A Modeling-based Approach for Non-standard Packing Problems[1]

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Abstract This chapter examines the problem of packing tetris-like items, orthogonally, with the possibility of rotations, into a convex domain, in the presence of additional conditions. An MILP (Mixed Integer Linear Programming) and an MINLP (Mixed Integer Nonlinear Programming) models, previously studied by the author (Fasano [8]), are surveyed. An efficient formulation of the objective function, aimed at maximizing the loaded cargo, is pointed out for the MILP model. The MINLP one, addressed to the relevant feasibility sub-problem, has been conceived to improve approximate solutions, as an intermediate step of a heuristic process. A space-indexed model is further introduced and the problem of approximating polygons by means of tetris-like items investigated. In both cases an MILP formulation has been adopted. An overall heuristic approach is proposed to provide effective solutions in practice. One chapter of this book focuses on the relevant computational aspects (Gliozzi et al. [9]).

Keywords: tetris-like items · Orthogonal packing · Convex domain · Additional/balancing conditions · Mixed Integer Linerar/Non-linear Programming · MILP/MINLP models · Global Optimization (GO) · Efficient formulation · Feasibility sub-problem · Space-indexed/grid-based-position paradigms · Polygon approximation · Heuristics

1 Introduction

This chapter summarizes and extends results descending from a long-lasting research effort aimed at solving complex three-dimensional packing problems arising in the space industry (Fasano [8]). In this challenging context, the relevant

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issues could hardly be considered applying a standard typology. Quite often, indeed, the operational scenarios to deal with are characterized by the presence of tricky geometries and complex additional conditions that can even be of global impact, such as in the case of balancing.

Often irregularly-shaped and of non-negligible dimensions, the objects involved cannot be realistically approximated in terms of single cuboids (i.e. rectangular parallelepipeds). Significant effort has therefore been addressed to allow for tetris-like items, i.e. objects consisting of clusters of mutually orthogonal (rectangular) parallelepipeds. Similarly, the domains (containers) to take account of are generally not box-shaped and often several internal volumes are not exploitable, since these correspond either to clearance/forbidden zones or actual holes. Additionally, separation planes (with no fixed position specified a priori) can partition the domain into sub-domains. Some items may be requested to assume pre-defined positions/orientations or are subject to placement restrictions, such as, for instance, the requirement of having a given side parallel or orthogonal to a specified direction.

In order to cope with overall conditions such as balancing, when necessary in addition to those mentioned above, a Global Optimization (GO) based view is highly desirable. This is essentially based on a modeling philosophy, as opposed to a pure algorithmic one, consisting of sequential procedures limited to local search.

A number of modeling-based works are present in the literature, although these are usually restricted to the case of box-shaped items (e.g. Cassioli and Locatelli [4], Chen et al. [6], Padberg [12], Pisinger and Sigurd [13]).

On the other hand, very interesting studies consider strongly irregularly-shaped objects, even though the adopted philosophy is mainly focused on local optimization (Stoyan and Chugay [14], Stoyan et al. 2012 [15], Egeblad et al. [7]).

This chapter emphasizes the solution of non-standard packing issues, in the context outlined above, by a GO approach. Mixed Integer Linear/Non-linear (MILP/MINLP) formulations have been conceived and a library of mathematical models set up. This supports ad hoc heuristics, implemented to obtain satisfactory, albeit probably sub-optimal (or at least non-optimal proven), solutions to a wide collection of real-world instances (Fasano [8]).

The general problem of placing tetris-like items orthogonally into a convex domain, without pair-wise intersection, so that the total volume loaded is maximized, is the main topic of this chapter.

Section 2 investigates a dedicated MILP model (Fasano [8]), specifically constructed to overcome the challenging computational difficulties that are typically associated with the problem in question, when formulated in terms of Mathematical Programming. It is, indeed, well known that, even when single parallelepipeds are involved (i.e. tetris-like items consisting of one component only), the relevant MILP models available in the specialist literature (e.g. Chen et al. [6], Padberg [12]) are very hard to solve. This holds also if a number of *valid inequalities* are purposely added. The model discussed in this section can be used to solve small-size instances, tout court. In addition, it can advantageously be adopted as a basic element of the above-mentioned heuristics that act recursively, following an overall *greedy* approach.

