

No. 2016-038

**ADJUSTABLE ROBUST STRATEGIES FOR FLOOD
PROTECTION**

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21 September 2016

ISSN 0924-7815
ISSN 2213-9532

Adjustable robust strategies for flood protection

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September 21, 2016

Abstract

Flood protection is of major importance to many flood-prone regions and involves substantial investment and maintenance costs. Modern flood risk management requires often to determine a cost-efficient protection strategy, i.e., one with lowest possible long run cost and satisfying flood protection standards imposed by the regulator throughout the entire planning horizon. There are two challenges that complicate the modeling: (i) *uncertainty* - many of the important parameters on which the strategies are based (e.g. the sea level rise) are uncertain, and will be known only in the future, and (ii) *adjustability* - decisions implemented at later time stages need to adapt to the realized uncertainty values. We develop a new mathematical model addressing both, based on recent advances in integer robust optimization and we implement it on the example of the Rhine Estuary - Drechtsteden area in the Netherlands. Our approach shows, among others, that (i) considering 40% uncertainty about the sea level rise leads to solution with a less than 10% increase in the total cost (ii) solutions taking the uncertainty into account are cheaper in the long run if the ‘bad scenarios’ for the uncertainty materialize, even if the ‘optimistic solutions’ are allowed to be fixed later on (iii) the optimal here-and-now investment decisions change when uncertainty and adjustability are considered.

Keywords: robust optimization, flood protection, adjustability, adaptivity

JEL codes: C61, Q54

1 Introduction

Managing flood risks is of vital importance for the Netherlands, a country which is for two-thirds flood prone. The areas at risk are protected by a large flood protection system, with dike rings consisting of dikes, dams, dunes, high grounds, and other water defense structures. Each year, about 1 billion euro is spent to maintain and improve this system.

Improving the flood defenses is necessary when flood protection standards are no longer met. This can be caused by for example a general deterioration of the flood defense, climate change but also because of changing protection standards. In the recent years, and within the context of the Dutch Delta Program, the Netherlands has been in the process of revising all its legal flood protection standards (Van Alphen 2015), since part of the old standards was still based on the advice of the first Delta Commission, installed after the last great flood disaster in the Netherlands of 1953 (Van Dantzig 1956).

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In the Deltra program, a novel, optimizing cost-benefit analysis (CBA) using operations research techniques was used to derive economically efficient ('optimal') flood protection standards, in which the discounted total cost of expected long-run investment and remaining flood damages was minimized (Brekelmans et al. 2012, Kind 2014, Eijgenraam et al. 2014, 2016). This CBA assumed dike reinforcements as flood protection measure and determined the optimal timing and size of the reinforcements, from which subsequently optimal flood protection standards were derived. It turned out that although the optimal timing and size of the reinforcements were dependent on the climate scenario assumed, the optimal standards were not. The optimal standards, however, are very sensitive to assumptions on e.g. the economic scenario, discount rate, investment costs and flood damages (Gauderis et al. 2013).

The updated standards, accepted by the Dutch government on July 5, 2016, are based on considerations over equity (a maximum tolerable probability for all individuals to lose his/her life because of flooding), efficiency (CBA) and social disruption (Van der Most et al. 2014). After the new flood protection standards become legally binding from 2017 and onwards, regional delta programs have the task of developing more detailed flood risk management strategies to meet those new flood protection standards. Those strategies are not necessarily restricted to dike reinforcements, but other measures are also considered.

In the lower reaches of the Rhine and Meuse rivers, the regional Delta Program *Rhine Estuary - Drechtsteden* studies alternatives to dike reinforcements, such as water storage, channel deepening, storm surge barriers, and room for the river measures - alternatives which lower (design) water levels and hence reduce the need to reinforce and heighten the dikes (Jeuken et al. 2013). To aid the regional program in developing and evaluating combined strategies, including proposal for timing of the investments, a 'planning kit' was developed (Kind et al. 2014). It turned out there was a large number of strategies possible and it was practically infeasible to find an optimal strategy by trial and error, even if uncertainties in climate scenarios were not considered.

There are two important challenges remaining in designing the optimal flood risk management strategies. First, it is important to incorporate climate change related uncertainties in the analysis explicitly. In the previous approaches, sensitivity analysis was used to show robustness, that is, the performance of the given solution was tested against different realizations of the uncertainty. Such an approach may (i) take a lot of computation time and for each scenario requires a detailed specification of the parameter evolution and (ii) may be infeasible already at the implementation stage - because parameter values different from the assumed ones are revealed between the decision-making moment and the implementation moment. This raises the challenge of *robustness to parameter uncertainty*. Second, it is important that the solutions are *adjustable* to the revealed values of uncertainties as opposed to *static* solutions. The static solutions, where the later-stage decisions do not adapt to the true outcome of, say, sea level rise speed in the first 30 years, may prove to be over-conservative and expensive. This raises the challenge of *adjustability to revealed uncertainties*.

In this paper, we develop a mathematical optimization model to determine optimal adaptation strategies that addresses both of the challenges above. This model is general and applicable in any area, and in our experiment we apply it to the Rhine Estuary area, where it improves upon the planning kit of (Kind et al. 2014). The issue of climate change related uncertainties is addressed by using Robust Optimization (RO, see Ben-Tal et al. (2009)). In this approach, instead of specifying a precise climate change trajectory, an uncertainty set of 'possible outcomes' of the unknown parameters is specified. Then, the problem is solved in such a way that the constraints (requirements) are satisfied by the decisions for every outcome of uncertainty within the uncertainty set.

The strict requirement that the decisions are feasible for all allowed outcomes of uncertainty makes RO the preferred methodology for safety-related optimization problem such as flood prevention. For an introduction and overview of techniques used in RO, we refer the

reader to the work by Ben-Tal et al. (2009), Bertsimas et al. (2011a), Gabrel et al. (2014) and references therein.

In order to allow the later-stage decisions adapt to the revealed uncertainties from earlier stage, we resort to an extension of RO - the Adjustable Robust Optimization (ARO). There, apart from requiring the constraints to hold for all outcomes of the uncertainty, the later stage decisions are formulated as functions of the uncertainties revealed before they are implemented, and the way these decisions adapt ('shape of the reaction') is also optimized.

ARO was initially developed to solve problems with continuous decision variables in Ben-Tal et al. (2004) where the concept of using affinely adjustable decision rules was introduced, extended later by Chen et al. (2012), Chen and Zhang (2012), Ben-Tal et al. (2009), and Bertsimas et al. (2011b). In the flood protecting problem, however, most of the decisions are binary variables determining whether and when a given measure (dike heightening, etc.) is applied. The first applications of ARO to integer recourse problems were Bertsimas and Caramanis (2007, 2010), and Vayanos et al. (2011), where the idea was to simply divide the uncertainty set into a dense grid of points and allow a different decision for each of them. Bertsimas and Georghiou (2015, 2014), and Hanasusanto et al. (2014) proposed to use specific decision rules for the integer variables whose 'shape' is optimized. However, these methodologies do not scale well with the size of the problem, which makes their application to problems like ours impossible.

The approach used in our paper is to construct multi-period adjustable integer decisions by means of multi-stage splitting of the uncertainty set into subsets. This is the approach taken in Postek and den Hertog (2016) and Bertsimas and Dunning (2014) where, having determined the splits of the uncertainty set, a different constant decision is applied for each part of the uncertainty set. The essence of this approach lies in determining the conditions that a splitting needs to satisfy in order to improve on the decisions' adaptivity. Since, however, in the model we consider it is not possible to determine the structure of the splits using the methods proposed by Postek and den Hertog (2016), our splitting strategy of the uncertainty set is pre-determined.

The structure of this paper is as follows. Section 2 gives the generic mathematical formulation of our problem without parameter uncertainty and adjustability. Section 3 defines how multi-stage parameter uncertainty is modeled and the corresponding adaptive decisions are constructed. Section 4 presents the results of numerical experiments for the Rhine - Meuse Estuary - Drechtsteden area.

2 Deterministic model

We consider the problem of constructing an optimal flood protection strategy. The aim of the strategy is to create a schedule when to take various measures that minimizes the present value of the measures' costs such that at each time moment and at each dike segment, the flood protection standards are satisfied. Hence, cost effectiveness is applied.

We assume that the flood protection system consists of N_s dike segments. The flood protection standards can be formulated in terms of *relative dike height* requirements - the height of the dike compared to the water level. The relative dike height can be improved by one of N_h dike heightenings of size $h \in \mathcal{H}$, and N_m large-scale measures such as, for example, changing the discharge distribution of a river (directing it via other river segments in the delta). Whereas a dike heightening affects only the relative dike height at a single segment, the large-scale measures affect more than one segment and its impact may differ throughout the time horizon after its implementation. We denote by $a_{m,s,\tau,t}$ the impact of implementing large-scale measure m at time τ on the relative dike height at segment s at time $t \geq \tau$. The flood protection standard requires that at each dike segment s and each

time $t \in \{0, 1, \dots, T\}$ the relative dike height is higher than or equal to the requirement $n_{s,t}$.

