Mixed-integer Programming Based Approaches for the Movement Planner Problem: Model, Heuristics and Decomposition

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1 Introduction

We formulate the Movement Planner Problem as a mixed-integer linear program (MILP). The novelty of our approach is that we integrate the strengths of the two previous formulations: we draw on the concept of segments in [Törnquist and Persson(2007)] and network representation in [Mu and Dessouky(2011)] so that unlike [Törnquist and Persson(2007)], we are able to model switches and sidings explicitly; contrary to [Mu and Dessouky(2011)], we do not introduce excessive variables on arcs not traveled on. To efficiently solve the problem, several formulation enhancement and heuristic variable fixing procedures are devised, followed by a rolling-horizon-based decomposition algorithm. The computational experiments on the three data sets provided show that our solution approaches consistently outperform the existing pure branch-and-cut algorithm of the commercial solver in identifying high quality solutions. For Data Set 1, we yield provably optimal solution in less than 10 seconds, achieving a 400 times reduction in computational time. For Data Set 2, a high quality solution is derived in less than 30 seconds; for Data Set 3, less than 3 minutes. The proposed algorithm substantially outstrip the benchmark algorithms in the tests both in solution quality and computational time.

2 Problem Formulation

In this paper a segment is defined as a collection of tracks (main tracks, sidings, switches, crossovers) between two adjacent nodes. Segments cover the territories in a way that a train must pass through every segment between its origin and destination and travel on one specific track within a given segment. The railway network in the data sets is divided into 53 segments in total according to the definition. Some dummy arcs, e.g., (48,48), (54,54), are added to facilitate the segmentation. Figure 1 presents

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some illustrative examples on how segments are set apart. Table 5 in the Appendix provides a complete list of 53 segments and the arcs each segment contains.

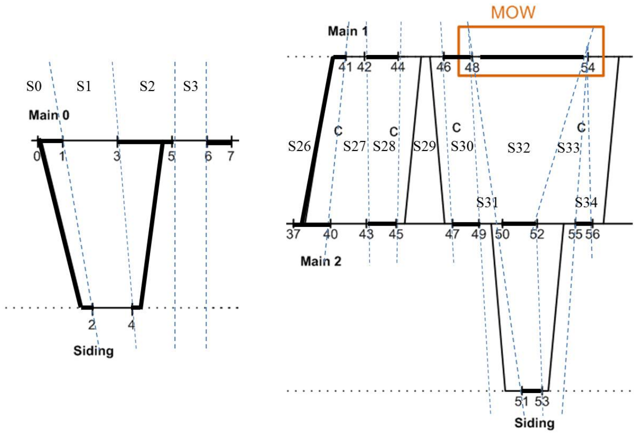

Figure 1: Examples of how the railway network is segmented, "S" stands for "Segment"

Based on the concept of segments, The input sets, data and parameters are denoted in Table 1 and Table 2; decision variables defined in Table 3. Based on the notation, the Movement Planner Problem is formulated as a mixed-integer linear program.

Notation	Definition
$\mathcal{T}$	set of trains, indexed by i
$\mathcal{T}^{\mathrm{E}}$	set of eastbound trains
$\mathcal{T}^{\mathrm{K}}$	set of Schedule Adherence trains
$\mathcal{T}^{\mathrm{H}}$	set of heavy trains
$\mathcal{T}^{\mathrm{L}}$	set of long trains
$\mathcal{T}^{\mathrm{I}}$	set of Inhalation Hazard trains
$\mathcal{G}$	set of segments, indexed by j, $\lvert\mathcal{G}\rvert=n$, j increases from the west to the east
$\mathcal{G}^{\mathrm{C}}$	set of segments that contain sidings
$\mathcal{B}_i$	set of segments occupied by train i
$\mathcal{C}_j$	set of trains that occupy segment j
$\mathcal{L}_j$	set of tracks for segment j, indexed by t
$\mathcal{V}_i$	set of nodes that train i can potentially travel on, the origin and destination nodes excluded
$\mathcal{E}(v), \mathcal{W}(\mathrm{v})$	set of tracks (arcs) linked to node v from the east (west) direction
$\mathcal{O}_i, \mathcal{D}_i$	set of tracks linked to the origin (destination) node of train i that can be potentially occupied
$\mathcal{F}_i$	set of segments that are occupied by train i and contain sidings shorter than the length of train i
$\mathcal{X}_i, \mathcal{Y}_i$	set of entry (exit) segments of the MOW railroads for train i
$\mathcal{N}_i$	set of segments whose exit node for train i has schedule adherence requirements
$\mathcal{U}_{i,j}$	set of unpreferred tracks at segment j for train i

Table 1: Input sets

Notation	Definition
ε	end time of the planning horizon
Δ^{P}	separation time, otherwise known as the minimum headway, 5 minutes in this case
Δ^{S}	schedule deviance threshold, 2 hours in this case
$\Delta^{\mathrm{TWT+}}, \Delta^{\mathrm{TWT-}}$	TWT deviance threshold for late (early) arrival, 3 (1) hours in this case
o_i	index of the origin segment for train i
d_i	index of the destination segment for train i, $d_i = n$ for eastbound trains, $d_i = 1$ for westbound trains
b_i	entry time at the origin for train i
$r_{i,j,t}$	required running time for train i to travel on track t of segment j
s_j	the index of the siding track at segment j
m_j	the index of the main track that has a complementary siding at segment j
w_j	the index of the track at segment j that has an MOW window
$\zeta_{t,t',j}$	equals 1 if track t and track t' are conflicting or if $t = t'$ at segment j, 0 otherwise
$\mathrm{SA}_{i,j}$	schedule adherence time for train i on segment j
TWT_i	terminal want time for train i
$\mathrm{MOW}_j^{\mathrm{begin}}, \mathrm{MOW}_j^{\mathrm{end}}$	MOW begin (end) time for segment j
c_i^{D}	delay penalty per time unit for train i
c^{S}	schedule deviance penalty cost per time unit
c^{TWT}	TWT deviance penalty per time unit
c^{U}	penalty for unpreferred track utilization per time unit
M	sufficiently large number

