismantel [2002] by a $\log n$ factor, and we derive an alternative variant of geometric scaling in the 0/1 case that does not require the describing system to be in equality form (Theorem 3.19). As a consequence, this shows that geometric scaling is (at least) as versatile as bit scaling, since for 0/1 polytopes geometric scaling is no worse than bit scaling (Corollaries 3.16 and 3.20). We also establish a simple improvement for bit scaling and geometric scaling over 0/1

polytopes, whenever a certain *width* (number of nonzero entries in any integral solution) is low (Theorem 3.22). We then continue to show that bit scaling can be arbitrarily worse compared to geometric scaling, by providing an example where $O(n)$ augmentations are sufficient for geometric scaling, but bit scaling can require an arbitrary number of augmentations (Section 4). Moreover, the number of bit scaling augmentations meets the theoretical upper bound up to a constant factor.

2. *Computational tests of various variants of scaling.* In the second part, we compare implementations of bit scaling, maximum-ratio augmentation (MRA), and geometric scaling. Additionally, we implemented a primal heuristic based on geometric scaling and a straightforward augmentation method that simply checks for an improving solution (see Section 5). The computations are performed on three different testsets. The results show that the augmentation methods use surprisingly few iterations. While MRA is relatively slow, bit scaling and geometric scaling perform well, but the application of bit scaling is limited to instances with different objective coefficients. It also turns out that the augmentation methods do not seem to be helpful for instances that can be solved in reasonable time, e.g., on MIPLIB 2010 benchmark instances. However, geometric scaling helps to find primal solutions of very good quality for very hard instances and outperforms the default settings. This advantage also carries over to the primal heuristic based on geometric scaling, which also performs very well.

1.3 Outline

In Section 2 we provide a brief summary of our notation and preliminaries. In Section 3 we then consider bit scaling and geometric scaling, review known results, and provide various improvements and comparisons. We then provide a worst-case example for bit scaling in Section 4, showing that geometric scaling can outperform bit scaling by an arbitrary factor. In Section 5 we discuss our implementations and in Section 6 provide a comprehensive set of computational results.

2 Preliminaries

Our goal is to solve

$$\max \{cx \mid Ax = b,\ l \le x \le u,\ x \in \mathbb{Z}^n\},$$

where $A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, and l, u, $c \in \mathbb{R}^n$. Note that we write xy for the inner product of two vectors $x, y \in \mathbb{R}^n$. By assumption l and u are finite, and thus $P := \{x \in \mathbb{R}^n \mid Ax = b,\ l \le x \le u\}$ is a polytope. We let $P_I := \operatorname{conv}(P \cap \mathbb{Z}^n)$ be the *integral hull* of P.

We will consider systems in equality form (except for the bounds), which is important since we apply potential functions to the above form. In Section 3.3, we will relax the equality form condition for the case that $P \subseteq [0,1]^n$ and effectively allow for arbitrary representations.

We will often work with *directions* $x - y$ induced by two vectors $x, y \in \mathbb{R}^n$. If $P \subseteq \mathbb{R}^n$ is a polyhedron, we say that $z \in \mathbb{R}^n$ is a *feasible direction* for $x \in P$ if $x + z \in P$. Moreover, z is an *augmenting direction* if $cz > 0$, and it is an *integer feasible direction* if $z \in \mathbb{Z}^n$.

We denote by $\mathbb{1}$ the all-one vector and write $[n] := \{1, \dots, n\}$ for $n \in \mathbb{Z}_+$. For a vector $x \in \mathbb{R}^n$, let $\operatorname{supp}(x) := \{j \in [n] \mid x_j \neq 0\}$ be the *support* of x. All logarithms in this paper will be to the basis 2. All other notation is standard and can be found in Schrijver [1986] and Nemhauser and Wolsey [1988], for example.

3 Scaling techniques

The idea of *scaling* is to replace a given optimization problem with a sequence of *augmentation problems*. The augmentation steps are controlled by means of an appropriate objective function using a potential function. This ensures a minimum relative progress in each augmentation and will lead to oracle-polynomial running times using an augmentation oracle.

3.1 Bit scaling

We first present the *bit scaling technique* for solving 0/1 programs (see Schulz et al. [1995]; also Edmonds and Karp [1972], Graham et al. [1995]). We want to maximize a linear function cx, with $c \in \mathbb{Z}^n$ over $P \cap \{0,1\}^n$ with maximum absolute value $\|c\|_\infty$. For the sake of exposition, and without loss of generality, we confine ourselves to $c \geq 0$ by applying suitable coordinate flips $x_i \mapsto 1 - x_i$. In the mixed-integer version of the algorithm (see Algorithm 4), however, we will deal with an arbitrary $c \in \mathbb{Z}^n$. The classical bit scaling algorithm is given in Algorithm 1.

Algorithm 1 Bit scaling

Input: Feasible solution x^0
Output: Optimal solution of $\max\{cx \mid x \in P \cap \mathbb{Z}^n\}$

```
  μ ← 2^⌈log C⌉, x̃ ← x^0
  repeat
      c^μ ← ⌊c/μ⌋
      compute x ∈ P integral with c^μ(x − x̃) > 0      ▷ improve w.r.t. c^μ approximation of c
      if there is no feasible solution then
          μ ← μ/2
      else
          x̃ ← x                                       ▷ update solution and repeat
      end if
  until μ < 1
  return x̃                                            ▷ return optimal solution
```

Scaling algorithms typically operate in phases: the algorithm improves the current objective function within a phase as long as possible and then goes to the next phase: we call *augmentation* the step where we compute $x \in P$ that improves the current objective function and *(scaling) phase* all steps that use the same scaling factor μ; bounds are typically given as a product of an upper bound on the number of phases and an upper bound on the number of augmentations per phase.

Schulz et al. [1995, Theorem 2] have proven that Algorithm 1 requires $O(n \log C)$ augmentation steps with $C := \|c\|_\infty + 1$. We now prove that the number of augmentation steps is bounded by $n \cdot (1 + \lceil \log C \rceil)$, and also characterize the absolute gap closed at each scaling phase. In Section 4 we prove that the bound on the number of augmentation steps is tight.

Lemma 3.1 (Bit scaling)**.** *Let $P \subseteq [0,1]^n$ be a polytope, and let $c \in \mathbb{Z}^n_+$ with $C := \|c\|_\infty + 1$ the largest absolute value of its components. Then Algorithm 1 solves the optimization problem $\max\{cx \mid x \in P \cap \mathbb{Z}^n\}$ with at most $n \cdot (1 + \lceil \log C \rceil)$ augmenting steps. Moreover, let $\tilde{x}$ be the solution at the end of scaling phase ℓ and $x^\star$ be an optimal solution for the original problem. Then the absolute gap $c(x^\star - \tilde{x})$ is bounded by*

$$2^{\lceil \log C \rceil - \ell} \cdot \max\{\mathbb{1}x \mid x \in P\}.$$

Proof. The algorithm applies at most $1 + \lceil \log C \rceil$ scaling phases, i.e., μ is halved at most $1 + \lceil \log C \rceil$ times. We will show that within each phase, we compute at most n augmenting directions.

Since c^μ is integral, within each phase, we improve the previous solution by at least one with respect to c^μ. Thus, it suffices to compare the initial solution of a given phase with the optimal solution of this phase. For this observe that for $\mu = 2^{\lceil \log C \rceil}$ we have $\|c^\mu\|_\infty \leq 1$, and hence the absolute gap between an optimal solution x^μ for c^μ and x^0 is at most $c^\mu(x^\mu - x^0) \leq n$, as $P \subseteq [0,1]^n$.

Next, we will show that if it holds that we need at most n augmenting directions in phase μ, then we need at most n augmenting directions in phase $\mu/2$. Observe that the objective for phase $\mu/2$ is $c' := \lfloor c/(\mu/2) \rfloor = \lfloor 2c/\mu \rfloor$, which satisfies $c' = 2c^\mu + \tilde{c}$ for some $\tilde{c} \in \{0,1\}^n$. If now x^μ (resp. x') is an optimal solution with respect to c^μ (resp. c') , we obtain

$$c'(x' - x^\mu) = \underbrace{2c^\mu(x' - x^\mu)}_{\leq 0} + \underbrace{\tilde{c}(x' - x^\mu)}_{\leq n} \leq n.$$

It remains to establish the bound on the gap. Let $x^\star$ be an optimal solution for the original objective function c, and let x^μ be the solution at the end of phase $\mu = 2^{\lceil \log C \rceil - \ell}$, i.e., x^μ is optimal for $\lfloor c/\mu \rfloor$. We can write $c = \lfloor c/\mu \rfloor \mu + r$ with $r \in \{0, \ldots, \mu - 1\}^n$ and obtain

$$c(x^\star - x^\mu) = (\lfloor c/\mu \rfloor \mu + r)(x^\star - x^\mu) = \underbrace{\lfloor c/\mu \rfloor \mu (x^\star - x^\mu)}_{\leq 0} + \underbrace{r(x^\star - x^\mu)}_{\leq \mu \max\{\mathbb{1}x \mid x \in P\}} \leq \mu \max\{\mathbb{1}x \mid x \in P\},$$

so the result follows. □

In particular, when each augmentation phase can be performed fast, then Algorithm 1 is fast. We would like to conclude this section with a few remarks:

Remark 3.2 (Efficacy of bit scaling for objective functions with two values). Note that the idea of bit scaling is somewhat lost if $c \in \{0, \gamma\}^n$ for some constant γ, since in this case only two phases are performed. In particular, the power of applying bit scaling can be reduced if the objective function cx is incorporated as a constraint $z_0 = cx$ and z_0 is maximized. Thus, bit scaling depends heavily on the formulation of the problem.

Remark 3.3 (General cost functions). We confined our discussion here to $c \in \mathbb{Z}^n$ (and $c \geq 0$), however it can be generalized to arbitrary $c \in \mathbb{Q}^n$ using a rounding scheme (via simultaneous Diophantine approximations) for c by Frank and Tardos [1987]. The same rounding scheme can be used to ensure that a number of augmentations polynomial in the dimension is always sufficient (see discussion after Corollary 4.4).

In the following, we restrict our discussion to $c \in \mathbb{Z}^n$ without loss of generality; our implementation works for arbitrary cost functions, for details, see Section 5.

Remark 3.4 (Bit scaling might revisit solutions). Bit scaling does not prevent feasible points from being revisited. This is because we change the objective function in a way that a non-optimal solution for a previous phase could become optimal for a later phase, i.e., the sequence of objective functions does not induce a unique ordering of the integral points. This undesirable behavior will be avoided by the method in the next section, and it is this revisiting (or cycling) phenomenon that is at the core of our worst-case example in Section 4.

3.2 Geometric scaling

While the analysis of the bit scaling algorithm in Section 3.1 is geared towards 0/1 polytopes, we will now present a more general scaling framework that can be used for integer programs. The generalization to the mixed integer case will be discussed in Section 5. The algorithms in this section are essentially identical to those in Schulz and Weismantel [2002], with minor modifications to use them in a framework where the augmentation steps are computed by means of a mixed-integer program. We will, however, provide a slightly improved analysis of the geometric scaling algorithm, shaving off a $\log n$ factor in comparison to the analysis in Schulz and Weismantel [2002]. This, in particular, establishes that for 0/1 polytopes bit scaling and geometric scaling have the same worst-case running time in terms of augmentation steps. However, as we will see in Section 4, there exist instances where geometric scaling requires significantly fewer augmentations than bit scaling (see Corollary 4.4).

Recall that we aim to solve $\max\{cx \mid x \in P \cap \mathbb{Z}^n\}$ for an objective function $c \in \mathbb{Z}^n$ and a polytope $P := \{x \in \mathbb{R}^n \mid Ax = b,\, l \le x \le u\}$ by a sequence of *augmentation steps*; the requirement of boundedness is important as the boundary will be used for a potential function. In Section 3.3 we use a different potential function, that does not require equality representations whenever $P \subseteq [0,1]^n$.

Let us consider a feasible solution $x \in P \cap \mathbb{Z}^n$. We will compute an augmenting direction $z \in \mathbb{Z}^n$ with $x + z \in P$ and $cz > 0$. In the following, we will only consider feasible directions z, i.e., those with $x + z \in P$, and we will simply call them *directions*. The (feasible) direction z is *exhaustive* for x if $x + 2z \notin P$. Note that an exhaustive direction is always nonzero, and by integrality, an integer feasible direction is exhaustive for P if and only if it is exhaustive for the integral hull P_I.

The following scaling algorithm can be understood as an analogue of interior point methods for integer programs: the objective function is augmented by a potential function, and the search for augmenting directions is very similar to the Newton directions obtained from the derivatives of the classical barrier function for linear programs (see, e.g., [Ben-Tal and Nemirovski, 2001, Section 4]).

Definition 3.5 (Potential function)**.** Let $P := \{x \in \mathbb{R}^n \mid Ax = b,\, l \le x \le u\}$ be a polytope. Then $\rho : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_+ \cup \{\infty\}$ is a *potential function for* P if for all integer feasible points $x \in P \cap \mathbb{Z}^n$ and feasible directions z for x:

1. $\rho(x,z) \in O(\operatorname{poly}(n))$,
2. $\rho(x,z) = \Omega(1/\operatorname{poly}(n))$ whenever z is exhaustive for x, and
3. $\rho(x, \alpha \cdot z) = \alpha \cdot \rho(x,z)$ for all $\alpha \ge 0$.

There are various appropriate potential functions. We now present one of the potential functions used in Schulz and Weismantel [2002]. For this, we use the standard notation to denote the positive and negative part of z by $z^+ \in \mathbb{Z}^n_+$ and $z^- \in \mathbb{Z}^n_+$, respectively, so that $z = z^+ - z^-$ and $z^+ z^- = 0$. We also use the standard convention $0 \cdot \infty = 0$ throughout the article.

Lemma 3.6 (Schulz and Weismantel [2002])**.** *Let $P = \{x \in \mathbb{R}^n \mid Ax = b,\, l \le x \le u\}$ be a polytope. Then*

$$\rho(x,z) := p(x)z^+ + n(x)z^-,$$

with

$$p(x)_j = \begin{cases} \frac{1}{u_j - x_j}, & \text{if } x_j < u_j \\ \infty, & \text{otherwise} \end{cases} \qquad \text{and} \qquad n(x)_j = \begin{cases} \frac{1}{x_j - l_j}, & \text{if } x_j > l_j \\ \infty, & \text{otherwise} \end{cases}$$

is a potential function such that

1. *$\rho(x,z) \le n$ for all integer feasible points x and feasible directions z;*
2. *$\rho(x,z) > \frac{1}{2}$ whenever z is exhaustive for x.*

Proof. Let $z = z^+ - z^-$ be an integer feasible direction and let x be integer feasible for P. We will show that for each $j \in [n]$ we have $p(x)_j z_j^+ + n(x)_j z_j^- \le 1$. Since $(p(x)_j z_j^+) \cdot (n(x)_j z_j^-) = 0$ by the definition of the positive and negative part, it suffices to prove that $p(x)_j z_j^+ \le 1$ and $n(x)_j z_j^- \le 1$.

Algorithm 2 Maximum-Ratio Augmentation (MRA)

Input: Integer feasible solution x^0, potential function ρ
Output: Optimal solution for $\max\{cx \mid x \in P \cap \mathbb{Z}^n\}$

```
x̃ ← x^0
repeat
    compute x ∈ argmax { c(x − x̃)/ρ(x̃, x − x̃) | c(x − x̃) > 0, x ∈ P ∩ Z^n }      ▷ MRA direction
    if there is no feasible solution then
        return x̃                                                                  ▷ solution is optimal
    else
        pick α ∈ Z_+ with α ≥ 1 so that z = α(x − x̃) is an exhaustive direction
        x̃ ← x + α(x − x̃)                                                         ▷ update solution and repeat
    end if
until x̃ is optimal
```

We consider the term $n(x)_j\, z_j^-$; the proof is analogous for $p(x)_j\, z_j^+$. Observe that whenever $x_j = l_j$ then $z_j^- = 0$, and hence $n(x)_j\, z_j^- = 0$ in this case. Thus, suppose that $x_j > l_j$. Then

$$n(x)_j\, z_j^- = \frac{z_j^-}{x_j - l_j} \leq 1,$$

because z is a (feasible) direction.

Now suppose that z is exhaustive for x, i.e., $x + z \in P$, but $x + 2z \notin P$. By definition, $\rho(x, z) \geq 0$. Moreover, since z is exhaustive, there exists $j \in [n]$ with either $x_j + 2z_j > u_j$, i.e., $z_j^+ > (u_j - x_j)/2$ or $x_j + 2z_j < l_j$, i.e., $z_j^- > (x_j - l_j)/2$. Hence, $p(x)_j\, z_j^+ > \frac{1}{2}$ in the former case or $n(x)_j\, z_j^- > \frac{1}{2}$ in the latter case.

Clearly, ρ as defined above is positively homogeneous in the direction z, i.e., Property 3 is satisfied. □

Next, we will show that if we can compute a direction that maximizes the ratio of the objective function value over the potential, we can solve the maximization problem $\max\{cx \mid x \in P \cap Z^n\}$ by a number of augmentations polynomial in n and $\log C$. This is achieved by the maximum-ratio augmentation (MRA) algorithm given in Algorithm 2, see [Schulz and Weismantel, 2002, Algorithm I]. Throughout, let $C := \|c\|_\infty$, $U := \max_{i \in [n]} u_i$, and $L := \min_{i \in [n]} l_i$.

Observe that we can obtain an exhaustive direction in Algorithm 2 from a maximum-ratio direction $z = x - \tilde{x}$ simply by scaling up. The scaled direction will remain an optimal solution to

$$\max\left\{ \frac{c(x - \tilde{x})}{\rho(\tilde{x}, x - \tilde{x})} \,\middle|\, c(\tilde{x} - x) > 0,\ x \in P \cap \mathbb{Z}^n \right\} \tag{1}$$

by Property 3 of Definition 3, since $\frac{c(\alpha z)}{\rho(x, \alpha z)} = \frac{cz}{\rho(x,z)}$. For a discussion of how to solve (1), see Section 5.6.

Remark 3.7. While a feature of the potential function in Lemma 3.6 is the guarantee that we do not leave the feasible region, we will not require this from a potential function in general (see Definition 3.5). In fact, feasibility will be ensured by (1), so that we potentially could use a more general class of potential functions, gaining some extra flexibility.

Remark 3.8 (Equality vs. inequality representation). It is important to observe that the potential function in Lemma 3.6 is defined with respect to the equality representation of P. In particular, we assign a potential to possible *slack variables* of inequalities, which ensures that we move 'inside' the feasible region. This is the reason why we require P to be bounded via $l \leq x \leq u$; see the discussion in [Schulz and Weismantel, 2002, Section 4].

In the case of polytopes $P \subseteq [0,1]^n$ we will relax this in Section 3.3 via a different potential function and provide essentially identical performance guarantees without adding slack variables.

In the remainder of this section, we will prove the guarantees for the potential function given in Lemma 3.6. However, we obtain polynomiality for any potential function. The results readily carry over by plugging in the performance values of Properties 1 and 2 of the considered potential function. We will first provide the classical analysis from Schulz and Weismantel [2002] for the MRA algorithm as it contains the main potential function argument that is used throughout the remainder of the article.

Theorem 3.9 (Optimization through maximum-ratio augmentation)**.** *Consider the polytope* $P = \{x \in \mathbb{R}^n \mid Ax = b,\, l \leq x \leq u\}$*, let* ρ *be the potential function from Lemma 3.6, and let* $x^0 \in P \cap \mathbb{Z}^n$*. Then Algorithm 2 solves the optimization problem* $\max\{cx \mid x \in P \cap \mathbb{Z}^n\}$ *with at most* $O(n \log(nC(U-L)))$ *computations of an MRA direction.*

Proof. Let $\tilde{x}$ be the solution given at the beginning of an iteration. Suppose an optimal solution x for (1) is found, and let $z = x - \tilde{x}$ be the exhaustive direction. Let $z^\star := x^\star - \tilde{x}$ be the direction from $\tilde{x}$ to the optimal solution $x^\star$ of the original problem. We have $\frac{cz}{\rho(\tilde{x},z)} \geq \frac{cz^\star}{\rho(\tilde{x},z^\star)}$ by the optimality of x. It follows that

$$cz \geq \frac{\rho(\tilde{x},z)}{\rho(\tilde{x},z^\star)} cz^\star \geq \frac{1}{2n} cz^\star,$$

since $\rho(\tilde{x},z^\star) \leq n$ and $\rho(\tilde{x},z) \geq \frac{1}{2}$ by Lemma 3.6. Thus, the computed direction recovers a $\frac{1}{2n}$ fraction of the objective value of the optimal direction $z^\star$. Moreover, the (absolute) gap $c(x^\star - x^0)$ between an optimal solution $x^\star$ and the initial solution x^0 is at most $K := n\,C\,(U-L)$. Thus, after ℓ rounds, the remaining gap is at most $(1-\frac{1}{2n})^\ell K$, and we want to estimate the number of iterations for which $(1-\frac{1}{2n})^\ell K \geq 1$ holds; once we drop below 1, we have reached an integer optimal solution and we are done. Taking the logarithm, we obtain

$$\ell \log(1-\tfrac{1}{2n}) + \log K \geq 0,$$

which can be bounded using $\log(1-\frac{1}{2n}) \leq -\frac{1}{2n}$. We obtain $\ell = O(n \log K)$, and the result follows. □

Unfortunately, Algorithm 2 computes an exact MRA direction at each iteration, which can be expensive and requires maximizing a ratio. We will now consider a scaling algorithm which approximates the maximum-ratio augmentation direction by a factor of 2 and hence has the same asymptotic running time.

The main idea of the *geometric scaling* algorithm (see Algorithm 3) is to only approximately compute an MRA direction, see [Schulz and Weismantel, 2002, Algorithm II]. For this observe that testing whether $\frac{cz}{\rho(\tilde{x},z)} \geq \mu$ is equivalent to testing $cz - \mu \cdot \rho(\tilde{x},z) \geq 0$. This is precisely the standard methodology of the barrier method to progressively tighten the complementary slackness residual.

Again, we can scale the direction to be exhaustive due to the homogeneity of the potential function. In order to establish a performance bound for Algorithm 3 we need the following simple observation.

Algorithm 3 Geometric scaling

Input: Integer feasible solution x^0, potential function ρ

Output: Optimal solution for $\max \{cx \mid x \in P \cap \mathbb{Z}^n\}$

```
μ ← 2C(U − L), x̃ ← x^0
repeat
    compute x ∈ P integral with c(x − x̃) − μ · ρ(x̃, x − x̃) > 0      ▷ approx. MRA direction
    if there is no feasible solution then
        μ ← μ/2
    else
        pick α ∈ Z_+ with α ≥ 1 so that z = α(x − x̃) is an exhaustive direction
        x̃ ← x̃ + α(x − x̃)                                          ▷ update solution and repeat
    end if
until μ < 1/n
return x̃                                                             ▷ return optimal solution
```

Observation 3.10. *Let $\tilde{x}$ be the last solution in the scaling phase for μ. Then for any integer feasible solution x, $\tilde{x}$ satisfies*

$$\frac{c(x-\tilde{x})}{\rho(\tilde{x}, x-\tilde{x})} \leq \mu,$$

i.e., whenever we enter a new scaling phase we have $c(x - \tilde{x}) \leq \mu\, n$ for the potential function in Lemma 3.6, which gives an upper bound on the remaining gap.

Moreover let us point out the following property:

Observation 3.11 (Geometric scaling never revisits a point)**.** *A feasible solution x in Algorithm 3 satisfies $c(x - \tilde{x}) - \mu \cdot \rho(\tilde{x}, x - \tilde{x}) > 0$ or equivalently,*

$$cx > c\tilde{x} + \mu \cdot \underbrace{\rho(\tilde{x}, x-\tilde{x})}_{\geq 0}.$$

Thus, geometric scaling produces solutions with strictly increasing cost with respect to the original objective function c and cannot revisit points.

The crucial advantage of the geometric scaling algorithm is that it uses the objective function to guide the search and hence we (potentially) obtain a speed-up over standard augmentation. For illustration purposes, we depict the behavior of the geometric scaling algorithm in Figure 1.

As with bit scaling and MRA, Algorithm 3 also requires only polynomially many augmentations with respect to the encoding length. First we bound the number of augmentations required per scaling phase.

Lemma 3.12 (Schulz and Weismantel [2002])**.** *Let $P = \{x \in \mathbb{R}^n \mid Ax = b, l \leq x \leq u\}$, let ρ be the potential function from Lemma 3.6, and let $x^0 \in P \cap \mathbb{Z}^n$ be an integer feasible solution. Then Algorithm 3 computes at most $4n$ approximate MRA directions between successive updates of μ.*

Proof. Let $y^0, y^1, \ldots$ be the points in P visited by the algorithm during the scaling phase for a given μ. In particular, y^0 is the current solution after the last update of μ. By Observation 3.10, we have

$$\frac{c(x^\star - y^0)}{\rho(y^0, x^\star - y^0)} \leq 2\mu,$$

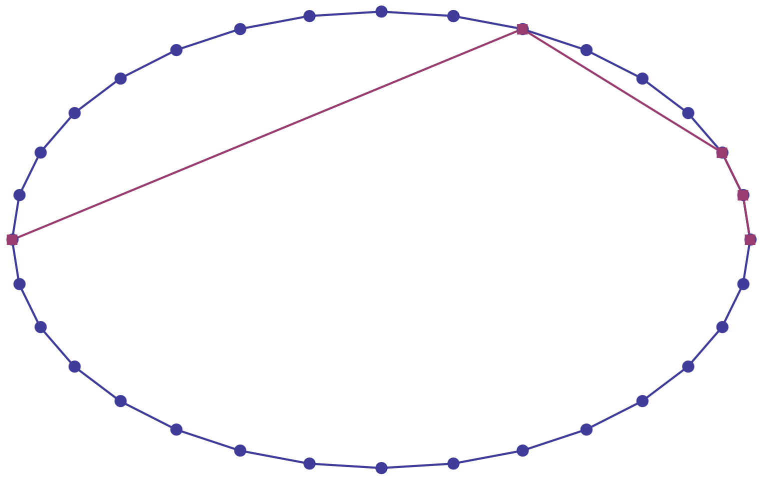

Figure 1: Illustration of the behavior of geometric scaling. The picture shows a polytope in $\mathbb{R}^2$. Our initial point is the leftmost point and we maximize $c = (1, 0)$ (i.e., the maximum is the rightmost point). A worst-case augmentation oracle might give the point with smallest improvement, which is always an adjacent vertex or jump between bottom and top. Standard augmentation with such a malicious worst-case oracle would now force us to visit either each point on the upper or lower path. Geometric scaling constructs shortcuts by controlling the objective function, leading to significant speed-ups.

where $x^\star$ is an integral optimal solution for the original problem. Now, consider any two consecutive iterates y^i and y^{i+1}. By definition of Algorithm 3, the $c(y^{i+1} - y^i) - \mu \cdot \rho(y^i, y^{i+1} - y^i) > 0$ holds. Moreover, as the direction $y^{i+1} - y^i$ is exhaustive, using Lemma 3.6, we have

$$c(y^{i+1} - y^i) > \mu \cdot \rho(y^i, y^{i+1} - y^i) \geq \frac{\mu}{2} \geq \frac{1}{4} \frac{c(x^\star - y^0)}{\rho(y^0, x^\star - y^0)} \geq \frac{1}{4n} c(x^\star - y^0),$$

hence we compute at most $4n$ approximate directions in each scaling phase. □

It is interesting to observe that in Lemma 3.12, in each scaling phase, we recover at least a $\frac{1}{4n}$ fraction of the improvement of the optimal direction $x^\star - y^0$ from the feasible solution y^0 at the beginning of the phase to the optimal solution $x^\star$. This is in contrast to Theorem 3.9, where the guaranteed improvement of $\frac{1}{2n}$ is with respect to two consecutive iterates only, i.e., we only guarantee to recover an $\frac{1}{2n}$ fraction of the improvement of $x^\star - y^{i-1}$, if we are in iteration i, which is potentially smaller than the one from direction $x^\star - y^0$.

We will now establish a bound on the number of required approximate MRA directions, which slightly improves the bound in Schulz and Weismantel [2002] by a $\log n$ factor. The key insight is that we can combine Observation 3.11 with Observation 3.10, to switch from the multiplicative regime to the additive regime, simply counting the remaining improvement steps.

Theorem 3.13 (Improved bound for geometric scaling)**.** *Let $P = \{x \in \mathbb{R}^n \mid Ax = b,\, l \leq x \leq u\}$, let ρ be the potential function from Lemma 3.6, and let $x^0 \in P \cap \mathbb{Z}^n$ be an integer feasible solution. Then Algorithm 3 solves the optimization problem* $\max\{cx \mid x \in P \cap \mathbb{Z}^n\}$ *with at most $O(n \log(C(U - L)))$ computations of approximate MRA directions.*

Proof. The algorithm initializes with $\mu = 2C(U - L)$. Hence after $\lceil \log(C(U - L)) \rceil + 1$ updates of μ, we have $\mu \leq 1$, and by Lemma 3.12 have computed at most $4n(\lceil \log(C(U - L)) \rceil + 1)$ approximate MRA directions in total. Let $\tilde{x}$ be the last solution computed by the algorithm in these first $\lceil \log(C(U - L)) \rceil + 1$ scaling phases.

We now switch to the additive regime and simply count the number of remaining improvements that are possible. As $\mu \leq 1$, Observation 3.10 implies

$$c(x^\star - \tilde{x}) \leq \mu \cdot \rho(\tilde{x}, x^\star - \tilde{x}) \leq n,$$

where $x^\star$ is an integral optimal solution with respect to c. Since all data is integral and by Observation 3.11, every approximate MRA direction leads to an improvement of the objective function by at least 1. It follows that no more than n solutions may be generated before obtaining a solution with cost $cx^\star$. Hence the algorithm terminates after computing at most $4n(\lceil \log(C(U-L)) \rceil + 1) + n$ approximate MRA directions. □

Remark 3.14 (Oracle calls vs. approximate MRA directions). In Theorem 3.13 and elsewhere we count the number of approximate MRA directions that we compute. That is slightly different than counting the number of calls to an approximate MRA oracle: we do not count the number of calls for which no approximate MRA direction exist for a given scaling factor μ and where μ is rescaled. However, note that this number of calls is dominated by the number of calls which do return improving directions. For example in Theorem 3.13, in the last phase where $\mu \leq 1$, we rescale at most $O(\log n) = o(n)$ times until $\mu < 1/n$. In fact, all our results also hold (up to constant factors) if we consider the number of oracle calls rather than approximate MRA directions.

Note that the bound in Theorem 3.13 is stronger than the one given in Theorem 3.9 for Algorithm 2. In fact, the above result implies the same worst-case bound for Algorithm 2: As Algorithm 3 may use a ratio-maximizing direction in each step, Algorithm 2 inherits any worst-case upper bounds proven for Algorithm 3. Thus we obtain the following improvement:

Corollary 3.15 (Improved bound for MRA). *Let $P = \{x \in \mathbb{R}^n \mid Ax = b,\, l \leq x \leq u\}$, ρ be the potential function from Lemma 3.6, and let $x^0 \in P \cap \mathbb{Z}^n$. Then Algorithm 2 solves the optimization problem* $\max\{cx \mid x \in P \cap \mathbb{Z}^n\}$ *with at most $O(n \log(C(U-L)))$ computations of an MRA direction.*

Moreover, in the case of $0/1$ polytopes where the description of the LP relaxation is in equality form, we obtain:

Corollary 3.16 (Worst-case performance for $0/1$ polytopes). *Let $P = \{x \in \mathbb{R}^n \mid Ax = b,\, 0 \leq x \leq \mathbb{1}\}$, ρ be the potential function from Lemma 3.6, and let $x^0 \in P \cap \{0,1\}^n$. Then Algorithms 2 and 3 both solve the optimization problem* $\max\{cx \mid x \in P \cap \{0,1\}^n\}$ *with at most $O(n \log C)$ augmentations.*

3.3 Geometric scaling for arbitrary polytopes in the 0/1 cube

We will now briefly explain how the setup from above can be changed in the case of polytopes $P = \{x \mid Ax \leq b\} \subseteq [0,1]^n$, i.e., not requiring equality form. Clearly, P can be written in equality form by adding slack variables. This, however, changes the ambient dimension, which affects all the bounds above. Moreover, slack variables do not necessarily have to be $0/1$ variables, complicating things further. Thus, we present a tailored analysis for problems in the above form with a particular potential function.

The following observation is crucial:

Observation 3.17 (Exhaustiveness for $0/1$ polytopes). *Every 0/1 solution $x \in P$ is a vertex of P, and, in particular, for each coordinate either $0 \leq x$ or $x \geq \mathbb{1}$ is tight, so that any direction $0 \neq z \in \{-1,0,1\}^n$ is always exhaustive. For the potential function $\rho(x,z) := \|z\|_1$, clearly $\rho(x,z) \leq n$, and $\rho(x,z) \geq 1$ whenever $z \neq 0$. Moreover, we do not need homogeneity, as no scaling of directions is required.*

We obtain the following lemma with a proof essentially identical to the one for Lemma 3.12.

Lemma 3.18. *Let $P = \{x \in \mathbb{R}^n \mid Ax \le b,\ 0 \le x \le \mathbb{1}\}$, let $\rho(x,z) = \|z\|_1$, and let $x^0 \in P \cap \{0,1\}^n$ be a feasible 0/1 solution. Then Algorithm 3 computes at most $2n$ approximate MRA directions between successive updates of μ.*

Proof. The beginning of the proof is as in Lemma 3.12, but now two consecutive iterates y^i, y^{i+1} within a scaling phase satisfy

$$c(y^{i+1} - y^i) \ge \mu \cdot \rho(y^i, y^{i+1} - y^i) \ge \mu \ge \frac{1}{2}\frac{c(x^\star - y^0)}{\rho(y^0, x^\star - y^0)} \ge \frac{1}{2n} c(x^\star - y^0),$$

by Observation 3.17, where $x^\star$ is an integral optimal solution with respect to c. Hence at most $2n$ approximate MRA directions are computed in each scaling phase. □

With this lemma we obtain the following version of Theorem 3.13 for *arbitrary* polytopes $P \subseteq [0,1]^n$. The proof follows exactly as in Theorem 3.13, but with Lemma 3.18 playing the role of Lemma 3.12.

Theorem 3.19. *Let $P = \{x \in \mathbb{R}^n \mid Ax \le b,\ 0 \le x \le \mathbb{1}\}$, let $\rho(x,z) = \|z\|_1$, and let $x^0 \in P \cap \{0,1\}^n$ be a feasible 0/1 solution. Then Algorithm 3 solves the optimization problem* $\max\{cx \mid x \in P \cap \{0,1\}^n\}$ *with at most $O(n \log C)$ computations of approximate MRA directions.*

In particular, the computation of the approximate MRA direction can be performed with a *single call* to an augmentation oracle as the resulting program with $\|\cdot\|_1$ as potential function can be phrased as an integer program. Thus, bit scaling and geometric scaling require essentially the same number of computations of augmenting steps (see Lemma 3.1). The following is a generalization of Corollary 3.16 in the case of $P \subseteq [0,1]^n$.

Corollary 3.20. *Let $P = \{x \in \mathbb{R}^n \mid Ax \le b,\ 0 \le x \le \mathbb{1}\} \subseteq [0,1]^n$ be a polytope and consider the potential function $\rho(x,z) := \|z\|_1$. Let $x^0 \in P \cap \{0,1\}^n$ be an arbitrary integral solution. Then Algorithms 2 and 3 both solve the optimization problem* $\max\{cx \mid x \in P \cap \{0,1\}^n\}$ *with at most $O(n \log C)$ augmentations.*

We will now give an intuition for the result above. In fact, it turns out that the geometric scaling algorithm (Algorithm 3) and bit scaling (Algorithm 1) are closely related:

Remark 3.21 (Relation between bit scaling and geometric scaling). Observe that the potential function from Lemma 3.6 is equivalent to the potential function $\rho(\tilde{x}, x - \tilde{x}) := |\operatorname{supp}(x - \tilde{x})|$ in the 0/1 case, provided we consider only feasible directions. Now consider a polytope $P \subseteq [0,1]^n$, an integral objective function $c \in \mathbb{Z}^n_+$ (which we can assume to be nonnegative by flipping), an integer feasible point $\tilde{x} \in P \cap \{0,1\}^n$, and $\mu = 2^\ell$ for some $\ell \in \mathbb{N}$. In Algorithm 3 we search for a direction z defined by $z = x - \tilde{x}$, where $x \in P$ is integer feasible so that

$$c(x - \tilde{x}) - \mu\,|\operatorname{supp}(x - \tilde{x})| > 0.$$

If we would now pick any coordinate $j \in [n]$, then the above stipulates that it is only beneficial to deviate from the $\tilde{x}_j$ value if $c_j > 2^\ell$. Writing $c = c^1 + c^0$ with $c^1 := \lfloor c/2^\ell \rfloor \cdot 2^\ell$ and $c^0 := c - c^1$, we obtain

$$c^1(x - \tilde{x}) + \underbrace{c^0(x - \tilde{x}) - 2^\ell\,|\operatorname{supp}(x - \tilde{x})|}_{\le 0} > 0.$$

Hence, $c^1(x - x_0) > 0$ is a necessary condition. Although this condition does not guarantee an improvement over $\frac{cx}{\rho(\tilde{x}, x - \tilde{x})}$, in each phase at most n augmentation steps are necessary (see the analysis in the proof of Lemma 3.1), leading virtually to the same overall running time as for Algorithm 3, however with the additional simplification of not explicitly having to consider the potential.