We define decision variables $x_{t,s,h} \in \{0, 1\}$ indicating whether the h -th dike heightening has been implemented at dike segment s at time t , and $y_{t,m} \in \{0, 1\}$ indicating whether large-scale measure m is implemented at time t . We denote the cost of dike heightening h for segment s by $p_{s,h}$, the cost of measure m by p_m , and we assume an inter-period discount rate $0 \leq d \leq 1$.

In this setting, the objective is to minimize the total discounted costs and the optimization problem is:

$$\begin{aligned} \min_{\mathbf{x}, \mathbf{y}} \quad & \sum_{t=1}^T \frac{1}{(1+d)^t} \left\{ \sum_{s=1}^{N_s} \sum_{h=1}^{N_h} p_{s,h} x_{t,s,h} + \sum_{m=1}^{N_m} p_m y_{t,m} \right\} \\ \text{s.t.} \quad & \sum_{\tau=0}^t \left\{ \sum_{h=1}^{N_h} h x_{\tau,s,h} + \sum_{m=1}^{N_m} a_{m,s,\tau,t} y_{\tau,m} \right\} \geq n_{s,t}, & \forall s, t & \quad (1a) \\ & \sum_{h=1}^{N_h} x_{t,s,h} \leq 1, & \forall t, s & \quad (1b) \\ & L_k(\mathbf{x}, \mathbf{y}) \leq 0, & \forall k = 1, \dots, K & \quad (1c) \\ & x_{t,s,h}, y_{t,m} \in \{0, 1\}, & \forall t, s, h, m, & \end{aligned}$$

where the constraints are:

- (1a) relative dike height constraint at segment s at time t - the total impact of the dike heightenings and the large-scale measures up to time t accommodates for the requirement $n_{s,t}$,
- (1b) for each dike segment s and time t , only one of N_h heightenings is implemented,
- (1c) K linear constraints - these may involve, for example (i) conditions how many times can a given large-scale measure be taken, (ii) which measures cannot be taken together, (iii) redundant constraints which the solution must satisfy and which improve the solution time.

3 Modeling uncertainty and adjustability

3.1 Introduction

In the previous section we considered the relative dike height requirements $n_{s,t}$ to be a deterministic quantity known in advance. In fact, $n_{s,t}$ is based on the protection standards but also on the sea level rise:

$$n_{s,t} = n_{s,0} + \beta_s \sum_{\tau=1}^t r_{\tau},$$

where $n_{s,0}$ is the initial requirement, r_{τ} is the sea level rise in time period $[\tau - 1, \tau]$, and β_s is the sensitivity of the dike segment s to the sea level rise - for example, sensitivity β_s of dike segments closer to the sea is higher than the dike segments situated higher along the rivers.

In this setting, the realized sea level rise may turn out to be different from the predicted values. Using only the forecast values of the sea level rise may lead to the solution becoming infeasible if small deviations from the given values occur (Ben-Tal et al. 2009). Secondly, even if the solution stays feasible, the future decisions are fixed and do not adjust to the outcomes of the uncertain sea level rise from the earlier periods. In the following, we discuss in detail the uncertainty structure w.r.t. the sea level rise and then develop the adjustable robust version of the deterministic model of Section 2.

3.2 Uncertainty structure

We divide the time horizon into intervals with points $0 = t_0 < t_1 < t_2 < \dots < t_{J-1} < t_J = T$ and assume that in each of the intervals $[t_{j-1}, t_j]$, $j = 1, \dots, J$ the sea level rise is equal to r_j which value is known only at time t_j . In this way (i) we allow the dynamics of the sea level rise to differ over time, (ii) it is possible to readjust the decisions at time points t_1, \dots, t_{J-1} after the value of the respective sea level rise is known.

Example 1 *Suppose that $T = 10$, and we define $J = 3$ with $t_1 = 3$ and $t_2 = 6$. We then have that the time horizon is divided into intervals: $[0, 3]$, $[3, 6]$ and $[6, 10]$. At time points t_1 and t_2 decisions can be readjusted, based on the revealed sea level rise speed in the periods $[0, 3]$ and $[3, 6]$, respectively.*

We assume that in each of the intervals $[t_{j-1}, t_j]$ the sea level rise belongs to an interval

$$r_j \in \mathcal{R}_j = [(1 - \rho)\bar{r}_j, (1 + \rho)\bar{r}_j], \quad j = 1, \dots, J,$$

where \bar{r}_j is the forecast value and parameter $0 \leq \rho \leq 1$ specifies the degree of uncertainty about r_j . For simplicity, we assume that $\bar{r}_j = \bar{r}$ for all j .

Let us denote the vector of uncertain sea level rises by $\mathbf{r} = (r_1, r_2, \dots, r_J)$ and assume that the sea level rise speed (the sea level grows linearly throughout the interval) is constant in each of the intervals $[t_{j-1}, t_j]$. Then, we model the relative dike height requirement as:

$$n_{s,t}(\mathbf{r}) = n_{s,0} + \beta_s \left(\sum_{j:t_j \leq t} r_j + \frac{t - t_{j(t)}}{t_{j(t)+1} - t_{j(t)}} r_{j(t)} \right), \quad (2)$$

where $n_{s,0}$ is the initial requirement in period 1, r_j the rise of the sea level throughout the interval $[t_{j-1}, t_j]$, and $j(t) = \min \{j : t_j \geq t\}$, that is, $j(t)$ is the smallest j such that t belongs to the interval $[t_j, t_{j+1}]$. Formula (2) means that the requirement $n_{s,t}(\mathbf{r})$ is equal to the sum of the initial requirement, the sea level rises from all the full intervals that passed until time t and the sea level rise in the interval containing t up to this time moment, multiplied by the sensitivity β_s .

3.3 Adjustability of decisions - applicability of adaptive splitting

Adjustability of integer decisions consists in its essence in splitting the uncertainty set into subsets and designing different later-stage decisions for each of these subsets. For example, one splits the set \mathcal{R}_1 into two sets $\mathcal{R}_1^1 = [(1 - \rho)\bar{r}_j, z]$ and $\mathcal{R}_1^2 = [z, (1 + \rho)\bar{r}_j]$, such that $\mathcal{R}_1^1 \cup \mathcal{R}_1^2 = \mathcal{R}_1$. Then, at time t_1 when r_j is observed and it is known whether it holds that $r_1 \in \mathcal{R}_1^1$ or $r_1 \in \mathcal{R}_1^2$. Then, one can differentiate the decisions implemented from time t_j on, whereas there can be only one possible decision implemented before t_j , since it is not known yet what r_1 is.

One of the crucial questions in this setting is how to choose the point z where the sets \mathcal{R}_j are split. Postek and den Hertog (2016) provide theoretical results on how to do it. This approach relies on the observation that for different constraints in the problem, different scenarios r_j are the worst-case scenarios, i.e., maximizing the left hand side. They prove that in order to improve on the worst-case objective function, these scenarios need to end up in different subsets \mathcal{R}_1^1 and \mathcal{R}_1^2 as a result of the splitting procedure. The splitting procedure is then applied iteratively, resulting in a more and more adjustable decision structure.

Remark 1 *Here, we make an important distinction that by the worst-case scenario we mean a realization of the uncertain parameter within the specified uncertainty set that is least favorable to the given set of decisions. This should be opposed to the ‘worst scenarios’ considered*

in the water management community, corresponding to a specific sea level rise trajectory prescribed by, for example, the government.

In the specific problem we consider, this method cannot be applied as (i) all the constraints of the problem depend on the uncertain parameter \mathbf{r} in the same way, that is, the highest the sea level rise, (ii) the uncertainty set is a box, i.e. we have that $\mathcal{R} = \mathcal{R}_1 \times \dots \times \mathcal{R}_J$. Combination of these two factors leads to the conclusions that it is always a single scenario \mathbf{r} that maximizes the right-hand side term $n_{s,t}(\mathbf{r})$ in the safety requirement constraint. This claim is proved in the later part of this paper as Proposition 1 and it implies that we need (i) another methodology of choosing how to split the uncertainty set \mathcal{R} than the one of Postek and den Hertog (2016), (ii) a different objective than simply the worst-case cost over the entire uncertainty set (total cost of the objective assuming that the flood protection policy is feasible for all $\mathbf{r} \in \mathcal{R}$). In the following subsections we outline the adjustability structure of our choice and demonstrate mathematically the need for a different objective function, which shall be the so-called *average worst-case objective*.

3.4 Adjustability of decisions - our approach

As for the splitting methodology, we assume that at t_1, t_2, \dots, t_j , depending whether the realized value r_j is such that $r_j \in \mathcal{R}_j^1 = [(1 - \rho)\bar{r}_j, \bar{r}_j]$ or $r_j \in \mathcal{R}_j^2 = [\bar{r}_j, (1 + \rho)\bar{r}_j]$, different decisions can be implemented in the periods to come. We choose to split the uncertainty sets \mathcal{R}_j^2 at points \bar{r}_j for its simplicity and lack of theoretical reasons justifying another splitting structure.