Table 2: Input data and parameters

Notation	Definition
$x_{i,j}^{\mathrm{entry}}, x_{i,j}^{\mathrm{exit}}$	entry (exit) time for train i at segment j
$q_{i,j,t}$	equals 1 if train i uses track t of segment j, 0 otherwise
$\gamma_{i,i',j}$	equals 1 if train i is scheduled earlier than train i' at segment j on the same or conflicting tracks, 0 otherwise
$\lambda_{i,i',j}$	equals 1 if train i is scheduled later than train i' at segment j on the same or conflicting tracks, 0 otherwise
$\mu_{i,i',j}$, $\mu'_{i,i',j}$	equals 1 if $\lambda_{i,i',j-1} - \lambda_{i,i',j+1} = 1$ ($\lambda_{i,i',j-1} - \lambda_{i,i',j+1} = -1$), 0 otherwise
$y_{i,i',j}$	equals 1 if train i and train i' have a meet-pass event at segment j, 0 otherwise
$z_{i,j}^{\mathrm{D}}$	delay for train i at segment j
$z_{i,j}^{\mathrm{S}}$	schedule deviance beyond 2 hours for train i at segment j
$z_i^{\mathrm{TWT+}}, z_i^{\mathrm{TWT-}}$	TWT deviance for late (early) arrival beyond the 4-hour window for train i
$z_{i,j}^{\mathrm{U}}$	unpreferred track time (if any) for train i at segment j
$\alpha_{i,j}$	equals 1 if train i exits segment j prior to the end of the planning horizon, 0 otherwise
$\beta_{i.j}$	equals 1 if $x_{i,j}^{\mathrm{entry}} \geq \mathrm{MOW}_j^{\mathrm{end}}$, 0 otherwise
$\xi_{i,j}$	equals $x_{i,j}^{\mathrm{exit}}$ if $\alpha_{i,j} = 0$, ε otherwise

Table 3: Decision variables

$$\text{Minimize} \quad \sum_{i \in \mathcal{T}} c_i^{\mathrm{D}} (x_{i,o_i}^{\mathrm{entry}} - b_i + \sum_{j \in \mathcal{B}_i} z_{i,j}^{\mathrm{D}}) + \sum_{i \in \mathcal{T}^{\mathrm{K}}} \sum_{j \in \mathcal{N}_i} c^{\mathrm{S}} z_{i,j}^{\mathrm{S}} + \sum_{i \in \mathcal{T}} c^{\mathrm{TWT}} (z_i^{\mathrm{TWT+}} + z_i^{\mathrm{TWT-}}) + \sum_{i \in \mathcal{T}} \sum_{j \in \mathcal{B}_i} c^{\mathrm{U}} z_{i,j}^{\mathrm{U}} \tag{1}$$

Subject to

Train moving constraints:

$$x_{i,j}^{\mathrm{exit}} = x_{i,j+1}^{\mathrm{entry}}, \qquad \forall i \in \mathcal{T}^{\mathrm{E}}, \forall j \in \mathcal{B}_i : j \neq n \tag{2}$$

$$x_{i,j}^{\mathrm{exit}} = x_{i,j-1}^{\mathrm{entry}}, \qquad \forall i \in \mathcal{T} \setminus \mathcal{T}^{\mathrm{E}}, \forall j \in \mathcal{B}_i : j \neq 1 \tag{3}$$

$$x_{i,o_i}^{\mathrm{entry}} \geq b_i, \qquad \forall i \in \mathcal{T} \tag{4}$$

$$x_{i,j}^{\mathrm{exit}} \geq x_{i,j}^{\mathrm{entry}} + \sum_{t \in \mathcal{L}_j} r_{i,j,t} q_{i,j,t}, \qquad \forall i \in \mathcal{T}, \forall j \in \mathcal{B}_i \tag{5}$$

Traffic network constraints:

$$\sum_{t\in\mathcal{O}_i} q_{i,o_i,t} = 1, \qquad \forall i \in \mathcal{T} \tag{6}$$

$$\sum_{t\in\mathcal{D}_i} q_{i,d_i,t} = 1, \qquad \forall i \in \mathcal{T} \tag{7}$$

$$\sum_{(t,j)\in\mathcal{E}(v)} q_{i,j,t} = \sum_{(t',j')\in\mathcal{W}(v)} q_{i,j',t'}, \qquad \forall i \in \mathcal{T}, \forall v \in \mathcal{V}_i \tag{8}$$

Nonconcurrency and headway constraints:

$$q_{i,j,t} + q_{i',j,t'} - 1 \le \lambda_{i,i',j} + \gamma_{i,i',j}, \qquad \forall i,i' \in \mathcal{C}_j, \forall j \in \mathcal{G}, \forall t,t' \in \mathcal{L}_j : i \ne i', \zeta_{t,t',j} = 1 \tag{9}$$

$$x^{\text{entry}}_{i',j} - x^{\text{exit}}_{i,j} \ge \Delta^{\text{P}} - M(1-\gamma_{i,i',j}), \qquad \forall i,i' \in \mathcal{C}_j, \forall j \in \mathcal{G}, \forall t,t' \in \mathcal{L}_j : i \ne i', \zeta_{t,t',j} = 1 \tag{10}$$

$$x^{\text{entry}}_{i,j} - x^{\text{exit}}_{i',j} \ge \Delta^{\text{P}} - M(1-\lambda_{i,i',j}), \qquad \forall i,i' \in \mathcal{C}_j, \forall j \in \mathcal{G}, \forall t,t' \in \mathcal{L}_j : i \ne i', \zeta_{t,t',j} = 1 \tag{11}$$

$$\lambda_{i,i',j} + \gamma_{i,i',j} \le 1, \qquad \forall i,i' \in \mathcal{C}_j, \forall j \in \mathcal{G} : i \ne i' \tag{12}$$