3.4 Improved bounds for structured 0/1 polytopes

When proving worst-case bounds for both bit scaling and geometric scaling, a crucial element is the $O(n)$ bound on the number of improvements made per scaling phase. In the case of bit scaling, this bound is due to the number of positive entries in the vector $x - \tilde{x}$ being at most n for any integral point $x, \tilde{x} \in P$. For geometric scaling, the bound arises from potential function values. In particular, the potential $\rho(x,z) := \|z\|_1$ is bounded from above by n. If this bound can be reduced for special polytopes, it would have direct consequences for worst-case bounds of either algorithm.

One condition that guarantees such a reduction is the following: Let $P \subseteq [0,1]^n$ be a polytope, and suppose there exists some function $f : \mathbb{Z}_+ \to \mathbb{Z}_+$ such that every integral point $x \in P$ has no more than $f(n)$ nonzero entries. In particular we are hoping for an $o(n)$ function, such as $\sqrt{n}$ or $\log n$. We then obtain the following improved worst-case bounds for both bit scaling and geometric scaling.

Theorem 3.22. *Let $c \in \mathbb{R}^n$ be a cost vector, $P \subseteq [0,1]^n$ a polytope, and let the potential function ρ be given as $\rho(x,z) := \|z\|_1$. Suppose there exists a function $f : \mathbb{Z}_+ \to \mathbb{Z}_+$ such that every integral point $x \in P$ has at most $f(n)$ nonzero entries. Then, given an initial solution $x^0 \in P$, Algorithms 1 and 3 solve the optimization problem* $\max\{cx \mid x \in P \cap \mathbb{Z}^n\}$ *after $O(f(n) \log C)$ augmentations.*

Proof. For Algorithm 1 the proof follows as for Lemma 3.1: suppose $x^\mu \in P$ optimizes $c^\mu = \lfloor c/\mu \rfloor$ over P, and we move to the next scaling phase (dividing μ by 2) in which we optimize over $c' = 2c^\mu + \tilde{c}$ for some $\tilde{c} \in \{0,1\}^n$. If we take x' to be an optimal solution with respect to c', we have

$$c'(x' - x^\mu) = \underbrace{2c^\mu(x' - x^\mu)}_{\leq 0} + \underbrace{\tilde{c}(x' - x^\mu)}_{\leq 2f(n)} \leq 2f(n),$$

since the direction $x' - x^\mu \in \{-1,0,1\}^n$ has at most $2f(n)$ positive entries. Hence, no more than $2f(n)$ improvements can be made in any of the $\lceil \log C \rceil + 1$ scaling phases.

In the case of Algorithm 3, we know that ρ is bounded above by $2f(n)$. From this point forward, the proof follows exactly as for Theorem 3.13, with requiring in total at most $8f(n)(\lceil \log(C) \rceil + 1) + 2f(n)$ approximate MRA directions. $\square$

Many well-studied polytopes satisfy the structural constraint from Theorem 3.22, especially those arising from graph-theoretic problems. For example, take the traveling salesman polytope $P \subseteq [0,1]^E$ on the complete graph with k nodes and $|E| = \binom{k}{2}$ edges. Even though the polytope is contained in a space of ambient dimension $\binom{k}{2}$, its integral points (corresponding to tours on the graph) contain exactly k nonzero entries, spanning a low dimensional subspace. Hence optimizing over P using either Algorithm 1 or Algorithm 3 can be done in $O(k \log C)$ augmentations, a factor-k improvement over the general $O(k^2 \log C)$ upper bound. Similar statements hold for the case of maximum weight matchings on a complete graph.

4 Worst-case example for bit scaling

We will now show that the upper bound in Lemma 3.1, on the number of augmentations necessary for bit scaling, is tight. For this we provide a family of polytopes $P_n \subseteq [0,1]^n$ and cost functions c^p so that the bit scaling method needs $\Omega(n \log \|c^p\|_\infty)$ augmentation steps in the worst case.

Each instance of this family is parametrized by two numbers, namely $k \in \mathbb{Z}_+$, which dictates the dimension $n := 8k - 2$ of the cube $[0,1]^n$, and $p \in \mathbb{Z}_+$, which controls how the objective function c^p is built, and, by construction, the number p of bit scaling phases that will be required to solve the instance.

4.1 Construction of the polytope

The polytope $P_n \subseteq [0,1]^n$ will be of the form

$$P_n = \operatorname{conv}\left(\left\{y^1, \dots, y^{2k}\right\}\right),$$

where the vectors $y^j \in \{0,1\}^n$ are defined in terms of vectors $y^{j,1} \in \{0,1\}^{k-1}, y^{j,2} \in \{0,1\}^{k-1}, y^{j,3} \in \{0,1\}^{3k}$, and $y^{j,4} \in \{0,1\}^{3k}$. With these four families of vectors defined, the full vector y^j is given by

$$y^j := \begin{pmatrix} y^{j,1} \\ y^{j,2} \\ y^{j,3} \\ y^{j,4} \end{pmatrix} \qquad \text{or equivalently} \qquad y^j_i := \begin{cases} y^{j,1}_i & \text{for } i \in \{1, \dots, k-1\}, \\ y^{j,2}_{i-k+1} & \text{for } i \in \{k, \dots, 2k-2\}, \\ y^{j,3}_{i-2k+2} & \text{for } i \in \{2k-1, \dots, 5k-2\}, \\ y^{j,4}_{i-5k+2} & \text{for } i \in \{5k-1, \dots, 8k-2\}. \end{cases}$$

The parts $y^{j,1}, y^{j,2}$ are defined in two batches. For the first batch with $j \in \{1, \dots, k\}$, we define

$$y^{j,1}_i := \begin{cases} 1 & \text{if } i \geq j, \\ 0 & \text{otherwise,} \end{cases} \qquad y^{j,2}_i := \begin{cases} 1 & \text{if } i < j, \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } i = 1, \dots, k-1.$$

For the second batch with $j \in \{k+1, \dots, 2k\}$, we define

$$y^{j,1}_i := \begin{cases} 1 & \text{if } i \geq j-k, \\ 0 & \text{otherwise,} \end{cases} \qquad y^{j,2}_i := \begin{cases} 1 & \text{if } i < j-k, \\ 0 & \text{otherwise,} \end{cases} \qquad \text{for } i = 1, \dots, k-1.$$

We define $y^{j,3}, y^{j,4}$ with $j \in \{1, \dots, 2k\}$ as follows

$$y^{j,3}_i := \begin{cases} 1 & \text{if } j \leq k, \\ 0 & \text{otherwise,} \end{cases} \qquad y^{j,4}_i := \begin{cases} 1 & \text{if } j > k, \\ 0 & \text{otherwise,} \end{cases} \qquad \text{for } i = 1, \dots, 3k.$$

See Figure 2 for an illustration.

4.2 Construction of the cost vector

The cost vector is defined inductively, keeping the mechanics of the bit scaling procedure in mind. We first define $c^0 := 0$, and for $\ell = 1, \dots, p$, we build $c^\ell = 2c^{\ell-1} + d^\ell$, for some vector $d^\ell \in \{0,1\}^n$ to be specified. We will find it convenient to construct $d^\ell = (d^{\ell,1}, d^{\ell,2}, d^{\ell,3}, d^{\ell,4})$ in terms of vectors $d^{\ell,1}, d^{\ell,2}, d^{\ell,3}$, and $d^{\ell,4}$ in the same manner as we did for the points y^j.

For $d^1 := c^1$, let

$$d^{1,1} := \mathbb{1}, \qquad d^{1,2} := 0, \qquad d^{1,3}_i := \begin{cases} 1 & \text{if } i \leq k, \\ 0 & \text{otherwise,} \end{cases} \text{for } i = 1, \dots, 3k, \qquad d^{1,4} := 0.$$

For $\ell \geq 2$, we set

$$d^{\ell,1} := 0, \qquad d^{\ell,2} := \mathbb{1}, \qquad d^{\ell,3} := \begin{cases} \mathbb{1} & \text{if } \ell \text{ is odd,} \\ 0 & \text{otherwise,} \end{cases} \qquad d^{\ell,4} := \begin{cases} \mathbb{1} & \text{if } \ell \text{ is even,} \\ 0 & \text{otherwise.} \end{cases}$$

In particular, after the first scaling phase, the contribution of the first $2(k-1)$ coordinates is the same for all y^j. In fact, we use the first $2(k-1)$ coordinates for the improvements steps within a scaling phase and the last $6k$ coordinates to switch between the phases; this will become clear soon. Note that for each $\ell > 1$, $\log \left\|c^\ell\right\|_\infty \in \Theta(\ell)$.

	$y^{j,1}$					$y^{j,2}$					$y^{j,3}$				$y^{j,4}$			
	1	2	3	$\cdots$	$k-1$	1	2	3	$\cdots$	$k-1$	1	2	$\cdots$	$3k$	1	2	$\cdots$	$3k$
y^1	1	1	1	$\cdots$	1	0	0	0	$\cdots$	0	1	1	$\cdots$	1	0	0	$\cdots$	0
y^2	0	1	1	$\cdots$	1	1	0	0	$\cdots$	0	1	1	$\cdots$	1	0	0	$\cdots$	0
y^3	0	0	1	$\cdots$	1	1	1	0	$\cdots$	0	1	1	$\cdots$	1	0	0	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
y^k	0	0	0	$\cdots$	0	1	1	1	$\cdots$	1	1	1	$\cdots$	1	0	0	$\cdots$	0
y^{k+1}	1	1	1	$\cdots$	1	0	0	0	$\cdots$	0	0	0	$\cdots$	0	1	1	$\cdots$	1
y^{k+2}	0	1	1	$\cdots$	1	1	0	0	$\cdots$	0	0	0	$\cdots$	0	1	1	$\cdots$	1
y^{k+3}	0	0	1	$\cdots$	1	1	1	0	$\cdots$	0	0	0	$\cdots$	0	1	1	$\cdots$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
y^{2k}	0	0	0	$\cdots$	0	1	1	1	$\cdots$	1	0	0	$\cdots$	0	1	1	$\cdots$	1
d^1	1	1	1	$\cdots$	1	0	0	0	$\cdots$	0	$d^{1,3}_1$	$d^{1,3}_2$	$\cdots$	$d^{1,3}_{3k}$	0	0	$\cdots$	0
d^2	0	0	0	$\cdots$	0	1	1	1	$\cdots$	1	0	0	$\cdots$	0	1	1	$\cdots$	1
d^3	0	0	0	$\cdots$	0	1	1	1	$\cdots$	1	1	1	$\cdots$	1	0	0	$\cdots$	0
d^4	0	0	0	$\cdots$	0	1	1	1	$\cdots$	1	0	0	$\cdots$	0	1	1	$\cdots$	1
d^5	0	0	0	$\cdots$	0	1	1	1	$\cdots$	1	1	1	$\cdots$	1	0	0	$\cdots$	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

Figure 2: Structure of y^j and d^ℓ; note that $d^{1,3}$ depends on k.

4.3 Lower bound on the number of augmentations

We will now derive a lower bound on the worst-case number of augmentations computed by the bit scaling algorithm when applied to a polytope P_n and cost vector c^p as defined in Section 4.1 and 4.2, respectively. We depict the overall structure of the construction in Figure 2, describing the points y^j and the "layers" d^ℓ of the cost function. Note how the columns in Figure 2 are divided into four segments. These four segments correspond to the four families of vectors used in defining y^j and d^ℓ. For example, the first group of columns in the y^j row depict the vector $y^{j,1}$, the second group of columns depict $y^{j,2}$, and so on.

The essence of the proof is the following: within a scaling phase, the algorithm may move to *any* solution with an improving cost with respect to vector $\lfloor c/\mu \rfloor$ (recall that μ is the scaling factor), no matter the magnitude of the improvement. In our construction, no matter the choice of k, ℓ, the bit scaling algorithm begins by optimizing over the cost vector c^1. The construction is such that $c^1 y^1 > c^1 y^2 > \cdots > c^1 y^{2k}$. Thus if the algorithm begins at initial solution y^{2k}, it may visit *all* of the $2k$ points in P_n, ending the initial phase at y^1.

In the second scaling phase, the algorithm optimizes over c^2. We will see that we have $c^2 y^1 < c^2 y^{2k} < c^2 y^{2k-1} < \cdots < c^2 y^{k+1}$. Thus, in this phase, the algorithm may take k augmentation steps before finishing at point y^{k+1}. In the third augmentation phase, while optimizing over c^3, we similarly have $c^3 y^{k+1} < c^3 y^k < \cdots < c^3 y^1$, giving another possible k augmentations within the phase.

The process continues in each subsequent scaling phase, with the algorithm having the opportunity to travel through each of the points $y^{2k}, y^{2k-1}, \ldots, y^{k+1}$ in even phases, and $y^k, y^{k-1}, \ldots, y^1$ in odd phases, as depicted in Figure 3. Since $k \approx n/8$, this implies a worst case $O(n)$ augmentations per scaling phase, meeting the upper bound from Lemma 3.1.

We now begin the formal proof. We will first show that in each phase ℓ, the first k points $y^1, \dots, y^k$ are ordered in a decreasing fashion by the objective function c^ℓ and similar for the second k points $y^{k+1}, \dots, y^{2k}$. In a second step we will then link the two groups.

Lemma 4.1 (Decreasing order within each group)**.** *Let c^ℓ, y^j be constructed as above. For any $\ell \geq 1$ and $j \in \{1, \dots, k-1\} \cup \{k+1, \dots, 2k-1\}$, we have*

$$c^\ell y^j = c^\ell y^{j+1} + 1.$$

Proof. The proof is by induction on ℓ, with base case $\ell = 1$. For $j \in \{1, \dots, 2k\}$, define $\alpha_{1,j} := d^{1,3} y^{j,3}$. For $j \in \{1, \dots, 2k\}$, we have

$$c^1 y^j = d^1 y^j = \underbrace{d^{1,1} y^{j,1}}_{=k-j} + \underbrace{d^{1,2} y^{j,2}}_{=0} + \underbrace{d^{1,3} y^{j,3}}_{=\alpha_{1,j}} + \underbrace{d^{1,4} y^{j,4}}_{=0} = k - j + \alpha_{1,j}.$$

By construction, we have $\alpha_{1,1} = \alpha_{1,2} = \cdots = \alpha_{1,k} = 1$ and $\alpha_{1,k+1} = \alpha_{1,k+2} = \cdots = \alpha_{1,2k} = 0$. Thus for $j \in \{1, \dots, k-1\} \cup \{k+1, \dots, 2k-1\}$, we can establish

$$c^1 y^j - c^1 y^{j+1} = k - j + \alpha_{1,j} - (k - (j+1) + \alpha_{1,j+1}) = 1,$$

as $\alpha_{1,j+1} = \alpha_{1,j}$.

Now assume $\ell \geq 2$. For $j \in \{1, \dots, k\}$ we can verify that

$$\begin{aligned} c^\ell y^j &= 2c^{\ell-1} y^j + d^\ell y^j \\ &= 2c^{\ell-1} y^j + \underbrace{d^{\ell,1} y^{j,1}}_{=0, \text{ as } d^{\ell,1}=0} + \underbrace{d^{\ell,2} y^{j,2}}_{=j-1} + \underbrace{d^{\ell,3} y^{j,3}}_{=:\alpha_\ell} + \underbrace{d^{\ell,4} y^{j,4}}_{=0, \text{ as } y^{j,4}=0 \text{ for } j \leq k} \\ &= 2c^{\ell-1} y^j + (j-1) + \alpha_\ell, \end{aligned}$$

where $\alpha_\ell = 3k$ if ℓ is odd, and otherwise $\alpha_\ell = 0$. Thus, for $j \in \{1, \dots, k-1\}$ we have

$$\begin{aligned} c^\ell y^j - c^\ell y^{j+1} &= 2c^{\ell-1} y^j + j - 1 + \alpha_\ell - (2c^{\ell-1} y^{j+1} + j + \alpha_\ell) \\ &= 2(\underbrace{c^{\ell-1} y^j - c^{\ell-1} y^{j+1}}_{=1, \text{ by induction}}) - 1 = 1. \end{aligned}$$

We can do a similar analysis for $\ell \geq 2$ and $j \in \{k+1, \dots, 2k\}$:

$$\begin{aligned} c^\ell y^j &= 2c^{\ell-1} y^j + d^\ell y^j \\ &= 2c^{\ell-1} y^j + \underbrace{d^{\ell,1} y^{j,1}}_{=0, \text{ as } d^{\ell,1}=0} + \underbrace{d^{\ell,2} y^{j,2}}_{=j-1} + \underbrace{d^{\ell,3} y^{j,3}}_{=0, \text{ as } y^{j,3}=0 \text{ for } j \geq k+1} + \underbrace{d^{\ell,4} y^{j,4}}_{=:\beta_\ell} \\ &= 2c^{\ell-1} y^j + (j-1) + \beta_\ell, \end{aligned}$$

where $\beta_\ell = 3k$ if ℓ is even, and $\beta_\ell = 0$ otherwise . As before we obtain that for $j = k+1, \dots, 2k-1$, $c^\ell y^j - c^\ell y^{j+1} = 1$ holds. □

Note that in the above argument the values of $d^{\ell,3}$, $d^{\ell,4}$ are irrelevant as they are eliminated in the difference of two consecutive points. However, they will become important as they enable the switching between and linking of the two groups $\{y^1, \dots, y^k\}$ and $\{y^{k+1}, \dots, y^{2k}\}$ as we will show now. To this end we prove the following lemma:

Lemma 4.2 (Decreasing intergroup ordering)**.** *For any $\ell \geq 1$, if ℓ is odd then $c^\ell y^k = c^\ell y^{k+1} + 1$, and if ℓ is even then $c^\ell y^{2k} = c^\ell y^1 + 1$.*

Proof. The proof is by alternating induction on the odd and even case. First observe that $c^1 y^k = c^1 y^{k+1} + 1$, which will be the start of our induction for the odd case:

$$c^1 y^k - c^1 y^{k+1} = \underbrace{d^{1,1}(y^{k,1} - y^{k+1,1})}_{=-(k-1)} + \underbrace{d^{1,2}(y^{k,2} - y^{k+1,2})}_{=0} + \underbrace{d^{1,3}(y^{k,3} - y^{k+1,3})}_{=k} + \underbrace{d^{1,4}(y^{k,4} - y^{k+1,4})}_{=0} = 1.$$

First, let $\ell \geq 1$ be even and suppose $c^{\ell-1} y^k = c^{\ell-1} y^{k+1} + 1$, which is satisfied in the case $\ell = 2$ by the above. Then, repeated application of Lemma 4.1 yields $c^{\ell-1} y^1 = c^{\ell-1} y^{2k} + 2k - 1$. Moreover, we have

$$c^\ell y^1 = 2c^{\ell-1} y^1 + d^\ell y^1 = 2c^{\ell-1} y^1 + \underbrace{d^{\ell,1} y^{1,1}}_{=0, \text{ as } \ell > 1} + \underbrace{d^{\ell,2} y^{1,2}}_{=0, \text{ as } y^{1,2} = 0} + \underbrace{d^{\ell,3} y^{1,3}}_{=0, \text{ as } \ell \text{ even}} + \underbrace{d^{\ell,4} y^{1,4}}_{=0, \text{ as } y^{1,4} = 0} = 2c^{\ell-1} y^1,$$

and

$$\begin{aligned} c^\ell y^{2k} &= 2c^{\ell-1} y^{2k} + d^\ell y^{2k} = 2c^{\ell-1} y^{2k} + \underbrace{d^{\ell,1} y^{2k,1}}_{=0, \text{ as } \ell > 1} + \underbrace{d^{\ell,2} y^{2k,2}}_{=k-1} + \underbrace{d^{\ell,3} y^{2k,3}}_{=0, \text{ as } \ell \text{ even}} + \underbrace{d^{\ell,4} y^{2k,4}}_{=3k} \\ &= 2c^{\ell-1} y^{2k} + (k-1) + 3k = 2c^{\ell-1} y^{2k} + 4k - 1. \end{aligned}$$

Thus, we obtain for the difference

$$\begin{aligned} c^\ell y^{2k} - c^\ell y^1 &= 2c^{\ell-1} y^{2k} + 4k - 1 - 2c^{\ell-1} y^1 \\ &= 2(\underbrace{c^{\ell-1} y^{2k} - c^{\ell-1} y^1}_{=1-2k, \text{ from above}}) + 4k - 1 = 2(1-2k) + 4k - 1 = 1. \end{aligned}$$

Now we consider the case where ℓ is odd, which is similar to the one above. Assume that $c^{\ell-1} y^{2k} = c^{\ell-1} y^1 + 1$, which we now know to hold for $\ell = 3$ by means of the argument for ℓ even case from above. Then, applying Lemma 4.1 in increasing and decreasing direction, we obtain $c^{\ell-1} y^k + 2k - 1 = c^{\ell-1} y^{k+1}$. We will show that $c^\ell y^k = c^\ell y^{k+1} + 1$. We have

$$c^\ell y^k = 2c^{\ell-1} y^k + \underbrace{d^{\ell,1} y^{k,1}}_{=0} + \underbrace{d^{\ell,2} y^{k,2}}_{=k-1} + \underbrace{d^{\ell,3} y^{k,3}}_{=3k} + \underbrace{d^{\ell,4} y^{k,4}}_{=0} = 2c^{\ell-1} y^k + 4k - 1$$

and

$$c^\ell y^{k+1} = 2c^{\ell-1} y^{k+1} + \underbrace{d^{\ell,1} y^{k+1,1}}_{=0} + \underbrace{d^{\ell,2} y^{k+1,2}}_{=0} + \underbrace{d^{\ell,3} y^{k+1,3}}_{=0} + \underbrace{d^{\ell,4} y^{k+1,4}}_{=0} = 2c^{\ell-1} y^{k+1},$$

so that

$$c^\ell y^k - c^\ell y^{k+1} = 2(\underbrace{c^{\ell-1} y^k - c^{\ell-1} y^{k+1}}_{=1-2k}) + 4k - 1 = 1. \qquad \square$$

With these last two lemmas in hand, we are ready to prove the worst-case lower bound. The proof describes the possible behavior of the bit scaling algorithm when given a polytope P_n and cost vector c^p, as depicted in Figure 3. The $\Omega(n \log \|c^p\|_\infty)$ lower bound proven here meets the upper bound established in Lemma 3.1, implying that the analysis is tight.

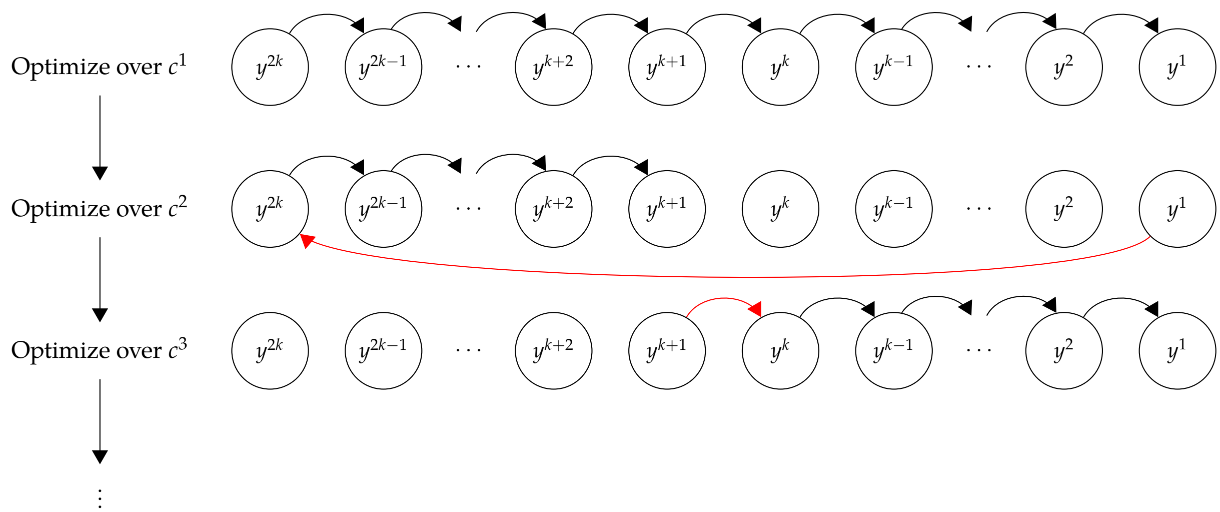

Figure 3: Points visited by the bit scaling algorithm in the worst case. Black arcs follow via Lemma 4.1, red arcs via Lemma 4.2.

Theorem 4.3. *Choose $k \geq 1$ and set $n := 8k - 2$. Let $P_n = \operatorname{conv}\left(\{y^1, \ldots, y^{2k}\}\right)$ be the polytope and c^p for some $p \geq 1$ the objective function as constructed above. Then the bit scaling algorithm optimizing c^p over P_n requires $\Omega(n \log \|c^p\|_\infty)$ augmentation steps in the worst case.*

Proof. By construction of c^p, the bit scaling algorithm optimizes over $c^1, c^2, \ldots, c^p$ in successive scaling phases. The algorithm begins by optimizing over c^1. By Lemmas 4.1 and 4.2 we have

$$c^1 y^{2k} < c^1 y^{2k-1} < \cdots < c^1 y^1.$$

Since an augmentation step moves to any point with improving cost, the algorithm may be forced to visit all $2k$ points when optimizing over c^1.

For $\ell \geq 2$ and ℓ even, y^1 maximizes $c^{\ell-1}$ over P_n and

$$c^\ell y^1 < c^\ell y^{2k} < c^\ell y^{2k-1} < \cdots < c^\ell y^{k+1},$$

so the bit scaling algorithm may visit all k points in $\{y^{k+1}, \ldots, y^{2k}\}$ in the ℓth scaling phase. Similarly, for $\ell \geq 2$ and ℓ odd, y^{k+1} maximizes $c^{\ell-1}$ over P_n and

$$c^\ell y^{k+1} < c^\ell y^k < c^\ell y^{k-1} < \cdots < c^\ell y^1,$$

so the algorithm may visit all k points in $\{y^1, \ldots, y^k\}$. Thus, for $\ell \in \{1, \ldots, p\}$, at least k augmentations may be necessary to optimize over c^ℓ. As $p = \lceil \log \|c^p\|_\infty \rceil$, this gives a total number of (at least)

$$k \lceil \log \|c^p\|_\infty \rceil = \frac{n+2}{8} \lceil \log \|c^p\|_\infty \rceil \in \Omega(n \log \|c^p\|_\infty)$$

augmentations necessary over the entire algorithm. □

Observe that in the example we have constructed above, we revisit the points y^j several times, which leads to the high worst-case number of augmentations. However, given the same polytope/cost vector pair (P_n, c^p), geometric scaling behaves differently. In particular, as shown in

Observation 3.11, the geometric scaling algorithm never revisits a point. Thus the number of augmentations necessary for geometric scaling is bounded by the number of vertices of P_n, which is $O(n)$. With a suitable choice of p (recall $\log \|c^p\|_\infty \approx p$), the number of augmentations calculated by the two methods can have an arbitrarily high difference. We thus have the following corollary.

Corollary 4.4. *For any $p \geq 1$, there exists a polytope $P \subseteq [0,1]^n$ with $n = 8k + 2$, $k \in \mathbb{Z}_{>0}$ and an objective function $c = c^p$, so that bit scaling computes $\Omega(n \log \|c^p\|_\infty) = \Omega(np)$ augmenting directions in the worst case, while geometric scaling needs $O(n)$ augmenting directions. In particular, the relative difference can be made arbitrarily large by choosing p appropriately.*

Theorem 4.3 is particularly interesting as it shows that the number of required augmentations for the bit scaling algorithm is unbounded for 0/1 polytopes. On the other hand, the rounding scheme of Frank and Tardos [1987] can be used to turn an arbitrary $c \in \mathbb{Q}^n$ into a vector $\bar{c} \in \mathbb{Z}^n$ with encoding length $O(n^3)$ in time polynomial in n and $\log \|c\|_\infty$ such that optimizing both vectors results in the same optimal solution. Thus, bit scaling requires at most $O(n^4)$ augmentations in the worst-case *with* preprocessing of the objective function. We obtain the same worst-case bound on the number of augmentations for geometric scaling.

5 Implementation

We implemented the discussed algorithms bit scaling (Algorithm 1), MRA (Algorithm 2), and geometric scaling (Algorithm 3) in C using the framework SCIP, see Achterberg [2009], SCIP. We also implemented a simple augmentation algorithm ("augment") that iteratively searches for augmenting directions. All methods run for arbitrary mixed-integer problems as described in the following sections. Generally, for each instance we run presolving and solve the root node, including cuts. We then start running the described algorithms, but keep all cuts and solutions found by heuristics so far, including those found during the augmentation iterations.

5.1 Solving the augmentation problems

The augmentation problem is solved as a MIP. In general, we add an objective cut $cx \geq cx^k + \delta$ with respect to the current objective c and last feasible iteration point x^k. We use $\delta = \lceil 2\varepsilon \cdot |cx^k| \rceil$ if the objective is *integral*, i.e., is guaranteed to yield integral values for all feasible solutions; we set $\delta = 2\varepsilon \cdot |cx^k|$ otherwise. Here $\varepsilon = 10^{-6}$ is the feasibility tolerance of SCIP. (The first solution is found without adding this constraint.) Note that continuous variables do not need to be treated differently, i.e., the approach works for arbitrary MIPs.

We then solve the MIP subproblem until we find an improving solution x^{k+1}. For any such solution, we try to exhaust the direction, by searching for the largest integral α such that $x^k + \alpha(x^{k+1} - x^k)$ is feasible.

The search for improving solutions can be incomplete: We first solve the root node of the subproblem and check whether we found an improving solution. If yes, we use this solution as an augmentation direction. Otherwise, we continue to solve the MIP until we find any feasible solution. It often happens that soon after finding some feasible solution, further solutions are found, e.g., by so-called exchange heuristics like 1-opt or crossover. We therefore continue the solution process until for a fixed number of nodes no further improving solution is found or the problem has been solved (this is called a "stall node limit" in SCIP).

Note that the last iteration has to be solved to optimality in all algorithms.

Algorithm 4 Bit Scaling Variant

Input: Feasible solution x_0
Output: Optimal solution for $\max \{cx \mid x \in P \cap \mathbb{Z}^n\}$
  $\mu \leftarrow 2^{\lceil \log C \rceil}$
  repeat
    set $c_0 \leftarrow \lfloor c/\mu \rfloor$
    compute $x_0 \in P$ integral with $c_0 x_0 = \max \{cx \mid x \in P_I\}$
    $\mu \leftarrow \mu/2$
  until $\mu < 1$
  return x_0 ▷ return optimal solution

5.2 Augment

For the basic augmentation method we proceed as follows: at each iteration with current best solution x^k, we add an objective cut. We then iteratively search for improving solutions until we prove infeasibility (in this case, x^k is optimal) or hit the time limit. Note that we solve the same problem as the original with an additional constraint. Since this constraint does not cut off any better solution, the dual bound obtained in each subproblem is valid for the original.

5.3 Bit scaling

In the bit scaling Algorithm 1, at each iteration, the problem is solved with the scaled objective function under the additional constraint that a solution must improve on the current one. The scaling factor is changed if no improving solution exists. In Algorithm 4, there is no additional constraint, but the optimal solution is computed at each iteration, rather than an improving solution; therefore, the scaling factor changes at each iteration. We have implemented both algorithms, as well as a variant of Algorithm 4 with an improving constraint (similar to Algorithm 1). This may alter the behavior of the MIP solver, but not the behavior of the algorithm itself.

Let us now point out further implementation issues. At the beginning of the algorithms, the objective function is replaced with the scaled version. In practice, the coefficients of the objective function may not be integer. Thus, an additional scaling may be needed to make the objective integral. For the variants where an improving constraint is used, such a constraint is added at the beginning of an iteration if a feasible solution is known. Between two iterations, we compare the objective functions and only solve the next iteration if the vectors are not equal up to a factor. Depending on the algorithm, we solve the iteration problem to optimality or use an incomplete search, as explained above. A new scaling factor is computed before each new iteration if the solution of the current iteration is optimal for the current objective function (note that this is systematically the case for Algorithm 4 and its variant).

5.4 Geometric scaling

Geometric scaling is implemented as described in Algorithm 3, using the function

$$\rho(\tilde{x}, x - \tilde{x}) := \|x - \tilde{x}\|_1, \tag{2}$$

i.e., $\rho(\tilde{x}, z) = \|z\|_1$. Note that for 0/1 problems this function is equal to $|\mathrm{supp}(x - \tilde{x})|$, which is a potential function, see Section 3.2. For general integer variables, ρ does not fulfill Part 1 of Definition 3.5. We nevertheless use this function, since it induces sparsity, is easy to handle, and also can be used for the MRA algorithm (see Section 5.6). Note that for general integer variables, we need to add artificial (continuous) variables that model the positive and negative parts.

Deviating from Algorithm 3, we start with μ equal to the smallest power of 2 larger than the value of any previously found solution; moreover, we make sure that the value of μ is not larger than 10^8. If the objective is not integral, we may need to solve one final problem, if $\mu < 1/n$. For each found solution, we try to exhaust the direction as described above.

5.5 Primal heuristic based on geometric scaling

It will turn out in our computational results that geometric scaling in general performs quite well. This motivates the implementation of a primal heuristic based on it. This heuristic is activated during an ordinary branch-and-cut run after a primal solution was found and for a certain number of nodes (by default 200) no further solution was found. It then runs the geometric scaling algorithm, but with a node limit for the individual subproblems that depends on the current number of nodes N. By default, we use $\min\{500, \max\{5000, 0.1\,N\}\}$. We also stop if the total number of nodes exceeds $0.6\,N$. In this way, the effort spent in this heuristic is limited, and one can still benefit from solutions found during the ordinary tree search.

5.6 MRA

The implementation of MRA (Algorithm 2) is based on $\rho(\tilde{x}, x - \tilde{x}) = \|x - \tilde{x}\|_1$, as well. Note that this function is convex in x. We then want to solve

$$\max\left\{\frac{c(x-\tilde{x})}{\rho(\tilde{x}, x-\tilde{x})} \mid c(x-\tilde{x}) > 0,\ x \in P,\ x \text{ integral}\right\}$$

where $\tilde{x}$ is some feasible solution. To solve this fractional program, we introduce a parameter $\mu \geq 0$ and check whether $c(x - \tilde{x}) \geq \mu \cdot \rho(\tilde{x}, x - \tilde{x})$ by maximizing $c(x - \tilde{x}) - \mu \cdot \rho(\tilde{x}, x - \tilde{x})$, which is concave in x. We then perform a binary search over μ, increasing μ if the objective value is positive and decreasing μ otherwise. To solve the inner optimization problem of maximizing $c(x - \tilde{x}) - \mu\,\rho(\tilde{x}, x - \tilde{x})$, we rewrite it as

$$\max\left\{cx - \tau \mid \tau \geq c\tilde{x} + \mu \cdot \rho(\tilde{x}, x-\tilde{x}),\ c(x-\tilde{x}) > 0,\ x \in P,\ x \text{ integral}\right\}.$$

(Note that: $cx - \tau \leq cx - c\tilde{x} - \mu \cdot \rho(\tilde{x}, x - \tilde{x}) = c(x - \tilde{x}) - \mu \cdot \rho(\tilde{x}, x - \tilde{x})$ for all feasible x.)

We solve this problem by iteratively generating subgradients for the convex function

$$f_\mu(x) := c\tilde{x} + \mu \cdot \rho(\tilde{x}, x - \tilde{x}).$$

Its subdifferential is

$$\partial f_\mu(x) = c - \mu\,\partial\,\|\cdot\|_1\,(x - \tilde{x}) = c - \mu\,\mathrm{sgn}(x - \tilde{x}),$$

where we define the set of vectors

$$\mathrm{sgn}(x)_j := \begin{cases} \{1\} & \text{if } x_j > 0 \\ [-1, 1] & \text{if } x_j = 0 \\ \{-1\} & \text{if } x_j < 0 \end{cases} \qquad \text{for all } j = 1, \dots, n.$$

For each subgradient $h \in \partial f_\mu(\tilde{x})$, we obtain the subgradient inequality

$$f_\mu(x) \geq f_\mu(\tilde{x}) + h(x - \tilde{x}).$$

Assuming that we have generated subgradients $h^1, \dots, h^k$ for points $x_1, \dots, x_k$, we solve

$$\max\left\{cx - \tau \mid \tau \geq f_\mu(x_i) + h^i(x - x_i),\ i = 1, \dots, k,\ c(x - x_0) > 0,\ x \in P,\ x \text{ integral}\right\}. \tag{3}$$

Let the optimal solution be x^{k+1}. We then compute a subgradient at x^{k+1} and check whether its subgradient inequality is violated. Note that each solution of (3) is a primal solution that can be stored and used for the original problem.