In other words, before time t_1 there is only one decision scenario to be implemented. From time t_1 and before t_2 there are two decision possibilities, between t_2 and t_3 four, and so on. This gives rise to the tree decision structure given in Figure 1.

As shown in Figure 1, $J-1$ split points result in a decision tree with 2^{J-1} paths - sequences of arrows from time 0 to T . We enumerate the paths using $(J-1)$ -tuples $\alpha \in \{1, 2\}^{J-1}$ which is illustrated by the following example.

Example 2 In Figure 1, $\alpha = (2, 2, 1)$ corresponds to the decision paths that branches ‘up’ at times t_1 and t_2 , and then ‘down’ at time t_3 .

We define that for each of the decision paths for the entire planning horizon $[0, T]$ there is a separate decision vector $x_{t,s,h,p}(\alpha)$ and $y_{t,m,p}(\alpha)$, where $t = 0, \dots, T$. Each of the decision paths is then uniquely identified with a trajectory of the sea level rise \mathbf{r} :

$$\alpha_j = \begin{cases} 1 & \text{if } r_j \in \mathcal{R}_j^1 \\ 2 & \text{if } r_j \in \mathcal{R}_j^2 \end{cases}, \quad j = 1, \dots, J-1.$$

In a similar way, we construct multi-period uncertainty sets for \mathbf{r} uniquely identifiable with α 's as:

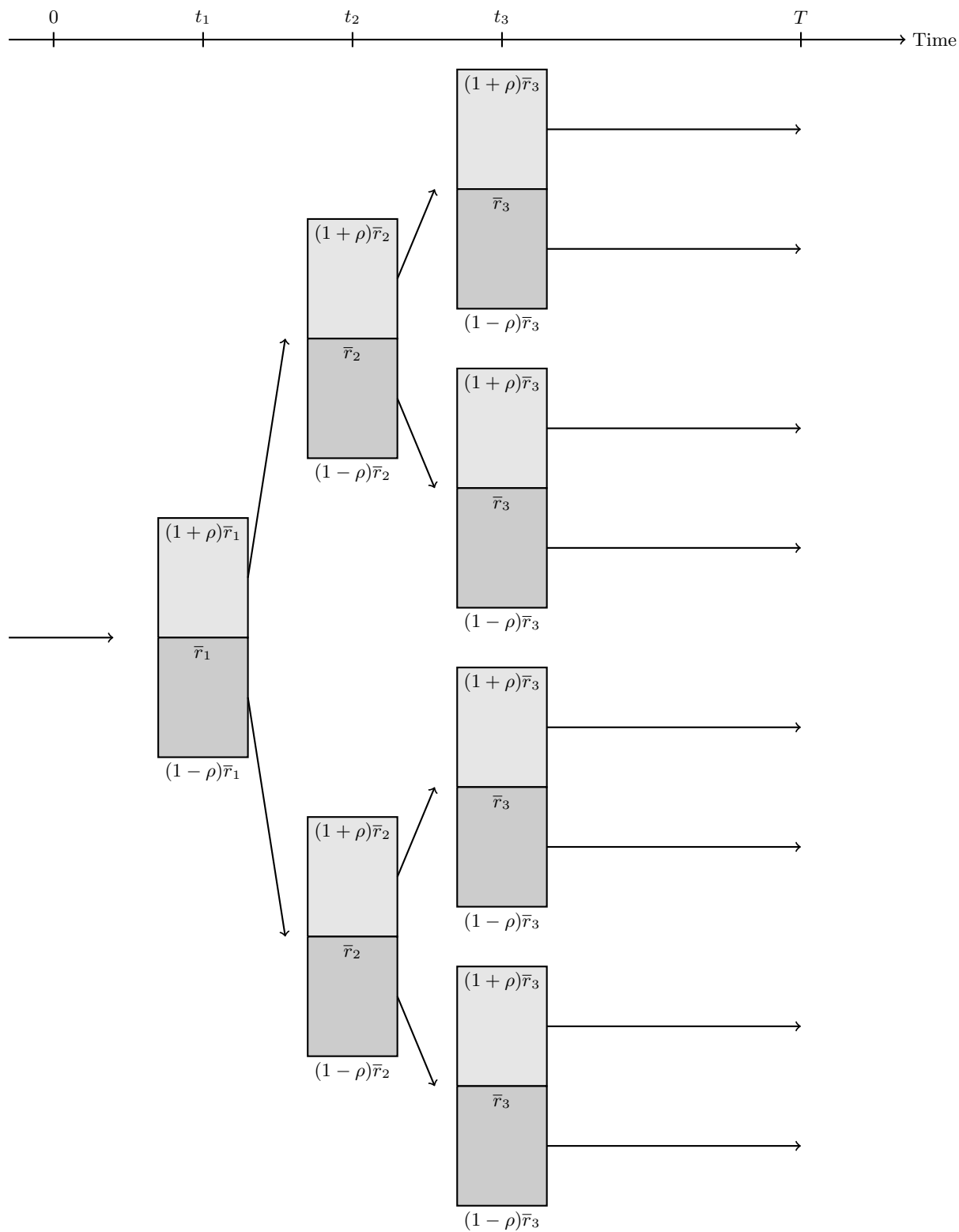
$$\mathcal{R}(\alpha) = \mathcal{R}_1^{\alpha_1} \times \mathcal{R}_2^{\alpha_2} \times \dots \times \mathcal{R}_{J-1}^{\alpha_{J-1}} \times \mathcal{R}_J.$$

The relation between the decisions $x_{t,s,h,p}(\alpha)$ and $y_{t,m,p}(\alpha)$ and the uncertainty sets $\mathcal{R}(\alpha)$ is that we require each of the decision paths to satisfy the relative dike requirement for each \mathbf{r} belonging to an uncertainty set corresponding to that path (adjustable robust version of constraint (1a)):

$$\sum_{\tau=0}^t \left\{ \sum_{h=1}^{N_h} h x_{\tau,s,h}(\alpha) + \sum_{m=1}^{N_m} a_{m,s,t,\tau} y_{t,m}(\alpha) \right\} \geq n_{s,t}(\mathbf{r}), \quad \forall s, t \quad \forall \mathbf{r} \in \mathcal{R}(\alpha). \quad (3)$$

However, as some of the decision paths from 0 to T coincide up to a certain time point, it is necessary to include *nonanticipativity constraints*, ensuring that decisions corresponding to

Figure 1: Adjustability scheme when $J = 4$. At times points t_1, t_2, t_3 it turns out whether the sea level rise belonged to $[(1 - \rho)\bar{r}_j, \bar{r}_j]$ (symbolized by the dark gray part of the box) or $[\bar{r}_j, (1 + \rho)\bar{r}_j]$ (symbolized by the light gray part of the box).



paths that are the same up to some point, are also the same, to prevent that decisions are based on information not available yet. These constraints can be formulated as:

$$x_{t,s,h}(\boldsymbol{\alpha}') = x_{t,s,h}(\boldsymbol{\alpha}''), \quad y_{t,m}(\boldsymbol{\alpha}') = y_{t,m}(\boldsymbol{\alpha}''), \quad \forall t : t < t_j, \quad \boldsymbol{\alpha}'_{1:j} = \boldsymbol{\alpha}''_{1:j}, \quad (4)$$

where $\boldsymbol{\alpha}_{1:j}$ denotes a subvector of vector $\boldsymbol{\alpha}$ consisting of its first j components. Nonanticipativity constraints are illustrated by the following example.

Example 3 *In the scheme of Figure 1 we have that the paths corresponding to the two top branches of the tree correspond to $\boldsymbol{\alpha}' = (2, 2, 2)$ and $\boldsymbol{\alpha}'' = (2, 2, 1)$. Thus we have that 2 is the largest j such that it holds that $\boldsymbol{\alpha}'_{1:j} = \boldsymbol{\alpha}''_{1:j}$. This, combined with (4) implies that $x_{t,s,h,p}(2, 2, 2) = x_{t,s,h,p}(2, 2, 1)$ and $y_{t,m,p}(2, 2, 2) = y_{t,m,p}(2, 2, 1)$ for $t < t_2$.*

3.5 Objective function

In the deterministic problem (1) there is only a single decision path and no uncertainty so that the objective function is simple and corresponds to the total discounted costs of the measures taken.

