Pricing wind: a revenue adequate, cost recovering uniform auction for electricity markets with intermittent generation*

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Abstract

With greater penetration of renewable generation, the uncertainty faced in electricity markets has increased substantially. Conventionally, generators are assigned a pre-dispatch quantity in advance of real time, based on estimates of uncertain quantities. Expensive real time adjustments then need to be made to ensure demand is met, as uncertainty takes on a realization. We propose a new stochastic-programming market clearing mechanism to optimize pre-dispatch quantities, given the uncertainties' probability distribution and the costs of real-time deviation. This model differs from similar mechanisms previously proposed in that pre-dispatch quantities are not subject to any network or other physical constraints; nor do they play a role in financial settlement. We establish revenue adequacy in each scenario (as opposed to "in expectation"), welfare enhancement and expected cost recovery (including deviation costs), for this market clearing mechanism. We also establish that this market clearing mechanism is social welfare optimizing.

Key words and phrases: stochastic programming, locational pricing, wind power, regulation.

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1 Introduction

The increasing penetration of intermittent renewable power generation (such as wind and solar) has led to much discussion of how to cope with the uncertainty these sources induce in electricity markets. Several authors (e.g. [3, 8, 12, 18, 2, 14]) have presented re-formulations of the traditional optimal power flow problem as two-stage stochastic programs; in the present paper, we aim to improve on these.

In models of this type, the first stage represents an initial dispatch computed in advance, with only probabilistic estimates of some quantities available. This could be thought of as a "day-ahead" dispatch, although the same ideas can be applied on shorter time scales. The second stage represents a regulation or balancing market operated in or near real-time.

Inasmuch as the initial dispatch may be modified later, it could be thought of as "non-physical"; however, the associated first-stage prices ("contract prices") may have real financial implications. In existing literature, the first stage is designed to be constrained with physical system constraints.

In general, these models exhibit revenue adequacy in expectation (see [8, 12, 18]). That is, the total expected payments made by consumers will equal or exceed the total expected compensation paid to generators. However, revenue adequacy need not occur in every scenario – an example is given in [12]. The model proposed in the present paper is free from this defect.

Another desirable property is cost recovery: the compensation paid for each dispatched energy tranche to the firm that offered it should equal or exceed the cost of producing it. The per-unit production cost may be taken (as e.g. in [8]) to coincide with the price at which the tranche was offered. Our model presents a uniform pricing model that is proved to have cost recovery in expectation, including the cost of any deviations.

We establish that our market clearing model is incentive compatible. We demonstrate here that the decisions made by an independent system operator, who clears the market based on maximizing expected total welfare align with those made by individual agents, as long as those agents are truthful (i.e. a competitive equilibrium is reached), as well as risk neutral.

One can also ask the usual questions asked of a stochastic program, i.e. do the above properties remain valid when the sample incorporated in the stochastic program is not the full underlying distribution? Revenue adequacy is maintained for our model even if we encounter a scenario that we had not included in the sample. Furthermore, we establish an asympthotic result that demonstrates expected cost recovery, as long as the chosen sample is "rich enough" in being reflective of the true distribution.

2 The new SP model

We assume a power system modelled by DC load flow approximation, consisting of a collection of nodes connected by lines. The lines are assumed lossless, so that if F is a vector of flows on the lines, the net power $\tau_n(F)$ imported into node n by F is a linear functional of F. The capabilities of the transmission system are represented by the requirement $F \in U$; we assume U to be a bounded convex polyhedral set with $0 \in U$.

The supply side of the market will comprise a finite collection of tranches, with tranche j consisting of a quantity g_j of energy offered at a node $n(j)$. Let $O(n)$ denote the set of tranches offered at node n.

2.1 The real-time problem.

Consider the following optimal power flow problem, to be thought of as representing real-time dispatch:

$$\begin{array}{llcll} [RT(x,d)]\text{:} & \min & \sum_j \left(c_j X_j + r_j^+ (X_j - x_j)_+ + r_j^- (X_j - x_j)_-\right) & & \\ & \text{s.t.} & \tau_n(F) + \sum_{j \in O(n)} X_j = d_n & \forall n & [\pi_n] \\ & & 0 \leq X_j \leq g_j & \forall j & \\ & & F \in U & & \end{array} \tag{1}$$

Here X_j represents the quantity of energy to accept (dispatch) from tranche j (for each j), and F the vector of associated line flows. The parameter $d = (d_n)$ gives the demand for energy at each node. The parameter $x = (x_j)$ reflects a non-standard offer structure: each offered tranche j has an associated per-unit cost (or ask price) c_j, with additional costs applying when the dispatch exceeds (r_j^+) or falls short of (r_j^-) the level x_j. Here x_j represents a previously established setpoint or pre-dispatch, and the objective terms including it represent the costs of making real-time adjustments away from this level. We assume $r_j^+ \geq 0$ and $r_j^- \geq 0$, but it is possible that $c_j < 0$.

