Evaluating the effect of environmental regulations on a closed-loop supply chain network: a variational inequality approach

E. Allevi[1], A. Gnudi[2], I.V. Konnov[3], and G. Oggioni[4]

Abstract

Global climate change has encouraged international and regional adoption of pollution taxes and carbon emission reduction policies. Europe has taken the leadership in environmental regulations by introducing the European Union Emissions Trading System (EU-ETS) in 2005 and by developing and promoting a set of policies destined to lower carbon emissions from transport sectors. These environmental policies have significantly affected the production choices of European energy and industrial sectors.

In this paper, we propose a closed-loop supply chain network model that includes raw material suppliers, manufacturers, consumers, and recovery centers. The objective of this paper is to formulate and optimize the equilibrium state of this closed-loop supply chain network assuming that manufacturers are subject to the EU-ETS and a carbon tax is imposed on truck transport. The model is optimized and solved by using the theory of variational inequalities.

The developed model is able to capture carbon regulation, recycling, transportation and technological factors within a unified framework and to capture their effects on production and CO_2 emission generation.

Our analysis shows that the combined application of the EU-ETS at manufacturers' tier and the carbon tax on truck transport implies additional costs for producers that reduce the production of goods. This has a positive outcome for environment since CO_2 emissions reduce as well. An increase of the efficiency level of the recycling process leads to an increase of the reusable raw material in the reverse supply chain. However, this production of reusable raw material can be negatively affected when a carbon tax is imposed on transport between recovery centers and manufacturers. Finally, the distance between couple of CLSC tiers plays a very important role. The lower is the distance covered by vehicles, the higher is the production of goods and the lower is the CO_2 emitted.

Keywords: Closed-loop supply chain network, environmental regulations, variational inequality approach.

[1]Department of Economics and Management, University of Brescia, via S. Faustino, 74/b, Brescia 25122, Italy. Email: elisabetta.allevi@unibs.it

[2]Department of Management, Economics and Quantitative Methods, University of Bergamo, via dei Caniana, 2, Bergamo 24127, Italy. Email: adriana.gnudi@unibg.it

[3]Department of System Analysis and Information Technologies, Kazan Federal University, ul. Kremlevskaya, 18, Kazan 420008, Russia. Email: konn-igor@yandex.ru

[4]**Corresponding author**. Department of Economics and Management, University of Brescia, via S. Faustino, 74/b, Brescia 25122, Italy. Phone: +390302988522. Email: giorgia.oggioni@unibs.it

1 Introduction

In recent years, a growing attention has been given to the impacts of climate change and carbon emissions on environment. At world-wide level, the Kyoto Protocol has been introduced in order to mitigate these effects. Other environmental policies have been applied, but only at regional level. In 2005, Europe introduced the European Union Emissions Trading System (EU-ETS) that is the first and largest cap-and-trade scheme applied at international level and covers more than half of the European annual carbon emissions. In particular, the EU-ETS imposes a cap on CO_2 emissions generated by power plants and industrial installations. Each ton of CO_2 emissions has to be covered by an allowance whose price is defined on a dedicated market. The EU-ETS has allowed Europe to maintain the Kyoto Protocol targets and now represents the main tool to achieve the ambitious target of 20% emission reductions by 2020 with respect to the 1990 level. This market has several objectives, which are strictly related to the Energy Roadmap 2050 strategic goals identified by the European Union (EU) with the aim of mitigating climate change and promoting a low carbon economy. The most ambitious EU target is an 80% decarbonization of the European energy systems by 2050 thanks to investments in RES, in the transport sector, and in efficient technologies.

The EU-ETS was initially subdivided in two distinct phases as indicated by Directive 2003/87/EC, but a third phase has been added by Directive 2009/29/EC. In the first two phases, that respectively covered the periods 2005-2007 and 2008-2012, allowances were mostly grandfathered to the involved sectors, but this created some economic distortions as indicated in [21] and [24]. The third phase, started in 2013, substantially modifies the EU-ETS by enlarging the involved sectors and significantly reducing the amount of grandfathered allowances. This is also the tendency of the announced fourth phase that is currently under discussion.[5] Among the sector currently involved in the EU-ETS there is also aviation.

According to European Commission[6], transport is the second biggest greenhouse gas emitting sector after energy and is responsible for around a quarter of European greenhouse gas emissions. Road transport significantly contributes to CO_2 emissions as well as maritime and aviation sectors. Since carbon emissions have been increasing for most of transport sectors, Europe has so far applied and intends to enhance the set of policies destined to lower emissions from road transport. These include strategies to reduce emissions from cars and vans, such as emission targets for new vehicles, and to limit fuel consumption and CO_2 emissions of heavy duty vehicles.

Production processes and transport are strictly related each others and are important tiers of a supply chain network. In addition to transport, there exist several factors that significantly affect the supply chain tiers and therefore the product flows at equilibrium. These can be classified into technological and environmental factors. The technological factors are represented by the relations between inputs and outputs of each couple of

[5]See the Proposal for a Directive of the European Parliament and of the council amending Directive 2003/87/EC to enhance cost-effective emission reductions and low-carbon investments available at http://ec.europa.eu/clima/policies/ets/revision/documentation_en.htm

[6]See http://ec.europa.eu/clima/policies/transport/index_en.htm

tiers. The environmental factors are strictly related to the growing recognition that issues of environmental pollution accompanying industrial development should be addressed simultaneously in the operational process of supply chain management. This increasing consciousness on environmental concerns can be reflected in the application of two groups of policies that can be either applied simultaneously or in a separated way. The first group of policies is related the legislative area and it is represented by regulations that limit the carbon emission generated in the whole supply chain. The second group of policies consists in the recycling of used products that is applied to make the production system more efficient and sustainable. In particular, some of the resources used in the production processes retain their basic physical and chemical properties during use and under the proper conditions can be recycled and reused. Businesses and organizations are increasingly modifying their supply chain models to increase sustainability in both their products and their processes. This leads to the development of closed-loop supply chain (CLSC) network where recovery centers are included in the production process. Improving the network of supply chains remains a challenging problem both at the academic research and the practical levels. From a modeling point of view, these environmental factors can be represented by the sorting and recycling capability of recovery centers in a CLSC network.

1.1 Paper aims

A CLCS network model is designed and managed to explicitly consider the reverse and forward supply chain activities over the entire life cycle of the product (see, e.g., [7],[8], [20], [27]). The CLSC system aims at maximizing the product value over its entire lifecycle with dynamic recovery of product parts over time. In line with European targets, product recovery and reuse improve industrial efficiency both from an economical and an environmental point of view. Moreover, the recycling process of used products (such as papers, glass, building wastes, electric, and electronic equipment) in a CLSC leads to a waste reduction and to an increasing awareness of climate change problems.

Parallel to the development of CLCS models that mainly focus on the recycling aspect, other academics concentrate their attention on the legislative intervention applied in a supply chain network to reduce carbon emissions. This has contributed to the develop of green-supply chain management (GrSCM) models (see [4], [9], [19], [23], [25], [28], [29]).

In this paper, we integrate all these environmental issues in one single model and we investigate how the joint application of carbon regulation (legislative intervention), recycling, technological and transport factors impact on the product flows of a CLSC network. More precisely, this paper aims at investigating the following issues, taking into account the technological and the material balance constraints that characterize each tier of the supply chain network:

Q1: How the application of carbon reduction policies both at production and transportation levels can affect the product flows and carbon emission generation in a CLSC;

Q2: How the application of environmental policies can affect the recycling process in the CLSC;

Q3: Whether the transport distance between couples of CLSC tiers can affect product flows when carbon regulations are applied.

To this scope, we develop a CLSC model where manufacturing processes are subject to a restrictive EU-ETS, namely allowances are assumed to be fully auctioned.[7] Moreover, in order to account for the European Commission's willingness to regulate carbon emissions from heavy-duty vehicles, we impose a carbon tax on CO_2 generated by trucks.

Unlike the papers quoted above, our model is based upon variational inequality theory, which facilitates the formulation of equilibrium problems that describe the interactions of several agents whose choices are subject to technological, economical and environmental constraints. The spatial control of such systems requires to consider not only the reactions of separate markets (or agents), but also technical constraints such as production capacity and mass balance constraints. Variational inequalities were developed and adopted to study various kinds of equilibria such as spatial equilibrium models ([14]), financial networks (see, e.g., [1], [15]) and transportation networks (see, e.g., [2], [22]). In addition to these applications, there exist papers that study supply chain management such as [16] and [31]. The variational inequality theory was also applied to analyze the effects of the application of a carbon tax in a power supply chain network (see [30]) or to model CLCS network for the electrical and electronic equipment waste Directive applied in Europe (WEEE Directive 2002/96/EC) as in [10], [17] and [26] or to develop sustainable supply chain models as in [18].

However, to the best of our knowledge, it is the first time that that variational inequality approach is applied to develop a model that takes into carbon regulation, recycling, transportation and technological factors within a unique framework. Moreover, the existing papers in the field of variational inequalities do not investigate the impacts of the combined application of the EU-ETS on manufacturing and of a carbon tax on transport in a CLCS network as we do (see Figure 1 for the considered environmental factors). We also notice that variational inequality models admit many efficient solution methods, which can be easily implemented (see [5] and [13]).

1.2 Contributions

Our model takes into account several issues (carbon regulation, recycling, endogenous demand) in a unified variational inequality model framework, while the recent literature usually threats them separately. The main contributions of this work are the following:

1. To analyze the application of carbon regulation both on manufacturing (EU-ETS) and transport and to evaluate the effect of their combined application on product flows;

2. To propose a possible policy, still under discussion at European level, that can be applied to limit CO_2 emissions of heavy-duty vehicles;

[7] Note that this assumption can be easily adjusted when modeling sectors that still receive free allowances in the third EU-ETS phase. For more details, see http://ec.europa.eu/clima/policies/ets/index_en.htm

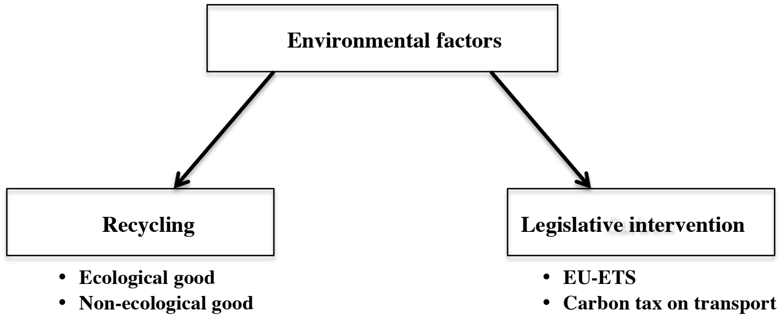


Figure 1: Considered environmental factors

3. To investigate the impacts of the joint application of legislative intervention and recycling in a complex CLSC network;

4. To develop an adequate variational inequality model of a CLSC subject to carbon reduction regulations.

The paper is organized as follows. Section 2 describes the proposed CLSC network under environmental regulations whose model is presented in Section 3. Section 4 derives the equilibrium conditions of the CLSC network model and illustrates the qualitative properties of the corresponding variational inequality model. Section 5 describes the algorithm used to implement the variational inequality model, while Section 6 is devoted to the descriptions of the results of the numerical experiments. Finally, Section 7 concludes the paper with some final remarks.

2 A CLSC network under environmental regulations

This paper aims at analyzing different environmental issues in one single CLSC network model. In particular, we combine the recycling aspects that are typical of a CLSC with environmental policies applied at different tiers of the supply chain. As indicated above, we assume that manufacturers are subject to the EU-ETS system and a carbon tax is imposed on CO_2 emissions generated by heavy-duty vehicles (trucks).

We propose a CLCS network where raw material suppliers, manufacturers and demand markets participate to the forward logistics; while, in the reverse logistics, recovery centers collect used but recyclable products from demand markets. These used products are thus dissembled and transformed in reusable raw materials that are sold back to manufacturers. Figure 2 illustrates this CLSC network.

We assume that, in this CLSC network, manufacturers produce two goods with the same end-use that only differ for the production processes and the raw material used. More precisely, one good is produced using reusable raw materials that manufacturers buy from recovery centers after the recycling process, while the other is produced using virgin (new) raw materials that manufacturers purchase from raw material suppliers. In

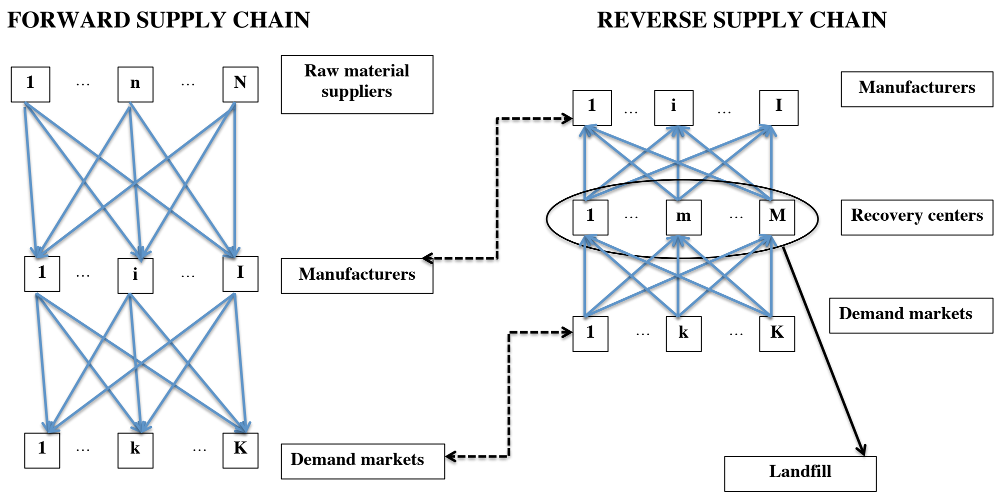


Figure 2: CLSC network

the following, we label the first good as “ecological" or, in a simpler way, as “eco”, while the second is denoted as “non-ecological” or, for the sake of simplicity, as “neco”.

We assume that, in the forward logistics, manufacturers produce two goods with the same end-use that differ for the production processes and the raw material used. More precisely, one good is produced using reusable raw materials that manufacturers buy from recovery centers after the recycling process, while the other is produced using virgin (new) raw materials that manufacturers purchase from raw material suppliers. We label the first good as “ecological” or, in a simpler way, as “eco”, while the second is denoted as “non-ecological” or, for the sake of simplicity, as “neco”.

Since the production processes applied to produce ecological and non-ecological goods are different, we assume that the production costs and thus the prices applied to the two goods are not the same. However, besides these production differences, the two goods are similar and can be consumed in the same way. An example of this can be represented by the recycled paper and the not-recycled paper products which have the same end-use but are characterized by different production processes and costs.

Manufacturers sell their goods to consumers located in different demand markets. Note that both manufacturers and consumers are involved in the reverse logistics in the following way. Recovery centers collect used but recyclable products at demand markets and transform them in reusable raw materials that are then purchased by manufacturers. In the transformation process operated in the recovery center, it may happen that part of the used products could not be recyclable. These useless disassembled materials are sent directly to the landfill.

The raw material suppliers and the recovery centers respectively sell virgin and reusable raw materials to manufacturers. It is also assumed that agents in one tier of this CLSC act independently of agents operating in another tier of the same supply chain. Moreover, inside their own tier, agents are assumed to operate in a non-cooperative fashion.

