Branch-and-Cut approaches for p-Cluster Editing

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Abstract

This paper deals with a variant of the well-known Cluster Editing Problem (CEP), more precisely, the p-CEP, in which a given input graph should be edited by adding and/or removing edges in such a way that p vertex-disjoint cliques (clusters) are generated with the minimum number of editions. We introduce several valid inequalities where some of them turned out to be very effective when implemented in branch-and-cut approaches over two mathematical formulations. Computational experiments were carried out over a set of instances available in the CEP literature. The results obtained show the efficiency of the approaches according to the value of p, the graph density and the ratio between p and the number of vertices.

Keywords: Cluster Editing, Valid Inequalities, Branch-and-cut

1. Introduction

The Cluster Editing Problem (CEP) can be defined as follows. Given a graph $G = (V, E)$, transform G into a vertex-disjoint union of cliques by inserting and deleting a minimum number of edges, i.e., by making a minimum number of *editions* in G. The cliques of a solution are referred to as *clusters* (a cluster with only one vertex is a *singleton*) and a graph

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where each component is a clique is a *cluster graph*. Figure 1 illustrates an example of a CEP instance and its optimal solution.

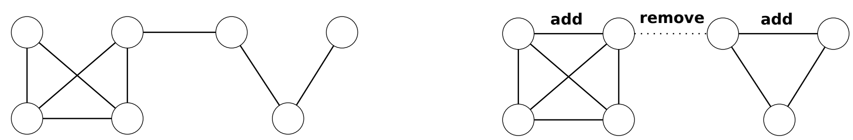


Figure 1: A CEP instance (left) and an optimal solution with 3 editions (right).

CEP is a $\mathcal{NP}$-Hard problem [29] with important applications in the fields of computational biology [6], machine learning [4], image processing [31] and data mining [7], to name a few. The problem is also APX-Hard [14], but constant factor approximation algorithms were given in [4], [2] and [14]. Moreover, it is known that CEP remains $\mathcal{NP}$-Hard even for graphs of maximum degree six and if each vertex is incident to at most four editions [24]. On the other hand, the problem is polynomial-time solvable in special classes of graphs such as paths and circles [8]. For more details on the complexity and applications of CEP, the reader is referred to the recent survey presented in [9].

The methods proposed in the literature to solve CEP range from *heuristics* [6, 28, 5] to *parameterized algorithms* [20, 21, 10, 13, 8, 27] and *exact methods* [12, 16]. Among these works, we highlight the one presented in [12], in which the authors managed to solve large instances (with hundreds of nodes) to optimality using a *branch-and-cut* over a mathematical formulation due to Grötschel and Wakabayashi [22]. It is worth mentioning that there is a great interest in exact CEP solutions in applications that arise in the field of computational biology [12].

In the last years, some CEP variants have been studied [24, 1, 11]. In this work, we are particularly interested in the p-Cluster Editing Problem (p-CEP) where the objective is to perform a minimum number of editions in a graph $G = (V, E)$ in order to transform it in a vertex-disjoint union of p cliques, $1 \leq p \leq |V|$ (1 and $|V|$ are trivial values for p). As mentioned in [18], p is an external requirement that must be satisfied for some practical settings (see for instance [3] and [19]). We now denote a p-CEP solution as a *p-cluster graph.*

p-CEP is $\mathcal{NP}$-Hard for every $p \geq 2$ [29], but it has a PTAS [18]. As in the general case, p-CEP was studied in the field of parameterized algorithms [23, 17]. In [15], the author extended the CEP formulation proposed in [22] in order to guarantee that the solution has exactly p cliques. However, in contrast to CEP, we are not aware of a work reporting exact solutions for large p-CEP instances.

Given the above, the main interest of this work is to devise an exact method for solving p-CEP. More precisely, we propose a branch-and-cut algorithm over a mathematical formulation from the literature that was enhanced by adding new valid inequalities for the problem. A new mathematical formulation, also implemented using a branch-and-cut approach, is introduced, but we show that this formulation is theoretically dominated by the one from the literature. Nevertheless, this new formulation appears to be more effective in practice than the other one for very small values of p.

The remainder of this work is organized as follows. Section 2 presents the mathematical formulations. Section 3 introduces the new valid inequalities. Section 4 describes the proposed branch-and-cut approaches. Section 5 contains the results of the computational experiments. Finally, Section 6 brings the concluding remarks.

2. Mathematical Formulations

Let $G = (V, E)$ be the input graph with vertex set $V = \{1, 2, ..., n\}$ and edge set E. Define x_{ij} as a binary variable associated to each pair of vertices $i \in V$ and $j \in V$, with $i < j$, such that $x_{ij} = 1$ if the edge $\{i, j\}$ is in the the solution and $x_{ij} = 0$, otherwise. Therefore, CEP can be formulated as follows.

$$\min \sum_{\{i,j\} \in E} (1 - x_{ij}) + \sum_{\{i,j\} \notin E} x_{ij} \tag{1}$$

$$
\begin{aligned}
\text{s.t.} \quad & -x_{ij} + x_{ik} + x_{jk} \leq 1, && \forall i, j, k \in V, i < j < k, && (2)\\
& x_{ij} - x_{ik} + x_{jk} \leq 1, && \forall i, j, k \in V, i < j < k, && (3)\\
& x_{ij} + x_{ik} - x_{jk} \leq 1, && \forall i, j, k \in V, i < j < k, && (4)\\
& x_{ij} \in \{0, 1\}, && \forall i, j \in V, i < j. && (5)
\end{aligned}
$$

This formulation was proposed in [22]. The objective function (1) minimizes the number of removals (first sum) and additions (second sum) of edges. Constraints (2)-(4) guarantee that the solution has no path with 3 vertices, a.k.a. P_3, as an induced subgraph (see Figure (2)). This is sufficient to ensure that the solution is a vertex-disjoint union of cliques. Throughout the paper, we will refer to (2)-(4) as *triangle inequalities*.

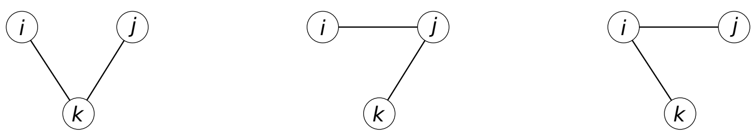


Figure 2: Induced subgraphs forbidden by (2)-(4) for each three vertices $i, j, k \in V$, respectively from the left to the right.

We now proceed to the p-CEP formulations.

2.1. Formulation F1

It is quite straightforward to formulate the p-CEP using three-index variables. Denote the clusters of a p-CEP solution as $C_1, C_2, ..., C_p$ and define the set $K = \{1, 2, ..., p\}$. Let w_{ijk} be a binary variable, where $w_{ijk} = 1$ if the edge $\{i, j\}$ belongs to C_k and $w_{ijk} = 0$, otherwise. Also, let z_{ik} be a binary variable, where $z_{ik} = 1$ if the vertex i belongs to C_k and $z_{ik} = 0$, otherwise. Formulation F1 can be written as follows.

$$
\min \sum_{\{i,j\} \in E} (1 - \sum_{t \in K} w_{ijt}) + \sum_{\{i,j\} \notin E} \sum_{t \in K} w_{ijt} \tag{6}
$$

$$
\begin{aligned}
\text{s.t.}\quad & w_{ijt} + 1 \geq z_{it} + z_{jt}, && \forall i,j \in V, i<j, \forall t \in K, && (7)\\
& w_{ijt} \leq z_{it}, && \forall i,j \in V, i<j, \forall t \in K, && (8)\\
& w_{ijt} \leq z_{jt}, && \forall i,j \in V, i<j, \forall t \in K, && (9)\\
& \sum_{t \in K} z_{it} = 1, && \forall i \in V, && (10)\\
& \sum_{i \in V} z_{it} \geq 1, && \forall t \in K, && (11)\\
& w_{ijt} \in \{0,1\}, && \forall i,j \in V, i<j, \forall t \in K, && (12)\\
& z_{it} \in \{0,1\}, && \forall i \in V, \forall t \in K. && (13)
\end{aligned}
$$

The objective function (6) minimizes the number of editions in the input graph. Constraints (7), (8) and (9) ensure that an edge $\{i,j\}$ belongs to C_k iff i and j are both in C_k, while constraints (10) and (11) impose that each vertex should belong to exactly one cluster and that each cluster has at least one vertex, respectively.

Formulation F1 may be rather intuitive, but it provides a very poor linear relaxation in practice, which in fact is the worst possible, i.e., 0.

Proposition 1. *The linear relaxation of F1 is 0.*

Proof. Consider the following solution for the linear program of F1 (LP-F1):

$$
s = \begin{cases} w_{ijt} = 1/p, \forall \{i,j\} \in E, \forall t \in K, \\ w_{ijt} = 0, \forall \{i,j\} \notin E, \forall t \in K, \\ z_{it} = 1/p, \forall i \in V, \forall t \in K. \end{cases}
$$

It is trivial to see that s has cost 0 and is feasible for LP-F1. □

Another drawback of F1 is its inherent symmetry, which negatively affects the performance of a branch-and-bound(cut) based procedure over such formulation.

In what follows, we discuss a better formulation for p-CEP.

2.2. Formulation F2

As shown in [15], if one defines another binary variable set y_i, $i \in V$, the CEP formulation presented in [22] can be adapted in order to ensure that the solution has exactly p clusters. In this case, $y_i = 1$ when the vertex i is a *leader of a cluster* and $y_i = 0$, otherwise. The leader of a cluster C is the vertex $i \in C$ with minimum index value. The complete p-CEP formulation proposed in [15] is given below.

$$\min \sum_{\{i,j\}\in E} (1 - x_{ij}) + \sum_{\{i,j\}\notin E} x_{ij} \tag{14}$$

$$\begin{aligned}
\text{s.t.} \quad & (2), (3), (4), & & \\
& y_1 = 1, & & (15)\\
y_j \leq & \frac{\sum_{i<j}(1 - x_{ij})}{j-1}, & \forall j \in V, j \geq 2, & \quad (16)\\
y_j \geq & 1 - \sum_{i<j} x_{ij}, & \forall j \in V, j \geq 2, & \quad (17)\\
& \sum_{j\in V} y_j = p, & & (18)\\
& x_{ij} \in \{0,1\}, & \forall i, j \in V, i < j, & \quad (19)\\
& y_i \in \{0,1\}, & \forall i \in V. & \quad (20)
\end{aligned}$$

We have made explicit in constraint (15) that $y_1 = 1$, but this directly follows from the definition of leader of a cluster. Constraint (18) determines that the solution has p clusters. Constraints (16) and (17) ensure that a vertex j is the leader of a cluster iff it is not connected to any vertex i such that $i < j$. In particular, constraints (16) enforce y_j to zero whenever j is connected to some i such that $i < j$, and constraints (17) enforce y_j to 1 when j is not connected to any i such that $i < j$.

Note that the constraints (16), which are clearly weak in a polyhedral sense, can be replaced by the following stronger inequalities:

$$y_j \leq 1 - x_{ij}, \forall i, j \in V, i < j, j \geq 2. \tag{21}$$

Furthermore, inequalities (2) can be lifted (see (28)), as we will later show in Section 3.1.

In the remainder of the paper, we focus on Formulation F2, defined by: (14), (3), (4), (28), (15), (21), (17), (18), (19) and (20). This formulation is not symmetric and has a stronger linear relaxation when compared to F1, as stated by the following proposition.

Proposition 2. *The linear relaxation of F2 is stronger than the one of F1.*

Proof. Since the linear relaxation of F1 is 0 (Proposition 1), the linear relaxation of F2 should be at least as strong as the one of F1. Therefore, we only have to show an instance where the linear linear relaxation of F1 is strictly stronger than the one of F2.

Consider the very small instance I: $V = \{0, 1\}$, $E = \{\{0, 1\}\}$, $p = 2$. For this instance, the linear program of F2 (LP-F2-I) can be written as follows.

$$\min \quad 1 - x_{01} \tag{22}$$

$$\text{s.t.} \quad y_0 = 1, \tag{23}$$

$$y_1 \leq 1 - x_{01}, \tag{24}$$

$$y_1 \geq 1 - x_{01}, \tag{25}$$

$$y_0 + y_1 = 2, \tag{26}$$

$$y_0, y_1, x_{01} \in [0, 1]. \tag{27}$$

Note that LP-F2-I has no triangle inequality because I has only two vertices. It can be seen that it is not possible to build a feasible solution of LP-F2-I with cost 0, since constraints (23)-(26) necessarily imply $x_{01} = 0$. Thus, the unique feasible solution of LP-F2-I is $x_{01} = 0$, $y_0 = 1$, $y_1 = 1$, with associated cost equal to 1. □

3. New Valid Inequalities

In this section we present some valid inequalities for F2. When possible, we also show how to adapt such inequalities for F1.

3.1. Lifted Triangle Inequalities

We will now show how inequalities (2) can be strengthen by adding variable y_k.

Proposition 3. *The so-called lifted triangle inequalities can be obtained by introducing variable y_k in the left-hand side of (2), that is:*

$$-x_{ij} + x_{ik} + x_{jk} + y_k \leq 1, \qquad \forall i, j, k \in V, i < j < k. \tag{28}$$

Proof. In order to prove that inequalities (28) are valid, we only have to consider the case where $y_k = 1$, because when $y_k = 0$ the inequalities become the original ones. If $y_k = 1$, then $x_{ik} = x_{jk} = 0$, since $y_k = 1$ means that the vertex k is not connected to any vertex with index less than k. Thus, the inequality becomes $x_{ij} \geq 0$, which is valid. It should be noted that, because of the role played by the indexes in the formulation, the same reasoning can not be applied to (3) and (4) in order to lift such inequalities. □

We remark once again that these inequalities are already considered to be part of F2, as mentioned in Section 2.2.

3.2. $|E|$-bounds Inequalities

The second valid inequalities are lower and upper bounds on the number of edges in a p-CEP solution, which we call *$|E|$-bounds inequalities.*

We start by characterizing a solution with the maximum number of edges. The upper bound follows the characterization.

Proposition 4. *Let $G' = (V', E')$ be a p-CEP solution. If G' has the maximum possible number of edges, then G' has at most one non-singleton cluster.*

Proof. Consider by contradiction that C_i and C_j are two non-singleton clusters of G' and suppose that $|C_i| \geq |C_j|$. Now move one vertex from C_j to C_i. We are then deleting $|C_j| - 1$ edges and adding $|C_i|$ edges. This increases the number of edges of G', since $|C_i| > |C_j| - 1$, which contradicts the fact that G' has the maximum possible number of edges. □

Corollary 1. *The maximum number of edges in a p-CEP solution* $G' = (V', E')$ *is* $\frac{(|V|-(p-1))(|V|-(p-1)-1)}{2}$.

We now proceed to the lower bound, which is very similar to Lemma 16 of [17].

Proposition 5. *Let* $G' = (V', E')$ *be a p-CEP solution. If* G' *has the minimum possible number of edges, then, for any pair of clusters* C_i, C_j *of* G'*, it holds that* $||C_i| - |C_j|| \leq 1$.

Proof. Suppose towards a contradiction that G' has two clusters C_i and C_j such that $||C_i| - |C_j|| > 1$, and w.l.o.g. suppose $|C_i| > |C_j|$. It suffices to see that moving one vertex from C_i to C_j decreases the number of edges of G'. Indeed, when moving one vertex from C_i to C_j, we are deleting $|C_i| - 1$ edges and adding $|C_j|$ edges to G'. Since $|C_i| - |C_j| > 1$, we are in fact deleting $|C_i| - |C_j| - 1 > 0$ edges, which contradicts the fact that G' has the minimum possible number of edges. □

Corollary 2. *The minimum number of edges in a p-CEP solution* $G' = (V', E')$ *is* $(|V| \bmod p)\frac{(q+1)q}{2} + (p - |V| \bmod p)\frac{q(q-1)}{2}$, *where* $q = \lfloor \frac{|V|}{p} \rfloor$.