[In our implementation, we actually solve the minimization version

$$\min\left\{-cx+\tau \mid \tau-h^i x \geq f_\mu(x_i)-h^i x_i,\ i=1,\dots,k,\ c(x-x_0)>0,\ x\in P,\ x \text{ integral}\right\},$$

for technical reasons.]

In the first iteration, the problem is unbounded, since τ is unbounded. We therefore start with x^0 (which is actually infeasible) and take the subgradient $h^0 = c \in \partial f_\mu(x^0)$. The inequality added to the optimization problem is then

$$\tau \geq cx^0 + c(x-x^0) = cx.$$

Thus, τ is bounded from below, if there exists a feasible $x \in P$, $c(x-x^0) > 0$, x integral.

6 Computational results

The algorithms were tested on a Linux cluster with 3.2 GHz Intel i3 processors with 8 GB of main memory and 4 MB of cache, running a single process at a time. We use SCIP 3.2.0 and CPLEX 12.6.1 as the LP-solver. SCIP runs with default settings, except that we turn off the "components" presolver, since it would decompose the problem into several runs, making a comparison more difficult.

We use the following testsets:

MIPLIB2010 The 87 benchmark instances from MIPLIB 2010[1], see Koch et al. [2011].

LB We use the testset of 29 instances from the "local branching" paper[2], see Fischetti and Lodi [2003]. This testset has also been used in Hansen et al. [2006]. The latter paper also contains improved results, which we will use below.

QUBO We use a testset of linearizations of 50 instances for quadratically unconstrained Boolean optimization (QUBO)[3], see Dash [2013], Dash and Puget [2015].

In an online supplement, we present details of the computations described in the following. We will first discuss results on the MIPLIB2010 test set. As it turns out, augmentation methods do not help to solve these instances, essentially because they are too easy. We then consider the very hard testsets LB and QUBO. Here, it will turn out that augmentation methods significantly improve on the default settings and produce primal solutions of very good quality.

6.1 Testset MIPLIB 2010

Table 1 shows a comparison of the four different augmentation methods and variants of these as well as for the default settings on the testset MIPLIB2010. In the table, "#nodes" and "time" give the shifted geometric means[4] of the total number of nodes (including subproblems) and the time (in seconds), respectively. Column "#run" presents the number of instances for which an

[1]available at http://miplib.zib.de/

[2]available at http://www.or.deis.unibo.it/research_pages/ORinstances/MIPs.html

[3]available at http://researcher.watson.ibm.com/researcher/files/us-sanjeebd/chimera-data.zip

[4]The shifted geometric mean of values $t_1,\dots,t_n$ is defined as $\left(\prod(t_i+s)\right)^{1/n} - s$ with shift s. We use a shift $s=10$ for time and $s=100$ for nodes in order to decrease the strong influence of the very easy instances in the mean values.

Table 1: Aggregated results of the different algorithms on testset MIPLIB 2010 (1 hour time limit, 87 instances)

name	#nodes	time	#run	#best	#improv.	#subprob.	#phases	#exhaust	prim-$\int$
bitscale	20116.2	934.93	59	67	3.7	8.4	4.8	0.0	57.4
MRA	16434.8	1819.46	83	32	411.4	428.6	418.7	8.1	207.2
geometric	5628.8	1632.31	83	65	6.2	23.6	17.4	0.0	49.8
augment	14793.5	924.71	83	63	12.9	12.9	12.9	0.0	54.5
bitscale-classic	25086.0	1120.62	59	62	4.1	11.0	6.9	0.0	60.4
bitscale-noimprove	20343.4	918.31	60	69	4.2	8.8	4.6	0.0	56.7
bitscale-complete	26902.6	1070.71	59	59	2.0	6.6	4.6	0.0	103.5
geom-no ℓ_1	6038.5	1651.26	83	65	6.4	23.6	17.2	0.0	49.5
geom-no cutoff	7648.7	2062.03	83	49	3.9	19.4	15.6	0.0	85.4
geom-8	8637.7	1313.55	83	69	5.7	12.8	7.1	0.0	40.7
geom-64	8680.9	1122.85	83	72	6.4	10.7	4.3	0.0	39.7
geom-256	8358.2	1128.40	83	69	7.1	10.9	3.7	0.0	39.7
geom-512	9895.4	1112.03	83	69	7.0	10.3	3.3	0.0	41.8
geom-1024	8392.5	1016.80	83	71	7.1	10.3	3.2	0.0	39.8
geom-heur	16858.8	741.52	68	71	1.3	33.6	32.4	0.0	30.4
geom-infer	21343.6	732.30	68	72	1.0	41.7	40.7	0.0	30.1
geom-heur-64	16158.0	683.18	68	73	1.7	9.9	8.1	0.0	29.3
default	15495.9	557.12	0	74	0.0	0.0	0.0	0.0	26.3

augmentation routine ran. Column "#best" refers to the number of times the best known primal solution value has been found. With respect to the augmentation methods, the columns "#improv.", "#subprob.", "#phases", and "#exhaust" refer to the average number of times an improved primal solution has been found, the number of subproblems (MIPs) solved, the number of phases, and the number of exhausting directions found, respectively. The number of phases refers to the number of subproblems solved with the same value of μ for bit and geometric scaling (in this case, #phases + #improv = #subprob); note that we count a possible search for the first primal solution as one phase. For MRA, we count each outer iteration as a phase; thus, the number of phases equals the number of Benders problems (1) solved. For `augment`, the number of phases equals the number of improving solutions and the number of subproblems.

Finally, the last column gives the *primal integral*, see Berthold [2013]. The primal integral is the value we obtain by integrating the gap between the current primal and best primal bound over time[5]. Thus, a smaller primal integral indicates a higher solution quality over time.

We can draw several conclusions from these experiments:

General Observations Among the 87 instances, four were solved before the end of the root node and no augmentation routine was applied.

The number of phases and subproblems is usually below 30, with the exception of MRA, which needs a larger number of phases and subproblems. Thus, it seems that in practice no long series of augmentation steps occur, but as the example of MRA shows, this would in principle be possible. Note, however, that the numbers are significantly smaller than the theoretical bounds.

The goal of the table is to illustrate the behavior of the augmentation procedures. However, we also add the results of the default settings for comparison. It turns out—as possibly expected—that these settings are faster and have a smaller primal integral than all stand-alone augmentation procedures on average.

Bitscaling Bitscaling only runs on 59 of the 87 instances, since the other instances have equal nonzero objective coefficients. Note that this somewhat reduces the corresponding averages, since the default settings are often faster.

[5] We define the gap between primal bound p and best primal bound b as $|p - b| / \max(|p|, |b|)$.

We compare four variants of bit scaling (bitscale, bitscale-classic, bitscale-noimprove, and bitscale-complete). The basic variant (bitscale) uses incomplete searches and an improving constraint. Recall that when performing incomplete searches, only some improving solution is searched for at each iteration, as opposed to the optimal solution. The objective function changes at each iteration.

The classic variant corresponds to Algorithm 1: each improving solution requires a solve, and the objective function is scaled only when the problem is infeasible (due to the improving constraint). This explains why this variant has the highest number of phases among all bit scaling variants. The classic variant outperformed the bitscale variant on only three instances out of the 59.

The noimprove variant is similar to bitscale, except that no improving constraint is added. The results are very close to bitscale, maybe slightly better. Depending on the instance one variant can be significantly faster than the other: indeed, bitscale can be 3.5 times as fast as noimprove, but also two times as slow. Note, however, that this might be the result of performance variability (see Koch et al. [2011]).

In the complete variant, each phase is solved to optimality. This variant thus requires fewer phases than bitscale. However, on this testset, the complete variant is never faster than bitscale and can perform up to 4.8 times slower.

Let us now compare the default bitscale variant to default SCIP. It is faster on 10 instances out of the 59 on which bitscale runs. While this shows that in some cases using bit scaling can be beneficial, default SCIP performs overall significantly better.

MRA For MRA, the difference between the number of subproblems and the number of phases is surprisingly small. This indicates that only very few subgradients are needed in MRA to compute the optimal inner value in (3): on average at most two subgradients are added in each phase. Note also that MRA generates a large number of improving solutions. Obviously, MRA takes a disadvantageous route through the feasible solutions.

When comparing the different variants, MRA is clearly the slowest, solves the fewest number of instances, and uses the largest number of phases. We currently do not have an explanation for this large difference.

Interestingly, MRA is the only variant for which the exhausting step actually was performed. Obviously the solutions produced by the other variants are always automatically exhaustive.

Geometric Scaling Next to the default settings, variant geometric solves the largest number of instances. It is faster than the default for two instances. However, it uses quite a number of phases and subproblems without finding an improving solution, in particular at the beginning.

It turns out that not using the ℓ_1-norm on general integer variables (variant geom-no ℓ_1), i.e., we ignore these variables in the objective function, does not make a difference. This can be explained by the fact that on average there are only a few general integer variables.

Interestingly, adding an objective constraint instead of using an objective cutoff (variant geom-no cutoff), worsens the results significantly. This might be due to the fact that SCIP stores suboptimal solutions and uses them to generate better ones, while infeasible solutions are not stored.

Finally, we consider different factors to update μ in Algorithm 3 (the default factor is 2). When increasing this factor, the running time generally decreases. The largest number of solutions with objective equal to the best value and the smallest primal integral appear for a factor of 64. The corresponding variant geom-64 is better than the default for three instances.

Augment Compared to the other variants, augment is surprisingly fast. It also usually takes few phases, but produces more improving solutions (with the exception of MRA). However, bitscale is not far behind. Moreover, most geometric scaling variants produce better solutions (#best) and smaller primal integrals, but they use more time on average.

Geometric scaling heuristic We also tested the heuristic based on geometric scaling (see Section 5.5), with the following settings: geom-heur, geom-heur-infer, and geom-heur-64. Here, geom-heur-infer uses inference branching instead of the default branching rule, which should lead to reduced times for the branching rules. Moreover, geom-heur-64 uses a factor of 64 to reduce μ, since this produced the best results for geometric scaling.

The running times and number of nodes are larger than the default. If we only consider the number of nodes in the main branch-and-bound tree on instances which were solved to optimality, variants geom-heur and geom-heur-64 reduce the number of nodes by about 10 % and 13 % relative to the default settings, respectively. However, this improvement is over-compensated by the overhead incurred by the heuristic. Moreover, because of the node limits, a larger number of subproblems could be treated, but the number of improving solutions is smaller relative to the other methods.

In total, we conclude that the heuristic does not help to improve the performance on the MIPLIB 2010 instances – essentially, most of these instances are "too easy".

As a general comment, note that the influence of heuristics on the performance is generally not too large: Berthold [2014] estimated the difference of the running time of SCIP using heuristics and not using any heuristics to be about 11 % on the MIPLIB 2010 benchmark testset. Moreover, a single heuristic has the disadvantage to "compete" against the other heuristics (42 in SCIP – not all of them active). On the other hand, the augmentation methods significantly benefit from good heuristics if an incomplete search is used.

6.2 LB testset

In the next experiment, we compare the results of different augmentation variants on the testset LB, see Table 2. We use default settings, augment, and bitscale. Moreover, we apply geometric scaling with a factor of 64 (geom-64), since this gave the best results on the MIPLIB2010 testset. Moreover, we again use the three variants of the geometric scaling heuristic (geom-heur, geom-heur-infer, and geom-heur-64).

The general picture of the augmentation methods is similar to the MIPLIB2010 testsets; for instance, the number of augmentation subproblems is generally small (see the online supplement for detailed results). The results show that variant geom-64 dominates augment and bitscale with respect to the number of instances for which the best solution among all variants was found. Bit scaling ran for 26 of the 29 instances. Due to the higher number of instances for which it is applied in comparison to the MIPLIB2010 testset, bitscale performs better than augment.

The default settings perform very favorably with 14 "best" solutions (better than geom-64), but are dominated by geom-heur, which finds the best value for 18 instances. For the LB testset, it seems to be essential to have access to the solutions generated by other heuristics and integer feasible LP solutions during the branch-and-cut algorithm. These can then be improved by geom-heur. Interestingly, increasing the μ reduction factor in the geometric scaling heuristic to 64 decreases the number of "best" solutions to 11 (see geom-heur-64). Obviously, the decrease of μ is too fast in order to produce good solutions on this testset. In fact, the number of phases in geom-heur and geom-heur-64 is on average much larger than for geom-64 (the averages are 41.9 for geom-heur and 14.3 for geom-heur-64 vs. 4.0 for geom-64). Note that geom-heur runs out of memory for the instance arki001.

Finally, these results are compared to the best values obtained in Fischetti and Lodi [2003] or Hansen et al. [2006] (presented in column "previous best"). These values are improved on nine instances by some variant and on five by geom-heur. Note that this is not even near a fair comparison, since the results in Fischetti and Lodi [2003] and Hansen et al. [2006] were obtained on different computers, as well as with different implementations and time limits. Moreover, in the meantime most values have been improved by other methods. Nevertheless, the results show that

Table 2: Best primal values for different variants on the testset LB (29 instances, 1 hour time limit). Column "previous best" gives the best value obtained in Fischetti and Lodi [2003] or Hansen et al. [2006]; all problems are minimization instances. For each instance, the best values among those obtained by the variants (excluding "previous best") are marked in black, otherwise the values are marked gray.

problem	default	augment	bitscale	geom-64	geom-heur	geom-heur-infer	geom-heur-64	previous best
A1C1S1	11,643.33	11,989.36	11,977.50	11,638.86	11,557.22	11,566.59	11,590.45	11,551.19
A2C1S1	10,983.28	11,422.77	11,115.34	11,040.72	10,897.77	10,994.27	10,909.95	10,889.14
arki001	7,580,813.05	7,581,527.87	7,580,813.05	7,582,202.93	—	7,580,814.51	7,580,813.05	7,580,889.44
B1C1S1	24,798.51	25,456.98	27,309.51	25,458.30	25,630.75	25,123.51	25,042.56	24,566.52
B2C1S1	25,763.12	27,253.74	26,592.19	26,167.32	26,412.44	25,926.61	26,002.11	26,073.78
biella1	3,065,005.78	3,065,005.78	3,065,005.78	3,065,005.78	3,065,005.78	3,065,005.78	3,065,005.78	3,070,810.15
core2536-691	689.00	689.00	689.00	689.00	689.00	689.00	689.00	690.00
core2586-950	970.00	972.00	1213.00	971.00	955.00	960.00	966.00	947.00
core4284-1064	1091.00	1100.00	3279.00	1080.00	1072.00	1073.00	1079.00	1065.00
core4872-1529	1580.00	1584.00	1769.00	1579.00	1546.00	1560.00	1575.00	1534.00
danoint	65.67	65.67	65.67	65.67	65.67	65.67	65.67	65.67
glass4	1,600,013,500.00	1,500,014,200.00	2,200,016,050.00	1,620,014,440.00	1,500,012,650.00	1,550,012,462.72	1,566,683,416.66	1,400,013,666.50
markshare1	7.00	9.00	32.00	12.00	10.00	10.00	10.00	7.00
markshare2	12.00	13.00	128.00	17.00	14.00	14.00	10.00	14.00
mkc	−559.11	−542.28	−557.56	−561.93	−562.93	−560.85	−561.33	−563.85
net12	214.00	214.00	214.00	214.00	214.00	214.00	214.00	214.00
NSR8K	127,262,743.24	68,351,187.10	2,176,184,843.46	21,415,513.00	127,262,743.24	127,262,743.24	127,262,743.24	20,780,430.00
nsrand_ipx	51,200.00	54,880.00	55,200.00	52,000.00	51,200.00	51,200.00	51,200.00	51,520.00
rail507	174.00	174.00	174.00	174.00	174.00	174.00	174.00	174.00
roll3000	12,890.00	12,899.00	13,380.00	12,904.00	12,890.00	12,890.00	12,890.00	12,890.00
seymour	425.00	425.00	425.00	424.00	424.00	425.00	424.00	423.00
sp97ar	663,515,230.72	726,599,877.76	674,470,726.72	662,299,239.68	674,213,859.52	664,157,022.72	673,642,038.40	666,368,944.96
sp97ic	435,258,209.12	450,307,285.28	430,937,067.04	439,446,697.12	434,570,609.44	432,663,431.84	439,022,248.00	429,892,049.60
sp98ar	530,322,047.84	551,452,928.96	532,671,408.48	530,242,941.12	530,437,736.32	530,489,389.92	530,251,516.00	530,916,867.40
sp98ic	451,409,231.04	465,544,414.56	455,081,136.48	450,843,038.08	450,519,098.72	449,226,843.52	453,626,659.52	449,226,843.52
swath	494.09	502.24	506.44	495.02	467.41	481.95	477.57	467.41
tr12-30	130,596.00	130,596.00	139,741.00	130,596.00	130,596.00	130,596.00	130,596.00	130,596.00
UMTS	30,094,335.00	30,091,967.00	30,091,457.00	30,092,333.00	30,093,479.00	30,092,081.00	30,091,738.00	30,139,634.00
van	5.09	5.59	5.35	6.12	5.09	5.09	5.09	4.84
#best:	13	6	8	10	18	10	11	

using geometric scaling inside a primal heuristic seems to be promising for obtaining high quality solutions for hard MIPs.

6.3 QUBO testset

Table 3 shows the best primal values of geometric scaling and the version that also uses the heuristic based on geometric scaling on the QUBO testset. For these instances, the other stand-alone augmentation methods do not perform well – and we skip their results here.

The three variants geom-heur, geom-heur-infer, and geom-heur-64 find significantly better primal solutions than the default settings. Moreover, their primal integral is significantly smaller. Among the three variants, geom-heur-infer performs slightly better than the other two. However, all three variants find the best solutions for some instances for which all other variants are not as good. As for the other two testsets, the geometric scaling heuristic uses more phases than geom-64, on average (geom-64: 3.2, geom-heur: 19.2, geom-heur-64: 4.8). The good performance comes from the fact that usually the other heuristics find good solutions, which can then easily be improved to even better solutions by the geometric scaling heuristic.

In any case, these results are surpassed by geom-64, which finds the largest number of best solutions and produces the smallest primal integral. These excellent results arise from the fact that very few phases are needed in order to arrive at a level of μ that helps to improve the primal solutions. Indeed, it is often the case that for a particular μ a series of improving solutions is found until the time limit is reached.

In summary, the QUBO instances show the excellent potential of geometric scaling. It is likely that extensive parameter tuning could help to even improve these results.

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Table 3: Best primal values and primal integral of the default settings and variants of the heuristic based on geometric scaling (50 instances, 1 hour time limit). For each instance, the best primal values are marked in black, otherwise the values are marked gray; all problems are minimization instances.

Problem	default		geom-64		geom-heur		geom-heur-infer		geom-heur-64	
	Primal	Prim-$\int$	Primal	Prim-$\int$	Primal	Prim-$\int$	Primal	Prim-$\int$	Primal	Prim-$\int$
chim8-4.1	−796	153.8	−822	58.9	−830	109.9	−788	188.3	−798	144.9
chim8-4.2	−776	192.6	−766	193.5	−794	60.5	−800	33.4	−806	17.8
chim8-4.3	−784	281.5	−840	145.7	−790	218.7	−790	218.6	−800	181.7
chim8-4.4	−806	291.1	−876	24.7	−828	203.5	−840	160.0	−852	107.2
chim8-4.5	−850	208.1	−882	58.0	−840	175.1	−828	223.3	−852	128.5
chim8-4.6	−790	218.4	−798	189.7	−840	140.9	−830	158.5	−836	132.6
chim8-4.7	−756	301.5	−810	134.2	−802	102.7	−824	188.6	−806	85.3
chim8-4.8	−786	248.0	−822	26.2	−796	125.8	−824	4.8	−814	68.7
chim8-4.9	−850	154.6	−810	265.7	−872	159.9	−802	291.8	−828	186.1
chim8-4.10	−810	349.5	−896	17.7	−812	341.1	−830	269.3	−828	280.2
chim8-4.11	−764	154.6	−768	113.3	−748	200.7	−790	68.6	−730	280.8
chim8-4.12	−746	306.5	−774	161.7	−786	105.7	−780	130.9	−808	8.2
chim8-4.13	−818	158.7	−848	17.8	−798	218.4	−788	259.6	−800	210.5
chim8-4.14	−806	63.7	−770	218.3	−818	36.4	−800	140.5	−792	124.6
chim8-4.15	−836	243.4	−896	14.9	−868	115.9	−880	67.8	−868	118.3
chim8-4.16	−834	164.7	−852	70.1	−814	205.4	−862	42.0	−818	194.2
chim8-4.17	−802	102.7	−732	376.1	−810	81.2	−816	157.7	−796	114.7
chim8-4.18	−856	63.4	−808	268.0	−868	15.2	−870	5.6	−870	11.8
chim8-4.19	−870	200.2	−906	35.3	−886	84.4	−876	126.0	−866	164.5
chim8-4.20	−818	249.5	−874	31.1	−878	51.5	−812	274.6	−850	120.3
chim8-4.21	−818	98.7	−816	120.5	−836	22.3	−830	47.6	−840	5.6
chim8-4.22	−816	122.7	−834	53.3	−820	110.9	−844	58.7	−834	48.3
chim8-4.23	−780	196.4	−824	39.6	−788	168.6	−768	249.4	−798	120.2
chim8-4.24	−840	222.8	−834	199.5	−862	87.4	−862	91.6	−880	123.4
chim8-4.25	−880	91.2	−872	115.8	−872	80.5	−858	136.8	−890	68.0
chim8-4.26	−796	190.6	−838	66.1	−792	205.4	−826	174.8	−788	220.9
chim8-4.27	−834	54.5	−844	36.3	−846	16.0	−834	57.0	−834	57.2
chim8-4.28	−782	82.4	−746	249.8	−792	34.2	−790	44.2	−798	11.4
chim8-4.29	−790	193.3	−820	74.4	−792	186.2	−834	94.3	−810	113.8
chim8-4.30	−842	200.1	−808	341.0	−870	87.4	−864	111.5	−890	42.6
chim8-4.31	−870	137.1	−890	58.8	−900	63.3	−882	198.9	−860	243.8
chim8-4.32	−790	192.8	−830	11.1	−824	144.3	−818	131.9	−822	160.7
chim8-4.33	−884	98.4	−830	275.1	−878	83.5	−896	51.4	−870	110.0
chim8-4.34	−890	24.9	−882	53.5	−872	80.2	−876	68.7	−860	126.7
chim8-4.35	−782	98.6	−798	84.0	−788	51.3	−774	112.7	−794	25.1
chim8-4.36	−788	133.1	−816	13.4	−792	111.2	−776	180.2	−780	164.1
chim8-4.37	−790	64.7	−798	12.1	−760	176.5	−798	72.7	−798	33.7
chim8-4.38	−806	349.6	−796	266.9	−856	154.8	−792	277.7	−840	184.2
chim8-4.39	−856	79.6	−866	20.7	−850	70.5	−846	86.8	−846	87.6
chim8-4.40	−790	82.9	−760	303.5	−788	79.8	−802	71.6	−800	38.9
chim8-4.41	−880	209.0	−890	36.7	−846	185.6	−836	223.6	−864	114.4
chim8-4.42	−658	190.7	−694	210.2	−678	89.6	−678	89.9	−684	58.8
chim8-4.43	−734	108.1	−740	86.0	−756	4.6	−742	70.2	−752	23.9
chim8-4.44	−742	145.7	−704	328.5	−772	174.5	−756	81.3	−764	48.2
chim8-4.45	−818	251.7	−846	26.3	−842	29.5	−842	24.6	−834	81.0
chim8-4.46	−854	154.6	−854	128.0	−840	152.5	−874	68.8	−832	179.0
chim8-4.47	−848	136.6	−856	66.1	−868	96.4	−834	145.0	−850	88.2
chim8-4.48	−826	94.0	−834	12.2	−812	98.6	−812	98.5	−832	18.1
chim8-4.49	−800	159.8	−820	15.1	−760	268.9	−746	328.3	−786	154.9
chim8-4.50	−822	114.2	−802	189.1	−814	124.9	−842	72.1	−838	39.9
#best	1		19		11		13		9	
AM prim-$\int$ (#50)		167.7		118.3		119.8		130.6		109.5
GM prim-$\int$ (#50)		148.0		73.5		94.6		100.0		79.4

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Online Supplement:
Solving MIPs via Scaling-based Augmentation

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This document provides supplementary material for the article "Solving MIPs via Scaling-based Augmentation". It gives detailed results on the computations therein. We refer to this article for details.

We recall that we use the testsets listed in Table 1. Table 2 gives an overview of the settings used. The experiments were run on a Linux cluster with 3.2 GHz Intel i3 processors, 8 GB of main memory, and 4 MB of cache, running a single process at a time. We use SCIP 3.2.0 and CPLEX 12.6.1 as the LP-solver. The different columns of the tables are explained in Table 3. At the bottom of each table, we give the Artihmetic Means (AM) and Geometric Means (GM) for most columns. Moreover, we present the Shifted Geometric Mean (SGM) for the running time and number of nodes. Recall that the shifted geometric mean of values $t_1, \ldots, t_n$ is defined as

$$\Big(\prod (t_i + s)\Big)^{1/n} - s$$

with shift s. We use a shift $s = 10$ for time and $s = 100$ for nodes in order to decrease the strong influence of the very easy instances in the mean values.

Table 1: Testsets for experiments

shortcut	explanation
MIPLIB2010	The 87 benchmark instances from MIPLIB 2010, available at `http://miplib.zib.de/`.
LB	Testset of 29 instances from the "local branching" paper, available at `http://www.or.deis.unibo.it/research_pages/ORinstances/MIPs.html`.
QUBO	Testset of linearizations of 50 instances for quadratically unconstrained boolean optimization (QUBO), available at `http://researcher.watson.ibm.com/researcher/files/us-sanjeebd/chimera-data.zip`.

Table 2: Settings for experiments

shortcut	explanation
default	SCIP with default settings
augment	basic augmentation procedure (Section 5.2)
bitscale	bit scaling procedure with incomplete search and improving constraint (Section 5.3)
bitscale-classic	change objective in bitscale when no improving solution exists
bitscale-noimprove	bitscale without improving constraint
bitscale-complete	bitscale with complete search
geometric	geometric scaling procedure with objective cutoff (Section 5.4)
geometric-no ℓ_1	geometric without ℓ_1 part for integer variables
geometric-no cutoff	geometric without objective cutoff, i.e., an improving constraint
geometric-8	geometric that divides μ by 8
geometric-64	geometric that divides μ by 64
geometric-256	geometric that divides μ by 256
geometric-512	geometric that divides μ by 512
geometric-1024	geometric that divides μ by 1024
geometric-heur	heursitc using geometric scaling (Section 5.5)
geometric-heur-inference	as geometric-heur, but using inference branching
geometric-heur-64	as geometric-heur, but dividing μ by 64
MRA	MRA (Section 5.6)

Table 3: Description of columns in results

shortcut	explanation
Name	instance name
# Nodes	number of nodes in main SCIP
Time	total time in seconds
Dual	final dual bound in main SCIP
Primal	final primal bound in main SCIP, including results from augmentation subproblems
Gap %	gap between dual bound (d) and primal bound (p): $100 \cdot |p - b| / \max(|p|, |b|)$
Prim-$\int$	primal integral
#impr	number of solution improvements in augmentation
#sub	number of subproblems
#phases	number of phases
#exh	number of time a proper exhausting solution was found
AM	Arithmetic mean
GM	Geometric mean
SGM	Shifted Geometric mean

List of Tables

Table 4: Statistics on MIPLIB 2010 testset (87 instances)

name	#vars	#binvars	#intvars	#conss	best primal
30n20b8	4628	4571	2	400	302
acc-tight5	1029	1029	0	2374	0
aflow40b	2726	1364	0	1440	1168
air04	7586	7586	0	607	56137
app1-2	26265	13000	0	52555	-41
ash608gpia-3col	3651	3651	0	24748	1e+20
bab5	21432	21432	0	4740	-106412
beasleyC3	1704	852	0	1153	754
biella1	7311	6110	0	1202	3.06501e+06
bienst2	449	35	0	520	54.6
binkar10_1	1443	170	0	825	6742.2
bley_xl1	787	787	0	9186	190
bnatt350	1738	1738	0	1768	0
core2536-691	11234	11234	0	1893	689
cov1075	120	120	0	637	20
csched010	1654	1457	0	260	408
danoint	513	56	0	656	65.6667
dfn-gwin-UUM	936	0	90	156	38752
eil33-2	4484	4484	0	32	934.008
eilB101	2715	2715	0	100	1216.92
enlight13	338	173	165	169	71
enlight14	392	200	192	196	1e+20
ex9	0	0	0	0	81
glass4	317	298	0	392	1.20001e+09
gmu-35-40	652	647	0	357	-2.40673e+06
iis-100-0-cov	100	100	0	3831	29
iis-bupa-cov	341	341	0	4803	36
iis-pima-cov	730	730	0	7196	33
lectsched-4-obj	2604	2498	106	4727	4
m100n500k4r1	500	500	0	100	-25
macrophage	2260	2260	0	3164	374
map18	15414	118	0	31212	-847
map20	15414	118	0	31212	-922
mcsched	1495	1495	0	1853	211913
mik-250-1-100-1	251	100	150	100	-66729
mine-166-5	709	709	0	6698	-5.66396e+08
mine-90-10	867	867	0	5814	-7.84302e+08
msc98-ip	12733	11910	0	14993	1.98395e+07
mspp16	4065	4065	0	524814	363
mzzv11	6542	6307	184	6616	-21718
n3div36	20602	20602	0	4453	130800
n3seq24	119856	119856	0	5950	52200
n4-3	3113	5	158	981	8993
neos-1109824	1520	1520	0	9979	378
neos-1337307	2840	2840	0	2023	-202319
neos-1396125	1158	129	0	1491	3000.05
neos13	1827	1815	0	17401	-95.4748
neos-1601936	3920	3570	0	3105	3
neos18	758	758	0	3290	16
neos-476283	11843	5544	0	9616	406.363
neos-686190	3660	3600	60	3658	6730
neos-849702	1692	1692	0	984	0
neos-916792	1361	708	0	1408	31.8704
neos-934278	8121	7354	0	8106	260
net12	12523	1113	0	12767	214
netdiversion	128968	128968	0	99483	242

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name	#vars	#binvars	#intvars	#conss	best primal
newdano	449	56	0	520	65.6667
noswot	120	75	20	171	-41
ns1208400	2596	2596	0	1981	2
ns1688347	1457	1457	0	3455	27
ns1758913	17824	17822	0	615190	-1454.67
ns1766074	100	90	0	110	1e+20
ns1830653	673	541	0	1400	20622
opm2-z7-s2	1896	1896	0	26691	-10280
pg5_34	2600	100	0	225	-14339.4
pigeon-10	390	360	0	525	-9000
pw-myciel4	1013	1012	1	4180	10
qiu	840	48	0	1192	-132.873
rail507	20698	20698	0	441	174
ran16x16	512	256	0	288	3823
reblock67	627	627	0	2271	-3.46306e+07
rmatr100-p10	7359	100	0	7260	423
rmatr100-p5	8784	100	0	8685	976
rmine6	1084	1084	0	7066	-457.186
rocII-4-11	1266	1184	0	3444	-6.65564
rococoC10-001000	2442	2442	0	576	11460
roll3000	819	584	114	1178	12890
satellites1-25	7102	7062	0	4160	-5
sp98ic	10876	10876	0	796	4.49145e+08
sp98ir	1557	869	688	1375	2.19677e+08
tanglegram1	34759	34759	0	68342	5182
tanglegram2	4714	4714	0	8980	443
timtab1	201	54	92	166	764772
triptim1	25380	15832	9548	15454	22.8681
unitcal_7	20297	2503	0	38656	1.96356e+07
vpphard	27488	27488	0	23418	5
zib54-UUE	5069	80	0	1761	1.0334e+07

Table 5: MIPLIB2010 testset: augment settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
30n20b8	176	210.32	302	302	0.00	174.40	3	3	3	0
acc-tight5	653	169.25	0	0	0.00	169.25	1	1	1	0
aflow40b	146698	1798.70	1168	1168	0.00	81.58	6	6	6	0
air04	36	91.11	56137	56137	0.00	5.00	5	5	5	0
app1-2	5111	3600.22	-164.137	1e+20	—	3600.22	0	0	0	0
ash608gpia-3col	6	31.90	1e+20	1e+20	—	31.90	1	1	1	0
bab5	23408	3600.35	-107401	-105869	1.43	430.90	3	3	3	0
beasleyC3	431334	3600.15	678.579	784	13.45	236.96	12	12	12	0
biella1	4214	1138.67	3.06501e+06	3.06501e+06	0.00	30.67	11	11	11	0
bienst2	170043	265.39	54.6	54.6	0.00	2.82	7	7	7	0
binkar10_1	151687	290.97	6742.2	6742.2	0.00	1.09	15	15	15	0
bley_xl1	9	305.12	190	190	0.00	272.87	8	8	8	0
bnatt350	8217	988.40	0	0	0.00	988.40	1	1	1	0
core2536-691	530	324.87	689	689	0.00	10.71	4	4	4	0
cov1075	804684	3600.30	17.2082	20	13.96	3.35	4	4	4	0
csched010	231047	3600.06	358.5	413	13.20	182.49	9	9	9	0
danoint	839840	3600.46	62.6887	65.6667	4.54	1.29	3	3	3	0
dfn-gwin-UUM	42771	176.57	38752	38752	0.00	17.46	16	16	16	0
eil33-2	1187	189.71	934.008	934.008	0.00	10.96	9	9	9	0
eilB101	9449	178.57	1216.92	1216.92	0.00	6.34	5	5	5	0
enlight13	1	0.01	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.02	1e+20	1e+20	—	0.02	0	0	0	0
ex9	1	65.32	81	81	0.00	65.30	0	0	0	0
glass4	4869068	3600.26	8.00003e+08	1.45001e+09	44.83	882.49	34	34	34	0
gmu-35-40	4023306	3601.02	-2.40692e+06	-2.40555e+06	0.06	2.53	14	14	14	0
iis-100-0-cov	101824	1999.62	29	29	0.00	5.56	4	4	4	0
iis-bupa-cov	88788	3600.06	26.6259	36	26.04	17.91	8	8	8	0
iis-pima-cov	11654	887.63	33	33	0.00	17.67	6	6	6	0
lectsched-4-obj	158838	2291.26	4	4	0.00	1818.23	33	33	33	0
m100n500k4r1	4337421	3600.00	-25	-24	4.00	144.96	5	5	5	0
macrophage	376316	3600.07	196.739	399	50.69	360.65	37	37	37	0
map18	703	1554.60	-847	-847	0.00	62.39	13	13	13	0
map20	1305	1265.57	-922	-922	0.00	26.79	11	11	11	0
mcsched	28676	382.74	211913	211913	0.00	5.03	213	213	213	0
mik-250-1-100-1	4074805	1339.25	-66729	-66729	0.00	0.10	5	5	5	0
mine-166-5	5633	841.75	-5.66396e+08	-5.66396e+08	0.00	45.20	47	47	47	0
mine-90-10	402252	1981.53	-7.84302e+08	-7.84302e+08	0.00	50.70	17	17	17	0
msc98-ip	3749	3600.14	1.97029e+07	2.44448e+07	19.40	735.15	2	2	2	0
mspp16	0	23.16	1e+20	1e+20	—	23.16	0	0	0	0
mzzv11	2895	1010.76	-21718	-21718	0.00	196.54	14	14	14	0
n3div36	41826	3601.30	120927	131400	7.97	261.27	15	15	15	0
n3seq24	7	3603.16	52000	85000	38.82	1456.61	2	2	2	0
n4-3	24887	766.93	8993	8993	0.00	52.97	18	18	18	0
neos-1109824	5916	187.42	378	378	0.00	20.33	7	7	7	0
neos-1337307	119539	3600.16	-203102	-202319	0.39	39.58	7	7	7	0
neos-1396125	15342	368.31	3000.05	3000.05	0.00	28.14	4	4	4	0
neos13	109	3600.10	-126.178	-67.8811	46.20	1200.67	32	32	32	0
neos-1601936	2512	2811.57	3	3	0.00	2521.13	3	3	3	0
neos18	5232	79.55	16	16	0.00	6.12	5	5	5	0
neos-476283	1385	2251.57	406.364	406.364	0.00	94.41	13	13	13	0
neos-686190	3119	263.24	6730	6730	0.00	34.11	8	8	8	0
neos-849702	54881	816.32	0	0	0.00	816.32	1	1	1	0
neos-916792	238999	3600.12	26.2832	32.1536	18.26	66.25	13	13	13	0
neos-934278	523	1457.40	260	260	0.00	151.10	8	8	8	0
net12	3839	1284.04	214	214	0.00	487.68	3	3	3	0
netdiversion	33	1553.44	242	242	0.00	788.69	19	19	19	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
newdano	1267645	3600.15	33.85	66.5	49.10	62.82	31	31	31	1
noswot	1411616	620.60	-41	-41	0.00	0.10	4	4	4	0
ns1208400	6005	1348.02	2	2	0.00	694.00	1	1	1	0
ns1688347	6758	1205.33	27	27	0.00	192.41	7	7	7	0
ns1758913	4	3601.87	-1454.67	-271.008	81.37	2969.09	2	2	2	0
ns1766074	891606	1540.42	1e+20	1e+20	—	1540.42	1	1	1	0
ns1830653	49793	1129.50	20622	20622	0.00	195.34	8	8	8	0
opm2-z7-s2	2196	3600.08	-12879.7	-9855	23.48	484.01	17	17	17	0
pg5_34	448614	3600.11	-14366.7	-14335.1	0.22	5.15	8	8	8	0
pigeon-10	4298879	3600.00	-10000	-9000	10.00	0.62	9	9	9	0
pw-myciel4	511550	3600.15	4	10	60.00	22.08	9	9	9	0
qiu	20524	310.78	-132.873	-132.873	0.00	166.42	20	20	20	0
rail507	1607	292.65	174	174	0.00	35.09	17	17	17	0
ran16x16	268260	374.49	3823	3823	0.00	4.06	7	7	7	0
reblock67	82835	470.70	-3.46306e+07	-3.46306e+07	0.00	23.73	18	18	18	0
rmatr100-p10	1668	290.74	423	423	0.00	7.02	6	6	6	0
rmatr100-p5	1121	333.95	976	976	0.00	9.73	6	6	6	0
rmine6	558187	3600.27	-461.364	-457.043	0.94	10.18	26	26	26	0
rocII-4-11	62666	1049.74	-6.65564	-6.65564	0.00	137.67	11	11	11	0
rococoC10-001000	435668	2654.69	11460	11460	0.00	54.11	25	25	25	0
roll3000	686713	3600.36	12290.5	12902	4.74	16.33	8	8	8	0
satellites1-25	5303	1539.73	-5	-5	0.00	906.00	7	7	7	0
sp98ic	55252	3600.23	4.44484e+08	4.66504e+08	4.72	196.50	23	23	23	0
sp98ir	7546	384.78	2.19677e+08	2.19677e+08	0.00	9.79	28	28	28	0
tanglegram1	125	3600.28	6	5182	99.88	307.04	4	4	4	0
tanglegram2	7	15.83	443	443	0.00	5.60	2	2	2	0
timtab1	1236838	939.47	764777	764777	0.00	27.57	31	31	31	0
triptim1	3	148.47	22.8681	22.8681	0.00	49.51	2	2	2	0
unitcal_7	28881	3600.25	1.95516e+07	1.9649e+07	0.50	144.16	20	20	20	0
vpphard	278	3600.40	0	128	—	3551.30	2	2	2	0
zib54-UUE	328640	3600.21	6.94952e+06	1.0334e+07	32.75	58.51	21	21	21	0
AM (# 87)	397164.1	1773.62			7.71	352.09	12.9	12.9	12.9	0.0
GM (# 87)	9099.9	851.17			2.15	54.51	6.9	6.9	6.9	1.0
SGM (# 87)	14793.5	924.71								

