Improving Public Transport Accessibility via Provision of a Dial-a-Ride Shuttle-Bus Service, Incorporating Passenger Travel-Mode Heterogeneity.

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Abstract

In many real-world transportation systems, passengers are often required to make a number of interchanges between different modes of transport. As cities continue to grow, a greater number of these connections tend to occur within centrally located Transport Hubs. In order to encourage the uptake of public transport in major cities, it is important for passengers to be able to connect between travel modes efficiently; thus it is desirable from a transport planning perspective to improve the synchronisation of connecting services at a Transport Hub. In this paper we consider the problem of designing shuttle-bus routes for passengers intending to connect with one of four different modes of transport at a Transport Hub; in such a way so as to minimise the total travel time and the missed connection costs, whilst incorporating time-of-day effects and passenger heterogeneity with respect to value-of-time. This problem is modelled as a capacitated Dial-a-Ride problem with time windows and solved via column generation and an efficient label setting algorithm. We compare the results of our approach with that of a classical Dial-a-Ride problem on 40 datasets reflecting different time-of-day passenger requests. Results indicate that our approach has the potential to achieve an average 48% improvement in total cost over the classical approach.

1 Introduction

As the world's major cities continue to expand, there is an increasingly urgent need to reduce traffic congestion and encourage the uptake of public transport. In a number of major cities, the public transport infrastructure has been unable to keep pace with the growth in the number of commuters; having typically developed in a radial pattern, in response to urban sprawl. Consequently, access to public transport has become increasingly difficult for commuters living greater distances from the city centre, with many instead opting for private motor-vehicle as their primary mode of transport. In March 2012, the Australian Bureau of Statistics (2012) reported that 78% of the Australian adult population opted for private motor-vehicle as their primary mode of transport, with only 16% making use of public transport. Moreover, of the 8 million people for whom private motor-vehicle is their preferred mode, only 23% included additional passengers as part of their trip. Similar figures have been observed internationally, with Eurostat (2014) reporting that in a recent analysis for the European Union, almost 83.3% of inland passenger transport was attributed to users opting for private motor-vehicle over public transport. Such reliance on the motor-vehicle, and in particular the increasing prevalence of single-person vehicles has led to increasing congestion in cities worldwide. Questions have been raised as to the sustainability of current levels, leading to an increased interest in ride-sharing schemes; however, as stated by Archetti et al. (2015), the underlying cause of the the low patronage for public transport has not yet been properly addressed. The economic and environmental benefits of improving the operation of public transport are substantial. In addition to reducing fuel consumption, carbon footprint and the number of cars on the road, it was estimated by APTA (2015) that in 2011 the public transportation system in the U.S. saved 865 million travel-time hours, and helped to prevent an increase of approximately $21 billion in congestion costs across 498 urban areas.

Recent studies by Buehler (2011) and Archetti et al. (2015) have identified that a greater focus on the needs of potential passengers is essential for changing current public perceptions of public transport and facilitating greater levels of patronage. A comparison of the modal split between Germany and the USA by Buehler (2011) highlighted a number of key factors that were considered barriers to switching to public transport; observing that even for those commuters living close to public transport services, a large number preferred to drive due to (i) the inability of being able to connect with the transport service easily as a result of inaccessibility (lack of safe footpaths, crosswalks or cycleways), and (ii) the low frequency of the public transport services offered, being limited to a few trains or buses per day with lengthy waiting time and poor connectivity between them. Furthermore, the dynamic nature of modern travel patterns have resulted in the expansion of demand-responsive-transportation, from services that were traditionally only utilised by the elderly and disabled, to those used by daily commuters as part of their journey to work. The results of a simulation study by Archetti et al. (2015) in which such an on-demand shuttle-bus service exists alongside conventional fixed bus services indicated that the on-demand service has the potential to attract a greater number of regular customers, due to its flexibility, decreased travel time and cost over a conventional fixed service; in addition to reducing congestion and pollution.

Therefore, to encourage the uptake of public transport in urban centres it is incumbent upon transport planners to enhance connectivity between different transport modes. In areas without an existing transport infrastructure, such as an existing rail line, buses are typically used to service the population. However, if required to serve an expansive area there is usually a trade-off between service frequency, area coverage and route length. This inflexibility may be considered undesirable to many potential passengers, resulting in additional car usage. Furthermore, in many cities, a number of transport modes converge at centrally located Transport Hubs, such as an airport or central railway station, offering passengers connections with aircraft, inter-city and suburban trains, buses, trams and ferries, with passengers wishing to travel from the outskirts of the city to the Transport Hub. An adaption of the shuttle-bus service proposed in Archetti et al. (2015) in which passengers are transferred to a Transport Hub, can be used to allow commuters in areas without existing transport infrastructure to connect easily with another mode at the Transport Hub, reducing the need for private motor-vehicle usage.

In this paper we propose to address this problem through the design of more flexible and frequent Dial-a-Ride shuttle-bus services whose destination is a Transport Hub. These services may be used to complement or replace an existing fixed-route bus service. We assume that passengers lodge their request at least one hour before they wish to travel, and specify a desired arrival time window for pick-up, their connecting mode and the latest time at which they can arrive at the Hub to make their outbound connection. We seek to design routes and allocate people to buses, in such a way so as to minimise the total travel time and missed connection cost. To ensure that such a service caters to the needs of potential passengers, we capture passenger heterogeneity via the inclusion of their desired time window for pick-up, and also their value-of-time. Since different modal connections are more important than others due to their frequency and cost of cancellation, e.g. an aircraft connection, compared with a suburban train connection. We attach a priority to each passenger's outbound connection according to the level of importance and penalise missed connections with greater priority more heavily.

1.1 The Vehicle Routing Problem with Time Windows

The Vehicle Routing Problem with Time Windows (VRPTW) was originally proposed in the seminal work of Solomon (1983). The VRPTW is the problem of designing routes for a set of vehicles to service a fixed number of customers in a minimal cost way and is a generalisation of the well-studied Vehicle Routing Problem (VRP), with the additional complexity of ensuring that each customer is served within a predefined time window. The vehicles are stationed at a central depot and each route begins and ends at the central depot, servicing a subset of the customers, subject to capacity and time-window constraints. Each customer is assigned to exactly one vehicle and visited exactly once.

For a detailed survey of the literature surrounding the VRPTW problem and its variants, the reader is referred to Desrosiers et al. (1995) and Desaulniers et al. (2002). Solving the VRPTW is of enormous practical importance, with many real-world logistical operations requiring the pick-up and delivery of goods within specified time windows, in a minimal cost way. Examples include school bus scheduling Fügenschuh (2011), air-cargo Abraham et al. (2012) and replenishment of perishable food items Coelho and Laporte (2013).

1.2 Variants of the Classical VRPTW

In recent years, the VRPTW has been modified and improved for specific real-world networks, with many variants and extensions proposed in the literature. One such variant is the Pickup Delivery Problem with Time Windows (PDPTW), which is a generalisation of the VRPTW as each customer request has its own pickup-delivery location pair. This location pairing necessitates the inclusion of pairing and precedence constraints; The former ensures that each pair is served by the same vehicle while the latter ensures that for each pair, pickup must occur prior to delivery. Examples of problems that can be modelled and solved as PDPTW include taxi dispatch modelling by Wang et al. (2009) and personnel transportation within a set of oil platforms by Velasco et al. (2009). For reviews of the literature surrounding the PDPTW problem and its variants, the reader is referred to Desaulniers et al. (2002) and Cordeau et al. (2007).