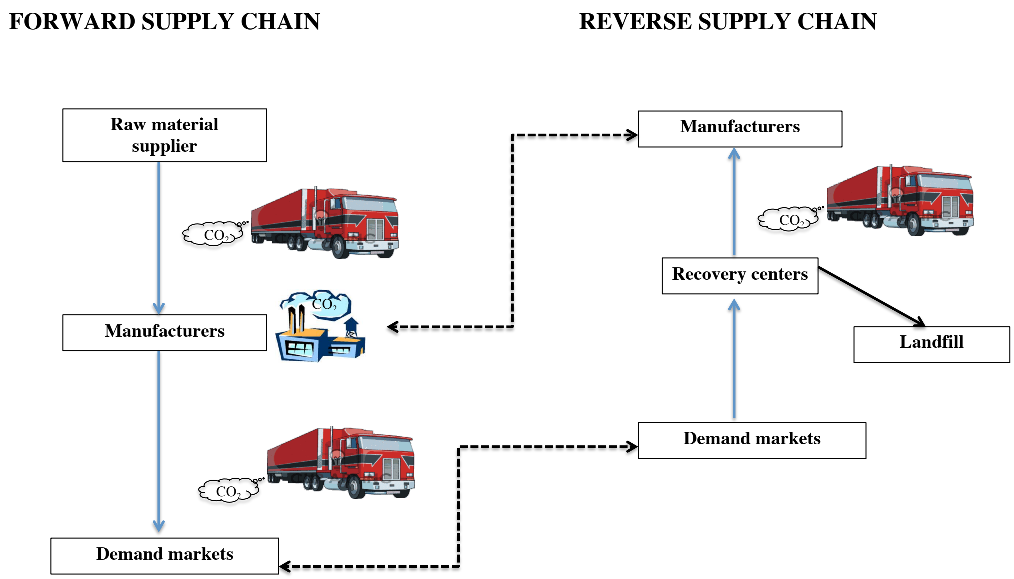


Figure 3: Environmental regulations applied in the CLSC network

As indicated above and here depicted in Figure 3, we assume that production activity at manufacturers' level is subject to the EU-ETS, implying a carbon charge to manufacturers that have to cover each ton of CO_2 emitted through an allowance. In addition, in order to model the European Commission's willingness to regulate carbon emissions from heavy-duty vehicles, we impose a carbon tax on CO_2 generated by trucks during the transport phase. Following [19], we suppose that trucks are only used in the following phases:

- transferring of virgin raw materials from suppliers to manufacturers (forward logistics);
- transferring of ecological and non-ecological goods from manufacturers to consumers at demand markets (forward logistics);
- transferring of reusable raw materials from recovery centers to manufacturers (reverse logistics).

As in Paksoy et al. (2011), we assume that transfers of used product from demand markets to recovery centers and of useless materials from recovery centers to landfill are conducted with vans of smaller dimensions that are not environmentally regulated.

3 Modeling the CLSC network under environmental regulations

In this section, we develop the CLSC network model with raw material suppliers, manufacturers, demand markets, and recovery centers as indicated in Figure 1. We first focus on the forward logistics and we optimize the behaviour of raw material suppliers, manufacturers, and consumers at demand markets. We then consider the reserve logistics and we describe the optimization problem solved by recovery centers and the role played by demand markets and manufacturers in the recycling process. The optimality conditions of each of the agents' problem are derived using the variational inequality theory. Finally, Section 4 defines the equilibrium conditions of the whole CLSC network model.

3.1 Notation

Indices

- N: number of raw material suppliers in the CLSC network, $n = \{1, ..., N\}$;
- V: number of virgin raw materials in the CLSC network, $v = \{1, ..., V\}$;
- R: number of reusable raw materials in the CLSC network, $r = \{1, ..., R\}$;
- I: number of manufacturers in the CLSC network, $i = \{1, ..., I\}$;
- K: number of demand markets in the CLSC network, $k = \{1, ..., K\}$;
- M: number of recovery centers in the CLSC network, $m = \{1, ..., M\}$;
- T: number of trucks in the CLSC network, $t = \{1, ..., T\}$.

Parameters: unit transportation costs in the forward chain

- ρ_{vnit}: unit truck transportation cost of virgin raw material v during the transportation from supplier n to manufacturer i using truck t;
- ρ_{rmit}: unit truck transportation cost of reusable raw material r during the transportation from recovery center m to manufacturer i using truck t;
- ρ_{ikt}: unit truck transportation costs of truck t during the transportation from manufacturer i to demand market k.

Parameters: unit transportation costs in the reverse chain (faced by recovery centers)

- ρ_{km}: unit transportation cost of used product from demand market k to recovery center m;
- ρ_m: unit transportation cost of useless disassembled materials from recovery center m to landfill.

Parameters: transportation capacity in the forward chain

- z_{nit}: transportation capacity of truck t from supplier n to manufacturer i;
- z_{mit}: transportation capacity of truck t from recovery center m to manufacturer i;
- z_{ikt}: transportation capacity of truck t from manufacturer i to demand market k;

Parameters: percentage rates in the reverse chain

- β_{rm}: percentage of reusable raw material r that recovery center m is able to extract from used product collected in the demand markets. Note that $(1 - \sum_{r=1}^{R} \beta_{rm})$ represents the proportion of useless materials got in the transformation process of used product that recovery center m is not able to recycle and thus it sends to landfill;
- $\bar{\beta}_m$: percentage of useless materials got in the transformation process of used product that recovery center m is not able to recycle and thus it sends to landfill. Note that $\bar{\beta}_m = 1 - \sum_{r=1}^{R} \beta_{rm}$;
- θ_k^{eco}: return ratio of the ecological good used in demand market k;
- θ_k^{neco}: return ratio of non-ecological good used in demand market k;

Parameters: subsidies

- s_m: unit of subsidy received by recovery center m for recycling used product collected at demand markets;

Parameters: landfill costs

- ω_m: cost per unit of useless disassembled material that recovery center m send to landfill.

Parameters: CO_2 regulations

- α: carbon tax on trucks;
- τ_t: carbon emission factor of truck t;
- p^{CO_2}: carbon price;
- ϕ_i^{eco}: carbon emission factor of manufacturer i for producing the ecological good;
- ϕ_i^{neco}: carbon emission factor of manufacturer i for producing the non-ecological good;

Parameters: output production capacity constraints

- $\bar{Q}_i^{eco}$: manufacturer i's production capacity of ecological good;
- $\bar{Q}_i^{neco}$: manufacturer i's production capacity of non-ecological good;

Additional parameters

- x_{ni}: distance (in Km) from supplier n to manufacturer i;
- x_{ik}: distance (in Km) from manufacturer i to demand market k;
- x_{mi}: distance (in Km) from recovery center m to manufacturer i;
- x_{km}: distance (in Km) from demand market k to recovery center m;
- x_m: distance (in Km) from recovery center m to landfill;

Variables: forward chain

- q_{rmit}: nonnegative amount of reusable raw material r that recovery center m provides to manufacturer i with truck t to produce the ecological good. We group the shipment of all the reusable raw materials into the column vector $Q^R \in \Re_+^{RMIT}$;
- q_{vnit}: nonnegative amount of virgin raw material v that supplier n provides to manufacturer i with truck t to produce the non-ecological good. We group the shipment of all the virgin raw materials into the column vector $Q^V \in \Re_+^{VNIT}$;
- q_{ikt}^{eco}: nonnegative amount of ecological good that manufacturer i sells and transfers to demand market k with truck t. We group the shipment of all the ecological goods from all manufacturers to all demand markets into the column vector $Q^{eco} \in R_+^{IKT}$;
- q_{ikt}^{neco}: nonnegative amount of non-ecological good that manufacturer i sells and transfers to demand market k with truck t. We group the shipment of all the non-ecological goods from all manufacturers to all demand markets into the column vector $Q^{neco} \in R_+^{IKT}$;

Variables: reserve chain

- q_{km}: nonnegative amount of recycling output collected in demand market k and sent to recovery center m. We group the volume of recycling products between all demand markets and recovery centers into the column vector $Q \in R_+^{KM}$.

Note that in the following variables with "*" indicate equilibrium solutions.

3.2 Raw material suppliers' behaviour and their optimality conditions

In the forward CLSC, suppliers provide virgin raw materials to manufacturers using trucks. Let c_{vn} denote the procurement cost of virgin raw material v faced by supplier n, which is a continuous and convex function and depends on the virgin raw material quantity. In particular, one has:

$$c_{vn} = c_{vn}\left(\sum_{i=1}^{I}\sum_{t=1}^{T} q_{vnit}\right) \quad \forall v,\ \forall n \tag{1}$$

The total profits that each raw material supplier aims at maximizing are given by:

$$Z_n = \sum_{v=1}^{V}\sum_{i=1}^{I}\sum_{t=1}^{T} p_{vni}^{*} \cdot q_{vnit} - \sum_{v=1}^{V} c_{vn}\left(\sum_{i=1}^{I}\sum_{t=1}^{T} q_{vnit}\right) - \sum_{v=1}^{V}\sum_{i=1}^{I}\sum_{t=1}^{T} x_{ni} \cdot \rho_{vnit} \cdot q_{vnit} + \tag{2}$$

$$-\alpha \cdot \sum_{i=1}^{I}\sum_{t=1}^{T} \tau_t \cdot x_{ni} \cdot \sum_{v=1}^{V} q_{vnit}$$

subject to

$$z_{nit} - \sum_{v=1}^{V} q_{vnit} \geq 0 \quad \forall n, \forall i, \forall t \qquad (\gamma_{nit}) \tag{3}$$

$$q_{vnit} \geq 0 \quad \forall v, \forall n, \forall i, \forall t \tag{4}$$

Profits (2) result from the difference between the revenues ($\sum_{v=1}^{V}\sum_{i=1}^{I}\sum_{t=1} p^*_{vni} \cdot q_{vnit}$) accruing from selling virgin raw materials q_{vnit} to manufacturers at optimal price p^*_{vni} and the costs represented by the raw material procurement burdens ($\sum_{v=1}^{V} c_{vn}(\sum_{i=1}^{I}\sum_{t=1}^{T} q_{vnit})$), the truck transportation charges ($\sum_{v=1}^{V}\sum_{i=1}^{I}\sum_{t=1}^{T} x_{ni} \cdot \rho_{vnit} \cdot q_{vnit}$) and the carbon tax related to transport ($\alpha \cdot \sum_{i=1}^{I}\sum_{t=1}^{T} \tau_t \cdot x_{ni} \cdot \sum_{v=1}^{V} q_{vnit}$). This tax α is imposed on each ton of CO_2 emitted by trucks when transferring virgin raw material from suppliers to manufacturers. The total amount of emission generated depends on the emission factor of each truck τ_t and is proportional to the quantity of raw materials transported ($\sum_{v=1}^{V} q_{vnit}$) and the covered distance (x_{ni}). Note that virgin raw material transportation is limited by constraint (3) and the non-negativity constraint (4) is imposed on variable q_{vnit}.

Let assume that virgin raw material suppliers compete in a non-cooperative fashion. The optimality conditions for all suppliers n can be thus described simultaneously using the following variational inequality: determine the solution $(Q^{V^*}, \gamma^*) \in \Re_+^{VNIT+NIT}$ satisfying:

$$\sum_{n=1}^{N}\sum_{v=1}^{V}\sum_{i=1}^{I}\sum_{t=1}^{T}\left[-p^*_{vni} + \frac{\partial c_{vn}\left(\sum_{i=1}^{I}\sum_{t=1}^{T} q^*_{vnit}\right)}{\partial q_{vnit}} + \rho_{vnit} \cdot x_{ni} + \ \alpha \cdot x_{ni} \cdot \tau_t + \gamma^*_{nit}\right] \times [q_{vnit} - q^*_{vnit}] + \tag{5}$$

$$+ \quad \sum_{n=1}^{N}\sum_{i=1}^{I}\sum_{t=1}^{T}\left[z_{nit} - \sum_{v=1}^{V} q^*_{vnit}\right] \times [\gamma_{nit} - \gamma^*_{nit}] \geq 0$$

$$\forall (Q^V, \gamma) \in \Re_+^{VNIT+NIT}$$

Note that γ_{nit} is the Lagrangian multiplier associated with constraint (3), while $\gamma \in \Re_+^{NIT}$.

3.3 Manufacturers' behaviour and their optimality conditions

In the forward CLSC, manufacturers produce both the ecological and the non-ecological goods that have the same end-use, but differ for the production processes and the raw material used. The ecological good is produced using reusable raw materials that are provided by recovery centers after the recycling process, while the the non-ecological is produced using virgin raw materials that manufacturers buy from raw material suppliers. Since the production processes applied to produce ecological and non-ecological goods are different, we assume that the production costs and thus the prices applied to the two goods are not the same. However, besides these production differences, the two goods are similar and can be consumed in the same way. We therefore introduce the following function defining the manufacturing costs.
Let c^{eco}_{ik} denote the manufacturing cost faced by manufacturer i for producing the ecological good q^{eco}_{ikt} using reusable raw material that is sold to demand market k. This cost is a

continuous and convex function and depends on q_{ikt}^{eco}. In particular, one has:

$$c_{ik}^{eco} = c_{ik}^{eco}\left(\sum_{t=1}^{T} q_{ikt}^{eco}\right) \quad \forall i,\ \forall k \tag{6}$$

Let c_{ik}^{neco} denote the manufacturing cost faced by manufacturer i for producing the non-ecological good q_{ikt}^{neco} using virgin raw materials that is sold to demand market k. This cost is a continuous and convex function and depends on q_{ikt}^{neco}. In particular, one has:

$$c_{ik}^{neco} = c_{ik}^{neco}\left(\sum_{t=1}^{T} q_{ikt}^{neco}\right) \quad \forall i,\ \forall k \tag{7}$$

We also assume that manufacturers face additional costs to further refine the reusable raw materials that they receive from recovery center. These costs are defined by the continuous and convex function c_{ri} that depends on the quantity of reusable raw material q_{rmit} that manufacturer i buy from recovery center m (see Section 3.5). In particular, one has:

$$c_{ri} = c_{ri}\left(\sum_{m=1}^{M}\sum_{t=1}^{T} q_{rmit}\right) \quad \forall i,\ \forall r \tag{8}$$

The total profits that each manufacturer i aims at maximizing are given by:

$$\begin{aligned} Z_i = {} & \sum_{k=1}^{K}\sum_{t=1}^{T} p_{ik}^{eco^*}\cdot q_{ikt}^{eco} + \sum_{k=1}^{K}\sum_{t=1}^{T} p_{ik}^{neco^*}\cdot q_{ikt}^{neco} - \sum_{v=1}^{V}\sum_{n=1}^{N} p_{vni}^{*}\cdot \sum_{t=1}^{T} q_{vnit} + \\ & -\sum_{r=1}^{R}\sum_{m=1}^{M} p_{rmi}^{*}\cdot\sum_{t=1}^{T} q_{rmit} - \sum_{k=1}^{K} c_{ik}^{eco}\left(\sum_{t=1}^{T} q_{ikt}^{eco}\right) - \sum_{k=1}^{K} c_{ik}^{neco}\left(\sum_{t=1}^{T} q_{ikt}^{neco}\right) + \\ & -\sum_{r=1}^{R} c_{ri}\left(\sum_{m=1}^{M}\sum_{t=1}^{T} q_{rmit}\right) - \sum_{t=1}^{T}\sum_{k=1}^{K} x_{ik}\cdot\rho_{ikt}\cdot(q_{ikt}^{eco}+q_{ikt}^{neco}) + \\ & -p^{CO_2}\cdot\sum_{t=1}^{T}\sum_{k=1}^{K}(\phi_i^{eco}\cdot q_{ikt}^{eco}+\phi_i^{neco}\cdot q_{ikt}^{neco}) - \alpha\cdot\sum_{t=1}^{T}\sum_{k=1}^{K} x_{ik}\cdot\tau_t\cdot(q_{ikt}^{eco}+q_{ikt}^{neco}) \end{aligned} \tag{9}$$

subject to

$$\bar{Q}_i^{eco} - \sum_{k=1}^{K}\sum_{t=1}^{T} q_{ikt}^{eco} \geq 0 \quad \forall i \qquad (\mu_i^{eco}) \tag{10}$$

$$\bar{Q}_i^{neco} - \sum_{k=1}^{K}\sum_{t=1}^{T} q_{ikt}^{neco} \geq 0 \quad \forall i \qquad (\mu_i^{neco}) \tag{11}$$

$$\sum_{k=1}^{K}\sum_{t=1}^{T} q_{ikt}^{eco} \leq \sum_{r=1}^{R}\sum_{m=1}^{M}\sum_{t=1}^{T} q_{rmit} \quad \forall i \qquad (\eta_i^{eco}) \tag{12}$$

$$\sum_{k=1}^{K}\sum_{t=1}^{T} q_{ikt}^{neco} \leq \sum_{v=1}^{V}\sum_{n=1}^{N}\sum_{t=1}^{T} q_{vnit} \quad \forall i \qquad (\eta_i^{neco}) \tag{13}$$

$$z_{ikt} - (q_{ikt}^{eco} + q_{ikt}^{neco}) \geq 0 \quad \forall i, \forall k, \forall t \qquad (\delta_{ikt}) \tag{14}$$

$$q_{vnit} \geq 0 \quad \forall v, \forall n, \forall i, \forall t \tag{15}$$

$$q_{rmit} \geq 0 \quad \forall r, \forall m, \forall i, \forall t \tag{16}$$

$$q_{ikt}^{eco} \geq 0 \quad \forall i, \forall k, \forall t \tag{17}$$

$$q_{ikt}^{neco} \geq 0 \quad \forall i, \forall k, \forall t \tag{18}$$

Objective function (9) states that manufacturers' profits are given by the difference between revenues accruing from selling the ecological ($\sum_{k=1}^{K}\sum_{t=1}^{T} p_{ik}^{eco^*} \cdot q_{ikt}^{eco}$) and the non-ecological ($\sum_{k=1}^{K}\sum_{t=1}^{T} p_{ik}^{neco^*} \cdot q_{ikt}^{neco}$) goods at prices $p_{ik}^{eco^*}$ and $p_{ik}^{eco^*}$ respectively to consumers in demand markets and all the operating, transportation and carbon costs. The operating costs are represented by the manufacturing costs ($\sum_{k=1}^{K} c_{ik}^{eco}(\sum_{t=1}^{T} q_{ikt}^{eco})$ and $\sum_{k=1}^{K} c_{ik}^{neco}(\sum_{t=1}^{T} q_{ikt}^{neco})$) and the charges to refine the reusable raw materials ($\sum_{r=1}^{R} c_{ri}(\sum_{m=1}^{M}\sum_{t=1}^{T} q_{rmit})$). The term $\sum_{t=1}^{T}\sum_{k=1}^{K} x_{ik} \cdot \rho_{ikt} \cdot (q_{ikt}^{eco} + q_{ikt}^{neco})$ indicates the total transportation costs.