Siding occupancy constraints:

$$\lambda_{i,i',j-1} - \lambda_{i,i',j+1} = \mu_{i,i',j} - \mu'_{i,i',j}, \qquad \forall i,i' \in \mathcal{C}_j, \forall j \in \mathcal{G}^{\text{C}} : i \ne i' \tag{13}$$

$$\mu_{i,i',j} + \mu'_{i,i',j} \le 1, \qquad \forall i,i' \in \mathcal{C}_j, \forall j \in \mathcal{G}^{\text{C}} : i \ne i' \tag{14}$$

$$y_{i,i',j} \ge \mu_{i,i',j} + \mu'_{i,i',j} + q_{i',j,m_j} - 1, \qquad \forall i,i' \in \mathcal{C}_j, \forall j \in \mathcal{G}^{\text{C}} : i \ne i' \tag{15}$$

$$2y_{i,i',j} \le \mu_{i,i',j} + \mu'_{i,i',j} + q_{i',j,m_j}, \qquad \forall i,i' \in \mathcal{C}_j, \forall j \in \mathcal{G}^{\text{C}} : i \ne i' \tag{16}$$

$$q_{i,j,s_j} \le \sum_{i'\in\mathcal{C}_j : i' \ne i} y_{i,i'j}, \qquad \forall i \in \mathcal{C}_j, \forall j \in \mathcal{G}^{\text{C}} \tag{17}$$

Special train constraints:

$$q_{i,j,s_j} = 0, \qquad \forall i \in \mathcal{T}^{\text{L}}, \forall j \in \mathcal{F}_i \tag{18}$$

$$q_{i,j,s_j} = 0, \qquad \forall i \in \mathcal{T}^{\text{I}}, \forall j \in \mathcal{B}_i \cap \mathcal{G}^{\text{C}} \tag{19}$$

$$q_{i,j,s_j} + q_{i',j,m_j} \le 2 - \mu_{i,i',j} - \mu'_{i,i',j}, \qquad \forall i \in \mathcal{C}_j \cap \mathcal{T}^{\text{H}}, i' \in \mathcal{C}_j \cap (\mathcal{T} \setminus \mathcal{T}^{\text{K}}), \forall j \in \mathcal{G}^{\text{C}} : i \ne i' \tag{20}$$

MOW constraints:

$$x^{\text{entry}}_{i,j} \ge \text{MOW}^{\text{end}}_j q_{i,j,w_j} - \text{MOW}^{\text{end}}_j (1-\beta_{i,j}), \qquad \forall i \in \mathcal{T}, \forall j \in \mathcal{X}_i \tag{21}$$

$$x^{\text{exit}}_{i,j} \le \text{MOW}^{\text{begin}}_j + M(1-q_{i,j,w_j}) + M\beta_{i,j'}, \qquad \forall i \in \mathcal{T}, \forall j \in \mathcal{Y}_i, \forall j' \in \mathcal{X}_i \tag{22}$$

Objective value related constraints:

$$\varepsilon - x^{\text{exit}}_{i,j} \le M\alpha_{i,j}, \qquad \forall i \in \mathcal{T}, j \in \mathcal{B}_i \tag{23}$$

$$x^{\text{exit}}_{i,j} - \varepsilon \le M(1-\alpha_{i,j}), \qquad \forall i \in \mathcal{T}, j \in \mathcal{B}_i \tag{24}$$

$$\xi_{i,j} \ge x^{\text{exit}}_{i,j} - M(1-\alpha_{i,j}), \qquad \forall i \in \mathcal{T}, j \in \mathcal{B}_i \tag{25}$$

$$\xi_{i,j} \ge \varepsilon(1-\alpha_{i,j}), \qquad \forall i \in \mathcal{T}, j \in \mathcal{B}_i \tag{26}$$

$$z^{\text{D}}_{i,j} \ge \xi_{i,j} - x^{\text{entry}}_{i,j} - \sum_{t\in\mathcal{L}_j} r_{i,j,t} q_{i,j,t}, \qquad \forall i \in \mathcal{T}, j \in \mathcal{B}_i \tag{27}$$

$$z^{\text{S}}_{i,j} \ge x^{\text{exit}}_{i,j} - \text{SA}_{i,j} - \Delta^{\text{S}} - M(1-\alpha_{i,j}), \qquad \forall i \in \mathcal{T}^{\text{K}}, \forall j \in \mathcal{N}_i \tag{28}$$

$$z^{\text{TWT+}}_i \ge x^{\text{exit}}_{i,d_i} - \text{TWT}_i - \Delta^{\text{TWT+}} - M(1-\alpha_{i,d_i}), \qquad \forall i \in \mathcal{T} \tag{29}$$

$$z^{\text{TWT-}}_i \ge -(x^{\text{exit}}_{i,d_i} - \text{TWT}_i + \Delta^{\text{TWT}-}) - M(1-\alpha_{i,d_i}), \qquad \forall i \in \mathcal{T} \tag{30}$$

$$z^{U}_{i,j} \ge \xi_{i,j} - x^{\text{entry}}_{i,j} - M(1 - \sum_{t\in\mathcal{U}_{i,j}} q_{i,j,t}), \qquad \forall i \in \mathcal{T}, \forall j \in \mathcal{B}_i \tag{31}$$