We can now write the $|E|$-bounds inequalities for F2 as

$$\sum_{i \in V} \sum_{\substack{j \in V \\ i<j}} x_{ij} \leq \frac{(|V| - (p-1))(|V| - (p-1) - 1)}{2} \tag{29}$$

$$\sum_{i \in V} \sum_{\substack{j \in V \\ i<j}} x_{ij} \geq (|V| \bmod p)\frac{(q+1)q}{2} + (p - |V| \bmod p)\frac{q(q-1)}{2}, \tag{30}$$

where $q = \lfloor \frac{|V|}{p} \rfloor$.

Analogously, for F1 we have

$$\sum_{i \in V} \sum_{\substack{j \in V \\ i<j}} \sum_{t \in K} w_{ijt} \leq \frac{(|V| - (p-1))(|V| - (p-1) - 1)}{2} \tag{31}$$

$$\sum_{i\in V}\sum_{\substack{j\in V\\ i<j}}\sum_{t\in K} w_{ijt} \geq (|V| \bmod p)\frac{(q+1)q}{2} + (p-|V| \bmod p)\frac{q(q-1)}{2}, \quad (32)$$

where $q = \lfloor \frac{|V|}{p} \rfloor$.

3.3. Tree inequalities

The maximum size of a cluster is $|V| - p + 1$, which occurs when there are $p-1$ singletons in the solution. Therefore, any vertex $v \in V$ has at most $|V| - p$ neighbors in the solution. That is:

$$\sum_{i<j} x_{ij} + \sum_{j<k} x_{jk} \leq |V| - p, \forall j \in V. \quad (33)$$

It can be seen that the edges used in an inequality of (33) induce a tree. In general, we can use any tree spanning the vertices $v \in V$ to produce inequalities similar to (33), as we show next.

Proposition 6. *Let $T = (V_T, E_T)$ be a tree such that $V_T = V$. Then the tree inequality*

$$\sum_{\{i,j\}\in E_T} x_{ij} \leq |V_T| - p$$

is valid for F2.

Proof. Let $G = (V, E)$ be a graph with $E = \emptyset$ and let $T = (V_T, E_T)$ be a tree such that $V = V_T$. Add the edges $\{i, j\} \in T$ to G. As each new edge $\{i, j\} \in T$ is added to G, the number of connected components of G necessarily decreases by one, because T has no cycle. Therefore, if $|E \cap E_T| = n$, then G has at most $|V| - n$ connected components. Thus, any p-CEP solution $G' = (V', E')$ has at most $|V'| - p$ edges of a tree T'. □

We can apply the same reasoning to F1. That is, let $T = (V_T, E_T)$ be a tree such that $V_T = V$, then the following inequality is valid for F1:

$$\sum_{\{i,j\}\in E_T} \sum_{t\in K} w_{ijt} \leq |V_T| - p \tag{34}$$

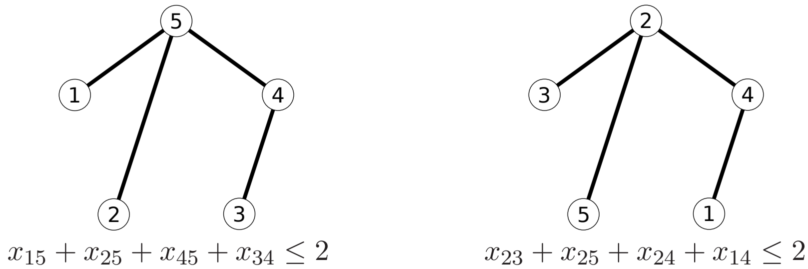


Figure 3: Some tree inequalities for an instance with $|V| = 5$ and $p = 3$.

3.4. Kite Inequalities

In this section we present the last set of valid inequalities proposed in this work.

Proposition 7. *Let $j, u, v, w \in V$ such that $u, v, w < j$. Then the kite inequality*

$$y_j \geq x_{uw} + x_{vw} - x_{uv} - \sum_{\substack{i<j \\ i\neq w}} x_{ij}$$

is valid for F2.

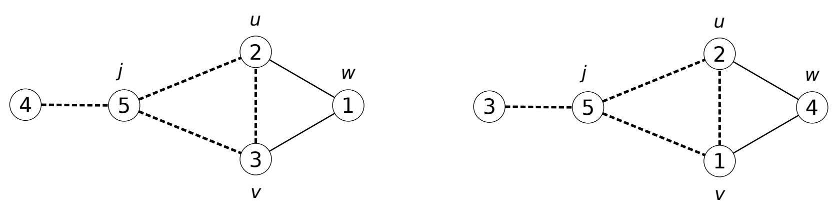
Before presenting the proof itself, we remark that, for ease of presentation, we slightly abuse the x_{ij} notation (where $i < j$) in this proposition, because no assumption can be made regarding the ordering relation between u, v and w.

Proof. The sum $x_{uw} + x_{vw} - x_{uv}$ is bounded from above by 1, because of the triangle inequalities. Thus, if $\sum_{\substack{i<j \\ w\neq j}} x_{ij} \geq 1$ the inequality is obviously valid. Assuming that $\sum_{\substack{i<j \\ w\neq j}} x_{ij} = 0$, we now obtain the inequality

$$y_j \geq x_{uw} + x_{vw} - x_{uv}.$$

If $x_{uw} + x_{vw} - x_{uv} = 1$, then the vertex w is connected to at least one of the vertices u and v. However, neither u nor v is connected to j, since we assumed that $\sum_{\substack{i<j \\ w \neq j}} x_{ij} = 0$. Therefore, from the triangle inequalities, j is not connected to w. All in all, if $\sum_{\substack{i<j \\ w \neq j}} x_{ij} = 0$ and $x_{uw} + x_{vw} - x_{uv} = 1$, the vertex j is not connected to any i such that $i < j$ and the variable y_j must assume value 1. □

Figure 4 shows two kite inequalities for vertex $j = 5$.



$y_5 \geq x_{12} + x_{13} - x_{23} - x_{25} - x_{35} - x_{45}$ $\qquad$ $y_5 \geq x_{14} + x_{24} - x_{12} - x_{15} - x_{25} - x_{35}$

Figure 4: Two kite inequalities for the vertex $j = 5$.

4. Branch-and-Cut Algorithms

We implemented a branch-and-cut algorithm over F2 (BC-F2) to solve p-CEP. We start BC-F2 with all constraints of F2 except the triangle inequalities, which are generated throughout the execution of the algorithm in a cutting plane fashion. Furthermore, we add the $|E|$-bounds inequalities to BC-F2 *a priori*, while tree and kite inequalities are separeted only at the root node.

We also implemented a branch-and-cut algorithm over F1 (BC-F1), which starts with all constraints of F1 and the $|E|$-bounds inequalities. The following cuts (triangle inequalities for F1) are also considered in BC-F1.

$$-\sum_{t\in K} w_{ijt} + \sum_{t\in K} w_{ikt} + \sum_{t\in K} w_{jkt} \leq 1, \qquad \forall i,j,k \in V, i<j<k, \tag{35}$$
$$\sum_{t\in K} w_{ijt} - \sum_{t\in K} w_{ikt} + \sum_{t\in K} w_{jkt} \leq 1, \qquad \forall i,j,k \in V, i<j<k, \tag{36}$$
$$\sum_{t\in K} w_{ijt} + \sum_{t\in K} w_{ikt} - \sum_{t\in K} w_{jkt} \leq 1, \qquad \forall i,j,k \in V, i<j<k. \tag{37}$$

As expected, BC-F1 has in general a poor performance, but it works surprisingly well for instances with very small values of p (see Section 5.4). In the following, we discuss the main components of BC-F1 and BC-F2.

4.1. Separation of Triangle Inequalities

As in [22] and [12], we separate the triangle inequalities by brute force. Although this separation procedure has a $\mathcal{O}(|V|^3)$ time complexity, it is reasonably fast for the size of the problems considered in this work. Of course, the algorithms would benefit from a more efficient separation procedure.

In both algorithms, we execute the separation procedure throughout the tree up to the 9th level and whenever an integer solution is found (the latter is required only for BC-F2). For fractional solutions, the separation procedure generates the 400 most violated inequalities (as in [22]). For integer solutions, the separation procedure generates all violated inequalities in case of $|V| \leq 100$. Otherwise, it generates at most α inequalities per violated edge. An edge $\{i,j\}$ is violated if it is not present in the solution and there exists a vertex k such that i, j and k induce a P_3. Preliminary experiments suggested $\alpha = 3$ for $100 < |V| \leq 150$, $\alpha = 2$ for $150 < |V| \leq 200$ and $\alpha = 1$ for $|V| > 200$.

4.2. Separation of Kite Inequalities

In order to determine a violated kite inequality for a vertex j, one should find three distinct vertices u, v and w that do not satisfy $y_j \geq x_{wu} + x_{wv} - x_{uv} - \sum_{i<j, i\neq w} x_{ij}$. Thus, in principle, the time required to separate the kite inequalities for all vertices is $\mathcal{O}(|V|^4)$. However, one can achieve $\mathcal{O}(|V|^3)$ time via dynamic programming.

Let $(\bar{x}, \bar{y})$ be a fractional solution of F2 and let $\bar{c}(w, u, v) = \bar{x}_{wu} + \bar{x}_{wv} - \bar{x}_{uv}$. Given a fixed w, the maximum violation of a kite inequality for a vertex j is obtained with vertices u and v in such a way that $\bar{x}_{wu} + \bar{x}_{wv} - \bar{x}_{uv}$ is maximized. Therefore, for vertices j and w with $j \geq 4$ and $w < j$, let $\mathcal{T}(j, w)$ be the maximum value $\bar{c}(w, u, v)$ such that $u < j$ and $v < j$. The value $\mathcal{T}(j, w)$ can be expressed by the following recursive expression.

$$\mathcal{T}(j, w) = \begin{cases} \max\{\mathcal{T}(j-1, w), \max\limits_{u}\{\bar{c}(w, j-1, u) \mid u < j-1, u \neq w\}\}, \\ \text{if } w < j-1 \text{ and } j \neq 4. \\ \\ \max\limits_{u,v}\{\bar{c}(w, u, v) \mid u, v < j \text{ and } u, v \neq w \text{ and } u < v\}, \\ \text{if } w = j-1 \text{ or } j = 4. \end{cases}$$

The dynamic programming algorithm to separate the kite inequalities computes all values $\mathcal{T}(j, w)$ by considering increasing values of j for a fixed value of w. Such computations require $\mathcal{O}(|V|^3)$ time, since there are $\mathcal{O}(|V|)$ values $\mathcal{T}(j, w)$ that demand $\mathcal{O}(|V|^2)$ time to be computed (if $w = j - 1$ or $j = 4$) and $\mathcal{O}(|V|^2)$ values $\mathcal{T}(j, w)$ that demand $\mathcal{O}(|V|)$ time to be computed (if $w < j - 1$ and $j \neq 4$). After computing the values $\mathcal{T}(j, w)$, we can determine the maximum violation of a kite inequality for a vertex j using the following expression.

$$\max\{0, \max_{w<j}\{\mathcal{T}(j, w) - \sum_{\substack{i<j \\ i \neq w}} \bar{x}_{ij} - \bar{y}_j\}\}.$$

The violated inequality itself can be determined by keeping track of vertices u and v associated to each value $\mathcal{T}(j, w)$, i.e., vertices u and v for which $\mathcal{T}(j, w) = \bar{c}(w, u, v)$.

4.3. Separation of Tree Inequalities

The tree inequalities, which are exponential in size, can be separated exactly in polynomial time by solving a well known graph optimization problem, as shown in the following.

Proposition 8. *The tree inequalities can be separated in polynomial time by solving the maximum spanning tree problem.*

Proof. The most violated tree inequality (if such a inequality exists) is defined by the edges of a tree T for which $\sum_{\{i,j\}\in T} \bar{x}_{ij}$ is maximized. Such a tree can be computed in polynomial time by Prim's [26] or Kruskal's algorithm [25]. □

4.4. Initial Primal Bound

We provide an initial primal (upper) bound for BC-F1 and BC-F2 in order to speed up the performance of the exact algorithms. Such bound is obtained by means of a heuristic algorithm in the spirit of Bastos et al. [5]. This algorithm, which was originally developed for CEP, combines GRASP (Greedy Randomized Adaptive Procedure) with a refinement method based on Set Partitioning.

5. Computational Experiments

The algorithms were coded in C++ using CPLEX 12.4. All experiments were executed in an Intel Core i7 with 3.4 GHz and with 8 GB of RAM, running Ubuntu 12.04 LTS operating system. The time limit imposed for each instance was 2 hours. Only a single thread was used in our testing. Detailed results of all instances are reported in Appendix.

In the following, we start by first describing the instances considered in this work. Next, we study the impact of the valid inequalities presented in Section 3. Furthermore, we evaluate the performance of BC-F2 according to the graph density and the ratio between p and the number of vertices. Finally, we report the results found by BC-F1 for small values of p.

5.1. Benchmark Instances

The graphs adopted in our experiments are the weighted complete graphs used in [12]. The vertices of these graphs represent more than 192000 protein sequences from the COG database [30] and the weights of the edges represent the similarities between the proteins. The reader is referred to [28] for more details about these graphs, which can be downloaded at `http://bio.informatik.uni-jena.de/data/` (file `biological_bielefeld.zip`).

Let $G = (V, E)$ be one of the weighted graphs mentioned above, and let w_{ij} be the weight of the edge $\{i, j\} \in E$. As we are not interested in edge weights, we transform G into the unweighted graph $G' = (V', E')$, where $V' = V$ and $E' = \{\{i, j\} \in E \mid w_{ij} > 0\}$. Also, to avoid a prohibitive number of experiments, we do not consider all possible values for p. Instead, we just execute the instances with the values $p \in \{2, 6, 10, ..., 2 + 4\lceil \frac{|V|-6}{4} \rceil\}$.

From the 3964 graphs derived from the COG database, we selected 27 graphs with up to 211 nodes. Table 1 describes the selected graphs. In this table, columns **Instance** and **|V|** contain the name and the size of the graph, respectively, and the column **Density** contains the density of the graph. The density of a graph $G = (V, E)$, denoted as d, is defined as the ratio between the number of existing edges and the maximum number of edges, i.e.,

$$d = \frac{2|E|}{|V|(|V| - 1)}. \tag{38}$$

Given the graphs of Table 1 and the selected values of p, the total number of p-CEP instances considered in our experiments is 892. To ease the presentation of our results, we have divided these 892 instances into 10 groups according to the value of $p/|V|$, where the *ith* group contains the instances for which $p/|V| \in (\frac{i-1}{10}, \frac{i}{10}]$ (see Table 2).

5.2. Impact of Valid Inequalities

In this section, we study the impact of the proposed valid inequalities, namely: $|E|$-bounds, kite and tree inequalities. We also study the effect of lifting the triangle inequalities.

5.2.1. Lifted Triangle Inequalities and |E|-bounds Inequalities

In order to analyze the impact of lifted triangle inequalities and $|E|$-bounds inequalities, we executed BC-F2 only at the root node using four different settings regarding the use or not of $|E|$-bounds inequalities and lifting. These four settings are defined in Table 3.

Table 1: Selected instances.

Instance	$\|V\|$	Density	Instance	$\|V\|$	Density
nr_1059_size_55	55	0.19	nr_369_size_60	60	0.26
nr_492_size_61	61	0.61	nr_438_size_72	72	0.26
nr_238_size_78	78	0.88	nr_522_size_85	85	0.44
nr_173_size_86	86	0.19	nr_978_size_92	92	0.76
nr_317_size_95	95	0.81	nr_630_size_102	102	0.14
nr_104_size_112	112	0.81	nr_413_size_127	127	0.45
nr_57_size_129	129	0.23	nr_606_size_130	130	0.91
nr_480_size_139	139	0.42	nr_537_size_144	144	0.68
nr_116_size_152	152	0.34	nr_593_size_157	157	0.49
nr_176_size_169	169	0.89	nr_459_size_171	171	0.11
nr_9_size_178	178	0.79	nr_200_size_179	179	0.42
nr_263_size_186	186	0.33	nr_365_size_196	196	0.14
nr_232_size_206	206	0.31	nr_279_size_209	209	0.54
nr_489_size_211	211	0.67			

Table 2: Interval for $p/|V|$ and size of each group of instances.