MINLP models (e.g. Cassioli and Locatelli [4]) have been built up for the *feasibility* sub-problem, derived from the general one, when a set of items need to be loaded (without any possibility of rejection, provided that the instance is feasible) and no *objective* function is assigned. Moreover, they can be adopted (Fasano [8]) to improve approximate solutions where intersections between items are admitted, 'minimizing' the overall overlap (actually this optimization target is attained only partially, through *surrogate* functions). An MINLP version, implemented for this specific case is summarized in Section 3.

An alternative formulation of the model reported in Section 2 (currently being looked into) is presented in Section 4. The relevant MILP model extends, in the case of tetris-like items and convex domains, previous formulations available in the literature, based on the discretization of the domain and often referred to as *space-indexed* or *grid-based-position* paradigms (e.g. Beasley [1], Hadjiconstantinou and Christofides [10]). All models presented in Sections 2, 3 and 4 are suitable for considering additional conditions, such as, for instance, specific loading requirements or balancing. Nevertheless, these aspects, albeit frequent in a number of real-world applications, are not considered in this chapter and the reader is referred to (Fasano [8]) for an extensive discussion (except the *space-indexed* formulation). Section 5 introduces the generation of (two-dimensional) covering tetris-like items, providing outer approximation of polygons. The issue of simplifying the representation of complex objects in such a way is a very interesting optimization problem per se, especially considering its potential applications. The three-dimensional extension is not surveyed in this chapter (since it is quite straightforward). Section 6 proposes a novel heuristic approach, mainly based on the MILP model presented in Section 2.

An extensive experimental analysis has recently been carried out, concerning the MILP model presented in Section 2. One chapter of this book (Gliozzi et al. [9]) reports and examine the computational results available to date, in depth, highlighting the advantages of the overall methodology suggested. Since this chapter is restricted to the computational aspects (assuming the relevant model as known) the present work serves also the scope of providing a topical framework. Fasano [8] offers an extensive bibliography, both on packing problems in general and on the more specific subjects considered here.

In order to state the general problem discussed in this chapter, the following definition is introduced.

A tetris-like item is a set of rectangular parallelepipeds positioned orthogonally, with respect to an (orthogonal) reference frame. This frame is called 'local' and each parallelepiped is a 'component'.

Hereinafter, 'tetris-like item' will usually be simply referred to as 'item', if no ambiguity occurs; similarly, 'rectangular parallelepipeds' are referred to as 'parallelepipeds'.

A set I of N items, together with a domain D, consisting of a (bounded) convex polyhedron, is considered (see Fig. 1). This is associated with a given orthogonal reference frame, indicated in the following as the main frame. The general problem is to place items into D, maximizing the loaded volume, considering the following positioning rules:

- *each local reference frame has to be positioned orthogonally, with respect to the main one* (*orthogonality* conditions);
- *for each item, each component has to be contained within D (domain* conditions);
- *the components of different items cannot overlap* (*non-intersection* conditions).