Variable restrictions:

$$x^{\text{entry}}_{i,j}, x^{\text{exit}}_{i,j}, \xi_{i,j}, z^{\text{D}}_{i,j}, z^{\text{U}}_{i,j} \ge 0, \qquad \forall i \in \mathcal{T}, \forall j \in \mathcal{B}_i \tag{32}$$

$$q_{i,j,t} \in \{0,1\}, \qquad \forall i \in \mathcal{T}, \forall j \in \mathcal{B}_i, \forall t \in \mathcal{L}_j \tag{33}$$

$$\gamma_{i,i',j}, \lambda_{i,i',j} \in \{0,1\}, \qquad \forall i,i' \in \mathcal{C}_j, \forall j \in \mathcal{G} : i \ne i' \tag{34}$$

$$\mu_{i,i',j}, \mu'_{i,i',j}, y_{i,i',j} \in \{0,1\}, \qquad \forall i,i' \in \mathcal{C}_j, \forall j \in \mathcal{G}^{\text{C}} : i \ne i' \tag{35}$$

$$z^{\text{S}}_{i,j} \ge 0, \qquad \forall i \in \mathcal{T}^{\text{K}}, \forall j \in \mathcal{N}_i \tag{36}$$

$$z^{\text{TWT+}}_i, z^{\text{TWT-}}_i \ge 0, \qquad \forall i \in \mathcal{T} \tag{37}$$

$$\alpha_{i,j} \in \{0,1\}, \qquad \forall i \in \mathcal{T}, \forall j \in \mathcal{B}_i \tag{38}$$

$$\beta_{i,j} \in \{0,1\}, \qquad \forall i \in \mathcal{T}, \forall j \in \mathcal{X}_i \tag{39}$$

The objective function (1) minimizes the total costs incurred by the train delay, schedule deviance over 2 hours for SA Trains, Terminal Want Time deviance beyond the 4-hour window and usage of unpreferred tracks.

Train moving constraints: Constraints (2 and 3) require that each train exits one segment before it seamlessly enters the next segment. This applies to both eastbound trains and westbound trains. Constraints (4) ensure that each train enters the railway network no earlier than its pre-specified entry time at the origin. Constraints (5) indicate that the time elapsed when the train occupies the assigned track at a given segment must be at least as long as the required running time.

Traffic network constraints: Constraints (6 and 7) prescribe each train starts from its origin node and arrives at its destination node. Constraints (8) are the traffic flow conversion equations that specify each train using any track that flows into a node must travel on one of the tracks that emanate from this node.

Nonconcurrency and headway constraints: The variables $\lambda_{i,i'j}$ and $\gamma_{i,i'j}$ are referred to as sequence variables as they specify which train occupies a segment earlier. To avoid collision, Constraints (9) denote that if two trains are using the same track or conflicting tracks at segment j, i.e., $q_{i,j,t} = q_{i',j,t'} = 1$, then these two events will not chronologically overlap, i.e., either $\gamma_{i,i',j}$ or $\lambda_{i,i',j}$ must assume the value of 1. Examples of two conflicting tracks at a given segment include a switch and its directly connected main track, a crossover and its directly connected main track, and two crossovers (to disallow "double crossover"). Following Constraints (9), either Constraints (10) become active if $\gamma_{i,i',j} = 1$, or Constraints (11) are active if $\lambda_{i,i',j} = 1$. Constraints (10) and Constraints (11) respect the operational rule of five-minute separation time between trains in all the three distinct cases specified in the problem statement. Constraints (12) ensure the chronological relationship between two trains when they occupy the same segment, i.e., $\gamma_{i,i',j}$ and $\lambda_{i,i',j}$ cannot be 1 simultaneously. For segments that only contain one track, or one switch and one main track, it is certain that $\gamma_{i,i',j} + \lambda_{i,i',j} = 1$, so in Constraints (9, 10 and 11), the term $q_{i,j,t} + q_{i',j,t'} - 1$ can be replaced by the constant 1.

Siding occupancy constraints: These sets of constraints enforce a train will not take the siding unless a meet-pass event occurs. This is equivalent to the expression that the sequence between two trains at the preceding switch segment and at the succeeding switch segment changes; in addition, there should be a train taking the complementary main track. Transformed to mathematical language, it is $|\lambda_{i,i',j-1} - \lambda_{i,i',j+1}| = 1$ and $q_{i',j,m_j} = 1$, where j is the index of the segment containing a siding. Constraints (13) and (14) imply that $\mu_{i,i'j} + \mu'_{i,i',j} = 1$ if and only if $|\lambda_{i,i',j-1} - \lambda_{i,i',j+1}| = 1$. Constraints (15) and (16) denote that $y_{i,i',j} = 1$ if and only if the two conditions are simultaneously satisfied. Constraints (17) specify the scenario where a train can be dispatched to sidings. Note that for single-track sidings, since $|\lambda_{i,i',j-1} - \lambda_{i,i',j+1}| = 1$ is sufficient for the definition of meet-pass events, Constraints (15) and (16) can be removed whereas $\mu_{i,i'j} + \mu'_{i,i',j}$ can be substituted for $y_{i,i',j}$ in Constraints (17).

Special train constraints: Constraints (18) and Constraints (19) enforce that long trains and the Inhalation Hazard train will not occupy sidings. Constraints (20) imply that if a heavy train and a Non-Schedule Adherence (NSA) train are undergoing a meet-pass event, it will not occur that the heavy train takes the siding and the NSA train holds the main track.

MOW constraints: Constraints (21 and 22) enforce that no trains use the MOW tracks during their pre-specified time windows. Note that the "entry segment of the MOW railroads" is the first segment a train will occupy if it is scheduled to enter this portion of the railroads. Likewise, the "exit segment of the MOW railroads" is the final segment a train will occupy before it leaves this portion of the railroads.

Objective value related constraints: These sets of constraints are introduced largely to prevent events outside the planning horizon from being considered in the objective function. Constraints (23 and 24) prescribe that $\alpha_{i,j} = 1$ if train i exits segment j prior to the end of the planning horizon. Constraints (25 and 26) imply that $\xi_{i,j}$ equals whichever is smaller of $x_{i,j}^{\text{exit}}$ and ε. Constraints (27) denote the delay for each train at any of its occupied segment. Constraints (28) denote the schedule adherence deviance beyond 2 hours for each train at its SA nodes. Note that in the definition of set $\mathcal{N}_i$, an exit node of a segment for a train is simply the end node of the segment from which the train leaves the segment. Constraints (29 and 30) denote the TWT deviance for late (early) arrival beyond the 4-hour window for each train. Constraints (31) denote the unpreferred track time for each train at any of its occupied segment. Note that it is sufficient to define $\gamma_{i,i',j}$ and $\lambda_{i,i',j}$ variables only for $i < i'$ as by definition $\gamma_{i',i,j} = 1 - \gamma_{i,i',j}$ and $\lambda_{i',i,j} = 1 - \lambda_{i,i',j}$. The redundant constraints involving these sequence variables can be eliminated accordingly.