In the adjustable case, however, each of the paths $\boldsymbol{\alpha}$ yields a different decision path that has a different total cost $C(\boldsymbol{\alpha})$:

$$C(\boldsymbol{\alpha}) = \sum_{t=0}^T \frac{1}{(1+d)^t} \left\{ \sum_{s=1}^{N_s} \sum_{h=1}^{N_h} p_{s,h} x_{t,s,h}(\boldsymbol{\alpha}) p_{s,h} + \sum_{m=1}^{N_m} p_m y_{t,m}(\boldsymbol{\alpha}) \right\},$$

and the objective function of the entire problem should somehow take into account the costs related to all the paths. A typical choice in Robust Optimization is to consider the worst-case cost as the objective function - in our case, the cost of the most expensive decision path. Suppose that indeed, for fixed $\mathbf{t} = (t_1, \dots, t_{J-1})$ we minimize the worst-case objective, i.e. the value of the objective function under the assumption that the worst-possible realization $\mathbf{r} \in \mathcal{R}$ is realized. The optimization problem is then:

$$\begin{aligned} \min_{x,y} \max_{\boldsymbol{\alpha} \in \{1,2\}^{n_z}} C(\boldsymbol{\alpha}) \\ \text{s.t. } \sum_{\tau=0}^t \left\{ \sum_{h=1}^{N_h} h x_{\tau,s,h}(\boldsymbol{\alpha}) + \sum_{m=1}^{N_m} a_{m,s,\tau,t} y_{t,m}(\boldsymbol{\alpha}) \right\} \geq n_{s,t}(\mathbf{r}), \quad \forall s, t \quad \forall \mathbf{r} \in \mathcal{R}(\boldsymbol{\alpha}) \end{aligned} \quad (5a)$$

$$n_{s,t}(\mathbf{r}) = n_{s,0} + \beta_s \left(\sum_{j:t_j \leq t} r_j + \frac{t - t_j(t)}{t_j(t)+1 - t_j(t)} r_{j(t)} \right) \quad (5b)$$

$$x_{t,s,h}(\boldsymbol{\alpha}') = x_{t,s,h}(\boldsymbol{\alpha}''), \quad y_{t,m}(\boldsymbol{\alpha}') = y_{t,m}(\boldsymbol{\alpha}''), \quad \forall t : t < t_j, \quad \boldsymbol{\alpha}'_{1:j} = \boldsymbol{\alpha}''_{1:j} \quad (5c)$$

$$\sum_{h=1}^{N_h} x_{t,s,h}(\boldsymbol{\alpha}) \leq 1, \quad \forall t, s, \quad \forall \boldsymbol{\alpha} \quad (5d)$$

$$L_k(\mathbf{x}(\boldsymbol{\alpha}), \mathbf{y}(\boldsymbol{\alpha})) \leq 0, \quad \forall k = 1, \dots, K, \quad \forall \boldsymbol{\alpha} \quad (5e)$$

$$x_{t,s,h}(\boldsymbol{\alpha}), y_{t,m}(\boldsymbol{\alpha}) \in \{0, 1\}, \quad \forall t, s, h, m, \quad \forall \boldsymbol{\alpha}. \quad (5f)$$

Let us denote by $WC(\mathbf{t})$ the optimal value to (5) for a given \mathbf{t} . There is no adjustability if $J = 1$ and we denote the objective value in this case by \overline{WC} .

As the following Proposition shows, it is not possible that $WC(\mathbf{t})$ is better than \overline{WC} .

Proposition 1 *For every \mathbf{t} it holds that $WC(\mathbf{t}) = \overline{WC}$.*

Proof. It follows easily that $WC(\mathbf{t}) \leq \overline{WC}$ from the fact that every feasible solution to the problem with $J = 1$ is also feasible to the problem with $J > 1$. Assume there exists \mathbf{t} such

that $\text{WC}(\mathbf{t}) < \overline{\text{WC}}$. We have that the optimal solution corresponding to $\boldsymbol{\alpha}^* = (2, 2, \dots, 2)$ is feasible for every

$$\mathcal{R}((2, 2, \dots, 2)) = [\bar{r}_1, (1 + \rho)\bar{r}_1] \times [\bar{r}_2, (1 + \rho)\bar{r}_2] \times \dots \times [\bar{r}_{J-1}, (1 + \rho)\bar{r}_{J-1}] \times \mathcal{R}_J.$$

However, since the only constraint dependent on \mathbf{r} is (5a), and its left hand side depends monotonically on components of \mathbf{r} , we have that the optimal solution corresponding to $\boldsymbol{\alpha}^*$ is feasible for all $\mathcal{R}(\boldsymbol{\alpha})$, $\boldsymbol{\alpha} \in \{1, 2\}^{J-1}$, and hence, for the entire uncertainty set:

$$\mathcal{R} = [(1 - \rho)\bar{r}_1, (1 + \rho)\bar{r}_1] \times [(1 - \rho)\bar{r}_2, (1 + \rho)\bar{r}_2] \times \dots \times [(1 - \rho)\bar{r}_{J-1}, (1 + \rho)\bar{r}_{J-1}] \times \mathcal{R}_J.$$

In other words, decisions feasible for the uncertainty set being a product of the ‘worse subintervals’ for r_j (containing the higher values) is feasible also for the ‘better intervals’ and hence, for the entire uncertainty set. But this implies that the decision sequence corresponding to $\boldsymbol{\alpha}^*$ is also feasible for problem with $J = 1$ (no adjustability) and, at the same time, gives a better objective value than $\overline{\text{WC}}$. This is a contradiction with the assumption that $\overline{\text{WC}}$ is the best objective value for problem with $J = 1$. \blacksquare

Proposition 1 shows that no improvement in the objective value is possible if the objective is the worst-case cost. More generally, it can be proven in a similar fashion that no adjustability schemes can lead to an improvement of the worst-case formulation. A simple consequence of this fact is, as we have already signaled, that the methods of adaptive splits of Postek and den Hertog (2016), in which the division of the uncertainty set \mathcal{R} is determined on the basis of solutions, cannot be applied. It is exactly for that reason that in our application the splits of the uncertainty sets $[(1 - \rho)\bar{r}_j, (1 + \rho)\bar{r}_j]$ into halves $[(1 - \rho)\bar{r}_j, \bar{r}_j]$ and $[\bar{r}_j, (1 + \rho)\bar{r}_j]$ is pre-determined.

As a consequence, we propose that instead of the worst-case cost of a decision path, the objective function is the average of the costs associated with all the paths:

$$\frac{1}{2^{J-1}} \sum_{\boldsymbol{\alpha} \in \{1, 2\}^{J-1}} C(\boldsymbol{\alpha}).$$

The weights $1/2^{J-1}$ correspond to the fact that each of the uncertainty subsets $\mathcal{R}(\boldsymbol{\alpha})$ has the same volume and that we assume both parts to be evenly likely (which happens if the underlying probability distribution is uniform). Secondly, due to the fact that each $C(\boldsymbol{\alpha})$ plays a role, it is almost sure that for the uncertainty subsets with smaller sea level increase, smaller later-stage investments are needed to meet the safety standards. In this way, we can expect increasing J to lead to better optimal values.

In the end, the adjustable problem we consider is:

$$\begin{aligned} \min_{x(\boldsymbol{\alpha}), y(\boldsymbol{\alpha})} \quad & \frac{1}{2^{J-1}} \sum_{\boldsymbol{\alpha} \in \{1, 2\}^{J-1}} C(\boldsymbol{\alpha}) \\ \text{s.t.} \quad & (5a) - (5f). \end{aligned} \tag{6}$$

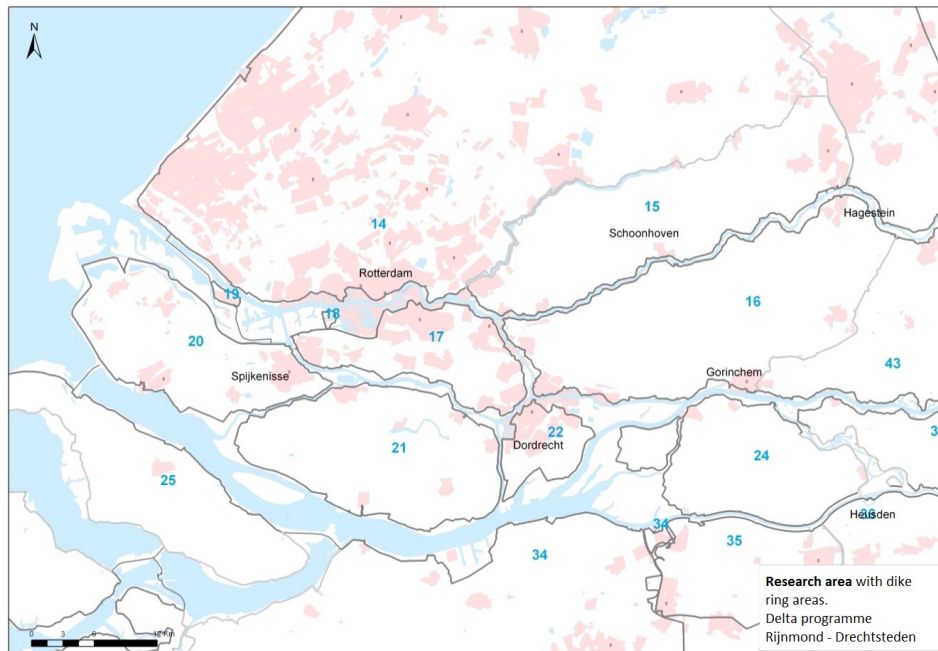
4 Numerical experiment

4.1 Region

In our numerical experiment we consider the Rhine - Meuse Estuary - Drechtsteden area illustrated in Figure 2, consisting of $N_s = 150$ dike segments. For each dike segment $N_h = 22$ dike heightenings are possible of size $\{0.1, 0.2, \dots, 2.0, 2.5, 3.0m\}$. Also there are $N_m = 14$ large scale water measures:

1. **Replacement of the Measlantbarrier.** This storm surge barrier is located at Hoek van Holland in the Nieuwe Waterweg. This barrier together with dikes and dunes protects the province Zuid-Holland against floods from sea. The storm surge barrier

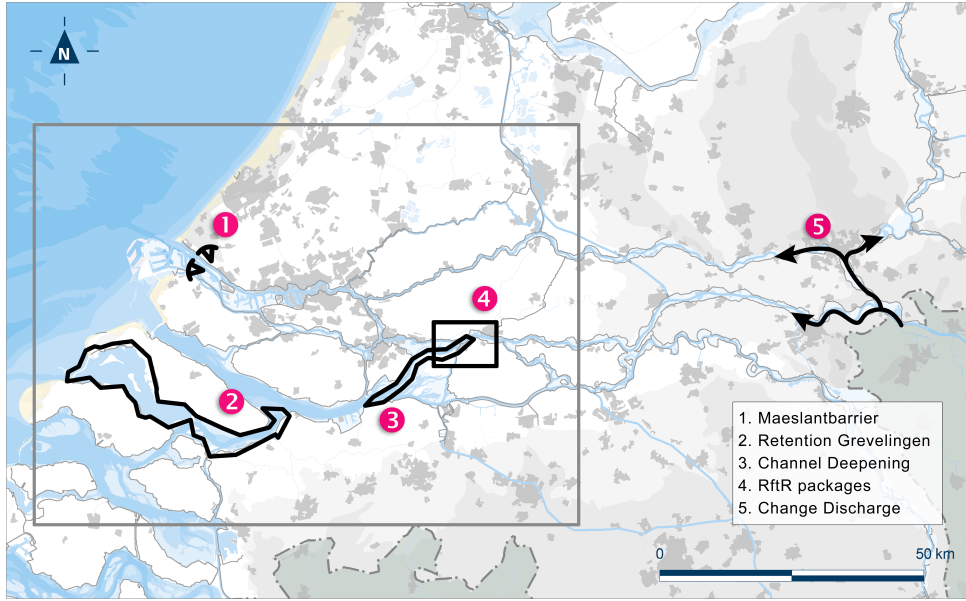
Figure 2: The Rhine - Meuse Estuary - Drechtsteden area. Thick lines stand for the dike segments, numbers denote the dike rings - areas surrounded by dikes or high grounds from all sides.



protects against high water and can be closed in emergencies. This measure can be implemented only from 2070 on.

2. **Improving the Maeslantbarrier** before 2070 and its replacement in 2070.
3. **Change the river discharge distribution via the IJssel river.** When the discharge via the Lek river becomes greater than $8000m^3/s$, more water is discharged via the IJssel river.
4. **Change the river discharge distribution via the Waal river.** When the discharge via the Lek river becomes between $8000m^3/s$ and $16000m^3/s$, more water is discharged via the Waal river.
5. **High water level Waal.** When the discharge of the Rhine is above $16000m^3/s$, more water will be discharged via the Waal river.
6. **Robert Koning.** In this measure extra space for the Merwede is created with a flood channel through the Land of Heusden and Altena. It is a green river, called Robert Koning, that is used when water levels exceed a certain height. It is expected that these high water levels occur once every 100 years.
7. **Room for the River packages (4 measures).** There are several measures that dig off areas around the river Merwede. The four measures that can be taken are the measures RvR klein 01, RvR klein 02*, MW007, and MW42.3. Two packages (RvR klein 01 and RvR klein 02*) combine several measures that dig off areas around Merwede.
8. **Grevelingen.** With this measure the possibility is created to store water at the Grevelingen lake.
9. **Spaargaren 1.** In this measure the seaside is closed by sluices at the Maeslantbarrier.
10. **Spaargaren 2.** Around the Nieuwe and Oude Maas rivers, sluices close the seaside with extra discharge capacity.
11. **Spaargaren 3.** The sluices close the seaside around the Nieuwe and Oude Maas rivers as Spaargaren 2, but without extra discharge capacity.

Figure 3: The Rhine - Meuse Estuary - Drechtsteden area with some of the large-scale measures. The box includes measures considered in the planning kit of Kind et al. (2014).



The big measures are illustrated in Figure 3.

4.2 Piping problem

An additional trait of the problem we consider is the so-called *piping problem* affecting some of the dike segments. If groundwater flows wash away the sand below a dike, a pipe through the dike can arise (Deltares 2014). The flood probability increases enormously and the piping problem needs to be solved instantly in order to reduce the flood probability significantly. This issue cannot be addressed by lowering the water level, but only by strengthening the dikes. Let us denote by \mathcal{S}^p the set of dike segments for which the piping problem exists.

The piping problem enters our problem in a way that for each $s \in \mathcal{S}^p$ it holds that (i) every dike heightening $h \in \mathcal{H}$ and (ii) solving the piping problem, have a fixed cost component c_s^f which is the same for both $p_{s,h}$ and r_s , such that:

$$p_{s,h} = c_s^f + p_{s,h}^v \quad r_s = c_s^f + r_s^v,$$

where $p_{s,h}^v$ is the variable cost of dike heightening depending on $h \in \mathcal{H}$, r_s is the total cost of solving the piping problem, and r_s^v is the remaining cost of solving the piping problem for dike segment s .

In our optimization model, we incorporate the piping issue in such a way that we assume that the piping problem, for dike segments s for which it exists, is solved at time $t = 0$. Thus, the fixed cost r_s is incurred at $t = 0$ for each $s \in \mathcal{S}^p$. In order to avoid double-counting of the fixed cost at $t = 0$ we assume that the cost of dike heightening h (if implemented) is equal to $p_{s,h}^v$ for that time period.

Since the total cost of solving the piping problem are relatively large compared to the cost of dike heightenings, in our numerical results we shall report on the total cost both with and without the cost of solving the piping problem.

4.3 Research questions and numerical results

The particular research questions that we subsequently answer in the coming sections, are as follows:

1. What is the nominal (nonrobust) solution and what the dangers are if uncertainty w.r.t. sea level rise is present?
2. What are the (adjustable) robust solutions, how do they change as more adjustability is allowed in the model?
3. How do the robust solutions differ from the nominal solution?
4. How do the robust solutions change when either the discount rate d or the the uncertainty level ρ changes?
5. How do the robust solutions change when an adjustability scheme different to that of Section 3 is applied? This question is considered, for reasons of brevity, in Appendix A.

In our numerical experiment, the planning horizon is 2020 - 2200, with the possibility to implement decisions every 10 years, which gives 19 decision stages in total. When talking about the results, we shall often refer by $t = 1$ to year 2020, $t = 1$ to year 2030 and so on.

In our numerical experiment, we shall consider equidistant splitting points (if any adjustability is present) t_j , $j = 1, \dots, J - 1$, to be defined as

$$t_j = jW,$$

where W is the length of the interval between t_{j-1} and t_j as measured in the ‘big’ time intervals of 10 years, chosen to be independent from j . In this way, $W = 3$ corresponds to differentiating the decisions every $3 \cdot 10 = 30$ years. We solve the optimization model for different values of $J - 1$ and W .

The sensitivities β_s and initial dike shortages $n_{s,0}$ are estimated from the data. For the robust models we assume that there is $\rho = 40\%$ uncertainty about the sea level rise speed, motivated by the Dutch KNMI uncertainty intervals for sea level rise speed.¹ We also solve a nonrobust (nominal) model where $\rho = 0\%$. The discount rate is assumed to be equal to $d = 5.5\%$. We assume that a dike segment should not be heightened more often than once in 30 years, which we denote as the *frequency constraint*.

Due to the size of the model and the number of binary variables, it is not expected that it is solved to full optimality within reasonable time limits. However, a particular feature of the model is that once the decisions w.r.t. the large-scale measures are fixed, the optimization problem related to the dike heightenings per each dike segment can be solved separately. For that reason, each time we reach the sub-optimal solution to the entire optimization model (which is the likely case as mixed-integer models of this size almost never are solved to full optimality), we run an extra step consisting in re-optimization of the dike decisions, calling it the *dike reoptimization step*.

The optimization model formulation has been coded using the C++ language and is solved with the Gurobi solver (Gurobi Optimization 2015). For each instance with given W and J we set time limit of 10h and the MIP optimality gap to 0.1%. Problems have been implemented using 16-core computing nodes of the Lisa computing cluster at SURF SARA computing center in Amsterdam.