Table 6: MIPLIB2010 testset: bitscale-classic settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
30n20b8	313	345.26	302	302	0.00	227.67	2	8	6	0
acc-tight5	413	95.50	0	0	0.00	42.90	0	0	0	0
aflow40b	454512	3600.00	1104.41	1168	5.44	89.45	11	18	7	0
air04	896	355.79	56137	56137	0.00	10.97	6	18	12	0
app1-2	9773	3600.03	-49.5462	-34	31.38	2167.03	0	0	0	0
ash608gpia-3col	13	34.35	1e+20	1e+20	—	34.35	0	0	0	0
bab5	59740	3600.00	-107385	-80619.4	24.92	1610.60	3	5	2	0
beasleyC3	363302	3600.00	684.743	773	11.42	393.80	2	5	3	0
biella1	125901	3600.07	3.06018e+06	3.1748e+06	3.61	165.36	4	9	5	0
bienst2	112679	151.67	54.6	54.6	0.00	2.19	0	0	0	0
binkar10_1	2170329	3600.02	6704.45	6742.2	0.56	13.01	19	41	22	0
bley_xl1	42	374.98	190	190	0.00	296.48	3	6	3	0
bnatt350	23940	1851.13	0	0	0.00	1851.00	0	0	0	0
core2536-691	260	135.38	689	689	0.00	9.82	2	3	1	0
cov1075	780240	3600.00	18.4175	20	7.91	2.46	0	0	0	0
csched010	562617	3600.00	360.552	486	25.81	982.17	7	10	3	0
danoint	927577	3600.00	63.7547	65.6667	2.91	6.10	0	0	0	0
dfn-gwin-UUM	371813	806.38	38752	38752	0.00	13.58	8	20	12	0
eil33-2	11093	1742.39	934.008	934.008	0.00	18.98	5	42	37	0
eilB101	125534	3033.44	1216.92	1216.92	0.00	69.78	8	49	41	0
enlight13	1	0.02	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.01	1e+20	1e+20	—	0.01	0	0	0	0
ex9	1	65.87	81	81	0.00	65.90	0	0	0	0
glass4	3059997	1893.90	1.20001e+09	1.20001e+09	0.00	464.54	9	12	3	0
gmu-35-40	3979540	3600.00	-2.40692e+06	-2.40597e+06	0.04	7.35	10	22	12	0
iis-100-0-cov	81999	1577.68	29	29	0.00	3.72	0	0	0	0
iis-bupa-cov	103564	3600.00	32.8733	36	8.69	8.13	0	0	0	0
iis-pima-cov	11775	779.03	33	33	0.00	15.70	0	0	0	0
lectsched-4-obj	4536	119.88	4	4	0.00	116.04	0	1	1	0
m100n500k4r1	4612271	3600.00	-25	-24	4.00	144.78	0	0	0	0
macrophage	724653	3600.00	282.287	381	25.91	75.69	0	0	0	0
map18	286	494.54	-847	-847	0.00	116.53	2	4	2	0
map20	367	358.71	-922	-922	0.00	65.83	3	5	2	0
mcsched	156912	1678.37	211913	211913	0.00	4.54	8	17	9	0
mik-250-1-100-1	10136690	3600.00	-70072.8	-66729	4.77	0.10	1	10	9	0
mine-166-5	21487	937.12	-5.66396e+08	-5.66396e+08	0.00	10.58	10	54	44	0
mine-90-10	756170	3600.00	-7.98557e+08	-7.84302e+08	1.79	11.22	20	35	15	0
msc98-ip	5116	3600.01	1.97029e+07	2.02358e+07	2.63	384.91	5	10	5	0
mspp16	0	23.13	1e+20	1e+20	—	23.13	0	0	0	0
mzzv11	2168	1226.48	-21718	-21718	0.00	396.60	9	19	10	0
n3div36	61373	3600.00	122120	134400	9.14	466.43	3	8	5	0
n4-3	41212	936.03	8993	8993	0.00	35.88	4	11	7	0
neos-1109824	11199	256.44	378	378	0.00	30.16	4	11	7	0
neos-1337307	85447	3600.00	-202943	-202319	0.31	3478.04	0	7	7	0
neos-1396125	248814	3281.62	3000.05	3000.05	0.00	117.00	2	22	20	0
neos13	2964	3571.70	-95.4748	-95.4748	0.00	350.18	14	22	8	0
neos-1601936	5178	3600.00	3	4	25.00	2663.37	5	13	8	0
neos18	5125	48.63	16	16	0.00	2.14	0	0	0	0
neos-476283	879	460.87	406.363	406.363	0.00	49.77	0	0	0	0
neos-686190	9617	610.95	6730	6730	0.00	138.46	8	17	9	0
neos-849702	26770	645.47	0	0	0.00	645.00	0	0	0	0
neos-916792	321573	3600.01	26.7767	33.9473	21.12	282.40	6	13	7	0
neos-934278	733	1342.89	260	260	0.00	179.35	8	11	3	0
net12	8313	2489.04	214	214	0.00	428.08	3	8	5	0
netdiversion	5	3601.33	232	4.60045e+06	99.99	3601.15	3	4	1	0
newdano	1411747	3600.00	55	65.6667	16.24	35.10	0	0	0	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
noswot	1136575	325.34	-41	-41	0.00	0.36	0	0	0	0
ns1208400	9919	1473.25	2	2	0.00	1473.00	0	0	0	0
ns1688347	4621	696.01	27	27	0.00	138.24	2	8	6	0
ns1758913	2	3602.51	-1454.67	-236.804	83.72	3023.05	0	1	1	0
ns1766074	889424	1018.05	1e+20	1e+20	—	1018.05	0	1	1	0
ns1830653	170365	3292.95	20622	20622	0.00	172.41	7	16	9	0
opm2-z7-s2	3949	3600.00	-12535.3	-10036	19.94	390.12	6	10	4	0
pg5_34	416948	3600.00	-14356.6	-14333	0.16	7.69	14	22	8	0
pigeon-10	5049010	3600.00	-10000	-9000	10.00	2.00	0	0	0	0
pw-myciel4	562891	3600.00	6.80769	10	31.92	24.84	0	0	0	0
qiu	351517	2469.12	-132.873	-132.873	0.00	84.70	3	27	24	0
rail507	653	126.39	174	174	0.00	16.76	2	3	1	0
ran16x16	715383	856.58	3823	3823	0.00	6.25	3	12	9	0
reblock67	1339055	3600.00	-3.57613e+07	-3.46306e+07	3.16	14.90	15	49	34	0
rmatr100-p10	1324	176.52	423	423	0.00	13.49	2	4	2	0
rmatr100-p5	361	129.08	976	976	0.00	11.03	1	3	2	0
rmine6	531296	3600.02	-461.111	-457.08	0.87	11.18	15	23	8	0
rocII-4-11	130360	2559.06	-6.65564	-6.65564	0.00	100.52	4	23	19	0
rococoC10-001000	992870	3600.00	10199.4	12308	17.13	295.98	4	9	5	0
roll3000	599578	3600.00	12395.3	12890	3.84	38.18	14	20	6	0
satellites1-25	4361	1543.51	-5	-5	0.00	896.40	4	11	7	0
sp98ic	115227	3600.00	4.46831e+08	4.58028e+08	2.44	121.24	9	18	9	0
sp98ir	26012	519.53	2.19677e+08	2.19677e+08	0.00	2.48	8	33	25	0
tanglegram1	109	866.89	5182	5182	0.00	101.42	0	0	0	0
tanglegram2	5	7.37	443	443	0.00	4.39	0	0	0	0
timtab1	5016513	3600.00	496517	764772	35.08	17.26	11	30	19	0
triptim1	43	3599.29	22.8681	22.8681	0.00	51.02	1	33	32	0
unitcal_7	64949	3600.32	1.95517e+07	1.98333e+07	1.42	1107.36	3	7	4	0
vpphard	1695	3600.22	0	50	—	3362.33	0	0	0	0
zib54-UUE	366941	3600.00	7.13872e+06	1.03635e+07	31.12	31.68	6	12	6	0
AM (# 86)	587225.9	2104.51			6.68	406.57	4.1	11.0	6.9	0.0
GM (# 86)	16456.7	1027.30			2.28	60.42	2.8	5.3	3.6	1.0
SGM (# 86)	25086.0	1120.62								

Table 7: MIPLIB2010 testset: bitscale-complete settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
30n20b8	401	294.58	302	302	0.00	228.00	1	6	5	0
acc-tight5	413	95.50	0	0	0.00	42.90	0	0	0	0
aflow40b	498997	3600.12	1099.36	1168	5.88	325.69	6	7	1	0
air04	857	269.80	56137	56137	0.00	10.36	6	12	6	0
app1-2	9618	3600.02	-49.5462	-34	31.38	2180.30	0	0	0	0
ash608gpia-3col	13	34.49	1e+20	1e+20	—	34.49	0	0	0	0
bab5	86670	3600.75	-107401	-45751.4	57.40	3600.75	0	2	2	0
beasleyC3	840733	3601.01	678.579	954	28.87	754.98	0	2	2	0
biella1	21784	3600.23	3.06009e+06	3.2679e+06	6.36	290.84	1	5	4	0
bienst2	112679	151.96	54.6	54.6	0.00	2.20	0	0	0	0
binkar10_1	2262317	3600.03	6703.7	6742.2	0.57	92.72	9	16	7	0
bley_xl1	44	365.26	190	190	0.00	295.39	3	3	0	0
bnatt350	23940	1850.17	0	0	0.00	1850.00	0	0	0	0
core2536-691	544	373.92	689	689	0.00	51.14	1	1	0	0
cov1075	774272	3600.00	18.4157	20	7.92	2.48	0	0	0	0
csched010	472301	3600.26	358.5	674	46.81	2138.20	1	3	2	0
danoint	925526	3600.00	63.7535	65.6667	2.91	6.10	0	0	0	0
dfn-gwin-UUM	473358	877.31	38752	38752	0.00	34.30	4	12	8	0
eil33-2	10858	1679.60	934.008	934.008	0.00	17.00	5	37	32	0
eilB101	121827	2755.84	1216.92	1216.92	0.00	52.61	4	41	37	0
enlight13	1	0.02	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.01	1e+20	1e+20	—	0.01	0	0	0	0
ex9	1	65.48	81	81	0.00	65.50	0	0	0	0
glass4	5830444	3606.20	8.00003e+08	1e+20	100.00	3606.20	0	1	1	0
gmu-35-40	7962279	3600.24	-2.40692e+06	-2.31299e+06	3.90	140.33	0	3	3	0
iis-100-0-cov	81999	1575.26	29	29	0.00	3.81	0	0	0	0
iis-bupa-cov	103921	3600.01	32.8791	36	8.67	8.13	0	0	0	0
iis-pima-cov	11775	777.23	33	33	0.00	15.64	0	0	0	0
lectsched-4-obj	3862	75.33	4	4	0.00	75.33	0	1	1	0
m100n500k4r1	4619627	3600.00	-25	-24	4.00	144.77	0	0	0	0
macrophage	720994	3600.00	282.254	381	25.92	75.75	0	0	0	0
map18	259	327.13	-847	-847	0.00	302.76	2	2	0	0
map20	312	269.92	-922	-922	0.00	246.68	2	2	0	0
mcsched	176834	1705.31	211913	211913	0.00	46.72	5	9	4	0
mik-250-1-100-1	10091700	3600.19	-70077.8	-66729	4.78	0.40	1	9	8	0
mine-166-5	21188	813.04	-5.66396e+08	-5.66396e+08	0.00	10.33	10	44	34	0
mine-90-10	915155	3600.04	-8.59772e+08	-7.843e+08	8.78	9.77	11	14	3	0
msc98-ip	6372	3600.21	1.97029e+07	2.02358e+07	2.63	303.24	3	5	2	0
mspp16	0	23.11	1e+20	1e+20	—	23.11	0	0	0	0
mzzv11	2597	891.42	-21718	-21718	0.00	406.18	8	10	2	0
n3div36	83100	3602.22	120927	418200	71.08	2477.17	1	5	4	0
n4-3	38164	821.41	8993	8993	0.00	81.12	2	7	5	0
neos-1109824	11702	230.72	378	378	0.00	29.56	4	7	3	0
neos-1337307	90111	3600.16	-203102	-201484	0.80	3600.16	0	7	7	0
neos-1396125	253260	3330.06	3000.05	3000.05	0.00	620.00	2	20	18	0
neos13	55554	3600.18	-126.178	-28.0396	77.78	2543.15	0	1	1	0
neos-1601936	10503	3600.10	3	536	99.44	3583.46	3	8	5	0
neos18	5125	48.93	16	16	0.00	2.14	0	0	0	0
neos-476283	879	461.71	406.363	406.363	0.00	49.80	0	0	0	0
neos-686190	9929	422.93	6730	6730	0.00	84.51	6	9	3	0
neos-849702	26770	644.17	0	0	0.00	644.00	0	0	0	0
neos-916792	363513	3600.26	26.2832	33.3481	21.19	302.27	3	6	3	0
neos-934278	155	325.02	260	260	0.00	307.47	3	3	0	0
net12	11615	3600.13	86.5388	214	59.56	3600.13	0	5	5	0
netdiversion	31	3601.30	232	638	63.64	3311.64	2	3	1	0
newdano	1402454	3600.00	55	65.6667	16.24	35.26	0	0	0	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
noswot	1136575	324.66	-41	-41	0.00	0.36	0	0	0	0
ns1208400	9919	1469.78	2	2	0.00	1470.00	0	0	0	0
ns1688347	4756	647.86	27	27	0.00	335.00	1	6	5	0
ns1758913	2	3602.49	-1454.67	-236.804	83.72	3023.03	0	1	1	0
ns1766074	889424	1010.53	1e+20	1e+20	—	1010.53	0	1	1	0
ns1830653	169355	2948.41	20622	20622	0.00	2948.41	0	9	9	0
opm2-z7-s2	8639	3600.15	-12879.7	-3685	71.39	2310.64	0	2	2	0
pg5_34	487197	3080.97	-14339.4	-14339.4	0.00	29.40	5	8	3	0
pigeon-10	5005460	3600.00	-10000	-9000	10.00	2.00	0	0	0	0
pw-myciel4	563936	3600.00	6.83333	10	31.67	24.83	0	0	0	0
qiu	341660	2392.61	-132.873	-132.873	0.00	191.00	2	24	22	0
rail507	734	86.35	174	174	0.00	59.03	1	1	0	0
ran16x16	1098536	1229.68	3823	3823	0.00	42.68	3	9	6	0
reblock67	1393695	3600.02	-3.80738e+07	-3.46306e+07	9.04	14.24	10	34	24	0
rmatr100-p10	903	93.36	423	423	0.00	43.72	1	2	1	0
rmatr100-p5	354	113.69	976	976	0.00	37.18	1	2	1	0
rmine6	572844	3600.35	-461.364	-457.053	0.93	23.39	6	8	2	0
rocII-4-11	108553	2186.80	-6.65564	-6.65564	0.00	96.00	3	19	16	0
rococoC10-001000	911520	3600.82	10149.4	17343	41.48	1233.48	1	5	4	0
roll3000	636548	3600.17	12290.5	12890	4.65	881.09	4	6	2	0
satellites1-25	2078	881.87	-5	-5	0.00	881.87	2	7	5	0
sp98ic	237789	3600.41	4.44484e+08	4.54677e+08	2.24	157.11	3	9	6	0
sp98ir	24392	465.33	2.19677e+08	2.19677e+08	0.00	2.26	6	25	19	0
tanglegram1	109	863.56	5182	5182	0.00	101.06	0	0	0	0
tanglegram2	5	7.36	443	443	0.00	4.39	0	0	0	0
timtab1	5042990	3600.07	481470	764797	37.05	69.41	6	17	11	0
triptim1	42	3599.19	22.8681	22.8681	0.00	49.82	1	32	31	0
unitcal_7	43033	3600.92	1.95516e+07	2.73473e+07	28.51	1183.37	2	5	3	0
vpphard	1695	3600.00	0	50	—	3362.13	0	0	0	0
zib54-UUE	384655	3600.16	6.94952e+06	1.0393e+07	33.13	63.01	4	6	2	0
AM (# 86)	681706.2	2090.44			12.91	679.28	2.0	6.6	4.6	0.0
GM (# 86)	17939.9	979.67			3.18	103.53	1.7	3.4	2.4	1.0
SGM (# 86)	26902.6	1070.71								

Table 8: MIPLIB2010 testset: bitscale-noimprove settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
30n20b8	81	239.10	302	302	0.00	190.47	2	6	4	0
acc-tight5	413	95.54	0	0	0.00	42.90	0	0	0	0
aflow40b	172011	1730.96	1168	1168	0.00	113.03	9	9	0	0
air04	749	229.37	56137	56137	0.00	8.50	7	12	5	0
app1-2	9761	3600.01	-49.5462	-34	31.38	2168.68	0	0	0	0
ash608gpia-3col	13	34.43	1e+20	1e+20	—	34.43	0	0	0	0
bab5	23116	3600.00	-107055	-106328	0.68	437.49	12	14	2	0
beasleyC3	410080	3600.51	678.579	874	22.36	495.67	1	4	3	0
biella1	30064	3600.00	3.06018e+06	3.06501e+06	0.16	105.13	12	19	7	0
bienst2	112679	151.65	54.6	54.6	0.00	2.21	0	0	0	0
binkar10_1	2177357	3600.00	6707.02	6742.2	0.52	1.49	13	32	19	0
bley_xl1	4	306.02	190	190	0.00	276.74	3	3	0	0
bnatt350	23940	1856.22	0	0	0.00	1856.00	0	0	0	0
core2536-691	207	85.57	689	689	0.00	15.56	1	1	0	0
cov1075	778475	3600.00	18.4172	20	7.91	2.46	0	0	0	0
csched010	303307	3600.22	358.5	482	25.62	922.17	5	8	3	0
danoint	927279	3600.00	63.7545	65.6667	2.91	6.10	0	0	0	0
dfn-gwin-UUM	189254	430.23	38752	38752	0.00	6.37	4	12	8	0
eil33-2	12412	1170.94	934.008	934.008	0.00	13.18	5	37	32	0
eilB101	112794	2235.72	1216.92	1216.92	0.00	23.30	8	41	33	0
enlight13	1	0.02	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.01	1e+20	1e+20	—	0.01	0	0	0	0
ex9	1	65.58	81	81	0.00	65.60	0	0	0	0
glass4	5394748	3603.43	8.00003e+08	2.38968e+09	66.52	1795.92	1	3	2	0
gmu-35-40	4740250	3604.48	-2.40692e+06	-2.40581e+06	0.05	1.68	9	16	7	0
iis-100-0-cov	81999	1580.87	29	29	0.00	3.81	0	0	0	0
iis-bupa-cov	102761	3600.00	32.8639	36	8.71	8.17	0	0	0	0
iis-pima-cov	11775	785.20	33	33	0.00	15.79	0	0	0	0
lectsched-4-obj	3862	75.38	4	4	0.00	75.38	0	1	1	0
m100n500k4r1	4631034	3600.00	-25	-24	4.00	144.77	0	0	0	0
macrophage	725644	3600.00	282.295	381	25.91	75.65	0	0	0	0
map18	277	308.13	-847	-847	0.00	286.25	2	2	0	0
map20	335	280.67	-922	-922	0.00	258.58	2	2	0	0
mcsched	36237	463.63	211913	211913	0.00	4.27	6	9	3	0
mik-250-1-100-1	9040137	3250.97	-66729	-66729	0.00	0.00	2	11	9	0
mine-166-5	28563	792.62	-5.66396e+08	-5.66396e+08	0.00	12.99	11	44	33	0
mine-90-10	764574	3600.00	-7.92854e+08	-7.84302e+08	1.08	9.55	21	29	8	0
msc98-ip	2457	3600.01	1.97029e+07	2.17849e+07	9.56	412.42	6	7	1	0
mspp16	0	23.19	1e+20	1e+20	—	23.19	0	0	0	0
mzzv11	2177	892.88	-21718	-21718	0.00	465.19	10	10	0	0
n3div36	87420	3602.80	120927	142800	15.32	348.32	6	10	4	0
n3seq24	23643	3612.52	52000	57000	8.77	508.23	2	9	7	0
n4-3	19905	445.46	8993	8993	0.00	19.08	4	7	3	0
neos-1109824	17769	267.31	378	378	0.00	23.86	4	7	3	0
neos-1337307	133030	3600.00	-203092	-202319	0.38	178.15	5	13	8	0
neos-1396125	655411	3600.01	1892.89	3000.05	36.90	110.00	4	12	8	0
neos13	2209	3579.10	-95.4748	-95.4748	0.00	358.33	9	19	10	0
neos-1601936	13040	3579.09	3	3	0.00	3562.91	5	8	3	0
neos18	5125	48.66	16	16	0.00	2.14	0	0	0	0
neos-476283	879	461.59	406.363	406.363	0.00	49.87	0	0	0	0
neos-686190	5212	336.06	6730	6730	0.00	85.20	5	9	4	0
neos-849702	26770	644.49	0	0	0.00	644.00	0	0	0	0
neos-916792	246925	3600.00	26.803	31.9331	16.06	100.28	13	17	4	0
neos-934278	461	600.91	260	260	0.00	566.18	3	3	0	0
net12	5324	2055.45	214	214	0.00	345.24	2	5	3	0
netdiversion	16	2861.05	242	242	0.00	2858.31	0	1	1	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
newdano	1403628	3600.00	55	65.6667	16.24	35.35	0	0	0	0
noswot	1136575	324.02	-41	-41	0.00	0.36	0	0	0	0
ns1208400	9919	1469.26	2	2	0.00	1469.00	0	0	0	0
ns1688347	2717	554.55	27	27	0.00	134.74	2	6	4	0
ns1758913	2	3602.30	-1454.67	-236.804	83.72	3022.85	0	1	1	0
ns1766074	889424	1003.26	1e+20	1e+20	—	1003.26	0	1	1	0
ns1830653	38619	1018.55	20622	20622	0.00	294.50	8	9	1	0
opm2-z7-s2	2426	1188.26	-10280	-10280	0.00	308.03	9	9	0	0
pg5_34	223980	1447.49	-14339.4	-14339.4	0.00	6.88	6	8	2	0
pigeon-10	5023200	3600.00	-10000	-9000	10.00	2.00	0	0	0	0
pw-myciel4	561802	3600.00	6.8	10	32.00	24.78	0	0	0	0
qiu	174964	1360.57	-132.873	-132.873	0.00	25.70	2	24	22	0
rail507	915	106.27	174	174	0.00	72.05	1	1	0	0
ran16x16	231340	286.96	3823	3823	0.00	16.35	5	9	4	0
reblock67	1214143	3600.07	-3.80738e+07	-3.46306e+07	9.04	14.15	20	48	28	0
rmatr100-p10	975	85.28	423	423	0.00	40.00	1	2	1	0
rmatr100-p5	350	112.09	976	976	0.00	36.73	1	2	1	0
rmine6	507586	3600.00	-458.475	-457.092	0.30	9.32	19	20	1	0
rocII-4-11	76404	1746.91	-6.65564	-6.65564	0.00	73.66	7	19	12	0
rococoC10-001000	92627	526.43	11460	11460	0.00	41.73	9	12	3	0
roll3000	254025	1435.09	12890	12890	0.00	176.98	6	7	1	0
satellites1-25	2755	604.12	-5	-5	0.00	604.12	2	7	5	0
sp98ic	95181	3600.00	4.46759e+08	4.5843e+08	2.55	122.35	11	17	6	0
sp98ir	24840	477.80	2.19677e+08	2.19677e+08	0.00	2.27	6	25	19	0
tanglegram1	109	868.00	5182	5182	0.00	101.44	0	0	0	0
tanglegram2	5	7.39	443	443	0.00	4.39	0	0	0	0
timtab1	5040485	3600.00	490942	764772	35.81	16.44	14	28	14	0
triptim1	31	3599.97	22.8681	22.8681	0.00	50.04	1	30	29	0
unitcal_7	13696	3600.01	1.956e+07	1.96356e+07	0.38	413.88	18	22	4	0
vpphard	1695	3600.00	0	50	—	3362.12	0	0	0	0
zib54-UUE	344254	3322.26	1.0334e+07	1.0334e+07	0.00	22.37	6	7	1	0
AM (# 87)	568536.1	1856.72			5.46	363.79	4.2	8.8	4.6	0.0
GM (# 87)	13102.2	839.14			1.93	56.68	2.7	4.3	2.4	1.0
SGM (# 87)	20343.4	918.31								

Table 9: MIPLIB2010 testset: bitscale settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
30n20b8	198	289.07	302	302	0.00	221.33	2	6	4	0
acc-tight5	413	95.57	0	0	0.00	42.90	0	0	0	0
aflow40b	176712	1933.56	1168	1168	0.00	95.31	9	9	0	0
air04	872	276.55	56137	56137	0.00	10.36	6	12	6	0
app1-2	9733	3600.16	-49.5462	-34	31.38	2171.20	0	0	0	0
ash608gpia-3col	13	34.36	1e+20	1e+20	—	34.36	0	0	0	0
bab5	34545	3600.00	-107148	-105595	1.45	754.10	6	8	2	0
beasleyC3	350670	3600.44	678.579	867	21.73	470.93	1	4	3	0
biella1	23949	3600.11	3.06009e+06	3.06501e+06	0.16	117.24	11	19	8	0
bienst2	112679	150.45	54.6	54.6	0.00	2.19	0	0	0	0
binkar10_1	2128294	3600.00	6707.02	6742.2	0.52	1.69	16	36	20	0
bley_xl1	40	367.60	190	190	0.00	296.79	3	3	0	0
bnatt350	23940	1856.94	0	0	0.00	1857.00	0	0	0	0
core2536-691	544	372.11	689	689	0.00	50.92	1	1	0	0
cov1075	776652	3600.00	18.4166	20	7.92	2.47	0	0	0	0
csched010	302116	3600.23	358.5	431	16.82	456.78	5	8	3	0
danoint	926500	3600.00	63.7541	65.6667	2.91	6.03	0	0	0	0
dfn-gwin-UUM	206169	468.98	38752	38752	0.00	4.31	3	12	9	0
eil33-2	10858	1677.87	934.008	934.008	0.00	16.99	5	37	32	0
eilB101	107914	2497.50	1216.92	1216.92	0.00	34.41	7	41	34	0
enlight13	1	0.01	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.02	1e+20	1e+20	—	0.02	0	0	0	0
ex9	1	65.30	81	81	0.00	65.30	0	0	0	0
glass4	5317569	3603.61	8.00003e+08	2.00002e+09	60.00	1442.46	1	3	2	0
gmu-35-40	3977248	3600.00	-2.40692e+06	-2.40597e+06	0.04	2.29	10	16	6	0
iis-100-0-cov	81999	1586.41	29	29	0.00	3.81	0	0	0	0
iis-bupa-cov	103250	3600.00	32.87	36	8.69	8.13	0	0	0	0
iis-pima-cov	11775	779.04	33	33	0.00	15.64	0	0	0	0
lectsched-4-obj	3862	75.43	4	4	0.00	75.43	0	1	1	0
m100n500k4r1	4636455	3600.00	-25	-24	4.00	144.77	0	0	0	0
macrophage	725079	3600.00	282.292	381	25.91	75.71	0	0	0	0
map18	259	326.46	-847	-847	0.00	302.11	2	2	0	0
map20	319	250.56	-922	-922	0.00	229.89	2	2	0	0
mcsched	31708	417.95	211913	211913	0.00	4.28	7	9	2	0
mik-250-1-100-1	10215258	3600.00	-70054.6	-66729	4.75	0.10	1	10	9	0
mine-166-5	19776	807.09	-5.66396e+08	-5.66396e+08	0.00	9.26	10	44	34	0
mine-90-10	830245	3600.00	-8.16331e+08	-7.84302e+08	3.92	14.50	17	20	3	0
msc98-ip	6299	3600.01	1.97029e+07	2.29354e+07	14.09	659.63	4	5	1	0
mspp16	0	23.07	1e+20	1e+20	—	23.07	0	0	0	0
mzzv11	1980	738.58	-21718	-21718	0.00	357.12	6	10	4	0
n3div36	41530	3600.87	120927	134600	10.16	188.40	5	10	5	0
n4-3	30711	619.60	8993	8993	0.00	36.32	4	7	3	0
neos-1109824	7477	188.07	378	378	0.00	30.05	4	7	3	0
neos-1337307	85212	3600.00	-202948	-202319	0.31	3486.04	0	7	7	0
neos-1396125	239993	3066.98	3000.05	3000.05	0.00	116.00	2	20	18	0
neos13	2369	3600.14	-126.178	-95.4748	24.33	356.10	11	21	10	0
neos-1601936	1610	1065.45	3	3	0.00	1062.52	4	8	4	0
neos18	5125	48.68	16	16	0.00	2.14	0	0	0	0
neos-476283	879	462.59	406.363	406.363	0.00	49.77	0	0	0	0
neos-686190	6016	323.52	6730	6730	0.00	76.72	6	9	3	0
neos-849702	26770	644.35	0	0	0.00	644.00	0	0	0	0
neos-916792	231435	3600.00	26.7262	32.2659	17.17	137.82	10	14	4	0
neos-934278	162	319.17	260	260	0.00	301.99	3	3	0	0
net12	3018	1292.76	214	214	0.00	343.15	2	5	3	0
netdiversion	33	3601.30	232	638	63.64	2901.23	3	4	1	0
newdano	1409382	3600.00	55	65.6667	16.24	35.14	0	0	0	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
noswot	1136575	324.13	-41	-41	0.00	0.36	0	0	0	0
ns1208400	9919	1472.64	2	2	0.00	1473.00	0	0	0	0
ns1688347	4613	684.59	27	27	0.00	138.24	2	6	4	0
ns1758913	2	3602.50	-1454.67	-236.804	83.72	3023.02	0	1	1	0
ns1766074	889424	1013.35	1e+20	1e+20	—	1013.35	0	1	1	0
ns1830653	30444	772.11	20622	20622	0.00	160.64	8	9	1	0
opm2-z7-s2	5361	3035.26	-10280	-10280	0.00	328.75	8	9	1	0
pg5_34	219619	1270.10	-14339.4	-14339.4	0.00	6.37	5	8	3	0
pigeon-10	5043592	3600.00	-10000	-9000	10.00	2.00	0	0	0	0
pw-myciel4	562698	3600.00	6.8	10	32.00	24.77	0	0	0	0
qiu	248033	1794.38	-132.873	-132.873	0.00	39.29	6	24	18	0
rail507	734	86.35	174	174	0.00	59.03	1	1	0	0
ran16x16	284419	359.53	3823	3823	0.00	4.20	3	9	6	0
reblock67	1421354	3600.00	-3.59743e+07	-3.46306e+07	3.73	14.57	16	44	28	0
rmatr100-p10	903	93.14	423	423	0.00	43.67	1	2	1	0
rmatr100-p5	354	113.57	976	976	0.00	37.15	1	2	1	0
rmine6	502938	3600.00	-457.972	-457.043	0.20	8.75	14	15	1	0
rocII-4-11	99917	2068.33	-6.65564	-6.65564	0.00	80.99	4	19	15	0
rococoC10-001000	121371	749.45	11460	11460	0.00	51.91	9	12	3	0
roll3000	627319	3168.11	12890	12890	0.00	317.99	6	7	1	0
satellites1-25	2078	881.20	-5	-5	0.00	881.20	2	7	5	0
sp98ic	73375	3600.01	4.46803e+08	4.5314e+08	1.40	161.48	11	17	6	0
sp98ir	21848	442.98	2.19677e+08	2.19677e+08	0.00	2.18	6	25	19	0
tanglegram1	109	867.35	5182	5182	0.00	101.42	0	0	0	0
tanglegram2	5	7.38	443	443	0.00	4.39	0	0	0	0
timtab1	5090038	3600.00	490942	764772	35.81	16.84	12	25	13	0
triptim1	42	3599.18	22.8681	22.8681	0.00	52.02	1	32	31	0
unitcal_7	25558	3600.01	1.96253e+07	1.96356e+07	0.05	673.59	5	9	4	0
vpphard	1695	3600.26	0	50	—	3362.37	0	0	0	0
zib54-UUE	361364	3600.22	6.94952e+06	1.0334e+07	32.75	11.01	5	7	2	0
AM (# 86)	581782.8	1873.66			6.23	371.44	3.7	8.4	4.8	0.0
GM (# 86)	13361.3	854.28			2.09	57.44	2.5	4.2	2.4	1.0
SGM (# 86)	20116.2	934.93								