3 Solution Approaches

The MILP problem is combinatorially difficult and solving it entails substantial computational efforts. Therefore, we reexamine the model and strengthen the formulation with several additional constraints without sacrificing optimality. One the other hand, obtaining the optimal solutions may not entirely justify the huge computational time required; a high quality solution usually suffices for practical purposes. Hence, we also develop several heuristic rules for variable fixing. In addition, we propose a rolling-horizon-based decomposition algorithm to "divide and conquer" the problem. Besides, big-M's are fine-tuned to tighten the formulation.

Before detailing various solution approaches, we first introduce $\underline{x}_{i,j}^{\text{exit}}$, a lower bound of $x_{i,j}^{\text{exit}}$, which will be referred to a number of times in the following text. It means the earliest possible time when train i leaves segment j and is calculated simply by summing the free flow running time on the shortest main track of every segment from the train's origin node plus the train's entry time.

$$\underline{x}_{i,j}^{\text{exit}} = b_i + \sum_{k=\min\{o_i,j\}}^{\max\{o_i,j\}} \min_{t \in \mathcal{M}_k} \{r_{i,k,t}\}, \qquad \forall i \in \mathcal{T}, \forall j \in \mathcal{B}_i \tag{40}$$

where $\mathcal{M}_k$ is the set of main tracks at segment k. In the single-track context, this lower bound is the time when the train will leave the segment if it always travels on the main track without any delay. In the double-track context, for the trains that must use crossovers at least once, this bound is strictly smaller than $x_{i,j}^{\text{exit}}$ since the lower bound naïvely assumes that a train can instantaneously "leap" from one main track to another.

3.1 Formulation Enhancement

1. Equalizing sequence variables $\lambda_{i,i'j}$ and $\gamma_{i,i'j}$ for adjacent segments

   For two geographically adjacent segments where no meet-pass events can occur, the sequence between the two trains on the two segments will remain unchanged. For instance, in the railway network provided, if train A were to occupy arc (5,6) earlier than train B does, then train A would also occupy arc (6,7) earlier. Expressed mathematically, this is

$$\lambda_{i,i',j} = \lambda_{i,i',j+1}, \gamma_{i,i',j} = \gamma_{i,i',j+1}, \qquad \forall i, i' \in \mathcal{C}_j, \forall j, j+1 \in \mathcal{G}' \tag{41}$$

   where $\mathcal{G}'$ is the set of single-track segments where no meet-pass events can occur. This includes segments 2-10, 12-22, 24-26, 41-49, 51-52 (See Appendix for the arcs affiliated to each segment).

2. No delays at intermediate nodes

   Although stopping at some intermediate nodes is permitted, there is really no incentive to do so. For example, on Main Track 0, it is indeed unnecessary to stop at node 8 for eastbound trains. A more adequate place to wait can be node 17 or node 18. Thus, inequality Constraints (5) for a number of segments can be revised to equality constraints. We do not intend to list all such segments, but note that whether a constraint can be revised is not only related to the territory, but also depends on the train's direction. Imposing zero delay on some nodes serves to eliminate duplicate scenarios, or alternative optima, thus achieving better computational performance.

3. Fixing MOW-related variables $\beta_{i,j}$

   If in Constraints (21), the lower bound of $x_{i,j}^{\text{entry}}$ (similarly calculated as the lower bound of $x_{i,j}^{\text{exit}}$) is no smaller than $\text{MOW}_j^{\text{end}}$, then by definition, $\beta_{i,j}$ must assume the value of 1. We can fix this portion of the MOW-related variables.

4. Fine-tuning big-M

   Customizing big-M for each constraint can make the formulation stronger than if big-M are assigned arbitrarily large values. We set up big-M based on the lower bound of $x_{i,j}^{\text{exit}}$. For example, In Constraints (10 and 11), big-M is given as the difference between the lower bound of $x_{i',j}^{\text{entry}}$ and that of $x_{i,j}^{\text{exit}}$ plus an allowance of 5 hours. Other big-M can be determined in a similar fashion.

3.2 Heuristic Variable Fixing Procedure

1. Sequence variable fixing for chronologically distant trains

   If two trains' lower bounds of $x_{i,j}^{\text{exit}}$ at a segment are different by more than 3 hours, then we assume the real chronological relationship between these two trains will not deviate from the lower bound relationship, and their sequence variables $\lambda_{i,i',j}$ $\gamma_{i,i',j}$ can be fixed accordingly.

2. Unattractive overtaking prohibition

   We prohibit unattractive overtakes by fixing the relevant sequence variables ($\lambda_{i,i',j}$ and $\gamma_{i,i',j}$) since some pass events are very unlikely to occur in an optimal solution. For example, if Train A and Train B are both eastbound trains, Train A is of higher priority train type, Train A's entry time is earlier than that of Train B, plus Train A's origin is to the east of that of Train B, then only in rare circumstances will Train B have the incentive to overtake Train A. In our setting, we fix the relevant sequence variables in such a way that a train always occupies any segment strictly earlier than another train moving in the same direction if its entry time is no later than another train, its type priority no lower, and its origin no farther from the destination. Note that if there is a tie between two trains in all three attributes, the sequence variables are left unfixed.

3. $\alpha_{i,j}$ fixing

   If in Constraints (23 and 24), the lower bound of $x^{\text{exit}}_{i,j}$ is larger than ε, then by definition, $\alpha_{i,j}$ must take the value of 0. While this is exact, we heuristically let $\alpha_{i,j} = 1$ if the lower bound is four hours earlier than the end time of the planning horizon ε.