4.3.1 Nominal solution and its shortcomings

Table 1 presents the results for the nominal solution, i.e. assuming that there is no uncertainty about the sea level rise, and Table 2 presents the corresponding implementation schedule of the large-scale measures. We observe that the model has been solved nearly to optimality, attaining the total cost of 1004 (excluding the piping cost). Of all the variable cost, the largest share are the here-and-now dike heightening cost, 587. We can clearly see

¹See the document (in Dutch) ‘Zeespiegelveranderingen in de toekomst’, at <http://www.knmi.nl/kennis-en-datacentrum/achtergrond/zeespiegelveranderingen-in-de-toekomst>.

Table 1: Results for the nominal solution. ‘Obj value’ - the optimal value (in millions of euros) of the optimization problem without piping cost; ‘Opt. gap’ - the MIP optimality gap after 10 hours of computation with dike reoptimization (without dike reoptimization); ‘Dike cost $t = 0$ ’ - total cost of dike heightenings implemented at $t = 0$

Nominal			
Obj value	Opt. gap	Obj value + piping	Dikes cost $t = 0$
1004.71	0.1% (0.1%)	3607.50	587.93

Table 2: Implementation schedules of the used large-scale measures for the nominal solution.

Measure	Implementation year
Replacing the Maeslantbarrier	2180
Grev100	2090
MW42 3	2060
MW007	2020

that the fixed cost of solving the piping problem (2602) dominate the cost of the entire solution, as they raise the total cost to the level of 3607.

Regarding the large-scale measures, in Table 2 we observe that only four of them are used: ‘Replacing the Maeslantbarrier’, ‘Grev100’, ‘MW42 3’ and ‘MW007’. This gives an indication that not all of the measures are relevant and are worth maintaining their readiness to use. This indication shall be confirmed in later experiments.

To investigate ‘how bad the nominal solution can become’, we check first for how many dikes and when are the safety standards violated if the sea level rise turns out to be the worst +40% value taken into account for the robust solutions. It turns out that throughout the planning horizon, the safety requirements are violated for 123 out of 150 dike segments. Figure 4 includes the histogram of the first time moments that for these 123 segments the safety standards are violated. We see that there is a ‘peak’ of the frequency of dike violations in time frame 2040-2080. On average, the first violation occurs in year 2087.

As an additional illustration of the advantages of taking into account the robustness and adjustability, we provide one more experiment. In this experiment, we shall begin with the nominal solution and re-optimize its decisions in the subsequent time periods to match the worst +40% value, so that it can be considered as an ‘on-the-spot fixing’ of the nominal solution. To be precise, the procedure is as follows.

1. At time $t = 0$ an optimal strategy is constructed with the assumption of no uncertainty in the sea level rise (using thus $\rho = 0$) and the here-and-now decisions are implemented.
2. However, at $t = 1$ the sea level rise between t_0 and t_1 turns out to be the highest possible one with +40% and at time t_1 a new strategy is constructed (using again $\rho = 0$) for time horizon $[t_1, T]$ and the here-and-now decisions for t_1 are implemented.
3. Step 2 is repeated in time periods $[t_1, t_2]$, $[t_2, t_3]$, $[t_{J-1}, t_J]$.

It is important to note that in this experiment it is possible (and it will occur) that for certain dikes segments the safety standards are violated. Another issue is that the constraint that a dike segment is heightened at most once in 30 years might not be possible to be satisfied.

Indeed, we observe that the problem to solve becomes infeasible in period $t = 9$ (year 2110), due to the necessity to increase the height of certain dikes immediately while not allowed because of the frequency constraint. Strikingly, already the solution of the optimization problem solved at $t = 1$ (year 2030, the ‘on-the-spot fixing’ model is still feasible) involves a major increase in the total cost - a value of 1686, 68% up compared to the value ‘promised’ by the nominal solution in Table 1. The evolution of the optimal cost from the optimization problem solved at each stage is given by the continuous line in Figure 5.

Because it is not possible to ‘keep fixing the nominal solution’ without heightening the dikes more often than once in 30 years, we implemented also a variant of this test without

Figure 4: Histogram of the first time moments of safety standards' violation for the 123 dike segments for which there is a violation (out of the total number of 150 dike segments), with +40% sea level rise increase compared to the nominal scenario.

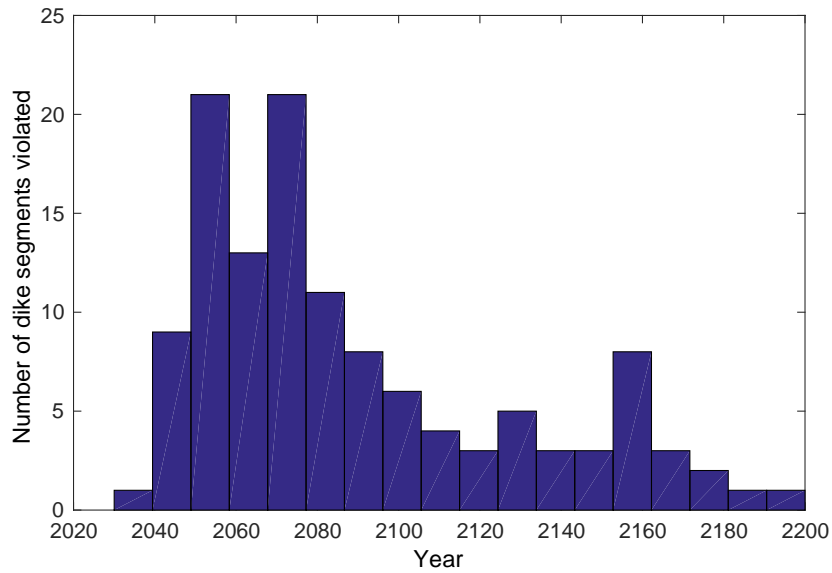


Figure 5: Increase of the total cost when the optimal solution is re-optimized due to the highest possible sea level rise realization. The continuous line corresponds to the case with frequency constraint (model infeasible in 2110) and without these constraints after 2020.

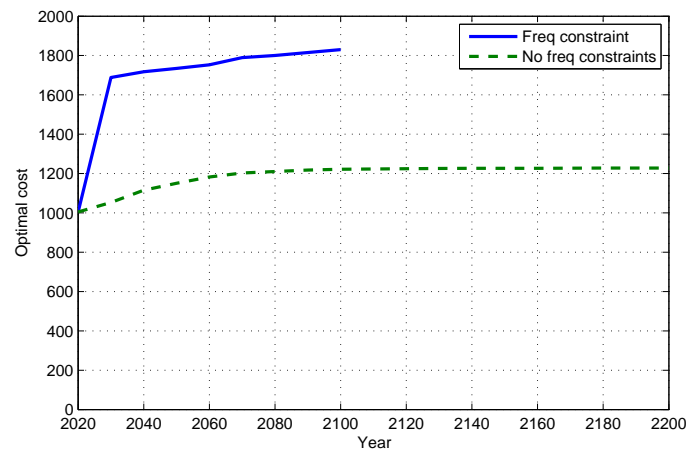


Table 3: Implementation schedules of the used large-scale measures for the nominal solutions and for the robust solutions with $W = 2$ and $J - 1 \in \{0, 2\}$. There is only one scenario for the nominal solution and for the robust solution with $J - 1 = 0$ (no adjustability) and four scenarios for $J - 1 = 2$.

Scenario	Nominal	$J - 1 = 0$ (no adjustability)	$W = 2$ and $J - 1 = 2$			
	-	-	1	2	3	4
Replacing the Maeslantbarrier	2180	2150	2110	2110	2110	2100
RvR kl01	-	-	2110	2130	2100	2090
Grev100	2090	2070	2080	2080	2080	2070
MW42 3	2060	2050	2050	2060	2050	2050
MW007	2020	2020	2020	2020	2020	2020

the frequency constraint at stages later than $t = 0$. It turns out that the total cost is equal to 1227, that is, 22% higher (see Figure 5) than assumed in Table 1. Also, in the course of such a strategy, 7 of the 14 large-scale measures are used - an increase from 4 measures assumed in the initial solution.

From this experiment we see that in presence of uncertainty, even if the nominal solution is allowed to be ‘fixed’ later on, the cost of this fixing may be substantial. As we shall see in the section to come, the total cost of the ‘fixed’ solution is higher than the cost of the robust solution, i.e., the one that takes uncertainty into account at the very beginning.

4.3.2 Robust solutions

Having studied the disadvantages of the nominal solution, we turn to the (adjustable) robust solutions with sea level rise uncertainty $\rho = 40\%$. We begin the discussion of the results with the changes in the large-scale measures implementations for different solutions. We show the solutions for the nominal model and the robust models for $W = 2$ with zero and two splits in Table 3. We see from it (second row) that for the non-adjustable solution the measure ‘RvR kl01’ is not used at all, whereas for the adjustable solution its year of implementation varies between 2090 and 2130. The most stable measures are ‘MW42 3’ implemented by all solutions in 2050, and ‘MW007’ which is implemented right away at the beginning of the planning horizon at 2020.