Note that $RT(x,d)$ is a linear program. It is a variant of the standard dispatch problem considered e.g. in [11]. We assume in this paper that $RT(x,d)$ is always feasible; this can be achieved by (for example) allowing unlimited load-shedding – which is effectively a high-priced offer of supply – at any node.

Now suppose that (X^*, F^*) is a primal optimal solution of $RT(x, d)$, and that (π_n) are optimal dual prices for the energy balance constraints (i.e. energy prices at the nodes). We will assume that energy is traded at the prices π_n with no side payments. That is, the consumers at node n collectively pay $\pi_n d_n$ for their demand d_n, and the provider of tranche j receives $\pi_{n(j)} X_j^*$ for the energy it supplies.

According to the Lagrangian Duality Theorem (see e.g. [17]), the Lagrangian

$$\begin{aligned} L(X,F) = \; & \textstyle\sum_j \left(c_j X_j + r_j^+ (X_j - x_j)_+ + r_j^- (X_j - x_j)_- \right) \\ & + \textstyle\sum_n \pi_n \left(d_n - \tau_n(F) - \sum_{j \in O(n)} X_j \right) \end{aligned} \tag{2}$$

is minimized, subject to the remaining constraints $0 \leq X_j \leq g_j \; \forall j$ and $F \in U$, at (X^*, F^*). This leads immediately to several useful observations.

2.2 Revenue adequacy.

Considering the terms containing F in (2), we see that $\sum_n \pi_n \tau_n(F)$ is maximized over $F \in U$ at F^*. By comparison with the feasible point $0 \in U$ (all zero flows), this gives $\sum_n \pi_n \tau_n(F^*) \geq 0$. But, since (X^*, F^*) satisfies the energy balance constraints in $RT(x, d)$, $\tau_n(F^*) = d_n - \sum_{j \in O(n)} X_j^*$. So we have

$$\sum_n \pi_n d_n \geq \sum_j \pi_{n(j)} X_j^*.$$

That is, the revenue raised from consumers is sufficient to pay the suppliers. Note that this result holds for all values of the parameters x and d; in particular, it is not necessary to choose the setpoints x in any particular way.

2.3 Pricing and dispatch of individual offers.

Considering the terms containing X_j in (2), we see that for each j,

$$(c_j - \pi_{n(j)}) X_j + r_j^+ (X_j - x_j)_+ + r_j^- (X_j - x_j)_-$$

is minimized over $0 \leq X_j \leq g_j$ at X_j^*. This reveals the relationship between the dispatch of a particular tranche and the energy price at its local node:

$$\begin{array}{ll} \text{if } \pi_{n(j)} < c_j - r_j^-, & \text{then } X_j^* = 0 \\ \text{if } \pi_{n(j)} = c_j - r_j^-, & \text{then } 0 \le X_j^* \le x_j \\ \text{if } c_j - r_j^- < \pi_{n(j)} < c_j + r_j^+, & \text{then } X_j^* = x_j \\ \text{if } \pi_{n(j)} = c_j + r_j^+, & \text{then } x_j \le X_j^* \le g_j \\ \text{if } \pi_{n(j)} > c_j + r_j^+, & \text{then } X_j^* = g_j. \end{array}$$

2.4 Supplier margins.

Assume that the objective of (1) reflects suppliers' actual costs. Then the margin (gross profit) made by the supplier of tranche j will be

$$\pi_{n(j)} X_j^* - c_j X_j^* - r_j^+ (X_j^* - x_j)_+ - r_j^- (X_j^* - x_j)_-.$$

In the light of the above result on X_j^*, this margin can be expressed as

$$m_j(x_j, \pi_{n(j)}) = \begin{cases} -r_j^- x_j, & \text{if } \pi_{n(j)} \le c_j - r_j^- \\ (\pi_{n(j)} - c_j) x_j, & \text{if } c_j - r_j^- \le \pi_{n(j)} \le c_j + r_j^+ \\ (\pi_{n(j)} - c_j - r_j^+) g_j + r_j^+ x_j, & \text{if } \pi_{n(j)} \ge c_j + r_j^+. \end{cases} \tag{3}$$

In particular, the margin is positive if $\pi_{n(j)} > c_j$ and negative if $\pi_{n(j)} < c_j$. The possibility of a negative margin (i.e. of the market accepting a supplier's offer in a way that fails to cover the supplier's costs) is in contrast to the standard DCOPF market formulation (see, e.g. [11]) in which supplier margins are always non-negative.