Table 10: MIPLIB2010 testset: default settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
30n20b8	2	154.41	302	302	0.00	133.00	0	0	0	0
acc-tight5	413	95.59	0	0	0.00	42.90	0	0	0	0
aflow40b	159639	1400.12	1168	1168	0.00	21.20	0	0	0	0
air04	7	20.26	56137	56137	0.00	3.82	0	0	0	0
app1-2	9561	3600.22	-49.5462	-34	31.38	2189.45	0	0	0	0
ash608gpia-3col	13	34.41	1e+20	1e+20	—	34.41	0	0	0	0
bab5	36338	3600.00	-107333	-106261	1.00	136.89	0	0	0	0
beasleyC3	632700	3600.00	696.77	761	8.44	39.92	0	0	0	0
biella1	6212	363.64	3.06501e+06	3.06501e+06	0.00	7.91	0	0	0	0
bienst2	112679	151.33	54.6	54.6	0.00	2.19	0	0	0	0
binkar10_1	207304	348.90	6742.2	6742.2	0.00	0.62	0	0	0	0
bley_xl1	1	279.33	190	190	0.00	270.83	0	0	0	0
bnatt350	4833	535.18	0	0	0.00	535.00	0	0	0	0
core2536-691	423	157.26	689	689	0.00	8.44	0	0	0	0
cov1075	771229	3600.00	18.4148	20	7.93	2.47	0	0	0	0
csched010	691797	3600.00	392.585	408	3.78	61.73	0	0	0	0
danoint	923979	3600.00	63.7525	65.6667	2.91	6.03	0	0	0	0
dfn-gwin-UUM	51637	91.86	38752	38752	0.00	1.73	0	0	0	0
eil33-2	705	53.48	934.008	934.008	0.00	4.15	0	0	0	0
eilB101	9171	147.39	1216.92	1216.92	0.00	6.40	0	0	0	0
enlight13	1	0.02	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.01	1e+20	1e+20	—	0.01	0	0	0	0
ex9	1	65.57	81	81	0.00	65.60	0	0	0	0
glass4	5608824	3600.02	9.00006e+08	1.55001e+09	41.94	875.28	0	0	0	0
gmu-35-40	5140870	3600.03	-2.40692e+06	-2.40619e+06	0.03	1.00	0	0	0	0
iis-100-0-cov	81999	1577.65	29	29	0.00	3.72	0	0	0	0
iis-bupa-cov	103758	3600.00	32.8746	36	8.68	8.13	0	0	0	0
iis-pima-cov	11775	779.10	33	33	0.00	15.67	0	0	0	0
lectsched-4-obj	4769	85.54	4	4	0.00	69.34	0	0	0	0
m100n500k4r1	4658377	3600.00	-25	-24	4.00	144.76	0	0	0	0
macrophage	725540	3600.00	282.294	381	25.91	75.71	0	0	0	0
map18	259	314.75	-847	-847	0.00	18.05	0	0	0	0
map20	407	297.36	-922	-922	0.00	10.42	0	0	0	0
mcsched	13338	128.94	211913	211913	0.00	1.67	0	0	0	0
mik-250-1-100-1	4488404	1598.66	-66729	-66729	0.00	0.00	0	0	0	0
mine-166-5	1850	41.81	-5.66396e+08	-5.66396e+08	0.00	23.80	0	0	0	0
mine-90-10	156489	862.21	-7.84302e+08	-7.84302e+08	0.00	23.54	0	0	0	0
msc98-ip	4425	1856.37	1.98395e+07	1.98395e+07	0.00	147.10	0	0	0	0
mspp16	0	23.06	1e+20	1e+20	—	23.06	0	0	0	0
mzzv11	1646	502.21	-21718	-21718	0.00	150.56	0	0	0	0
n3div36	86168	3600.00	123357	131000	5.83	51.11	0	0	0	0
n3seq24	25469	3600.01	52000	52200	0.38	364.60	0	0	0	0
n4-3	56405	738.36	8993	8993	0.00	8.50	0	0	0	0
neos-1109824	11639	141.97	378	378	0.00	5.11	0	0	0	0
neos-1337307	140587	3600.00	-202437	-202319	0.06	91.11	0	0	0	0
neos-1396125	101582	425.40	3000.05	3000.05	0.00	5.40	0	0	0	0
neos13	41254	1846.51	-95.4748	-95.4748	0.00	138.04	0	0	0	0
neos-1601936	3615	1873.46	3	3	0.00	1792.63	0	0	0	0
neos18	5125	48.69	16	16	0.00	2.14	0	0	0	0
neos-476283	879	462.62	406.363	406.363	0.00	49.68	0	0	0	0
neos-686190	7319	110.68	6730	6730	0.00	26.01	0	0	0	0
neos-849702	26770	644.73	0	0	0.00	645.00	0	0	0	0
neos-916792	319681	3600.00	28.1256	32.0737	12.31	57.70	0	0	0	0
neos-934278	272	572.89	260	260	0.00	78.84	0	0	0	0
net12	4058	1560.42	214	214	0.00	194.63	0	0	0	0
netdiversion	3	463.89	242	242	0.00	350.98	0	0	0	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
newdano	1407271	3600.00	55	65.6667	16.24	35.17	0	0	0	0
noswot	1136575	324.10	-41	-41	0.00	0.35	0	0	0	0
ns1208400	9919	1469.29	2	2	0.00	1469.00	0	0	0	0
ns1688347	1568	264.22	27	27	0.00	85.89	0	0	0	0
ns1758913	2	3600.17	-1454.67	-376.035	74.15	2815.77	0	0	0	0
ns1766074	921546	1248.86	1e+20	1e+20	—	1248.86	0	0	0	0
ns1830653	29703	392.98	20622	20622	0.00	80.39	0	0	0	0
opm2-z7-s2	2827	713.58	-10280	-10280	0.00	120.64	0	0	0	0
pg5_34	233992	1303.66	-14339.4	-14339.4	0.00	1.56	0	0	0	0
pigeon-10	5054099	3600.00	-10000	-9000	10.00	1.90	0	0	0	0
pw-myciel4	564325	3600.00	6.83333	10	31.67	24.73	0	0	0	0
qiu	14178	85.38	-132.873	-132.873	0.00	16.42	0	0	0	0
rail507	1036	100.33	174	174	0.00	10.33	0	0	0	0
ran16x16	328398	297.91	3823	3823	0.00	0.41	0	0	0	0
reblock67	148401	405.35	-3.46306e+07	-3.46306e+07	0.00	10.72	0	0	0	0
rmatr100-p10	933	80.22	423	423	0.00	4.22	0	0	0	0
rmatr100-p5	391	103.61	976	976	0.00	3.45	0	0	0	0
rmine6	686744	3600.00	-457.459	-457.186	0.06	5.35	0	0	0	0
rocII-4-11	16051	509.39	-6.65564	-6.65564	0.00	104.16	0	0	0	0
rococoC10-001000	155823	821.13	11460	11460	0.00	5.27	0	0	0	0
roll3000	753349	3600.00	12790.7	12890	0.77	6.65	0	0	0	0
satellites1-25	3091	886.37	-5	-5	0.00	533.20	0	0	0	0
sp98ic	212900	3600.00	4.47518e+08	4.51004e+08	0.77	50.96	0	0	0	0
sp98ir	6107	65.04	2.19677e+08	2.19677e+08	0.00	2.54	0	0	0	0
tanglegram1	109	862.33	5182	5182	0.00	100.86	0	0	0	0
tanglegram2	5	7.36	443	443	0.00	4.31	0	0	0	0
timtab1	926793	544.67	764772	764772	0.00	7.92	0	0	0	0
triptim1	1	87.13	22.8681	22.8681	0.00	51.80	0	0	0	0
unitcal_7	18512	1685.75	1.96356e+07	1.96356e+07	0.00	96.59	0	0	0	0
vpphard	1687	3600.02	0	50	—	3364.27	0	0	0	0
zib54-UUE	443700	3600.00	9.39988e+06	1.0334e+07	9.04	9.46	0	0	0	0
AM (# 87)	442899.3	1373.74			3.42	221.86	0.0	0.0	0.0	0.0
GM (# 87)	8702.1	500.27			1.58	26.33	1.0	1.0	1.0	1.0
SGM (# 87)	15495.9	557.12								

Table 11: MIPLIB2010 testset: geom-1024 settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
30n20b8	43	438.51	302	302	0.00	243.03	8	12	4	0
acc-tight5	653	169.34	0	0	0.00	169.34	1	1	0	0
aflow40b	148606	3419.42	1168	1168	0.00	75.61	8	12	4	0
air04	4	74.01	56137	56137	0.00	3.79	5	9	4	0
app1-2	5113	3600.23	-164.137	1e+20	—	3600.23	0	1	1	0
ash608gpia-3col	6	32.06	1e+20	1e+20	—	32.06	0	1	1	0
bab5	22033	3600.01	-106727	-106410	0.30	405.62	9	12	3	0
beasleyC3	442959	3600.00	683.054	758	9.89	28.70	7	10	3	0
biella1	2526	2087.87	3.06501e+06	3.06501e+06	0.00	13.76	22	28	6	0
bienst2	111679	697.32	54.6	54.6	0.00	7.63	9	12	3	0
binkar10_1	159959	1082.99	6742.2	6742.2	0.00	0.99	7	11	4	0
bley_xl1	2	289.43	190	190	0.00	267.97	2	5	3	0
bnatt350	8217	988.83	0	0	0.00	988.83	1	1	0	0
core2536-691	2	334.28	689	689	0.00	8.82	4	8	4	0
cov1075	788312	3600.00	17.2146	20	13.93	1.98	1	3	2	0
csched010	90720	3600.04	358.634	408	12.10	211.24	13	16	3	0
danoint	405063	3600.00	62.705	65.6667	4.51	39.81	7	10	3	0
dfn-gwin-UUM	30301	206.53	38752	38752	0.00	3.54	4	8	4	0
eil33-2	382	208.32	934.008	934.008	0.00	9.03	8	12	4	0
eilB101	4078	319.39	1216.92	1216.92	0.00	6.13	4	8	4	0
enlight13	1	0.02	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.02	1e+20	1e+20	—	0.02	0	0	0	0
ex9	1	65.69	81	81	0.00	65.70	0	0	0	0
glass4	3582571	3602.16	8.00003e+08	1.50001e+09	46.67	815.26	22	25	3	0
gmu-35-40	2985176	3603.19	-2.40692e+06	-2.40603e+06	0.04	1.52	9	12	3	0
iis-100-0-cov	176372	3600.00	17.5309	29	39.55	6.17	3	6	3	0
iis-bupa-cov	1043	3600.00	26.6265	37	28.04	104.17	1	4	3	0
iis-pima-cov	47404	3402.27	33	33	0.00	30.58	2	5	3	0
lectsched-4-obj	1	84.73	4	4	0.00	71.10	6	10	4	0
m100n500k4r1	3787886	3601.42	-25	-24	4.00	146.32	3	6	3	0
macrophage	408215	3600.00	225.746	378	40.28	61.04	11	14	3	0
map18	294	1292.05	-847	-847	0.00	3.80	2	5	3	0
map20	322	1136.63	-922	-922	0.00	3.90	2	5	3	0
mcsched	21402	557.61	211913	211913	0.00	3.58	7	11	4	0
mik-250-1-100-1	1718263	1634.54	-66729	-66729	0.00	0.10	1	5	4	0
mine-166-5	889	902.76	-5.66396e+08	-5.66396e+08	0.00	65.31	53	58	5	0
mine-90-10	126624	2662.04	-7.84302e+08	-7.84302e+08	0.00	47.97	26	31	5	0
msc98-ip	311	3600.00	1.97029e+07	1.98395e+07	0.69	259.31	13	17	4	0
mspp16	0	23.23	1e+20	1e+20	—	23.23	0	0	0	0
mzzv11	2484	811.41	-21718	-21718	0.00	184.49	8	11	3	0
n3div36	30903	3600.00	120927	131400	7.97	100.55	10	13	3	0
n3seq24	20782	3600.00	52000	52200	0.38	153.21	2	6	4	0
n4-3	16864	1073.66	8993	8993	0.00	18.42	9	13	4	0
neos-1109824	4096	352.78	378	378	0.00	13.85	5	9	4	0
neos-1337307	116015	3600.01	-203095	-202319	0.38	39.26	5	9	4	0
neos-1396125	59161	529.70	3000.05	3000.05	0.00	34.44	4	8	4	0
neos13	6	1952.02	-95.4748	-95.4748	0.00	250.98	8	11	3	0
neos-1601936	2	1605.99	3	3	0.00	1431.90	9	13	4	0
neos18	3810	40.39	16	16	0.00	9.97	2	5	3	0
neos-476283	402	1042.63	406.363	406.363	0.00	73.34	5	9	4	0
neos-686190	1491	649.37	6730	6730	0.00	125.25	16	20	4	0
neos-849702	54881	817.56	0	0	0.00	817.56	1	1	0	0
neos-916792	68376	3600.01	26.7499	31.9586	16.30	81.67	9	12	3	0
neos-934278	7023	3579.45	260	260	0.00	59.80	10	14	4	0
net12	3155	1984.55	214	214	0.00	434.84	2	6	4	0
netdiversion	2	984.69	242	242	0.00	639.67	2	7	5	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
newdano	1102043	3600.00	34.1842	68	49.73	128.90	3	5	2	0
noswot	1186573	1230.69	-41	-41	0.00	0.39	6	9	3	0
ns1208400	2	758.08	2	2	0.00	694.00	1	4	3	0
ns1688347	1020	751.23	27	27	0.00	160.25	4	7	3	0
ns1758913	2	3603.53	-1454.67	-911.307	37.35	2937.86	1	3	2	0
ns1766074	891606	1532.05	1e+20	1e+20	—	1532.05	0	1	1	0
ns1830653	15716	978.70	20622	20622	0.00	118.68	4	8	4	0
opm2-z7-s2	354	3600.02	-12595	-10274	18.43	286.09	15	18	3	0
pg5_34	23268	3600.00	-14366.2	-14339.4	0.19	3.17	8	12	4	0
pigeon-10	4657593	3601.58	-10000	-9000	10.00	1.40	1	3	2	0
pw-myciel4	331879	3600.00	4	10	60.00	38.56	6	9	3	0
qiu	14758	438.69	-132.873	-132.873	0.00	95.27	13	17	4	0
rail507	85809	3600.00	172.146	174	1.07	14.25	4	7	3	0
ran16x16	204115	739.94	3823	3823	0.00	0.90	4	8	4	0
reblock67	51209	765.46	-3.46306e+07	-3.46306e+07	0.00	25.15	21	26	5	0
rmatr100-p10	562	464.82	423	423	0.00	12.78	4	8	4	0
rmatr100-p5	366	440.09	976	976	0.00	9.61	2	6	4	0
rmine6	15096	3600.00	-461.364	-457.174	0.91	5.84	15	18	3	0
rocII-4-11	4633	772.70	-6.65564	-6.65564	0.00	93.69	7	10	3	0
rococoC10-001000	99190	1617.74	11460	11460	0.00	7.22	8	12	4	0
roll3000	319451	3600.01	12395.3	12890	3.84	15.83	17	20	3	0
satellites1-25	2463	649.23	-5	-5	0.00	332.40	3	6	3	0
sp98ic	141168	3600.01	4.44772e+08	4.52073e+08	1.62	48.20	6	9	3	0
sp98ir	1548	365.76	2.19677e+08	2.19677e+08	0.00	11.99	31	36	5	0
tanglegram1	66	2288.65	5182	5182	0.00	103.62	2	6	4	0
tanglegram2	4	24.28	443	443	0.00	9.34	2	6	4	0
timtab1	584273	1797.23	764772	764772	0.00	12.22	16	20	4	0
triptim1	2	350.03	22.8681	22.8681	0.00	50.81	2	5	3	0
unitcal_7	20432	3600.01	1.956e+07	1.96356e+07	0.38	141.48	10	13	3	0
vpphard	1243	3600.00	0	5	—	2752.70	10	13	3	0
zib54-UUE	50662	3600.00	7.13872e+06	1.0334e+07	30.92	8.38	7	12	5	0
AM (# 87)	290505.7	1856.46			5.05	252.30	7.1	10.3	3.2	0.0
GM (# 87)	3844.1	938.05			1.82	39.85	4.5	7.5	3.0	1.0
SGM (# 87)	8392.5	1016.80								

Table 12: MIPLIB2010 testset: geom-256 settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
30n20b8	2	269.20	302	302	0.00	189.43	4	8	4	0
acc-tight5	653	169.47	0	0	0.00	169.47	1	1	0	0
aflow40b	131595	3196.06	1168	1168	0.00	63.99	5	10	5	0
air04	4	115.65	56137	56137	0.00	4.55	6	11	5	0
app1-2	5121	3600.24	-164.137	1e+20	—	3600.24	0	1	1	0
ash608gpia-3col	6	32.07	1e+20	1e+20	—	32.07	0	1	1	0
bab5	27046	3600.00	-106712	-106285	0.40	410.93	6	9	3	0
beasleyC3	278140	3600.00	684.743	760	9.90	35.10	9	12	3	0
biella1	2526	1813.36	3.06501e+06	3.06501e+06	0.00	20.14	17	24	7	0
bienst2	111679	623.38	54.6	54.6	0.00	14.66	13	16	3	0
binkar10_1	158131	1290.64	6742.2	6742.2	0.00	1.24	7	12	5	0
bley_xl1	2	472.34	190	190	0.00	308.99	4	8	4	0
bnatt350	8217	988.88	0	0	0.00	988.88	1	1	0	0
core2536-691	120	724.19	689	689	0.00	8.26	5	9	4	0
cov1075	682130	3600.00	17.2085	20	13.96	1.07	2	5	3	0
csched010	4500	3600.05	358.634	409	12.31	204.37	17	20	3	0
danoint	576675	3599.99	62.705	65.6667	4.51	8.71	5	8	3	0
dfn-gwin-UUM	28964	370.07	38752	38752	0.00	5.58	10	15	5	0
eil33-2	382	187.22	934.008	934.008	0.00	9.93	5	9	4	0
eilB101	4078	259.69	1216.92	1216.92	0.00	4.46	3	7	4	0
enlight13	1	0.01	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.02	1e+20	1e+20	—	0.02	0	0	0	0
ex9	1	65.37	81	81	0.00	65.40	0	0	0	0
glass4	1638684	3600.44	8.00003e+08	1.50002e+09	46.67	923.02	12	15	3	0
gmu-35-40	4546805	3604.91	-2.40692e+06	-2.40623e+06	0.03	0.98	8	12	4	0
iis-100-0-cov	204543	3600.00	17.5309	29	39.55	4.92	2	5	3	0
iis-bupa-cov	109554	3600.00	27.1183	39	30.47	281.84	1	4	3	0
iis-pima-cov	44834	2897.56	33	33	0.00	22.97	2	5	3	0
lectsched-4-obj	1	74.81	4	4	0.00	61.12	10	14	4	0
m100n500k4r1	4204602	3601.24	-25	-24	4.00	146.34	3	6	3	0
macrophage	435558	3600.00	225.746	381	40.75	91.44	10	13	3	0
map18	294	1352.12	-847	-847	0.00	3.80	2	5	3	0
map20	322	1112.02	-922	-922	0.00	3.80	2	5	3	0
mcsched	11806	562.97	211913	211913	0.00	4.09	7	12	5	0
mik-250-1-100-1	2127587	2719.78	-66729	-66729	0.00	0.00	1	6	5	0
mine-166-5	889	566.82	-5.66396e+08	-5.66396e+08	0.00	62.68	27	33	6	0
mine-90-10	126624	3433.25	-7.84302e+08	-7.84302e+08	0.00	16.12	18	24	6	0
msc98-ip	18864	3326.91	1.98395e+07	1.98395e+07	0.00	62.14	5	11	6	0
mspp16	0	23.25	1e+20	1e+20	—	23.25	0	0	0	0
mzzv11	606	678.16	-21718	-21718	0.00	184.42	8	11	3	0
n3div36	26951	3600.00	120927	131400	7.97	101.41	8	12	4	0
n3seq24	12343	3600.02	52000	52200	0.38	141.64	2	6	4	0
n4-3	13543	3600.00	7978.2	9003	11.38	9.28	5	21	16	0
neos-1109824	25405	489.90	378	378	0.00	11.14	5	9	4	0
neos-1337307	119344	3600.00	-203095	-202319	0.38	39.65	12	16	4	0
neos-1396125	53510	389.46	3000.05	3000.05	0.00	22.09	4	8	4	0
neos13	6	2749.68	-95.4748	-95.4748	0.00	297.61	17	20	3	0
neos-1601936	2	2529.63	3	3	0.00	1778.99	11	15	4	0
neos18	5859	63.07	16	16	0.00	6.81	6	9	3	0
neos-476283	457	2028.57	406.363	406.363	0.00	88.74	9	13	4	0
neos-686190	2093	701.82	6730	6730	0.00	86.65	11	16	5	0
neos-849702	54881	815.48	0	0	0.00	815.48	1	1	0	0
neos-916792	31313	3600.02	26.6586	31.8704	16.35	65.17	12	16	4	0
neos-934278	4397	3556.15	260	260	0.00	62.53	8	13	5	0
net12	2594	1943.07	214	214	0.00	428.29	3	7	4	0
netdiversion	212	2686.88	242	242	0.00	1622.40	1	5	4	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
newdano	28488	3600.00	34.1842	66.25	48.40	118.20	6	9	3	0
noswot	1780061	3521.83	-41	-41	0.00	0.15	2	5	3	0
ns1208400	1421	838.49	2	2	0.00	694.00	1	4	3	0
ns1688347	586	752.88	27	27	0.00	156.95	5	9	4	0
ns1758913	2	3602.57	-1454.67	-236.804	83.72	3023.10	0	2	2	0
ns1766074	891606	1546.02	1e+20	1e+20	—	1546.02	0	1	1	0
ns1830653	16970	1886.74	20622	20622	0.00	230.27	9	14	5	0
opm2-z7-s2	288	3600.00	-12535.3	-10278	18.01	238.05	28	31	3	0
pg5_34	26672	3600.00	-14366	-14339.4	0.19	3.23	11	16	5	0
pigeon-10	4354915	3602.07	-10000	-9000	10.00	1.60	1	3	2	0
pw-myciel4	394912	3600.01	4	10	60.00	15.13	6	9	3	0
qiu	15068	463.84	-132.873	-132.873	0.00	69.73	12	16	4	0
rail507	75364	3600.00	172.146	174	1.07	16.37	2	6	4	0
ran16x16	204115	848.84	3823	3823	0.00	2.12	9	13	4	0
reblock67	51209	910.81	-3.46306e+07	-3.46306e+07	0.00	19.23	25	31	6	0
rmatr100-p10	562	320.28	423	423	0.00	17.61	3	7	4	0
rmatr100-p5	366	508.57	976	976	0.00	12.20	2	7	5	0
rmine6	204198	3600.00	-461.11	-457.155	0.86	5.77	9	13	4	0
rocII-4-11	4450	1161.64	-6.65564	-6.65564	0.00	180.11	8	11	3	0
rococoC10-001000	80850	2553.96	11460	11460	0.00	19.54	20	25	5	0
roll3000	634942	3600.00	12395.3	12890	3.84	8.86	4	7	3	0
satellites1-25	3625	1020.71	-5	-5	0.00	229.00	4	8	4	0
sp98ic	1757	3600.00	4.44771e+08	4.56069e+08	2.48	84.77	9	12	3	0
sp98ir	1994	308.42	2.19677e+08	2.19677e+08	0.00	8.33	21	27	6	0
tanglegram1	46	3306.30	5182	5182	0.00	116.68	4	9	5	0
tanglegram2	4	24.24	443	443	0.00	5.43	1	6	5	0
timtab1	610297	2398.04	764772	764772	0.00	23.60	25	30	5	0
triptim1	2	405.70	22.8681	22.8681	0.00	50.11	2	6	4	0
unitcal_7	6167	3600.00	1.95558e+07	1.96356e+07	0.41	138.63	11	15	4	0
vpphard	819	3600.01	0	6	—	3134.75	12	15	3	0
zib54-UUE	5076	3600.00	7.13872e+06	1.0334e+07	30.92	33.85	16	22	6	0
AM (# 87)	290129.8	2023.77			5.73	273.82	7.1	10.9	3.7	0.0
GM (# 87)	4189.6	1043.02			1.90	39.67	4.7	7.9	3.4	1.0
SGM (# 87)	8358.2	1128.40								

Table 13: MIPLIB2010 testset: geom-512 settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
30n20b8	2	370.23	302	302	0.00	220.07	6	10	4	0
acc-tight5	653	169.52	0	0	0.00	169.52	1	1	0	0
aflow40b	185538	3385.71	1168	1168	0.00	70.93	8	12	4	0
air04	6	107.82	56137	56137	0.00	4.37	6	11	5	0
app1-2	5121	3601.08	-164.137	1e+20	—	3601.08	0	1	1	0
ash608gpia-3col	6	31.88	1e+20	1e+20	—	31.88	0	1	1	0
bab5	5569	3600.00	-106719	-106410	0.29	404.53	9	12	3	0
beasleyC3	277752	3600.00	683.054	758	9.89	25.72	8	11	3	0
biella1	2526	1661.07	3.06501e+06	3.06501e+06	0.00	15.32	14	20	6	0
bienst2	123059	622.04	54.6	54.6	0.00	7.17	9	12	3	0
binkar10_1	160753	672.33	6742.2	6742.2	0.00	0.79	9	13	4	0
bley_xl1	2	432.54	190	190	0.00	292.42	5	9	4	0
bnatt350	8217	990.80	0	0	0.00	990.80	1	1	0	0
core2536-691	96	277.73	689	689	0.00	8.77	5	9	4	0
cov1075	620482	3600.00	17.2146	20	13.93	3.51	2	5	3	0
csched010	9011	3600.00	358.634	410	12.53	211.32	20	23	3	0
danoint	472024	3600.00	62.705	65.6667	4.51	83.55	4	7	3	0
dfn-gwin-UUM	47260	262.56	38752	38752	0.00	4.73	8	12	4	0
eil33-2	382	223.22	934.008	934.008	0.00	9.77	7	11	4	0
eilB101	4078	374.85	1216.92	1216.92	0.00	9.10	7	11	4	0
enlight13	1	0.01	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.01	1e+20	1e+20	—	0.01	0	0	0	0
ex9	1	65.26	81	81	0.00	65.30	0	0	0	0
glass4	645011	3600.00	8.00004e+08	1.48001e+09	45.95	911.66	16	19	3	0
gmu-35-40	4400229	3604.98	-2.40692e+06	-2.40604e+06	0.04	1.28	14	17	3	0
iis-100-0-cov	189356	3600.00	17.5309	29	39.55	17.42	4	7	3	0
iis-bupa-cov	84621	3600.00	27.1183	36	24.67	30.19	2	5	3	0
iis-pima-cov	38592	2433.65	33	33	0.00	18.18	2	5	3	0
lectsched-4-obj	1	283.61	4	4	0.00	251.27	10	14	4	0
m100n500k4r1	3825892	3601.28	-25	-24	4.00	145.97	2	5	3	0
macrophage	372825	3600.00	225.746	375	39.80	35.60	14	17	3	0
map18	294	1327.69	-847	-847	0.00	3.80	2	5	3	0
map20	322	1156.16	-922	-922	0.00	3.80	2	5	3	0
mcsched	13460	674.84	211913	211913	0.00	2.39	5	10	5	0
mik-250-1-100-1	2334826	1919.22	-66729	-66729	0.00	0.10	1	5	4	0
mine-166-5	889	520.59	-5.66396e+08	-5.66396e+08	0.00	62.21	27	32	5	0
mine-90-10	126624	3283.75	-7.84302e+08	-7.84302e+08	0.00	34.65	23	28	5	0
msc98-ip	11358	5059.41	1.98395e+07	1.98395e+07	0.00	142.06	13	19	6	0
mspp16	0	23.31	1e+20	1e+20	—	23.31	0	0	0	0
mzzv11	978	698.26	-21718	-21718	0.00	184.55	8	11	3	0
n3div36	144134	3600.00	120927	131000	7.69	65.58	4	7	3	0
n3seq24	18700	3600.01	52000	52200	0.38	209.42	1	5	4	0
n4-3	16652	1006.71	8993	8993	0.00	13.90	10	14	4	0
neos-1109824	2438	292.00	378	378	0.00	17.59	8	12	4	0
neos-1337307	137780	3600.01	-203095	-202319	0.38	39.37	5	9	4	0
neos-1396125	51137	425.11	3000.05	3000.05	0.00	14.57	3	7	4	0
neos13	6	2173.57	-95.4748	-95.4748	0.00	220.88	11	14	3	0
neos-1601936	2	1222.44	3	3	0.00	1173.39	9	13	4	0
neos18	11500	77.36	16	16	0.00	8.96	5	8	3	0
neos-476283	10521	3600.02	406.247	406.6	0.09	194.10	1	3	2	0
neos-686190	2123	547.35	6730	6730	0.00	78.16	15	19	4	0
neos-849702	54881	816.24	0	0	0.00	816.24	1	1	0	0
neos-916792	235364	3600.00	26.7228	31.9775	16.43	85.55	8	11	3	0
neos-934278	4652	3575.27	260	260	0.00	52.16	5	9	4	0
net12	2958	2075.96	214	214	0.00	466.70	3	7	4	0
netdiversion	211	3187.33	242	242	0.00	1933.70	1	5	4	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
newdano	1019007	3600.00	34.4666	68.7143	49.84	163.94	3	5	2	0
noswot	1610719	3600.43	-43	-41	4.65	0.19	3	6	3	0
ns1208400	727	804.22	2	2	0.00	694.00	1	4	3	0
ns1688347	727	819.89	27	27	0.00	194.05	7	10	3	0
ns1758913	2	3602.61	-1454.67	-236.804	83.72	3023.11	0	2	2	0
ns1766074	891606	1536.22	1e+20	1e+20	—	1536.22	0	1	1	0
ns1830653	14676	1136.62	20622	20622	0.00	139.08	5	9	4	0
opm2-z7-s2	3	3600.00	-11660.7	-10269	11.94	173.56	17	20	3	0
pg5_34	98423	3600.00	-14366.2	-14339.4	0.19	3.13	7	11	4	0
pigeon-10	4337577	3602.30	-10000	-9000	10.00	1.60	1	3	2	0
pw-myciel4	403315	3600.01	4	10	60.00	31.75	6	9	3	0
qiu	15068	485.83	-132.873	-132.873	0.00	88.44	15	19	4	0
rail507	81429	3600.00	172.146	174	1.07	13.71	5	8	3	0
ran16x16	204115	844.41	3823	3823	0.00	1.73	5	9	4	0
reblock67	51209	1009.85	-3.46306e+07	-3.46306e+07	0.00	33.29	11	16	5	0
rmatr100-p10	562	366.06	423	423	0.00	11.59	3	7	4	0
rmatr100-p5	366	421.94	976	976	0.00	10.74	2	6	4	0
rmine6	91703	3600.01	-461.364	-457.173	0.91	4.87	8	12	4	0
rocII-4-11	4699	900.71	-6.65564	-6.65564	0.00	106.11	8	11	3	0
rococoC10-001000	174503	1943.40	11460	11460	0.00	14.69	18	22	4	0
roll3000	294673	3600.00	12395.3	12890	3.84	14.00	16	19	3	0
satellites1-25	15025	2855.38	-5	-5	0.00	282.00	2	6	4	0
sp98ic	142353	3600.00	4.44772e+08	4.50886e+08	1.36	52.14	6	9	3	0
sp98ir	1670	392.92	2.19677e+08	2.19677e+08	0.00	9.67	29	35	6	0
tanglegram1	85	3600.02	77	5182	98.51	121.90	4	8	4	0
tanglegram2	4	29.02	443	443	0.00	10.24	3	7	4	0
timtab1	535038	2319.03	764772	764772	0.00	13.75	24	29	5	0
triptim1	2	445.57	22.8681	22.8681	0.00	51.91	2	6	4	0
unitcal_7	12991	3600.01	1.95595e+07	1.96356e+07	0.39	141.70	9	13	4	0
vpphard	182	3600.01	0	13	—	3371.62	6	9	3	0
zib54-UUE	127624	3600.00	7.13872e+06	1.0334e+07	30.92	53.71	23	28	5	0
AM (# 87)	284953.6	1999.44			6.64	274.24	7.0	10.3	3.3	0.0
GM (# 87)	4813.4	1029.48			1.95	41.82	4.6	7.6	3.1	1.0
SGM (# 87)	9895.4	1112.03								

Table 14: MIPLIB2010 testset: geom-64 settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
30n20b8	2	300.03	302	302	0.00	193.58	5	10	5	0
acc-tight5	653	169.36	0	0	0.00	169.36	1	1	0	0
aflow40b	105712	3600.00	1105.43	1168	5.36	60.46	3	9	6	0
air04	4	65.98	56137	56137	0.00	3.93	2	8	6	0
app1-2	5113	3600.25	-164.137	1e+20	—	3600.25	0	1	1	0
ash608gpia-3col	6	32.00	1e+20	1e+20	—	32.00	0	1	1	0
bab5	24731	3600.00	-107176	-106095	1.01	422.36	8	12	4	0
beasleyC3	401618	3600.00	683.054	761	10.24	43.96	6	9	3	0
biella1	2526	1524.83	3.06501e+06	3.06501e+06	0.00	16.22	13	21	8	0
bienst2	106950	731.86	54.6	54.6	0.00	6.82	6	10	4	0
binkar10_1	155242	894.95	6742.2	6742.2	0.00	0.83	10	16	6	0
bley_xl1	2	297.68	190	190	0.00	268.23	2	7	5	0
bnatt350	8217	988.88	0	0	0.00	988.88	1	1	0	0
core2536-691	148	395.32	689	689	0.00	8.10	5	10	5	0
cov1075	722892	3600.00	17.2085	20	13.96	4.42	3	6	3	0
csched010	208493	3600.00	358.642	417	13.99	197.68	7	10	3	0
danoint	94442	3600.00	62.705	65.6667	4.51	10.33	6	10	4	0
dfn-gwin-UUM	30167	488.99	38752	38752	0.00	8.53	2	8	6	0
eil33-2	382	209.84	934.008	934.008	0.00	16.29	4	9	5	0
eilB101	4078	328.13	1216.92	1216.92	0.00	7.57	4	9	5	0
enlight13	1	0.02	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.01	1e+20	1e+20	—	0.01	0	0	0	0
ex9	1	65.20	81	81	0.00	65.20	0	0	0	0
glass4	21487	2313.53	1.20001e+09	1.20001e+09	0.00	524.21	17	26	9	0
gmu-35-40	4408316	3603.85	-2.40692e+06	-2.40562e+06	0.05	1.88	7	11	4	0
iis-100-0-cov	32730	3600.00	17.5309	29	39.55	7.96	4	8	4	0
iis-bupa-cov	95226	3600.00	27.1183	36	24.67	16.08	3	6	3	0
iis-pima-cov	38592	3474.29	33	33	0.00	20.64	2	6	4	0
lectsched-4-obj	507299	3600.00	4	24	83.33	3014.65	4	7	3	0
m100n500k4r1	3623716	3601.53	-25	-24	4.00	145.92	3	6	3	0
macrophage	412091	3600.00	225.746	376	39.96	78.20	17	21	4	0
map18	294	1654.95	-847	-847	0.00	3.80	2	6	4	0
map20	322	1352.03	-922	-922	0.00	3.80	2	6	4	0
mcsched	21402	706.18	211913	211913	0.00	2.98	8	14	6	0
mik-250-1-100-1	1718263	2913.74	-66729	-66729	0.00	0.15	1	7	6	0
mine-166-5	889	387.26	-5.66396e+08	-5.66396e+08	0.00	49.82	18	25	7	0
mine-90-10	126624	3168.73	-7.84302e+08	-7.84302e+08	0.00	7.96	11	18	7	0
msc98-ip	6462	4301.89	1.98395e+07	1.98395e+07	0.00	56.65	10	18	8	0
mspp16	0	23.38	1e+20	1e+20	—	23.38	0	0	0	0
mzzv11	978	811.13	-21718	-21718	0.00	183.54	8	12	4	0
n3div36	136658	3600.00	120927	131000	7.69	66.62	4	8	4	0
n3seq24	14408	3600.01	52000	52200	0.38	210.65	1	6	5	0
n4-3	16668	1550.55	8993	8993	0.00	14.89	10	16	6	0
neos-1109824	16509	437.27	378	378	0.00	15.97	5	10	5	0
neos-1337307	142271	3600.01	-203095	-202319	0.38	39.40	5	10	5	0
neos-1396125	61901	525.43	3000.05	3000.05	0.00	18.41	5	10	5	0
neos13	6	3579.47	-95.4748	-95.4748	0.00	531.23	27	31	4	0
neos-1601936	2	2076.83	3	3	0.00	1318.92	6	11	5	0
neos18	7477	100.97	16	16	0.00	11.60	6	10	4	0
neos-476283	450	1447.87	406.363	406.363	0.00	93.69	7	12	5	0
neos-686190	1770	648.30	6730	6730	0.00	81.43	10	16	6	0
neos-849702	54881	816.32	0	0	0.00	816.32	1	1	0	0
neos-916792	101950	3600.00	26.6586	31.8704	16.35	45.34	13	17	4	0
neos-934278	2	492.49	260	260	0.00	52.74	7	13	6	0
net12	2308	2601.18	214	214	0.00	474.23	3	8	5	0
netdiversion	212	2690.68	242	242	0.00	1624.71	1	6	5	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
newdano	897904	3600.00	34.4666	66.6364	48.28	73.60	6	9	3	0
noswot	791956	687.61	-41	-41	0.00	0.14	4	7	3	0
ns1208400	727	839.81	2	2	0.00	695.00	1	5	4	0
ns1688347	995	443.12	27	27	0.00	137.23	2	6	4	0
ns1758913	2	3602.50	-1454.67	-236.804	83.72	3023.05	0	2	2	0
ns1766074	891606	1541.20	1e+20	1e+20	—	1541.20	0	1	1	0
ns1830653	15820	1190.27	20622	20622	0.00	86.02	5	11	6	0
opm2-z7-s2	4	3600.04	-12535.3	-10280	17.99	152.33	15	19	4	0
pg5_34	48571	3600.00	-14366	-14339.4	0.19	4.11	5	10	5	0
pigeon-10	3743788	3601.66	-10000	-9000	10.00	1.60	1	3	2	0
pw-myciel4	158521	3600.00	4	10	60.00	20.24	4	8	4	0
qiu	14758	429.80	-132.873	-132.873	0.00	59.92	18	23	5	0
rail507	85590	3600.01	172.146	174	1.07	17.29	5	9	4	0
ran16x16	204115	921.49	3823	3823	0.00	1.56	6	11	5	0
reblock67	51209	1069.57	-3.46306e+07	-3.46306e+07	0.00	24.24	24	31	7	0
rmatr100-p10	562	436.10	423	423	0.00	8.36	5	10	5	0
rmatr100-p5	366	598.48	976	976	0.00	10.00	4	10	6	0
rmine6	16040	3600.00	-461.11	-457.158	0.86	5.60	9	14	5	0
rocII-4-11	6609	1142.18	-6.65564	-6.65564	0.00	195.57	8	12	4	0
rococoC10-001000	103986	2725.63	11460	11460	0.00	22.42	19	25	6	0
roll3000	96140	3600.00	12395.3	12890	3.84	10.89	12	17	5	0
satellites1-25	2539	1237.41	-5	-5	0.00	194.00	3	8	5	0
sp98ic	18858	3600.00	4.44548e+08	4.49468e+08	1.09	53.80	10	14	4	0
sp98ir	1594	325.79	2.19677e+08	2.19677e+08	0.00	4.12	15	23	8	0
tanglegram1	20	3600.01	77	5182	98.51	125.85	5	10	5	0
tanglegram2	4	35.38	443	443	0.00	10.40	3	9	6	0
timtab1	583033	2364.93	764772	764772	0.00	23.78	19	25	6	0
triptim1	2	445.59	22.8681	22.8681	0.00	49.71	2	7	5	0
unitcal_7	2896	3600.01	1.95558e+07	1.96356e+07	0.41	143.44	11	16	5	0
vpphard	104	3600.01	0	36	—	3432.98	5	8	3	0
zib54-UUE	291529	3600.00	7.13872e+06	1.0334e+07	30.92	33.77	16	22	6	0
AM (# 87)	246820.6	1984.04			7.15	297.08	6.4	10.7	4.3	0.0
GM (# 87)	4190.8	1040.14			1.97	39.66	4.4	8.1	3.9	1.0
SGM (# 87)	8680.9	1122.85								