Group	$p/\|V\|$ interval	Size
1	$(0.0, 0.1]$	88
2	$(0.1, 0.2]$	92
3	$(0.2, 0.3]$	89
4	$(0.3, 0.4]$	91
5	$(0.4, 0.5]$	89
6	$(0.5, 0.6]$	90
7	$(0.6, 0.7]$	90
8	$(0.7, 0.8]$	85
9	$(0.8, 0.9]$	89
10	$(0.9, 1.0]$	88

Table 3: Different settings involving $|E|$-bounds and lifted triangle inequalities.

Setting	$\|E\|$-bounds inequalities	Lifted triangle inequalities
S1		
S2		✓
S3	✓	
S4	✓	✓

Figures 5 and 6 present the results obtained by BC-F2 using the four settings defined in Table 3. In these figures, the X-axis contains the instances per group. In Figure 5, the Y-axis contains the average root node gap, while in Figure 6 the Y-axis contains the average time (in seconds) spent at the root node. It can be seen that both lifted triangle and $|E|$-bounds inequalities significantly improve the gap and the time spent at the root node in almost all groups. The impact of the $|E|$-bounds inequalities is quite evident in group 10, in which all instances were solved to optimality at the root node after their addition.

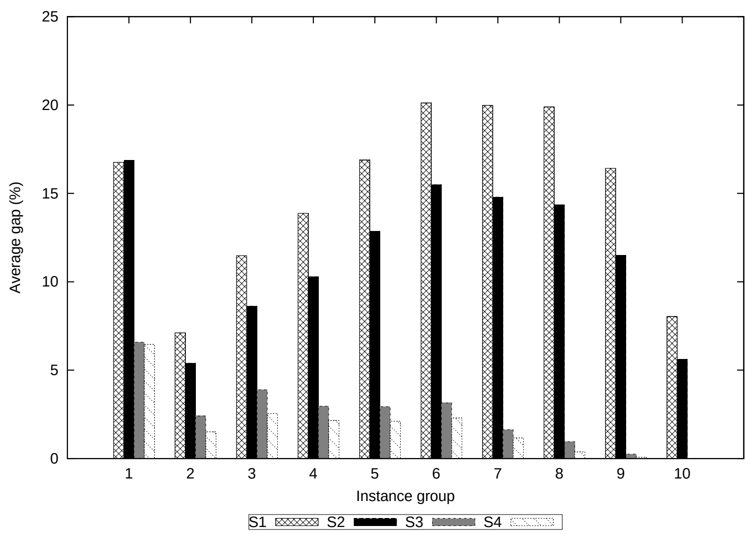


Figure 5: Average root node gap for each group of instances in all settings.

5.2.2. *Tree and Kite inequalities*

With a view of evaluating the practical impact of tree and kite inequalities, we executed BC-F2 only at the root node using three different settings regarding the use or not of tree and kite inequalities:

- **BC-F2:** BC-F2 without tree and kite inequalities.
- **BC-F2 + kite:** BC-F2 considering kite inequalities at the root node. At each iteration, we add the most violated kite inequality for each

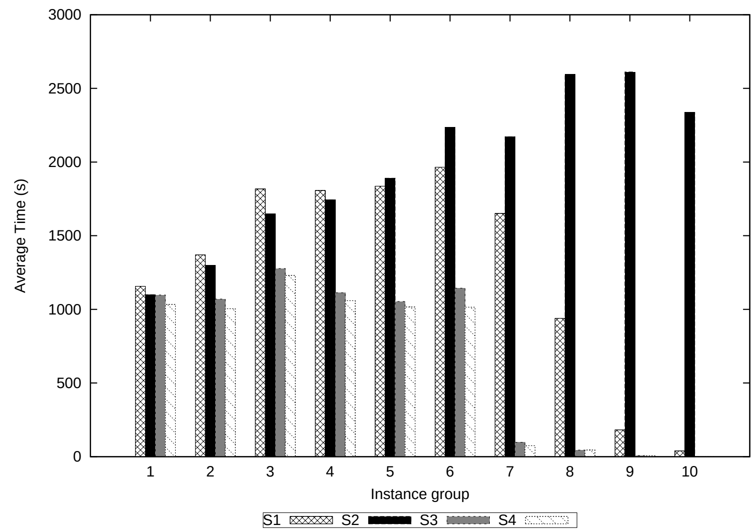


Figure 6: Average time spent at root node for each group of instances in all settings.

vertex. The kite inequalities are separated in a dynamic programming fashion (see Section 4.2).

- **BC-F2 + tree:** BC-F2 considering tree inequalities at the root node. At each iteration, we add the most violated tree inequality, which is separated by means of Kruskal's algorithm.

Table 4 contains the results obtained for each group of instances with these 3 different settings. In this table, **Root Gap** contais the average root node gap, **Root Time** represents the average time (in seconds) spent at the root node and **Separ. Time** corresponds to the average time (in seconds) spent separating cuts at the root node. It can be clearly seen that the overhead of separating the kite inequalities does not justify their use in practice. For example, when considering the kite inequalities, the average time spent separating cuts in the instances of group 1 increases from 126.9 to 2134.94 seconds without any improvement in the average gap. Moreover, although the tree inequalities were capable of improving the average root gap for some groups, such improvement is negligible when compared to the overhead of separating them. Therefore, tree and kite inequalities were disregarded from BC-F2.

Table 4: Impact of tree and kite inequalities at the root node.

Group	BC-F2			BC-F2 + kite			BC-F2 + tree		
	Root Gap	Root Time	Separ. Time	Root Gap	Root Time	Separ. Time	Root Gap	Root Time	Separ. Time
1	6.46	1033.27	126.90	11.64	2365.20	2134.94	6.37	1191.83	467.15
2	1.51	1004.07	49.29	5.37	1639.94	1377.31	1.72	1424.34	498.30
3	2.54	1230.29	10.75	7.81	2104.47	1843.52	3.06	1758.21	433.16
4	2.15	1059.12	2.82	3.63	2050.49	1669.20	2.37	1390.88	158.84
5	2.11	1015.64	1.08	2.53	1942.76	1662.14	2.15	1160.50	32.59
6	2.29	1015.27	0.55	2.37	1820.56	1312.30	2.17	1102.10	4.15
7	1.16	74.68	0.11	1.25	522.59	439.75	1.05	148.10	0.29
8	0.38	44.87	0.04	0.38	227.17	166.42	0.32	20.99	0.06
9	0.08	5.77	0.01	0.08	49.57	44.63	0.05	128.24	0.04
10	0.00	0.19	0.00	0.00	0.09	0.00	0.00	0.13	0.00

5.3. Impact of $p/|V|$ and graph density

In this section we study the impact of $p/|V|$ and graph density in the perfomance of BC-F2. To analyze the impact of graph density (d), we defined three classes: **sparse**, graphs with $d \leq 1/3$; **medium**, graphs with $1/3 < d \leq 2/3$; and **dense**, graphs with $d > 2/3$.

Tables 5 and 6 show detailed average results obtained by BC-F2 for each group of instances and for each graph (i.e., results for a specific graph with different values of p), respectively. In these two tables, **Root Gap (%)** represents the average root node gap, **Root Time (s)** contains the average time (in seconds) spent at the root node, **#((3)+(4)+(28))** is the average number of triangle inequalities added, **Tree Size** is the average number of nodes opened, **Time (s)** indicates the average total computational time and **Gap (%)** represents the average final gap obtained.

From the results presented in Table 5 we can observe that group 1 (for which $p/|V| \leq 0.1$) is clearly the most difficult. For example, the average final gap for group 1 is more than two times greater than the average final gap of group 3, the one with the second highest average final gap. On the other hand, groups with high values of $p/|V|$ (specially groups 8, 9 and 10) presented the best values for final gap and total computational time. In

Table 5: Detailed average results for each group of instances.

Group	Root Gap (%)	Root Time (s)	#((3)+(4)+(28))	Tree Size	Time (s)	Gap (%)
1	6.46	1033.27	35024.05	1817.16	4214.31	5.53
2	1.51	1004.07	27895.87	1598.53	2019.92	0.90
3	2.54	1230.29	28084.26	304.15	2373.02	2.19
4	2.15	1059.12	26252.74	2260.20	2595.15	1.74
5	2.11	1015.64	24920.92	716.49	2149.23	1.80
6	2.29	1015.27	21682.00	1724.93	2229.63	1.83
7	1.16	74.68	5194.79	4077.34	941.41	0.58
8	0.38	44.87	2490.00	2212.31	409.63	0.15
9	0.08	5.77	682.29	455.30	177.88	0.04
10	0.00	0.19	0.00	1.00	0.19	0.00

Table 6: Detailed average results for each graph.

Graph	d	Root Gap (%)	Root Time (s)	#((3)+(4)+(28))	Tree Size	Time (s)	Gap (%)
nr_1059_size_55	0.19	2.04	1.78	2310.00	5152.00	694.94	1.01
nr_369_size_60	0.27	2.46	3.25	3756.93	2203.33	965.86	1.60
nr_492_size_61	0.62	2.07	0.38	1467.27	60.27	9.09	0.00
nr_438_size_72	0.26	0.99	5.06	4650.22	1956.33	653.08	0.28
nr_238_size_78	0.88	0.11	0.12	1865.11	1.32	1.26	0.00
nr_522_size_85	0.44	1.97	14.38	11879.33	6335.29	1176.15	1.23
nr_173_size_86	0.19	2.41	8.30	6816.29	7777.81	1060.63	1.31
nr_978_size_92	0.76	0.89	0.42	4632.13	344.09	315.52	0.25
nr_317_size_95	0.82	0.22	0.31	4804.71	1.00	0.31	0.22
nr_630_size_102	0.15	1.90	8.61	5352.20	6626.40	945.43	1.23
nr_104_size_112	0.82	0.07	139.86	9172.71	7.75	390.43	0.05
nr_413_size_127	0.45	3.54	55.44	15411.84	2100.38	2448.09	2.62
nr_57_size_129	0.24	1.37	25.87	9286.66	2953.94	1096.76	0.86
nr_606_size_130	0.92	0.11	0.92	1775.28	1.75	4.69	0.00
nr_480_size_139	0.42	1.41	39.18	9250.11	4288.63	1103.90	1.07
nr_537_size_144	0.68	0.51	66.01	12367.28	309.72	623.47	0.38
nr_116_size_152	0.35	1.30	73.49	13582.53	172.95	664.22	1.18
nr_593_size_157	0.49	1.00	938.44	27992.46	77.28	1740.86	0.90
nr_176_size_169	0.90	0.18	6.95	2584.95	30.38	178.16	0.10
nr_459_size_171	0.11	2.07	46.41	11310.86	5744.67	1426.44	1.47
nr_9_size_178	0.79	0.19	800.54	21232.30	15.07	988.31	0.16
nr_200_size_179	0.43	1.30	467.79	23250.13	516.27	2077.94	1.08
nr_263_size_186	0.34	7.27	699.19	30215.80	329.57	4057.47	6.27
nr_365_size_196	0.15	1.42	315.55	21104.43	1363.02	2442.24	1.16
nr_232_size_206	0.31	6.21	214.30	23357.14	1163.98	4004.55	5.55
nr_279_size_209	0.54	1.69	4257.44	43088.12	1.02	4261.07	1.69
nr_489_size_211	0.68	1.79	3641.74	48304.57	1.15	3805.58	1.79

particular, all instances with $p/|V| > 0.9$ (group 10) were solved at the root node without the addition of any triangle inequality.

Figure 7 shows the average time spent by BC-F2 for each density class only considering instances solved to optimality (693 out of 892 instances).

Three facts arise from Table 6 and Figure 7: (i) graphs with density around 0.5 (medium) are the most time-consuming in the majority of the groups; (ii) sparse graphs are the easiest ones for small values of $p/|V|$; and (iii) dense graphs are the easiest ones in general.

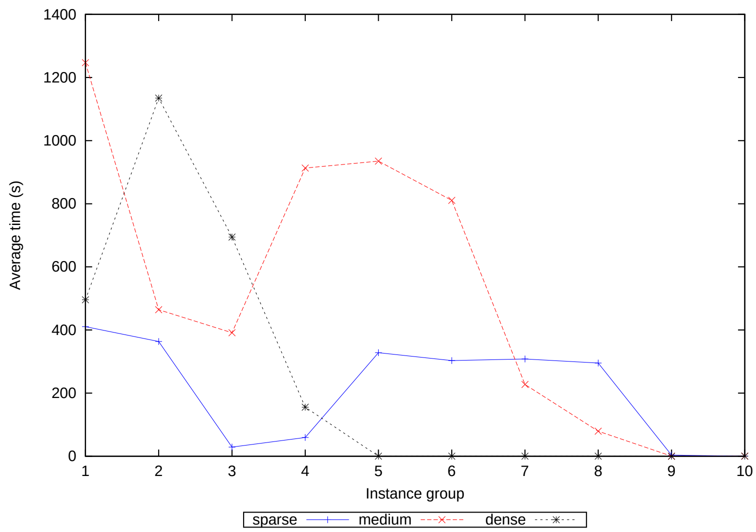


Figure 7: Average time spent by BC-F2 in the instances solved to optimality for each density class.

5.4. BC-F1 for small values of p

This section presents a comparison between BC-F1 and BC-F2 for small values of p. Our goal is to show that BC-F1 outperforms BC-F2, in practice, for some particular values of p, specially for $p = 2$ and $p = 3$. For the sake of comparison, we executed BC-F1 and BC-F2 for all graphs and for $p \in \{2, 3, 4, 5\}$.

Table 7 presents the results obtained by BC-F1 and BC-F2 for the instances with $p \in \{2, 3, 4, 5\}$. For each value of $p \in \{2, 3, 4, 5\}$, there are two columns in the table: **Gap F1 (%)** and **Gap F2 (%)**, which contain, respectively, the final gaps obtained by BC-F1 and BC-F2 for the instances with the specific value of p. For each column, we have in the last two lines

Table 7: Final gap obtained by BC-F1 and BC-F2 for small values of p.

Graph	p = 2		p = 3		p = 4		p = 5	
	Gap F1 (%)	Gap F2 (%)	Gap F1 (%)	Gap F2 (%)	Gap F1 (%)	Gap F2 (%)	Gap F1 (%)	Gap F2 (%)
nr_1059_size_55	0.00	3.91	0.00	0.00	2.23	9.97	12.98	13.58
nr_369_size_60	0.00	15.03	20.72	23.89	24.17	22.01	19.08	15.23
nr_492_size_61	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
nr_438_size_72	0.00	1.29	15.63	23.50	18.13	15.61	14.04	9.97
nr_238_size_78	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
nr_522_size_85	0.00	25.65	8.63	10.98	4.88	3.26	2.24	0.64
nr_173_size_86	0.00	1.20	5.57	10.29	21.41	20.90	28.45	24.89
nr_978_size_92	0.00	5.74	0.00	0.00	0.00	0.00	0.00	0.00
nr_317_size_95	0.00	5.35	0.00	0.00	0.00	0.00	0.00	0.00
nr_630_size_102	1.09	2.58	6.34	6.51	5.59	3.42	12.47	11.29
nr_104_size_112	0.00	1.35	0.00	0.00	0.00	0.00	0.00	0.00
nr_413_size_127	0.43	10.42	6.88	9.29	0.00	0.98	0.00	0.00
nr_57_size_129	4.32	5.28	32.65	33.04	36.85	35.25	29.82	27.71
nr_606_size_130	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
nr_480_size_139	3.18	31.64	18.60	19.83	12.13	10.51	7.35	5.81
nr_537_size_144	0.00	13.22	2.35	3.02	0.99	0.42	3.27	0.00
nr_116_size_152	22.91	27.91	41.85	44.21	32.40	30.82	22.81	21.29
nr_593_size_157	0.16	30.50	14.95	15.97	13.67	9.11	22.10	5.23
nr_176_size_169	0.00	4.05	0.00	0.00	0.00	0.00	1.27	0.00
nr_459_size_171	0.34	0.19	1.75	1.68	1.74	1.51	7.87	7.80
nr_9_size_178	0.00	4.74	5.77	1.04	19.88	0.00	40.10	0.00
nr_200_size_179	8.60	14.87	9.69	8.30	1.92	0.00	11.27	0.00
nr_263_size_186	13.30	13.45	1.81	0.58	0.00	0.00	0.00	0.00
nr_365_size_196	4.48	3.80	20.81	19.25	27.06	21.08	34.54	22.32
nr_232_size_206	16.31	15.88	0.61	0.00	3.86	0.00	0.00	0.00
nr_279_size_209	29.60	20.27	43.02	9.77	56.11	2.10	67.98	0.74
nr_489_size_211	22.67	19.76	52.52	7.40	55.89	4.85	68.62	3.55
Avg	4.71	9.69	11.48	9.20	12.55	7.10	15.04	6.29
Avg ($\|V\| < 200$)	2.45	8.85	8.91	9.64	9.29	7.70	11.23	6.90

the average gap in the column (**Avg**) and the average gap for instances with $|V| < 200$ (**Avg** ($|V| < 200$)).