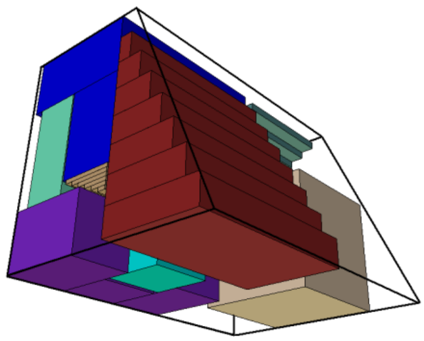

Fig. 1 Tetris-like item packing into a convex domain

2 Direct MILP Formulation

An MILP model for the general problem stated in Section 1 is described next, expanding on some aspects not pointed out in its previous discussion (Fasano [8]). Recalling the basic concepts introduced there, the main orthogonal reference frame has origin O and axes w_β, $\beta \in \{1,2,3\} = B$. It is assumed, without loss of generality, that the whole domain D is entirely contained inside its first octant. Similarly, each local reference frame, associated to every item, is chosen so that all item components lie within its first octant. Its origin coordinates, with respect to the main reference frame, are denoted by $o_{\beta i}$. The set Ω of all (24 possible) orthogonal rotations, admissible for any local reference frame, with respect to the main one, is introduced.

The set of components of a generic item *i* is denoted by C_i. For each item *i*, the set E_{hi} of all (8) vertices associated with each of its components *h* is defined. An extension of this set is obtained by adding to E_{hi} the geometrical center of component *h*. This extended set is denoted by $\widehat{E}_{hi}$. For each item *i* and each possible orthogonal orientation $\omega \in \Omega$, the following binary (0-1) variables are introduced:

$\chi_i \in \{0,1\}$, with $\chi_i = 1$ if item *i* is chosen; $\chi_i = 0$ otherwise;

$\vartheta_{\omega i} \in \{0,1\}$, with $\vartheta_{\omega i} = 1$ if item *i* is chosen and it has the orthogonal orientation $\omega \in \Omega$; $\vartheta_{\omega i} = 0$ otherwise.

The *orthogonality* conditions can be expressed as follows:

$$\forall i \in I \qquad \sum_{\omega \in \Omega} \vartheta_{\omega i} = \chi_i , \tag{1}$$

$$\forall \beta \in B, \forall i \in I, \forall h \in C_i, \forall \eta \in \widehat{E}_{hi} \qquad w_{\beta\eta hi} = o_{\beta i} + \sum_{\omega \in \Omega} W_{\omega\beta\eta hi} \vartheta_{\omega i} . \tag{2}$$

Here $w_{\beta\eta hi}$ ($\forall \eta \in \widehat{E}_{hi}$) are the vertex coordinates of component *h*, with respect to the main reference frame, or its geometrical center ($\eta = 0$), relative to item *i*; $W_{\omega\beta\eta hi}$ are the projections on the axes w_β of the coordinate differences between points $\eta \in \widehat{E}_{hi}$ and the origin of the local reference frame, corresponding to orientation ω of item *i*.

The *domain* conditions are expressed as follows.

$$\forall \beta \in B, \forall i \in I, \forall h \in C_i, \forall \eta \in E_{hi} \qquad w_{\beta\eta hi} = \sum_{\gamma \in V} V_{\beta\gamma} \lambda_{\gamma\eta hi} , \tag{3}$$

$$\forall i \in I, \forall h \in C_i, \forall \eta \in E_{hi} \quad \sum_{\gamma \in V} \lambda_{\gamma\eta hi} = \chi_i \tag{4}$$

Here *V* is the set of vertices delimiting *D*, $V_{\beta\gamma}$ are their coordinates (with respect to the main reference frame) and $\lambda_{\gamma\eta hi}$ are non-negative variables. These conditions correspond to the well-known necessary and sufficient conditions for a point to belong to a convex domain.

The *non-intersection* conditions are represented by the constraints shown below, see (Fasano [8]) for more details:

$$\forall \beta \in B, \forall i, j \in I / i < j, \forall h \in C_i, \forall k \in C_j \tag{5-1}$$

$$w_{\beta 0hi} - w_{\beta 0kj} \geq \frac{1}{2} \sum_{\omega \in \Omega} \left(L_{\omega \beta hi} \vartheta_{\omega i} + L_{\omega \beta kj} \vartheta_{\omega j} \right) - D_\beta \left(1 - \sigma^{+}_{\beta hkij} \right),$$

$$\forall \beta \in B, \forall i, j \in I / i < j, \forall h \in C_i, \forall k \in C_j \tag{5-2}$$