Note also from (3) that the supplier's margin $m_j(x_j, \pi_{n(j)})$ is uniquely determined (provided the local nodal price $\pi_{n(j)}$ is uniquely determined) even in degenerate cases where X_j^* is not uniquely determined. Such a case may occur, for example, if two identical tranches are offered at the same node.

2.5 The dual real-time problem.

Now consider the dual $DRT(x, d)$ of the linear program $RT(x, d)$. When we express $RT(x, d)$ as a linear program in standard form, the parameters x and d are constraint right-hand-sides; thus they appear in $DRT(x, d)$ only as objective coefficients. That is, the feasible set S of $DRT(x, d)$ does not depend on x or d. From this observation we can derive a result which will be useful later in the paper, and may also be of independent interest.

Lemma 2.1 *There exists a constant C, depending only on the feasible set S, such that for any x, the set*

$$\{d : RT(x,d) \text{ has unique energy price duals satisfying } |\pi_n| \leq C\}$$

has full Lebesgue measure.

Remark. This result offers reassurance that the prices generated by the proposed mechanism are both well-determined and bounded – unless the system is exceptionally unlucky in the demands experienced at its nodes.

Proof. Since S is a (possibly unbounded) polyhedral set with finitely many vertices, we can choose C so that $\|v\|_\infty \leq C$ for all vertices v of S. The optimal set of any problem $DRT(x,d)$ will be a face f (possibly just a single vertex) of S; if the energy price variables (π_n) are uniquely determined (i.e. constant on f), then they will satisfy $|\pi_n| \leq C$. It therefore suffices to show that for a fixed x, the set

$$\Delta_x = \{d : RT(x,d) \text{ has non-unique energy price duals } (\pi_n)\}$$

has Lebesgue measure zero.

For each face f of S on which the (π_n) variables are not constant, choose points v_f, w_f differing in at least one π_n component. When $DRT(x,d)$ has such a face f as its optimal set, its objective row is orthogonal to $v_f - w_f$. Since this objective row includes the demands d_n as coefficients corresponding to the variables π_n, we have $a_f^T d + b_f = 0$ for some non-zero vector a_f and scalar b_f. So

$$\Delta_x \subseteq \bigcup_f \left\{d : a_f^T d + b_f = 0\right\}.$$

Since S has only finitely many faces, the latter set is a finite union of hyperplanes, with Lebesgue measure zero. ∎

2.6 The stochastic programming problem.

We turn now to the problem of selecting optimal setpoints x prior to solution of the real-time problem (1). Let $J(x,d)$ denote the optimal value of the problem $RT(x,d)$. Consider the first-stage problem

$$[FP(\mu)]: \quad \min_x \quad E\left[J(x,D)\right] \tag{4}$$

where D is a random vector of demands with distribution μ. Note that this is an unconstrained optimization: we place no *a priori* constraints on the decision variable x.

We can bring (1) and (4) together as the two-stage stochastic programming problem

$$\begin{array}{llll} [SP]:\ \min & E\left[\sum_j \left(c_j X_j + r_j^+(X_j - x_j)_+ + r_j^-(X_j - x_j)_-\right)\right] & & \\ \text{s.t.} & \tau_n(F) + \sum_{j\in O(n)} X_j = D_n & \forall n \text{ w.p.1} & [\pi_n] \\ & 0 \le X_j \le g_j & \forall j \text{ w.p.1} & \\ & F \in U & \text{w.p.1} & \end{array} \tag{5}$$

Here the demands D_n, the second-stage decision variables X_j and F, and the second-stage dual variables π_n are all random variables, i.e. have values depending on an outcome ω selected from a sample space Ω. To simplify notation, we follow the usual convention of suppressing the dependence on ω for random quantities; thus e.g. F rather than $F(\omega)$.