We can observe that BC-F1 is capable of significantly improving the gaps in most instances with $p = 2$ and $p = 3$. More precisely, BC-F1 improved the gap in 18 out of 27 instances with $p = 2$ and in 11 out of 27 instances with $p = 3$. Also, by using BC-F1 we were capable of finding 12 new optimal solutions for these values of p. On the other hand, for $p \geq 4$, BC-F2 clearly outperformed BC-F1, presenting a better average performance in almost all cases.

6. Concluding remarks

In this paper we addressed the p-Cluster Editing Problem, a $\mathcal{NP}$-hard problem that consists of transforming an input graph G into a vertex-disjoint union of exactly p cliques by editing (adding or removing) the smallest possible number of edges. In particular, we studied two mathematical formulations for the problem: a new one based on three-index variables (Formulation F1) and another one from the literature (Formulation F2). From a theoretical point of view, we showed that F1 has a poor linear relaxation and is theoretically dominated by F2. In addition, we proposed new families of valid inequalities for both formulations. From a practical point of view, we implemented branch-and-cut algorithms over F1 and F2, denoted respectively by BC-F1 and BC-F2, but we focused on BC-F2 because of its highly superior performance.

We also analyzed the impact of the new valid inequalities on the performance of BC-F2 according to the graph density and the ratio between p and the number of vertices using 892 instances adapted from the literature. The results suggested that both graph density and the ratio between p and the number of vertices have an impact in the performance of BC-F2. For example, sparse and dense graphs seem to be in general easier than graphs with density around 0.5, as it happens in the classical Cluster Editing [5]. On the other hand, the experiments showed that regardless of the graph density: (i) instances with small values of $p/|V|$ are the most difficult to solve; and (ii) instances with large values of $p/|V|$ are the easiest to solve. As we expected, BC-F1 has in general a poor performance when compared to BC-F2. However, it works surprisingly well for instances with very small values of p, outperforming BC-F2 in almost all instances with $p = 2$ and $p = 3$. Overall, the algorithms could solve 78.92% of all the instances, and 90.43% of the instances with $|V| \leq 178$.

7. Acknowledgements

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Appendix. Detailed results for all instances.

Table 8: Results for the graph nr_1059_size_55 ($|V| = 55, d = 0.19$).

p	Root LB	Root Time (s)	#((3)+ (4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2*	480.14	8.65	5236	57	55.54	513.00	**513**	0.00
6	115.29	3.81	3094	62100	2507.14	119.35	133	10.26
10	112.00	0.61	1392	1	0.61	112.00	**112**	0.00
14	108.00	0.11	1415	1	0.11	108.00	**108**	0.00
18	113.00	0.05	1418	1	0.05	113.00	**113**	0.00
22	121.00	0.06	1369	1	0.06	121.00	**121**	0.00
26	135.00	0.17	1448	1	0.17	135.00	**135**	0.00
30	150.00	0.49	1513	20	0.65	153.00	**153**	0.00
34	168.16	2.25	1486	9	3.46	171.00	**171**	0.00
38	188.23	2.19	1449	2431	16.54	199.00	**199**	0.00
42	209.00	0.22	1213	1	0.22	209.00	**209**	0.00
46	243.00	0.06	0	1	0.06	243.00	**243**	0.00
50	273.00	0.04	0	1	0.04	273.00	**273**	0.00
54	287.00	0.04	0	1	0.04	287.00	**287**	0.00

*Found using BC-F1.

Table 9: Results for the graph nr_369_size_60 ($|V| = 60, d = 0.27$).

p	Root LB	Root Time (s)	#((3)+ (4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2*	518.23	17.06	5514	5938	4467.20	614.00	**614**	0.00
6	251.58	5.30	4001	27990	7200.00	256.91	282	8.90
10	247.51	4.67	4621	54	18.07	250.00	**250**	0.00
14	247.00	1.52	4340	1	1.52	247.00	**247**	0.00
18	251.00	1.99	4278	1	1.99	251.00	**251**	0.00
22	257.00	2.06	4252	1	2.06	257.00	**257**	0.00
26	263.22	3.58	4294	3	3.93	265.00	**265**	0.00
30	273.56	3.74	4352	5	4.60	275.00	**275**	0.00
34	286.00	8.32	4525	57	34.24	290.00	**290**	0.00
38	301.00	1.26	3239	1	1.26	301.00	**301**	0.00
42	308.92	1.69	3190	94	20.01	331.00	**331**	0.00
46	369.00	0.06	0	1	0.06	369.00	**369**	0.00
50	419.00	0.05	0	1	0.05	419.00	**419**	0.00
54	453.00	0.05	0	1	0.05	453.00	**453**	0.00
58	471.00	0.05	0	1	0.05	471.00	**471**	0.00

*Found using BC-F1.

Table 10: Results for the graph nr_492_size_61 ($|V| = 61, d = 0.62$).

p	Root LB	Root Time (s)	#((3)+(4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2	91.75	1.88	6618	148	128.80	112.00	**112**	0.00
6	91.96	1.39	3912	13	1.56	94.00	**94**	0.00
10	113.48	1.06	3481	670	4.05	124.00	**124**	0.00
14	134.91	0.89	3674	62	1.41	138.00	**138**	0.00
18	196.00	0.18	4324	1	0.18	196.00	**196**	0.00
22	348.00	0.04	0	1	0.04	348.00	**348**	0.00
26	498.00	0.03	0	1	0.03	498.00	**498**	0.00
30	632.00	0.04	0	1	0.04	632.00	**632**	0.00
34	750.00	0.03	0	1	0.03	750.00	**750**	0.00
38	852.00	0.03	0	1	0.03	852.00	**852**	0.00
42	938.00	0.03	0	1	0.03	938.00	**938**	0.00
46	1008.00	0.03	0	1	0.03	1008.00	**1008**	0.00
50	1062.00	0.03	0	1	0.03	1062.00	**1062**	0.00
54	1100.00	0.05	0	1	0.05	1100.00	**1100**	0.00
58	1122.00	0.04	0	1	0.04	1122.00	**1122**	0.00

Table 11: Results for the graph nr_438_size_72 ($|V| = 72, d = 0.26$).

p	Root LB	Root Time (s)	#((3)+(4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2*	681.19	41.24	8716	59	357.01	708.00	**708**	0.00
6	248.34	19.88	8908	31896	4455.68	253.30	263	3.69
10	244.00	0.64	4436	1	0.64	244.00	**244**	0.00
14	243.00	0.17	4299	1	0.17	243.00	**243**	0.00
18	248.00	0.20	4277	1	0.20	248.00	**248**	0.00
22	261.91	3.38	4460	89	9.58	266.00	**266**	0.00
26	276.00	0.85	4415	1	0.85	276.00	**276**	0.00
30	295.00	0.96	4576	1	0.96	295.00	**295**	0.00
34	315.56	1.60	4642	3	1.77	317.00	**317**	0.00
38	341.22	8.10	6275	61	42.69	348.00	**348**	0.00
42	368.13	9.13	5955	272	35.09	375.00	**375**	0.00
46	392.50	2.94	4246	8	5.70	395.00	**395**	0.00
50	424.88	1.84	2947	3	1.88	427.00	**427**	0.00
54	499.00	0.05	0	1	0.05	499.00	**499**	0.00
58	565.00	0.04	0	1	0.04	565.00	**565**	0.00
62	615.00	0.05	0	1	0.05	615.00	**615**	0.00
66	649.00	0.05	0	1	0.05	649.00	**649**	0.00
70	667.00	0.06	0	1	0.06	667.00	**667**	0.00

*Found using BC-F1.

Table 12: Results for the graph nr_238_size_78 ($|V| = 78, d = 0.88$).

p	Root LB	Root Time (s)	#((3)+ (4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2	145.00	0.68	13953	7	22.34	148.00	**148**	0.00
6	163.00	0.16	8748	1	0.16	163.00	**163**	0.00
10	323.00	0.42	12736	1	0.42	323.00	**323**	0.00
14	571.00	0.04	0	1	0.04	571.00	**571**	0.00
18	821.00	0.06	0	1	0.06	821.00	**821**	0.00
22	1055.00	0.07	0	1	0.07	1055.00	**1055**	0.00
26	1273.00	0.05	0	1	0.05	1273.00	**1273**	0.00
30	1475.00	0.06	0	1	0.06	1475.00	**1475**	0.00
34	1661.00	0.07	0	1	0.07	1661.00	**1661**	0.00
38	1831.00	0.06	0	1	0.06	1831.00	**1831**	0.00
42	1985.00	0.06	0	1	0.06	1985.00	**1985**	0.00
46	2123.00	0.05	0	1	0.05	2123.00	**2123**	0.00
50	2245.00	0.05	0	1	0.05	2245.00	**2245**	0.00
54	2351.00	0.07	0	1	0.07	2351.00	**2351**	0.00
58	2441.00	0.06	0	1	0.06	2441.00	**2441**	0.00
62	2515.00	0.06	0	1	0.06	2515.00	**2515**	0.00
66	2573.00	0.07	0	1	0.07	2573.00	**2573**	0.00
70	2615.00	0.06	0	1	0.06	2615.00	**2615**	0.00
74	2641.00	0.06	0	1	0.06	2641.00	**2641**	0.00

Table 13: Results for the graph nr_522_size_85 ($|V| = 85, d = 0.44$).

p	Root LB	Root Time (s)	#((3)+ (4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2*	909.00	416.64	12800	1	416.64	909.00	**909**	0.00
6	608.95	7.73	17764	388	530.42	618.00	**618**	0.00
10	610.59	5.17	17975	7	16.21	614.00	**614**	0.00
14	615.07	6.72	17962	19	53.81	621.00	**621**	0.00
18	624.18	9.77	18005	27	61.80	630.00	**630**	0.00
22	636.67	9.68	18013	45	77.83	643.00	**643**	0.00
26	651.89	13.08	18483	2093	635.37	667.00	**667**	0.00
30	671.17	9.13	17857	2258	1130.22	687.00	**687**	0.00
34	701.00	9.29	17761	20	33.56	705.00	**705**	0.00
38	737.00	8.99	16132	1	8.99	737.00	**737**	0.00
42	797.14	35.72	15033	3	43.34	799.00	**799**	0.00
46	875.55	38.32	14415	89	231.59	883.00	**883**	0.00
50	959.72	57.46	13663	1110	891.42	977.00	**977**	0.00
54	1085.00	49.15	11495	25800	6584.21	1085.00	**1085**	0.00
58	1203.00	0.76	3999	99862	7200.00	1203.00	1205	0.17
62	1305.00	0.08	0	1	0.08	1305.00	**1305**	0.00
66	1391.00	0.06	0	1	0.06	1391.00	**1391**	0.00
70	1461.00	0.07	0	1	0.07	1461.00	**1461**	0.00
74	1515.00	0.07	0	1	0.07	1515.00	**1515**	0.00
78	1553.00	0.05	0	1	0.05	1553.00	**1553**	0.00
82	1575.00	0.06	0	1	0.06	1575.00	**1575**	0.00

*Found using BC-F1.

Table 14: Results for the graph nr_173_size_86 ($|V| = 86, d = 0.19$).

p	Root LB	Root Time (s)	#((3)+(4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2*	1156.07	74.22	12000	151	839.81	1171.00	**1171**	0.00
6	268.68	29.81	10358	17015	6651.71	271.94	334	18.58
10	264.50	33.91	9454	21449	2712.61	265.20	276	3.91
14	261.00	0.89	4719	1	0.89	261.00	**261**	0.00
18	257.00	0.80	4575	1	0.80	257.00	**257**	0.00
22	255.00	0.24	4592	1	0.24	255.00	**255**	0.00
26	257.00	0.23	4478	1	0.23	257.00	**257**	0.00
30	261.00	0.20	4484	1	0.20	261.00	**261**	0.00
34	267.00	0.20	4271	1	0.20	267.00	**267**	0.00
38	282.13	2.00	4323	138	7.14	287.00	**287**	0.00
42	298.00	0.28	4507	1	0.28	298.00	**298**	0.00
46	324.33	1.66	4466	3	1.90	327.00	**327**	0.00
50	361.48	7.36	5256	120	36.37	372.00	**372**	0.00
54	398.46	11.43	5572	54983	3844.77	430.00	**430**	0.00
58	434.99	10.18	4900	60951	1546.87	452.93	471	3.84
62	471.16	9.02	4622	8472	259.74	486.00	**486**	0.00
66	500.15	3.20	3984	31	9.01	508.00	**508**	0.00
70	568.00	0.05	0	1	0.05	568.00	**568**	0.00
74	626.00	0.06	0	1	0.06	626.00	**626**	0.00
78	668.00	0.07	0	1	0.07	668.00	**668**	0.00
82	694.00	0.06	0	1	0.06	694.00	**694**	0.00

*Found using BC-F1.

Table 15: Results for the graph nr_978_size_92 ($|V| = 92, d = 0.76$).

p	Root LB	Root Time (s)	#((3)+ (4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2*	344.00	23.43	5200	1	23.43	344.00	**344**	0.00
6	312.82	2.12	18947	15	4.10	318.00	**318**	0.00
10	338.50	1.41	19504	2568	21.14	356.00	**356**	0.00
14	364.50	1.46	21846	4161	29.64	378.00	**378**	0.00
18	431.00	0.62	25718	1	0.62	431.00	**431**	0.00
22	697.00	0.08	0	1	0.08	697.00	**697**	0.00
26	971.00	0.06	0	1	0.06	971.00	**971**	0.00
30	1229.00	0.09	0	1	0.09	1229.00	**1229**	0.00
34	1471.00	0.08	0	1	0.08	1471.00	**1471**	0.00
38	1697.00	0.06	0	1	0.06	1697.00	**1697**	0.00
42	1907.00	0.09	0	1	0.09	1907.00	**1907**	0.00
46	2101.00	0.09	0	1	0.09	2101.00	**2101**	0.00
50	2279.00	0.06	0	1	0.06	2279.00	**2279**	0.00
54	2441.00	0.08	0	1	0.08	2441.00	**2441**	0.00
58	2587.00	0.08	0	1	0.08	2587.00	**2587**	0.00
62	2717.00	0.06	0	1	0.06	2717.00	**2717**	0.00
66	2831.00	0.07	0	1	0.07	2831.00	**2831**	0.00
70	2929.00	0.07	0	1	0.07	2929.00	**2929**	0.00
74	3011.00	0.06	0	1	0.06	3011.00	**3011**	0.00
78	3077.00	0.08	0	1	0.08	3077.00	**3077**	0.00
82	3127.00	0.07	0	1	0.07	3127.00	**3127**	0.00
86	3161.00	0.06	0	1	0.06	3161.00	**3161**	0.00
90	3179.00	0.09	0	1	0.09	3179.00	**3179**	0.00

*Found using BC-F1.

