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# Exact algorithms for bi-objective ring tree problems with reliability measures

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ABSTRACT

We introduce bi-objective models for ring tree network design with a focus on network reliability within telecommunication applications. Our approaches generalize the capacitated ring tree problem (CRTP) which asks for a partially reliable topology that connects customers with different security requirements to a depot node by combined ring and tree graphs. While the CRTP aims at optimizing the edge installation costs, we propose four alternative, reliability-oriented objective functions. We study the case of service interruptions due to single-edge failures, and consider the overall number of tree customers and tree edges, the maximal number of subtree customers, and the maximal number of tree hops from rings as additional measures. To model the corresponding novel bi-objective problems, we develop mathematical multi-commodity flow formulations and identify relationships between the new objectives. For identifying the Pareto fronts, we apply an $\epsilon$-constraint method based on integer programming. The computational efficiency is increased by employing local search heuristics in order to tighten upper bounds and by valid inequalities to strengthen lower bounds in the subproblems. In a computational study we report results, illustrate solution network topologies and extensively analyze the algorithm performance for instances from the literature.



## 1. Introduction

### 1.1. Motivation

In telecommunication network design, the minimization of costs for constructing infrastructure is of major importance since it typically represents the predominant cost factor. However, alternative measures such as, e.g., coverage, lead times or stability are of high practical relevance in order to provide a competitive service level to the customers. Especially the loss of connection for a subset of customers is a serious incident that telecommunication providers seek to avoid by implementing reliable network structures. Due to high infrastructure cost, in particular in wire-based networks, it might not be feasible to install complete redundancy in practice. Thus, a compromise between cost- and reliability-oriented policy is required.

In this work, the failure of a single connection and the implications for the communication are considered within the initial design of the network. In our approach we group customers by their valuation (or need) of a highly stable network service. Customers that require this augmented stability (type 2) will be willing to pay an increased charge and receive special consideration by being connected redundantly. For the remaining customers it is assumed that simple connectivity is sufficient (type 1). A combinatorial optimization model which follows this approach is the NP-hard capacitated ring tree problem (CRTP) Hill (2015); Hill et al. (2017); Hill and Voß (2016, 2017). The solution of a CRTP is a hybrid structure called ring tree, which is a connected graph, consisting of rings that intersect in a single depot node and trees that can be either rooted at this depot or at one of the ring nodes. Type 2 customers have to be located on rings in a solution and thus remain connected in the case of any single edge failure. Type 1 customers may either be tree or ring nodes and non-customer (Steiner) nodes may be used as intermediate nodes anywhere within the ring trees. Moreover, practically relevant capacity restrictions that limit the number of customers in a ring tree as well as the number of ring trees are imposed.

In the CRTP it is assumed that the elevation of the reliability for the type 1 customers can be accomplished by their inclusion in the rings. Since this implies a significant cost increase it is necessary to identify partial reliability improvements for selected customers while measuring the rise of network costs. The following

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