Table 15: MIPLIB2010 testset: geom-8 settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
30n20b8	6	351.70	302	302	0.00	188.47	5	14	9	0
acc-tight5	653	169.73	0	0	0.00	169.73	1	1	0	0
aflow40b	121919	3600.00	1108.39	1168	5.10	69.27	6	14	8	0
air04	4	114.78	56137	56137	0.00	3.75	4	15	11	0
app1-2	5099	3600.23	-164.137	1e+20	—	3600.23	0	1	1	0
ash608gpia-3col	6	31.83	1e+20	1e+20	—	31.83	0	1	1	0
bab5	16529	3600.00	-106972	-105826	1.07	411.80	6	12	6	0
beasleyC3	489449	3600.00	683.054	756	9.65	18.33	7	12	5	0
biella1	2468	2810.53	3.06501e+06	3.06501e+06	0.00	12.19	19	34	15	0
bienst2	123059	1425.81	54.6	54.6	0.00	15.55	8	15	7	0
binkar10_1	159959	1462.21	6742.2	6742.2	0.00	0.91	8	18	10	0
bley_xl1	2	335.42	190	190	0.00	270.09	4	12	8	0
bnatt350	8217	990.50	0	0	0.00	990.50	1	1	0	0
core2536-691	286	427.01	689	689	0.00	8.05	5	14	9	0
cov1075	679545	3600.14	17.2082	20	13.96	3.54	2	7	5	0
csched010	208450	3600.04	358.642	417	13.99	241.58	9	14	5	0
danoint	573979	3600.00	62.705	65.6667	4.51	57.24	5	10	5	0
dfn-gwin-UUM	47260	655.72	38752	38752	0.00	4.70	5	15	10	0
eil33-2	382	345.64	934.008	934.008	0.00	16.98	6	15	9	0
eilB101	4078	492.18	1216.92	1216.92	0.00	6.98	4	13	9	0
enlight13	1	0.02	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.02	1e+20	1e+20	—	0.02	0	0	0	0
ex9	1	65.25	81	81	0.00	65.20	0	0	0	0
glass4	5443206	3605.34	8.00003e+08	1.90002e+09	57.89	1359.79	5	11	6	0
gmu-35-40	15776	3600.00	-2.40692e+06	-2.40634e+06	0.02	3.56	6	13	7	0
iis-100-0-cov	18028	3600.00	17.5309	29	39.55	16.41	4	10	6	0
iis-bupa-cov	92074	3600.01	27.1183	36	24.67	16.81	2	7	5	0
iis-pima-cov	9535	3600.00	27.0874	33	17.92	19.26	2	9	7	0
lectsched-4-obj	508310	3600.00	4	24	83.33	3014.68	4	9	5	0
m100n500k4r1	3282965	3601.05	-25	-24	4.00	145.99	2	7	5	0
macrophage	126086	3600.00	225.746	377	40.12	85.84	16	22	6	0
map18	294	2252.27	-847	-847	0.00	3.80	2	8	6	0
map20	322	1880.62	-922	-922	0.00	3.80	2	8	6	0
mcsched	21402	1072.24	211913	211913	0.00	4.08	9	20	11	0
mik-250-1-100-1	1440068	3600.12	-70077.8	-66729	4.78	0.15	1	10	9	0
mine-166-5	889	485.98	-5.66396e+08	-5.66396e+08	0.00	50.17	18	31	13	0
mine-90-10	18548	3600.00	-7.98557e+08	-7.84302e+08	1.79	8.11	13	26	13	0
msc98-ip	11592	5292.19	1.98395e+07	1.98395e+07	0.00	35.13	8	22	14	0
mspp16	0	23.16	1e+20	1e+20	—	23.16	0	0	0	0
mzzv11	900	926.83	-21718	-21718	0.00	183.44	8	14	6	0
n3div36	47413	3600.00	120927	132800	8.94	198.21	3	10	7	0
n3seq24	13306	3600.00	52000	52200	0.38	204.37	3	12	9	0
n4-3	17367	2082.23	8993	8993	0.00	10.58	10	20	10	0
neos-1109824	10584	470.88	378	378	0.00	17.01	6	14	8	0
neos-1337307	108032	3600.01	-203095	-202319	0.38	39.33	5	14	9	0
neos-1396125	61901	702.90	3000.05	3000.05	0.00	18.75	5	14	9	0
neos13	6	3135.68	-95.4748	-95.4748	0.00	537.67	16	23	7	0
neos-1601936	2	1445.77	3	3	0.00	557.97	11	20	9	0
neos18	6822	154.39	16	16	0.00	10.90	5	12	7	0
neos-476283	516	3408.55	406.363	406.363	0.00	114.78	11	20	9	0
neos-686190	1750	669.91	6730	6730	0.00	89.39	8	18	10	0
neos-849702	54881	817.39	0	0	0.00	817.39	1	1	0	0
neos-916792	111589	3600.00	26.7257	32.4216	17.57	154.73	7	12	5	0
neos-934278	582	3581.21	260	260	0.00	55.55	9	19	10	0
net12	10	3600.00	91.6402	214	57.18	473.57	3	12	9	0
netdiversion	212	2691.93	242	242	0.00	1626.26	1	10	9	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
newdano	1133344	3600.00	34.4666	69.6	50.48	211.38	2	6	4	0
noswot	980629	755.09	-41	-41	0.00	0.11	3	8	5	0
ns1208400	1422	948.59	2	2	0.00	694.00	1	7	6	0
ns1688347	995	616.49	27	27	0.00	154.60	2	9	7	0
ns1758913	2	3607.12	-1454.67	-236.804	83.72	3026.91	0	3	3	0
ns1766074	891606	1541.01	1e+20	1e+20	—	1541.01	0	1	1	0
ns1830653	14676	1784.52	20622	20622	0.00	118.66	4	14	10	0
opm2-z7-s2	14	3600.00	-12706.7	-10278	19.11	165.68	14	20	6	0
pg5_34	92655	3599.99	-14366.2	-14339.4	0.19	3.80	5	13	8	0
pigeon-10	3685512	3602.72	-10000	-9000	10.00	1.21	2	5	3	0
pw-myciel4	132272	3600.01	4	10	60.00	15.86	3	9	6	0
qiu	14748	584.97	-132.873	-132.873	0.00	69.51	19	27	8	0
rail507	88622	3600.00	172.146	174	1.07	11.94	5	12	7	0
ran16x16	204115	1369.75	3823	3823	0.00	1.56	4	13	9	0
reblock67	51209	1351.54	-3.46306e+07	-3.46306e+07	0.00	15.17	16	29	13	0
rmatr100-p10	562	643.06	423	423	0.00	9.23	5	14	9	0
rmatr100-p5	366	863.84	976	976	0.00	10.05	4	14	10	0
rmine6	8407	3600.00	-461.11	-457.155	0.86	4.72	8	16	8	0
rocII-4-11	6587	1281.33	-6.65564	-6.65564	0.00	255.88	6	12	6	0
rococoC10-001000	90284	3064.01	11460	11460	0.00	5.90	10	20	10	0
roll3000	257486	3600.00	12395.3	12890	3.84	11.34	8	15	7	0
satellites1-25	2830	1435.06	-5	-5	0.00	448.60	5	13	8	0
sp98ic	12496	3600.00	4.44548e+08	4.49468e+08	1.09	51.45	11	17	6	0
sp98ir	1594	393.12	2.19677e+08	2.19677e+08	0.00	5.01	10	25	15	0
tanglegram1	318	3600.00	77	5182	98.51	139.36	3	11	8	0
tanglegram2	4	42.10	443	443	0.00	8.48	2	12	10	0
timtab1	535064	2866.50	764772	764772	0.00	11.14	12	23	11	0
triptim1	2	568.12	22.8681	22.8681	0.00	50.42	2	10	8	0
unitcal_7	16845	3600.01	1.956e+07	1.96356e+07	0.38	140.89	8	16	8	0
vpphard	703	3600.01	0	8	—	3210.93	6	11	5	0
zib54-UUE	104507	3600.00	7.13872e+06	1.0334e+07	30.92	7.54	4	15	11	0
AM (# 87)	255128.7	2189.64			8.82	304.49	5.7	12.8	7.1	0.0
GM (# 87)	4467.5	1220.61			2.30	40.68	4.2	9.8	6.0	1.0
SGM (# 87)	8637.7	1313.55								

Table 16: MIPLIB2010 testset: geom-heur-64 settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
30n20b8	2	153.83	302	302	0.00	133.00	0	0	0	0
acc-tight5	537	143.15	0	0	0.00	42.90	1	4	3	0
aflow40b	130830	1346.46	1168	1168	0.00	24.29	4	8	4	0
air04	7	20.33	56137	56137	0.00	3.92	0	0	0	0
app1-2	4108	3600.08	-50.1048	-37	26.15	2137.81	1	3	2	0
ash608gpia-3col	13	34.05	1e+20	1e+20	—	34.05	0	0	0	0
bab5	29515	3600.00	-107353	-106206	1.07	139.20	2	10	8	0
beasleyC3	463869	3600.00	697.69	759	8.08	30.79	3	13	10	0
biella1	10957	848.13	3.06501e+06	3.06501e+06	0.00	7.95	3	12	9	0
bienst2	204062	518.69	54.6	54.6	0.00	12.37	0	24	24	0
binkar10_1	219120	393.02	6742.2	6742.2	0.00	0.55	3	15	12	0
bley_xl1	1	279.73	190	190	0.00	271.52	0	0	0	0
bnatt350	4833	536.02	0	0	0.00	536.00	0	0	0	0
core2536-691	423	156.91	689	689	0.00	8.43	0	0	0	0
cov1075	757618	3600.00	18.4044	20	7.98	5.69	0	6	6	0
csched010	618784	3600.00	390.154	408	4.37	66.91	1	15	14	0
danoint	891706	3600.00	63.7674	65.6667	2.89	8.56	2	4	2	0
dfn-gwin-UUM	53579	103.00	38752	38752	0.00	1.73	0	3	3	0
eil33-2	986	78.80	934.008	934.008	0.00	4.12	0	3	3	0
eilB101	7721	166.06	1216.92	1216.92	0.00	5.35	2	4	2	0
enlight13	1	0.02	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.02	1e+20	1e+20	—	0.02	0	0	0	0
ex9	1	65.57	81	81	0.00	65.60	0	0	0	0
glass4	1387075	1204.16	1.20001e+09	1.20001e+09	0.00	263.33	11	83	72	0
gmu-35-40	4929951	3600.03	-2.40691e+06	-2.40592e+06	0.04	1.99	2	16	14	0
iis-100-0-cov	88614	1742.71	29	29	0.00	4.26	1	4	3	0
iis-bupa-cov	101254	3600.00	32.7782	36	8.95	8.13	0	3	3	0
iis-pima-cov	11696	847.33	33	33	0.00	15.44	1	3	2	0
lectsched-4-obj	10034	175.86	4	4	0.00	123.73	4	11	7	0
m100n500k4r1	4577218	3600.00	-25	-24	4.00	145.26	0	7	7	0
macrophage	529023	3600.00	281.292	380	25.98	87.84	2	16	14	0
map18	435	549.13	-847	-847	0.00	18.05	0	3	3	0
map20	589	500.11	-922	-922	0.00	10.49	0	3	3	0
mcsched	34846	415.95	211913	211913	0.00	1.93	2	17	15	0
mik-250-1-100-1	4495441	1608.76	-66729	-66729	0.00	0.00	0	3	3	0
mine-166-5	2287	57.52	-5.66396e+08	-5.66396e+08	0.00	23.80	0	8	8	0
mine-90-10	71688	437.55	-7.84302e+08	-7.84302e+08	0.00	17.11	1	24	23	0
msc98-ip	5865	1848.60	1.98395e+07	1.98395e+07	0.00	112.65	1	9	8	0
mspp16	0	23.07	1e+20	1e+20	—	23.07	0	0	0	0
mzzv11	1907	711.36	-21718	-21718	0.00	150.56	0	6	6	0
n3div36	66852	3600.00	122493	131000	6.49	106.48	2	23	21	0
n3seq24	25140	3600.00	52000	52200	0.38	364.24	0	5	5	0
n4-3	69332	1036.36	8993	8993	0.00	9.28	2	10	8	0
neos-1109824	12333	164.10	378	378	0.00	5.11	0	3	3	0
neos-1337307	120440	3600.00	-202457	-202319	0.07	91.11	0	10	10	0
neos-1396125	133535	1316.37	3000.05	3000.05	0.00	5.40	0	23	23	0
neos13	1516	3600.00	-126.178	-85.0352	32.61	653.24	24	27	3	0
neos-1601936	10008	3600.01	3	5	40.00	2343.05	4	8	4	0
neos18	8133	71.02	16	16	0.00	2.14	0	3	3	0
neos-476283	879	463.89	406.363	406.363	0.00	49.81	0	1	1	0
neos-686190	16263	401.75	6730	6730	0.00	57.11	2	12	10	0
neos-849702	26770	645.76	0	0	0.00	646.00	0	0	0	0
neos-916792	328885	3600.01	28.2925	32.063	11.76	43.77	6	11	5	0
neos-934278	272	617.96	260	260	0.00	83.57	0	0	0	0
net12	4665	1751.68	214	214	0.00	187.08	1	4	3	0
netdiversion	3	463.26	242	242	0.00	349.98	0	0	0	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
newdano	1371124	3600.00	54	65.6667	17.77	44.81	0	25	25	0
noswot	1018454	295.12	-41	-41	0.00	0.11	1	3	2	0
ns1208400	9919	1469.63	2	2	0.00	1470.00	0	0	0	0
ns1688347	3217	361.09	27	27	0.00	98.95	1	7	6	0
ns1758913	2	3600.17	-1454.67	-376.035	74.15	2815.77	0	0	0	0
ns1766074	921546	1260.82	1e+20	1e+20	—	1260.82	0	0	0	0
ns1830653	30610	504.26	20622	20622	0.00	94.63	4	8	4	0
opm2-z7-s2	5017	3014.57	-10280	-10280	0.00	127.10	9	16	7	0
pg5_34	220784	1748.17	-14339.4	-14339.4	0.00	1.45	2	18	16	0
pigeon-10	5121069	3600.00	-10000	-9000	10.00	2.40	1	13	12	0
pw-myciel4	425346	3600.00	7	10	30.00	25.79	1	4	3	0
qiu	26785	214.31	-132.873	-132.873	0.00	29.71	1	5	4	0
rail507	1433	182.50	174	174	0.00	10.90	1	4	3	0
ran16x16	356491	361.16	3823	3823	0.00	0.50	0	8	8	0
reblock67	111976	398.20	-3.46306e+07	-3.46306e+07	0.00	12.67	5	21	16	0
rmatr100-p10	1483	148.38	423	423	0.00	4.29	0	3	3	0
rmatr100-p5	729	234.25	976	976	0.00	3.47	0	3	3	0
rmine6	615649	3600.01	-457.549	-457.186	0.08	5.18	1	20	19	0
rocII-4-11	15354	571.14	-6.65564	-6.65564	0.00	89.28	1	8	7	0
rococoC10-001000	316865	2277.99	11460	11460	0.00	8.25	6	38	32	0
roll3000	671221	3600.00	12757	12890	1.03	10.14	5	28	23	0
satellites1-25	5015	1025.75	-5	-5	0.00	577.00	1	4	3	0
sp98ic	127611	3600.00	4.47215e+08	4.5291e+08	1.26	60.16	5	18	13	0
sp98ir	7985	158.13	2.19677e+08	2.19677e+08	0.00	2.97	4	11	7	0
tanglegram1	109	866.01	5182	5182	0.00	101.40	0	0	0	0
tanglegram2	5	7.36	443	443	0.00	4.31	0	0	0	0
timtab1	1172817	920.78	764772	764772	0.00	19.61	9	89	80	0
triptim1	1	85.41	22.8681	22.8681	0.00	50.00	0	0	0	0
unitcal_7	26238	2912.75	1.96356e+07	1.96356e+07	0.00	97.47	4	16	12	0
vpphard	2124	3600.78	0	36	—	3342.52	1	3	2	0
zib54-UUE	445771	3600.04	9.42791e+06	1.0334e+07	8.77	9.16	1	6	5	0
AM (# 87)	385107.7	1504.79			3.72	229.05	1.7	9.9	8.1	0.0
GM (# 87)	9126.8	619.37			1.65	29.32	1.5	5.2	4.4	1.0
SGM (# 87)	16158.0	683.18								

Table 17: MIPLIB2010 testset: geom-heur-infer settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
30n20b8	2	154.24	302	302	0.00	133.00	0	0	0	0
acc-tight5	1736	179.72	0	0	0.00	42.80	0	8	8	0
aflow40b	194453	1842.98	1168	1168	0.00	27.85	1	46	45	0
air04	7	20.37	56137	56137	0.00	3.83	0	0	0	0
app1-2	9135	3600.03	-50.4056	-34	32.55	2185.27	0	9	9	0
ash608gpia-3col	13	34.27	1e+20	1e+20	—	34.27	0	0	0	0
bab5	38761	3600.00	-107336	-106212	1.05	137.50	0	27	27	0
beasleyC3	490872	3600.01	698.004	761	8.28	52.02	3	51	48	0
biella1	37606	1607.49	3.06501e+06	3.06501e+06	0.00	8.01	1	112	111	0
bienst2	182344	438.62	54.6	54.6	0.00	8.64	0	71	71	0
binkar10_1	291555	533.43	6742.2	6742.2	0.00	0.66	3	98	95	0
bley_xl1	1	277.65	190	190	0.00	268.93	0	0	0	0
bnatt350	4833	536.49	0	0	0.00	536.00	0	0	0	0
core2536-691	423	156.81	689	689	0.00	8.43	0	0	0	0
cov1075	777395	3600.00	18.4404	20	7.80	2.03	1	11	10	0
csched010	418140	3600.01	377.491	408	7.48	125.21	2	136	134	0
danoint	908442	3600.00	63.7306	65.6667	2.95	11.70	0	36	36	0
dfn-gwin-UUM	52120	95.75	38752	38752	0.00	1.74	0	11	11	0
eil33-2	3021	61.74	934.008	934.008	0.00	4.15	0	8	8	0
eilB101	32071	300.61	1216.92	1216.92	0.00	7.74	2	45	43	0
enlight13	1	0.02	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.03	1e+20	1e+20	—	0.03	0	0	0	0
ex9	1	65.44	81	81	0.00	65.40	0	0	0	0
glass4	4450230	3600.01	8.32438e+08	1.50001e+09	44.50	902.73	12	244	232	0
gmu-35-40	5459575	3600.02	-2.4069e+06	-2.4066e+06	0.01	0.75	9	228	219	0
iis-100-0-cov	90117	1769.22	29	29	0.00	4.66	0	17	17	0
iis-bupa-cov	101061	3600.00	32.7762	36	8.96	8.14	0	10	10	0
iis-pima-cov	23312	1408.95	33	33	0.00	21.73	0	21	21	0
lectsched-4-obj	9626	142.34	4	4	0.00	104.96	5	30	25	0
m100n500k4r1	4592101	3600.00	-25	-24	4.00	145.31	0	22	22	0
macrophage	664946	3600.00	281.708	381	26.06	97.10	2	50	48	0
map18	546	575.13	-847	-847	0.00	18.10	0	10	10	0
map20	618	452.40	-922	-922	0.00	10.39	0	10	10	0
mcsched	45666	479.87	211913	211913	0.00	1.99	0	92	92	0
mik-250-1-100-1	4488541	1603.70	-66729	-66729	0.00	0.00	0	11	11	0
mine-166-5	3185	54.34	-5.66396e+08	-5.66396e+08	0.00	23.86	0	16	16	0
mine-90-10	555099	2651.29	-7.84302e+08	-7.84302e+08	0.00	24.24	4	148	144	0
msc98-ip	5699	1836.80	1.98395e+07	1.98395e+07	0.00	109.42	1	40	39	0
mspp16	0	23.04	1e+20	1e+20	—	23.04	0	0	0	0
mzzv11	4825	758.66	-21718	-21718	0.00	150.60	0	18	18	0
n3div36	97455	3600.00	123211	131000	5.95	72.40	0	83	83	0
n3seq24	25193	3600.02	52000	52200	0.38	364.07	0	5	5	0
n4-3	72444	1030.30	8993	8993	0.00	10.73	0	31	31	0
neos-1109824	12212	147.81	378	378	0.00	5.11	0	11	11	0
neos-1337307	112011	3600.00	-202460	-202319	0.07	91.41	0	39	39	0
neos-1396125	142476	990.64	3000.05	3000.05	0.00	5.40	0	51	51	0
neos13	50059	3600.00	-97.9752	-95.4748	2.55	267.51	1	65	64	0
neos-1601936	14519	3600.00	3	5	40.00	2734.44	4	24	20	0
neos18	8053	64.99	16	16	0.00	2.14	0	11	11	0
neos-476283	879	463.19	406.363	406.363	0.00	49.71	0	1	1	0
neos-686190	31460	457.12	6730	6730	0.00	48.91	0	81	81	0
neos-849702	26770	643.77	0	0	0.00	644.00	0	0	0	0
neos-916792	305683	3600.00	28.0792	32.0127	12.29	45.01	1	30	29	0
neos-934278	272	590.62	260	260	0.00	81.26	0	0	0	0
net12	4013	1634.37	214	214	0.00	183.06	1	12	11	0
netdiversion	3	462.17	242	242	0.00	349.98	0	0	0	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
newdano	810762	3600.00	48	66.4545	27.77	56.93	0	70	70	0
noswot	1207071	360.95	-41	-41	0.00	0.44	0	34	34	0
ns1208400	9919	1471.73	2	2	0.00	1472.00	0	0	0	0
ns1688347	3611	397.23	27	27	0.00	90.40	0	31	31	0
ns1758913	2	3600.15	-1454.67	-376.035	74.15	2815.85	0	0	0	0
ns1766074	921546	1255.36	1e+20	1e+20	—	1255.36	0	0	0	0
ns1830653	37825	604.53	20622	20622	0.00	79.00	1	32	31	0
opm2-z7-s2	4664	1697.43	-10280	-10280	0.00	117.37	1	28	27	0
pg5_34	458234	2797.51	-14339.4	-14339.4	0.00	2.01	1	109	108	0
pigeon-10	5028352	3600.00	-10000	-9000	10.00	2.80	5	56	51	0
pw-myciel4	525653	3600.00	6.4	10	36.00	37.50	0	20	20	0
qiu	24655	172.94	-132.873	-132.873	0.00	32.23	1	11	10	0
rail507	11707	316.31	174	174	0.00	10.86	0	29	29	0
ran16x16	347875	331.47	3823	3823	0.00	0.44	0	21	21	0
reblock67	172829	536.35	-3.46306e+07	-3.46306e+07	0.00	11.98	1	138	137	0
rmatr100-p10	1750	170.65	423	423	0.00	4.29	0	10	10	0
rmatr100-p5	589	130.10	976	976	0.00	3.55	0	7	7	0
rmine6	645894	3600.00	-457.49	-457.178	0.07	5.40	0	67	67	0
rocII-4-11	84922	1463.15	-6.65564	-6.65564	0.00	209.31	0	86	86	0
rococoC10-001000	329557	2183.00	11460	11460	0.00	12.50	2	147	145	0
roll3000	647155	3600.00	12741	12890	1.16	18.56	5	204	199	0
satellites1-25	6404	1201.16	-5	-5	0.00	569.00	1	28	27	0
sp98ic	220650	3600.00	4.47466e+08	4.49145e+08	0.37	26.64	4	52	48	0
sp98ir	16390	142.03	2.19677e+08	2.19677e+08	0.00	2.79	2	46	44	0
tanglegram1	109	863.10	5182	5182	0.00	100.90	0	0	0	0
tanglegram2	5	7.31	443	443	0.00	4.31	0	0	0	0
timtab1	1334835	992.29	764772	764772	0.00	16.58	8	173	165	0
triptim1	1	85.28	22.8681	22.8681	0.00	50.00	0	0	0	0
unitcal_7	37916	3392.80	1.96356e+07	1.96356e+07	0.00	97.54	3	72	69	0
vpphard	4626	3600.34	0	43	—	3326.54	2	12	10	0
zib54-UUE	428043	3600.00	9.37067e+06	1.0334e+07	9.32	9.60	0	68	68	0
AM (# 87)	438679.7	1594.48			4.18	238.06	1.0	41.7	40.7	0.0
GM (# 87)	12118.7	663.98			1.70	30.12	1.3	15.2	14.9	1.0
SGM (# 87)	21343.6	732.30								

Table 18: MIPLIB2010 testset: geom-heur settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
30n20b8	2	153.60	302	302	0.00	132.00	0	0	0	0
acc-tight5	2193	225.29	0	0	0.00	42.90	0	8	8	0
aflow40b	129608	1447.93	1168	1168	0.00	24.90	3	26	23	0
air04	7	20.31	56137	56137	0.00	3.82	0	0	0	0
app1-2	5719	3600.02	-49.5462	-34	31.38	2182.78	0	10	10	0
ash608gpia-3col	13	34.07	1e+20	1e+20	—	34.07	0	0	0	0
bab5	34489	3600.00	-107343	-106212	1.05	138.54	0	29	29	0
beasleyC3	401337	3600.00	696.945	764	8.78	50.65	2	24	22	0
biella1	6288	647.82	3.06501e+06	3.06501e+06	0.00	7.93	2	42	40	0
bienst2	190764	535.46	54.6	54.6	0.00	10.99	0	76	76	0
binkar10_1	299243	594.48	6742.2	6742.2	0.00	0.60	2	88	86	0
bley_xl1	1	277.81	190	190	0.00	269.52	0	0	0	0
bnatt350	4833	536.69	0	0	0.00	537.00	0	0	0	0
core2536-691	423	157.51	689	689	0.00	8.44	0	0	0	0
cov1075	771913	3600.00	18.4389	20	7.81	2.02	1	11	10	0
csched010	398790	3600.00	379.862	408	6.90	105.66	6	91	85	0
danoint	886135	3600.00	63.7157	65.6667	2.97	11.89	0	37	37	0
dfn-gwin-UUM	53916	111.24	38752	38752	0.00	1.74	0	13	13	0
eil33-2	1274	114.71	934.008	934.008	0.00	4.16	0	11	11	0
eilB101	13526	363.52	1216.92	1216.92	0.00	9.80	3	26	23	0
enlight13	1	0.01	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.02	1e+20	1e+20	—	0.02	0	0	0	0
ex9	1	65.61	81	81	0.00	65.60	0	0	0	0
glass4	3916667	3600.01	8.50004e+08	1.55001e+09	45.16	908.73	7	268	261	0
gmu-35-40	4982147	3600.03	-2.40692e+06	-2.4063e+06	0.03	0.89	1	84	83	0
iis-100-0-cov	90249	1862.94	29	29	0.00	9.25	0	18	18	0
iis-bupa-cov	101771	3600.00	32.8462	36	8.76	8.16	0	10	10	0
iis-pima-cov	17661	1446.00	33	33	0.00	17.67	0	22	22	0
lectsched-4-obj	4546	130.85	4	4	0.00	95.64	5	30	25	0
m100n500k4r1	4574038	3600.00	-25	-24	4.00	145.62	0	22	22	0
macrophage	468247	3600.00	279.125	378	26.16	88.16	6	52	46	0
map18	476	1022.56	-847	-847	0.00	18.02	0	12	12	0
map20	626	835.84	-922	-922	0.00	10.40	0	12	12	0
mcsched	44622	735.79	211913	211913	0.00	2.47	0	95	95	0
mik-250-1-100-1	4490051	1604.92	-66729	-66729	0.00	0.00	0	12	12	0
mine-166-5	2922	87.10	-5.66396e+08	-5.66396e+08	0.00	23.79	0	28	28	0
mine-90-10	72381	480.90	-7.84302e+08	-7.84302e+08	0.00	17.29	1	88	87	0
msc98-ip	5956	2106.68	1.98395e+07	1.98395e+07	0.00	116.48	1	36	35	0
mspp16	0	23.15	1e+20	1e+20	—	23.15	0	0	0	0
mzzv11	3187	935.83	-21718	-21718	0.00	150.60	0	22	22	0
n3div36	77284	3600.14	123022	131000	6.09	77.42	0	84	84	0
n3seq24	25177	3600.01	52000	52200	0.38	364.86	0	5	5	0
n4-3	61536	945.71	8993	8993	0.00	10.05	1	33	32	0
neos-1109824	12450	175.23	378	378	0.00	5.11	0	13	13	0
neos-1337307	105012	3600.00	-202467	-202319	0.07	91.81	0	41	41	0
neos-1396125	136796	1466.78	3000.05	3000.05	0.00	5.30	0	64	64	0
neos13	11451	3600.21	-98.4144	-95.1549	3.31	413.22	8	42	34	0
neos-1601936	5021	1933.39	3	3	0.00	1555.32	6	32	26	0
neos18	5460	64.55	16	16	0.00	2.14	0	11	11	0
neos-476283	879	465.81	406.363	406.363	0.00	49.58	0	1	1	0
neos-686190	15220	403.81	6730	6730	0.00	60.99	1	42	41	0
neos-849702	26770	645.40	0	0	0.00	645.00	0	0	0	0
neos-916792	268301	3600.00	27.9851	32.0467	12.67	64.75	1	43	42	0
neos-934278	272	581.63	260	260	0.00	80.48	0	0	0	0
net12	3133	1435.42	214	214	0.00	197.42	1	10	9	0
netdiversion	3	463.86	242	242	0.00	350.98	0	0	0	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
newdano	705725	3600.00	47	66.4545	29.27	57.54	0	72	72	0
noswot	1197318	368.50	-41	-41	0.00	0.41	0	35	35	0
ns1208400	9919	1471.75	2	2	0.00	1472.00	0	0	0	0
ns1688347	2142	370.26	27	27	0.00	93.26	1	21	20	0
ns1758913	2	3600.15	-1454.67	-376.035	74.15	2815.94	0	0	0	0
ns1766074	921546	1266.27	1e+20	1e+20	—	1266.27	0	0	0	0
ns1830653	41200	804.35	20622	20622	0.00	93.27	1	33	32	0
opm2-z7-s2	5290	3600.01	-10343.1	-10272	0.69	135.51	4	62	58	0
pg5_34	310077	2434.63	-14339.4	-14339.4	0.00	2.27	2	84	82	0
pigeon-10	5003964	3600.00	-10000	-9000	10.00	2.96	5	56	51	0
pw-myciel4	526572	3600.00	6.45991	10	35.40	40.96	0	20	20	0
qiu	17537	141.18	-132.873	-132.873	0.00	30.60	1	8	7	0
rail507	1315	185.41	174	174	0.00	10.79	1	15	14	0
ran16x16	352136	361.93	3823	3823	0.00	0.49	0	23	23	0
reblock67	116268	438.73	-3.46306e+07	-3.46306e+07	0.00	11.92	4	59	55	0
rmatr100-p10	1395	241.53	423	423	0.00	4.31	0	11	11	0
rmatr100-p5	523	257.24	976	976	0.00	3.44	0	9	9	0
rmine6	622631	3600.10	-457.49	-457.178	0.07	5.47	0	65	65	0
rocII-4-11	44033	1364.12	-6.65564	-6.65564	0.00	209.47	1	61	60	0
rococoC10-001000	241069	2136.87	11460	11460	0.00	9.73	1	125	124	0
roll3000	523488	3600.00	12779.6	12890	0.86	11.17	4	113	109	0
satellites1-25	4077	1025.15	-5	-5	0.00	568.00	1	11	10	0
sp98ic	191637	3600.00	4.47534e+08	4.50687e+08	0.70	42.85	4	22	18	0
sp98ir	6547	151.88	2.19677e+08	2.19677e+08	0.00	2.73	3	21	18	0
tanglegram1	109	863.65	5182	5182	0.00	101.27	0	0	0	0
tanglegram2	5	7.32	443	443	0.00	4.31	0	0	0	0
timtab1	967759	729.62	764772	764772	0.00	18.23	11	112	101	0
triptim1	1	86.47	22.8681	22.8681	0.00	51.00	0	0	0	0
unitcal_7	27506	3591.83	1.96356e+07	1.96356e+07	0.00	98.03	3	76	73	0
vpphard	4128	3600.01	0	5	—	2058.82	5	18	13	0
zib54-UUE	423312	3600.00	9.36068e+06	1.0334e+07	9.42	10.05	0	71	71	0
AM (# 87)	402299.1	1575.17			3.75	212.53	1.3	33.6	32.4	0.0
GM (# 87)	9541.9	673.79			1.63	30.42	1.4	13.9	13.5	1.0
SGM (# 87)	16858.8	741.52								