Let (x^*, X^*, F^*) be optimal for (5); then x^* is optimal for (4) (see [3] or [5]). As with the real-time problem, Lagrangian duality (for this case see [15]) can be used to establish properties of the solution. The Lagrangian of (5)

$$\begin{array}{rl} L(x, X, F) = & E\left[\sum_j \left(c_j X_j + r_j^+(X_j - x_j)_+ + r_j^-(X_j - x_j)_-\right)\right] \\ & +E\left[\sum_n \pi_n \left(D_n - \tau_n(F) - \sum_{j\in O(n)} X_j\right)\right] \end{array} \tag{6}$$

is minimized, subject to the remaining constraints $0 \le X_j \le g_j\ \forall j$ w.p.1 and $F \in U$ w.p.1, at (x^*, X^*, F^*). This leads immediately to some further useful observations.

2.7 Optimal setpoints are quantiles of real-time dispatch.

Considering the terms containing x_j in (6), we see that

$$E\left[r_j^+(X_j^* - x_j)_+ + r_j^-(X_j^* - x_j)_-\right]$$

is minimized over the unconstrained variable x_j at x_j^*. If $r_j^+ = r_j^- = 0$ this is a trivial result, but otherwise the problem is a variant of the well known "News Vendor" stochastic optimization. The result follows (see, e.g. [15, 7]) that x_j^* is the $\frac{r_j^+}{r_j^+ + r_j^-}$ quantile of the probability distribution of X_j^*.

2.8 Expected supplier margins.

Considering the terms containing x_j or X_j in (6), we see that

$$E\left[(c_j - \pi_{n(j)})X_j + r_j^+(X_j - x_j)_+ + r_j^-(X_j - x_j)_-\right]$$

is minimized, subject to $0 \leq X_j \leq g_j$ w.p.1 (and x_j unconstrained), at (x_j^*, X_j^*). It follows by comparison with the feasible solution $x_j = X_j = 0$ that

$$E\left[(c_j - \pi_{n(j)})X_j^* + r_j^+(X_j^* - x_j^*)_+ + r_j^-(X_j^* - x_j^*)_-\right] \leq 0.$$

That is, the *expected* margin made by the supplier of tranche j (see section 2.4) is non-negative. This result offers some solace to suppliers: while this market model may deliver them negative margins on occasion, their margins are at least non-negative in expectation.

2.9 Uplift payments

The possibility of negative margins suggests a need for uplift payments to suppliers to ensure that their costs are recovered. The most straightforward approach would be a simple uplift payment in each market trading period equal to the negative part of the supplier's margin on each tranche. Alternatively, the result of Section 2.8 suggests a way to make the uplift payments smaller: aggregate the margin over multiple market trading periods and make an uplift payment equal to the negative part of the total. As the number of trading periods aggregated over increases, the required uplift payments should tend to zero.

It should be noted that the latter approach is not entirely consistent with the ideas of [**?**] regarding appropriate incentives for longer-term investment. However further discussion of this point is beyond the scope of the present paper.

2.10 Competitive equilibrium.

The Lagrangian (6) offers a way to regard the optimum as the solution to a competitive game. It may be expressed as

$$\begin{aligned} L(x, X, F) = \quad & -\sum_j E\left[(\pi_{n(j)} - c_j)X_j - r_j^+(X_j - x_j)_+ - r_j^-(X_j - x_j)_-\right] \\ & -E\left[\sum_n \pi_n \tau_n(F)\right] + E\left[\sum_n \pi_n D_n\right]. \end{aligned} \tag{7}$$

Consider a game in which each energy tranche j is offered by an agent A_j desiring to maximize the supplier margin on that tranche, while an additional agent

Firm	cost of gen.	ramp up	ramp down	capacity
F1	10	1	-1	50
F2	20	5	0.001	80
I1	0.01	1	0.001	20 in scen 1 50 in scen 2

Table 1: Offer costs (in $) and quantities (in MW) for all firms.

A_{trans} controls the transmission system and desires to maximize the transmission loss and constraint rental $\sum_n \pi_n \tau_n(F)$. If the game is perfectly competitive and all agents are risk-neutral (so that expected values will serve as their objectives) then (7) shows that the optimum (x^*, X^*, F^*) is an equilibrium for the game. In economic language: the competitive solution is social-welfare maximizing.

The reader may wonder if a similar result could be proved when the agents in the market are risk averse. In this case, if the market for trading risk is complete (i.e. there are enough instruments such as hedge contracts available to agents), then the results obtained in [13] can be applied. In this case, we can demonstrate that the risk adjusted system position will coincide with the risk-averse, competitive equilibrium.