$$w_{\beta 0kj} - w_{\beta 0hi} \geq \frac{1}{2} \sum_{\omega \in \Omega} \left(L_{\omega \beta hi} \vartheta_{\omega i} + L_{\omega \beta kj} \vartheta_{\omega j} \right) - D_\beta \left(1 - \sigma^{-}_{\beta hkij} \right),$$

$$\forall i, j \in I / i < j, \forall h \in C_i, \forall k \in C_j \tag{6}$$

$$\sum_{\beta \in B} \left(\sigma^{+}_{\beta hkij} + \sigma^{-}_{\beta hkij} \right) \geq \chi_i + \chi_j - 1,$$

$$\forall i, j \in I / i < j, \forall h \in C_i, \forall k \in C_j \tag{7-1}$$

$$\sum_{\beta \in B} \left(\sigma^{+}_{\beta hkij} + \sigma^{-}_{\beta hkij} \right) \leq \chi_i,$$

$$\forall i, j \in I / i < j, \forall h \in C_i, \forall k \in C_j \tag{7-2}$$

$$\sum_{\beta \in B} \left(\sigma^{+}_{\beta hkij} + \sigma^{-}_{\beta hkij} \right) \leq \chi_j.$$

Here the constants D_β are the sides (respectively parallel to the main reference frame axes) of the parallelepiped, of minimum dimensions, containing *D*; $w_{\beta 0hi}$ and $w_{\beta 0kj}$ are the center coordinates, with respect to the main reference frame, of components *h* and *k* of items *i* and *j* respectively; $L_{\omega \beta hi}$ and $L_{\omega \beta kj}$ are their side projections on the w_β axes, corresponding to the orientation ω; $\sigma^{+}_{\beta hkij}$ and $\sigma^{-}_{\beta hkij} \in \{0,1\}$.

The constraints (7-1) and (7-2) have been introduced with the purpose of *tightening* the model (they are not taken account of in the following). It is worth noticing that, in some particular situations, the above *non-intersection* constraints ((5-1), (5-2) and (6)) should be properly complemented, in order to avoid solutions that could hardly be considered as appropriate in practice (see Fasano [8]). Nonetheless, these aspects will be omitted here.

The most straightforward formulation relevant to the *objective* function, to maximize the volume loaded, is the following:

$$max \sum_{i \in I} V_i \chi_i \, , \tag{8}$$

where V_i represents the volume of item *i*.

The formulation represented by expressions (1) to (8) (with possible variants regarding the constraints) is notoriously inefficient, even when restricted to single parallelepipeds only, and the situation tends to become even worse when tetris-like items are involved.

The following expression has thus been suggested (Fasano [8]) as a promising alternative to (8):

$$max \sum_{i \in I, h \in C_i} \frac{V_{hi}}{\sum_{\alpha \in A} L_{\alpha hi}} \left(\sum_{\substack{\beta \in B, \\ \omega \in \Omega}} L_{\omega\beta hi} \vartheta_{\omega i} \right), \tag{9}$$

where $L_{\alpha hi}$, $\alpha \in \{1,2,3\} = A$, are the sides of the generic component *h* of item i. It is assumed, without loss of generality, that $L_{1hi} \le L_{2hi} \le L_{3hi}$.

As easily seen, the functions (8) and (9) are equivalent for any *integer-feasible* solution. Indeed, the following implications hold:

$$\forall i \in I, \forall h \in C_i \quad \chi_i = 0 \Leftrightarrow \frac{\sum_{\substack{\beta \in B, \\ \omega \in \Omega}} L_{\omega\beta hi} \vartheta_{\omega i}}{\sum_{\alpha \in A} L_{\alpha hi}} = 0 \, , \tag{10-1}$$

$$\forall i \in I, \forall h \in C_i \quad \chi_i = 1 \Leftrightarrow \frac{\sum_{\substack{\beta \in B, \\ \omega \in \Omega}} L_{\omega\beta hi} \vartheta_{\omega i}}{\sum_{\alpha \in A} L_{\alpha hi}} = 1 . \tag{10-2}$$

Both derive from (1), the second, in particular, is true in virtue of the fact that, in any *integer-feasible* solution: $\forall i \in I / \chi_i = 1, \ \exists! \omega \in \Omega / \vartheta_{\omega i} = 1$.