We assume that manufacturers face carbon costs both at production and transport levels. More precisely, manufacturers' production activity is subject to the EU-ETS. For this reason, each ton of CO_2 emission generated in the production process has to be covered by an allowance. The carbon costs due to the EU-ETS are expressed by the term $p^{CO_2} \cdot \sum_{t=1}^{T}\sum_{k=1}^{K}(\phi_i^{eco} \cdot q_{ikt}^{eco} + \phi_i^{neco} \cdot q_{ikt}^{neco})$ where p^{CO_2} indicates the exogenous allowance price and $\sum_{t=1}^{T}\sum_{k=1}^{K}(\phi_i^{eco} \cdot q_{ikt}^{eco} + \phi_i^{neco} \cdot q_{ikt}^{neco})$ stands for the total carbon emissions generated in producing ecological and non-ecological goods. As for suppliers that use trucks to transfer virgin raw materials, manufacturers are subject to a carbon tax α when transferring ecological and non-ecological goods to consumers located in the demand markets. The total amount of emission generated depends on the emission factor of each truck τ_t and is proportional to the quantity of transported goods ($q_{ikt}^{eco} + q_{ikt}^{neco}$) and the covered distance (x_{ik}).

Note that the manufacturers have to consider a series of constraints while maximizing their profits. In addition to the variable non-negativity constraints (15)-(18), they have to account for the production capacity constraints (10)-(11); the production balance constraints (13)-(12) and the truck transportation constraint (14).

Let assume that manufacturers behave in a non-cooperative fashion. Therefore, the optimality conditions for all manufacturers i can be described simultaneously using the following variational inequality: determine the solution $(Q^{V^*}, Q^{R^*}, Q^{eco^*}, Q^{neco^*}, \mu^{eco^*}, \mu^{neco^*}, \eta^{eco^*}, \eta^{neco^*}, \delta^*) \in \Re_+^{VNIT+RMIT+IKT+IKT+I+I+I+I+IKT}$ satisfying

$$\sum_{v=1}^{V}\sum_{n=1}^{N}\sum_{i=1}^{I}\sum_{t=1}^{T}\left[p_{vni}^* - \eta_i^{eco^*}\right] \times [q_{vnit} - q_{vnit}^*] + \tag{19}$$

$$+\sum_{r=1}^{R}\sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{t=1}^{T}\left[p_{rmi}^* + \frac{\partial c_{ri}\left(\sum_{m=1}^{M}\sum_{t=1}^{T} q_{rmit}^*\right)}{\partial q_{rmit}} - \eta_i^{neco^*}\right] \times [q_{rmit} - q_{rmit}^*] +$$

$$+\sum_{i=1}^{I}\sum_{i=k}^{K}\sum_{t=1}^{T}\left[-p_{ik}^{eco^*}+\frac{\partial c_{ik}^{eco}\left(\sum_{t=1}^{T} q_{ikt}^{eco^*}\right)}{\partial q_{ikt}^{eco}}+p^{CO_2}\cdot\phi_i^{eco}+x_{ik}\cdot\rho_{ikt}+\alpha\cdot x_{ik}\cdot\tau_t+\right.$$

$$\left.+\mu_i^{eco^*}+\eta_i^{eco^*}+\delta_{ikt}^*\right]\times[q_{ikt}^{eco}-q_{ikt}^{eco^*}]+$$

$$+\sum_{i=1}^{I}\sum_{i=k}^{K}\sum_{t=1}^{T}\left[-p_{ik}^{neco^*}+\frac{\partial c_{ik}^{neco}\left(\sum_{t=1}^{T} q_{ikt}^{neco^*}\right)}{\partial q_{ikt}^{neco}}+p^{CO_2}\cdot\phi_i^{neco}+x_{ik}\cdot\rho_{ikt}+\alpha\cdot x_{ik}\cdot\tau_t+\right.$$

$$\left.+\mu_i^{neco^*}+\eta_i^{neco^*}+\delta_{ikt}^*\right]\times[q_{ikt}^{neco}-q_{ikt}^{neco^*}]+$$

$$+\sum_{i=1}^{I}\left[\bar{Q}_i^{eco}-\sum_{k=1}^{K}\sum_{t=1}^{T} q_{ikt}^{eco^*}\right]\times[\mu_i^{eco}-\mu_i^{eco^*}]+$$

$$+\sum_{i=1}^{I}\left[\bar{Q}_i^{neco}-\sum_{k=1}^{K}\sum_{t=1}^{T} q_{ikt}^{neco^*}\right]\times[\mu_i^{neco}-\mu_i^{neco^*}]+$$

$$+\sum_{i=1}^{I}\left[\sum_{r=1}^{R}\sum_{n=1}^{N}\sum_{t=1}^{T} q_{rmit}^*-\sum_{k=1}^{K}\sum_{t=1}^{T} q_{ikt}^{eco^*}\right]\times[\eta_i^{eco}-\eta_i^{eco^*}]+$$

$$+\sum_{i=1}^{I}\left[\sum_{v=1}^{V}\sum_{n=1}^{N}\sum_{t=1}^{T} q_{vnit}^*-\sum_{k=1}^{K}\sum_{t=1}^{T} q_{ikt}^{neco^*}\right]\times[\eta_i^{neco}-\eta_i^{neco^*}]+$$

$$+\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[z_{ikt}-(q_{ikt}^{eco^*}+q_{ikt}^{neco^*})\right]\times[\delta_{ikt}-\delta_{ikt}^*]\geq 0$$

$$\forall(Q^V,Q^R,Q^{eco},Q^{neco},\mu^{eco},\mu^{neco},\eta^{eco},\eta^{neco},\delta)\in\Re_+^{VNIT+RMIT+IKT+IKT+I+I+I+I+IKT}$$

Note that μ_i^{eco}, μ_i^{neco}, η_i^{eco}, η_i^{neco}, and δ_{ikt} are the Lagrangian multipliers of constraints (10), (11), (13), (12), and (14) respectively, while μ^{eco}, μ^{neco}, η^{eco}, η^{neco} are column vectors of all manufacturers' multipliers and $\delta \in \Re_+^{IKT}$.

3.4 Consumers' behaviour and demand market equilibrium conditions

Consumers participate to both the forward and the reverse CLCS. In the forward CLCS, consumers decide the quantity of ecological and non-ecological goods that they desire to buy from each manufacturer i and how much they are willing to pay for them. In the reverse CLSC, consumers provide used products that has to be sent to recovery centers.

In the forward CLSC, the equilibrium conditions for consumers at the demand market k are as defined in Nagurney et al. (2002). In particular, let d_k^{eco} and p_k^{eco} respectively denote the quantity of ecological good, produced with reusable raw material, demanded by consumers at demand market k and the price that they are willing to pay. Let assume that demand d_k^{eco} is a continuous function of price p_k^{eco} and that $p^{eco} \in R_+^K$. We define:

$$d_k^{eco} = d_k^{eco}(p^{eco}) \quad \forall k \tag{20}$$

In a similar way, let d_k^{neco} and p_k^{neco} respectively denote the quantity of non-ecological good, produced using virgin raw material, demanded by consumers at demand market k and the respective price that they are willing to pay. Let assume that demand d_k^{neco} is a continuous function of price p_k^{neco} and that $p^{neco} \in R_+^K$. We define:

$$d_k^{neco} = d_k^{neco}(p^{neco}) \quad \forall k \tag{21}$$

In addition, we assume that there are transaction costs related to the purchase of the ecological and non-ecological goods. Let tc_{ik}^{eco} and tc_{ik}^{neco} denote the transaction costs between manufacturer i and consumers at demand market k associated with the purchase of ecological q_{ikt}^{eco} and non-ecological q_{ikt}^{neco} goods respectively. These transaction costs are assumed to be continuous and positive and are defined as follows: for all manufacturers $i = 1, ..., I$

$$tc_{ik}^{eco} = tc_{ik}^{eco}\left(\sum_{t=1}^{T} q_{ikt}^{eco}\right) \quad \forall i,\ \forall k \tag{22}$$

$$tc_{ik}^{neco} = tc_{ik}^{neco}\left(\sum_{t=1}^{T} q_{ikt}^{neco}\right) \quad \forall i,\ \forall k \tag{23}$$

On this basis, (24) and (25) represent the equilibrium conditions for consumers at the demand market k that buy the ecological goods. In particular, for all manufacturers $i = 1, ..., I$, it holds that:

$$p_{ik}^{eco^*} + tc_{ik}^{eco}\left(\sum_{t=1}^{T} q_{ikt}^{eco^*}\right) \begin{cases} = p_k^{eco^*} & \text{if} \quad \sum_{t=1}^{T} q_{ikt}^{eco^*} > 0 \\ \geq p_k^{eco^*} & \text{if} \quad \sum_{t=1}^{T} q_{ikt}^{eco^*} = 0 \end{cases} \tag{24}$$

$$d_k^{eco}(p^{eco^*}) \begin{cases} = \sum_{i=1}^{I} \sum_{t=1}^{T} q_{ikt}^{eco^*} & \text{if} \quad p_k^{eco^*} > 0 \\ \leq \sum_{i=1}^{I} \sum_{t=1}^{T} q_{ikt}^{eco^*} & \text{if} \quad p_k^{eco^*} = 0 \end{cases} \tag{25}$$

where $p_{ik}^{eco^*}$ is the price applied by manufacturers to the ecological goods (see Section 3.3). Condition (24) indicates that consumers at demand market k buy the ecological good from manufacturer i if the charged price $p_{ik}^{eco^*}$ increased by the transaction costs tc_{ik}^{eco} does not exceed the price $p_k^{eco^*}$ that they are willing to pay for the ecological good. On the other side, condition (25) states that, if price $p_k^{eco^*}$ is positive, then the ecological consumed at demand market k corresponds exactly to the consumers' demand in that market.

In a similar way, (26) and (27) correspond to the equilibrium conditions for consumers at the demand market k that purchase the non-ecological goods. In particular, for all manufacturers $i = 1, ..., I$, it holds that:

$$p_{ik}^{neco^*} + tc_{ik}^{neco}\left(\sum_{t=1}^{T} q_{ikt}^{neco^*}\right) \begin{cases} = p_k^{neco^*} & \text{if} \quad \sum_{t=1}^{T} q_{ikt}^{neco^*} > 0 \\ \geq p_k^{neco^*} & \text{if} \quad \sum_{t=1}^{T} q_{ikt}^{neco^*} = 0 \end{cases} \tag{26}$$

$$
d_k^{neco}(p^{neco^*}) \begin{cases} = \sum_{i=1}^{I} \sum_{t=1}^{T} q_{ikt}^{neco^*} & \text{if} \quad p_k^{neco^*} > 0 \\ \\ \leq \sum_{i=1}^{I} \sum_{t=1}^{T} q_{ikt}^{neco^*} & \text{if} \quad p_k^{neco^*} = 0 \end{cases} \tag{27}
$$

where $p_{ik}^{neco^*}$ is the price applied by manufacturers to the non-ecological goods (see Section 3.3).

Consumers also participate to the reverse CLCS network. After the consumption process, both the ecological and non-ecological goods, or part of them, can be recycled and reused. We assume that these used products are collected by consumers and sent to recovery centers in order to be transformed in reusable raw materials. For each unit of used product collected in demand market k and sent to recovery centers (independently of the raw materials used to produce them), consumers receive a buy-back price equal to p_{km}^* from recovery centers. Note that transportation costs are faced by recovery centers. Consumers' aversion is modeled by a continuous function $a_k(Q)$ that depends on the amount of used product returned to all recovery centers. The larger is the quantity of collected products, the more a recovery center has to offer as a buy-back price. The increase of the buy-back price represents an incentive for consumers to recycle. This means that the buy-back price p_{km}^* splits consumers into two groups: those who are willing to recycle and return used products to recovery centers and those are not. In addition, the amount that recovery centers have to pay-back to consumers at demand markets not only depend on the quantity of used product they desire to collect, but also on the quantity collected by their competitors (see [10]).

To sum up consumers' behaviour in the reverse CLSC can be defined by condition (28) subject to constraint (29).

$$
a_k(Q^*) \begin{cases} = p_{km}^* & \text{if} \quad q_{km}^* > 0 \\ \\ \geq p_{km}^* & \text{if} \quad q_{km}^* = 0 \end{cases} \tag{28}
$$

subject to

$$
\sum_{m=1}^{M} q_{km} \leq \sum_{i=1}^{I} \sum_{t=1}^{T} \theta_k^{eco} \cdot q_{ikt}^{eco} + \sum_{i=1}^{I} \sum_{t=1}^{T} \theta_k^{neco} \cdot q_{ikt}^{neco} \quad (\sigma_k) \tag{29}
$$

Condition (28) state that consumers at demand market k will choose to return a volume of product corresponding to the value of the buy-back price. Constraint (29) imposes that the quantity of outputs returned to recovery centers by consumers at demand market k is not greater than the sum of the ecological and non-ecological goods purchased by manufactures in the forward chain multiplied by the respective return ratios.

Considering consumers' behaviour both in the forward and the reverse CLSC, their optimality conditions at demand markets k are defined as follows: determine $(Q^{eco^*}, Q^{neco^*}, Q^*, p^{eco^*}, p^{neco^*}, \sigma^*) \in \Re_+^{IKT+IKT+KM+K+K+K}$ satisfying

$$\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[p_{ik}^{eco^*}+tc_{ik}^{eco}\left(\sum_{t=1}^{T}q_{ikt}^{eco^*}\right)-p_k^{eco^*}-\theta_k^{eco}\cdot\sigma_k^*\right]\times[q_{ikt}^{eco}-q_{ikt}^{eco^*}]+ \tag{30}$$

$$+\quad\sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[p_{ik}^{neco^*}+tc_{ik}^{neco}\left(\sum_{t=1}^{T}q_{ikt}^{neco^*}\right)-p_k^{neco^*}-\theta_k^{neco}\cdot\sigma_k^*\right]\times[q_{ikt}^{neco}-q_{ikt}^{neco^*}]+$$

$$+\quad\sum_{k=1}^{K}\sum_{m=1}^{M}[a_k(Q^*)-p_{km}^*+\sigma_k^*]\times[q_{km}-q_{km}^*]+$$

$$+\quad\sum_{k=1}^{K}\left[\sum_{i=1}^{I}\sum_{t=1}^{T}q_{ikt}^{eco^*}-d_k^{eco}(p^{eco^*})\right]\times[p_k^{eco}-p_k^{eco^*}]+$$

$$+\quad\sum_{k=1}^{K}\left[\sum_{i=1}^{I}\sum_{t=1}^{T}q_{ikt}^{neco^*}-d_k^{neco}(p^{neco^*})\right]\times[p_k^{neco}-p_k^{neco^*}]+$$

$$+\quad\sum_{k=1}^{K}\left[\sum_{i=1}^{I}\sum_{t=1}^{T}\theta_k^{eco}\cdot q_{ikt}^{eco^*}+\sum_{i=1}^{I}\sum_{t=1}^{T}\theta_k^{neco}\cdot q_{ikt}^{neco^*}-\sum_{m=1}^{M}q_{km}^*\right]\times[\sigma_k-\sigma_k^*]\geq 0$$

$$\forall(Q^{eco},Q^{neco},Q,p^{eco},p^{neco},\sigma)\in\Re_+^{IKT+IKT+KM+K+K+K}$$

Note that σ_k is the Lagrangian multiplier associated with constraint (29), while $\sigma\in\Re_+^K$.