Table 16: Results for the graph nr_317_size_95 ($|V| = 95, d = 0.82$).

p	Root LB	Root Time (s)	#((3)+ (4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2*	338.00	31.50	6000	1	31.50	338.00	**338**	0.00
6	314.00	0.24	17397	1	0.24	314.00	**314**	0.00
10	332.00	0.65	17934	1	0.65	332.00	**332**	0.00
14	440.00	0.66	22153	1	0.66	440.00	**440**	0.00
18	666.00	1.95	37307	1	1.95	666.00	**666**	0.00
22	946.00	0.05	0	1	0.05	946.00	**946**	0.00
26	1232.00	0.05	0	1	0.05	1232.00	**1232**	0.00
30	1502.00	0.05	0	1	0.05	1502.00	**1502**	0.00
34	1756.00	0.05	0	1	0.05	1756.00	**1756**	0.00
38	1994.00	0.05	0	1	0.05	1994.00	**1994**	0.00
42	2216.00	0.05	0	1	0.05	2216.00	**2216**	0.00
46	2422.00	0.05	0	1	0.05	2422.00	**2422**	0.00
50	2612.00	0.05	0	1	0.05	2612.00	**2612**	0.00
54	2786.00	0.05	0	1	0.05	2786.00	**2786**	0.00
58	2944.00	0.05	0	1	0.05	2944.00	**2944**	0.00
62	3086.00	0.05	0	1	0.05	3086.00	**3086**	0.00
66	3212.00	0.05	0	1	0.05	3212.00	**3212**	0.00
70	3322.00	0.05	0	1	0.05	3322.00	**3322**	0.00
74	3416.00	0.05	0	1	0.05	3416.00	**3416**	0.00
78	3494.00	0.05	0	1	0.05	3494.00	**3494**	0.00
82	3556.00	0.05	0	1	0.05	3556.00	**3556**	0.00
86	3602.00	0.05	0	1	0.05	3602.00	**3602**	0.00
90	3632.00	0.05	0	1	0.05	3632.00	**3632**	0.00
94	3646.00	0.08	0	1	0.08	3646.00	**3646**	0.00

*Found using BC-F1.

Table 17: Results for the graph nr_630_size_102 ($|V| = 102, d = 0.15$).

p	**Root LB**	**Root Time (s)**	**#((3)+ (4)+(28))**	**Tree Size**	**Total Time (s)**	**LB**	**UB**	**Gap (%)**
2*	1903.64	684.72	18363	248	7200.00	1940.54	1962	1.09
6	419.93	46.33	12464	8401	7200.00	423.08	509	16.88
10	316.46	35.37	9340	22640	4516.72	320.01	355	9.86
14	312.41	15.60	7145	18900	2083.67	316.00	**316**	0.00
18	307.67	1.44	5332	12	1.53	309.00	**309**	0.00
22	308.00	1.01	4707	1	1.01	308.00	**308**	0.00
26	312.00	0.70	4587	1	0.70	312.00	**312**	0.00
30	318.00	1.86	5090	3	1.91	319.00	**319**	0.00
34	326.00	1.46	4786	2	1.54	327.00	**327**	0.00
38	334.25	2.41	4771	3	2.54	336.00	**336**	0.00
42	346.00	2.07	4805	1	2.07	346.00	**346**	0.00
46	356.00	3.25	4851	10	4.68	358.00	**358**	0.00
50	368.00	1.35	4766	1	1.35	368.00	**368**	0.00
54	384.00	2.47	4789	266	24.44	389.00	**389**	0.00
58	400.00	1.03	4653	1	1.03	400.00	**400**	0.00
62	418.00	2.17	4874	516	27.02	425.00	**425**	0.00
66	434.72	1.52	4462	3	1.55	436.00	**436**	0.00
70	458.94	7.12	5455	38	16.88	465.00	**465**	0.00
74	485.46	7.67	4810	44835	1780.33	507.00	**507**	0.00
78	513.54	5.49	3977	69259	764.21	524.00	531	1.32
82	541.00	0.55	2801	3	2.10	543.00	**543**	0.00
86	615.00	0.07	0	1	0.07	615.00	**615**	0.00
90	673.00	0.07	0	1	0.07	673.00	**673**	0.00
94	715.00	0.08	0	1	0.08	715.00	**715**	0.00
98	741.00	0.06	0	1	0.06	741.00	**741**	0.00

*Found using BC-F1.

Table 18: Results for the graph nr_104_size_112 ($|V| = 112, d = 0.82$).

p	Root LB	Root Time (s)	#((3)+ (4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2*	830.00	160.94	8400	1	160.94	830.00	**830**	0.00
6	812.00	21.55	15002	1	21.55	812.00	**812**	0.00
10	817.00	18.23	12578	1	18.23	817.00	**817**	0.00
14	885.00	29.86	12878	1	29.86	885.00	**885**	0.00
18	1061.00	86.69	19042	1	86.69	1061.00	**1061**	0.00
22	1275.72	135.93	26076	3	138.07	1277.00	**1277**	0.00
26	1523.00	144.42	27766	1	144.42	1523.00	**1523**	0.00
30	1789.00	275.31	30309	1	275.31	1789.00	**1789**	0.00
34	2059.00	763.77	33871	1	763.77	2059.00	**2059**	0.00
38	2337.00	2252.33	40581	1	2252.33	2337.00	**2337**	0.00
42	2609.00	0.11	0	1	0.11	2609.00	**2609**	0.00
46	2883.00	0.10	0	1	0.10	2883.00	**2883**	0.00
50	3141.00	0.09	0	1	0.09	3141.00	**3141**	0.00
54	3383.00	0.11	0	1	0.11	3383.00	**3383**	0.00
58	3609.00	0.11	0	1	0.11	3609.00	**3609**	0.00
62	3819.00	0.09	0	1	0.09	3819.00	**3819**	0.00
66	4013.00	0.11	0	1	0.11	4013.00	**4013**	0.00
70	4191.00	0.11	0	1	0.11	4191.00	**4191**	0.00
74	4353.00	0.09	0	1	0.09	4353.00	**4353**	0.00
78	4499.00	0.10	0	1	0.10	4499.00	**4499**	0.00
82	4629.00	0.10	0	1	0.10	4629.00	**4629**	0.00
86	4743.00	0.09	0	1	0.09	4743.00	**4743**	0.00
90	4841.00	0.09	0	1	0.09	4841.00	**4841**	0.00
94	4923.00	0.11	0	1	0.11	4923.00	**4923**	0.00
98	4989.00	0.08	0	1	0.08	4989.00	**4989**	0.00
102	5039.00	0.09	0	1	0.09	5039.00	**5039**	0.00
106	5073.00	0.09	0	1	0.09	5073.00	**5073**	0.00
110	5091.00	0.08	0	1	0.08	5091.00	**5091**	0.00

*Found using BC-F1.

Table 19: Results for the graph nr_413_size_127 ($|V| = 127, d = 0.45$).

p	Root LB	Root Time (s)	#((3)+ (4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2*	1535.36	7200.00	31116	1	7200.00	1535.36	1542	0.43
6	785.00	1.54	29242	1	1.54	785.00	**785**	0.00
10	786.00	1.76	21907	1	1.76	786.00	**786**	0.00
14	804.96	18.20	24565	37	52.74	812.00	**812**	0.00
18	827.65	18.72	24123	1473	503.15	840.00	**840**	0.00
22	850.00	11.96	20318	1	11.96	850.00	**850**	0.00
26	919.78	162.96	33960	3143	7200.00	954.70	962	0.76
30	1003.49	121.27	31370	2047	7200.00	1040.26	1118	6.95
34	1087.20	131.03	31228	2299	7200.00	1126.40	1252	10.03
38	1170.90	107.18	28188	5512	7200.00	1211.62	1366	11.30
42	1254.61	92.92	35429	6312	7200.00	1292.11	1464	11.74
46	1338.56	97.45	32246	9100	7200.00	1370.42	1544	11.24
50	1423.34	117.55	28078	10750	6099.82	1447.60	1606	9.86
54	1507.24	112.17	27582	10399	6857.05	1534.64	1652	7.10
58	1590.85	100.51	26668	10237	7017.13	1608.69	1682	4.36
62	1674.95	115.68	28860	5480	6587.37	1692.00	**1692**	0.00
66	1784.00	107.14	24484	1	107.14	1784.00	**1784**	0.00
70	1978.00	10.12	13892	3	697.19	1980.00	**1980**	0.00
74	2200.00	0.12	0	1	0.12	2200.00	**2200**	0.00
78	2406.00	0.12	0	1	0.12	2406.00	**2406**	0.00
82	2596.00	0.10	0	1	0.10	2596.00	**2596**	0.00
86	2770.00	0.11	0	1	0.11	2770.00	**2770**	0.00
90	2928.00	0.11	0	1	0.11	2928.00	**2928**	0.00
94	3070.00	0.12	0	1	0.12	3070.00	**3070**	0.00
98	3196.00	0.10	0	1	0.10	3196.00	**3196**	0.00
102	3306.00	0.11	0	1	0.11	3306.00	**3306**	0.00
106	3400.00	0.11	0	1	0.11	3400.00	**3400**	0.00
110	3478.00	0.11	0	1	0.11	3478.00	**3478**	0.00
114	3540.00	0.10	0	1	0.10	3540.00	**3540**	0.00
118	3586.00	0.11	0	1	0.11	3586.00	**3586**	0.00
122	3616.00	0.11	0	1	0.11	3616.00	**3616**	0.00
126	3630.00	0.18	0	1	0.18	3630.00	**3630**	0.00

*Found using BC-F1.

Table 20: Results for the graph nr_57_size_129 ($|V| = 129, d = 0.24$).

p	Root LB	Root Time (s)	#((3)+ (4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2*	2123.84	566.51	18134	152	7200.00	2145.25	2242	4.32
6	639.27	127.04	23037	2655	7200.00	642.66	777	17.29
10	634.32	236.16	19906	1050	7200.00	636.38	653	2.55
14	630.32	66.44	14698	8019	3595.56	633.00	**633**	0.00
18	627.00	14.34	9747	1	14.34	627.00	**627**	0.00
22	628.00	11.73	9567	1	11.73	628.00	**628**	0.00
26	632.00	8.97	10575	1	8.97	632.00	**632**	0.00
30	638.00	9.90	10843	1	9.90	638.00	**638**	0.00
34	647.01	21.78	11566	7	25.41	649.00	**649**	0.00
38	660.00	15.87	9398	1	15.87	660.00	**660**	0.00
42	675.43	16.46	11497	69	37.98	680.00	**680**	0.00
46	692.28	21.60	10126	134	57.89	696.00	**696**	0.00
50	714.09	21.75	11729	123	165.26	724.00	**724**	0.00
54	741.11	23.47	9873	12261	2383.85	760.00	**760**	0.00
58	767.67	21.98	11655	276	77.03	774.00	**774**	0.00
62	800.10	22.59	12659	5	27.34	802.00	**802**	0.00
66	838.26	19.13	9603	7921	2273.98	868.00	**868**	0.00
70	875.74	26.87	11070	33111	3281.49	902.85	916	1.44
74	913.84	16.94	8727	28720	1429.56	930.07	940	1.06
78	950.75	11.36	7865	3	11.81	952.00	**952**	0.00
82	1004.76	15.87	9160	3	16.07	1006.00	**1006**	0.00
86	1078.00	11.27	10853	1	11.27	1078.00	**1078**	0.00
90	1204.12	38.26	11207	3	39.43	1206.00	**1206**	0.00
94	1350.00	0.12	0	1	0.12	1350.00	**1350**	0.00
98	1484.00	0.10	0	1	0.10	1484.00	**1484**	0.00
102	1602.00	0.10	0	1	0.10	1602.00	**1602**	0.00
106	1704.00	0.13	0	1	0.13	1704.00	**1704**	0.00
110	1790.00	0.10	0	1	0.10	1790.00	**1790**	0.00
114	1860.00	0.11	0	1	0.11	1860.00	**1860**	0.00
118	1914.00	0.11	0	1	0.11	1914.00	**1914**	0.00
122	1952.00	0.10	0	1	0.10	1952.00	**1952**	0.00
126	1974.00	0.11	0	1	0.11	1974.00	**1974**	0.00

*Found using BC-F1.

Table 21: Results for the graph nr_606_size_130 ($|V| = 130, d = 0.92$).

p	Root LB	Root Time (s)	#((3)+(4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2	108.00	17.16	37968	25	137.76	112.00	**112**	0.00
6	108.00	0.45	5126	1	0.45	108.00	**108**	0.00
10	443.00	8.26	13715	1	8.26	443.00	**443**	0.00
14	907.00	0.13	0	1	0.13	907.00	**907**	0.00
18	1365.00	0.12	0	1	0.12	1365.00	**1365**	0.00
22	1807.00	0.14	0	1	0.14	1807.00	**1807**	0.00
26	2233.00	0.12	0	1	0.12	2233.00	**2233**	0.00
30	2643.00	0.12	0	1	0.12	2643.00	**2643**	0.00
34	3037.00	0.12	0	1	0.12	3037.00	**3037**	0.00
38	3415.00	0.13	0	1	0.13	3415.00	**3415**	0.00
42	3777.00	0.13	0	1	0.13	3777.00	**3777**	0.00
46	4123.00	0.14	0	1	0.14	4123.00	**4123**	0.00
50	4453.00	0.14	0	1	0.14	4453.00	**4453**	0.00
54	4767.00	0.14	0	1	0.14	4767.00	**4767**	0.00
58	5065.00	0.14	0	1	0.14	5065.00	**5065**	0.00
62	5347.00	0.14	0	1	0.14	5347.00	**5347**	0.00
66	5613.00	0.13	0	1	0.13	5613.00	**5613**	0.00
70	5863.00	0.13	0	1	0.13	5863.00	**5863**	0.00
74	6097.00	0.14	0	1	0.14	6097.00	**6097**	0.00
78	6315.00	0.13	0	1	0.13	6315.00	**6315**	0.00
82	6517.00	0.14	0	1	0.14	6517.00	**6517**	0.00
86	6703.00	0.13	0	1	0.13	6703.00	**6703**	0.00
90	6873.00	0.13	0	1	0.13	6873.00	**6873**	0.00
94	7027.00	0.13	0	1	0.13	7027.00	**7027**	0.00
98	7165.00	0.12	0	1	0.12	7165.00	**7165**	0.00
102	7287.00	0.13	0	1	0.13	7287.00	**7287**	0.00
106	7393.00	0.12	0	1	0.12	7393.00	**7393**	0.00
110	7483.00	0.12	0	1	0.12	7483.00	**7483**	0.00
114	7557.00	0.12	0	1	0.12	7557.00	**7557**	0.00
118	7615.00	0.11	0	1	0.11	7615.00	**7615**	0.00
122	7657.00	0.12	0	1	0.12	7657.00	**7657**	0.00
126	7683.00	0.11	0	1	0.11	7683.00	**7683**	0.00

Table 22: Results for the graph nr_480_size_139 ($|V| = 139, d = 0.42$).

p	Root LB	Root Time (s)	#((3)+(4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2*	1038.57	1078.97	27635	150	7200.00	1288.69	1331	3.18
6	745.19	114.78	21436	1295	7200.00	747.45	774	3.43
10	740.81	28.67	14076	4493	5477.54	746.00	**746**	0.00
14	740.00	14.70	11975	1	14.70	740.00	**740**	0.00
18	741.00	11.68	10450	1	11.68	741.00	**741**	0.00
22	746.00	10.39	11539	1	10.39	746.00	**746**	0.00
26	754.00	11.83	11924	2	11.88	755.00	**755**	0.00
30	767.00	11.33	12431	1	11.33	767.00	**767**	0.00
34	791.90	28.25	12543	8	50.38	795.00	**795**	0.00
38	820.79	30.16	13090	179	157.20	837.00	**837**	0.00
42	854.47	32.08	12809	32326	7200.00	883.35	887	0.41
46	890.99	29.20	11756	54766	5108.68	906.74	925	1.97
50	924.27	25.52	11774	56158	4385.29	943.00	**943**	0.00
54	967.00	11.60	9427	1	11.60	967.00	**967**	0.00
58	1063.69	31.18	13963	3	32.35	1065.00	**1065**	0.00
62	1221.00	34.67	16194	1	34.67	1221.00	**1221**	0.00
66	1424.36	73.83	21162	3	78.77	1427.00	**1427**	0.00
70	1668.35	231.76	24827	2	241.11	1669.00	**1669**	0.00
74	1915.00	534.72	27667	1	534.72	1915.00	**1915**	0.00
78	2165.00	38.02	19038	2	861.82	2165.00	**2165**	0.00
82	2403.00	0.16	0	1	0.16	2403.00	**2403**	0.00
86	2625.00	0.12	0	1	0.12	2625.00	**2625**	0.00
90	2831.00	0.14	0	1	0.14	2831.00	**2831**	0.00
94	3021.00	0.14	0	1	0.14	3021.00	**3021**	0.00
98	3195.00	0.12	0	1	0.12	3195.00	**3195**	0.00
102	3353.00	0.15	0	1	0.15	3353.00	**3353**	0.00
106	3495.00	0.15	0	1	0.15	3495.00	**3495**	0.00
110	3621.00	0.14	0	1	0.14	3621.00	**3621**	0.00
114	3731.00	0.14	0	1	0.14	3731.00	**3731**	0.00
118	3825.00	0.15	0	1	0.15	3825.00	**3825**	0.00
122	3903.00	0.13	0	1	0.13	3903.00	**3903**	0.00
126	3965.00	0.15	0	1	0.15	3965.00	**3965**	0.00
130	4011.00	0.15	0	1	0.15	4011.00	**4011**	0.00
134	4041.00	0.12	0	1	0.12	4041.00	**4041**	0.00
138	4055.00	0.30	0	1	0.30	4055.00	**4055**	0.00

*Found using BC-F1.