The Dial-a-Ride Problem (DARP) is a generalisation of the PDPTW in which a person is transported instead of an item, with each passenger having an individual time-window for pick-up and ride-time constraint. The inclusion of passenger-centric constraints and requirements means that the DARP differs from most classical vehicle routing problems in so far as it is often necessary, particularly in medical applications to reduce user inconvenience and waiting time, which is balanced by the objective of minimisation of operating costs. It was noted by Pacquette et al. (2013) that customers' perceptions of quality-of-service are sensitive to (i) an inconvenient arrival time at the destination, (ii) the waiting time at the pickup node, (iii) the ratio of realised trip time to the time of the most-direct trip. According to the review paper by Cordeau and Laporte (2007), the DARP has traditionally been applied in the context of door-to-door transportation and services for the elderly or disabled; examples include the study by Borndörfer et al. (1999) involving the provision of services for the disabled, the Dial-a-Flight Problem by Espinoza et al. (2008) and Parragh (2011) who proposes a heterogeneous patient allocation model for the Austrian Red Cross. Such services may operate in a static way, with all requests known prior to operation, or in a dynamic way, with requests gradually revealed throughout the day. However, as noted by Borndörfer et al. (1999), the distinction between static and dynamic DARPs is often blurred in practice due to the cancellation of requests, with transport operators allowing the introduction of new requests in a solution designed for a static problem. Moreover, Cordeau and Laporte (2003) note that dynamic DARPs are often rare in practice, as at least a subset of requests are known in advance.

1.3 Multiple Trips and Inclusion of Heterogeneity

One of the drawbacks of both the classical VRP and the VRPTW is the implicit assumption that each vehicle performs only a single routing over a specified planning period (e.g. one working day). To address this issue, a number of authors have proposed modifications to the classical VRP, allowing for multiple trips for each vehicle and by extension, driver within the given work-period. Such a scenario is often useful in practice as it allows greater utilisation of the both vehicles and drivers, and thus provides significant cost savings. This problem has been formulated by Taillard et al. (1996) and Brandão and Mercer (1997) and solved using Tabu search heuristics. Olivera and Viera (2007) propose an adaptive memory algorithm, and Salhi and Petch (2007) employ a hybrid genetic algorithm for its solution. More recently, Azi et al. (2010) proposed an exact algorithm for the VRPTW with multiple use of vehicles. The authors use a set-partitioning formulation and solve the problem using column generation. Their objective is to maximise profit whilst simultaneously minimising total distance. Crainic et al. (2012) extend this idea to the VRPTW and propose a multi-zone, multi-trip VRPTW in which customers are divided into zones according to common attributes, such as geographical proximity, delivery times, type of product etc. Moreover the authors place a restriction on the allowable waiting time at each supply point and the number of times each vehicle can enter each zone. Ceselli et al. (2009) solve a real-world, multi-trip vehicle routing problem with multiple depots with driver maximum duty-length constraints. The model includes heterogeneity in terms of costs associated with each vehicle route and dependency on route factors such as total number of pallets carried, total weights, volume, value, distance and number of stops. A dynamic programming algorithm is used to solve the subproblem in a column generation framework. Seixas and Mendes (2013) propose a variant on the VRPTW in which the authors consider a heterogeneous fleet and restrictions on driver working hours. The fleet is heterogeneous in terms of both capacity and speed, and the cost of the vehicle is a variable depending only on the total distance travelled and the vehicle type. The vehicles are also able to perform multiple trips in a work day. The authors propose two heuristics (a constructive heuristic and a Tabu search) that are used to construct an initial feasible solution allowing a warm-start initialisation for the column generation algorithm. Liu et al. (2015) propose a realistic DARP that simultaneously considers multiple trips, heterogeneous vehicles, multiple request types and configurable vehicle capacity. The authors formulate two-mixed integer programming models, and introduce 8 classed of valid inequalities to improve the bounds of the branch-and-cut algorithm. Parragh (2011) proposes a DARP for heterogeneous users and vehicles motivated by patient transportation in the Austrian Red Cross. The patient is classified as requiring a set, stretcher or wheelchair and an accompanying person may also be present. The transportation requirements of the passengers are matched the configuration of the vehicle to which they are assigned.

1.4 Synchronisation Approaches

In classical VRPTWs, the vehicles routes are independent of one another and therefore route-cost factors accumulate individually along routes. For problems in which certain tasks are required to be synchronised (either temporally or spatially), the interaction between vehicles can be critical. Despite the practical implications of the quality of synchronisation, the topic has yet to be developed deeply in the literature. A recent study by Drexl (2012) classified the different forms of synchronisation in terms of task, operation, movement, load and resource synchronisation, and provides a detailed summary of specific modelling issues and a review of the different forms of synchronisation in the literature. For a detailed summary, the reader is referred to Drexl (2012) and the references therein. A categorisation of different VRPTW models according to temporal and spatial constraints is provided by Mankowska et al. (2011). The authors propose a model for synchronising two vehicles at a fixed set of points using precedence constraints. Although these models include synchronisation, the heterogeneity of items has not yet been considered.

1.5 Exact Solution Approaches

Traditional solution approaches for the VRPTW and DARP have centred around heuristics such as the insertion heuristic by Toth and Vigo (1997). Cordeau and Laporte (2007) propose a tabu search heuristic for the static DARP with route duration constraints that evaluates the capacity of delaying the departure time from the depot to help improve solution feasibility. Pacquette et al. (2013) formulate a metaheuristic, integrating a reference point method for multicriteria optimisation within a tabu search algorithm.

The last few years have witnessed a growing body of literature focusing on the development of exact solution methods, and exact methods used in combination with efficient heuristics. Exact algorithms used to solve the VRPTW and its variants are primarily based on branch-and-cut and branch-and-price algorithms, used in combination with an efficient labelling algorithm to solve the resource constrained shortest path problem. Recent contributions include Ropke et al. (2007) who propose a branch and cut algorithm for the PDPTW based on a two-index MIP model and applied it solve the DARP effectively. Baldacci et al. (2011) also propose a pickup and delivery problem with time windows, and present a new exact algorithm for the based on a set-partitioning-like integer formulation, and a heuristic bounding procedure that finds a near-optimal dual solution of the LP-relaxation. The pickup and delivery problem was extended to include shuttle routes by Masson et al. (2014) for which trips between pick up and delivery are split into a pickup and shuttle legs, capturing network structures with a large set of pickup points and a restricted set of delivery points. Solutions are obtained using a branch-cut-and-price algorithm in conjunction with a pricing subproblem solved via an Elementary Shortest Path Problem with Resource Constraints (ESPPRC), using a labelling algorithm enhanced with efficient dominance rules. A DARP for heterogeneous users was proposed by Parragh (2011). Heterogeneity is introduced by way of individual medical requirements, and the matching of users to vehicles according to the ability of the vehicle to accommodate their needs. The author develops a branch and cut formulation and metaheuristic and analyzes the effect of vehicle waiting time on quality of service. Instances with up to 40 requests are solved to optimality. A mixed-integer model for the PDPTW with heterogeneous fleet was proposed by Qu and Bard (2015). An exact method is introduced based on branch and price and cut, the master problem is solved by column generation and a labelling subproblem to find a lower bound. Subset-row inequalities are applied to the variables of the master problem to improve the lower bound.