3.5 Recovery centers' behaviour and their optimality conditions

Recovery centers operate in the reverse CLSC. As explained in Section 3.4, recovery centers pay a buy-back price p_{km}^* to consumers for the used and recyclable products that are collected at demand markets k. This represents a cost for recovery centers that is proportional to the collected amount of used products.

Once received these used products from the demand markets, recovery centers have to transform them in order to get reusable raw materials that can be sold to manufacturers. Let c_{rm} denote the recycling costs faced by recovery center m to transform the used product collected at demand markets k into reusable raw material r. These costs are continuous and convex and are proportional to the amount of used products that the recovery center receives from consumers and are given by:

$$c_{rm}=c_{rm}\left(\beta_{rm}\cdot\sum_{k=1}^{K}q_{km}\right)\quad\forall r,\ \forall m \tag{31}$$

The recycling activity is subsidized and, for each unit of used product collected at demand markets, recovery centers receive a subsidy equal to s_m. Not all of the collected used products can be transformed into reusable raw materials, i.e. part of the disassembled materials obtained in the transformation process cannot be used again and thus is sent to a landfill. The amount of useless materials that are sent to landfill corresponds to the difference between the used products collected at demand markets and the total amount of reusable raw materials that recovery centers obtain in the recycling process:

$$\bar{\beta}_m\cdot\sum_{k=1}^{K}q_{km}=(1-\sum_{r=1}^{R}\beta_{rm})\cdot\sum_{k=1}^{K}q_{km}$$

As indicated in constraint (33), the transformation process of used product allows recovery centers to get reusable raw materials q_{rmit} that are then sold to manufacturers at price p^*_{rmi} (compare Section 3.3).

Each recovery center m aims at maximizing its profits as defined below:

$$Z_m = \sum_{r=1}^{R}\sum_{i=1}^{I} p^*_{rmi} \cdot \sum_{t=1}^{T} q_{rmit} + s_m \cdot \sum_{k=1}^{K} q_{km} - \sum_{k=1}^{K} p^*_{km} \cdot q_{km} + \tag{32}$$

$$-\sum_{r=1}^{R} c_{rm}\left(\beta_{rm} \cdot \sum_{k=1}^{K} q_{km}\right) - \omega_m \cdot \bar{\beta}_m \cdot \sum_{k=1}^{K} q_{km} +$$

$$-\sum_{t=1}^{T}\sum_{i=1}^{I} x_{mi} \cdot \rho_{rmit} \cdot \sum_{r=1}^{R} q_{rmit} - \alpha \cdot \sum_{t=1}^{T}\sum_{i=1}^{I} x_{mi} \cdot \tau_t \cdot \sum_{r=1}^{R} q_{rmit} +$$

$$-\sum_{k=1}^{K} x_{km} \cdot \rho_{km} \cdot q_{km} - x_m \cdot \rho_m \cdot \bar{\beta}_m \cdot \sum_{k=1}^{K} q_{km}$$

subject to

$$\sum_{i=1}^{I}\sum_{t=1}^{T} q_{rmit} \leq \beta_{rm} \cdot \sum_{k=1}^{K} q_{km} \quad \forall r, \forall m \quad (\varphi_{rm}) \tag{33}$$

$$z_{mit} - \sum_{r=1}^{R} q_{rmit} \geq 0 \quad \forall m, \forall i, \forall t \qquad (\lambda_{mit}) \tag{34}$$

$$q_{rmit} \geq 0 \quad \forall r, \forall m, \forall i, \forall t \tag{35}$$

$$q_{km} \geq 0 \quad \forall k, \forall m \tag{36}$$

Profits (32) are given by the difference between the revenues ($\sum_{r=1}^{R}\sum_{i=1}^{I} p^*_{rmi} \cdot \sum_{t=1}^{T} q_{rmit}$) arising from selling to manufacturers reusable raw materials ($\sum_{t=1}^{T} q_{rmit}$) at price p^*_{rmi} plus subsidies ($s_m \cdot \sum_{k=1}^{K} q_{km}$) and costs. These costs are associated to the transformation and recycling process ($\sum_{r=1}^{R} c_{rm}(\beta_{rm} \cdot \sum_{k=1}^{K} q_{km})$), the buy-back price ($\sum_{k=1}^{K} p^*_{km} \cdot q_{km}$), the landfill charges ($\omega_m \cdot \bar{\beta}_m \cdot \sum_{k=1}^{K} q_{km}$) and all transportation fees, including CO_2 taxes ($\alpha \cdot \sum_{t=1}^{T}\sum_{i=1}^{I} x_{mi} \cdot \tau_t \cdot \sum_{r=1}^{R} q_{rmit}$). In maximizing their profits, recovery centers have to account for some operating constraints. Constraint (33) defines the amount of reusable raw materials that recovery center obtain after the transformation process of used products. Constraint (34) imposes a transportation limit on trucks used to transfer reusable raw materials from recovery center m to manufacturer i. Finally, non-negativity constraints (35)-(36) are imposed on problem variables.

Let assume that recovery centers behave in a non-cooperative fashion. Assuming that the recycling costs faced by recovery center are a continuous and convex function, the optimality conditions of all recovery centers can be described simultaneously using the following variational inequality: determine $(Q^{R^*}, Q^*, \varphi^*, \lambda^*) \in \Re_+^{RMIT+KM+RM+MIT}$ satisfying

$$\sum_{k=1}^{K}\sum_{m=1}^{M}\left[-s_m+p^*_{km}+\sum_{r=1}^{R}\frac{\partial c_{rm}(\beta_{rm}\cdot\sum_{k=1}^{K}q^*_{km})}{\partial q_{km}}+\omega_m\cdot\bar{\beta}_m+x_{km}\cdot\rho_{km}+\right. \tag{37}$$

$$\left.+x_m\cdot\rho_m\cdot\bar{\beta}_m-\beta_{rm}\cdot\varphi^*_{rm}\right]\times[q_{km}-q^*_{km}]+$$

$$+\quad\sum_{r=1}^{R}\sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{t=1}^{T}[-p^*_{rmi}+x_{mi}\cdot\rho_{rmit}+\alpha\cdot x_{mi}\cdot\tau_t+\varphi^*_{rm}+\lambda^*_{mit}]\times[q_{rmit}-q^*_{rmit}]+$$

$$+\quad\sum_{r=1}^{R}\sum_{m=1}^{M}\left[\beta_{rm}\cdot\sum_{k=1}^{K}q^*_{km}-\sum_{i=1}^{I}\sum_{t=1}^{T}q^*_{rmit}\right]\times[\varphi_{rm}-\varphi^*_{rm}]+$$

$$+\quad\sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{t=1}^{T}\left[z_{mit}-\sum_{r=1}^{R}q^*_{rmit}\right]\times[\lambda_{mit}-\lambda^*_{mit}]\geq 0$$

$$\forall(Q^R,Q,\varphi,\lambda)\in\Re_+^{RMIT+KM+RM+MIT}$$

Note that φ_{rm} and λ_{mit} are the Lagrangian multipliers associated with constraint (33) and (34) respectively, while $\varphi\in\Re_+^{RM}$ and $\lambda\in\Re_+^{MIT}$.

4 Equilibrium conditions and qualitative properties of the CLSC network model

In the forward logistics, in equilibrium, the shipments of virgin and reusable raw materials respectively from suppliers and recovery centers to manufacturers must be equal to the shipments that manufacturers accept from virgin raw material suppliers and recovery center respectively. In a similar way, the amounts of ecological and non-ecological goods purchased by consumers at demand markets must be equal to the amounts sold by manufacturers. In the reverse logistics, in equilibrium, the amount of used products that consumers at demand markets sell to recovery centers must be equal to the amount bought by recovery centers. Analogously, the amount of useless materials that recovery centers send to landfill must be equal to the amount that landfill accept. The equilibrium shipment and price pattern in the CLSC must satisfy the sum of variational inequalities (5), (19), (30), and (37) in order to formalize the agreements between the tiers. This is formally stated in Definition 1.

Definition 1 *(Equilibrium). The equilibrium state for the closed-loop supply chain network is the one where flows between reverse and forward tiers coincide and satisfy the sum of variational inequalities (5), (19), (30), and (37).*

The variational inequality that governs equilibrium conditions according to Definition 1 is now derived.

Theorem 1 *(Variational Inequality Formulation of the closed-loop supply chain network). The equilibrium condition non capital governing the closed-loop supply chain network with material transformation factors, recycled and new (virgin) materials according to Definition 1 coincides with the solution to the following variational inequality:*

Find $(Q^{R^*}, Q^{V^*}, Q^{eco^*}, Q^{neco^*}, Q^*, p^{eco^*}, p^{neco^*}, \gamma^*, \mu^{eco^*}, \mu^{neco^*}, \eta^{eco^*}, \eta^{neco^*}, \delta^*, \sigma^*, \varphi^*, \lambda^*)$
$\in \Re_+^{RMIT+VNIT+IKT+IKT+KM+K+K+NIT+I+I+I+I+IKT+K+RM+MIT}$ such that:

$$\begin{aligned}
& \sum_{r=1}^{R}\sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{t=1}^{T}\left[\frac{\partial c_{ri}\left(\sum_{m=1}^{M}\sum_{t=1}^{T} q^*_{rmit}\right)}{\partial q_{rmit}} + x_{mi}\cdot\rho_{rmit} + \alpha\cdot x_{mi}\cdot\tau_t + \varphi^*_{rm} + \lambda^*_{mit} - \eta_i^{eco^*}\right] \times [q_{rmit} - q^*_{rmit}] + \qquad (38)\\
+ \quad & \sum_{n=1}^{N}\sum_{v=1}^{V}\sum_{i=1}^{I}\sum_{t=1}^{T}\left[\frac{\partial c_{vn}\left(\sum_{i=1}^{I}\sum_{t=1}^{T} q^*_{vnit}\right)}{\partial q_{vnit}} + x_{ni}\cdot\rho_{vnit} + \alpha\cdot x_{ni}\cdot\tau_t + \gamma^*_{nit} - \eta_i^{neco^*}\right] \times [q_{vnit} - q^*_{vnit}] +\\
+ \quad & \sum_{i=1}^{I}\sum_{i=k}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_{ik}^{eco}\left(\sum_{t=1}^{T} q_{ikt}^{eco^*}\right)}{\partial q_{ikt}^{eco}} + \frac{\partial tc_{ik}^{eco}\left(\sum_{t=1}^{T} q_{ikt}^{eco^*}\right)}{\partial q_{ikt}^{eco}} + p^{CO_2}\cdot\phi_i^{eco} + x_{ik}\cdot\rho_{ikt} + \alpha\cdot x_{ik}\cdot\tau_t + \mu_i^{eco^*} + \eta_i^{eco^*} + \right.\\
& \qquad\qquad \left. + \delta^*_{ikt} - p_k^{eco^*} - \theta_k^{eco}\cdot\sigma_k^*\right] \times [q_{ikt}^{eco} - q_{ikt}^{eco^*}] +\\
+ \quad & \sum_{i=1}^{I}\sum_{i=k}^{K}\sum_{t=1}^{T}\left[\frac{\partial c_{ik}^{neco}\left(\sum_{t=1}^{T} q_{ikt}^{neco^*}\right)}{\partial q_{ikt}^{neco}} + \frac{\partial tc_{ik}^{neco}\left(\sum_{t=1}^{T} q_{ikt}^{neco^*}\right)}{\partial q_{ikt}^{neco}} + p^{CO_2}\cdot\phi_i^{neco} + x_{ik}\cdot\rho_{ikt} + \alpha\cdot x_{ik}\cdot\tau_t + \mu_i^{neco^*} + \eta_i^{neco^*} + \right.\\
& \qquad\qquad \left. + \delta^*_{ikt} - p_k^{neco^*} - \theta_k^{neco}\cdot\sigma_k^*\right] \times [q_{ikt}^{neco} - q_{ikt}^{neco^*}] +\\
+ \quad & \sum_{k=1}^{K}\sum_{m=1}^{M}\left[a_k(Q^*) + \sigma_k^* - s_m + \sum_{r=1}^{R}\frac{\partial c_{rm}(\beta_{rm}\cdot\sum_{k=1}^{K} q^*_{km})}{\partial q_{km}} + \omega_m\cdot\bar{\beta}_m + x_{km}\cdot\rho_{km} + \right.\\
& \qquad\qquad \left. + x_m\cdot\rho_m\cdot\bar{\beta}_m - \beta_{rm}\cdot\varphi_{rm}\right] \times [q_{km} - q^*_{km}] +\\
+ \quad & \sum_{k=1}^{K}\left[\sum_{i=1}^{I}\sum_{t=1}^{T} q_{ikt}^{eco^*} - d_k^{eco}(p^{eco^*})\right] \times [p_k^{eco} - p_k^{eco^*}] + \sum_{k=1}^{K}\left[\sum_{i=1}^{I}\sum_{t=1}^{T} q_{ikt}^{neco^*} - d_k^{neco}(p^{neco^*})\right] \times [p_k^{neco} - p_k^{neco^*}] +\\
+ \quad & \sum_{n=1}^{N}\sum_{i=1}^{I}\sum_{t=1}^{T}\left[z_{nit} - \sum_{v=1}^{V} q^*_{vnit}\right] \times [\gamma_{nit} - \gamma^*_{nit}] +\\
+ \quad & \sum_{i=1}^{I}\left[\bar{Q}_i^{eco} - \sum_{k=1}^{K}\sum_{t=1}^{T} q_{ikt}^{eco^*}\right] \times [\mu_i^{eco} - \mu_i^{eco^*}] + \sum_{i=1}^{I}\left[\bar{Q}_i^{neco} - \sum_{k=1}^{K}\sum_{t=1}^{T} q_{ikt}^{neco*}\right] \times [\mu_i^{neco} - \mu_i^{neco^*}] +\\
+ \quad & \sum_{i=1}^{I}\left[\sum_{r=1}^{R}\sum_{n=1}^{N}\sum_{t=1}^{T} q^*_{rmit} - \sum_{k=1}^{K}\sum_{t=1}^{T} q_{ikt}^{eco^*}\right] \times [\eta_i^{eco} - \eta_i^{eco^*}] + \sum_{i=1}^{I}\left[\sum_{v=1}^{V}\sum_{n=1}^{N}\sum_{t=1}^{T} q^*_{vnit} - \sum_{k=1}^{K}\sum_{t=1}^{T} q_{ikt}^{neco^*}\right] \times [\eta_i^{neco} - \eta_i^{neco^*}] +\\
+ \quad & \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T}\left[z_{ikt} - (q_{ikt}^{eco^*} + q_{ikt}^{neco^*})\right] \times [\delta_{ikt} - \delta^*_{ikt}] + \sum_{k=1}^{K}\left[\sum_{i=1}^{I}\sum_{t=1}^{T}\theta_k^{eco}\cdot q_{ikt}^{eco^*} + \sum_{i=1}^{I}\sum_{t=1}^{T}\theta_k^{neco}\cdot q_{ikt}^{neco^*} - \sum_{m=1}^{M} q^*_{km}\right] \times [\sigma_k - \sigma_k^*] +\\
+ \quad & \sum_{r=1}^{R}\sum_{m=1}^{M}\left[\beta_{rm}\cdot\sum_{k=1}^{K} q^*_{km} - \sum_{i=1}^{I}\sum_{t=1}^{T} q^*_{rmit}\right] \times [\varphi_{rm} - \varphi^*_{rm}] + \sum_{m=1}^{M}\sum_{i=1}^{I}\sum_{t=1}^{T}\left[z_{mit} - \sum_{r=1}^{R} q^*_{rmit}\right] \times [\lambda_{mit} - \lambda^*_{mit}] \geq 0
\end{aligned}$$