Table 23: Results for the graph nr_537_size_144 ($|V| = 144, d = 0.68$).

p	Root LB	Root Time (s)	#((3)+(4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2*	1517.00	1226.58	14800	1	1226.58	1517.00	**1517**	0.00
6	1307.54	102.95	28856	3	103.18	1309.00	**1309**	0.00
10	1312.00	99.82	25709	10	102.09	1313.00	**1313**	0.00
14	1319.50	191.61	27457	5	193.75	1321.00	**1321**	0.00
18	1346.23	118.45	27667	1042	1652.75	1368.00	**1368**	0.00
22	1378.31	200.92	39091	6641	7200.00	1400.02	1406	0.43
26	1410.77	199.59	41606	3364	4616.59	1424.00	**1424**	0.00
30	1448.09	187.98	35457	7	212.76	1450.00	**1450**	0.00
34	1506.00	147.00	30033	1	147.00	1506.00	**1506**	0.00
38	1632.00	106.67	39047	1	106.67	1632.00	**1632**	0.00
42	1864.00	192.28	50565	1	192.28	1864.00	**1864**	0.00
46	2187.67	628.50	51629	3	713.57	2190.00	**2190**	0.00
50	2560.00	0.20	0	1	0.20	2560.00	**2560**	0.00
54	2930.00	0.19	0	1	0.19	2930.00	**2930**	0.00
58	3284.00	0.19	0	1	0.19	3284.00	**3284**	0.00
62	3622.00	0.19	0	1	0.19	3622.00	**3622**	0.00
66	3944.00	0.20	0	1	0.20	3944.00	**3944**	0.00
70	4250.00	0.19	0	1	0.19	4250.00	**4250**	0.00
74	4540.00	0.20	0	1	0.20	4540.00	**4540**	0.00
78	4814.00	0.18	0	1	0.18	4814.00	**4814**	0.00
82	5072.00	0.20	0	1	0.20	5072.00	**5072**	0.00
86	5314.00	0.20	0	1	0.20	5314.00	**5314**	0.00
90	5540.00	0.19	0	1	0.19	5540.00	**5540**	0.00
94	5750.00	0.19	0	1	0.19	5750.00	**5750**	0.00
98	5944.00	0.18	0	1	0.18	5944.00	**5944**	0.00
102	6122.00	0.18	0	1	0.18	6122.00	**6122**	0.00
106	6284.00	0.18	0	1	0.18	6284.00	**6284**	0.00
110	6430.00	0.17	0	1	0.17	6430.00	**6430**	0.00
114	6560.00	0.15	0	1	0.15	6560.00	**6560**	0.00
118	6674.00	0.15	0	1	0.15	6674.00	**6674**	0.00
122	6772.00	0.17	0	1	0.17	6772.00	**6772**	0.00
126	6854.00	0.16	0	1	0.16	6854.00	**6854**	0.00
130	6920.00	0.16	0	1	0.16	6920.00	**6920**	0.00
134	6970.00	0.15	0	1	0.15	6970.00	**6970**	0.00
138	7004.00	0.15	0	1	0.15	7004.00	**7004**	0.00
142	7022.00	0.15	0	1	0.15	7022.00	**7022**	0.00

*Found using BC-F1.

Table 24: Results for the graph nr_116_size_152 ($|V| = 152, d = 0.35$).

p	Root LB	Root Time (s)	#((3)+(4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2*	1883.78	1929.01	41600	75	7200.00	1950.35	2530	22.91
6	902.69	270.04	40612	700	7200.00	904.19	1059	14.62
10	897.16	425.22	44319	1108	7200.00	898.81	919	2.20
14	891.91	71.62	25000	127	430.66	894.00	**894**	0.00
18	890.00	23.88	18373	1	23.88	890.00	**890**	0.00
22	894.00	25.58	17465	1	25.58	894.00	**894**	0.00
26	897.50	30.00	20901	3	30.17	899.00	**899**	0.00
30	906.00	32.27	17059	1	32.27	906.00	**906**	0.00
34	916.00	27.91	16703	1	27.91	916.00	**916**	0.00
38	930.00	33.22	16130	29	55.68	933.00	**933**	0.00
42	946.00	28.33	18500	1	28.33	946.00	**946**	0.00
46	962.00	40.84	16269	893	435.78	968.00	**968**	0.00
50	978.00	23.81	14344	1	23.81	978.00	**978**	0.00
54	996.00	39.62	15064	2513	619.10	1004.00	**1004**	0.00
58	1014.00	29.85	17457	1	29.85	1014.00	**1014**	0.00
62	1032.50	31.84	18120	368	162.49	1040.00	**1040**	0.00
66	1052.10	49.34	20969	93	138.12	1055.00	**1055**	0.00
70	1083.00	23.52	14375	1	23.52	1083.00	**1083**	0.00
74	1219.00	44.96	16675	1	44.96	1219.00	**1219**	0.00
78	1384.33	224.14	23336	3	244.48	1385.00	**1385**	0.00
82	1576.86	137.86	26384	2	140.15	1577.00	**1577**	0.00
86	1789.00	159.10	23121	1	159.10	1789.00	**1789**	0.00
90	2011.76	949.91	31624	3	961.59	2013.00	**2013**	0.00
94	2251.00	0.22	0	1	0.22	2251.00	**2251**	0.00
98	2477.00	0.19	0	1	0.19	2477.00	**2477**	0.00
102	2687.00	0.20	0	1	0.20	2687.00	**2687**	0.00
106	2881.00	0.20	0	1	0.20	2881.00	**2881**	0.00
110	3059.00	0.18	0	1	0.18	3059.00	**3059**	0.00
114	3221.00	0.19	0	1	0.19	3221.00	**3221**	0.00
118	3367.00	0.18	0	1	0.18	3367.00	**3367**	0.00
122	3497.00	0.17	0	1	0.17	3497.00	**3497**	0.00
126	3611.00	0.18	0	1	0.18	3611.00	**3611**	0.00
130	3709.00	0.18	0	1	0.18	3709.00	**3709**	0.00
134	3791.00	0.16	0	1	0.16	3791.00	**3791**	0.00
138	3857.00	0.18	0	1	0.18	3857.00	**3857**	0.00
142	3907.00	0.17	0	1	0.17	3907.00	**3907**	0.00
146	3941.00	0.15	0	1	0.15	3941.00	**3941**	0.00
150	3959.00	0.17	0	1	0.17	3959.00	**3959**	0.00

*Found using BC-F1.

Table 25: Results for the graph nr_593_size_157 ($|V| = 157, d = 0.49$).

p	**Root LB**	**Root Time (s)**	**#((3)+ (4)+(28))**	**Tree Size**	**Total Time (s)**	**LB**	**UB**	**Gap (%)**
2*	2454.16	7200.00	25600	1	7200.00	2454.16	2458	0.16
6	1694.41	973.02	61020	295	7200.00	1695.96	1750	3.09
10	1688.88	773.00	46715	1043	7200.00	1692.68	1698	0.31
14	1686.05	278.77	35302	7	374.54	1688.00	**1688**	0.00
18	1688.00	186.88	32531	1	186.88	1688.00	**1688**	0.00
22	1691.51	238.81	38356	3	241.96	1693.00	**1693**	0.00
26	1700.00	278.99	35191	1	278.99	1700.00	**1700**	0.00
30	1708.50	278.19	37537	3	281.05	1710.00	**1710**	0.00
34	1724.00	347.61	39217	44	720.95	1729.00	**1729**	0.00
38	1740.00	336.96	38983	15	369.03	1742.00	**1742**	0.00
42	1759.00	207.77	32949	251	661.17	1768.00	**1768**	0.00
46	1779.00	347.44	32888	1185	2823.52	1786.00	**1786**	0.00
50	1812.34	483.71	35999	3	508.36	1814.00	**1814**	0.00
54	1902.00	816.02	45211	1	816.02	1902.00	**1902**	0.00
58	2032.00	1306.39	44969	1	1306.39	2032.00	**2032**	0.00
62	2204.00	1944.43	52584	1	1944.43	2204.00	**2204**	0.00
66	2392.00	1413.30	48025	1	1413.30	2392.00	**2392**	0.00
70	2595.64	3475.86	59842	2	3481.25	2596.00	**2596**	0.00
74	2810.00	1892.00	53519	1	1892.00	2810.00	**2810**	0.00
78	3032.00	1979.39	55337	1	1979.39	3032.00	**3032**	0.00
82	3264.02	3623.68	55213	17	5410.69	3268.00	**3268**	0.00
86	3496.57	7200.00	65041	1	7200.00	3496.57	3524	0.78
90	3758.00	7200.00	64032	1	7200.00	3758.00	3778	0.53
94	4020.00	18.53	23262	2	7200.00	4020.00	4022	0.05
98	4266.00	0.15	0	1	0.15	4266.00	**4266**	0.00
102	4496.00	0.15	0	1	0.15	4496.00	**4496**	0.00
106	4710.00	0.15	0	1	0.15	4710.00	**4710**	0.00
110	4908.00	0.55	0	1	0.55	4908.00	**4908**	0.00
114	5090.00	0.15	0	1	0.15	5090.00	**5090**	0.00
118	5256.00	0.15	0	1	0.15	5256.00	**5256**	0.00
122	5406.00	0.16	0	1	0.16	5406.00	**5406**	0.00
126	5540.00	0.15	0	1	0.15	5540.00	**5540**	0.00
130	5658.00	0.15	0	1	0.15	5658.00	**5658**	0.00
134	5760.00	0.15	0	1	0.15	5760.00	**5760**	0.00
138	5846.00	0.15	0	1	0.15	5846.00	**5846**	0.00
142	5916.00	0.15	0	1	0.15	5916.00	**5916**	0.00
146	5970.00	0.15	0	1	0.15	5970.00	**5970**	0.00
150	6008.00	0.15	0	1	0.15	6008.00	**6008**	0.00
154	6030.00	0.16	0	1	0.16	6030.00	**6030**	0.00

*Found using BC-F1.

Table 26: Results for the graph nr_176_size_169 ($|V| = 169, d = 0.90$).

p	**Root LB**	**Root Time (s)**	**#((3)+ (4)+(28))**	**Tree Size**	**Total Time (s)**	**LB**	**UB**	**Gap (%)**
2*	371.00	120.12	8565	1	120.12	371.00	**371**	0.00
6	339.00	2.08	13432	1	2.08	339.00	**339**	0.00
10	337.00	1.96	10791	1	1.96	337.00	**337**	0.00
14	687.00	9.02	20899	1	9.02	687.00	**687**	0.00
18	1271.00	261.13	38964	1	261.13	1271.00	**1271**	0.00
22	1867.00	0.58	0	1	0.58	1867.00	**1867**	0.00
26	2449.00	0.19	0	1	0.19	2449.00	**2449**	0.00
30	3015.00	0.20	0	1	0.20	3015.00	**3015**	0.00
34	3565.00	0.20	0	1	0.20	3565.00	**3565**	0.00
38	4099.00	0.21	0	1	0.21	4099.00	**4099**	0.00
42	4617.00	0.21	0	1	0.21	4617.00	**4617**	0.00
46	5119.00	0.22	0	1	0.22	5119.00	**5119**	0.00
50	5605.00	0.22	0	1	0.22	5605.00	**5605**	0.00
54	6075.00	0.23	0	1	0.23	6075.00	**6075**	0.00
58	6529.00	0.23	0	1	0.23	6529.00	**6529**	0.00
62	6967.00	0.24	0	1	0.24	6967.00	**6967**	0.00
66	7389.00	0.23	0	1	0.23	7389.00	**7389**	0.00
70	7795.00	0.24	0	1	0.24	7795.00	**7795**	0.00
74	8185.00	0.24	0	1	0.24	8185.00	**8185**	0.00
78	8559.00	0.24	0	1	0.24	8559.00	**8559**	0.00
82	8917.00	0.24	0	1	0.24	8917.00	**8917**	0.00
86	9259.00	0.24	0	1	0.24	9259.00	**9259**	0.00
90	9585.00	0.24	0	1	0.24	9585.00	**9585**	0.00
94	9895.00	0.24	0	1	0.24	9895.00	**9895**	0.00
98	10189.00	0.24	0	1	0.24	10189.00	**10189**	0.00
102	10467.00	0.24	0	1	0.24	10467.00	**10467**	0.00
106	10729.00	0.23	0	1	0.23	10729.00	**10729**	0.00
110	10975.00	0.23	0	1	0.23	10975.00	**10975**	0.00
114	11205.00	0.23	0	1	0.23	11205.00	**11205**	0.00
118	11419.00	0.23	0	1	0.23	11419.00	**11419**	0.00
122	11617.00	0.22	0	1	0.22	11617.00	**11617**	0.00
126	11799.00	0.21	0	1	0.21	11799.00	**11799**	0.00
130	11965.00	0.21	0	1	0.21	11965.00	**11965**	0.00
134	12115.00	0.20	0	1	0.20	12115.00	**12115**	0.00
138	12249.00	0.20	0	1	0.20	12249.00	**12249**	0.00
142	12367.00	0.19	0	1	0.19	12367.00	**12367**	0.00
146	12469.00	0.19	0	1	0.19	12469.00	**12469**	0.00
150	12555.00	0.17	0	1	0.17	12555.00	**12555**	0.00
154	12625.00	0.17	0	1	0.17	12625.00	**12625**	0.00
158	12679.00	0.17	0	1	0.17	12679.00	**12679**	0.00
162	12717.00	0.17	0	1	0.17	12717.00	**12717**	0.00
166	12739.00	0.17	0	1	0.17	12739.00	**12739**	0.00

*Found using BC-F1.