Since *objective* functions (8) and (9) are equivalent, they give rise to the same optimal (or sub-optimal) integer solutions. Nonetheless, quite different behaviours occur when dealing with (partial or total) *LP-relaxations* of the MILP model (as usually utilized by the solvers), making the choice for the second one highly preferable. Some considerations follow, in support of this point […]

4 Grid-based Position MILP Model

The *space-indexed* approach (e.g. Beasley [1], Hadjiconstantinou and Christofides [10]) can be advantageously reconsidered to include operational scenarios that are quite frequent in practice. Relevant extensions, albeit still addressed to box-shaped items and domains, are aimed at allowing for additional conditions, such as *stability* and *load bearing* (cf. Junqueira et al. [11]). This section focuses instead on a *grid-based-position* MILP model, conceived as an alternative to the one discussed in Section 2, focusing on the orthogonal packing of tetris-like items, inside a convex region.

The given domain (of Section 1) is discretized, so that it is associated with a set of internal points whose coordinates are supposed to be integer. The main reference frame, still defined as in Section 1, thus becomes a unit-cube grid, whose node coordinates are indicated as $(n_1, n_2, n_3) \in D$. Tetris-like items are grouped on a typology basis. The set of all types τ is denoted by T.

The following assumptions relevant to each tetris-like item are made:

- *the local reference frame has a pre-fixed orientation (orthogonal with respect to the main one);*
- *the local reference frame origin can only be positioned on grid points; all component vertices have integer coordinates.*

Remark 4.1 It should be observed that the prefixed orientation assumption does not represent an actual limitation. Orthogonal rotations of the same object can, indeed, simply be considered by introducing a set of pre-oriented items (one for each possible orthogonal orientation).

For each type τ, the sub-set of grid points in which the local frame origin can be positioned (so that the corresponding item is entirely inside the domain *D*) is introduced. It is denoted hereinafter by D_τ.

The binary variables $\chi_{\tau n_1 n_2 n_3} \in \{0,1\}$ are then defined, with the following meaning:

$\chi_{\tau n_1 n_2 n_3} = 1$	if one item of type τ is positioned with its local reference origin in the grid node of coordinates (n_1, n_2, n_3);
$\chi_{\tau n_1 n_2 n_3} = 0$	otherwise.

A possible modeling of the general problem (of Section 1) is shown next, considering the *orthogonality*, *domain* and *non-intersection* conditions.

[…]

As for the model discussed in Section 2, also in this case additional conditions, such as balancing, could quite easily be introduced. They are, however, not taken into account here. It should, moreover, be observed, that the *grid-based position* model, as formulated in this section is (at least) theoretically susceptible to extensions contemplating any irregularly-shaped item type. In such cases, the above mentioned pre-processing phase should be carried out appropriately.

5 An MILP Approach for the Tetris-like Approximation of Irregular Items

6 Heuristics

An overall modeling-based heuristic methodology has been developed to tackle real-world scenarios, generally consisting of large-scale instances, characterized by tricky geometries dealt with by tetris-like approximations, in the presence of additional conditions such as balancing. In (Fasano [8]) a range of models and procedures were discussed in a general framework, providing the basis to build alternative solution strategies. A novel and promising approach, representing the objective of ongoing research, is, instead, discussed here (see Gliozzi et al. [9] for experimental results). Prior to proceeding with the topical discussion, the basic concept of *abstract configuration* (Fasano [8]) is recalled, providing the following two definitions.