$\forall(Q^R, Q^V, Q^{eco}, Q^{neco}, Q, p^{eco}, p^{neco}, \gamma, \mu^{eco}, \mu^{neco}, \eta^{eco}, \eta^{neco}, \delta, \sigma, \varphi, \lambda) \in K$

where
$\Omega \equiv \{(Q^R, Q^V, Q^{eco}, Q^{neco}, Q, p^{eco}, p^{neco}, \gamma, \mu^{eco}, \mu^{neco}, \eta^{eco}, \eta^{neco}, \delta, \sigma, \varphi, \lambda) | (Q^R, Q^V, Q^{eco}, Q^{neco}, Q, p^{eco}, p^{neco},$
$\gamma, \mu^{eco}, \mu^{neco}, \eta^{eco}, \eta^{neco}, \delta, \sigma, \varphi, \lambda) \in \Re_+^{RMIT+VNIT+IKT+IKT+KM+K+K+NIT+I+I+I+I+IKT+K+RM+MIT}\}$

Proof. See [16]. □

Note that variational inequality (38) can be rewritten in the standard form (see [14]) as follows: find $X^* \in \Omega$, such that:

$$\langle F(X^*), X - X^* \rangle \geq 0, \quad \forall X \in \Omega \tag{39}$$

where

$X \equiv (Q^R, Q^V, Q^{eco}, Q^{neco}, Q, p^{eco}, p^{neco}, \gamma, \mu^{eco}, \mu^{neco}, \eta^{eco}, \eta^{neco}, \delta, \sigma, \varphi, \lambda)$ with

$X \in \Re_{+}^{VMIT+RNIT+IKT+IKT+KM+K+K+NIT+I+I+I+I+IKT+K+RM+MIT}$ and

$F(X) \equiv (F_{rmit}, F_{vnit}, F_{ikt}^{eco}, F_{ikt}^{neco}, F_{km}, F_{k}^{eco}, F_{k}^{neco}, F_{nit}, F_{i}^{\mu,eco}, F_{i}^{\mu,neco}, F_{i}^{\eta,eco}, F_{i}^{\eta,neco}, F_{ikt}, F_{k}, F_{rm}, F_{imt})$ with the specific component of $F(X)$ being given by the respective functional terms preceding the multiplication signs in (38).

For the sake of simplicity, in the following we will indicate the standard variational inequality (39) when referring to variational inequality (38) defining the equilibrium of our model.

Following [16], it is possible to retrieve prices $p_{vni}^*, p_{rmi}^*, p_{ik}^{eco^*}, p_{ik}^{neco^*}$, and p_{km}^* for $v = 1, ..., V;\ \ r = 1, ..., R;\ \ n = 1, ..., N;\ \ i = 1, ..., I;\ \ k = 1, ..., K;\ \ m = 1, ..., M$ from the solutions of variational inequality (38) (or (5), (19), (30), and (37)).
For instance, consider the price p_{vni}^* $(v = 1, ..., V;\ \ n = 1, ..., N;\ \ i = 1, ..., I)$ in problem (2)-(4). Since the objective function (2) is continuously differentiable and concave and the feasible set is convex, the Karush-Kuhn-Tucker conditions associated to the optimization problem (2)-(4) are as follows:

$$\left[-p_{vni}^* + \frac{\partial c_{vn}\left(\sum_{i=1}^{I}\sum_{t=1}^{T} q_{vnit}^*\right)}{\partial q_{vnit}} + x_{ni}\cdot\rho_{vnit} + \alpha\cdot x_{ni}\cdot\tau_t + \gamma_{nit}^*\right] \geq 0; \tag{40}$$
$$\left[-p_{vni}^* + \frac{\partial c_{vn}\left(\sum_{i=1}^{I}\sum_{t=1}^{T} q_{vnit}^*\right)}{\partial q_{vnit}} + x_{ni}\cdot\rho_{vnit} + \alpha\cdot x_{ni}\cdot\tau_t + \gamma_{nit}^*\right] q_{vnit}^* = 0; \quad q_{vnit}^* \geq 0$$

$$\left[z_{nit} - \sum_{v=1}^{V} q_{vnit}^*\right] \geq 0; \quad \left[z_{nit} - \sum_{v=1}^{V} q_{vnit}^*\right]\gamma_{nit}^* = 0; \quad \gamma_{nit}^* \geq 0 \tag{41}$$

for $v = 1, ..., V,\ \ n = 1, ..., N,\ \ i = 1, ..., I,\ \ t = 1, ..., T.$

Complementarity condition (40) has the following interpretation: if for $v = 1, ..., V,\ \ n = 1, ..., N,\ \ i = 1, ..., I,\ \ t = 1, ..., T$, it holds that $q_{vnit}^* > 0$ then

$$p_{vni}^* = \frac{\partial c_{vn}\left(\sum_{i=1}^{I}\sum_{t=1}^{T} q_{vnit}^*\right)}{\partial q_{vnit}} + x_{ni}\cdot\rho_{vnit} + \alpha\cdot x_{ni}\cdot\tau_t + \gamma_{nit}^* \tag{42}$$

A similar method can be applied to obtain prices $p_{rmi}^*, p_{ik}^{eco^*}, p_{ik}^{neco^*}$, and p_{km}^* by using variational inequality (38) (or (19), (30), and (37)).

As indicated by [14], a variational inequality admits at least one solution if the feasible region Ω is compact and convex and the function F in (39) is continuous on Ω. In our model, we can guarantee the continuity of function F in (39), but not the compactness of the feasible region Ω. For this reason, we cannot derive existence of a solution simply from the assumption of continuity of the functions. However, as illustrated in [16], it is possible to introduce weaker conditions to guarantee solution existence.

Let:
$\Omega_b = \{(Q^R, Q^V, Q^{eco}, Q^{neco}, Q, p^{eco}, p^{neco}, \gamma, \mu^{eco}, \mu^{neco}, \eta^{eco}, \eta^{neco}, \delta, \sigma, \varphi, \lambda) | 0 \leq Q^R \leq b^R;$

$0 \leq Q^V \leq b^V;\ 0 \leq Q^{eco} \leq b^{eco};\ 0 \leq Q^{neco} \leq b^{neco};\ 0 \leq Q \leq b_{km};\ 0 \leq p^{eco} \leq b_p^{eco};\ 0 \leq p^{neco} \leq b_p^{neco};$
$0 \leq \gamma \leq b_\gamma;\ 0 \leq \mu^{eco} \leq b_\mu^{eco};\ 0 \leq \mu^{neco} \leq b_\mu^{neco};\ 0 \leq \eta^{eco} \leq b_\eta^{eco};\ 0 \leq \eta^{neco} \leq b_\eta^{neco};\ 0 \leq \delta \leq b_\delta;$
$0 \leq \sigma \leq b_\sigma;\ 0 \leq \varphi \leq b_\varphi;\ 0 \leq \lambda \leq b_\lambda\},$

where

$$b = (b^R, b^V, b^{eco}, b^{neco}, b_{km}, b_p^{eco}, b_p^{neco}, b_\gamma, b_\mu^{eco}, b_\mu^{neco}, b_\eta^{eco}, b_\eta^{neco}, b_\delta, b_\sigma, b_\varphi, b_\lambda) \geq 0,$$

and

$$0 \leq Q^R \leq b^R;\ 0 \leq Q^V \leq b^V;\ 0 \leq Q^{eco} \leq b^{eco};\ 0 \leq Q^{neco} \leq b^{neco};\ 0 \leq Q \leq b_{km};\ 0 \leq p^{eco} \leq b_p^{eco};$$
$$0 \leq p^{neco} \leq b_p^{neco};\ 0 \leq \gamma \leq b_\gamma;\ 0 \leq \mu^{eco} \leq b_\mu^{eco};\ 0 \leq \mu^{neco} \leq b_\mu^{neco};\ 0 \leq \eta^{eco} \leq b_\eta^{eco};\ 0 \leq \eta^{neco} \leq b_\eta^{neco};$$
$$0 \leq \delta \leq b_\delta; 0 \leq \sigma \leq b_\sigma;\ 0 \leq \varphi \leq b_\varphi;\ 0 \leq \lambda \leq b_\lambda$$

means that

$$q_{rmit} \leq b^R;\ q_{vnit} \leq b^V;\ q_{ikt}^{eco} \leq b^{eco};\ q_{ikt}^{neco} \leq b^{neco};\ q_{km} \leq b_{km};\ p_k^{eco} \leq b_p^{eco};\ p_k^{neco} \leq b_p^{neco};$$
$$\gamma_{nit} \leq b_\gamma;\ \mu_i^{eco} \leq b_\mu^{eco};\ \mu_i^{neco} \leq b_\mu^{neco};\ \eta_i^{eco} \leq b_\eta^{eco};\ \eta_i^{neco} \leq b_\eta^{neco};\ \delta_{ikt} \leq b_8; \sigma_k \leq b_\sigma;\ \varphi_{rm} \leq b_\varphi;$$
$$\lambda_{mit} \leq b_\lambda\ \forall v, r, n, i, k, m, t.$$

Under these assumptions, Ω_b is a bounded, closed, convex subset of $\Re^{RMIT+VNIT+IKT+IKT+KM+K+K+NIT+I+I+I+I+IKT+K+RM+MIT}$ and the following variational inequality (43)

$$\langle F(X^b), X - X^b \rangle \geq 0, \quad \forall X^b \in \Omega_b \tag{43}$$

admits at least one solution $X^b \in \Omega_b$ from the standard theory of variational inequalities, since Ω_b is compact and F is continuous.

Moreover, following [11], we then have:

Lemma 1 *Variational inequality (38) admits a solution if and only if there exists a $b > 0$ such that variational inequality (43) admits a solution in Ω_b with*

$$Q^R < b^R;\quad Q^V < b^V;\quad Q^{eco} < b^{eco};\quad Q^{neco} < b^{neco};\quad Q < b_{km};\quad p^{eco} < b_p^{eco};\quad p^{neco} < b_p^{neco};$$

$$\gamma < b_\gamma;\quad \mu^{eco} < b_\mu^{eco};\quad \mu^{neco} < b_\mu^{neco};\quad \eta^{eco} < b_\eta^{eco};\quad \eta^{neco} < b_\eta^{neco};\quad \delta < b_\delta;\quad \sigma < b_\sigma;$$

$$\varphi < b_\varphi;\quad \lambda < b_\lambda.$$

Theorem 2 *(Existence). Suppose that there exist positive constants A, B, C with $C > A$, such that:*

$$
\begin{aligned}
&\frac{\partial c_{ri}\left(\sum_{m=1}^{M}\sum_{t=1}^{T} q_{rmit}\right)}{\partial q_{rmit}} + x_{mi}\cdot\rho_{rmit} + \alpha\cdot x_{mi}\cdot\tau_t + \varphi_{rm} + \lambda_{mit} - \eta_i^{eco} \geq A, \quad \forall Q^R \quad with \quad q_{rmit} \geq B \quad \forall r,m,i,t \\
&\frac{\partial c_{vn}\left(\sum_{i=1}^{I}\sum_{t=1}^{T} q_{vnit}\right)}{\partial q_{vnit}} + x_{ni}\cdot\rho_{vnit} + \alpha\cdot x_{ni}\cdot\tau_t + \gamma_{nit} - \eta_i^{eco} \geq A, \quad \forall Q^V \quad with \quad q_{vnit} \geq B \quad \forall r,n,i,t \\
&\frac{\partial c_{ik}^{eco}\left(\sum_{t=1}^{T} q_{ikt}^{eco}\right)}{\partial q_{ikt}^{eco}} + \frac{\partial tc_{ik}^{eco}\left(\sum_{t=1}^{T} q_{ikt}^{eco}\right)}{\partial q_{ikt}^{eco}} + p^{CO_2}\cdot\phi_i^{eco} + x_{ik}\cdot\rho_{ikt} + \alpha\cdot x_{ik}\cdot\tau_t + \mu_i^{eco} + \eta_i^{eco} + \\
&\qquad\qquad +\delta_{ikt} - p_k^{eco} - \theta_k^{eco}\cdot\sigma_k \geq M, \quad \forall Q^{eco} \quad with \quad q_{ikt}^{eco} \geq B \quad \forall i,k,t \\
&\frac{\partial c_{ik}^{neco}\left(\sum_{t=1}^{T} q_{ikt}^{neco}\right)}{\partial q_{ikt}^{neco}} + \frac{\partial tc_{ik}^{neco}\left(\sum_{t=1}^{T} q_{ikt}^{neco}\right)}{\partial q_{ikt}^{neco}} + p^{CO_2}\cdot\phi_i^{neco} + x_{ik}\cdot\rho_{ikt} + \alpha\cdot x_{ik}\cdot\tau_t + \mu_i^{neco} + \eta_i^{neco} + \\
&\qquad\qquad +\delta_{ikt} - p_k^{neco} - \theta_k^{neco}\cdot\sigma_k \geq M, \quad \forall Q^{neco} \quad with \quad q_{ikt}^{neco} \geq B \quad \forall i,k,t \\
&a_k(Q) + \theta_k^{neco}\cdot\sigma_k - s_m + \sum_{r=1}^{R}\frac{\partial c_{rm}(\beta_{rm}\cdot\sum_{k=1}^{K} q_{km})}{\partial q_{km}} + \omega_m\cdot\bar{\beta}_m + x_{km}\cdot\rho_{km} + x_m\cdot\rho_m\cdot\bar{\beta}_m - \beta_{rm}\cdot\varphi_{rm} \geq A, \quad \forall Q \quad with \quad q_{km} \geq B \quad \forall k,m \\
&tc_{ik}^{eco}\left(\sum_{t=1}^{T} q_{ikt}^{eco}\right) \geq A, \quad \forall Q^{eco} \quad with \quad q_{ikt}^{eco} \geq B \quad \forall i,k,t \\
&tc_{ik}^{neco}\left(\sum_{t=1}^{T} q_{ikt}^{neco}\right) \geq A, \quad \forall Q^{neco} \quad with \quad q_{ikt}^{neco} \geq B \quad \forall i,k,t \\
&a_k(Q) \geq A, \quad \forall Q \quad with \quad q_{km} \geq B \quad \forall k,m \\
&d_k^{eco}(p^{eco}) \leq B, \quad \forall p^{eco} \quad with \quad p_k^{eco} > C \quad \forall k \\
&d_k^{neco}(p^{neco}) \leq B, \quad \forall p^{neco} \quad with \quad p_k^{neco} > C \quad \forall k
\end{aligned}
\tag{44}
$$

Then variational inequality (38), or equivalently variational inequality (39), admits at least one solution.

Under the condition of Theorem 2, it is possible to construct $b^R, b^V, b^{eco}, b^{neco}, b_{km}$, $b_p^{eco}, b_p^{neco}, b_\gamma, b_\mu^{eco}, b_\mu^{neco}, b_\eta^{eco}, b_\eta^{neco}, b_\delta, b_\sigma, b_\varphi, b_\lambda$ large enough so that variational inequality (43) will satisfy the boundedness condition described in Lemma 1. Uniqueness is another important property that one has to assure in order to guarantee the converge of an algorithm applied to solve any variational inequality.