3.3 Decomposition Algorithm

Motivated by the fact that chronologically distant trains are relatively independent and impact each other insignificantly, we propose a rolling-horizon-based decomposition algorithm. First, the trains are ordered by their entry time. Then, in each iteration i, we optimize a subproblem that only schedules trains whose entry time is earlier than a time threshold δ_i set for this iteration. After the subproblem's optimal solution is obtained, for the trains that arrive at their destinations prior to the threshold, all the corresponding variables (which track to take, the entry and exit time at each segment, etc) are fixed to their optimal solution values, including the sequence variables with "future trains." For those trains that have not arrived, we only fix variables corresponding to events that end no later than $\delta_i - \delta^r$, with δ^r being the rollback time. An event is simply the resource request a train has of a segment, from the train's entry to its exit. Thus, trains are dynamically dispatched in each iteration until all the trains are scheduled. This is referred to as rolling-horizon-based decomposition because in each iteration, we derive complete schedules for the trains considered as if there were no other trains to be dispatched later, but for the trains that have not arrived, we rollback for a certain period, and only the schedules prior to the time $\delta_i - \delta^r$ are actually adopted, before we progress to the next rolling horizon.

Decomposition Algorithm

Order the trains by their entry time, indexed by t.
Set the number of iterations m, indexed by i, and the threshold δ_i. Set $\delta_m = \varepsilon$.
Set $i = 1$.
DO WHILE $i \leq m$
  Solve the subproblem that dispatches trains with $b_t < \delta_i$.
  IF $i = m$, **BREAK**
  FOR all trains dispatched
    IF $x^{\text{exit}}_{t,d_t} \leq \delta_i$,
      Fix all the variables for train t.
    ELSE
      Fix all the variables for events with $x^{\text{exit}}_{t,j} \leq \delta_i - \delta^r$.
  ENDFOR
  $i = i + 1$
ENDDO
RETURN dispatching solutions

An important set of parameters that can substantially affect the performance of this generic algorithm is the time threshold, δ_i, or more essentially the interval between them, $\delta_{i+1} - \delta_i$, named the "stepsize". We develop an adaptive stepsize scheme, in which the stepsize is set to a certain duration, for example, 1 hour, at the beginning of each iteration. The optimization routine is invoked to solve the subproblem if new trains are included within the current stepsize; otherwise we directly fix variables within this stepwise without optimization, and increment the stepsize by another hour until new trains, if any, are identified.

4 Computational Results

We evaluate our problem formulation and the solution approaches with the three data sets provided. All the computational tests are performed on a PC with 2.40 GHz CPU and 4GB RAM. The implementation is coded in C++ invoking the commercial solver ILOG CPLEX 12.1 for the integer program. All the solver parameters are at their default settings except for the 1 hour limit on the CPU time.

	Decomposition		Heuristic variable fixing		Formulation enhancement		Original model	
Data Set	Obj. ($)	Time (s)	Obj. ($)	Time (s)	Obj. ($)	Time (s)	Obj. ($)	Time (s)
1	844.706	9.86	844.706	169.57	856.165	3600	867.216	3600
2	4077.65	26.91	-*	-	-	-	-	-
3	7049.25	147.17	10935.6	3600†	-	-	-	-

*: "-" implies no integer solution has been found in 1 hour.

†: "3600" means the underlying model has not been solved to optimality within the 1 hour time limit.

Table 4: Computational results of different solution approaches on the three data sets

Table 4 reports the objective value ("Obj.") in dollars and the CPU time ("Time") in seconds for the computational tests of different solution approaches on the three data sets. For a fair comparison, big-M's are already fine-tuned in the light of Subsection 3.1 and remain the same for all the solution approaches compared. As the names suggest, "Heuristic variable fixing", "Formulation enhancement", and "Original model" report the tests that apply their respective methodologies, but without decomposition. "Heuristic variable fixing" tests are based on formulation enhancement but also use heuristics. Likewise, the "Decomposition" tests embed both formulation enhancement and heuristic variable fixing procedures into the decomposition algorithm. As for the decomposition algorithm, we have performed extensive testing on the choice of the stepsize and the rollback ratio, defined as $\frac{\delta^r}{\delta_{i+1}-\delta_i}$, and report the results with the best computational performance. For Data Set 1, the stepsize is set to 4 hour, and for Data Set 2 and Data Set 3, it is 1 hour. The rollback ratio is 0.5 for Data Set 1 and Data Set 3, and 0.1 for Data Set 2. The selection of both the stepsize and the rollback ratio is a tradeoff between solution quality and computational time, and our general recommendation is that the stepsize should be chosen in such a way that there are six to seven trains within one stepsize, if at all possible; a rule of thumb for the rollback ratio is an interval between 0.1 and 0.5. The take-away message is that a bit of rollback usually benefits the solution quality as it offers a second chance to "undo the past", but too much of it backfires in terms of computational time. We find that given the stepsize, tweaking the rollback ratio within a small interval does not dramatically impact the performance.

The computational results clearly demonstrate the efficacy of our solution approaches. For Data Set 1, although within 1 hour, neither the enhanced model nor the original model can be solved to optimality, the enhanced model returns a better solution. And after one hour the optimality gap for the enhanced model is 26.5%, but is 31.6% for the original model. In fact, the enhanced model can be solved to optimality in 4525.15 seconds (some 1 hr 15 min) and the optimal solution is 844.706. This solution is exactly what the heuristic variable fixing procedure yields, but in substantially less CPU time (less than 3 minutes). The decomposition algorithm produces the same optimal solution, but even faster (less than 10 seconds). For Data Set 1, the decomposition algorithm is able to speed up the solution process by a factor of at least 400 while still obtaining the provably optimal solution. For Data Set 2, only the decomposition algorithm is able to find integer solutions within the time limit, and it only takes less than 30 seconds. For Data Set 3, the hardest instance, despite the original model and the enhanced model's continual failure to find integer solutions, the heuristic variable fixing procedure is able to identify integer solutions within the time limit, but its quality is a far cry from the solution produced by the decomposition algorithm (more than 50% worse off). Furthermore, it only takes the decomposition algorithm less than 3 minutes to find the solution. Note that without decomposition, even the first feasible integer solution cannot be found until more than ten minutes later.

