A Study of Three-Period Ramp-Up Polytope

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We study the polyhedron of the unit commitment problem, and consider a relaxation involving the ramping constraints. We study the three-period ramp-up polytope, and describe the convex-hull using a new class of inequalities.

1. Introduction

The unit commitment (UC) problem is a very important problem in the power industry [1]. It seeks to minimize system-wide operational costs of power generators by providing an optimal schedule of power production while meet the demand and other technical constraints. By the combinatorial nature of the operational constraints, UC problem is difficult to solve to optimality for practical large-scale instances [2]. In this project, we consider a relaxation of UC problem with ramping constraints and production limits. Ramping constraints are used to control the change in production level for a generator from one period to the next. Damci-Kurt et al (2013) [2] present the the complete description of two-period ramping relaxation convex hull, and also describe a few classes of multi-period inequalities. We give the complete description of three-period ramp-up polytope, show that this description includes one new inequality which cannot be obtained as a special case of previously known multi-period ineqaulities. This new inequality can be used to further strengthen the ramping formulation of multi-period case. Independently, Pan and Guan (2015) [3] study the integrated ramping and minimum up/down time polytope, and also describe the convex hull of the three-period version of their polytope.

2. Notations

Parameters:

l : minimum production level

u : maximum production level

δ : maximum change in production from one operating period to the next

$\bar{u}$: maximum start-up production level

T : minimum up time

t : minimum down time

Decision Variables:

x_i : whether operate generator in period i, $i = 1, 2, 3$

s_i : whether start up generator in period i, $i = 2, 3$

p_i : production level in period i, $i = 1, 2, 3$

3. Two-period ramp-up polytope

The two-period ramp-up polytope is given as

$$s_2 \geq 0 \tag{1}$$

$$s_2 \leq x_2 \tag{2}$$

$$x_1 + s_2 \leq 1 \tag{3}$$

$$x_1 + s_2 \geq x_2 \tag{4}$$

$$p_1 \geq l x_1 \tag{5}$$

$$p_2 \geq l x_2 \tag{6}$$

$$p_1 \geq u x_1 \tag{7}$$

$$p_2 \leq u x_2 - (u - \bar{u}) s_2 \tag{8}$$

$$p_2 - p_1 \leq (l + \delta) x_2 + (\bar{u} - l - \delta) s_2 - l x_1 \tag{9}$$

4. Three-period ramp-up polytope

We now first consider the most generous case with $T = t = 1$, in which generators can be shut down right after the set up and vice versa. Here, we assume $u \geq l + 2\delta$ and $u \geq \bar{u} + \delta$.

Then we can show that the following four constraints with $p_3 - p_1$ as left-hand side are valid:

$$p_3 - p_1 \leq -lx_1 + (2\delta + l)x_3 + (\bar{u} - l - 2\delta)s_3 \tag{10}$$

$$p_3 - p_1 \leq -lx_1 + (\delta + l - \bar{u})x_2 + (\bar{u} + \delta)x_3 - (\delta + l - \bar{u})s_2 - \delta s_3 \tag{11}$$

$$p_3 - p_1 \leq -lx_1 + (l + 2\delta)x_3 + (\bar{u} - l - \delta)s_2 + (\bar{u} - l - 2\delta)s_3 \tag{12}$$

$$p_3 - p_1 \leq -lx_1 + (\bar{u} + \delta)x_3 - \delta s_3 \tag{13}$$

From Table 1, we can see that constraints (10) and (11) are valid when $\bar{u} \leq l + \delta$ and constraint (12), (13) are valid when $l + \delta \leq \bar{u}$. When $\bar{u} = l + \delta$, these four constraints are equal. In fact, these four constraints coincide with the multi-period facets proposed by Pelin Damci-Kurt, et al (2013) [2]. Constraint (10) corresponds to "constraint (22)" when $S = \{3\}$ and $l + \delta \geq \bar{u}$; Constraint (11) corresponds to the case when $S = \{2, 3\}$ and $l + \delta \geq \bar{u}$. Constraint (12) corresponds to "constraint (21)" and constraint (13) corresponds to "constraint (22)" when $S = \{3\}$ (or $S = \{2, 3\}$) and $l + \delta \leq \bar{u}$.