1.6 Summary of Contributions and Outline of Paper

The aim in this work is to extend the existing literature of Dial-a-Ride Problems to include the benefits of passenger-centric measures of quality-of-service and passenger heterogeneity with respect to value-of-time and priority of outbound mode so as to enhance connectivity with other transport modes at a Transport Hub. In relation to providing a demand-responsive service that suits the needs of the individual customer, existing extensions of the DARP and its variants suffer from a number of key deficiencies, which require modification to allow for the shuttle-bus routing system we propose.

Firstly, existing VRPTW models focus only on optimising the *current routing* for the given passenger requests and do not consider the effect of the routing on the passenger's ability to connect with another transport mode as part of their onward journey. That is, current models are not able to account for the potential multi-modal aspect of each passenger's trip. Consequently, the complications caused by a routing containing passengers who miss their connections at the Hub is also ignored. As mentioned in the introduction, poor connectivity and infrequent services negatively affect customer's perceptions and willingness to use public transport. Therefore, in our proposed model, we optimise routes in such a way as to minimise the travel time cost of the current routing, but additionally consider the needs of passengers by minimising the cost of missed outbound connections at the Transport Hub.

Secondly, the majority of DARP models assume passengers are homogeneous with respect to value-of-time and by extension, priority of outbound mode. In the context of providing an attractive service for commuters, the heterogeneity of onward connection must be considered. For example, missing an aircraft connection at the Transport Hub would be substantially more costly than missing a bus connection. Traditionally, the objective of the DARP models has usually been to minimise quantities from the end of the operator such as total route time, total cost, or the number of vehicles required. Even when individual requirements of passengers are taken into account such as in Parragh (2011), they involve only the current routing and do not consider an onward connection. However, in order to ensure greater uptake of services, a balance must be struck between the operator's objectives and providing enhanced passenger connectivity and goodwill. For example, a passenger with high priority who misses their connection at the Hub is unlikely to patronise such a service again. Moreover, although some authors such as Pacquette et al. (2013) have addressed different objectives, they do not consider heterogeneity of customers. We propose to extend the ideas of Ceselli et al. (2009), to differentiate between different passengers according to priority of outbound mode at the Hub, and include passenger-centric measures of quality-of-service.

Thirdly, solution approaches to-date have typically utilised a combination of exact and heuristic techniques. We propose an efficient label setting algorithm for the resource constrained shortest path problem to be used within a column generation framework. Our approach is novel as it considers both time cost and missed connection cost quantities, and seeks to minimise the reduced cost of the weighted sum of these.

The key ingredients of our approach are (i) incorporation of passenger-centric measures of quality-of-service via the minimisation of the weighted sum of both travel time and missed connection cost, (ii) the inclusion of passenger heterogeneity with respect to value-of-time and outbound priority, and (iii) an exact solution approach that makes use of column generation and an efficient label setting algorithm.

In Section 2.1 we describe our proposed problem, assumptions and application. In Section 2.2 we outline the Master Problem, and the way in which time and missed-connection cost are calculated. In Section 3 we describe our computational approach, and in 3.1 describe our pricing subproblem in detail, providing an Algorithm for its efficient solution. Computational results are presented in Section 4, and we conclude our analysis in Section 5, with some some suggestions for future research.

2 Dial-a-Ride with Heterogeneous Passengers

In this section we describe the problem formulation for our proposed Dial-a-Ride Problem with Heterogeneous Passengers (DARP-HP). We begin by defining the problem and describe our master problem and the method in which the travel time and missed connection costs are defined and calculated. We then define our pricing subproblem, and an efficient label-setting algorithm for its solution, in the following section. The master and subproblem are used within a column generation framework.

2.1 Problem Description

We define a Transportation Hub to be a transport interchange that provides connections with a combination of different modal services. Such services may include: aircraft, inter-city rail, bus, tram and ferry services to name a few. We consider a neighbourhood of given size (e.g. an area of 10km radius, or according to spatially relevant topography) surrounding the Hub as depicted in Figure 1 below.

Of the passengers travelling to the Transport Hub, a certain proportion wish to make an onward connection with one of the modes that connect with the Hub. Additionally, the perceived cost of missing the connection at the Hub, will depend on a traveller's value-of-time and outbound modal choice. For example, it is likely to be far most costly to miss an aircraft connection, than to miss a connection with a regular bus service. For simplicity we assume that a traveller's value-of-time is likely to be correlated with their choice of outbound connection mode at the Hub, and so we attach a priority to each passenger, based on their outbound modal choice. This is depicted in Figure 1 below.

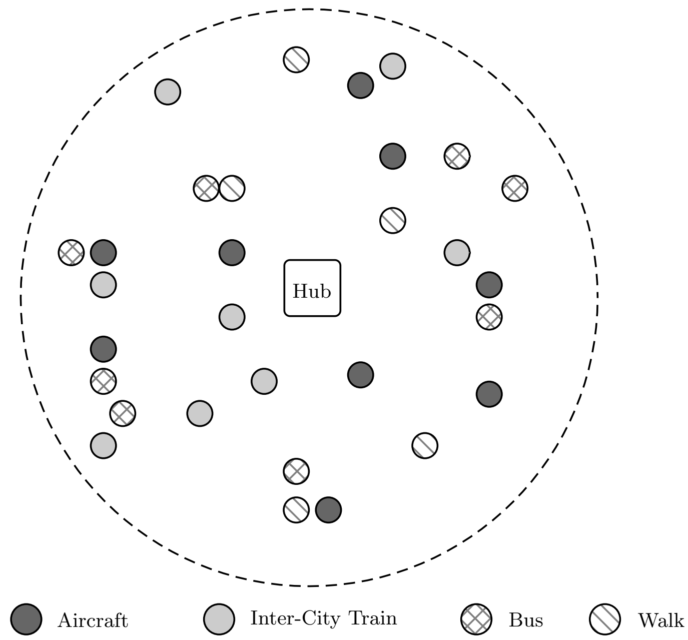

Figure 1: An example of a selection of requests for passengers (represented as dots) within a 10km radius of the Transport Hub. The pattern associated with each passenger corresponds with their intended outbound connection mode choice at the Transport Hub.

It is assumed that the area surrounding the Hub suffers from a lack of transport services or infrequent, inflexible public transport (feeder) services, which influence commuters to avoid public transport and travel to the Hub (or onward) by private motor-vehicle. In order to facilitate the uptake of public transport both to and from the Hub, we propose to design a dial-a-ride, shuttle-bus service to connect passengers with the Hub whilst minimising a weighted sum of travel time and missed connection cost. Passengers wishing to use the service are required to register their request at least one hour before departure, and specify a (i) time-window for pick up, (ii) an outbound connection mode at the Hub and (iii) the departure time of their service at the Hub. We seek a minimal weighted-sum cost assignment of vehicles to passengers, so as to ensure each passenger is covered exactly once, by one vehicle and is picked-up within their defined time-window. The weighted-sum costs will represent the cost of the total time cost and passenger missed connection cost over the relevant time period.