5 Concluding Remarks

In this study, the Movement Planner Problem is tackled with a mixed-integer programming formulation that explicitly captures many real-life constraints. Based on the mathematical model, the proposed decomposition algorithm, with the embedded formulation enhancement and variable fixing procedures, accelerates the solution process by a factor of over 400 for Data Set 1 and is able to deliver high quality solutions efficiently for Data Set 2 and Data Set 3, meeting the operational requirements of real-time train dispatching. Future research following the decomposition algorithm can be focused on a more sophisticated and intelligent selection of the stepsize and rollback ratio. How these parameters affect the dynamics of the solution quality and computational time can be investigated to attain better understanding of the problem characteristics.

References

[Mu and Dessouky(2011)] Mu, S., Dessouky, M., August 2011. Scheduling freight trains traveling on complex networks. Transportation Research Part B: Methodological 45 (7), 1103–1123.

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Appendices

A A List of Segments

segment ID	arcs		segment ID	arcs			
0	(0,1)	(0,2)	27	(41,42)	(40,43)		
1	(1,3)	(2,4)	28	(42,44)	(43,45)		
2	(3,5)	(4,5)	29	(44,46)	(44,47)	(45,47)	(45,46)
3	(5,6)		30	(46,48)	(47,49)		
4	(6,7)		31	(49,50)	(49,51)	(48,48)	
5	(7,8)		32	(50,52)	(51,53)	(48,54)	
6	(8,10)		33	(52,55)	(53,55)	(54,54)	
7	(10,12)		34	(55,56)	(54,54)		
8	(12,13)		35	(54,57)	(54,58)	(56,58)	(56,57)
9	(13,14)		36	(57,59)	(58,60)		
10	(14,15)	(14,16)	37	(59,61)	(60,62)		
11	(15,17)	(16,18)	38	(61,63)	(62,64)		
12	(17,19)	(18,19)	39	(63,65)	(64,66)		
13	(19,20)		40	(65,67)	(66,68)		
14	(20,21)		41	(67,69)	(68,69)		
15	(21,22)		42	(69,70)			
16	(22,25)		43	(70,71)			
17	(25,26)		44	(71,72)			
18	(26,27)		45	(72,74)			
19	(27,28)		46	(74,76)			
20	(28,30)		47	(76,77)			
21	(30,31)		48	(77,78)			
22	(31,32)	(31,33)	49	(78,79)	(78,80)		
23	(32,34)	(33,35)	50	(79,81)	(80,82)		
24	(34,36)	(35,36)	51	(81,38)	(82,38)		
25	(36,37)		52	(38,39)			
26	(37,40)	(37,41)					

Table 5: Arcs and the segments to which they are assigned

B Solution Statistics

Data Set 1	Total Cost:	$844.706								
	Delay Time (sec)	Arrive at (sec)	Arrive Time (sec)	Required SA (sec)	SA diff (sec)	SA Penalty (> 2 hrs) (sec)	Required TWT (sec)	TWT diff (sec)	TWT Penalty (outside 4 hr window) (sec)	Unpreferred Track (sec)
Train A1	0									0
		node 37	2398.500	3600	1201.500	0				
		node 39	4945.500	7800	2854.500	0	5400	454.500	0	
Train A2	0									0
		node 37	6124.740	7800	1675.260	0				
		node 39	8805.789	12000	3194.211	0	9000	194.211	0	
Train B2	0									0
		node 37	13621.760	15600	1978.240	0				
		node 39	16618.235	19800	3181.765	0	16800	181.765	0	
Train C1	0									0
		node 37	17598.000	19200	1602.000	0				
		node 39	20994.000	24600	3606.000	0	21600	606.000	0	
Train C2	0									0
		node 37	25026.430	27000	1973.570	0				
		node 39	28665.000	33000	4335.000	0	28800	135.000	0	
Train E1	54.980									0
		node 37	34245.980	36000	1754.020	0				
		node 39	39339.980	44400	5060.020	0	39000	-339.980	0	
Train B3	0									0
		node 37	38725.540	40200	1474.430	0				
		node 39	41619.886	45000	3380.114	0	42000	380.114	0	
Train B1	2766.430									502.597
		node 37	12889.916	12600	-289.916	0				
		node 0	17666.104	17400	-266.104	0	13800	-3866.104	0	
Train D2	1245.990									680.440
		node 37	20796.099	21600	803.901	0				
		node 0	25500.490	28200	2699.510	0	23400	-2100.490	0	
Train D1	2330.570									737.143
		node 37	28953.429	29400	446.571	0				
		node 0	33522.000	36600	3078.000	0	31200	-2322.000	0	
Train D3	1483.040									631.837
		node 37	33325.710	35400	2074.290	0				
		node 0	39433.660	41400	1966.340	0	37200	-2233.660	0	
Train F1	39.980									0
		node 37	63124.300	57600	O.P.H.*					
		node 0	74088.900	75000	O.P.H.*		63000	O.P.H.*		