Constraints of the types

$$w_{\beta 0hi} - w_{\beta 0kj} \geq \frac{1}{2}\sum_{\omega\in\Omega}\left(L_{\omega\beta hi}\vartheta_{\omega i} + L_{\omega\beta kj}\vartheta_{\omega j}\right),$$
$$w_{\beta 0kj} - w_{\beta 0hi} \geq \frac{1}{2}\sum_{\omega\in\Omega}\left(L_{\omega\beta hi}\vartheta_{\omega i} + L_{\omega\beta kj}\vartheta_{\omega j}\right),$$

corresponding to either $\sigma^{+}_{\beta hkij}=1$ *or* $\sigma^{-}_{\beta hkij}=1$ *in (5-1) and (5-2) , respectively, are called relative position constraints.*

Given a set of N items and the corresponding N_C *pairs of components belonging to different items, an abstract configuration consists of* N_C *relative position constraints, exactly one for each pair, giving rise to a feasible solution in any unbounded domain.*

A method to extract an *abstract configuration* from any approximate solution, with intersections between items, has been shown (Fasano [8]): this subject is not discussed here, referring to the cited work. As previously, the whole process discussed in this section is essentially based on the following modules: *Initialization*, *Packing*, *Item-exchange* and *Hole-filling*. In the versions investigated here, they are based on the MILP model presented in Section 2. In the following, the heuristic overall logic is outlined first and then the basic modules are considered.

6.1 Overall Logic

As in the heuristics looked into in the previous work, the search algorithm consists of a recursive procedure that, at each step, activates one of the above mentioned modules. An *abstract configuration* is generated at each step tentatively improving the previous one; the best-so-far solution is retrieved when the current step does not meet its objective. The search process is terminated when a satisfactory, albeit non-optimal proven solution (in terms of loaded volume) is found. Since for real-world instances the computational task is quite demanding, at each step, only sub-optimal solutions are sought, interrupting the optimization on the basis of suitable stopping rules.

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6.2 Use of the General MILP Model

The *Initialization* module, in the version considered here, focuses on the use of a specific *LP-relaxation* of the general MILP model of Section 2. As the relevant sub-problem is expressed in terms of *feasibility*, all variables χ_i ($\forall i \in I$) are set to 1. The $l_{\beta h i}$ variables, introduced in Section 2, are reconsidered instead. These are not defined any longer as $l_{\beta h i} = \sum_{\omega \in \Omega} L_{\omega \beta h i} \vartheta_{\omega i}$, but simply as continuous variables subject to the following bounds:

$$\forall \beta \in B, \forall i \in I, \forall h \in C_i \quad L_{1hi} \le l_{\beta hi} \le L_{3hi}. \tag{27}$$

Here, as previously specified, L_{1hi} and L_{3hi} represent the sides associated with *h*, of minimum and maximum length respectively. The *non-intersection* conditions (5-1) and (5-2) and the *objective* function (9) are rewritten as follows:

$$\forall \beta \in B, \forall i, j \in I / i < j, \forall h \in C_i, \forall k \in C_j \tag{28-1}$$
$$w_{\beta 0hi} - w_{\beta 0kj} \geq \frac{1}{2} \sum_{\omega \in \Omega} \left(l_{\beta hi} + l_{\beta kj} \right) - D_\beta \left(1 - \sigma^{+}_{\beta hkij} \right),$$

$$\forall \beta \in B, \forall i, j \in I / i < j, \forall h \in C_i, \forall k \in C_j \tag{28-2}$$
$$w_{\beta 0kj} - w_{\beta 0hi} \geq \frac{1}{2} \sum_{\omega \in \Omega} \left(l_{\beta hi} + l_{\beta kj} \right) - D_\beta \left(1 - \sigma^{-}_{\beta hkij} \right),$$

$$max \sum_{i \in I, h \in C_i} \frac{V_{hi}}{\sum_{\alpha \in A} L_{\alpha hi}} \left(\sum_{\beta \in B} l_{\beta hi} \right). \tag{29}$$

[…]

7 Conclusion

Non-standard packing problems that involve non-box-shaped items and domains, in the presence of additional constraints, are usually very tough to solve. This chapter, extending the author’s previous work, discusses the issue of placing tetris-like items orthogonally into a convex domain. A Global Optimization point of view, focused on MILP/MINLP formulations, is looked into for the purpose of providing models that are suitable for treating additional loading restriction rules and global conditions such as balancing.