2.2 The Master Problem

Feasible routes begin and end their journey at the Transport Hub (depot), and are represented as columns of a $m \times n$ binary matrix A, where m corresponds to the number of passengers in the given time period and n the number of feasible routings. The (i, j)th element of A is takes the value one if passenger i is contained within route j and zero otherwise.

The decision variable x_j takes the value one if route j is included in the optimal solution and zero otherwise. Thus the master problem may be stated as:

$$\begin{aligned} \text{Minimise:} \quad & \mathbf{c}^T\mathbf{x} \\ \text{subject to:} \quad & A\mathbf{x} = \mathbf{e} \\ & \mathbf{x} \in \{0,1\}^n \end{aligned} \tag{1}$$

where $\mathbf{e}$ is a m-dimensional vector of ones.

In practice there may be an extremely large number of feasible routes, and so column generation is used to generate the most beneficial columns (routes). We record the total travel time (TTT) for the round-trip and assign a dollar cost per unit of travel time. For the missed connection costs, we record for each passenger the number of minutes that their assigned vehicle arrives after their specified departure time (for their outbound mode), and attribute a value of zero if they arrive before departure. This value is then for each passenger multiplied by their respective priority cost, and the total across all passengers is known as the Missed Connection Cost (MCC). We assign a dollar cost per unit of time, and for the following weighted sum c_j for column (route) j:

$$c_j = TTT + \lambda(MCC),$$

where $\lambda \in \mathbb{R}^+$ is a penalty (tuning) parameter.

2.2.1 Time and Missed Connection Cost Calculation

We now describe the way in which time cost and missed connection cost are calculated in the pricing subproblem. The aim of the subproblem is to determine a feasible route with minimal reduced cost, to add to the master problem. We propose to do this efficiently via a label setting algorithm as used in Dunbar et al. (2012) and Dunbar et al. (2014). Since we seek to minimise a weighted sum of two quantities, namely total travel time and missed connection cost, we are required to keep track of both the partial time cost and partial missed connection cost along each path for dominance checking and pruning of unviable paths. However, the final calculation of missed connection cost depends only on the total travel time for the given path.

We denote by π, a path from source to sink, in which the both the source and the sink correspond to the Transport Hub. Let $\pi(i)$ denote an ordered collection of nodes in the routing path π, truncated so that node i is the final node in the list. Define $\pi^-(i)$ to be the predecessor of node i in path π. Let $t_{\pi^-(j);\, j}$ be the time-cost incurred by travelling to node j from its predecessor node $\pi^-(j)$ in path π. Let w_i be the dual multiplier for node i, and l_i and u_i denote the lower and upper bounds of the time window for node i.

We additionally associate with each passenger the specified time of their outbound departure at the Hub, or more specifically, the adjusted departure time relative to the start time of the vehicle route in which they are contained, denoted by d_i. If for example, the departure time at the Hub for passenger i was 09:30 and the vehicle route in which they are contained, departs from the Hub (source node) at 09:00, the adjusted departure time for passenger i is given by $d_i = 30$. In addition, we attach a corresponding penalty cost p_i for missing the specified outbound connection mode, weighted according to the priority of the connection. We penalise positive deviations (late arrivals) from the specified departure time in a linear fashion, with early arrivals receiving no penalty. This is captured via the max function in the equation below.

We calculate the time cost c_r^T and missed connection cost c_r^M via the following equations

$$c_r^T = \sum_{j \in \pi \setminus \{s\}} t_{\pi^-(j);\, j}, \tag{2}$$

$$c_r^M = \sum_{j \in \pi \setminus \{s,t\}} \max\{p_j(c_r^T - d_j), 0\}. \tag{3}$$

Figure 2 below provides an illustration of the attributes associated with each node, for one possible route. The shading patterns denote the type of modal connection each passenger will make at the Hub. For such an example route, the total time cost and missed connection cost is given by

$$c_r^T = t_{si} + t_{ij} + t_{jk} + t_{kt},$$
$$c_r^M = \max\{p_i(c_r^T - d_i), 0\} + \max\{p_j(c_r^T - d_j), 0\} + \max\{p_k(c_r^T - d_k), 0\},$$

where each of the p_i, p_j, p_k depend on the outbound travel mode.

We further restrict the route time to a maximum of T_{max} mins to ensure frequency of service. Thus the routing subproblem is to determine the minimal reduced cost path satisfying the time window constraints and route duration constraints. This is determined via the solution of the following problem.

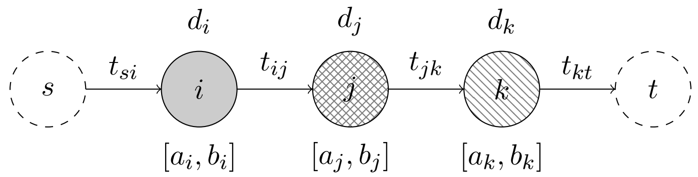

Figure 2: An illustration of the attributes associated with each node in an example route.

$$\text{Minimise: } \sum_{j\in\pi\setminus\{s\}} t_{\pi^-(j);j} + \lambda \sum_{i\in\pi\setminus\{s,t\}} \max\left\{p_i\left[\left(\sum_{j\in\pi\setminus\{s\}} t_{\pi^-(j);j}\right) - d_i\right], 0\right\} - \sum_{j\in\pi\setminus\{s,t\}} w_j \tag{4}$$

$$\text{Subject to: } \quad l_i \leq \sum_{j\in\pi(i)\setminus\{s\}} t_{\pi^-(j);\,j} \leq u_i, \quad \text{for all passengers } \; i,$$

$$\sum_{j\in\pi\setminus\{s\}} t_{\pi^-(j);\,j} \leq T_{max},$$

$$\pi \text{ is a path from } s \text{ to } t,$$

where $\lambda \in \mathbb{R}^+$ is a penalty parameter. Upon obtaining a solution to this problem, the minimising path forms a column A_j of the matrix A in the master problem. The corresponding path is assigned a cost of $c_j = c_r^T + \lambda c_r^M$. The minimising path is solved by an efficient label setting algorithm, which is outlined in the following section.

3 Computational Approach

We now outline our label setting solution algorithm, augmented by a notion of label dominance, modified from related problems in Desrochers and Soumis (1988) and Dumitrescu and Boland (2003), that works efficiently in the cases tested.

3.1 Solving the Pricing Subproblem

It is assumed that passengers request their trip via a phone or online booking service at least one hour prior to specified departure and are allowed to choose a desired time-window for pick up. We topologically sort the passengers from earliest to latest based on the start-time of their time window. In the event two or more passengers request the same start time, we give ordering precedence to the one with smaller time window. If one or more requests have the same start time and time-window size, we assign a precedence ordering to those passengers based on outbound connection mode priority. This is performed as a pre-processing step. Let $G = (\mathcal{N}, \mathcal{A})$ be a graph consisting of an ordered collection of nodes $\{s, i_1, i_2, \ldots, i_n, t\}$ in $\mathcal{N}$, with (s, i_q), $(i_q, t) \in \mathcal{A}$, for $q = 1, \ldots, n$ and $(i_p, i_q) \in \mathcal{A}$ for all $p = 1, \ldots, n$, $q = 1, \ldots, n$ such that $p < q$. For $i \in \pi$, let $\pi(i)$ denote the ordered collection of nodes in the path π, truncated so that the final node in the list is i; we will also refer to $\pi(i)$ as a *partial-path*. We define $T_{\pi(i)} = \sum_{j\in\pi(i)} t_{\pi(j)}$ to be the total time required to reach passenger i in path π.