*: O.P.H. stands for outside planning horizon

Table 6: Solution statistics for Data Set 1

Data Set 2	Total Cost:	$4077.65								
	Delay Time (sec)	Arrive at (sec)	Arrive Time (sec)	Required SA (sec)	SA diff (sec)	SA Penalty (> 2 hrs) (sec)	Required TWT (sec)	TWT diff (sec)	TWT Penalty (outside 4 hr window) (sec)	Unpreferred Track (sec)
Train A1	20.308									940.500
		node 37	2418.810	3600	1181.190	0				
		node 39	4938.808	7800	2861.192	0	5400	461.192	0	
Train E1	3230.730									0
		node 37	6418.235	6600	181.765	0				
		node 39	12926.180	14400	1473.820	0	9600	-3326.180	0	
Train D2	373.269									0
		node 37	1855.380	3600	1744.620	0				
		node 39	6147.115	9600	3452.885	0	6600	452.885	0	
Train C2	0									0
		node 37	198.000	0	-198.000	0				
		node 39	3594.000	5400	1806.000	0	3600	6	0	
Train B1	1509.010									264.706
		node 37	6877.060	4800	-2077.060	0				
		node 39	11520.190	9000	-2520.190	0	9600	-1920.190	0	
Train A2	744.601									0
		node 37	10598.195	11400	801.805	0				
		node 39	13463.022	15600	2136.978	0	12600	-863.022	0	
Train A3	462.244									225.000
		node 37	17736.966	18000	263.034	0				
		node 39	20608.744	22200	1591.256	0	19800	-808.744	0	
Train F1	782.830									0
		node 37	26029.300	29400	3370.700	0				
		node 39	33769.193	41400	7630.807	0	33600	-169.193	0	
Train B2	1180.760									281.250
		node 37	32978.880	26400	-6578.880	0				
		node 39	36308.882	31200	-5108.882	0	35400	-908.882	0	
Train C1	3679.980									0
		node 37	40031.900	26400	-13631.900	-6431.900				
		node 39	43297.300	31800	O.P.H.*		39000	O.P.H.*		
Train D1	775.750									0
		node 37	41268.250	43800	2531.750	0				
		node 39	57984.000	51000	O.P.H.*		44400	O.P.H.*		
Train E2	4681.430									0
		node 37	3088.740	1800	-1288.740	0				
		node 0	11995.710	10800	-1195.710	0	7200	-4795.710	0	
Train E3	6647.890									884.571
		node 37	13415.890	9000	-4415.890	0				
		node 0	19638.750	18000	-1638.750	0	12000	-7638.750	0	
Train B3	3997.710									552.857
		node 37	11002.406	11400	397.594	0				
		node 0	14791.290	16800	2008.710	0	10800	-3991.290	0	
Train F2	6353.610									0
		node 37	26730.210	30000	3269.790	0				
		node 0	45310.800	51000	O.P.H.*		36000	O.P.H.*		
Train C3	2643.710									552.857
		node 37	24873.710	20400	-4473.710	0				
		node 0	28763.000	25800	-2963.000	0	25200	-3563.000	0	
Train E4	1063.450									0
		node 37	27881.120	30000	2118.880	0				
		node 0	33374.622	37800	4425.378	0	32400	-974.622	0	
Train A4	5289.020									465.564
		node 37	35962.110	31800	-4162.110	0				
		node 0	45658.100	36600	O.P.H.*		39000	O.P.H.*		

*: O.P.H. stands for outside planning horizon

Table 7: Solution statistics for Data Set 2

Data Set 3	Total Cost:	$7049.25								
	Delay Time (sec)	Arrive at (sec)	Arrive Time (sec)	Required SA (sec)	SA diff (sec)	SA Penalty (> 2 hrs) (sec)	Required TWT (sec)	TWT diff (sec)	TWT Penalty (outside 4 hr window) (sec)	Unpreferred Track (sec)
Train B3	1718.280									252.809
		node 37	4745.913	-3000	-7745.913	-545.913				
		node 39	7841.310	1800	-6041.310	0	6000	-1841.310	0	
Train C2	2235.980									0
		node 37	5662.410	0	-5662.410	0				
		node 39	9300.980	6000	-3300.980	0	7200	-2100.980	0	
Train D1	2764.440									0
		node 37	6454.440	4200	-2254.440	0				
		node 39	10372.901	10200	-172.901	0	7800	-2572.900	0	
Train E2	1083.000									450.000
		node 37	16191.000	18000	1809.000	0				
		node 39	22602.000	26400	3798.000	0	21000	-1602.000	0	
Train E3	1732.750									0
		node 37	21900.935	21000	-900.935	0				
		node 39	27235.480	28800	1564.520	0	24000	-3235.480	0	
Train F2	9956.160									0
		node 37	44141.200	37200	O.P.H.*					
		node 39	55150.500	51000	O.P.H.*		42000	O.P.H.*		
Train A5	5112.000									0
		node 37	36697.500	32400	-4297.500	0				
		node 39	39244.500	36600	-2644.500	0	34200	-5044.500	0	
Train E4	9801.040									0
		node 37	51082.800	40800	O.P.H.*					
		node 39	56742.800	49800	O.P.H.*		43800	O.P.H.*		
Train D2	5039.500									0
		node 37	45324.900	42000	O.P.H.*					
		node 39	57389.000	48000	O.P.H.*		43800	O.P.H.*		
Train A1	0									0
		node 37	967.500	2400	1432.500	0				
		node 39	3514.500	6000	2485.500	0	4200	685.500	0	
Train B2	0									409.091
		node 39	2981.250	4800	1818.750	0	3000	18.750	0	
Train E1	2610.570									982.857
		node 37	9792.571	10200	407.429	0				
		node 0	16222.000	19200	2978.000	0	13200	-3022.000	0	
Train F1	8977.480									1769.14
		node 37	22513.480	18000	-4513.480	0				
		node 0	33478.050	34800	1321.950	0	23400	-10078.050	0	
Train A3	154.286									730.286
		node 37	21054.860	22800	1745.140	0				
		node 0	23796.000	27000	3204.000	0	24000	204.000	0	
Train B4	7059.000									552.857
		node 37	29638.790	16800	-12838.790	-5638.790				
		node 0	39915.430	21600	-18315.430	-11115.430	32400	-7515.430	0	
Train C3	4595.950									903.673
		node 37	37560.796	30000	-7560.796	-360.796				
		node 0	41573.100	36000	-5573.100	0	37200	-4373.100	0	
Train A4	1612.200									465.564
		node 37	37086.790	33600	-3486.790	0				
		node 0	40459.710	38400	-2059.710	0	39000	-1459.710	0	
Train A2	1958.410									442.286
		node 37	1656.000	2400	744.000	0				
		node 0	6049.550	6600	550.450	0	4200	-1849.550	0	
Train C1	6387.520									0
		node 0	9754.370	6000	-3754.370	0	4200	-5554.370	0	
Train B1	0									0
		node 0	1496.104	1800	303.896	0	1200	-296.104	0	

*: O.P.H. stands for outside planning horizon

Table 8: Solution statistics for Data Set 3