3 Illustrative examples

In this section we lay out two simple examples to illustrate the properties of our model.

3.1 Single node example

Consider a single node system where two firm and one intermittent suppliers offer in energy and a deterministic demand of 100MW. The intermittent supplier (firm I1) faces two equally likely production scenarios, one low and one high. The cost of generation, ramp up and down costs as well as capacities of generation are provided in Table 1. The dispatch problem can then be formulated as

$$
\begin{aligned}
\text{[Ex1]} \quad \min \sum_{\omega \in \{1,2\}} \frac{1}{2} \Big(& 10X_1^\omega + 20X_2^\omega + 0.01X_3^\omega \\
& +1(X_1^\omega - x_1)_+ + 1(X_1^\omega - x_1)_- \\
& +5(X_2^\omega - x_2)_+ + 0.001(X_2^\omega - x_2)_- \\
& +1(X_3^\omega - x_3)_+ + 0.001(X_3^{\omega_1} - x_3)_- \Big)
\end{aligned}
$$

$$
\begin{aligned}
\text{s/t} \qquad X_1^\omega + X_2^\omega + X_3^\omega &= 100 \quad \omega \in \{1,2\} \\
0 \le X_1^\omega &\le 50 \quad \omega \in \{1,2\} \\
0 \le X_2^\omega &\le 80 \quad \omega \in \{1,2\} \\
0 \le X_3^1 &\le 20 \\
0 \le X_3^1 &\le 50 \\
x_i &\ge 0 \quad i \in \{1,2,3\}
\end{aligned}
$$

The solution to this problem is outlined in Table 2.

firm	advisory position	real time generation
F1	50	50 in scen. 1 50 in scen. 2
F2	30	30 in scen. 1 0 in scen. 2
I1	50	20 in scen. 1 50 in scen. 2

Table 2: Advisory and real time generation (in MW) for the firms.

For this problem the optimal objective is \$800.38, and the probability adjusted second stage duals are 20.001 and 11 for first and second scenarios respectively. Table 3 outlines the revenues, costs and profits of the generators in this simple example.

This example demonstrates the mechanics of the proposed market clearing and settlement schemes. It is clear here that the we maintain revenue adequacy in each scenario and that generators recover cost in expectation. We will also point out that if a "physical" first stage constraint, such as proposed in [12] is added to the model that welfare decreases. Here if we ensure that that first stage advisory

firm and scenario	revenue	cost	profit	expected profit
F1 scen 1	1000.05	500	500.05	275.025
F1 scen 2	550	500	50	
F2 scen 1	600.03	600	0.03	0
F2 scen 2	0	0.03	-0.03	
I1 scen 1	400.02	0.23	399.79	474.645
I1 scen 2	550	0.5	549.5	

Table 3: Computation of payments (in $) to the generators.

position must meet demand, i.e. $\sum_i x_i = 100$, then the optimal dispatch cost increases to $815.37.

3.2 Two node example

The example depicted in Figure 1 is taken from [12]. It has two inelastic demand scenarios (this can be thought of as demand net of renewable generation) which, although they agree as to the total load, differ markedly in the location of the load. The Thermal generation offer is completely inflexible (requiring $X_i(\omega_1) = X_i(\omega_2) = x_i$), while the Hydros are completely flexible with indicated deviation costs. The line is lossless.

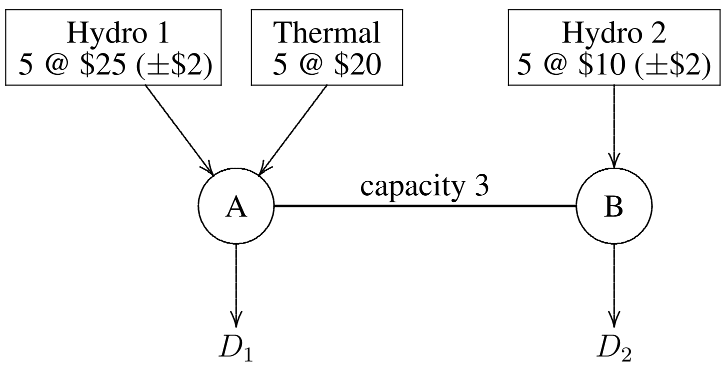

Figure 1: A two-node system with random loads.