Similarly, we define the partial-missed connection cost $M_{\pi(i)} = \sum_{j\in\pi(i)} \max\{p_j \left(T_{\pi(i)} - d_j\right), 0\}$. Note that this does not accumulate independently, but is calculated according to the corresponding value of $T_{\pi(i)}$ at each node i. Finally, we define $W_{\pi(i)} = \sum_{j\in\pi(i)} w_j$ to be the sum of the dual values up to an including node i along path $\pi(i)$. Using this terminology, it is possible to re-write the subproblem (4) as:

$$\begin{aligned} \text{Minimise:} \quad & \left(T_{\pi(t)} + \lambda M_{\pi(t)}\right) - W_{\pi(t)} \\ \text{Subject to:} \quad & l_i \le T_{\pi(i)} \le u_i, \quad \forall\, i \in \mathcal{N} \\ & T_{\pi(t)} \le T_{max} \\ & \pi \text{ is a path from } s \text{ to } t, \end{aligned} \tag{5}$$

In order to keep track of these non-linear, inter-dependent quantities we are required to keep track of both the accumulated time, the corresponding missed connection cost, and the reduced cost. In order to ensure we generate only those labels that are likely to appear in an optimal solution, an efficient method for pruning paths is required. This motivates the following dominance condition for the given labels.

Definition 3.1. *(Dominance Condition) The triple (label)*

$$\left(T_{\pi(i)} + \lambda M_{\pi(i)} - W_{\pi(i)},\ T_{\pi(i)},\ M_{\pi(i)}\right),$$

is said to dominate $\left(T_{\eta(i)} + \lambda M_{\eta(i)} - W_{\eta(i)},\ T_{\eta(i)},\ M_{\eta(i)}\right)$ *if:*

$$\begin{aligned} T_{\pi(i)} + \lambda M_{\pi(i)} - W_{\pi(i)} &\le T_{\eta(i)} + \lambda M_{\eta(i)} - W_{\eta(i)}, \quad && \textit{and} \\ T_{\pi(i)} &\le T_{\eta(i)}, && \textit{and} \\ M_{\pi(i)} &\le M_{\eta(i)}, \end{aligned}$$

and the labels are not identical.

Lemma 3.2. *Let* ω *be a path from* j *to* k*, where* $(i,j) \in \mathcal{A}$*. If* $(T_{\pi(i)}+\lambda M_{\pi(i)}-W_{\pi(i)},\ T_{\pi(i)},\ M_{\pi(i)})$ *dominates* $(T_{\eta(i)}+\lambda M_{\eta(i)}-W_{\eta(i)},\ T_{\eta(i)},\ M_{\eta(i)})$*, then* $(T_{\{\pi(i),\omega\}}+\lambda M_{\{\pi(i),\omega\}}-W_{\{\pi(i),\omega\}},\ T_{\{\pi(i),\omega\}},\ M_{\{\pi(i),\omega\}})$ *dominates* $(T_{\{\eta(i),\omega\}} + \lambda M_{\{\eta(i),\omega\}} - W_{\{\eta(i),\omega\}},\ T_{\{\eta(i),\omega\}},\ M_{\{\eta(i),\omega\}})$*.*

Proof: We show that this is true if i connects to j by a single arc (the arc ω consists of a single node $\{j\}$); the result then follows by induction. We have that:

$$\begin{aligned} T_{\{\pi(i),j\}} &= T_{\pi(i)} + t_{i;j}, \\ T_{\{\eta(i),j\}} &= T_{\eta(i)} + t_{i;j}, \end{aligned}$$

Thus since $T_{\pi(i)} \le T_{\eta(i)}$ we also have that $T_{\{\pi(i),j\}} \le T_{\{\eta(i),j\}}$. For the missed connection costs we have $M_{\pi(i)} = \sum_{k\in\pi(i)} \max\{p_k \left(T_{\pi(i)} - d_k\right), 0\}$. Recall that the fixed values $p_k, d_k \ge 0$ for all k, and thus

$$\begin{aligned} M_{\{\pi(i),j\}} &= \sum_{k\in\{\pi(i),j\}} \max\left\{p_k \left(T_{\{\pi(i),j\}} - d_k\right), 0\right\}, \\ M_{\{\eta(i),j\}} &= \sum_{k\in\{\eta(i),j\}} \max\left\{p_k \left(T_{\{\eta(i),j\}} - d_k\right), 0\right\}. \end{aligned}$$

But by non-negativity of time along an arc, $T_{\{\pi(i),j\}} \geq T_{\pi(i)}$ and $T_{\{\eta(i),j\}} \geq T_{\eta(i)}$, we have that $M_{\{\pi(i),j\}} \geq M_{\pi(i)}$ and $M_{\{\eta(i),j\}} \geq M_{\eta(i)}$. Thus $M_{\{\pi(i),j\}} \leq M_{\{\eta(i),j\}}$, since $M_{\pi(i)} \leq M_{\eta(i)}$. Now since $\lambda \in \mathbb{R}^+$,

$$T_{\{\pi(i),j\}} + \lambda M_{\{\pi(i),j\}} - W_{\{\pi(i),j\}} = T_{\pi(i)} + t_{ij} + \lambda \left(\sum_{k\in\{\pi(i),j\}} \max\left\{p_k \left((T_{\pi(i)} + t_{ij}) - d_k\right), 0\right\} \right) - W_{\pi(i)} - w_j,$$

$$T_{\{\eta(i),j\}} + \lambda M_{\{\eta(i),j\}} - W_{\{\eta(i),j\}} = T_{\eta(i)} + t_{ij} + \lambda \left(\sum_{k\in\{\eta(i),j\}} \max\left\{p_k \left((T_{\eta(i)} + t_{ij}) - d_k\right), 0\right\} \right) - W_{\eta(i)} - w_j,$$

and thus $T_{\{\pi(i),\omega\}} + \lambda M_{\{\pi(i),\omega\}} - W_{\{\pi(i),\omega\}} \leq T_{\{\eta(i),\omega\}} + \lambda M_{\{\eta(i),\omega\}} - W_{\{\eta(i),\omega\}}$, and we are finished. ■

In our labelling algorithm described below, we may therefore at each node only create labels for those paths that are not dominated by any path at that node. We call such labels *efficient*.

Definition 3.3. *(Efficient Label) A label $(T_{\pi(i)} + \lambda M_{\pi(i)} - W_{\pi(i)},\ T_{\pi(i)},\ M_{\pi(i)})$ at node i is said to be efficient if it is not dominated by any other label at node i. A path $\pi(i)$ is said to be efficient if the label it corresponds to at node i is efficient.*

3.2 Algorithm

Algorithm 3.1: Label Setting Algorithm for DARP-HP

1. *Initialisation:*
   Set $I_s = \{s\}$ and $I_i = \emptyset$ for all $i \in \mathcal{N}\backslash\{s\}$.
   Set $L_i = \emptyset$ for each $i \in \mathcal{N}$.