Table 1: 3-period Ramping Up Constraint (1)

x_1	x_2	x_3	s_2	s_3	$\overline{p_3 - p_1}$	RHS of (10)	RHS of (11)	RHS of (12)	RHS of (13)
1	1	1	0	0	2δ	2δ	2δ	2δ	$\bar{u} + \delta - l$
1	0	0	0	0	$-l$	$-l$	$-l$	$-l$	$-l$
1	1	0	0	0	$-l$	$-l$	$\delta - \bar{u}$	$-l$	$-l$
1	0	1	0	1	$\bar{u} - l$	$\bar{u} - l$	$\bar{u} - l$	$\bar{u} - l$	$\bar{u} - l$
0	1	1	1	0	$\bar{u} + \delta$	$l + 2\delta$	$\bar{u} + \delta$	$\bar{u} + \delta$	$\bar{u} + \delta$
0	1	0	1	0	0	0	0	$\bar{u} - l - \delta$	0
0	0	1	0	1	$\bar{u}$	$\bar{u}$	$\bar{u}$	$\bar{u}$	$\bar{u}$

And in fact, when $u \leq l + 2\delta$ and $u \leq \bar{u} + \delta$, all the above inequalities are dominated by $p_3 - p_1 \leq ux_3 - (u - \bar{u})s_3 - lx_1$, which is a sum of two inequalities that have been already defined in a two-period polytope: $p_3 \leq ux_3 - (u - \bar{u})s_3$ and $p_1 \geq lx_1$.

Another new inequality with left-hand side as p_3 can be derived as

$$p_3 \leq (u - \bar{u} - \delta)x_2 + (\bar{u} + \delta)x_3 - (u - \bar{u} - \delta)s_2 - \delta s_3 \tag{14}$$

Constraint (14) is valid when $u \geq \bar{u} + \delta$, which is the same as "constraint (23)" in [2] with $j = 1$. And this constraint is dominated by $p_3 \leq ux_3 - (u - \bar{u})s_3$ when $u \leq \bar{u} + \delta$. If $\bar{u} + \delta < u < l + 2\delta$, constraint (14) dominates constraint (11).

In addition, there is a new inequality not covered by aforementioned multi-period facets that takes the form as

$$np_3-(n-i)p_1 \leq -(n-i)lx_1+[2(n-i)\delta+iu+(n-i)l]x_3-[2(n-i)\delta+iu+(n-i)l-n\bar{u}]s_3 \quad (15)$$

The right-hand side of (15) is shown in Table 2. We see that the (15) is valid when $n(\bar{u}+\delta) \leq 2(n-i)\delta + iu + (n-i)l$, which is equivalent to

$$n(\bar{u}-\delta-l) \leq i(u-l-2\delta)$$

When $\bar{u} > l+\delta$ and $u > l+\delta$, the above inequality is satisfied at equality by setting $n = u-l-2\delta$ and $i = \bar{u}-\delta-l$. Then it becomes

$$(u-l-2\delta)p_3-(u-\bar{u}-\delta)p_1 \leq -l(u-\bar{u}-\delta)x_1+(\delta+\bar{u})(u-2\delta-l)x_3-\delta(u-2\delta-l)s_3 \quad (16)$$

Constraint (16) is dominated by (10) if $\bar{u} < l+\delta$. And when $\bar{u} = l+\delta$, it is equivalent to (10)$\sim$(13). When $\bar{u} > l+\delta$, we propose that constraint (16) is a facet.

Table 2: 3-period Ramping Up Constraint

x_1	x_2	x_3	s_2	s_3	$\overline{np_3-(n-i)p_1}$	RHS of (15)
1	1	1	0	0	$2(n-i)\delta+iu$	$2(n-i)\delta+iu$
1	0	0	0	0	$-(n-i)l$	$-(n-i)l$
1	1	0	0	0	$-(n-i)l$	$-(n-i)l$
1	0	1	0	1	$n\bar{u}-(n-i)l$	$n\bar{u}-(n-i)l$
0	1	1	1	0	$n(\bar{u}+\delta)$	$2(n-i)\delta+iu+(n-i)l$
0	1	0	1	0	0	0
0	0	1	0	1	$n\bar{u}$	$n\bar{u}$

Proposition 1. *When $\bar{u} > l+\delta$ and $u \geq \bar{u}+\delta$, constraint (16) is a facet of the 3-period ramping-up polytope.*

Proof. We prove the facet by finding 8 affinely independent points, as listed in Table 3. Denote these points as $\mathbf{s}^i$, and we need to show

$$\sum_{i=1}^{8} \lambda_i \mathbf{s}^i = \mathbf{0} \quad \sum_{i=1}^{8} \lambda_i = 0$$

holds if and only if $\lambda_i = 0$. To see this, first, from the coefficients of x_1, x_2, x_3, s_1, s_2, we find

$$\lambda_3 = \lambda_7 = -\lambda_4 = -\lambda_5 \quad \lambda_2 = \lambda_8 = -\lambda_6$$

Then from the coefficients of p_1, we have

$$\bar{u}\lambda_8 + (u - l - 2\delta)\lambda_4 = 0$$

Besides, from the coefficients of p_3, we have

$$(u - \bar{u} - \delta)\lambda_4 - \bar{u}\lambda_8 = 0$$

As $u > l + 2\delta$ and $u \geq \bar{u} + \delta$, it follows that $\lambda_4 = \lambda_8 = 0$. Hence $\lambda_i = 0$ for $i = 2, \ldots, 8$. Then $\lambda_1 = 0$. So they are affinely independent. Since the dimension of this polytope is eight, this will be enough to prove the facet. □