The cost and dike heightening implementation characteristics of the robust solutions are given in Table 4 on page 17 (including also the nominal solution for ease of comparison) and we comment on it in a column-by-column basis, beginning the discussion on the example of models with $W = 2$, comparing it later to the nominal solution. Note that for all values of W the first row (solution with $J - 1 = 0$) is the same - this is because this is the solution with no adjustability.

In the first column, we can see that, as the degree of adjustability grows with the number of splits, the objective function value decreases. Already for one split we observe a drop of -3.73% , equivalent to approximately 45 million euro. For most adjustable solutions in Table 4 with three splits we have a decrease in the average worst-case cost at the level of -6.35% . The second column provides the information about the remaining optimality gap of the MIP solver. For example, for three splits we can observe a remaining optimality gap 0.43% which without the reoptimization step would have been equal to 0.52% . Roughly speaking, this means that there is uncertainty whether the given solution can be improved by another 4.4 million euro.

In the third column the objective value of the optimization problem is added to the fixed cost of solving the piping problem, which dominates the cost structure. We can see that then, the relative improvements in the total cost due to adjustability oscillate between 1–2%.

The fourth and fifth columns provide information about the worst-case cost for the best and the worst α . That is, they provide the range to which the worst-case cost fall depending on the trajectory of the sea level rise. For the no adjustability case there is no gap between

the two as there is only one scenario. But already for the single split solution, we can see that the ‘best scenario’ gives worst-case cost of 1106 compared to 1206 of the worst scenario. The difference between the two, equal to 100 million euro, informs us about the uncertainty in the total cost due to uncertainty about the sea level growth. This difference takes about $100/1200 \approx 8.3\%$ of the total variable cost for $J - 1 = 1$. For the three splits solution we can see that the difference between the worst and best scenarios is even bigger, with values 1044 and 1207, respectively.

An important observation is that the worst-case scenario cost does not change that much for different number of splits, showing that minimization of the average worst-case objective does not lead to the minimization of the optimistic scenarios at the expense of much bigger riskiness, so that their average would go down.

The last three columns inform on the changes in the here-and-now dike heightening decisions. We can see that with adjustability, the decisions are different for 11-12 dike segments, with the average difference oscillating between 0.1–0.2m. The resulting change in the here-and-now dike heightening cost is equally small, compare for example (last column) 606 for three splits and 606 for no split.

With respect to the impact of different values of W we can observe that $W = 2$ and $W = 3$ provide slightly higher improvements in the total cost than $W = 4$. This can be linked to the fact that if the first differentiation of the total costs is done only after 40 years, then the only later-stage decisions that differ for scenarios α are the ones with discount factor at least $1.055^{-40} \approx 0.1175$. That is, the differentiation of decisions comes ‘too late’ to lead to substantial changes in the total cost.

4.3.3 Comparison of the robust and nominal solutions

As for the nominal solution, we can clearly see in Table 4 that the objective function value in the first column is significantly smaller than for the robust models: compare 1004 for the nominal solution to 1201 for the robust solutions without adjustability. We note however, that if the nominal solution is to be fixed ‘on the spot’ later, the cost grows sharply beyond the cost of the worst scenarios for the robust solution, as illustrated in Figure 5.

Due to lack of the need to accommodate for the uncertain sea level rise, the here-and-now investment decisions for the dikes differ for 20 dike segments, with the average difference of 0.19m (lower heightenings), with a total cost of 587 at $t = 0$, instead of 606 for the robust solution - about 3% less than the robust solution. In Table 3 we see that the nominal solution applies all the large-scale measures at least as late as the robust solutions.

Overall, we can say that incorporating 40% uncertainty in the sea level rise leads only to only about 8.3% change of the expected total worst-case cost, and a 3-4% change in the here-and-now cost. These numbers seem to be rather small compared to the guarantees gained - recall what happens if the nominal solution is to be fixed later.

4.3.4 Robust solutions when the key parameters change

Conclusions of the experiments up to now rely on the value of the two crucial parameters: (i) the discount rate d , and (ii) the uncertainty parameter ρ , whose values can be seen as somewhat arbitrary. We want to validate our results by considering the model with changes in these parameters and investigate what happens to the results of Table 4 if we change the discount rate from 5.5% to 2.5%, or change the uncertainty about the sea level rise from 40% to 60%. To make this comparison concise, we focus only on the models with $W = 3$. Table 6 on page 23 presents the results.

In the first part of the table we can see the solutions for the changed discount rate. Lower discount rate implies that (i) the overall discounted cost grows higher, (ii) the cost of

Table 4: Results for different values of W . ‘Obj value’ - the optimal value (in millions of euros) of the optimization problem without piping cost (change w.r.t. no adjustability in %); ‘Opt. gap’ - the MIP optimality gap after 10 hours of computation with dike reoptimization (without dike reoptimization); ‘Best scenario’ - worst-case cost for the best scenario α ; ‘Worst scenario’ - worst-case cost for the worst scenario α ; ‘# dikes different’ - number of dikes for which the here-and-now decision is different from the non-adjustable robust solution; ‘Dike difference $t = 0$ ’ - the average differences of dike heightenings at $t = 0$ compared to the non-adjustable robust solution (for dikes for which the decision differs); ‘Dike cost $t = 0$ ’ - total cost of dike heightenings implemented at $t = 0$

		Nominal															
		Robustness with $W = 2$					Robustness with $W = 3$										
Splits ($J - 1$)	Obj value	Opt. gap	Obj value + piping	Best scenario	Worst scenario	# dikes different $t = 0$	Dike difference $t = 0$ (m)	Dikes cost $t = 0$	Splits ($J - 1$)	Obj value	Opt. gap	Obj value + piping	Best scenario	Worst scenario	# dikes different $t = 0$	Dike difference $t = 0$ (m)	Dikes cost $t = 0$
-	1004.71	0.1% (0.1%)	3607.50	-	-	20	0.19	587.93									
Splits ($J - 1$)		Robustness with $W = 2$															
0 (no adjustability)	Obj value	Opt. gap	Obj value + piping	Best scenario	Worst scenario	# dikes different $t = 0$	Dike difference $t = 0$ (m)	Dikes cost $t = 0$									
	1201.36	0.11% (0.11%)	3804.36	1201.35	1201.35	-	-	606.88									
1	1156.57 (-3.73%)	0.20% (0.23%)	3759.57 (-1.18%)	1106.98	1206.16	11	0.14	606.59									
2	1133.33 (-5.66%)	0.29% (0.32%)	3736.33 (-1.79%)	1061.78	1208.06	11	0.15	604.46									
3	1125.08 (-6.35%)	0.43% (0.52%)	3728.08 (-2.01%)	1044.49	1207.61	11	0.15	606.39									
4	1121.92 (-6.61%)	0.88% (0.97%)	3724.92 (-2.09%)	1037.25	1207.91	10	0.14	606.82									
Splits ($J - 1$)		Robustness with $W = 3$															
0 (no adjustability)	Obj value	Opt. gap	Obj value + piping	Best scenario	Worst scenario	# dikes different $t = 0$	Dike difference $t = 0$ (m)	Dikes cost $t = 0$									
	1201.35	0.1% (0.1%)	3804.35	1201.35	1201.35	-	-	606.87									
1	1147.68 (-4.46%)	0.1% (0.1%)	3750.68 (-1.43%)	1092.16	1203.20	9	0.16	603.42									
2	1135.06 (-5.84%)	0.22% (0.23%)	3738.06 (-1.77%)	1062.46	1205.60	10	0.15	602.91									
3	1132.56 (-6.07%)	0.35% (0.37%)	3735.56 (-1.84%)	1058.00	1206.23	9	0.16	603.05									
4	1131.97 (-6.12%)	0.43% (0.46%)	3734.97 (-1.85%)	1056.32	1206.95	9	0.16	602.79									
Splits ($J - 1$)		Robustness with $W = 4$															
0 (no adjustability)	Obj value	Opt. gap	Obj value + piping	Best scenario	Worst scenario	# dikes different $t = 0$	Dike difference $t = 0$ (m)	Dikes cost $t = 0$									
	1201.35 (0.00%)	0.1% (0.1%)	3804.35	1201.35	1201.35	-	-	606.87									
1	1154.88 (-3.87%)	0.1% (0.1%)	3757.88 (-1.22%)	1105.56	1204.12	5	0.55	602.42									
2	1147.96 (-4.44%)	0.13% (0.14%)	3750.96 (-1.40%)	1092.40	1204.43	5	0.65	602.17									
3	1147.36 (-4.49%)	0.20% (0.20%)	3750.36 (-1.42%)	1091.05	1204.78	5	0.65	602.17									
4	1147.23 (-4.51%)	0.19% (0.19%)	3750.23 (-1.42%)	1091.09	1204.84	5	0.65	602.17									

Table 5: Results for changed discount rate (top part), uncertainty level (middle part), and (splitting strategy). Terminology the same as in Table 4.