2. *Selection of the label to be treated:*
   if $\bigcup_{i\in\mathcal{N}}(I_i\backslash L_i) = \emptyset$ **then go to** Step 4; all efficient labels have been generated.
   else choose $i \in \mathcal{N}$ and $\pi(i) \in I_i\backslash L_i$ so that $T_{\pi(i)} + \lambda M_{\pi(i)} - W_{\pi(i)}$ is minimal.

3. *Treatment of label* $(T_{\pi(i)} + \lambda M_{\pi(i)} - W_{\pi(i)}\ , T_{\pi(i)},\ M_{\pi(i)})$
   forall $(i, j) \in \mathcal{A}$
   if $(T_{\{\pi(i),j\}} + \lambda M_{\{\pi(i),j\}} - W_{\{\pi(i),j\}},\ T_{\{\pi(i),j\}},\ M_{\{\pi(i),j\}})$ is not dominated by $(T_{\{\eta(i),j\}} + \lambda M_{\{\eta(i),j\}} - W_{\{\eta(i),j\}},\ T_{\{\eta(i),j\}},\ M_{\{\eta(i),j\}})$ for any $\eta(j) \in I_j$ **and** $T_{\{\pi(i),j\}} \leq T_{max}$
   then set $I_j = I_j \cup \{\pi(i), j\}$
   end do
   Set $L_i := L_i \cup \{\pi(i)\}$**.**
   Go to Step 2.

4. **Return** $\arg\min_{\pi(t)\in I_t}(T_{\pi(t)} + \lambda M_{\pi(t)} - W_{\pi(t)},\ T_{\pi(t)},\ M_{\pi(t)})$**.**

We now describe the label setting algorithm we use to solve the problem (5). At node $i \in \mathcal{N}$, the current collection of labels is denoted by I_i and the current collection of treated labels by L_i. Because the dominance condition does not allow identical labels at a node i, each label in I_i will correspond to a unique path ($\pi(i)$, for example) for the source node s to the sink node t. For brevity therefore, we will denote individual elements of I_i and M_i as path such as $\pi(i)$.

4 Numerical Results

To evaluate the effectiveness of our DARP-HP over the traditional DARP with homogeneous passengers, we generate and compare routing solutions for each problem, according to the objective of each. More specifically, our proposed DARP-HP whose objective is to minimise the weighted sum of time and missed connection cost, is solved via a column generation framework with master problem (1) and pricing subproblem (4), and solved efficiently using the label-setting algorithm (Algorithm 3.1). The results of our proposed DARP-HP are then compared with the results of the classical DARP whose objective is simply to obtain a minimum time-cost routing.

In order to compare the quality of our solutions across different time-of-day travel patterns, we generated 4 spatially-random dataset types to capture likely time-of-day travel patterns. These reflect possible passenger compositions throughout the morning peak, middle of the day and afternoon peak, and are summarised in Table 1 below.

Time of day	Dataset Type	Connection Types
6am & 6pm	Business	15 Aircraft, 10 Inter-City, 5 Bus, 0 Walk
7am & 5pm	Inter-City	5 Aircraft, 20 Inter-City, 5 Bus, 0 Walk
8am & 3pm	School	5 Aircraft, 5 Inter-City, 15 Bus, 5 Walk
11am & 2pm	Balanced	10 Aircraft, 10 Inter-City, 10 Bus, 0 Walk

Table 1: The different dataset types, and the connection types contained within each.

For each of these datasets, we simulated 10 random instances with different passenger outbound connection departure times at the Hub, to produce 40 datasets in total. Each dataset consists of 30 passengers, and captures a range of requests which must be served within the given time horizon. The solution obtained using our DARP-HP approach occasionally requires additional vehicles, to that obtained in the solution of the classical approach, and so we assume an additional cost of $40 per additional vehicle, per 20 minute trip. We determined through direct experimentation that setting $\lambda = 0.5$, obtained the best trade-off between minimisation of time and missed connection cost.

In the tables that follow, we refer to the solution generated using the classical DARP approach as the Base Case, which is denoted by B. The solution obtained using our DARP-HP approach is denoted by $TTMC$. We compare the following quantites:-

(i) Time Cost (c^T),

(ii) Missed Connection Cost (c^M),

(iii) Total Weighted Cost ($c^T + \lambda c^M$) + Cost of Additional Vehicles ($40 per vehicle, per 20 minutes),

(iv) The Percentage Improvement of TTMC over B.

All computational results were obtained using C++ and SCIP (Achterberg (2009)), and solved with CPLEX 12.1 on a 2.0GHz PC with 8GB of RAM. The IP was solved at the root node using column generation and did not require any further branching.

	Time Cost		Missed Connection Cost		Total Weighted + Add. Vehicle Cost		% Improvement Over B
Instance	B	TTMC	B	TTMC	B	TTMC	%
1	88	92	350	0	263	132	49.81
2	88	109	350	100	263	239	9.13
3	88	93	500	0	338	173	48.82
4	88	89	1300	200	738	229	68.97
5	88	90	600	0	388	170	56.12
6	88	89	900	0	538	129	76.02
7	88	90	320	0	248	170	31.45
8	88	97	1200	0	688	217	68.46
9	88	93	170	0	173	133	23.12
10	88	97	850	0	513	177	65.55
Average	88	94	654	30	415	177	57.37

Table 2: Results for the Business Dataset.

	Time Cost		Missed Connection Cost		Total Weighted + Add. Vehicle Cost		% Improvement Over B
Instance	B	TTMC	B	TTMC	B	TTMC	%
1	86	90	50	0	111	130	−17.12
2	86	90	50	0	111	90	18.92
3	86	95	100	0	136	135	0.74
4	86	90	250	0	211	130	38.39
5	86	90	25	0	99	130	−31.98
6	86	89	200	0	186	129	30.65
7	86	96	155	0	164	136	16.82
8	86	101	750	0	461	181	60.74
9	86	98	525	0	349	178	48.92
10	86	94	525	0	349	134	61.55
Average	86	93.3	263	0	218	137	36.87

Table 3: Results for the Inter-City Dataset.

	Time Cost		Missed Connection Cost		Total Weighted + Add. Vehicle Cost		% Improvement Over B
Instance	B	TTMC	B	TTMC	B	TTMC	%
1	89	91	160	0	169	91	46.15
2	89	89	110	0	144	89	38.19
3	89	90	160	0	169	90	46.75
4	89	90	80	0	129	90	30.23
5	89	90	45	0	112	90	19.28
6	89	91	230	0	204	91	55.39
7	89	95	910	0	544	95	82.54
8	89	90	150	0	164	90	45.12
9	89	90	310	0	244	90	63.12
10	89	92	135	0	157	92	41.21
Average	89	90	229	0	204	90	55.68

Table 4: Results for the School Dataset.

	Time Cost		Missed Connection Cost		Total Weighted + Add. Vehicle Cost		% Improvement Over B
Instance	B	TTMC	B	TTMC	B	TTMC	%
1	86	89	50	0	111	89	19.82
2	86	87	50	0	111	87	21.62
3	86	89	50	0	111	89	19.82
4	86	93	20	0	96	93	3.13
5	86	88	100	0	136	88	35.29
6	86	95	90	0	131	175	−3.36
7	86	106	850	40	511	246	51.86
8	86	104	850	40	511	164	67.91
9	86	97	300	0	236	137	41.95
10	86	108	1100	636	238	92	62.57
Average	86	95.5	343	18	258	141	45.40

Table 5: Results for the Balanced Dataset.