Table 19: MIPLIB2010 testset: geom-no ℓ_1 settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
30n20b8	4	397.67	302	302	0.00	183.88	4	28	24	0
acc-tight5	653	169.57	0	0	0.00	169.57	1	1	0	0
aflow40b	41944	3600.00	1105.43	1168	5.36	120.36	8	26	18	0
air04	6	235.03	56137	56137	0.00	3.68	6	36	30	0
app1-2	5110	3600.26	-164.137	1e+20	—	3600.26	0	1	1	0
ash608gpia-3col	6	31.94	1e+20	1e+20	—	31.94	0	1	1	0
bab5	18032	3600.00	-107128	-105905	1.14	419.43	8	23	15	0
beasleyC3	89556	3600.00	683.054	780	12.43	224.17	8	20	12	0
biella1	2008	3600.02	3.06018e+06	3.06501e+06	0.16	15.17	28	64	36	0
bienst2	123059	2183.61	54.6	54.6	0.00	25.73	6	23	17	0
binkar10_1	158132	2258.02	6742.2	6742.2	0.00	0.95	14	40	26	0
bley_xl1	2	361.84	190	190	0.00	272.60	5	26	21	0
bnatt350	8217	989.61	0	0	0.00	989.61	1	1	0	0
core2536-691	90	421.53	689	689	0.00	7.07	8	33	25	0
cov1075	767491	3600.00	17.2146	20	13.93	1.40	3	14	11	0
csched010	193022	3600.00	358.642	415	13.58	217.88	7	20	13	0
danoint	91008	3600.00	62.705	65.6667	4.51	65.49	4	17	13	0
dfn-gwin-UUM	46517	2897.45	38752	38752	0.00	4.84	8	36	28	0
eil33-2	382	716.62	934.008	934.008	0.00	25.73	4	29	25	0
eilB101	4078	1076.35	1216.92	1216.92	0.00	11.80	3	28	25	0
enlight13	1	0.02	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.02	1e+20	1e+20	—	0.02	0	0	0	0
ex9	1	65.68	81	81	0.00	65.70	0	0	0	0
glass4	4868779	3602.65	8.00003e+08	1.90002e+09	57.89	1356.03	5	19	14	0
gmu-35-40	2089700	3600.83	-2.40692e+06	-2.40526e+06	0.07	6.60	6	23	17	0
iis-100-0-cov	45519	3600.00	17.5309	29	39.55	15.02	2	15	13	0
iis-bupa-cov	60458	3600.01	27.1183	36	24.67	36.38	2	14	12	0
iis-pima-cov	8716	3600.00	26.7488	33	18.94	12.36	3	18	15	0
lectsched-4-obj	1	1152.90	4	4	0.00	750.13	9	31	22	0
m100n500k4r1	3430143	3600.66	-25	-24	4.00	145.10	3	15	12	0
macrophage	359805	3600.00	225.746	391	42.26	192.53	10	24	14	0
map18	3	3600.01	-910.697	-847	6.99	3.80	1	13	12	0
map20	12	3600.01	-982.638	-922	6.17	3.80	1	15	14	0
mcsched	21402	2576.94	211913	211913	0.00	6.76	11	42	31	0
mik-250-1-100-1	4532612	3600.00	-70072.8	-66729	4.77	0.00	1	16	15	0
mine-166-5	889	664.64	-5.66396e+08	-5.66396e+08	0.00	35.14	10	45	35	0
mine-90-10	160147	3600.00	-7.96659e+08	-7.84302e+08	1.55	9.37	11	30	19	0
msc98-ip	750	3600.01	1.97029e+07	1.98408e+07	0.70	84.75	10	32	22	0
mspp16	0	23.03	1e+20	1e+20	—	23.03	0	0	0	0
mzzv11	491	1345.73	-21718	-21718	0.00	162.29	3	17	14	0
n3div36	118517	3600.00	120927	138200	12.50	525.56	2	19	17	0
n3seq24	3674	3600.02	52000	52200	0.38	208.13	5	29	24	0
n4-3	7385	3600.00	7978.2	8993	11.28	52.77	18	30	12	0
neos-1109824	11610	1299.35	378	378	0.00	11.17	5	27	22	0
neos-1337307	69	3600.00	-203095	-202319	0.38	39.21	5	29	24	0
neos-1396125	58462	1547.81	3000.05	3000.05	0.00	14.88	3	28	25	0
neos13	6	3588.10	-95.4748	-95.4748	0.00	843.30	19	29	10	0
neos-1601936	16524	3600.01	3	8	62.50	2353.05	11	28	17	0
neos18	5651	275.20	16	16	0.00	9.15	3	20	17	0
neos-476283	2	3601.38	406.245	406.364	0.03	144.83	15	37	22	0
neos-686190	1769	1002.20	6730	6730	0.00	28.83	7	35	28	0
neos-849702	54881	816.85	0	0	0.00	816.85	1	1	0	0
neos-916792	58028	3600.00	26.5993	32.0355	16.97	141.17	9	22	13	0
neos-934278	7572	3582.20	260	260	0.00	51.43	9	31	22	0
net12	6	3600.00	91.6402	214	57.18	450.29	3	20	17	0
netdiversion	212	2654.86	242	242	0.00	1606.24	1	26	25	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
newdano	78491	3600.01	34.4666	70.75	51.28	265.10	1	11	10	0
noswot	2292738	3600.00	-43	-41	4.65	0.83	4	16	12	0
ns1208400	2	962.91	2	2	0.00	694.00	1	15	14	0
ns1688347	1016	1220.35	27	27	0.00	166.56	3	21	18	0
ns1758913	2	3608.21	-1454.67	-236.804	83.72	3027.80	0	6	6	0
ns1766074	891606	1528.46	1e+20	1e+20	—	1528.46	0	1	1	0
ns1830653	831	3600.00	11622	20622	43.64	178.83	4	28	24	0
opm2-z7-s2	279	3600.00	-12706.7	-10279	19.11	136.44	10	27	17	0
pg5_34	44053	3600.00	-14366.2	-14339.4	0.19	4.38	9	26	17	0
pigeon-10	3337764	3601.72	-10000	-9000	10.00	1.47	3	10	7	0
pw-myciel4	135497	3600.00	4	10	60.00	30.74	6	20	14	0
qiu	14988	1189.71	-132.873	-132.873	0.00	40.47	9	31	22	0
rail507	1538	3600.01	172.146	174	1.07	12.74	7	26	19	0
ran16x16	204115	3359.12	3823	3823	0.00	4.49	6	30	24	0
reblock67	51209	3012.36	-3.46306e+07	-3.46306e+07	0.00	22.60	17	52	35	0
rmatr100-p10	562	1428.60	423	423	0.00	11.69	8	32	24	0
rmatr100-p5	366	2082.89	976	976	0.00	25.54	6	32	26	0
rmine6	32532	3600.00	-461.364	-457.155	0.91	5.67	13	31	18	0
rocII-4-11	4476	1982.48	-6.65564	-6.65564	0.00	478.11	3	17	14	0
rococoC10-001000	64517	3600.00	10199.4	11460	11.00	9.73	11	34	23	0
roll3000	73218	3600.00	12395.3	12897	3.89	19.62	9	25	16	0
satellites1-25	4140	2918.67	-5	-5	0.00	560.80	6	26	20	0
sp98ic	46	3600.00	4.44771e+08	4.5847e+08	2.99	185.71	10	21	11	0
sp98ir	2094	872.30	2.19677e+08	2.19677e+08	0.00	4.58	8	49	41	0
tanglegram1	47	3600.02	77	5207	98.52	153.32	8	28	20	0
tanglegram2	4	118.67	443	443	0.00	24.49	8	34	26	0
timtab1	317926	3600.00	490942	764774	35.81	44.92	30	49	19	0
triptim1	2	270.40	22.8681	22.8681	0.00	49.61	2	23	21	0
unitcal_7	6858	3600.01	1.95558e+07	1.96356e+07	0.41	145.65	13	33	20	0
vpphard	1217	3600.01	0	12	—	3272.19	7	19	12	0
zib54-UUE	41625	3600.00	7.13872e+06	1.0334e+07	30.92	8.31	8	36	28	0
AM (# 87)	288239.9	2516.06			10.09	318.54	6.4	23.6	17.2	0.0
GM (# 87)	2606.6	1540.58			2.86	49.55	4.5	17.1	13.0	1.0
SGM (# 87)	6038.5	1651.26								

Table 20: MIPLIB2010 testset: geom-no cutoff settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
30n20b8	2	492.22	302	302	0.00	176.33	3	27	24	0
acc-tight5	653	169.24	0	0	0.00	169.24	1	1	0	0
aflow40b	131741	3600.00	1113.48	1204	7.52	375.07	4	17	13	0
air04	6	539.34	56137	56137	0.00	4.21	7	37	30	0
app1-2	5121	3600.98	-164.137	1e+20	—	3600.98	0	1	1	0
ash608gpia-3col	6	32.02	1e+20	1e+20	—	32.02	0	1	1	0
bab5	12264	3600.00	-107132	-104935	2.05	492.35	4	18	14	0
beasleyC3	417163	3600.00	684.743	804	14.83	624.35	1	11	10	0
biella1	7970	3600.00	3.06045e+06	3.2582e+06	6.07	237.69	7	25	18	0
bienst2	160014	2038.82	54.6	54.6	0.00	30.93	5	22	17	0
binkar10_1	167739	2738.09	6743.24	6743.24	0.00	1.97	10	36	26	0
bley_xl1	2	466.14	190	190	0.00	280.58	3	24	21	0
bnatt350	8217	990.22	0	0	0.00	990.22	1	1	0	0
core2536-691	175	1761.12	689	689	0.00	9.89	5	30	25	0
cov1075	719696	3600.00	17.2229	20	13.89	0.76	1	12	11	0
csched010	135155	3600.04	358.625	453	20.83	537.24	4	15	11	0
danoint	268897	3600.00	62.705	65.6667	4.51	81.87	3	15	12	0
dfn-gwin-UUM	57670	2672.32	38752	38752	0.00	12.31	5	33	28	0
eil33-2	724	2170.86	934.008	934.008	0.00	88.44	4	29	25	0
eilB101	8270	3405.89	1216.92	1216.92	0.00	24.41	3	28	25	0
enlight13	1	0.01	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.01	1e+20	1e+20	—	0.01	0	0	0	0
ex9	1	65.47	81	81	0.00	65.50	0	0	0	0
glass4	3625546	3601.36	8.00003e+08	1.90002e+09	57.90	1347.37	4	18	14	0
gmu-35-40	1390793	3600.65	-2.40692e+06	-2.40596e+06	0.04	2.98	4	23	19	0
iis-100-0-cov	23772	3600.00	17.5309	29	39.55	4.98	3	16	13	0
iis-bupa-cov	40459	3600.00	27.1183	36	24.67	35.39	3	15	12	0
iis-pima-cov	23353	3600.00	27.0874	33	17.92	16.05	3	17	14	0
lectsched-4-obj	544547	3600.00	4	103	96.12	3548.83	1	12	11	0
m100n500k4r1	2486480	3600.00	-25	-24	4.00	224.50	3	15	12	0
macrophage	95378	3600.00	225.746	507	55.47	949.16	1	14	13	0
map18	2	3600.26	-910.697	-847	6.99	3.80	1	9	8	0
map20	2	3600.01	-982.638	-922	6.17	3.80	1	9	8	0
mcsched	10840	2814.83	211913	211913	0.00	9.48	5	36	31	0
mik-250-1-100-1	2209990	3601.85	-70077.8	-66669	4.86	6.26	1	16	15	0
mine-166-5	4122	1005.05	-5.66396e+08	-5.66396e+08	0.00	61.39	15	50	35	0
mine-90-10	13797	3600.01	-8.17887e+08	-7.84302e+08	4.11	42.96	8	31	23	0
msc98-ip	6460	3600.00	1.97029e+07	2.28378e+07	13.73	530.39	2	14	12	0
mspp16	0	23.14	1e+20	1e+20	—	23.14	0	0	0	0
mzzv11	1270	3600.15	-21920.1	-21718	0.92	182.32	5	13	8	0
n3div36	15664	3600.00	120927	147600	18.07	1504.18	1	16	15	0
n3seq24	2	3650.03	52000	91200	42.98	1583.36	0	15	15	0
n4-3	14866	3600.00	7978.2	8993	11.28	22.99	6	26	20	0
neos-1109824	134381	3600.00	347.714	378	8.01	15.67	3	20	17	0
neos-1337307	25790	3600.00	-203094	-202319	0.38	39.87	5	27	22	0
neos-1396125	56199	1415.17	3000.05	3000.05	0.00	24.74	3	28	25	0
neos13	3	3600.03	-126.178	-66.8793	47.00	1499.18	31	40	9	0
neos-1601936	4152	3600.00	3	144	97.92	3528.94	10	23	13	0
neos18	28825	2482.57	16	16	0.00	4.23	2	19	17	0
neos-476283	271	3600.01	406.29	406.567	0.07	518.81	3	16	13	0
neos-686190	228688	3600.00	5538.66	20690	73.23	2429.85	1	12	11	0
neos-849702	54881	816.11	0	0	0.00	816.11	1	1	0	0
neos-916792	32396	3600.02	26.6687	32.6196	18.24	476.30	5	15	10	0
neos-934278	134	3600.00	259.5	261	0.57	115.12	6	27	21	0
net12	135	3600.01	91.6402	214	57.18	526.47	3	19	16	0
netdiversion	8	3600.03	233.889	610	61.66	2719.23	1	26	25	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
newdano	308331	3600.00	34.4666	70.75	51.28	296.94	1	10	9	0
noswot	966888	3600.64	-43	-41	4.65	9.83	3	15	12	0
ns1208400	144	3600.00	0	2	—	694.00	1	13	12	0
ns1688347	853	3600.00	23	27	14.81	517.51	3	19	16	0
ns1758913	2	3602.37	-1454.67	-236.804	83.72	3022.91	0	5	5	0
ns1766074	891606	1541.61	1e+20	1e+20	—	1541.61	0	1	1	0
ns1830653	12454	3600.01	11622	20622	43.64	288.54	4	21	17	0
opm2-z7-s2	7	3600.00	-12331.4	-10279	16.64	607.89	12	27	15	0
pg5_34	14776	3600.00	-14366	-14339.4	0.19	7.05	5	25	20	0
pigeon-10	1198653	3600.00	-10000	-9000	10.00	2.26	3	10	7	0
pw-myciel4	12304	3600.00	4	10	60.00	59.00	4	16	12	0
qiu	12922	1430.55	-132.873	-132.873	0.00	49.64	5	27	22	0
rail507	19711	3600.01	172.239	177	2.69	78.81	1	15	14	0
ran16x16	154382	3600.00	3575.3	3823	6.48	17.40	2	23	21	0
reblock67	16161	3600.01	-3.64261e+07	-3.46306e+07	4.93	32.51	9	38	29	0
rmatr100-p10	539	3600.00	363.874	423.249	14.03	26.93	3	54	51	0
rmatr100-p5	350	2394.27	976	976	0.00	21.66	3	29	26	0
rmine6	236020	3600.00	-461.142	-456.949	0.91	8.84	7	24	17	0
rocII-4-11	23132	1528.32	-6.65564	-6.65564	0.00	268.20	3	17	14	0
rococoC10-001000	78985	3600.00	10199.4	11464	11.03	15.58	5	22	17	0
roll3000	106462	3600.00	12395.3	12928	4.12	32.73	8	23	15	0
satellites1-25	204	3600.30	-20	-5	75.00	1694.00	3	15	12	0
sp98ic	12359	3600.00	4.44729e+08	4.82849e+08	7.89	1039.77	2	9	7	0
sp98ir	4762	3151.25	2.19677e+08	2.19677e+08	0.00	10.01	7	48	41	0
tanglegram1	51	3600.00	77	5375	98.57	228.87	4	21	17	0
tanglegram2	4	91.73	443	443	0.00	6.94	1	27	26	0
timtab1	401561	3600.00	490942	764777	35.81	84.27	8	24	16	0
triptim1	2	2502.37	22.8681	22.8681	0.00	49.94	2	23	21	0
unitcal_7	1079	3600.01	1.956e+07	1.96527e+07	0.47	183.86	9	26	17	0
vpphard	92	3600.00	0	209	—	3554.24	1	11	10	0
zib54-UUE	97528	3600.01	7.13872e+06	1.0393e+07	31.31	34.81	4	30	26	0
AM (# 87)	203988.7	2887.33			16.29	518.87	3.9	19.4	15.6	0.0
GM (# 87)	3432.7	1927.53			4.83	85.45	2.8	14.1	11.6	1.0
SGM (# 87)	7648.7	2062.03								

Table 21: MIPLIB2010 testset: geometric settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
30n20b8	2	390.36	302	302	0.00	178.96	4	28	24	0
acc-tight5	653	169.29	0	0	0.00	169.29	1	1	0	0
aflow40b	41789	3600.00	1105.43	1168	5.36	120.46	8	26	18	0
air04	6	235.04	56137	56137	0.00	3.68	6	36	30	0
app1-2	5113	3600.24	-164.137	1e+20	—	3600.24	0	1	1	0
ash608gpia-3col	6	32.01	1e+20	1e+20	—	32.01	0	1	1	0
bab5	17996	3600.01	-107128	-105905	1.14	420.44	8	23	15	0
beasleyC3	86328	3600.00	683.054	780	12.43	224.91	8	20	12	0
biella1	2022	3600.01	3.06018e+06	3.06501e+06	0.16	15.08	28	64	36	0
bienst2	123059	2190.98	54.6	54.6	0.00	25.74	6	23	17	0
binkar10_1	158132	2276.04	6742.2	6742.2	0.00	0.94	14	40	26	0
bley_xl1	2	361.16	190	190	0.00	271.67	5	26	21	0
bnatt350	8217	988.98	0	0	0.00	988.98	1	1	0	0
core2536-691	90	421.46	689	689	0.00	7.07	8	33	25	0
cov1075	768226	3600.00	17.2146	20	13.93	1.38	3	14	11	0
csched010	193533	3600.00	358.642	415	13.58	217.32	7	20	13	0
danoint	92558	3600.00	62.705	65.6667	4.51	65.54	4	17	13	0
dfn-gwin-UUM	47260	1653.38	38752	38752	0.00	6.01	9	37	28	0
eil33-2	382	710.82	934.008	934.008	0.00	25.49	4	29	25	0
eilB101	4078	1074.16	1216.92	1216.92	0.00	11.80	3	28	25	0
enlight13	1	0.02	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.02	1e+20	1e+20	—	0.02	0	0	0	0
ex9	1	65.64	81	81	0.00	65.60	0	0	0	0
glass4	4863507	3602.61	8.00003e+08	1.90002e+09	57.89	1356.18	5	19	14	0
gmu-35-40	2055343	3600.82	-2.40692e+06	-2.40526e+06	0.07	6.62	6	23	17	0
iis-100-0-cov	45736	3600.00	17.5309	29	39.55	15.04	2	15	13	0
iis-bupa-cov	60606	3600.01	27.1183	36	24.67	36.49	2	14	12	0
iis-pima-cov	8934	3600.01	26.7488	33	18.94	12.36	3	18	15	0
lectsched-4-obj	1	108.15	4	4	0.00	71.02	9	31	22	0
m100n500k4r1	3456830	3600.67	-25	-24	4.00	145.09	3	15	12	0
macrophage	358797	3600.00	225.746	391	42.26	192.64	10	24	14	0
map18	3	3600.00	-910.697	-847	6.99	3.80	1	13	12	0
map20	17	3600.01	-982.638	-922	6.17	3.80	1	15	14	0
mcsched	21402	2574.70	211913	211913	0.00	6.75	11	42	31	0
mik-250-1-100-1	929097	3600.14	-70077.8	-66729	4.78	3.35	2	20	18	0
mine-166-5	889	664.48	-5.66396e+08	-5.66396e+08	0.00	35.14	10	45	35	0
mine-90-10	160489	3600.00	-7.96659e+08	-7.84302e+08	1.55	9.47	11	30	19	0
msc98-ip	772	3600.00	1.97029e+07	1.98408e+07	0.70	84.21	10	32	22	0
mspp16	0	23.13	1e+20	1e+20	—	23.13	0	0	0	0
mzzv11	6	1546.87	-21718	-21718	0.00	183.44	8	22	14	0
n3div36	118508	3600.00	120927	138200	12.50	525.56	2	19	17	0
n3seq24	3649	3600.02	52000	52200	0.38	207.71	5	29	24	0
n4-3	1167	3600.00	7978.2	8993	11.28	11.90	7	31	24	0
neos-1109824	11610	1297.15	378	378	0.00	11.18	5	27	22	0
neos-1337307	62	3600.00	-203095	-202319	0.38	39.31	5	29	24	0
neos-1396125	58462	1546.28	3000.05	3000.05	0.00	14.94	3	28	25	0
neos13	6	3584.83	-95.4748	-95.4748	0.00	843.16	19	29	10	0
neos-1601936	16498	3600.00	3	8	62.50	2353.08	11	28	17	0
neos18	5651	275.99	16	16	0.00	9.19	3	20	17	0
neos-476283	2	3603.42	406.245	406.364	0.03	144.64	15	37	22	0
neos-686190	2194	2045.27	6730	6730	0.00	340.14	7	35	28	0
neos-849702	54881	818.90	0	0	0.00	818.90	1	1	0	0
neos-916792	58279	3600.01	26.5993	32.0355	16.97	140.97	9	22	13	0
neos-934278	7752	3582.39	260	260	0.00	52.52	9	31	22	0
net12	5	3600.00	91.6402	214	57.18	451.29	3	20	17	0
netdiversion	212	2658.54	242	242	0.00	1608.57	1	26	25	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
newdano	72485	3600.01	34.4666	70.75	51.28	265.09	1	11	10	0
noswot	965935	2622.46	-41	-41	0.00	0.37	4	17	13	0
ns1208400	2	961.85	2	2	0.00	693.00	1	15	14	0
ns1688347	1016	1220.02	27	27	0.00	166.56	3	21	18	0
ns1758913	2	3608.24	-1454.67	-236.804	83.72	3027.84	0	6	6	0
ns1766074	891606	1541.34	1e+20	1e+20	—	1541.34	0	1	1	0
ns1830653	1035	3600.00	11622	20622	43.64	178.58	4	28	24	0
opm2-z7-s2	198	3600.00	-12706.7	-10279	19.11	143.32	10	27	17	0
pg5_34	42823	3600.00	-14366.2	-14339.4	0.19	4.32	9	26	17	0
pigeon-10	3310439	3601.71	-10000	-9000	10.00	1.47	3	10	7	0
pw-myciel4	23887	3600.00	4	10	60.00	34.73	5	20	15	0
qiu	14988	1187.93	-132.873	-132.873	0.00	40.46	9	31	22	0
rail507	1298	3600.00	172.146	174	1.07	12.73	7	26	19	0
ran16x16	204115	3347.25	3823	3823	0.00	4.50	6	30	24	0
reblock67	51209	3020.91	-3.46306e+07	-3.46306e+07	0.00	22.55	17	52	35	0
rmatr100-p10	562	1427.90	423	423	0.00	11.53	8	32	24	0
rmatr100-p5	366	2081.25	976	976	0.00	25.45	6	32	26	0
rmine6	30727	3600.01	-461.364	-457.155	0.91	5.71	13	31	18	0
rocII-4-11	4476	1978.72	-6.65564	-6.65564	0.00	477.20	3	17	14	0
rococoC10-001000	54485	3600.01	10199.4	11460	11.00	9.75	11	34	23	0
roll3000	478823	3600.00	12395.3	12897	3.89	14.81	9	25	16	0
satellites1-25	4140	2913.85	-5	-5	0.00	559.40	6	26	20	0
sp98ic	59	3600.00	4.44771e+08	4.5847e+08	2.99	185.41	10	21	11	0
sp98ir	1654	895.32	2.19677e+08	2.19677e+08	0.00	10.15	9	50	41	0
tanglegram1	49	3600.02	77	5207	98.52	153.32	8	28	20	0
tanglegram2	4	118.98	443	443	0.00	24.49	8	34	26	0
timtab1	361140	3600.00	496517	764777	35.08	28.91	13	35	22	0
triptim1	2	1015.02	22.8681	22.8681	0.00	52.15	2	23	21	0
unitcal_7	6870	3600.01	1.95558e+07	1.96356e+07	0.41	146.62	13	33	20	0
vpphard	1192	3600.01	0	12	—	3273.02	7	19	12	0
zib54-UUE	41556	3600.00	7.13872e+06	1.0334e+07	30.92	8.33	8	36	28	0
AM (# 87)	234689.3	2501.69			10.03	314.23	6.2	23.6	17.4	0.0
GM (# 87)	2340.2	1522.03			2.81	49.80	4.5	17.2	13.2	1.0
SGM (# 87)	5628.8	1632.31								

Table 22: MIPLIB2010 testset: MRA settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
30n20b8	1129	653.60	302	302	0.00	299.10	9	21	14	0
acc-tight5	653	169.43	0	0	0.00	169.43	1	1	0	0
aflow40b	170128	3600.07	1099.36	1688	34.87	1326.74	41	57	49	0
air04	2432	3600.06	55648.7	56297	1.15	40.40	33	46	40	0
app1-2	5121	3601.01	-164.137	1e+20	—	3601.01	0	0	0	0
ash608gpia-3col	6	32.19	1e+20	1e+20	—	32.19	0	1	0	0
bab5	15826	3600.29	-107401	-90216	16.00	3600.29	1	2	1	0
beasleyC3	313715	3600.40	678.579	954	28.87	754.85	0	1	1	0
biella1	8309	3600.04	3.06009e+06	3.63251e+06	15.76	679.81	127	138	133	15
bienst2	12380	3600.00	33.6315	79.2457	57.56	1263.99	12211	12265	12229	4
binkar10_1	1464772	3600.13	6703.7	6776.35	1.07	20.67	96	123	109	11
bley_xl1	571	646.42	190	190	0.00	300.39	17	45	26	0
bnatt350	8217	990.46	0	0	0.00	990.46	1	1	0	0
core2536-691	613	3600.13	688.476	690	0.22	175.49	85	109	96	0
cov1075	447895	3600.18	17.2082	20	13.96	1.04	4	18	11	0
csched010	238249	3600.06	358.5	575	37.65	1152.27	11	26	18	0
danoint	228629	3600.00	62.6887	79.4944	21.14	773.93	651	687	668	1
dfn-gwin-UUM	561742	3600.02	35046.4	42028	16.61	390.57	39	48	43	0
eil33-2	5272	502.19	934.008	934.008	0.00	33.99	13	24	19	0
eilB101	202375	3600.03	1112.8	1216.92	8.56	196.83	31	48	40	0
enlight13	1	0.01	71	71	0.00	0.00	0	0	0	0
enlight14	1	0.02	1e+20	1e+20	—	0.02	0	0	0	0
ex9	1	65.34	81	81	0.00	65.30	0	0	0	0
glass4	138637	3600.00	8.00003e+08	2.61458e+09	69.40	2207.92	6817	6818	6817	15
gmu-35-40	4051826	3600.33	-2.40692e+06	-2.40429e+06	0.11	4.48	35	44	40	0
iis-100-0-cov	161339	3600.04	16.9879	29	41.42	4.41	6	20	13	0
iis-bupa-cov	77293	3600.02	26.6259	37	28.04	274.12	7	26	15	0
iis-pima-cov	40433	3600.04	26.7475	33	18.95	78.14	8	23	15	0
lectsched-4-obj	473424	3600.62	4	281	98.58	3600.62	1	3	1	0
m100n500k4r1	1827719	3600.01	-25	-24	4.00	144.51	6	19	13	0
macrophage	469336	3600.07	196.739	442	55.49	560.55	15	45	26	0
map18	463	3600.17	-910.697	-0.00102706	100.00	3585.57	410	462	434	14
map20	494	3600.18	-982.638	-0.0010215	100.00	3585.08	445	494	469	14
mcsched	79547	3600.03	193781	211913	8.56	14.80	300	319	310	0
mik-250-1-100-1	5266851	3600.39	-70077.8	-66589	4.98	9.37	1	2	2	0
mine-166-5	26041	1747.32	-5.66396e+08	-5.66396e+08	0.00	97.79	154	155	155	1
mine-90-10	320360	3600.03	-8.59772e+08	-7.76081e+08	9.73	126.33	140	141	141	0
msc98-ip	682	3600.14	1.97029e+07	2.64351e+07	25.47	928.32	2	3	3	0
mspp16	0	23.09	1e+20	1e+20	—	23.09	0	0	0	0
mzzv11	61821	3601.09	-21961.4	-13778	37.26	1470.03	35	62	45	0
n3div36	2136	3600.30	120927	151800	20.34	683.11	18	25	22	0
n3seq24	95	3603.12	52000	69600	25.29	1355.64	4	21	12	0
n4-3	307194	3600.68	7880.04	15085	47.76	1461.36	5	19	11	0
neos-1109824	242921	3600.10	346	378	8.47	77.72	36	49	42	0
neos-1337307	17109	3600.08	-203102	-202219	0.43	135.60	19	39	27	0
neos-1396125	9202	290.48	3000.05	3000.05	0.00	33.27	7	32	19	0
neos13	1785	3600.11	-126.178	-95.1494	24.59	348.87	637	684	661	271
neos-1601936	865	3600.04	3	1162	99.74	3594.41	5	17	10	0
neos18	7364	110.99	16	16	0.00	4.46	5	29	14	0
neos-476283	133	3605.96	406.245	561.832	27.69	1155.54	75	132	93	23
neos-686190	283193	3600.65	5204.92	20690	74.84	3600.65	1	4	1	0
neos-849702	54881	817.21	0	0	0.00	817.21	1	1	0	0
neos-916792	332710	3600.06	26.2832	33.3937	21.29	342.80	83	133	103	7
neos-934278	854	3600.08	259.5	366	29.10	1825.10	87	115	98	0
net12	2133	610.79	214	214	0.00	434.31	4	14	8	0
netdiversion	8	3600.98	232	4.90044e+06	100.00	3600.80	0	1	1	0

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Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
newdano	17245	3600.01	33.85	79.4523	57.40	635.04	3435	3492	3454	0
noswot	2131810	1381.42	-41	-41	0.00	0.94	11	38	19	0
ns1208400	5037	1018.15	2	2	0.00	1018.15	1	2	1	0
ns1688347	2497	500.69	27	27	0.00	177.18	9	26	17	0
ns1758913	39	3601.04	-1454.67	-236.804	83.72	3021.82	0	39	20	0
ns1766074	891606	1529.06	1e+20	1e+20	—	1529.06	0	1	0	0
ns1830653	157537	2751.36	20622	20622	0.00	410.35	19	20	19	0
opm2-z7-s2	2727	3600.09	-12879.7	-9768	24.16	321.95	85	105	95	0
pg5_34	8394	3600.05	-14366.7	-12056.9	16.08	705.66	8367	8395	8380	264
pigeon-10	1130814	3600.19	-10000	-9000	10.00	0.80	9	18	14	0
pw-myciel4	98007	806.08	10	10	0.00	13.99	11	43	20	0
qiu	26151	356.48	-132.873	-132.873	0.00	25.39	40	75	56	0
rail507	3190	3600.18	172.146	177	2.74	211.61	57	77	67	0
ran16x16	1778638	3600.05	3557.11	4122	13.70	324.89	12	25	19	0
reblock67	1072336	3600.02	-3.80738e+07	-3.32779e+07	12.60	178.22	41	42	42	0
rmatr100-p10	2778	886.22	423	423	0.00	35.16	82	113	97	30
rmatr100-p5	2884	1707.34	976.002	976.002	0.00	38.09	42	67	55	17
rmine6	417984	3600.06	-461.364	-454.673	1.45	48.37	130	163	146	0
rocII-4-11	99306	2038.13	-6.65564	-6.65564	0.00	323.55	32	82	56	0
rococoC10-001000	281812	3600.11	10149.4	17140	40.79	1224.86	25	66	34	0
roll3000	137553	3600.02	12290.5	13920	11.71	349.53	6	26	13	0
satellites1-25	1969	1297.08	-5	-5	0.00	563.00	8	23	16	0
sp98ic	32966	3600.26	4.44484e+08	5.41851e+08	17.97	678.27	51	52	52	0
sp98ir	152320	3600.03	2.16745e+08	2.2161e+08	2.20	136.76	75	76	76	0
tanglegram1	130	3600.26	6	5207	99.88	343.76	13	45	26	0
tanglegram2	130	263.81	443	443	0.00	19.38	50	87	61	0
timtab1	4190675	3600.00	481470	1.00736e+06	52.20	927.10	380	392	384	21
triptim1	3	3600.21	22.8681	22.8701	0.01	50.11	0	2	1	0
unitcal_7	57	3600.22	1.95516e+07	2.02134e+07	3.27	377.95	33	41	36	0
vpphard	64	3600.39	0	230	—	3600.39	1	2	1	0
zib54-UUE	122366	3600.03	6.94952e+06	1.14397e+07	39.25	371.62	25	41	32	0
AM (# 87)	353056.4	2726.62			20.97	801.36	411.4	428.6	418.7	8.1
GM (# 87)	10995.0	1698.74			6.48	207.16	17.2	30.4	23.2	1.6
SGM (# 87)	16434.8	1819.46								

Table 23: LB testset: default settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
A1C1S1	314367	3600.01	10100.5	11643.3	13.25	120.49	0	0	0	0
A2C1S1	353224	3600.00	9528.77	10983.3	13.24	79.01	0	0	0	0
arki001	1436818	3600.00	7.5802e+06	7.58081e+06	0.01	11.89	0	0	0	0
B1C1S1	279431	3600.00	19925.4	24798.5	19.65	225.30	0	0	0	0
B2C1S1	187782	3600.01	19018.8	25763.1	26.18	130.11	0	0	0	0
biella1	12098	673.98	3.06501e+06	3.06501e+06	0.00	15.21	0	0	0	0
core2536-691	423	156.76	689	689	0.00	8.43	0	0	0	0
core2586-950	3454	3600.01	936.577	970	3.45	100.45	0	0	0	0
core4284-1064	1713	3600.02	1054.52	1091	3.34	145.72	0	0	0	0
core4872-1529	1151	3600.01	1512.14	1580	4.29	146.30	0	0	0	0
danoint	955861	3600.00	64.0232	65.6667	2.50	8.69	0	0	0	0
glass4	5662724	3600.03	9.00006e+08	1.60001e+09	43.75	478.34	0	0	0	0
markshare1	26445351	3600.07	9.09495e-13	7	—	2484.38	0	0	0	0
markshare2	23745563	3600.10	1.81899e-12	12	—	1518.07	0	0	0	0
mkc	523523	3600.00	-564.922	-559.112	1.03	45.90	0	0	0	0
net12	3152	1279.05	214	214	0.00	241.76	0	0	0	0
NSR8K	64	3600.04	1.75034e+07	1.27263e+08	86.25	3295.63	0	0	0	0
nsrand_ipx	335908	1736.42	51200	51200	0.00	43.09	0	0	0	0
rail507	1036	100.30	174	174	0.00	10.37	0	0	0	0
roll3000	416498	2286.15	12890	12890	0.00	7.11	0	0	0	0
seymour	76909	3600.00	414.776	425	2.41	28.64	0	0	0	0
sp97ar	135166	3600.00	6.55917e+08	6.63515e+08	1.15	78.81	0	0	0	0
sp97ic	156872	3600.00	4.24946e+08	4.35258e+08	2.37	126.36	0	0	0	0
sp98ar	154349	3600.00	5.28302e+08	5.30322e+08	0.38	38.29	0	0	0	0
sp98ic	203527	3600.00	4.47482e+08	4.51409e+08	0.87	62.74	0	0	0	0
swath	284206	3600.00	389.537	494.093	21.16	236.17	0	0	0	0
tr12-30	1290035	3600.00	130541	130596	0.04	0.44	0	0	0	0
UMTS	1123262	3600.00	3.00542e+07	3.00943e+07	0.13	4.35	0	0	0	0
van	725	3600.02	2.00969	5.08845	60.50	333.24	0	0	0	0
AM (# 29)	2210523.9	3070.10			10.55	345.73	0.0	0.0	0.0	0.0
GM (# 29)	87831.4	2497.14			3.18	70.46	1.0	1.0	1.0	1.0
SGM (# 29)	92779.4	2519.04								

Table 24: LB testset: bitscale settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
A1C1S1	130121	3600.00	8479.24	11977.5	29.21	282.02	17	23	6	0
A2C1S1	202586	3600.00	9300.12	11115.3	16.33	299.03	9	17	8	0
arki001	1454725	3600.00	7.58021e+06	7.58081e+06	0.01	11.79	0	0	0	0
B1C1S1	261334	3600.00	15077.3	27309.5	44.79	473.46	3	13	10	0
B2C1S1	145444	3600.00	15390	26592.2	42.13	260.79	6	14	8	0
biella1	24698	3600.00	3.06138e+06	3.06501e+06	0.12	72.41	13	30	17	0
core2536-691	544	374.18	689	689	0.00	51.18	1	1	0	0
core2586-950	1951	3600.15	935.922	1213	22.84	790.42	0	1	1	0
core4284-1064	1613	3600.64	1054.05	3279	67.85	2440.46	0	1	1	0
core4872-1529	496	3600.35	1511.46	1769	14.56	505.68	0	1	1	0
danoint	955572	3600.00	64.023	65.6667	2.50	8.76	0	0	0	0
glass4	5805746	3602.86	8.00003e+08	2.20002e+09	63.64	1311.21	1	3	2	0
markshare1	24910555	3609.86	9.09495e-13	32	—	3271.50	4	9	5	0
markshare2	23241823	3611.42	1.81899e-12	128	—	3357.50	3	10	7	0
mkc	251390	3600.00	-564.75	-557.564	1.27	105.72	5	8	3	0
net12	4960	1686.63	214	214	0.00	151.21	2	5	3	0
NSR8K	3	3600.69	1.75011e+07	2.17618e+09	99.20	3579.09	1	3	2	0
nsrand_ipx	670407	3600.21	50497	55200	8.52	266.35	1	7	6	0
rail507	734	86.45	174	174	0.00	59.13	1	1	0	0
roll3000	762513	3600.15	12233.9	13380	8.57	156.23	5	7	2	0
seymour	76927	3600.00	414.776	425	2.41	28.64	0	0	0	0
sp97ar	53194	3600.00	6.54134e+08	6.74471e+08	3.02	112.21	4	10	6	0
sp97ic	104223	3600.00	4.23107e+08	4.30937e+08	1.82	93.31	7	12	5	0
sp98ar	64684	3600.00	5.27644e+08	5.32671e+08	0.94	67.49	8	13	5	0
sp98ic	138752	3600.00	4.46426e+08	4.55081e+08	1.90	115.65	5	11	6	0
swath	158008	3600.00	379.657	506.436	25.03	350.81	6	10	4	0
tr12-30	1127397	3600.12	130225	139741	6.81	235.87	0	10	10	0
UMTS	739548	3600.00	2.99907e+07	3.00915e+07	0.33	5.90	13	15	2	0
van	444	3600.02	1.73065	5.35326	67.67	575.77	1	2	1	0
AM (# 29)	2113461.8	3302.54			18.33	656.57	4.0	8.2	4.2	0.0
GM (# 29)	64561.9	2853.04			5.71	199.77	2.6	5.0	3.0	1.0
SGM (# 29)	75046.4	2874.15								