An efficient heuristic procedure, aimed at finding satisfactory solutions to real-world instances, is proposed. This approach will be the objective of future investigation, focused on the MILP/MINLP search strategies.

The issue of covering irregularly-shaped objects with tetris-like items consisting of a given number of components of minimum total volume, itself, leads to a non-trivial optimization problem. Insights on its two-dimensional version, relevant to the optimal outer approximation of polygons, are provided. A further contribution appearing in this book is dedicated to the computational aspects relevant to the MILP model discussed in this chapter.

References

1. Beasley, J.E.: An exact two-dimensional non-guillotine cutting tree search procedure. Oper. Res. **33** (1), 49-64 (1985)
2. Birgin, E.G., Lobato, R.D.: Orthogonal packing of identical rectangles within isotropic convex regions. Computers and Industrial Engineering **59** (4), 595-602 (2010)
3. Birgin, E., Martinez, J., Nishihara, F.H.,Ronconi, D.P.: Orthogonal packing of rectangular items within arbitrary convex regions by nonlinear optimization. Computers & Operations Research **33** (12), 3535-3548 (2006)
4. Cassioli, A., Locatelli, M.: A heuristic approach for packing identical rectangles in convex regions. Computers and Operations Research **38** (9), 1342-1350 (2011)
5. Castellazzo, A., Fasano, G. Pintér, J.D.: An MINLP Formulation for The Container Loading Problems: An Experimental Analysis. Working paper, Thales Alenia Space (2015)
6. Chen, C.S., Lee, S.M., Shen, Q.S.: An analytical model for the container loading problem. European Journal of Operational Research **80**, 68-76 (1995)
7. Egeblad, J., Nielsen, B. K., Odgaard, A.: Fast neighborhood search for two-and three-dimensional nesting problems. European Journal of Operational Research **183** (3),1249-1266 (2007)
8. Fasano, G.: Solving Non-standard Packing Problems by Global Optimization and Heuristics. SpringerBriefs in Optimization**,** Springer Science + Business Media, New York (2014)
9. Gliozzi, S., Castellazzo, A., Fasano, G.: Container Loading Problem MIP-based Heuristics Solved by CPLEX: An Experimental Analysis. In: Fasano, G., Pintér, J.D., (eds.) Optimized Packings and Their Applications. Springer Optimization and Its Applications, Springer Science + Business Media, New York (2015)
10. Hadjiconstantinou, E., Christofides, N.: An exact algorithm for general, orthogonal, two-dimensional knapsack problems. European Journal of Operational Research **83** (1), 39-56 (1995)
11. Junqueira, L., Morabito, R., Yamashita, D.S., Yanasse, H.H.: Optimization Models for the Three-Dimensional Container Loading Problem with Practical Constraints. In: Fasano, G. and Pintér, J.D. (eds.) Modeling and Optimization in Space Engineering, pp. 271-294. Springer Science + Business Media, New York (2013)
12. Padberg M.W.: Packing small boxes into a big box. Office of Naval Research, N00014-327, New York University (1999)
13. Pisinger, D., Sigurd, M.: The two-dimensional bin packing problem with variable bin sizes and costs. Discrete Optimization **2** (2), 154-167 (2005)
14. Stoyan, Y.G., Chugay, A.M.: Packing cylinders and rectangular cuboids with distances between them into a given region. European Journal of Operational Research **197**, 446-455 (2009)

15. Stoyan, Y.G., Zlotnik, M. V., Chugay, A. M.: Solving an optimization packing problem of circles and non-convex polygons with rotations into a multiply connected region. Journal of the Operational Research Society **63** (3), 379-391 (2012)