	Time Cost	Missed Connection Cost	Total Cost
Business	-6.705	95.41	57.37
Inter-City	-8.49	100.00	36.87
School	-1.35	100.00	55.67
Balanced	-11.16	94.75	45.40
Average	-6.93	97.54	48.83

Table 6: Average percentage improvement of TTMC over the B approach.

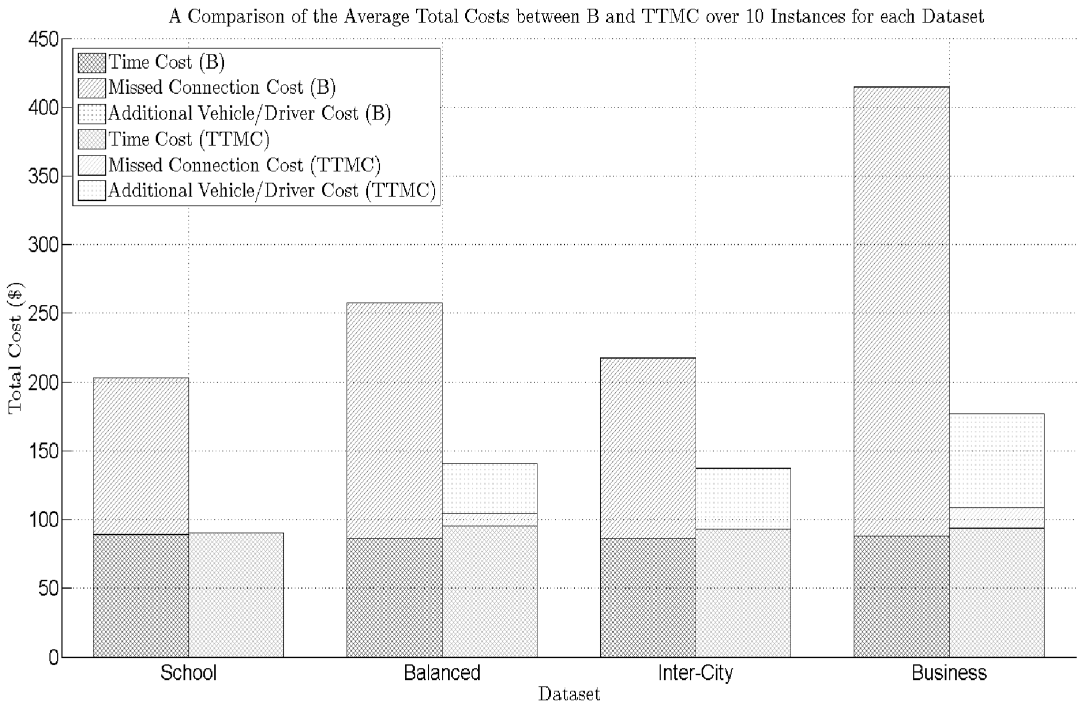

Figure 3: A comparison of the average total costs over 10 instances, between B and TTMC for each dataset.

5 Discussion and Conclusions

Our DARP-HP methodology for the inclusion of multi-modal connectivity and passenger value-of-time has clear advantages over the classical DARP approach, in which only objective(s) from the viewpoint of the operator are taken into account. It may be observed that across all four dataset types in Figure 3 and Table 6, that on average, our proposed DARP-HP universally improves over the classical DARP in terms of both missed connection cost and total cost. In particular, over all 4 dataset types, the average reduction in missed connection cost was between 94% and 100%. Moreover, if the operator is willing to accept an increase to total travel time of around 7% (to be shared across the routes, not necessarily for an individual route), our DARP-HP approach has the potential to outperform the classical DARP approach by producing an average decrease of 48.83% in total costs, even after the costs of one/two additional vehicles costs are taken into account.

In terms of additional vehicles, the Business dataset required an additional two vehicles on average, the Inter-City and the Balanced dataset required only one additional vehicle on average, and the School dataset did not require any additional vehicles. Therefore, for different passenger compositions during different times of the day, it is not only possible to use exactly the same number of vehicles as for the classical DARP approach, but also to re-design the routes in such a way as to completely eliminate the possibility of missed connections, for only a minimal (1.3%) increase to total travel time cost. Such a scenario would be viewed upon favourably by both the operator and potential passengers; in terms of both operating cost and retaining the patronage of customers.

It is interesting to note that our model was not able to completely eliminate all missed connections for all instances in the Business and Inter-City instances. This is most probably due to the larger number of passengers with higher priority, namely Aircraft and Inter-City Train connections at the Hub. However, in the 2-3 instances in which missed connection cost remains, the cost is substantially less than for the classical DARP approach. For the instances in which improvement over the classical DARP was negative, the cost of operating additional vehicles in order to reduce missed connection cost was in these cases, not beneficial. However, these accounted for only a small number of datasets, as in 37/40 instances the inclusion of missed connection costs was beneficial.

In this proof of concept study, we have limited our analysis to minimising missed connection cost as one potential quality-of-service indicator from the viewpoint of potential passengers. There are however, a selection of potential measures that may be also useful to include, such as:- waiting time at the Hub, trip duration (for longer time periods) and alternative missed connection cost functions, such as a heaviside step function, to replicate the all-or-nothing cost of missing different types of connections, rather than a simplified linear increase with time.

Moreover, this work is amenable and directly applicable to the perishable-good delivery problem in which certain goods have greater priority for arrival than others. Possible extensions also include a robust version, taking into account time-of-day traffic information for link travel time between passengers, and incorporation of uncertainty and delay propagation along routes. Future work will explore this possibility.

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References

A. K. Abraham, B. C. Jos, and G. S. Mangaluthu. The Pickup and Delivery Vehicle Routing Problem for Perishable Goods in Air-Cargo Industry. *International Journal of Emerging Technology and Advanced Engineering*, 2012.

T. Achterberg. SCIP: Solving constraint integer programs. *Mathematical Programming Computation*, 1(1):1–41, 2009. `http://mpc.zib.de/index.php/MPC/article/view/4`.

APTA. Public Transportation Benefits, 2015. URL www.apta.com.

C. Archetti, M. Grazia Speranza, and D. Weyland. On-demand public transportation. *Submitted*, 2015.

Australian Bureau of Statistics. 4602.0.55.002 - environmental issues: Waste management, transport and motor vehicle usage, 2012. URL www.abs.gov.au.

N. Azi, M. Gendreau, and J.-Y. Potvin. An Exact Algorithm for a Vehicle Routing Problem with Time Windows and Multiple Use of Vehicles. *European Journal of Operational Research*, 202:756–763, 2010.

R. Baldacci, E. Bartolini, and A. Mingozzi. An exact algorithm for the pickup and delivery problem with time windows. *Operations Research*, 59(2):414–426, 2011. doi: 10.1287/opre.1100.0881.

R. Borndörfer, M. Grötschel, D. Klostermeier, and C. Küttner. Telebus Berlin: Vehicle Scheduling in a Dial-a-Ride System. In *Lecture Notes in Economics and Mathematical Systems*, Proceedings of the 7th International Workshop on Computer-Aided Transit Scheduling, pages 391–422. Springer, 1999.