Table 25: LB testset: geom-64 settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
A1C1S1	220226	3600.00	8316.05	11638.9	28.55	72.38	9	12	3	0
A2C1S1	153342	3600.00	8545.7	11040.7	22.60	117.85	11	14	3	0
arki001	656020	3600.00	7.5799e+06	7.5822e+06	0.03	16.04	6	10	4	0
B1C1S1	86928	3600.00	15077.3	25458.3	40.78	224.33	12	18	6	0
B2C1S1	53573	3600.00	15390	26167.3	41.19	222.98	8	11	3	0
biella1	3330	1785.38	3.06501e+06	3.06501e+06	0.00	12.19	16	24	8	0
core2536-691	148	396.21	689	689	0.00	8.09	5	10	5	0
core2586-950	2189	3600.00	935.922	971	3.61	109.43	4	8	4	0
core4284-1064	1438	3600.15	1054.05	1080	2.40	120.00	4	8	4	0
core4872-1529	22	3600.00	1511.46	1579	4.28	152.98	5	9	4	0
danoint	57608	3600.00	62.7143	65.6667	4.50	15.98	8	12	4	0
glass4	442757	3600.00	8.00003e+08	1.62001e+09	50.62	1457.82	11	15	4	0
markshare1	7171735	3608.93	9.09495e-13	12	—	2884.36	6	9	3	0
markshare2	10988649	3621.39	1.81899e-12	17	—	2030.34	9	12	3	0
mkc	78869	3600.00	-565.51	-561.926	0.63	19.11	9	11	2	0
net12	2966	3440.20	214	214	0.00	189.74	4	9	5	0
NSR8K	2	3600.31	1.75011e+07	2.14155e+07	18.28	836.07	4	9	5	0
nsrand_ipx	427422	3600.00	50518.4	52000	2.85	71.75	8	13	5	0
rail507	85755	3600.00	172.146	174	1.07	17.19	5	9	4	0
roll3000	754688	3600.00	12432.3	12904	3.66	11.36	7	11	4	0
seymour	20984	3600.00	408.393	424	3.68	27.79	5	9	4	0
sp97ar	30271	3600.12	6.53863e+08	6.62299e+08	1.27	51.36	14	18	4	0
sp97ic	6453	3600.00	4.23107e+08	4.39447e+08	3.72	195.95	7	10	3	0
sp98ar	14726	3600.01	5.26435e+08	5.30243e+08	0.72	65.88	18	22	4	0
sp98ic	6986	3600.00	4.44916e+08	4.50843e+08	1.31	99.18	8	11	3	0
swath	173498	3600.00	379.657	495.017	23.30	235.55	8	11	3	0
tr12-30	220967	3600.00	130263	130596	0.25	0.65	11	17	6	0
UMTS	167514	3600.00	2.99907e+07	3.00923e+07	0.34	11.01	10	15	5	0
van	738	3600.02	1.73065	6.1185	71.71	754.70	0	2	2	0
AM (# 29)	752751.9	3422.51			11.43	345.97	8.0	12.0	4.0	0.0
GM (# 29)	26475.8	3252.38			3.81	81.66	7.1	11.1	3.9	1.0
SGM (# 29)	33148.0	3264.23								

Table 26: LB testset: geom-heur settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
A1C1S1	156347	3600.03	10002.3	11557.2	13.45	87.11	7	46	39	0
A2C1S1	139584	3600.00	9347.15	10897.8	14.23	68.67	12	64	52	0
arki001	0	0.00	1e+20	1e+20	—	—	0	0	0	0
B1C1S1	137165	3600.00	19159.9	25630.8	25.25	241.61	3	39	36	0
B2C1S1	89768	3600.00	17667.1	26412.4	33.11	237.46	9	66	57	0
biella1	6538	769.93	3.06501e+06	3.06501e+06	0.00	14.43	5	46	41	0
core2536-691	423	157.18	689	689	0.00	8.43	0	0	0	0
core2586-950	5566	3600.46	936.406	955	1.95	73.19	4	18	14	0
core4284-1064	1511	3600.36	1054.4	1072	1.64	100.17	5	22	17	0
core4872-1529	999	3600.01	1512.14	1546	2.19	125.41	6	22	16	0
danoint	948986	3600.00	64.421	65.6667	1.90	18.88	2	28	26	0
glass4	4395253	3600.01	8.32438e+08	1.50001e+09	44.50	387.77	9	232	223	0
markshare1	26490451	3600.07	9.09495e-13	10	—	2563.42	0	43	43	0
markshare2	24942932	3600.09	1.81899e-12	14	—	1361.95	0	63	63	0
mkc	246286	3600.00	-564.95	-562.932	0.36	22.78	11	25	14	0
net12	3607	1562.91	214	214	0.00	241.88	0	11	11	0
NSR8K	64	3600.01	1.75034e+07	1.27263e+08	86.25	3294.72	0	0	0	0
nsrand_ipx	464485	3600.00	50908.9	51200	0.57	54.84	6	81	75	0
rail507	1315	185.03	174	174	0.00	10.79	1	15	14	0
roll3000	670009	3600.00	12829.5	12890	0.47	9.88	1	54	53	0
seymour	58795	3600.01	414.489	424	2.24	18.15	3	16	13	0
sp97ar	87656	3600.00	6.56034e+08	6.74214e+08	2.70	111.48	3	23	20	0
sp97ic	132314	3600.00	4.25189e+08	4.34571e+08	2.16	66.81	4	23	19	0
sp98ar	69715	3600.00	5.2791e+08	5.30438e+08	0.48	65.80	7	53	46	0
sp98ic	135531	3600.87	4.47259e+08	4.50519e+08	0.72	48.51	5	39	34	0
swath	215978	3600.00	388.001	467.407	16.99	261.63	4	44	40	0
tr12-30	1238981	3600.00	130516	130596	0.06	0.51	1	85	84	0
UMTS	453462	3600.00	3.00443e+07	3.00935e+07	0.16	3.87	3	120	117	0
van	606	3600.02	1.97137	5.08845	61.26	333.84	0	7	7	0
AM (# 28)	2181940.2	3181.32			11.17	351.25	4.0	45.9	41.9	0.0
GM (# 28)	62466.2	2659.74			3.17	73.05	3.0	28.5	25.3	1.0
SGM (# 28)	66162.5	2678.83								

Table 27: LB testset: geom-heur-infer settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
A1C1S1	202628	3600.00	10047.4	11566.6	13.13	97.21	3	59	56	0
A2C1S1	200313	3600.00	9302.49	10994.3	15.39	116.62	9	86	77	0
arki001	914794	3600.00	7.58033e+06	7.58081e+06	0.01	12.35	10	344	334	0
B1C1S1	151092	3600.00	19046.1	25123.5	24.19	257.98	4	42	38	0
B2C1S1	149573	3600.00	18579.6	25926.6	28.34	175.95	2	70	68	0
biella1	27068	1161.16	3.06501e+06	3.06501e+06	0.00	14.42	3	59	56	0
core2536-691	423	156.33	689	689	0.00	8.42	0	0	0	0
core2586-950	7465	3600.01	936.531	960	2.44	79.71	3	16	13	0
core4284-1064	4246	3600.01	1054.43	1073	1.73	106.91	5	20	15	0
core4872-1529	2038	3600.01	1512.14	1560	3.07	133.47	3	18	15	0
danoint	824948	3600.00	63.7933	65.6667	2.85	14.94	0	51	51	0
glass4	5276805	3600.02	9.00006e+08	1.55001e+09	41.94	410.52	3	211	208	0
markshare1	26557136	3600.07	9.09495e-13	10	—	2563.40	0	43	43	0
markshare2	24862811	3600.09	1.81899e-12	14	—	1361.36	0	63	63	0
mkc	356135	3600.00	-564.75	-560.852	0.69	29.97	8	27	19	0
net12	3746	1533.74	214	214	0.00	243.04	0	11	11	0
NSR8K	64	3600.00	1.75034e+07	1.27263e+08	86.25	3296.19	0	0	0	0
nsrand_ipx	435171	2575.34	51200	51200	0.00	29.71	6	73	67	0
rail507	11708	315.90	174	174	0.00	10.86	0	29	29	0
roll3000	567812	3180.36	12890	12890	0.00	11.81	0	125	125	0
seymour	67536	3600.00	414.34	425	2.51	30.95	3	41	38	0
sp97ar	82372	3600.00	6.55901e+08	6.64157e+08	1.24	90.04	9	66	57	0
sp97ic	159319	3600.00	4.2513e+08	4.32663e+08	1.74	55.89	2	30	28	0
sp98ar	114080	3600.00	5.28068e+08	5.30489e+08	0.46	39.97	6	77	71	0
sp98ic	223138	3600.00	4.47446e+08	4.49227e+08	0.40	26.40	5	54	49	0
swath	264198	3600.00	388.026	481.953	19.49	214.14	2	50	48	0
tr12-30	1297099	3600.00	130528	130596	0.05	0.48	0	87	87	0
UMTS	814096	3600.00	3.00555e+07	3.00921e+07	0.12	2.98	4	157	153	0
van	1570	3600.02	1.86409	5.08845	63.37	333.07	0	7	7	0
AM (# 29)	2192392.6	3162.86			10.67	336.89	3.1	66.1	63.0	0.0
GM (# 29)	97173.7	2730.83			3.06	66.06	2.5	38.0	35.4	1.0
SGM (# 29)	101791.8	2747.74								

Table 28: LB testset: geom-heur-64 settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
A1C1S1	187382	3600.00	10128.3	11590.5	12.62	70.19	3	13	10	0
A2C1S1	197405	3600.00	9492.38	10909.9	12.99	69.29	5	15	10	0
arki001	1214159	3600.00	7.5803e+06	7.58081e+06	0.01	12.15	6	43	37	0
B1C1S1	141973	3600.00	19271.2	25042.6	23.05	190.65	3	14	11	0
B2C1S1	64252	3600.00	17835.4	26002.1	31.41	236.30	10	28	18	0
biella1	11680	1004.62	3.06501e+06	3.06501e+06	0.00	15.35	3	14	11	0
core2536-691	423	157.16	689	689	0.00	8.43	0	0	0	0
core2586-950	3134	3600.00	936.118	966	3.09	90.84	1	5	4	0
core4284-1064	898	3600.42	1054.4	1079	2.28	121.08	4	8	4	0
core4872-1529	896	3600.00	1512.14	1575	3.99	143.31	1	5	4	0
danoint	934928	3600.01	64.2627	65.6667	2.14	14.02	2	5	3	0
glass4	5385256	3600.02	9.25007e+08	1.56668e+09	40.96	469.06	11	76	65	0
markshare1	25764997	3600.08	9.09495e-13	10	—	2565.57	2	22	20	0
markshare2	24654307	3600.07	1.81899e-12	10	—	743.68	1	34	33	0
mkc	308071	3600.00	-564.929	-561.326	0.64	28.80	10	17	7	0
net12	4801	1810.43	214	214	0.00	241.76	0	3	3	0
NSR8K	64	3600.01	1.75034e+07	1.27263e+08	86.25	3295.17	0	0	0	0
nsrand_ipx	572158	3600.00	50909	51200	0.57	71.49	5	37	32	0
rail507	1433	182.37	174	174	0.00	10.77	1	4	3	0
roll3000	667394	3600.00	12789.8	12890	0.78	9.45	5	28	23	0
seymour	62429	3600.00	414.501	424	2.24	18.90	3	6	3	0
sp97ar	45897	3600.01	6.55763e+08	6.73642e+08	2.65	104.54	6	14	8	0
sp97ic	109899	3600.00	4.25114e+08	4.39022e+08	3.17	92.72	3	10	7	0
sp98ar	87915	3600.00	5.28013e+08	5.30252e+08	0.42	26.69	7	24	17	0
sp98ic	127523	3600.00	4.4722e+08	4.53627e+08	1.41	60.02	5	18	13	0
swath	215272	3600.00	388.431	477.565	18.66	225.73	7	26	19	0
tr12-30	1273572	3600.00	130532	130596	0.05	0.52	0	22	22	0
UMTS	761859	3600.00	3.00644e+07	3.00917e+07	0.09	5.86	5	31	26	0
van	623	3600.01	1.97137	5.08845	61.26	332.73	0	2	2	0
AM (# 29)	2165537.9	3212.25			10.71	319.86	3.8	18.1	14.3	0.0
GM (# 29)	70454.3	2724.80			3.26	68.14	2.9	11.6	8.8	1.0
SGM (# 29)	74586.6	2743.64								

Table 29: QUBO testset: default settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
chim8-4.10	648180	3600.00	-1202	-810	32.61	349.51	0	0	0	0
chim8-4.11	676368	3600.00	-1121	-764	31.85	154.63	0	0	0	0
chim8-4.12	730401	3600.00	-1140	-746	34.56	306.45	0	0	0	0
chim8-4.13	695240	3600.00	-1165	-818	29.79	158.69	0	0	0	0
chim8-4.14	639119	3600.01	-1165	-806	30.82	63.73	0	0	0	0
chim8-4.15	668688	3600.00	-1269	-836	34.12	243.45	0	0	0	0
chim8-4.16	591363	3600.00	-1133	-834	26.39	164.71	0	0	0	0
chim8-4.17	609060	3600.00	-1093	-802	26.62	102.74	0	0	0	0
chim8-4.18	422767	3600.00	-1162.5	-856	26.37	63.44	0	0	0	0
chim8-4.19	703389	3600.00	-1228.17	-870	29.16	200.21	0	0	0	0
chim8-4.1	771936	3600.00	-1158	-796	31.26	153.80	0	0	0	0
chim8-4.20	853232	3600.00	-1227	-818	33.33	249.46	0	0	0	0
chim8-4.21	463857	3600.02	-1110	-818	26.31	98.70	0	0	0	0
chim8-4.22	821578	3600.00	-1241	-816	34.25	122.65	0	0	0	0
chim8-4.23	714016	3600.00	-1184	-780	34.12	196.42	0	0	0	0
chim8-4.24	672659	3600.00	-1151.5	-840	27.05	222.79	0	0	0	0
chim8-4.25	709647	3600.02	-1214	-880	27.51	91.18	0	0	0	0
chim8-4.26	734488	3600.00	-1228	-796	35.18	190.58	0	0	0	0
chim8-4.27	624205	3600.00	-1152	-834	27.60	54.54	0	0	0	0
chim8-4.28	612601	3600.00	-1132.5	-782	30.95	82.38	0	0	0	0
chim8-4.29	716359	3600.00	-1186.67	-790	33.43	193.31	0	0	0	0
chim8-4.2	621465	3600.00	-1103.5	-776	29.68	192.62	0	0	0	0
chim8-4.30	536177	3600.00	-1145.33	-842	26.48	200.06	0	0	0	0
chim8-4.31	791325	3600.00	-1242	-870	29.95	137.13	0	0	0	0
chim8-4.32	865918	3600.00	-1226	-790	35.56	192.83	0	0	0	0
chim8-4.33	664187	3600.01	-1189.5	-884	25.68	98.37	0	0	0	0
chim8-4.34	569663	3600.00	-1184.5	-890	24.86	24.93	0	0	0	0
chim8-4.35	628308	3600.00	-1111.17	-782	29.62	98.58	0	0	0	0
chim8-4.36	733903	3600.00	-1192	-788	33.89	133.09	0	0	0	0
chim8-4.37	673627	3600.00	-1082	-790	26.99	64.70	0	0	0	0
chim8-4.38	703562	3600.00	-1145	-806	29.61	349.63	0	0	0	0
chim8-4.39	546777	3600.01	-1201	-856	28.73	79.57	0	0	0	0
chim8-4.3	664381	3600.00	-1123	-784	30.19	281.50	0	0	0	0
chim8-4.40	653650	3600.00	-1114	-790	29.08	82.93	0	0	0	0
chim8-4.41	484170	3600.01	-1173	-880	24.98	208.96	0	0	0	0
chim8-4.42	571781	3600.00	-1029.5	-658	36.09	190.73	0	0	0	0
chim8-4.43	892696	3600.00	-1125	-734	34.76	108.10	0	0	0	0
chim8-4.44	518871	3600.00	-1057	-742	29.80	145.67	0	0	0	0
chim8-4.45	574756	3600.01	-1142	-818	28.37	251.72	0	0	0	0
chim8-4.46	655568	3600.00	-1194.5	-854	28.51	154.63	0	0	0	0
chim8-4.47	814916	3600.00	-1196	-848	29.10	136.63	0	0	0	0
chim8-4.48	671200	3600.00	-1138.67	-826	27.46	93.99	0	0	0	0
chim8-4.49	759863	3600.00	-1176.33	-800	31.99	159.79	0	0	0	0
chim8-4.4	574884	3600.00	-1150.33	-806	29.93	291.07	0	0	0	0
chim8-4.50	633082	3600.00	-1142	-822	28.02	114.15	0	0	0	0
chim8-4.5	772508	3600.00	-1210	-850	29.75	208.06	0	0	0	0
chim8-4.6	570657	3600.00	-1112	-790	28.96	218.40	0	0	0	0
chim8-4.7	519471	3600.00	-1137	-756	33.51	301.46	0	0	0	0
chim8-4.8	742776	3600.00	-1154	-786	31.89	247.96	0	0	0	0
chim8-4.9	693587	3600.00	-1208	-850	29.64	154.62	0	0	0	0
AM (# 50)	663657.6	3600.00			30.13	167.73	0.0	0.0	0.0	0.0
GM (# 50)	655578.6	3600.00			29.98	147.98	1.0	1.0	1.0	1.0
SGM (# 50)	655679.9	3610.00								

Table 30: QUBO testset: geom-64 settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-$\int$	#impr	#sub	#phases	#exh
chim8-4.10	196015	3600.00	-1268	-896	29.34	17.69	8	11	3	0
chim8-4.11	268683	3600.00	-1187	-768	35.30	113.25	5	8	3	0
chim8-4.12	292255	3600.00	-1206	-774	35.82	161.68	5	8	3	0
chim8-4.13	192246	3600.00	-1231	-848	31.11	17.80	7	10	3	0
chim8-4.14	214758	3600.00	-1231	-770	37.45	218.33	6	9	3	0
chim8-4.15	152705	3600.00	-1329	-896	32.58	14.85	6	9	3	0
chim8-4.16	143306	3600.00	-1199	-852	28.94	70.07	12	16	4	0
chim8-4.17	196313	3600.00	-1159	-732	36.84	376.12	6	9	3	0
chim8-4.18	152540	3600.00	-1222.5	-808	33.91	268.02	6	9	3	0
chim8-4.19	189747	3600.00	-1294.17	-906	29.99	35.28	10	13	3	0
chim8-4.1	204592	3600.00	-1224	-822	32.84	58.91	5	8	3	0
chim8-4.20	209850	3600.00	-1299	-874	32.72	31.09	10	13	3	0
chim8-4.21	296035	3600.00	-1168.5	-816	30.17	120.46	8	12	4	0
chim8-4.22	282367	3600.00	-1301	-834	35.90	53.31	6	9	3	0
chim8-4.23	120964	3600.00	-1250	-824	34.08	39.57	11	14	3	0
chim8-4.24	162852	3600.00	-1217.5	-834	31.50	199.48	6	10	4	0
chim8-4.25	8794	3600.00	-1274	-872	31.55	115.80	9	12	3	0
chim8-4.26	50917	3600.00	-1294	-838	35.24	66.06	12	15	3	0
chim8-4.27	172147	3600.00	-1218	-844	30.71	36.28	8	11	3	0
chim8-4.28	189833	3600.00	-1198.5	-746	37.76	249.80	7	10	3	0
chim8-4.29	366220	3600.00	-1252.67	-820	34.54	74.38	9	12	3	0
chim8-4.2	215463	3600.00	-1169.5	-766	34.50	193.50	7	10	3	0
chim8-4.30	264561	3600.00	-1211.33	-808	33.30	341.05	5	8	3	0
chim8-4.31	164735	3600.00	-1308	-890	31.96	58.76	8	11	3	0
chim8-4.32	260500	3600.00	-1292	-830	35.76	11.09	8	11	3	0
chim8-4.33	270667	3600.00	-1255.5	-830	33.89	275.15	5	8	3	0
chim8-4.34	163164	3600.00	-1249.5	-882	29.41	53.55	12	17	5	0
chim8-4.35	183583	3600.00	-1177.17	-798	32.21	83.99	9	12	3	0
chim8-4.36	299805	3600.00	-1258	-816	35.14	13.44	6	9	3	0
chim8-4.37	267413	3600.00	-1148	-798	30.49	12.11	6	9	3	0
chim8-4.38	338694	3600.00	-1211	-796	34.27	266.92	6	10	4	0
chim8-4.39	324773	3600.00	-1266	-866	31.60	20.70	8	12	4	0
chim8-4.3	70610	3600.00	-1189	-840	29.35	145.74	9	12	3	0
chim8-4.40	47456	3600.00	-1180	-760	35.59	303.52	12	16	4	0
chim8-4.41	198664	3600.00	-1244	-890	28.46	36.67	11	14	3	0
chim8-4.42	14964	3600.00	-1095.5	-694	36.65	210.25	8	11	3	0
chim8-4.43	249406	3600.00	-1191	-740	37.87	86.04	5	8	3	0
chim8-4.44	337753	3600.00	-1122	-704	37.25	328.52	6	9	3	0
chim8-4.45	202939	3600.00	-1206	-846	29.85	26.31	9	12	3	0
chim8-4.46	107529	3600.00	-1254.5	-854	31.93	127.97	7	10	3	0
chim8-4.47	210310	3600.00	-1262	-856	32.17	66.09	8	11	3	0
chim8-4.48	208566	3600.00	-1204.67	-834	30.77	12.20	6	9	3	0
chim8-4.49	222838	3600.00	-1242.33	-820	34.00	15.08	6	9	3	0
chim8-4.4	214306	3600.00	-1215.33	-876	27.92	24.66	10	13	3	0
chim8-4.50	254204	3600.00	-1208	-802	33.61	189.08	9	12	3	0
chim8-4.5	21701	3600.00	-1276	-882	30.88	58.00	11	14	3	0
chim8-4.6	198869	3600.00	-1178	-798	32.26	189.74	6	9	3	0
chim8-4.7	122949	3600.00	-1203	-810	32.67	134.23	11	14	3	0
chim8-4.8	212342	3600.00	-1220	-822	32.62	26.20	8	11	3	0
chim8-4.9	142054	3600.00	-1274	-810	36.42	265.71	6	9	3	0
AM (# 50)	197079.1	3600.00			33.02	118.31	7.8	11.0	3.2	0.0
GM (# 50)	165289.6	3600.00			32.92	73.47	7.5	10.7	3.1	1.0
SGM (# 50)	165454.4	3610.00								

Table 31: QUBO testset: geom-heur settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
chim8-4.10	668938	3600.03	-1202	-812	32.45	341.10	2	13	11	0
chim8-4.11	581297	3600.00	-1121	-748	33.27	200.66	4	15	11	0
chim8-4.12	650689	3600.00	-1140	-786	31.05	105.69	3	14	11	0
chim8-4.13	716084	3600.02	-1165	-798	31.50	218.45	4	15	11	0
chim8-4.14	546393	3600.01	-1165	-818	29.79	36.39	7	43	36	0
chim8-4.15	691768	3600.00	-1269	-868	31.60	115.85	2	13	11	0
chim8-4.16	575766	3600.00	-1133	-814	28.16	205.43	2	13	11	0
chim8-4.17	376936	3600.00	-1099	-810	26.30	81.23	6	37	31	0
chim8-4.18	450236	3600.01	-1162.5	-868	25.33	15.19	3	14	11	0
chim8-4.19	618858	3600.00	-1228.17	-886	27.86	84.45	3	14	11	0
chim8-4.1	355630	3600.00	-1164	-830	28.69	109.88	9	42	33	0
chim8-4.20	589196	3600.00	-1233	-878	28.79	51.51	6	36	30	0
chim8-4.21	443920	3600.00	-1110	-836	24.68	22.32	2	13	11	0
chim8-4.22	725467	3600.00	-1241	-820	33.92	110.87	3	26	23	0
chim8-4.23	708370	3600.01	-1184	-788	33.45	168.59	3	14	11	0
chim8-4.24	379117	3600.00	-1157.5	-862	25.53	87.43	3	35	32	0
chim8-4.25	491530	3600.00	-1214	-872	28.17	80.46	5	27	22	0
chim8-4.26	679170	3600.00	-1228	-792	35.50	205.45	4	15	11	0
chim8-4.27	484468	3600.00	-1152	-846	26.56	16.03	3	30	27	0
chim8-4.28	560329	3600.00	-1132.5	-792	30.07	34.16	3	26	23	0
chim8-4.29	682706	3600.02	-1186.67	-792	33.26	186.18	3	14	11	0
chim8-4.2	497916	3600.01	-1104	-794	28.08	60.53	4	15	11	0
chim8-4.30	456818	3600.00	-1145.83	-870	24.07	87.38	2	25	23	0
chim8-4.31	458565	3600.00	-1248	-900	27.88	63.35	8	40	32	0
chim8-4.32	603844	3600.00	-1226	-824	32.79	144.28	7	40	33	0
chim8-4.33	493844	3600.00	-1189.5	-878	26.19	83.50	2	36	34	0
chim8-4.34	535333	3600.01	-1184.5	-872	26.38	80.16	4	27	23	0
chim8-4.35	582888	3600.00	-1111.17	-788	29.08	51.30	3	14	11	0
chim8-4.36	673071	3600.00	-1192	-792	33.56	111.25	3	14	11	0
chim8-4.37	620487	3600.00	-1082	-760	29.76	176.53	4	15	11	0
chim8-4.38	316563	3600.00	-1153.33	-856	25.78	154.76	8	41	33	0
chim8-4.39	522388	3600.00	-1202	-850	29.28	70.48	2	14	12	0
chim8-4.3	595230	3600.00	-1123	-790	29.65	218.74	3	14	11	0
chim8-4.40	572739	3600.00	-1114	-788	29.26	79.79	5	28	23	0
chim8-4.41	536184	3600.01	-1173	-846	27.88	185.65	3	14	11	0
chim8-4.42	514907	3600.01	-1029.5	-678	34.14	89.63	3	14	11	0
chim8-4.43	789877	3600.00	-1131	-756	33.16	4.58	2	13	11	0
chim8-4.44	401479	3600.00	-1062	-772	27.31	174.52	7	35	28	0
chim8-4.45	487644	3600.00	-1142	-842	26.27	29.46	4	28	24	0
chim8-4.46	585643	3600.00	-1194.5	-840	29.68	152.47	4	39	35	0
chim8-4.47	475914	3600.00	-1202	-868	27.79	96.40	5	37	32	0
chim8-4.48	539025	3600.00	-1138.67	-812	28.69	98.61	2	14	12	0
chim8-4.49	729177	3600.00	-1176.33	-760	35.39	268.89	3	14	11	0
chim8-4.4	517554	3600.01	-1150.83	-828	28.05	203.50	5	28	23	0
chim8-4.50	591489	3600.01	-1142	-814	28.72	124.92	4	15	11	0
chim8-4.5	782453	3600.00	-1206	-840	30.35	175.13	2	13	11	0
chim8-4.6	456645	3600.00	-1114	-840	24.60	140.89	7	35	28	0
chim8-4.7	486747	3600.00	-1137	-802	29.46	102.72	2	13	11	0
chim8-4.8	705697	3600.00	-1154	-796	31.02	125.84	2	14	12	0
chim8-4.9	393437	3600.00	-1214	-872	28.17	159.86	8	41	33	0
AM (# 50)	558008.5	3600.00			29.37	119.87	4.0	23.2	19.2	0.0
GM (# 50)	546145.9	3600.00			29.23	94.56	3.6	20.9	17.1	1.0
SGM (# 50)	546248.2	3610.00								

Table 32: QUBO testset: geom-heur-infer settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
chim8-4.10	641380	3600.00	-1202	-830	30.95	269.25	3	26	23	0
chim8-4.11	603218	3600.00	-1122	-790	29.59	68.61	6	49	43	0
chim8-4.12	676734	3600.00	-1140	-780	31.58	130.87	2	13	11	0
chim8-4.13	703750	3600.00	-1165	-788	32.36	259.55	3	14	11	0
chim8-4.14	594877	3600.00	-1165	-800	31.33	140.51	7	52	45	0
chim8-4.15	622063	3600.01	-1269	-880	30.65	67.85	2	14	12	0
chim8-4.16	504043	3600.00	-1133	-862	23.92	41.95	5	34	29	0
chim8-4.17	488938	3600.00	-1099	-816	25.75	157.74	5	38	33	0
chim8-4.18	452301	3600.00	-1162.5	-870	25.16	5.64	2	14	12	0
chim8-4.19	612063	3600.00	-1228.17	-876	28.67	126.05	3	14	11	0
chim8-4.1	699599	3600.00	-1158	-788	31.95	188.29	2	25	23	0
chim8-4.20	790931	3600.00	-1233	-812	34.14	274.57	2	13	11	0
chim8-4.21	441843	3600.00	-1110.5	-830	25.26	47.56	1	13	12	0
chim8-4.22	700548	3600.01	-1241	-844	31.99	58.69	6	40	34	0
chim8-4.23	796701	3600.00	-1184	-768	35.14	249.37	3	14	11	0
chim8-4.24	538526	3600.00	-1151.5	-862	25.14	91.58	4	35	31	0
chim8-4.25	502765	3600.00	-1214	-858	29.32	136.81	3	14	11	0
chim8-4.26	639759	3600.00	-1228	-826	32.74	174.83	8	41	33	0
chim8-4.27	593281	3600.00	-1152	-834	27.60	56.99	0	24	24	0
chim8-4.28	587242	3600.00	-1132.5	-790	30.24	44.21	4	27	23	0
chim8-4.29	669365	3600.00	-1186.67	-834	29.72	94.26	6	40	34	0
chim8-4.2	497679	3600.00	-1104	-800	27.54	33.38	4	15	11	0
chim8-4.30	522569	3600.00	-1145.83	-864	24.60	111.50	4	27	23	0
chim8-4.31	702875	3600.00	-1242	-882	28.99	198.92	4	28	24	0
chim8-4.32	782241	3600.00	-1226	-818	33.28	131.91	9	42	33	0
chim8-4.33	495357	3600.00	-1195.5	-896	25.05	51.41	6	53	47	0
chim8-4.34	536256	3600.00	-1184.5	-876	26.04	68.67	3	28	25	0
chim8-4.35	598831	3600.00	-1111.17	-774	30.34	112.68	2	13	11	0
chim8-4.36	679045	3600.00	-1192	-776	34.90	180.17	2	14	12	0
chim8-4.37	527982	3600.00	-1084	-798	26.38	72.68	6	51	45	0
chim8-4.38	608647	3600.00	-1145	-792	30.83	277.69	3	26	23	0
chim8-4.39	528434	3600.00	-1202	-846	29.62	86.81	1	13	12	0
chim8-4.3	599760	3600.00	-1123	-790	29.65	218.56	4	15	11	0
chim8-4.40	581406	3600.00	-1114	-802	28.01	71.58	7	49	42	0
chim8-4.41	550930	3600.01	-1173	-836	28.73	223.64	3	15	12	0
chim8-4.42	529435	3600.00	-1029.5	-678	34.14	89.89	3	14	11	0
chim8-4.43	846694	3600.00	-1127	-742	34.16	70.22	1	13	12	0
chim8-4.44	518633	3600.00	-1057	-756	28.48	81.33	2	13	11	0
chim8-4.45	539668	3600.00	-1142	-842	26.27	24.65	4	27	23	0
chim8-4.46	595488	3600.00	-1194.5	-874	26.83	68.77	8	37	29	0
chim8-4.47	705747	3600.00	-1196	-834	30.27	145.01	2	14	12	0
chim8-4.48	554583	3600.00	-1138.67	-812	28.69	98.54	2	14	12	0
chim8-4.49	797962	3600.00	-1172.33	-746	36.37	328.29	2	14	12	0
chim8-4.4	593530	3600.00	-1150.33	-840	26.98	160.02	4	27	23	0
chim8-4.50	500548	3600.00	-1142	-842	26.27	72.05	5	38	33	0
chim8-4.5	732693	3600.00	-1210	-828	31.57	223.33	1	13	12	0
chim8-4.6	538737	3600.00	-1113	-830	25.43	158.46	6	39	33	0
chim8-4.7	466062	3600.00	-1137	-824	27.53	188.63	5	30	25	0
chim8-4.8	647110	3600.00	-1154	-824	28.60	4.82	3	26	23	0
chim8-4.9	707988	3600.00	-1208	-802	33.61	291.83	2	13	11	0
AM (# 50)	606936.3	3600.00			29.45	130.63	3.7	25.5	21.8	0.0
GM (# 50)	599181.4	3600.00			29.28	100.00	3.2	22.5	19.2	1.0
SGM (# 50)	599282.7	3610.00								

Table 33: QUBO testset: geom-heur-64 settings

Name	#Nodes	Time	Dual	Primal	Gap%	Prim-∫	#impr	#sub	#phases	#exh
chim8-4.10	655674	3600.00	-1202	-828	31.11	280.24	2	6	4	0
chim8-4.11	713040	3600.00	-1121	-730	34.88	280.77	3	5	2	0
chim8-4.12	554692	3600.00	-1140	-808	29.12	8.23	4	6	2	0
chim8-4.13	694592	3600.00	-1165	-800	31.33	210.46	3	8	5	0
chim8-4.14	562265	3600.01	-1165	-792	32.02	124.65	4	9	5	0
chim8-4.15	699032	3600.00	-1269	-868	31.60	118.32	1	3	2	0
chim8-4.16	579761	3600.00	-1133	-818	27.80	194.23	1	3	2	0
chim8-4.17	501720	3600.00	-1093	-796	27.17	114.73	5	12	7	0
chim8-4.18	400750	3600.00	-1168.5	-870	25.55	11.81	1	9	8	0
chim8-4.19	617644	3600.00	-1228.17	-866	29.49	164.47	3	5	2	0
chim8-4.1	708124	3600.00	-1158	-798	31.09	144.86	2	4	2	0
chim8-4.20	775061	3600.00	-1233	-850	31.06	120.27	2	4	2	0
chim8-4.21	433205	3600.00	-1110.5	-840	24.36	5.64	1	4	3	0
chim8-4.22	729032	3600.00	-1241	-834	32.80	48.29	2	4	2	0
chim8-4.23	648122	3600.01	-1184	-798	32.60	120.23	4	6	2	0
chim8-4.24	560123	3600.00	-1151.5	-880	23.58	123.37	3	13	10	0
chim8-4.25	322724	3600.00	-1220	-890	27.05	67.98	4	21	17	0
chim8-4.26	800044	3600.00	-1228	-788	35.83	220.88	3	5	2	0
chim8-4.27	586875	3600.00	-1152	-834	27.60	57.24	0	6	6	0
chim8-4.28	615006	3600.02	-1132.5	-798	29.54	11.35	3	7	4	0
chim8-4.29	679717	3600.01	-1186.67	-810	31.74	113.78	2	6	4	0
chim8-4.2	508332	3600.00	-1104	-806	26.99	17.83	4	9	5	0
chim8-4.30	419096	3600.00	-1151.33	-890	22.70	42.62	5	16	11	0
chim8-4.31	664695	3600.00	-1242	-860	30.76	243.80	3	8	5	0
chim8-4.32	769964	3600.00	-1226	-822	32.95	160.70	7	14	7	0
chim8-4.33	555694	3600.00	-1189.5	-870	26.86	110.02	1	6	5	0
chim8-4.34	578712	3600.00	-1184.5	-860	27.40	126.70	2	4	2	0
chim8-4.35	555844	3600.01	-1111.17	-794	28.54	25.07	1	6	5	0
chim8-4.36	665037	3600.00	-1192	-780	34.56	164.12	3	5	2	0
chim8-4.37	479001	3600.00	-1084	-798	26.38	33.73	3	13	10	0
chim8-4.38	527598	3600.00	-1151	-840	27.02	184.16	7	19	12	0
chim8-4.39	532019	3600.01	-1202	-846	29.62	87.59	2	4	2	0
chim8-4.3	470481	3600.01	-1123	-800	28.76	181.69	1	6	5	0
chim8-4.40	601378	3600.00	-1114	-800	28.19	38.87	6	11	5	0
chim8-4.41	506063	3600.01	-1174	-864	26.41	114.40	3	6	3	0
chim8-4.42	514523	3600.01	-1029.5	-684	33.56	58.75	2	4	2	0
chim8-4.43	772715	3600.01	-1131	-752	33.51	23.90	1	6	5	0
chim8-4.44	518210	3600.00	-1057	-764	27.72	48.24	3	7	4	0
chim8-4.45	482937	3600.00	-1142	-834	26.97	81.03	5	11	6	0
chim8-4.46	607464	3600.01	-1194.5	-832	30.35	179.02	3	8	5	0
chim8-4.47	615460	3600.00	-1196	-850	28.93	88.21	2	10	8	0
chim8-4.48	487288	3600.00	-1138.67	-832	26.93	18.14	3	9	6	0
chim8-4.49	721638	3600.00	-1176.33	-786	33.18	154.86	3	5	2	0
chim8-4.4	532357	3600.00	-1150.83	-852	25.97	107.22	4	6	2	0
chim8-4.50	453698	3600.00	-1144	-838	26.75	39.93	2	12	10	0
chim8-4.5	706094	3600.00	-1210	-852	29.59	128.49	1	3	2	0
chim8-4.6	520598	3600.00	-1113	-836	24.89	132.55	4	11	7	0
chim8-4.7	457664	3600.00	-1137	-806	29.11	85.28	3	6	3	0
chim8-4.8	650488	3600.01	-1154	-814	29.46	68.65	6	11	5	0
chim8-4.9	666989	3600.00	-1208	-828	31.46	186.07	2	4	2	0
AM (# 50)	587584.8	3600.00			29.26	109.49	2.9	7.7	4.8	0.0
GM (# 50)	577241.2	3600.00			29.10	79.41	2.5	6.9	4.0	1.0
SGM (# 50)	577343.1	3610.00								