Table 27: Results for the graph nr_459_size_171 ($|V| = 171, d = 0.11$).

p	Root LB	Root Time (s)	#((3)+ (4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2	5681.69	715.77	70806	42	7200.00	5684.95	5696	0.19
6	1052.23	232.04	49319	697	7200.00	1054.66	1210	12.84
10	549.60	163.09	28809	3649	7200.00	552.44	694	20.40
14	545.19	214.45	33253	3071	7200.00	547.53	596	8.13
18	540.82	261.57	23977	3854	7200.00	542.60	560	3.11
22	537.24	16.95	11069	1808	7200.00	539.00	543	0.74
26	533.08	5.53	9027	11	14.53	535.00	**535**	0.00
30	532.00	2.69	8618	1	2.69	532.00	**532**	0.00
34	535.00	3.10	8567	1	3.10	535.00	**535**	0.00
38	539.00	2.68	8617	1	2.68	539.00	**539**	0.00
42	543.00	2.17	8679	1	2.17	543.00	**543**	0.00
46	547.50	3.14	8540	3	3.28	549.00	**549**	0.00
50	556.00	3.13	8519	1	3.13	556.00	**556**	0.00
54	563.00	2.47	8393	3	2.55	564.00	**564**	0.00
58	571.50	3.20	7841	3	3.30	573.00	**573**	0.00
62	587.00	3.49	8506	1	3.49	587.00	**587**	0.00
66	600.00	2.16	8676	1	2.16	600.00	**600**	0.00
70	618.27	5.26	8874	3	6.27	620.00	**620**	0.00
74	641.80	13.36	8849	244	80.87	649.00	**649**	0.00
78	663.73	7.89	8529	21	10.75	666.00	**666**	0.00
82	689.00	6.07	8380	1	6.07	689.00	**689**	0.00
86	715.42	19.34	9126	4	27.44	718.00	**718**	0.00
90	744.94	21.63	9320	1091	296.12	759.00	**759**	0.00
94	774.07	22.89	9545	25442	1846.97	788.91	813	2.96
98	806.26	26.61	9360	27138	1650.20	820.55	836	1.85
102	840.32	27.41	9457	630	224.30	851.00	**851**	0.00
106	877.83	28.67	9295	135	135.83	889.00	**889**	0.00
110	913.40	24.35	9205	33308	4312.80	929.80	945	1.61
114	948.47	27.01	10092	25838	1261.17	958.42	988	2.99
118	985.53	18.79	9140	32898	2291.35	996.00	1004	0.80
122	1019.00	9.38	8009	20	14.69	1024.00	**1024**	0.00
126	1066.21	19.51	8637	258	242.96	1084.00	**1084**	0.00
130	1108.64	18.13	8535	29412	2587.45	1128.20	1154	2.24
134	1151.03	20.18	9274	27317	1681.43	1163.38	1202	3.21
138	1193.34	17.18	7672	29619	1329.14	1201.11	1228	2.19
142	1226.84	10.61	7070	440	49.47	1240.00	**1240**	0.00
146	1288.00	12.53	6782	48	35.65	1292.00	**1292**	0.00
150	1382.00	0.21	0	1	0.21	1382.00	**1382**	0.00
154	1460.00	0.19	0	1	0.19	1460.00	**1460**	0.00
158	1522.00	0.19	0	1	0.19	1522.00	**1522**	0.00
162	1568.00	0.19	0	1	0.19	1568.00	**1568**	0.00
166	1598.00	0.19	0	1	0.19	1598.00	**1598**	0.00
170	1612.00	0.45	0	1	0.45	1612.00	**1612**	0.00

Table 28: Results for the graph nr_9_size_178 ($|V| = 178, d = 0.79$).

p	**Root LB**	**Root Time (s)**	**#((3)+(4)+(28))**	**Tree Size**	**Total Time (s)**	**LB**	**UB**	**Gap (%)**
2*	2066.00	3708.91	20400	1	3708.91	2066.00	**2066**	0.00
6	1953.34	941.93	49756	3	943.55	1955.00	**1955**	0.00
10	1961.00	310.28	40261	2	310.40	1962.00	**1962**	0.00
14	1978.29	396.24	33085	97	774.45	1986.00	**1986**	0.00
18	1998.00	733.38	47145	316	1908.65	2005.00	**2005**	0.00
22	2023.45	822.61	47154	3	828.31	2025.00	**2025**	0.00
26	2111.00	958.69	46874	1	958.69	2111.00	**2111**	0.00
30	2293.00	1184.41	56842	1	1184.41	2293.00	**2293**	0.00
34	2565.00	1532.96	71532	1	1532.96	2565.00	**2565**	0.00
38	2913.00	2741.29	86069	1	2741.29	2913.00	**2913**	0.00
42	3321.00	3493.09	78728	1	3493.09	3321.00	**3321**	0.00
46	3695.00	7200.00	103981	1	7200.00	3695.00	3767	1.91
50	4217.00	7200.00	116285	1	7200.00	4217.00	4235	0.43
54	4723.00	7200.00	108796	1	7200.00	4723.00	4725	0.04
58	5213.00	0.31	0	1	0.31	5213.00	**5213**	0.00
62	5687.00	0.32	0	1	0.32	5687.00	**5687**	0.00
66	6145.00	0.31	0	1	0.31	6145.00	**6145**	0.00
70	6587.00	0.32	0	1	0.32	6587.00	**6587**	0.00
74	7013.00	0.32	0	1	0.32	7013.00	**7013**	0.00
78	7423.00	0.33	0	1	0.33	7423.00	**7423**	0.00
82	7817.00	0.32	0	1	0.32	7817.00	**7817**	0.00
86	8195.00	0.33	0	1	0.33	8195.00	**8195**	0.00
90	8557.00	0.33	0	1	0.33	8557.00	**8557**	0.00
94	8903.00	0.33	0	1	0.33	8903.00	**8903**	0.00
98	9233.00	0.32	0	1	0.32	9233.00	**9233**	0.00
102	9547.00	0.33	0	1	0.33	9547.00	**9547**	0.00
106	9845.00	0.32	0	1	0.32	9845.00	**9845**	0.00
110	10127.00	0.32	0	1	0.32	10127.00	**10127**	0.00
114	10393.00	0.31	0	1	0.31	10393.00	**10393**	0.00
118	10643.00	0.33	0	1	0.33	10643.00	**10643**	0.00
122	10877.00	0.31	0	1	0.31	10877.00	**10877**	0.00
126	11095.00	0.30	0	1	0.30	11095.00	**11095**	0.00
130	11297.00	0.29	0	1	0.29	11297.00	**11297**	0.00
134	11483.00	0.29	0	1	0.29	11483.00	**11483**	0.00
138	11653.00	0.28	0	1	0.28	11653.00	**11653**	0.00
142	11807.00	0.27	0	1	0.27	11807.00	**11807**	0.00
146	11945.00	0.27	0	1	0.27	11945.00	**11945**	0.00
150	12067.00	0.27	0	1	0.27	12067.00	**12067**	0.00
154	12173.00	0.26	0	1	0.26	12173.00	**12173**	0.00
158	12263.00	0.25	0	1	0.25	12263.00	**12263**	0.00
162	12337.00	0.25	0	1	0.25	12337.00	**12337**	0.00
166	12395.00	0.23	0	1	0.23	12395.00	**12395**	0.00
170	12437.00	0.24	0	1	0.24	12437.00	**12437**	0.00
174	12463.00	0.24	0	1	0.24	12463.00	**12463**	0.00

*Found using BC-F1.

Table 29: Results for the graph nr_200_size_179 ($|V| = 179, d = 0.43$).

p	Root LB	Root Time (s)	#((3)+ (4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2*	1935.35	6021.64	41878	3	7200.00	2016.22	2206	8.60
6	1519.00	113.59	38304	1	113.59	1519.00	**1519**	0.00
10	1542.00	200.61	36312	471	2140.70	1555.00	**1555**	0.00
14	1565.00	202.00	34938	1	202.00	1565.00	**1565**	0.00
18	1593.00	240.35	33482	1	240.35	1593.00	**1593**	0.00
22	1629.00	437.16	38385	91	1262.79	1643.00	**1643**	0.00
26	1665.00	390.24	37225	4076	7200.00	1685.50	1700	0.85
30	1701.00	297.53	36892	1307	2639.56	1717.00	**1717**	0.00
34	1736.41	263.87	30810	3	267.98	1738.00	**1738**	0.00
38	1780.79	371.77	35121	3	374.62	1782.00	**1782**	0.00
42	1838.22	283.31	32059	3	285.82	1840.00	**1840**	0.00
46	1906.28	347.84	34736	25	942.33	1918.00	**1918**	0.00
50	1974.12	403.35	33473	797	7200.00	2003.44	2034	1.50
54	2042.31	517.72	36850	1274	7200.00	2065.72	2154	4.10
58	2107.56	497.72	38215	2800	7200.00	2118.66	2252	5.92
62	2180.04	648.26	41607	1860	7200.00	2191.23	2334	6.12
66	2253.76	766.53	43103	1555	7200.00	2261.92	2400	5.75
70	2321.28	622.24	43315	1984	7200.00	2326.09	2450	5.06
74	2392.67	334.74	34066	2683	7200.00	2399.83	2482	3.31
78	2464.96	397.06	36699	4153	7200.00	2469.39	2492	0.91
82	2562.15	368.43	36441	3	375.74	2564.00	**2564**	0.00
86	2730.30	525.35	42230	3	535.13	2732.00	**2732**	0.00
90	2973.43	724.20	41878	3	862.92	2976.00	**2976**	0.00
94	3256.00	1023.67	49560	1	1023.67	3256.00	**3256**	0.00
98	3549.64	2712.91	51948	3	3034.67	3552.00	**3552**	0.00
102	3848.59	7200.00	65112	1	7200.00	3848.59	3854	0.14
106	4146.00	0.23	0	1	0.23	4146.00	**4146**	0.00
110	4432.00	0.22	0	1	0.22	4432.00	**4432**	0.00
114	4702.00	0.23	0	1	0.23	4702.00	**4702**	0.00
118	4956.00	0.24	0	1	0.24	4956.00	**4956**	0.00
122	5194.00	0.22	0	1	0.22	5194.00	**5194**	0.00
126	5416.00	0.24	0	1	0.24	5416.00	**5416**	0.00
130	5622.00	0.23	0	1	0.23	5622.00	**5622**	0.00
134	5812.00	0.22	0	1	0.22	5812.00	**5812**	0.00
138	5986.00	0.24	0	1	0.24	5986.00	**5986**	0.00
142	6144.00	0.23	0	1	0.23	6144.00	**6144**	0.00
146	6286.00	0.22	0	1	0.22	6286.00	**6286**	0.00
150	6412.00	0.24	0	1	0.24	6412.00	**6412**	0.00
154	6522.00	0.23	0	1	0.23	6522.00	**6522**	0.00
158	6616.00	0.22	0	1	0.22	6616.00	**6616**	0.00
162	6694.00	0.23	0	1	0.23	6694.00	**6694**	0.00
166	6756.00	0.23	0	1	0.23	6756.00	**6756**	0.00
170	6802.00	0.22	0	1	0.22	6802.00	**6802**	0.00
174	6832.00	0.23	0	1	0.23	6832.00	**6832**	0.00
178	6846.00	0.51	0	1	0.51	6846.00	**6846**	0.00

*Found using BC-F1.

Table 30: Results for the graph nr_263_size_186 ($|V| = 186, d = 0.34$).

p	**Root LB**	**Root Time (s)**	**#((3)+(4)+(28))**	**Tree Size**	**Total Time (s)**	**LB**	**UB**	**Gap (%)**
2*	3342.99	7200.00	39824	1	7200.00	3342.99	3856	13.30
6	979.00	37.28	37448	1	37.28	979.00	**979**	0.00
10	1019.00	121.50	38375	1	121.50	1019.00	**1019**	0.00
14	1081.32	560.64	40975	13	1159.06	1087.00	**1087**	0.00
18	1159.31	712.53	44074	123	3780.03	1181.00	**1181**	0.00
22	1238.44	701.28	43099	173	4473.31	1267.00	**1267**	0.00
26	1328.53	1698.11	52050	136	7200.00	1330.91	1393	4.46
30	1405.45	835.70	47057	300	7200.00	1427.28	1561	8.57
34	1500.69	979.27	42978	198	7200.00	1512.14	1713	11.73
38	1584.56	2277.99	49869	177	7200.00	1584.66	1858	14.71
42	1683.49	2366.37	46679	53	7200.00	1683.56	1939	13.17
46	1770.32	1052.20	41307	700	7200.00	1776.39	2045	13.13
50	1866.68	1380.55	43085	300	7200.00	1874.45	2135	12.20
54	1955.99	1046.56	41414	907	7200.00	1959.31	2209	11.30
58	2048.39	1205.71	48155	166	7200.00	2048.51	2255	9.16
62	2144.48	943.16	39263	483	7200.00	2145.23	2300	6.73
66	2243.67	669.15	39074	900	7200.00	2247.42	2318	3.05
70	2336.19	777.43	38965	33	1815.59	2345.00	**2345**	0.00
74	2433.06	697.39	39640	541	7200.00	2454.66	2527	2.86
78	2534.63	923.21	38853	366	7200.00	2536.82	2746	7.62
82	2629.00	1041.42	46871	400	7200.00	2629.64	3097	15.09
86	2732.26	1273.08	53000	50	7200.00	2732.41	3083	11.37
90	2834.29	1043.47	41754	300	7200.00	2834.79	3221	11.99
94	2937.96	1261.91	53572	400	7200.00	2938.46	3442	14.63
98	3039.59	1258.73	48775	52	7200.00	3039.98	3680	17.39
102	3142.10	1235.48	47411	137	7200.00	3143.63	3854	18.43
106	3242.71	2011.75	52319	28	7200.00	3242.79	4049	19.91
110	3286.00	312.48	44579	2094	7200.00	3293.00	4205	21.69
114	3397.50	240.47	41501	3440	7200.00	3407.00	4303	20.82
118	3520.51	836.85	47021	2570	7200.00	3537.00	3730	5.17
122	3737.00	0.26	0	1	0.26	3737.00	**3737**	0.00
126	3987.00	0.26	0	1	0.26	3987.00	**3987**	0.00
130	4221.00	0.26	0	1	0.26	4221.00	**4221**	0.00
134	4439.00	2027.01	47370	3	2450.38	4439.00	**4439**	0.00
138	4641.00	0.29	0	1	0.29	4641.00	**4641**	0.00
142	4827.00	0.26	0	1	0.26	4827.00	**4827**	0.00
146	4997.00	0.26	0	1	0.26	4997.00	**4997**	0.00
150	5151.00	0.26	0	1	0.26	5151.00	**5151**	0.00
154	5289.00	0.29	0	1	0.29	5289.00	**5289**	0.00
158	5411.00	0.29	0	1	0.29	5411.00	**5411**	0.00
162	5517.00	0.28	0	1	0.28	5517.00	**5517**	0.00
166	5607.00	0.28	0	1	0.28	5607.00	**5607**	0.00
170	5681.00	0.26	0	1	0.26	5681.00	**5681**	0.00
174	5739.00	0.27	0	1	0.27	5739.00	**5739**	0.00
178	5781.00	0.26	0	1	0.26	5781.00	**5781**	0.00
182	5807.00	0.26	0	1	0.26	5807.00	**5807**	0.00

*Found using BC-F1.