Theorem 3 *(Strict monotonicity) The vector function F of variational inequality (38) is strictly monotone, with respect to $X = (Q^R, Q^V, Q^{eco}, Q^{neco}, Q, p^{eco}, p^{neco})$, that is*

$$
\langle F(X') - F(X''), X' - X'' \rangle > 0, \quad \forall X', X'' \in \Omega, \quad X' \neq X'' \tag{45}
$$

under these two conditions:

1. *One of the families of convex function $c_{vn},\ c_{ik}^{eco},\ c_{ik}^{neco},\ c_{ri},\ c_{rm}\quad \forall v,n,r,i,k,m$ is a family of strictly convex functions;*
2. $tc_{ik}^{eco}\left(\sum_{t=1}^{T} q_{ikt}^{eco}\right)\ (\forall i,k);\quad tc_{ik}^{neco}\left(\sum_{t=1}^{T} q_{ikt}^{neco}\right)\ (\forall i,k);\quad a_k(Q)\ (\forall k,m);$ $-\,d_k^{eco}\ (\forall k)$ *and* $-\,d_k^{neco}\ (\forall k)$ *are strictly monotone.*

Theorem 4 *(Uniqueness) Under the conditions of Theorem 3, there must exist a unique reusable raw material supply pattern (Q^R), a unique virgin raw material supply pattern (Q^V), unique production patterns (Q^{eco} and Q^{neco}), a unique retail shipment (consumption) pattern (Q), and unique demand price vectors (p^{eco} and p^{neco}) satisfying* *the equilibrium conditions of the closed-loop supply chain network. In other words, if the variational inequality (38) admits a solution, that should be the only solution in* $(Q^R, Q^V, Q^{eco}, Q^{neco}, Q, p^{eco}, p^{neco})$.

5 Algorithm and input data

We apply the combined relaxation method proposed by Konnov in [12] and [13] to compute the equilibrium solution of the proposed CLSC network model. The convergence of this algorithm is assured if the function F is continuous and possesses certain (generalized) monotonicity type properties. In this method, the first part of each iteration serves for computing parameters of a hyperplane that strictly separates the considered iteration and the solution set via solving the so called auxiliary variational inequality problem. This part also includes an Armijo-Goldstein type line search procedure with choosing the step-size as $\theta_k = 2^{-i_k}$. The second part consists in making the projection on this hyperplane and on the feasible set. In this way, a monotone decrease of the distance is obtained. The stopping criterion was defined as follows: the absolute value of two successive iterations is less than a selected accuracy.

The algorithm has been implemented in Matlab with the help of the following computer environment: OS 64 bit MacBook Pro; CPU Intel Core i7 2.66 GHz; Memory 8 GB. The following results are obtained with the value for the stop criterion equal to 10^{-4}. The average of the number of the main iterations for different settings of the parameters is 25 and the average of the execution time is 0.103 seconds.

The CLSC network used for the numerical experiments consists of two raw material suppliers ($n = 1, 2$, where $N = 2$), two manufacturers ($i = 1, 2$, where $I = 2$), two demand markets ($k = 1, 2$, where $K = 2$), and two recovery centers ($m = 1, 2$ where $M = 2$). In the forward CLSC, suppliers provides two types of virgin raw materials ($v = 1, 2$ where $V = 2$) to manufacturers. In the reverse CLSC, manufacturers can buy two types of reusable raw materials ($r = 1, 2$ where $R = 2$) from recovery centers. Finally, for sake of simplicity, transport limit of the truck owned by each agent in the different tiers has been imposed higher than production capacity in order to make feasible any transportation flow. It is assumed that the production of the ecological good is more costly than the non-ecological goods because it is obtained with a more environmental-friendly process.

Model functions used in the numerical experiments are set as follows:

1. Procurement cost functions for virgin raw material faced by suppliers n

$$c_{1n} = 2.5 \left(\sum_{i=1}^{2} \sum_{t=1}^{T} q_{1nit} \right)^2 + \sum_{i=1}^{2} \sum_{t=1}^{T} q_{1nit} + 2, \quad \forall n, v = 1$$

$$c_{2n} = 1.5 \left(\sum_{i=1}^{2} \sum_{t=1}^{T} q_{2nit} \right)^2 + 0.5 \sum_{i=1}^{2} \sum_{t=1}^{T} q_{2nit} + 1 \quad \forall n, v = 2$$

2. Production costs faced by manufacturers i

$$c_{1k}^{eco} = 3 \left(\sum_{t=1}^{T} q_{1kt}^{eco} \right)^2 + \sum_{t=1}^{T} q_{1kt}^{eco} + 5 \quad \forall k, i = 1$$

$$c_{2k}^{eco} = 2.5\left(\sum_{t=1}^{T} q_{2kt}^{eco}\right)^2 + 1.5\sum_{t=1}^{T} q_{2kt}^{eco} + 2 \quad \forall k, i = 2$$

$$c_{1k}^{neco} = 3.5\left(\sum_{t=1}^{T} q_{1kt}^{neco}\right)^2 + 2\sum_{t=1}^{T} q_{1kt}^{neco} + 2 \quad \forall k, i = 1$$

$$c_{2k}^{neco} = 4\left(\sum_{t=1}^{T} q_{2kt}^{neco}\right)^2 + 1.5\sum_{t=1}^{T} q_{2kt}^{neco} + 1 \quad \forall k, i = 2$$

3. Additional costs to refine reusable raw materials faced by manufacturers i

$$c_{1i} = 3\left(\sum_{m=1}^{2}\sum_{t=1}^{T} q_{1mit}\right)^2 + 2\left(\sum_{m=1}^{2}\sum_{t=1}^{T} q_{1mit}\right) \quad \forall i, r = 1$$

$$c_{2i} = 3.5\left(\sum_{m=1}^{2}\sum_{t=1}^{T} q_{2mit}\right)^2 + 2.5\left(\sum_{m=1}^{2}\sum_{t=1}^{T} q_{2mit}\right) \quad \forall i, r = 2$$

4. Demand functions of consumers at demand markets k

$$d_1^{eco}(p^{eco}) = -p_1^{eco} - 0.3p_2^{eco} \quad k = 1$$
$$d_1^{neco}(p^{neco}) = -p_1^{neco} - 0.2p_2^{neco} \quad k = 1$$
$$d_2^{eco}(p^{eco}) = -0.3p_1^{eco} - p_2^{eco} \quad k = 2$$
$$d_2^{neco}(p^{neco}) = -0.2p_1^{neco} - p_2^{neco} \quad k = 2$$

5. Transaction costs faced by consumers at demand market k

$$tc_{ik}^{eco} = 0.1\left(\sum_{t=1}^{T} q_{ikt}^{eco}\right) \quad \forall i, k$$

$$tc_{ik}^{neco} = 0.1\left(\sum_{t=1}^{T} q_{ikt}^{neco}\right) \quad \forall i, k$$

6. Consumers' aversion function

$$a_k(Q) = 0.5\sum_{m=1}^{2} q_{km} + 5 \quad \forall k$$

7. Recycling cost functions for recovery centers m

$$c_{1m} = 2\left(\beta_{1m} \cdot \sum_{k=1}^{2} q_{1m}\right)^2 + 3, \quad \forall m, r = 1$$

$$c_{2m} = 3\left(\beta_{2m} \cdot \sum_{k=1}^{2} q_{2m}\right)^2 + 2 \quad \forall m, r = 2$$

6 Numerical experiments and discussion

In order to answer to the questions defined in Section 1.1 we conduct sensitivity analyses on the following parameters:

- p^{CO_2}: CO_2 price;
- α: carbon tax applied to transport by trucks as indicated in Section 2;
- β_{rm}: percentage of reusable raw material that recovery centers extract from used product collected in the demand markets;
- x_{ni}: distance in Km between raw material suppliers and manufacturers covered by trucks subject to carbon tax;
- x_{ik}: distance in Km between manufactures and demand markets covered by trucks subject to carbon tax;
- x_{mi}: distance in Km between recovery centers and manufacturers covered by trucks subject to carbon tax.

The other parameters are assumed to be fixed. Transportation cost and the distances covered in the reverse supply chain are defined as indicated in Table 1. Truck and transportation costs in the reverse CLSC are taken from [6], the values of the recycling subsidy s_m and of the landfill costs ω_m are as in [31].

Truck transportation costs forward chain		
ρ_{vnit}	0.35 €/ton-Km	$\forall r, n, i, t$
ρ_{rmit}	0.35 €/ton-Km	$\forall r, n, i, t$
ρ_{ikt}	0.35 €/ton-Km	$\forall i, k, t$
Transportation costs in the reverse chain (faced by recovery centers)		
ρ_{km}	0.14 €/ton-Km	$\forall k, m$
ρ_{rmit}	0.14 €/ton-Km	$\forall r, n, i, t$
ρ_{ikt}	0.14 €/ton-Km	$\forall i, k, t$
Distance in the reverse chain		
x_{km}	50 Km	$\forall n, i$
x_m	10 Km	$\forall i, k$

Table 1: Parameter values

Taking as reference the report by European Commission [3], we set the carbon emission rates ϕ_i^{eco} and ϕ_i^{neco} of the manufacture of ecological and non-ecological goods respectively equal to 0.6 and to 0.7 ton of CO_2 per ton of good produced. The parameter τ_t corresponding to emissions rate of heavy-duty vehicles is set equal to 622 g CO_2/ton-Km.[8] The subsidy s_m paid by recovery center to consumers that collect used goods in the demand markets is assumed to be equal to 6 €/ton and, finally, ω_m that is the cost of sending useless disassembled material to the landfill is set equal to 2 €/ton. Finally, parameters θ^{eco} and θ^{neco}, defining the return ratio of the ecological and non-ecological goods in the demand markets, are both set equal to 0.6.

[8] See `https://www.politesi.polimi.it/bitstream/10589/26301/3/2011_10_CICCARELLO.pdf`

6.1 Evaluating the EU-ETS impacts

In order to evaluate the EU-ETS impacts on the production of the ecological and the non-ecological goods, we conduct a sensitivity analysis on the CO_2 price. We consider a set of *Cases* where p^{CO_2} is defined as indicated in Table 2 and the carbon tax α is always set equal to zero.

EU-ETS Cases		
Case 0	$p^{CO_2} = 0$	$\alpha = 0$
Case 1	$p^{CO_2} = 10$	$\alpha = 0$
Case 2	$p^{CO_2} = 20$	$\alpha = 0$
Case 3	$p^{CO_2} = 30$	$\alpha = 0$
Case 4	$p^{CO_2} = 40$	$\alpha = 0$
Case 5	$p^{CO_2} = 50$	$\alpha = 0$

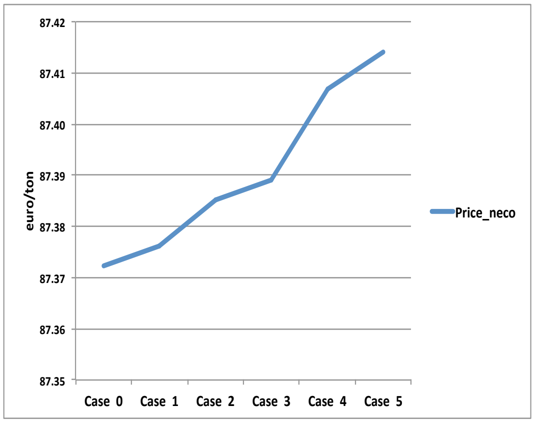
Table 2: p^{CO_2} and α values in *Case 0*, *Case 1*, *Case 2*, *Case 3*, *Case 4*, and *Case 5*

In particular, we compare a reference case where there is no application of any environmental regulations (*Case 0* in Table 2) to situations where the EU-ETS is applied in a progressively stringent way. We consider different values for the allowance price, starting from a p^{CO_2} value equal to 10 €/ton CO_2, that corresponds to the price currently registered on the market[9], and we then progressively increase it up to a value of 50 €/ton CO_2. Allowance prices of 30 and 40 €/ton CO_2 are reasonable targets for a proper functioning of the EU-ETS system and similar values have been considered by the European Commission in its studies. We also analyze a price of 50 €/ton CO_2 to investigate a particular stringent application of the EU-ETS system. In all these *Cases*, it is assumed that $\beta_{rm} = 0.4\ \forall r, m$, while x_{ni}, x_{ik}, and x_{mi} are equal to 100 Km. Since the EU-ETS is applied only at manufacturing level, we concentrate our analysis on its impacts on the production of ecological and non-ecological goods.

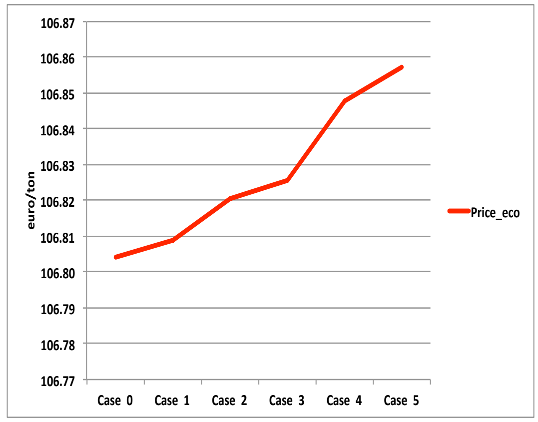
The EU-ETS application leads to an increase of the production costs since manufacturers have to buy allowances to cover the CO_2 emissions generated in the production process. The increase of production costs is proportional to allowance price because the higher is the allowance price and the higher are the carbon costs faced by manufacturers. This has as effects *(i)* the increase of the selling prices applied to ecological and non-ecological goods (see Figure 4) and *(ii)* the fall of the manufacturers' production of both the with respect to the corresponding levels in *Case 0* (see Figure 5). Moreover, the comparison between the results of *Case 0* and *Cases 1-5* shows that the production of the ecological goods decreases more than that of the non-ecological goods since the former are obtained with a more expensive production process (see Table 3 for the reduction in percentage).

Notice that the reduction of the produced quantities due to the EU-ETS has a positive impact on environment because it implies a cut of CO_2 emissions generated in the manufacturing processes. This production drop implies also a cut of the emission generated in the transportation phases since the good quantities to be transferred reduce (see Table 4).

[9]See `https://www.eex.com/en/`



(a)



(b)

Figure 4: EU-ETS impact on non-ecological and ecological good prices

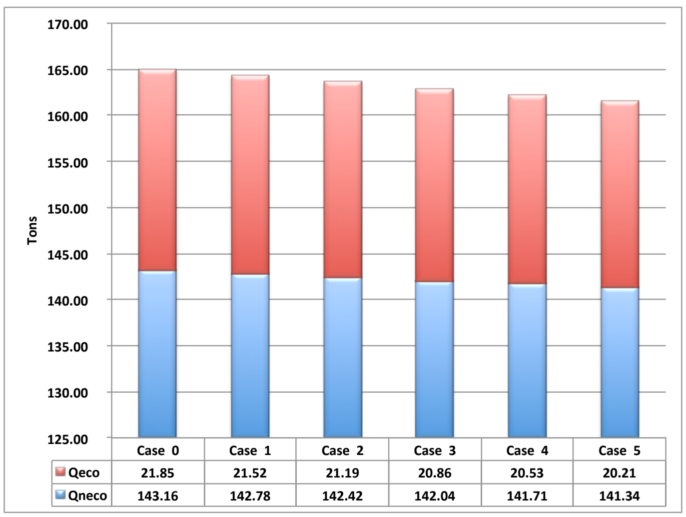


Figure 5: EU-ETS impact on the (total) quantity of non-ecological and ecological good produced

	Case 0	**Case 1**	**Case 2**	**Case 3**	**Case 4**	**Case 5**
$\sum_{i=1}^{2}\sum_{k=1}^{2}\sum_{t=1}^{2} q_{ikt}^{neco}$	-	-0.27%	-0.52%	-0.79%	-1.02%	-1.27%
$\sum_{i=1}^{2}\sum_{k=1}^{2}\sum_{t=1}^{2} q_{ikt}^{eco}$	-	-1.51%	3.02%	-4.52%	-6.02%	-7.51%

Table 3: Reduction of the quantity of ecological and non-ecological good produced under the EU-ETS compared to *Case 0* (%)

6.2 Evaluating the impacts of the application of a carbon tax on (truck) transport

To the aim of evaluating the impacts of a possible application of a carbon tax on tuck transport, we conduct a sensitivity analysis on the parameter α. We consider a set of

ton CO_2	Case 0	Case 1	Case 2	Case 3	Case 4	Case 5
CO_2 emissions from manufacturing	113.32	112.86	112.40	111.94	111.51	111.06
CO_2 emissions from transport	21.48	21.44	21.40	21.36	21.33	21.29
Total CO_2 emissions	134.81	134.30	133.81	133.30	132.84	132.35
CO_2 reduction from manufacturing compared to Case 0	-	-0.41%	-0.81%	-1.22%	-1.60%	-1.99%
CO_2 reduction in transport compared to Case 0	-	-0.20%	-0.38%	-0.58%	-0.72%	-0.91%
Global CO_2 reduction compared to Case 0	-	-0.38%	-0.74%	-1.12%	-1.46%	-1.82%

Table 4: CO_2 emissions generated in the manufacturing and transport phases in *Cases 0-5*

Cases where the values of α are as indicated in Table 5. In these *Cases*, we assume that the EU-ETS does not apply and we impose that p^{CO_2} is equal to zero. In all these *Cases*, it is assumed that $\beta_{rm} = 0.4 \; \forall r, m$, while x_{ni}, x_{ik}, and x_{mi} are equal to 100 Km. Note that the carbon tax is applied on the transport of reusable raw materials, virgin raw material, ecological and non-ecological goods. For these reason, we concentrate our analysis on these CLSC products.