Table 3: Affinely Independent Points of Constraint (16)

	x_1	x_2	x_3	s_2	s_3	p_1	p_2	p_3
$\mathbf{s}^1$	0	0	0	0	0	0	0	0
$\mathbf{s}^2$	1	0	0	0	0	l	0	0
$\mathbf{s}^3$	1	1	0	0	0	l	$l+\delta$	0
$\mathbf{s}^4$	1	1	1	0	0	$u-2\delta$	$u-\delta$	u
$\mathbf{s}^5$	0	1	0	1	0	0	0	0
$\mathbf{s}^6$	1	0	1	0	1	l	0	$\bar{u}$
$\mathbf{s}^7$	0	1	1	1	0	0	$\bar{u}$	$\bar{u}+\delta$
$\mathbf{s}^8$	0	0	1	0	1	$\bar{u}$	0	0

Then we can summarize as our results and give the proof to our proposed representation of three-period ramp-up polytope. With the general assumption that $u \geq \bar{u} + \delta$, we can discuss two cases: $\bar{u} \leq l + \delta$ and $\bar{u} > l + \delta$.

4.1 $\bar{u} \leq l + \delta$

Assume $u \geq l + 2\delta$, and we give the following propositions.

Proposition 2. *When $\bar{u} < l + \delta$, the convex hull is given by constraints (10), (11), (14) and other constraints from two-period ramp-up polytope.*

Proposition 3. *When $\bar{u} = l + \delta$, the convex hull is given by one of constraints (10)∼(13), (24) and other constraints from two-period ramp-up polytope.*

Notice that there is actually no new inequalities derived for this case. And the proof for these two propositions can be referred to Kai Pan and Yongpei Guan (2015) [3], and thus omitted here.

4.2 $\bar{u} > l + \delta$ and $u \geq \bar{u} + \delta$

Proposition 4. *When $T = t = 1$, the convex hull is given as $conv(Q^{>}_{1,1})$:*

$$s_2 \geq 0, s_3 \geq 0 \tag{17}$$

$$s_2 \leq x_2, s_3 \leq x_3 \tag{18}$$

$$x_1 + s_2 \leq 1, x_2 + s_3 \leq 1 \tag{19}$$

$$x_1 \geq x_2 - s_2, x_2 \geq x_3 - s_3 \tag{20}$$

$$p_1 \geq lx_1, p_2 \geq lx_2, p_3 \geq lx_3 \tag{21}$$

$$p_1 \leq ux_1 \tag{22}$$

$$p_2 \leq ux_2 - (u - \bar{u})s_2 \tag{23}$$

$$p_3 \leq ux_3 - (u - \bar{u})s_3 \tag{24}$$

$$p_3 \leq (\bar{u} + \delta)x_3 + (u - \bar{u} - \delta)(x_2 - s_2) - \delta s_3 \tag{25}$$

$$p_2 - p_1 \leq (l + \delta)x_2 + (\bar{u} - l - \delta)s_2 - lx_1 \tag{26}$$

$$p_3 - p_2 \leq (l + \delta)x_3 + (\bar{u} - l - \delta)s_3 - lx_2 \tag{27}$$

$$p_3 - p_1 \leq (l + 2\delta)x_3 - lx_1 + (\bar{u} - l - \delta)s_2 + (\bar{u} - l - 2\delta)s_3 \tag{28}$$

$$(u - l - 2\delta)p_3 - (u - \bar{u} - \delta)p_1 \leq (\bar{u} + \delta)(u - l - 2\delta)x_3 - l(u - \bar{u} - \delta)x_1 - \delta(u - l - 2\delta)s_3 \tag{29}$$

We find a new inequality (29) in this case. The polytope for cases when T and t are given other values can be found in the paper by Pan and Guan [3]. We observe that constraint (29) joins the polytope when T jumps down from 2 to 1. The proof sketch for these propositions are given as follows.

Proof. The validity of each constraint has been discussed in previous sections. The facet-defining proof for the new inequality has already been given while the facet-defining proof for remaining inequalities are referred to [2]. It is easy to show the full dimensionality of these polyhedral representations. Then we can simply borrow the proof scheme developed by Pan and Guan [3] to show that all extreme points are integral. The basic idea is to develop a system of integral points to represent all points on the faces by their convex combinations. Then all extreme points (also extreme points on faces) will be integral. Thus the convex hull proof is complete.

□

5. Conclusion

In this paper, we focus on the derivation of the complete description of three-period ramping polytope in the unit commitment problem. Compared with the facet-defining inequalities already given in the literature, one new inequality is firstly discovered, and this equality is also found independently by Pan and Guan [3]. A proof sketch is presented although some details are omitted. We believe that the new inequality can be used to strengthen the existing formulations of unit commitment problem and reduce the computation time.

References

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