Table 31: Results for the graph nr_365_size_196 ($|V| = 196, d = 0.15$).

p	Root LB	Root Time (s)	#((3)+(4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2	6768.96	1204.43	90981	28	7200.00	6768.98	7036	3.80
6	1717.52	4520.66	67738	21	7200.00	1717.56	2182	21.29
10	1251.76	722.82	37499	833	7200.00	1252.63	1487	15.76
14	1246.43	844.55	34928	1094	7200.00	1247.91	1300	4.01
18	1242.25	527.35	26226	1847	7200.00	1243.71	1254	0.82
22	1237.74	132.12	21554	5	147.00	1240.00	**1240**	0.00
26	1234.49	99.56	20050	6	103.33	1236.00	**1236**	0.00
30	1234.11	89.98	23184	3	94.86	1236.00	**1236**	0.00
34	1240.00	91.77	19889	1	91.77	1240.00	**1240**	0.00
38	1246.00	103.63	20421	1	103.63	1246.00	**1246**	0.00
42	1252.00	86.41	20355	1	86.41	1252.00	**1252**	0.00
46	1260.00	99.53	20660	1	99.53	1260.00	**1260**	0.00
50	1269.00	87.77	20263	1	87.77	1269.00	**1269**	0.00
54	1279.00	102.25	20492	1	102.25	1279.00	**1279**	0.00
58	1289.89	110.95	20246	34	182.99	1293.00	**1293**	0.00
62	1303.30	112.37	19096	5	121.71	1305.00	**1305**	0.00
66	1318.57	119.21	19985	14	152.07	1321.00	**1321**	0.00
70	1335.00	111.62	23857	18	189.68	1337.00	**1337**	0.00
74	1353.00	147.68	21104	105	382.94	1357.00	**1357**	0.00
78	1371.38	158.76	20385	3	168.56	1373.00	**1373**	0.00
82	1390.67	137.48	19794	5483	2361.52	1399.00	**1399**	0.00
86	1408.71	133.24	23240	3	143.77	1410.00	**1410**	0.00
90	1430.67	143.41	20666	1595	1160.04	1441.00	**1441**	0.00
94	1452.24	120.47	22841	826	582.65	1458.00	**1458**	0.00
98	1474.06	137.25	19792	15	348.48	1478.00	**1478**	0.00
102	1496.00	141.73	23049	13370	7200.00	1505.00	1508	0.20
106	1518.00	104.32	22021	1	104.32	1518.00	**1518**	0.00
110	1550.38	391.32	21828	35	1141.19	1558.00	**1558**	0.00
114	1587.48	350.93	26092	2596	7200.00	1605.70	1617	0.70
118	1624.32	945.38	28028	7578	7200.00	1634.87	1661	1.57
122	1661.33	282.59	25130	13606	6199.84	1670.00	1679	0.54
126	1698.00	199.21	23607	1	199.21	1698.00	**1698**	0.00
130	1743.55	397.51	21916	63	1505.36	1752.00	**1752**	0.00
134	1791.39	408.45	22072	1100	7200.00	1806.31	1831	1.35
138	1840.52	489.28	24803	2017	7200.00	1848.22	1908	3.13
142	1891.18	422.68	22722	1136	7200.00	1898.16	1943	2.31
146	1939.68	327.98	22833	2696	7200.00	1952.26	1954	0.09
150	1988.19	231.10	19383	18	300.44	1990.00	**1990**	0.00
154	2015.24	163.97	18988	290	2704.02	2044.00	**2044**	0.00
158	2086.00	208.32	17948	1197	7200.00	2112.29	2134	1.02
162	2232.00	249.53	18451	9132	7200.00	2232.00	2240	0.36
166	2362.00	0.64	0	1	0.64	2362.00	**2362**	0.00
170	2476.00	0.24	0	1	0.24	2476.00	**2476**	0.00
174	2574.00	0.25	0	1	0.25	2574.00	**2574**	0.00
178	2656.00	0.26	0	1	0.26	2656.00	**2656**	0.00
182	2722.00	0.27	0	1	0.27	2722.00	**2722**	0.00
186	2772.00	0.25	0	1	0.25	2772.00	**2772**	0.00
190	2806.00	0.25	0	1	0.25	2806.00	**2806**	0.00
194	2824.00	0.26	0	1	0.26	2824.00	**2824**	0.00

Table 32: Results for the graph nr_232_size_206 ($|V| = 206, d = 0.31$).

p	Root LB	Root Time (s)	#((3)+(4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2	4153.16	928.92	64147	14	7200.00	4153.61	4938	15.88
6	687.00	3.01	24217	1	3.01	687.00	**687**	0.00
10	713.18	16.20	21009	1245	782.12	731.00	**731**	0.00
14	739.36	25.35	20985	2540	830.52	753.00	**753**	0.00
18	768.00	26.92	19739	1	26.92	768.00	**768**	0.00
22	824.00	27.98	23931	1	27.98	824.00	**824**	0.00
26	914.05	171.76	32868	119	953.60	926.00	**926**	0.00
30	1004.87	201.58	35392	1907	7200.00	1039.93	1096	5.12
34	1099.88	259.67	34470	1374	7200.00	1138.05	1258	9.54
38	1196.10	559.21	37631	788	7200.00	1227.59	1404	12.57
42	1286.94	237.15	31910	1274	7200.00	1304.46	1534	14.96
46	1384.03	291.15	32218	2000	7200.00	1406.14	1648	14.68
50	1476.96	368.44	38639	1618	7200.00	1495.53	1746	14.35
54	1570.51	306.01	36771	2255	7200.00	1588.47	1828	13.10
58	1667.51	373.89	38202	2001	7200.00	1681.22	1894	11.23
62	1758.41	331.63	36433	2001	7200.00	1764.55	1944	9.23
66	1850.65	214.84	33514	2800	7200.00	1858.23	1978	6.05
70	1947.00	127.72	31993	7282	7200.00	1952.86	1996	2.16
74	2065.00	241.17	35306	1464	7200.00	2076.49	2123	2.19
78	2197.00	222.33	32663	3156	7200.00	2233.66	2373	5.87
82	2329.00	243.76	37527	1823	7200.00	2365.50	2604	9.16
86	2461.00	260.96	38145	863	7200.00	2487.34	2818	11.73
90	2593.00	235.75	35634	2700	7200.00	2642.84	3016	12.37
94	2725.00	263.68	39509	1001	7200.00	2743.68	3198	14.21
98	2857.00	242.79	33148	1700	7200.00	2896.84	3364	13.89
102	2989.00	242.09	37425	1300	7200.00	3025.29	3514	13.91
106	3121.00	290.30	39777	1100	7200.00	3153.00	3648	13.57
110	3253.00	226.98	30426	1676	7200.00	3290.56	3766	12.62
114	3385.00	240.90	34544	1961	7200.00	3422.59	3868	11.52
118	3517.00	233.58	36503	3045	7200.00	3548.02	3954	10.27
122	3649.00	306.46	37218	2604	7200.00	3672.15	4024	8.74
126	3781.00	611.97	41331	2131	7200.00	3790.54	4078	7.05
130	3913.00	1127.48	44403	1496	7200.00	3920.37	4116	4.75
134	4045.00	1462.64	43586	2105	7200.00	4046.66	4138	2.21
138	4203.00	0.27	0	1	0.27	4203.00	**4203**	0.00
142	4469.00	0.29	0	1	0.29	4469.00	**4469**	0.00
146	4719.00	0.28	0	1	0.28	4719.00	**4719**	0.00
150	4953.00	0.27	0	1	0.27	4953.00	**4953**	0.00
154	5171.00	0.27	0	1	0.27	5171.00	**5171**	0.00
158	5373.00	0.28	0	1	0.28	5373.00	**5373**	0.00
162	5559.00	0.27	0	1	0.27	5559.00	**5559**	0.00
166	5729.00	0.33	0	1	0.33	5729.00	**5729**	0.00
170	5883.00	0.32	0	1	0.32	5883.00	**5883**	0.00
174	6021.00	0.33	0	1	0.33	6021.00	**6021**	0.00
178	6143.00	0.27	0	1	0.27	6143.00	**6143**	0.00
182	6249.00	0.30	0	1	0.30	6249.00	**6249**	0.00
186	6339.00	0.27	0	1	0.27	6339.00	**6339**	0.00
190	6413.00	0.27	0	1	0.27	6413.00	**6413**	0.00
194	6471.00	0.27	0	1	0.27	6471.00	**6471**	0.00
198	6513.00	0.27	0	1	0.27	6513.00	**6513**	0.00
202	6539.00	0.29	0	1	0.29	6539.00	**6539**	0.00

Table 33: Results for the graph nr_279_size_209 ($|V| = 209, d = 0.54$).

p	**Root LB**	**Root Time (s)**	**#((3)+(4)+(28))**	**Tree Size**	**Total Time (s)**	**LB**	**UB**	**Gap (%)**
2	3811.27	7200.00	59053	1	7200.00	3811.27	4780	20.27
6	3775.00	5447.40	59490	1	5447.40	3775.00	**3775**	0.00
10	3751.99	7200.00	71609	1	7200.00	3751.99	3783	0.82
14	3777.47	7200.00	71919	1	7200.00	3777.47	3807	0.78
18	3822.90	7012.64	70240	2	7200.00	3824.75	3839	0.37
22	3819.24	7200.00	68302	1	7200.00	3819.24	3857	0.98
26	3811.14	7200.00	72130	1	7200.00	3811.14	3877	1.70
30	3874.97	7200.00	59832	1	7200.00	3874.97	3931	1.43
34	3922.09	7200.00	67147	1	7200.00	3922.09	3966	1.11
38	3859.45	7200.00	64266	1	7200.00	3859.45	3978	2.98
42	3925.44	7200.00	56564	1	7200.00	3925.44	4010	2.11
46	4001.08	7200.00	58215	1	7200.00	4001.08	4102	2.46
50	4073.74	7200.00	56559	1	7200.00	4073.74	4196	2.91
54	4066.03	7200.00	54634	1	7200.00	4066.03	4272	4.82
58	4148.16	7200.00	57523	1	7200.00	4148.16	4330	4.20
62	4199.18	7200.00	59098	1	7200.00	4199.18	4366	3.82
66	4278.88	7200.00	68350	1	7200.00	4278.88	4384	2.40
70	4367.56	7200.00	69512	1	7200.00	4367.56	4408	0.92
74	4492.95	7200.00	77192	1	7200.00	4492.95	4528	0.77
78	4552.73	7200.00	78314	1	7200.00	4552.73	4744	4.03
82	4822.87	7200.00	84076	1	7200.00	4822.87	4946	2.49
86	5033.81	7200.00	83292	1	7200.00	5033.81	5174	2.71
90	5238.20	7200.00	89060	1	7200.00	5238.20	5428	3.50
94	5502.50	7200.00	85578	1	7200.00	5502.50	5702	3.50
98	5612.00	7200.00	91557	1	7200.00	5612.00	6012	6.65
102	6050.00	7200.00	95797	1	7200.00	6050.00	6334	4.48
106	6472.00	7200.00	85094	1	7200.00	6472.00	6654	2.74
110	6878.00	7200.00	85641	1	7200.00	6878.00	6988	1.57
114	7268.00	7200.00	76321	1	7200.00	7268.00	7324	0.76
118	7642.00	7200.00	88008	1	7200.00	7642.00	7672	0.39
122	8000.00	7200.00	76209	1	7200.00	8000.00	8006	0.07
126	8342.00	0.51	0	1	0.51	8342.00	**8342**	0.00
130	8668.00	0.50	0	1	0.50	8668.00	**8668**	0.00
134	8978.00	0.49	0	1	0.49	8978.00	**8978**	0.00
138	9272.00	0.47	0	1	0.47	9272.00	**9272**	0.00
142	9550.00	0.47	0	1	0.47	9550.00	**9550**	0.00
146	9812.00	0.47	0	1	0.47	9812.00	**9812**	0.00
150	10058.00	0.46	0	1	0.46	10058.00	**10058**	0.00
154	10288.00	0.45	0	1	0.45	10288.00	**10288**	0.00
158	10502.00	0.43	0	1	0.43	10502.00	**10502**	0.00
162	10700.00	0.43	0	1	0.43	10700.00	**10700**	0.00
166	10882.00	0.42	0	1	0.42	10882.00	**10882**	0.00
170	11048.00	0.43	0	1	0.43	11048.00	**11048**	0.00
174	11198.00	0.39	0	1	0.39	11198.00	**11198**	0.00
178	11332.00	0.38	0	1	0.38	11332.00	**11332**	0.00
182	11450.00	0.34	0	1	0.34	11450.00	**11450**	0.00
186	11552.00	0.36	0	1	0.36	11552.00	**11552**	0.00
190	11638.00	0.35	0	1	0.35	11638.00	**11638**	0.00
194	11708.00	0.34	0	1	0.34	11708.00	**11708**	0.00
198	11762.00	0.33	0	1	0.33	11762.00	**11762**	0.00
202	11800.00	0.34	0	1	0.34	11800.00	**11800**	0.00
206	11822.00	0.34	0	1	0.34	11822.00	**11822**	0.00

Table 34: Results for the graph nr_489_size_211 ($|V| = 211, d = 0.68$).

p	Root LB	Root Time (s)	#((3)+ (4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
2	3533.60	7200.00	62283	1	7200.00	3533.60	4404	19.76
6	3929.99	7200.00	82882	1	7200.00	3929.99	3972	1.06
10	3934.74	6075.19	70714	2	7200.00	3934.82	3940	0.13
14	3934.00	6767.53	74903	1	6767.53	3934.00	**3934**	0.00
18	3111.98	7200.00	51610	1	7200.00	3111.98	3938	20.98
22	3940.18	7072.69	77935	3	7117.23	3942.00	**3942**	0.00
26	3829.07	7200.00	71706	1	7200.00	3829.07	3948	3.01
30	3945.01	7200.00	84377	1	7200.00	3945.01	3956	0.28
34	3957.54	7200.00	84539	1	7200.00	3957.54	3966	0.21
38	3974.14	6720.23	76256	3	7150.70	3976.00	**3976**	0.00
42	4014.49	5323.14	65006	3	5389.08	4016.00	**4016**	0.00
46	4118.00	3477.32	59021	1	3477.32	4118.00	**4118**	0.00
50	4252.00	5877.36	77917	1	5877.36	4252.00	**4252**	0.00
54	4383.37	7200.00	77250	1	7200.00	4383.37	4486	2.29
58	4602.40	7200.00	87780	1	7200.00	4602.40	4758	3.27
62	4836.40	7200.00	102405	1	7200.00	4836.40	5022	3.70
66	5206.92	7200.00	103817	1	7200.00	5206.92	5376	3.15
70	5513.47	7200.00	106007	1	7200.00	5513.47	5748	4.08
74	5516.00	7200.00	104539	1	7200.00	5516.00	6138	10.13
78	6058.00	7200.00	113897	1	7200.00	6058.00	6548	7.48
82	6584.00	7200.00	111632	1	7200.00	6584.00	6970	5.54
86	7094.00	7200.00	114131	1	7200.00	7094.00	7382	3.90
90	7588.00	7200.00	103908	1	7200.00	7588.00	7788	2.57
94	8066.00	7200.00	124367	1	7200.00	8066.00	8188	1.49
98	8528.00	7200.00	124572	1	7200.00	8528.00	8614	1.00
102	8974.00	7200.00	96832	1	7200.00	8974.00	9024	0.55
106	9404.00	7200.00	103373	1	7200.00	9404.00	9434	0.32
110	9818.00	7200.00	101553	1	7200.00	9818.00	9826	0.08
114	10216.00	186.74	44930	2	7200.00	10216.00	10218	0.02
118	10598.00	0.47	0	1	0.47	10598.00	**10598**	0.00
122	10964.00	0.46	0	1	0.46	10964.00	**10964**	0.00
126	11314.00	0.29	0	1	0.29	11314.00	**11314**	0.00
130	11648.00	0.45	0	1	0.45	11648.00	**11648**	0.00
134	11966.00	0.29	0	1	0.29	11966.00	**11966**	0.00
138	12268.00	0.29	0	1	0.29	12268.00	**12268**	0.00
142	12554.00	0.30	0	1	0.30	12554.00	**12554**	0.00
146	12824.00	0.29	0	1	0.29	12824.00	**12824**	0.00
150	13078.00	0.29	0	1	0.29	13078.00	**13078**	0.00
154	13316.00	0.30	0	1	0.30	13316.00	**13316**	0.00
158	13538.00	0.39	0	1	0.39	13538.00	**13538**	0.00
162	13744.00	0.29	0	1	0.29	13744.00	**13744**	0.00
166	13934.00	0.29	0	1	0.29	13934.00	**13934**	0.00
170	14108.00	0.30	0	1	0.30	14108.00	**14108**	0.00
174	14266.00	0.29	0	1	0.29	14266.00	**14266**	0.00
178	14408.00	0.29	0	1	0.29	14408.00	**14408**	0.00
182	14534.00	0.29	0	1	0.29	14534.00	**14534**	0.00

Table 35: Results for the graph nr_489_size_211 (Continued).

p	Root LB	Root Time (s)	#((3)+ (4)+(28))	Tree Size	Total Time (s)	LB	UB	Gap (%)
186	14644.00	0.32	0	1	0.32	14644.00	**14644**	0.00
190	14738.00	0.29	0	1	0.29	14738.00	**14738**	0.00
194	14816.00	0.29	0	1	0.29	14816.00	**14816**	0.00
198	14878.00	0.29	0	1	0.29	14878.00	**14878**	0.00
202	14924.00	0.29	0	1	0.29	14924.00	**14924**	0.00
206	14954.00	0.29	0	1	0.29	14954.00	**14954**	0.00
210	14968.00	0.70	0	1	0.70	14968.00	**14968**	0.00