Carbon tax impacts		
Case 0	$p^{CO_2} = 0$	$\alpha = 0$
Case 6	$p^{CO_2} = 0$	$\alpha = 10$
Case 7	$p^{CO_2} = 0$	$\alpha = 20$
Case 8	$p^{CO_2} = 0$	$\alpha = 30$
Case 9	$p^{CO_2} = 0$	$\alpha = 40$
Case 10	$p^{CO_2} = 0$	$\alpha = 50$

Table 5: p^{CO_2} and α values in *Case 0*, *Case 6*, *Case 7*, *Case 8*, *Case 9*, and *Case 10*

Similarly to the analysis conducted in Section 6.1, we consider the reference *Case 0* where no environmental regulation is applied and we then compare it with *Cases 6-10* where a progressively increasing carbon tax is imposed on truck transport of virgin raw materials, reusable raw materials, ecological and non-ecological goods. Figure 6 compare the amounts of these products produced and exchanged in the different *Cases*. This analysis shows that the amounts of (virgin and reusable) raw materials and (ecological and non-ecological) goods slightly decrease compared to *Case 0* even when the carbon tax becomes very stringent like in *Case 10*. This also implies a limited reduction of the CO_2 emission generated in the CLSC network (see Table 6), leading to a negative impact on environment. More specifically, the emission levels in *Cases 6-10* are lower but very close to that of *Case 0* and higher than those reached in *Cases 1-5* (compare Tables 4 and 6). This means that a carbon tax imposed on transport that is not accompanied by CO_2 regulations on manufacturing process does not have a strong positive effect on environment since the level

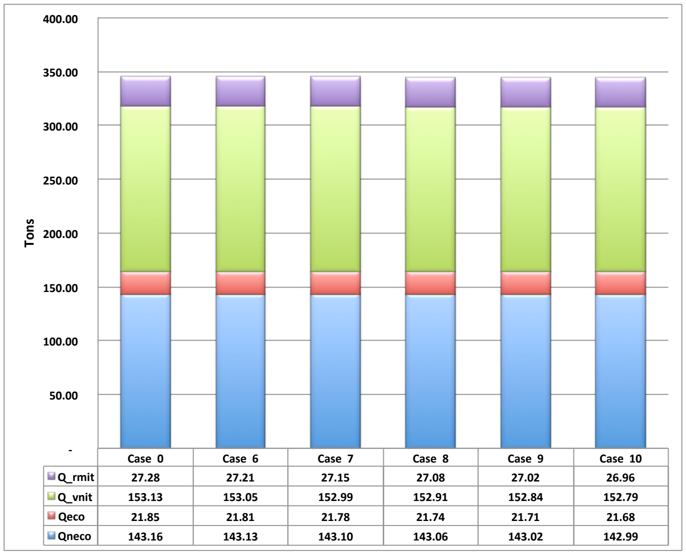


Figure 6: Impact of a carbon tax applied to truck transport on the CLSC products

of global emissions generated does not reduce in a significative way. This is due to the fact that, in absence of carbon limits on manufacturing process, like those imposed by the EU-ETS, manufacturers' production decisions are less affected and they are not induced to deeply modify the quantities of ecological and non-ecological goods produced (see Figure 6) as well as those of the employed virgin and reusable raw materials.

ton CO_2	**Case 0**	**Case 6**	**Case 7**	**Case 8**	**Case 9**	**Case 10**
CO_2 emissions from manufacturing	113.32	113.28	113.23	113.19	113.14	113.10
CO_2 emissions from transport	21.48	21.47	21.46	21.45	21.43	21.42
Total CO_2 emissions	134.81	134.75	134.69	134.63	134.57	134.52
CO_2 reduction in manufacturing compared to Case 0	-	-0.04%	-0.08%	-0.12%	-0.16%	-0.20%
CO_2 reduction in transport compared to Case 0	-	-0.06%	-0.11%	-0.18%	-0.24%	-0.29%
Global CO_2 reduction compared to Case 0	-	-0.04%	-0.08%	-0.13%	-0.17%	-0.21%

Table 6: CO_2 emissions generated in the manufacturing and transport phases in *Case 0* and *Cases 6-10*

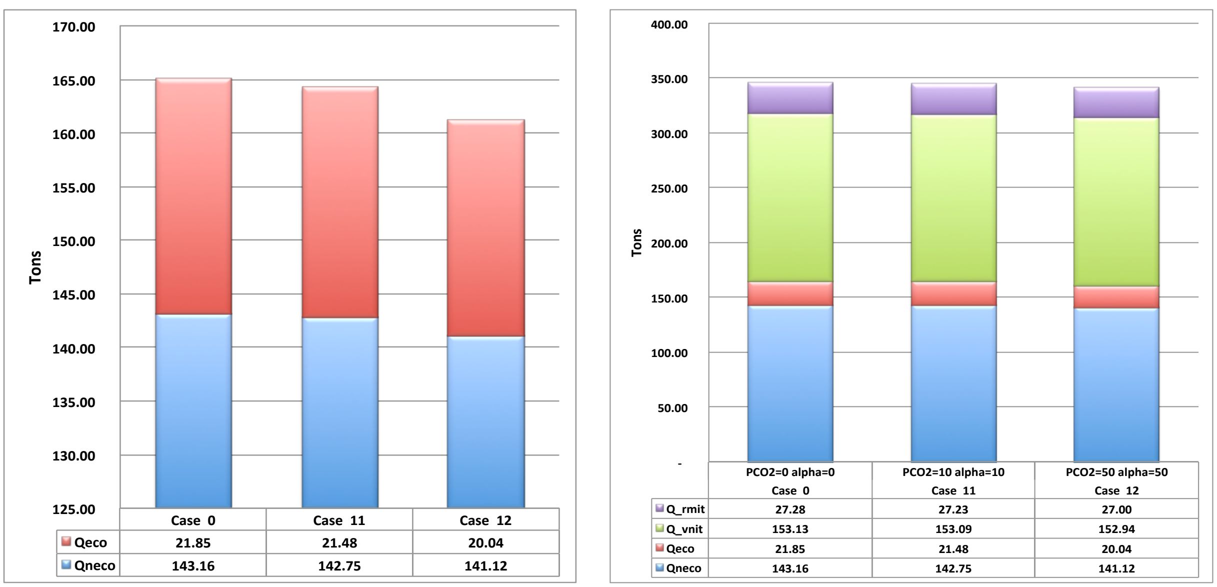
6.3 Evaluating the impacts of the combined application of the EU-ETS and of a carbon tax on transport

Considering the results of the analysis conducted in Section 6.2, we here study the effects of the joint application of the EU-ETS and a carbon tax on the CLSC product flows. For this reason, we compare *Case 0* with the *Cases 11* and *12* listed in Table 7. In all these *Cases*, it is assumed that $\beta_{rm} = 0.4 \; \forall r, m$, while x_{ni}, x_{ik}, and x_{mi} are equal to 100 Km.

EU-ETS and carbon tax impacts		
Case 0	$p^{CO_2} = 0$	$\alpha = 0$
Case 11	$p^{CO_2} = 10$	$\alpha = 10$
Case 12	$p^{CO_2} = 50$	$\alpha = 50$

Table 7: p^{CO_2} and α values in *Case 0*, *Case 11*, and *Case 12*

As indicated in Figures 7a and 7b the combined application of the EU-ETS and the carbon tax on truck transport in *Case 11* and *Case 12* leads to a reduction with respect to *Case 0* of the CLSC products, namely reusable raw materials, virgin raw materials, ecological and non-ecological goods. Reductions are more significant in *Case 12* where carbon policies are more stringent. In addition, comparing *Case 11* in Figure 7 with *Case 1* and *Case 6* respectively in Figure 5 and in Figure 6, one can see that the joint imposition of carbon restrictions on both manufacturing and transport implies a higher reduction of the quantities of CLSC products compared to situations where these environmental policies are applied separately. This also applies when comparing *Case 12* in Figure 7 with *Case 5* in Figure 5 and *Case 10* in Figure 6.



(a) (b)

Figure 7: Impact of the jointly application of the EU-ETS and a carbon tax on the CLSC products

On the other side, this means that the combined application of the EU-ETS and the carbon tax is beneficial for the environment because their joint implementation leads to a higher cut of carbon emissions as illustrated in Figure 8.

Figure 8: Comparison of the total CO_2 emission level in different *Cases*

Table 8 reports in more details the CO_2 emission reductions both at manufacturing and transport levels in *Case 11* and *Case 12* compared to *Case 0*.

ton CO_2	**Case 0**	**Case 11**	**Case 12**
CO_2 emissions from manufacturing	113.32	112.82	110.81
CO_2 emissions from transport	21.48	21.43	21.22
Total CO_2 emissions	134.81	134.25	132.03
CO_2 reduction from manufacturing compared to Case 0	-	-0.45%	-2.22%
CO_2 reduction in transport compared to Case 0	-	-0.25%	-1.25%
Global CO_2 reduction compared to Case 0	-	-0.42%	-2.06%

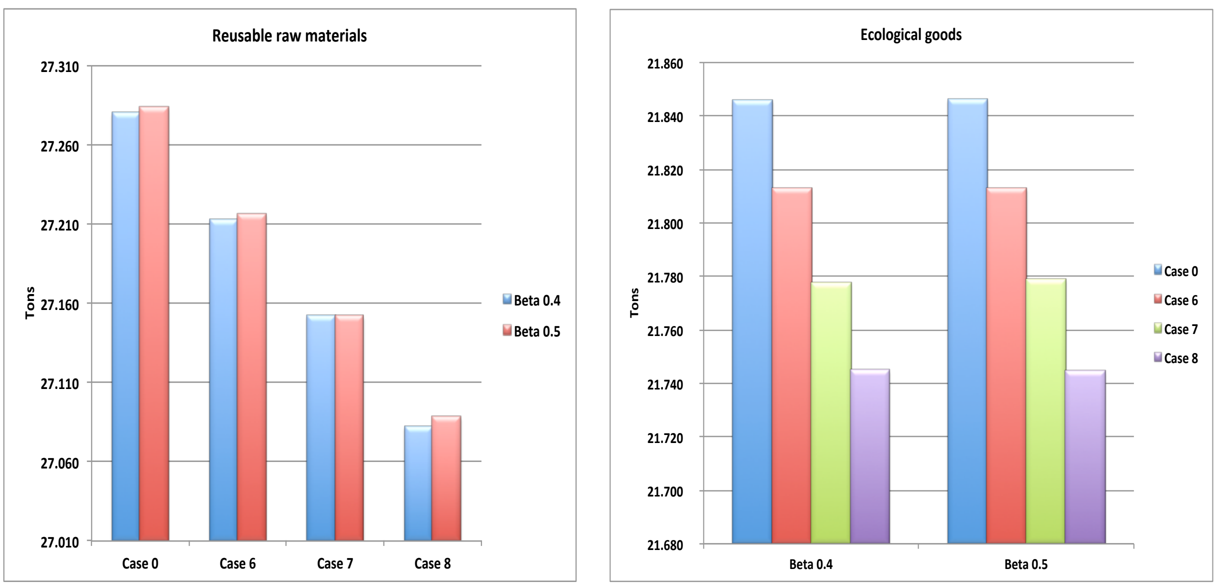
Table 8: CO_2 emissions generated in the manufacturing and transport phases in *Case 0*, *Case 11*, and *Case 12*

6.4 Evaluating the impacts of environmental policies on the recycling process in the CLSC network

Another important aspect of the CLSC is represented by the recycling process operated by recovery centers in the reserve supply chain. In our model, the output of the recycling process is represented by the reusable raw materials that are then employed by manufacturers to produce the ecological goods. The production of reusable raw materials can be directly affected by the application of the carbon tax imposed on the truck transport from recovery centers to manufacturers. For this reason, we select some of the carbon tax *Cases* presented in Section 6.2 in order to evaluate the changes in reusable raw materials and ecological goods production under this environmental restriction.

In addition to the environmental policies, the recycling process is affected by the recycling rate β_{rm}, namely the ability of recovering center to transform the used products collected at demand markets into reusable raw materials. To this aim we compare two possible values of this parameter:

- $\beta_{rm} = 0.4 \ \forall r, m$
- $\beta_{rm} = 0.5 \ \forall r, m.$



(a) (b)

Figure 9: Impact of the application of a carbon tax on transport on reusable raw materials and ecological goods

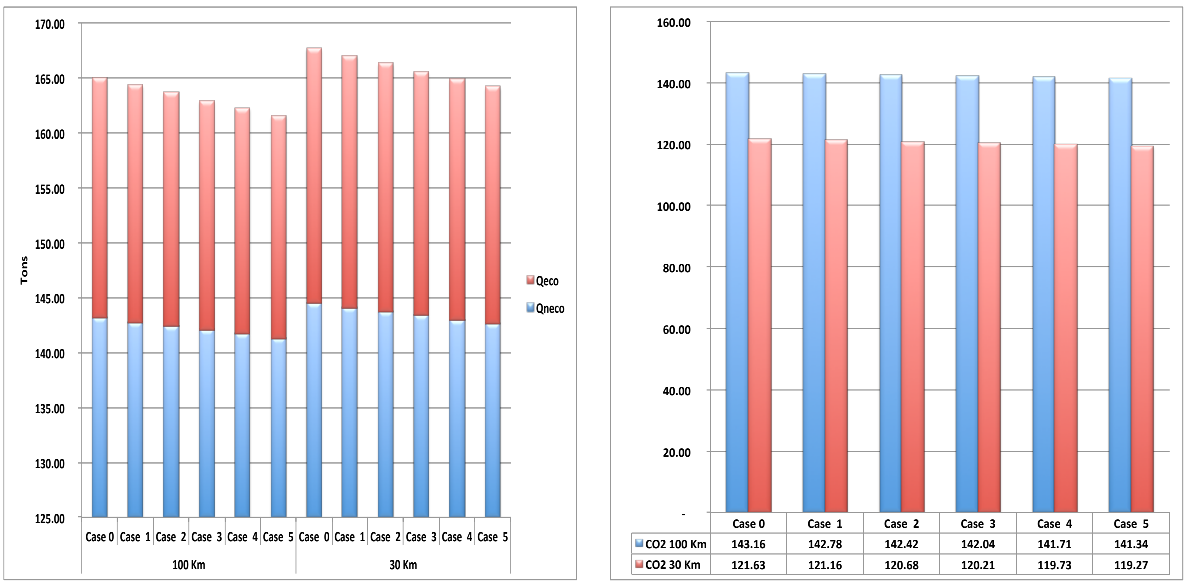
Figure 9 describes the impacts of the application of a transport carbon tax on reusable raw materials (Figure 9a) and ecological good (Figure 9b) production. In particular, we compare *Case 0* with *Case 6*, *Case 7*, *Case 8* and we consider $\beta_{rm} = 0.4$ and $\beta_{rm} = 0.5$. We recall that the carbon tax is applied to the transport of reusable raw materials from recovery centers to manufacturers and from manufacturers to demand markets.

A first remark is that a higher recycling rate (e.g. $\beta_{rm} = 0.5$) leads to an increase of the production of reusable raw materials (see Figure 9a). However, the application of a transport carbon tax has a negative effect on the production of reusable raw materials.