J. Brandão and A. Mercer. A Tabu Search Algorithm for the Multi-Trip Vehicle Routing and Scheduling Problem. *European Journal of Operational Research*, 100:180–191, 1997.

R. Buehler. Determinants of Transport Mode Choice: A Comparison of Germany and the USA. *Journal of Transport Geography*, 19:644–657, 2011.

A. Ceselli, G. Righini, and M. Salani. A Column Generation Algorithm for a Vehicle Routing Problem with Economies of Scale and Additional Constraints. *Transportation Science*, 43(1):56–69, 2009.

L. C. Coelho and G. Laporte. Optimal Joint Replenishment and Delivery of Perishable Products. Technical report, Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation, March 2013.

J.-F. Cordeau and G. Laporte. The Dial-a-Ride Problem (DARP): Variants, modeling issues and algorithms. *4OR: Quarterly Journal of the Belgian, French and Italian Operations Research Societies*, pages 89–101, 2003.

J.-F. Cordeau and G. Laporte. The Dial-a-Ride Problem: Models and Algorithms. *Annals of Operations Research*, 153:29–46, 2007.

J.-F Cordeau, G. Laporte, J.-Y Potvin, and M. W. P. Savelsbergh. Transportation on demand. In *In Transportation*, pages 429–466. North-Holland, 2007.

T. G. Crainic, Y. Gajpal, and M. Gendreau. Multi-Zone, Multi-Trip Vehicle Routing Problem with Time Windows. Technical report, Interuniversity Research Centre on Enterprise Networks, Logistics and Transportation, 2012.

G. Desaulniers, A. Erdmann, M. M. Solomon, and F. Soumis. The VRP with Pickup and Delivery. The vehicle routing problem, SIAM Monographs on Discrete Mathematics and Applications, pages 225–242. Philadelphia, 2002.

M. Desrochers and F. Soumis. A Generalized Permanent Labelling Algorithm for the Shortest Path Problem with Time Windows. *INFOR: Information Systems and Operations Research*, 26(3):191–212, 1988.

J. Desrosiers, Y. Dumas, M. M. Solomon, and F. Soumis. *Time Constrained Routing and Scheduling*, volume 8 of *Network Routing, Handbooks in Operations Research and Management Science*, pages 35–139. Elsevier Science, Amsterdam, 1995.

M. Drexl. Synchronization in Vehicle Routing: A Survey of VRPs with Multiple Synchronization Constraints. *Transportation Science*, 46(3):297–316, 2012.

I. Dumitrescu and N. Boland. Improved Pre-processing, Labelling and Scaling Algorithms for the Weight-Constrained Shortest Path Problem. *Networks*, 42(3):135–153, 2003.

M. Dunbar, G. Froyland, and C. L. Wu. Robust Airline Schedule Planning: Minimizing Propagated Delay in an Integrated Routing and Crewing Framework. *Transportation Science*, 46(2):204–216, 2012.

M. Dunbar, G. Froyland, and C. L. Wu. An Integrated Scenario-Based Approach for Robust Aircraft Routing, Crew Pairing and Re-Timing. *Computers & Operations Research*, 45: 68–86, 2014.

D. Espinoza, R. Garcia, M. Goycoolea, G. L. Nemhauser, and M. W. P. Savelsbergh. Per-Seat, On-Demand Air Transportation Part I: Problem Description and an Integer Multicommodity Flow Model. *Transportation Science*, 42(3):263–278, 2008.

Eurostat. Passenger transport statistics, 2014. URL `http://ec.europa.eu/eurostat/statistics-explained/`.

A. Fügenschuh. A Set-Partitioning Reformulation of a School Bus Scheduling Problem. *Journal of Scheduling*, 14:307–318, 2011.

M. Liu, Z. Luo, and A. Lim. A branch-and-cut algorithm for a realistic dial-a-ride problem. *Transportation Research Part B*, 2015. In Press.

D. S. Mankowska, C. Biereirth, and F. Meisel. Modelling the Synchronization of Transport Means in Logistics Service operations. In *Computational Logistics: Second International Conference, ICCL*, pages 74–85. Springer-Verlag Berlin Heidelberg, 2011.

R. Masson, S. Ropke, F. Lehuede, and O. Peton. A branch-and-cut-and-price approach for the pickup and delivery problem with shuttle routes. *European Journal of Operations Research*, 236:849–862, 2014.

A. Olivera and O. Viera. Adaptive Memory Programming for the Vehicle Routing Problem with Multiple Trips. *Computers and Operations Research*, 34(1):28–47, 2007.

J. Pacquette, J.-F. Cordeau, G. Laporte, and M. M. B Pascoal. Combining Multicriteria Analysis and Tabu Search for Dial-a-Ride Problems. *Transportation Research Part B: Methodological*, 52:1–16, 2013.

S. Parragh. Introducing heterogeneous users and vehicles into models and algorithms for the dial-a-ride problem. *Transportation Research Part C*, 19:912–930, 2011.

Y. Qu and J.F. Bard. A branch-and-price-and-cut algorithm for heterogeneous pickup and delivery problems with configurable vehicle capacity. *Transportation Science*, 49(2):254–270, 2015.

S. Ropke, J.-F. Cordeau, and G. Laporte. Models and branch-and-cut algorithms for pick up and delivery problems with time windows. *Networks*, 49(4):258–272, 2007.

S. Salhi and R. J. Petch. A GA-Based Heuristic for the Vehicle Routing Problem with Multiple Trips. *Journal of Mathematical Modelling and Algorithms*, 6(4):591–613, 2007.

M. P. Seixas and A. B. Mendes. Column Generation for a Multitrip Vehicle Routing Problem with Time Windows, Driver Work Hours, and Heterogeneous Fleet. *Mathematical Problems in Engineering*, pages 1–14, 2013. http://dx.doi.org/10.1155/2013/824961.

M. Solomon. *Vehicle Routing and Scheduling with Time Window Constraints: Models and Algorithms*. PhD thesis, Dept. of Decision Sciences, University of Pennsylvania, 1983.

E. Taillard, G. Laporte, and M. Gendreau. The vehicle routing problem with multiple use of vehicles. *Journal of the Operational Research Society*, 47:1065–1070, 1996.

P. Toth and D. Vigo. Heuristic algorithms for the handicapped persons transportation problem. *Transportation Science*, 31(1):60–71, 1997.

N. Velasco, P. Castagliola, P. Dejax, C. Gueret, and C. Prins. A memetic algorithm for a pick-up and delivery problem by helicopter. In Francisco Babtista Pereira and Jorge Tavares, editors, *Bio-inspired Algorithms for the Vehicle Routing Problem*, volume 161 of *Studies in Computational Intelligence*, pages 173–190. Springer Berlin Heidelberg, 2009. ISBN 978-3-540-85151-6. doi: 10.1007/978-3-540-85152-3_8. URL `http://dx.doi.org/10.1007/978-3-540-85152-3_8`.

H. Wang, D.-H Lee, and R. Cheu. PDPTW Based Taxi Dispatch Modeling for Booking Service. In *Proceedings of the 5th International Conference on Natural Computation*, ICNC'09, pages 242–247, Piscataway, NJ, USA, 2009. IEEE Press. ISBN 978-1-4244-4542-4. URL `http://dl.acm.org/citation.cfm?id=1797096.1797146`.