The optimal advisory position for this problem is given by $x^* = (0, 3, 5)$, that is, 3 units from the Thermal and all 5 units of Hydro 2. The optimal real time

Scenario	probability	D_1	D_2
ω_1	0.6	2	6
ω_2	0.4	7	1

Table 4: Scenarios for the two-node problem.

dispatches for this example are given by $X^*(\omega_1) = (0, 3, 5)$ for scenario 1 and $X^*(\omega_2) = (1, 3, 4)$ for the second scenario. Furthermore the nodal prices are given by $\pi_A^{\omega_1} = \pi_B^{\omega_1} = 15.333333$ for scenario 1, and $\pi_A^{\omega_2} = 27$, and $\pi_B^{\omega_2} = 8$ for scenario 2. For this example, the total payment in scenario one is \$122.667 which is precisely what is collected from the consumer. In scenario two, the payment to generators adds up to \$140.00 while the collected revenues from demand is \$197.00. The difference between the collected revenues and payments to the generators is entirely due to congestion rent in this scenario. Clearly the settlement mechanism is revenue adequate in each scenario. We note that if physical constraints are imposed on the advisory position and the settlement scheme follows what is suggested in [12] then collected revenues in scenario one will be insufficient to cover the participant payments.

4 Continuity properties

The probability distribution of the demands D in (4) is inevitably a modelling approximation to the real world. In practice it is likely to be a discrete distribution, consisting of a finite collection of scenarios with attached probabilities. In this section we consider the possibility of the actual demand vector being drawn from a "true" probability distribution different from the one used in (4) to determine setpoints x. In particular, this affects the result of section 2.8: expected supplier margins are non-negative *if* the setpoints have been chosen optimally for the demand distribution that will actually be experienced.

Proposition 4.1 *Let $\mu_1, \mu_2, \ldots$ be probability distributions of demand, and suppose that the μ_n converge in distrbution to a limiting distribution μ, with the supports of μ and the μ_n all lying within a bounded set. Let x^{n*} be a minimizer of $FP(\mu_n)$. Then any limit point x^* of $\{x^{n*}\}$ is a minimizer of $FP(\mu)$.*

Remark. Our intended interpretation of this result casts μ in the role of the true distribution of demand, and the μ_n as modelling approximations thereof. The result suggests that the minimizers of $FP(\mu_n)$ may be regarded as approximations to a minimizer of $FP(\mu)$.

Proof. Apply Theorem 12 of Chapter 9 in [1]. The uniform bound on the supports of μ_n, μ and the fact that $J(\cdot, D)$ is convex and finite-valued with probability 1, ensure that the required hypotheses are satisfied. ∎

We turn now to the supplier margins considered in sections 2.4 and 2.8. Let $M_j(x, d)$ be the margin made by the supplier of tranche j in problem $RT(x, d)$; recall that this margin may be expressed as $m_j(x_j, \pi_{n(j)})$, where m_j is the (continuous) function given in (3).

Proposition 4.2 *Let the demand vector D be drawn from a probability distribution μ which has bounded support and is absolutely continuous with respect to Lebesgue measure. Then $E\left[M_j(x, D)\right]$ is continuous in x.*

Remark. Combining Propositions 4.1 and 4.2 offers comfort to suppliers even when the modelled distribution of demand differs from the true one. The expected supplier margin – which according to Section 2.8 is non-negative when the demand distribution is modelled exactly – will vary continuously as the modelled demand distribution (and associated optimal setpoints) are varied away from this ideal.

Proof. Let the sequence (x^k) converge to x. Let the demand vector d be such that $RT(x, d)$ has uniquely determined prices (π_n). (According to Lemma 2.1, the random vector D will take on such a value with probability 1.) As noted in Section 2.5, the feasible set S of $DRT(x, d)$ does not depend on either x or d. It follows from this that for sufficiently large k, $DRT(x^k, d)$ will have its optimum on the same face of S as $DRT(x, d)$; in particular, the energy prices (π_n) will be the same for $RT(x^k, d)$ as they are for $RT(x, d)$. We then have

$$m_j(x_j^k, \pi_{n(j)}) \to m_j(x_j, \pi_{n(j)})$$

by continuity of m_j. That is, $M_j(x^k, d) \to M_j(x, d)$. This gives $M_j(x^k, D) \to M_j(x, D)$ with probability 1. The result follows by the dominated convergence theorem, with Lemma 2.1 providing the necessary boundedness for the duals. ∎

The result of Proposition 4.2 does not hold if the demand distribution is discrete rather than continuous.

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