More precisely, recovery centers that are in charge of transferring these materials to manufacturers decrease their production as soon as the carbon tax is applied in order to decrease their costs. This production drop of reusable raw materials is proportional to the carbon tax (see Figure 9a). As already described in Section 6.2, transport carbon tax negatively affect the production of the ecological goods as well (see Figure 9b).

6.5 Evaluating the impacts of transport distance between couples of CLSC tiers on product flows in presence of carbon regulations

In order to detect the impacts of the transportation costs on the CLSC flows under environmental policies, we conduct a sensitivity analysis on the distances x_{ni}, x_{ik}, and x_{mi} covered by trucks. More precisely, we compare the results obtained by setting the distance covered by trucks x_{ni}, x_{ik}, and x_{mi} equal to 100 Km and 30 Km under *Cases 0-12*. In all considered *Cases*, it is assumed that $\beta_{rm} = 0.4 \ \forall r, m$.



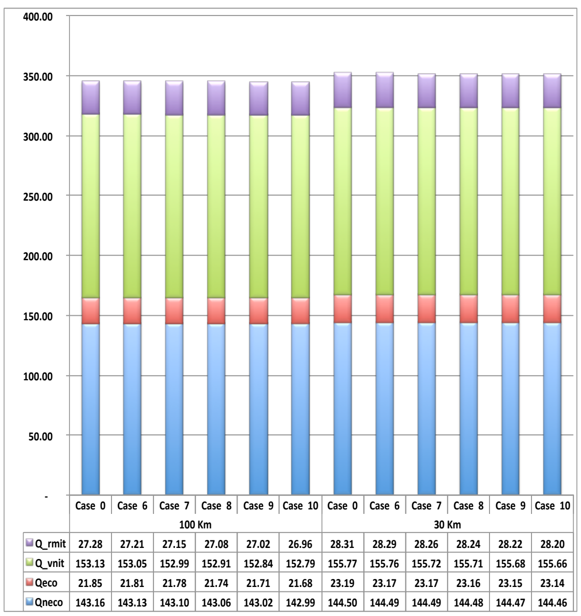
(a) (b)

Figure 10: Impacts of transport distance on ecological and non-ecological goods and emissions under the EU-ETS application

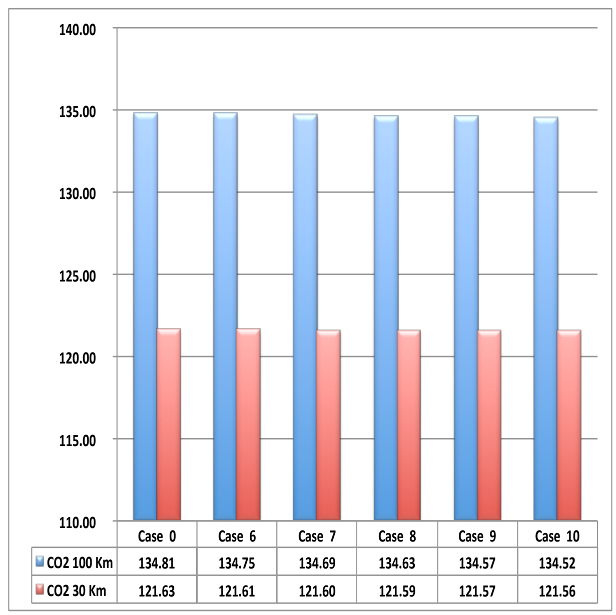
Figures 10, 11, and 12 shows the impacts of x_{ni}, x_{ik}, and x_{mi} variations on CLSC products and carbon emissions under environmental regulations. More precisely, Figure 10a shows the variation in the production of ecological and non-ecological goods when only the EU-ETS applies (*Cases 0-5*), Figure 11a illustrates how the CLSC product flows change when only the transport carbon tax is introduced (*Cases 0 and 6-10*) and, finally, Figure 12a depicts the variation of the CLSC flows when both environmental policies apply (*Cases 0 and 11-12*). Figures 10b, 11b, and 12b illustrate the amount of carbon emission globally generated in the CLSC under the different transport distance and environmental policy assumptions.

Considering the impacts on CLSC product flows, Figures 10a, 11a, and 12a show a similar trend: when the distance between tiers reduces passing from 100 Km to 30 Km, the transportation costs reduce and then there is an increase of the quantity of (virgin and reusable) raw materials, ecological and non-ecological goods due to a reduction of the transportation costs.

The reduction of transport distance has also a positive effect on CO_2 emissions that decrease significantly when the distance covered by trucks is equal to 30 Km (compare Table 9 with Tables 4, 6, and 8 respectively in Section 6.1, Section 6.2, and Section 6.3). This happens independently of the environmental policy applied (see Figure 10b, 11b, and 12b). In other words, the distance among the CLSC tiers has significant impacts on environment and product flows. This means that the optimization of transport is indispensable for reducing CO_2 emissions.



(a)



(b)

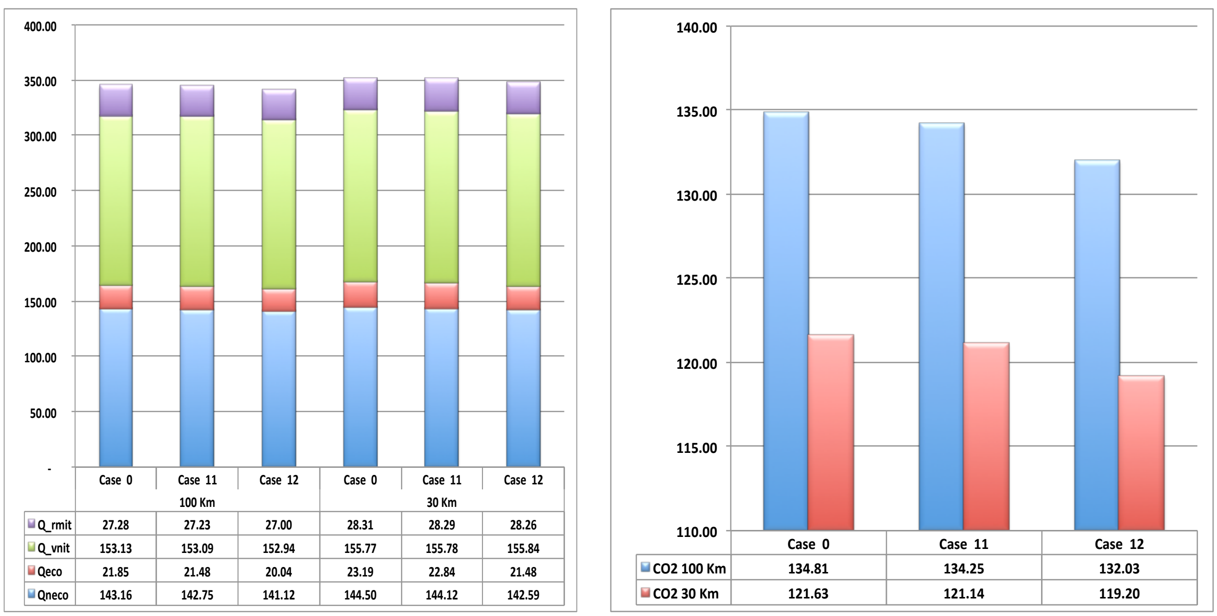
Figure 11: Impacts of transport distance on goods and emissions under the different transport carbon tax

7 Conclusions

it is the first time that that variational inequality approach is applied to develop a model that takes into carbon regulation, recycling, transportation and technological factors within a unique framework. Moreover, the existing papers in the field of variational inequalities do not investigate as we do (see Figure 1 for the considered environmental factors). We

ton CO_2	Case 0	Case 1	Case 2	Case 3	Case 4	Case 5
CO_2 100 Km	143.16	142.78	142.42	142.04	141.71	141.34
CO_2 30 Km	121.63	121.16	120.68	120.21	119.73	119.27
Reduction in %	-15%	-15%	-15%	-15%	-16%	-16%
ton CO_2	**Case 0**	**Case 6**	**Case 7**	**Case 8**	**Case 9**	**Case 10**
CO_2 100 Km	134.81	134.75	134.69	134.63	134.57	134.52
CO_2 30 Km	121.63	121.61	121.60	121.59	121.57	121.56
Reduction in %	-10%	-10%	-10%	-10%	-10%	-10%
ton CO_2	**Case 0**	**Case 11**	**Case 12**			
CO_2 100 Km	134.81	134.25	132.03			
CO_2 30 Km	121.63	121.14	119.20			
Reduction in %	-10%	-10%	-10%			

Table 9: CO_2 emissions generated globally produced



(a) (b)

Figure 12: Impacts of transport distance on goods and emissions under the EU-ETS and a transport carbon tax

also notice that variational inequality models admit many efficient solution methods, which can be easily implemented

In this paper, we analyze the effects of the application of environmental regulations on a CLSC network using a variational inequality approach. In this CLSC network, manufacturers produce both an ecological good with the reusable raw materials provided by the recovery centers and a non-ecological good using virgin raw materials bought from the suppliers. Ecological and non-ecological goods are assumed to have the same end-use. The developed model is able to take into carbon regulation, recycling, transportation and technological factors within a unique framework. The innovative aspect of the paper is that we investigate the impacts of the combined application of the EU-ETS on manufacturing and of a carbon tax on transport in a CLCS network.

The EU-ETS represents an environmental policy already applied, while the carbon tax applied on trucks can can envisage the European willingness to regulate CO_2 emissions generated by transport. Considering the issues indicated in Section 1.1 our analysis shows that:

Q1: The application of the EU-ETS only that affects the manufacturing process increases production costs. This implies a decrease of the quantity of ecological and non-ecological goods produced and an increase of their (selling) prices. However, the goal of the EU-ETS application is reached because the total amount of CO_2 emission generated reduces (Section 6.1);

Q1: The introduction of a carbon tax on truck transport only leads to slight decrease of the raw materials and goods produced even when the carbon tax becomes very stringent. This also implies that the reduction of the CO_2 emission generated in the CLSC network is lower than in the cases where the EU-ETS applies (Section 6.2);

Q1: The combined application of the EU-ETS and the carbon tax has a positive impacts on environment since the registered cut of CO_2 emissions is higher with respect to situation where these environmental policies are applied separately. On the other side, this positive effects is accompanied by a more significant drop of the quantity of good produced (Section 6.3);

Q2: A higher recycling rate leads to an increase of the amount of reusable raw material produced under all the assumption of environmental policy application (Section 6.4);

Q2: Assuming the application of the carbon tax on the transport from recovery centers to manufacturers leads to a reduction of the reusable raw materials produced (Section 6.4);

Q3: The distance between couples of CLSC tiers plays a strategic role. Reducing the distance covered by trucks is beneficial both for environment and CLSC players because it leads to an increase of the production and to a reduction of the CO_2 emissions. This means that optimizing the transport is extremely important for tackling climate change (Section 6.5).

References

[1] Barbagallo, A., Daniele, P., Giuffrè, S., Maugeri, A. (2014) Variational approach for a general financial equilibrium problem: the Deficit Formula, the Balance Law and the Liability Formula A path to the economy recovery. *European Journal of Operational Research*, **237(1)**, 231-244.

[2] Daniele, P., Maugeri, A., Oettli, W. (1999) Time-dependent traffic equilibria. *Journal of Optimization Theory and Applications*, **103**, 543-555.

[3] European Commission Directorate General for Environment, McKinsey, Ecofys (2006) *EU-ETS Review. Report on international competitiveness.* Available at `http://ec.europa.eu/clima/policies/ets/docs/report_int_competitiveness_20061222_en.pdf`

[4] Elhedhli, S., Merrick R. (2012) Green supply chain network design to reduce carbon emissions. *Transportation Research Part D*, 17:370-379.

[5] Facchinei, F., & Pang, J.-S., 2003. *Finite-Dimensional Variational Inequalities and Complementarity Problems.* Springer-Verlag, Berlin (two volumes).

[6] Gao, N., Ryan, S.M. (2014) Robust design of a closed-loop supply chain network for uncertain carbon regulations and random product flows. *EURO Journal on Transportation and Logistics*, 3(1):5-34.

[7] Govindan, K., Soleimanib, H., Kannan, D. (2015) Reverse logistics and closed-loop supply chain: A comprehensive review to explore the future. *European Journal of Operational Research*, 240(3):603-626.

[8] Guide, V.D.R., Van Wassenhove, L.N. (2008) The Evolution of Closed-Loop Supply Chain Research. *Faculty & Research Working Paper, 2008/07/TOM /ISIC.*

[9] Gupta S., Van Palsule-Desai, O.D. (2011) Sustainable supply chain management: review and research opportunities. *IIMB Management Review*, 23(4):234-245.

[10] Hammond, D., Beullens, P. (2007) Closed-loop supply chain network equilibrium under legislation. *European Journal of Operational Research*, 183:895-908.

[11] Kinderlehrer, D., Stampacchia, G. (1980) *An introduction to Variational Inequalities and their Application.* Academic Press, New York.

[12] Konnov, I.V. (1993) Combined relaxation methods for finding equilibrium points and solving related problems. *Russian Mathematics (Iz. VUZ)*, 3:44-51.

[13] Konnov, I.V. (2001) *Combined Relaxation Methods for Variational Inequalities.* Springer-Verlag.

[14] Nagurney, A. (1999) *Network Economics: A Variational Inequality Approach.* Kluwer, Dordrecht.

[15] Nagurney, A. (2003) *Innovations in financial and economic networks.* Edward Elgar Publishing, Inc.

[16] Nagurney, A., Dong, J., Zhang, D. (2002) A supply chain network equilibrium model. *Transportation Research Part E*, 38:281-303.

[17] Nagurney, A., Toyasaki, F. (2005) Reverse supply chain management and electronic waste recycling: a multitiered network equilibrium framework for e-cycling. *Transportation Research Part E*, 41(1):1-28.

[18] Nagurney, A., Liu, Z., Woolley, T. (2007) Sustainable Supply Chain and Transportation Networks. *International Journal of Sustainable Transportation*, 1(1), 29-51.

[19] Paksoy, T., Ozceylan, E., Weber, G.W. (2011) A multi objective model for optimization of a green supply chain network. *Global Journal of Technology & Optimization*, 2(2):84-96.

[20] Qiang, Q., Ke, K., Anderson, T., Dong, J. (2013). The closed-loop supply chain network with competition, distribution channel investment, and uncertainties. *Omega*, 41(2):186-194.

[21] Reinaud, J. (2009) Trade, competitiveness and carbon leakage: challenges and opportunities. *Energy, Environment and Development Programme Paper*, 09-01.

[22] Scrimali, L. (2004) Variational Inequalities and optimal equilibrium distributions in transportation networks. *Mathematical Inequalities and Applications*, **7(3)**, 439-451.

[23] Seuring, S. (2013) A review of modeling approaches for sustainable supply chain management. *Decision Support Systems*, 54(4):1513-1520.

[24] Sijm, J.P.M., Neuhoff, K., Chen, Y. (2006) Cost pass thought and windfall profits in the power sector. *Climate Policy*, 5: 61-78.

[25] Sivrastava, S.K. (2007) Green supply-chain management: A state-of-the-art literature review. *International Journal of Management Reviews*, 9(1):53-80.

[26] Toyasaki, F., Daniele, P., Wakolbinger, T. (2014) A variational inequality formulation of equilibrium models for end-of-life products with nonlinear constraints. *European Journal of Operational Research*, 236:340-350.

[27] Wang, H.-F., Hsu, H.-W. (2010) A closed-loop logistic model with a spanning-tree based genetic algorithm. *Computers & Operations Research*, 37:376-389.

[28] Wang, F., Lai, X., Shi, N. (2011) A multi-objective optimization for green supply chain network design. *Decision Support Systems*, 51:262-269.

[29] Wang, G., Gunasekaran, A. (2015) Modeling and analysis of sustainable supply chain dynamics. *Annals of Operations Research*, 51:262-269.

[30] Wu, K., Nagurney, A., Liu, Z., Stranlund J.K. (2006) Modeling generator power plant portfolios and pollution taxes in electric power supply chain networks: A transportation network equilibrium transformation. *Transportation Research Part D*, 11:171-190.

[31] Yang, G., Wang, Z., Li, X. (2009) The optimization of the closed-loop supply chain network. *Transportation Research Part E*, 45:16-28.