Splits ($J-1$)		$W = 3$ and discount rate 2.5%						
(no adjustability)	Obj value	Opt. gap	Obj value + piping	Best scenario	Worst scenario	# dikes different $t = 0$	Dike difference $t = 0$ (m)	Dikes cost $t = 0$
0	2291.73	4894.73	0.92 (0.92%)	2291.73	2291.73	0	0.0	940.46
1	2203.23 (-3.86%)	4806.23 (-1.80%)	2.56 (2.68%)	2109.58	2296.88	14	0.19	919.43
2	2146.97 (-6.31%)	4749.97 (-2.95%)	3.52 (4.29%)	1990.36	2305.81	22	0.2	902.84
Splits ($J-1$)		$W = 3$ and uncertainty 60%						
(no adjustability)	Obj value	Opt. gap	Obj value + piping	Best scenario	Worst scenario	# dikes different $t = 0$	Dike difference $t = 0$ (m)	Dikes cost $t = 0$
0	1298.21	0.09% (0.10%)	3901.21	1298.21	1298.21	-	-	632.21
1	1226.74 (-5.50%)	0.10% (0.10%)	3829.74 (-1.83%)	1151.84	1301.59	4	0.2	620.98
2	1207.61 (-6.97%)	0.19% (0.20%)	3810.61 (-2.32%)	1111.67	1302.09	4	0.2	620.98
3	1204.05 (-7.25%)	0.29% (0.30%)	3807.05 (-2.41%)	1104.14	1302.89	4	0.2	620.98
4	1203.16 (-7.32%)	0.39% (0.40%)	3806.16 (-2.43%)	1102.50	1302.55	4	0.8	620.98

later-stage decisions grow relatively to the here-and-now decisions and hence, adjustability should become more important. This is indeed visible, for example, in the number of the dike segments for which the adjustable solutions involve different here-and-now decisions, which is 14 for a single split and 22 for two splits. This contrasts with the case $d = 5.5\%$ for which only 11–12 dike segments required different decisions because of the adjustability. Surprisingly, the adjustable decisions in this case require smaller here-and-now investment cost, which is reflected in the last column.

Also, we would like to highlight here the impact of the discount rate itself - the nonadjustable solutions of the first part of Table 6 implies a different here-and-now decision for 43 dike segments compared to the nonadjustable solution of Table 4, with an average difference of 0.42m, typically implying a larger here-and-now heightening. This is logical as with the lower discount rate, the here-and-now decisions become relatively cheaper.

The second part of the table includes the solutions for $\rho = 60\%$. Here, only four dike segments require a different here-and-now decision when adjustability is applied. Also, in the last column we see that in this setting the cost of the here-and-now dike heightening decisions are lower - 620 of $J - 1 = 1$ compared to 632 of $J - 1 = 0$.

4.3.5 Numerical experiment - results overview

Here, we summarize the conclusions of the numerical experiment of Section 4 including direct links to the tables and figures. These are as follows:

- not taking the uncertainty into account and assuming a ‘moderate’ sea level rise speed leads to solutions for which the safety standards are mostly violated within the period of about 40 years (see Figure 4) if the worst-case happens;
- not taking the uncertainty into account and assuming a ‘moderate’ sea level, combined with ‘on the spot’ fixing of the situation leads to rapidly growing total cost of the solution and/or violating some of the constraints (safety requirements and *frequency constraints*, stating how often can a single dike segment be heightened), see Figure 5. In fact, the total cost of the ‘on the spot problem fixing’ strategy are higher than the maximum cost of the strategy taking into account robustness from the very beginning;
- considering 40% uncertainty in the sea level rise results in only 8.3% difference in the total project cost (see Table 4), equivalent to about 200 million euro;
- adjustability allows for lower-cost solutions particularly when the realized uncertain sea level rise in the early periods is low, see Tables 4 and 6;
- adjustability and here-and-now decisions for dike segments play a much more important role when the discount rate is low (see the first part of Table 6, where 22 dikes require a different here-and-now decision, compared to 10 dike segments in Table 4);
- only a small part of the large-scale measures (5 out of 14) are used by the optimal solutions and their moment of implementation is relatively stable (see Table 3);
- a different set splitting strategy for adjustability (see Appendix A), based on sums of sea level increase, does not change the solutions significantly.

5 Conclusion

In this paper we have considered the problem of finding the optimal flood protection strategy in a long-term horizon. In particular, the aim was to address two challenges: (i) one related to explicitly taking into account of the uncertainty related to the future sea level rise and (ii) the second one, requiring the future decisions to adjust to the revealed uncertainties from earlier

periods. Both challenges have been addressed by applying the tools of integer-adjustable robust optimization as proposed in Postek and den Hertog (2016).

The numerical experiments reveal that taking uncertainty into account early leads to solutions that, compared to the nominal solutions based on ‘moderate’ sea level rise scenario, do not require expensive adjustments at later time stages, if the uncertainty deviates from the moderate scenario. At the same time, adjustability of decisions ensures that cheaper decisions are made when the realized uncertainty is low.

Also, taking into account uncertainty results in different here-and-now decisions for some of the dike segments, which facilitate better adjustments in the future. This becomes even more important when the assumed discount rate is low (and the relative costs of the later-stage decisions are higher). In the end, we conclude that only a small part of the available large-scale measures are used in the optimal solutions.

Regarding future research, it is important to consider the problem of finding an optimal solution on a larger-scale region including also dike segments whose safety standards are driven by the (uncertain) river flow rather than by the sea. Furthermore, the model can be extended by taking into account costs arising in case a certain area is flooded.

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A Experiment - alternative splitting proposal

The results for the adjustable robust solutions up to now have been based on the splitting methodology presented in Section 3. In it, we have been differentiating the decisions based on whether r_1 fell into the upper or lower half of the corresponding interval, and the same for the later r_j .

Alternatively, one can argue that the splitting (adjustability) at time t_j should be based on the outcome of the sum $\sum_{s=1}^j r_s$. This is because the total water level depends on the sum of the sea level rises in the subsequent time periods rather than on each of them

separately. Each of the uncertainty subsets $\mathcal{R}(\boldsymbol{\alpha})$ is identified then, instead with the sequence of inequalities in the following sequence (as in Section 3)

$$r_1 (\geq \text{ or } \leq) \bar{r}_1, \quad r_1 (\geq \text{ or } \leq) \bar{r}_2, \quad r_3 (\geq \text{ or } \leq) \bar{r}_2, \quad \dots$$

with the sequence of inequalities in the following sequence:

$$r_1 (\geq \text{ or } \leq) \bar{r}_1, \quad r_1 + r_2 (\geq \text{ or } \leq) \bar{r}_1 + \bar{r}_2, \quad r_1 + r_2 + r_3 (\geq \text{ or } \leq) \bar{r}_1 + \bar{r}_2 + \bar{r}_2, \quad \dots$$

Again, the objective function is a weighted average of the cost corresponding to each of the 2^{J-1} scenarios. However, in this setting it is no longer true that all uncertainty subsets $\mathcal{R}(\boldsymbol{\alpha})$ are equally probable if one assumes uniform probability distribution of \mathbf{r} over \mathcal{R} instead, the volume of each of the uncertainty subsets needs to be computed. In other words, with $J - 1 = 2$ the probability of the scenario $\boldsymbol{\alpha} = (2, 2)$ is equal to the proportion $p(\boldsymbol{\alpha})$ of the volume of the polytope

$$\mathcal{R}(\boldsymbol{\alpha}) = \{ \mathbf{r} : (1 - \rho)\bar{r}_j \leq r_j \leq (1 + \rho)\bar{r}_j, \quad j = 1, 2, 3, \quad r_1 \geq \bar{r}_1, \quad r_1 + r_2 \geq \bar{r}_1 + \bar{r}_2, \}$$

to the volume of the entire uncertainty set \mathcal{R} . We have computed these proportions $p(\boldsymbol{\alpha})$ using a Monte Carlo simulation and we note here only that these proportions (probabilities) give relatively more weight to the extreme uncertainty subsets - the ones where the sums of sea level rises is consistently in the ‘lower part’ or ‘upper part’, respectively.

The results of the experiment with a changed splitting strategy are given in the last part of Table 6. Obviously, since the objective function is weighted with different probabilities than in the original case, we should rather compare the new splitting strategy based on the last five columns. Regarding the worst-case costs in the worst and best scenario, they are nearly the same as the ones for $W = 3$ in Table 4, with the worst-scenario only slightly better than in Table 4. This difference is most likely accountable to the new probabilities assigning more weight to the extreme scenarios. We also observe that the number of dikes with different $t = 0$ decisions is rather stable, between 6 and 9. Overall, we conclude that in this parameter setting a different splitting strategy does not lead to significantly different solution, especially in the first time period.

Table 6: Results for the changed splitting strategy. Terminology the same as in Table 4.

Splits ($J - 1$)	Obj value		Opt. gap	Obj value + piping		Best scenario		Worst scenario		# dikes different $t = 0$	Dike difference $t = 0$ (m)	Dike costs $t = 0$
	1201.35	(-4.46%)		0.1% (0.1%)	3804.35	1201.35	1201.35	1201.35	1201.35			
0 (no adjustability)	1201.35		0.1% (0.1%)	3804.35	1201.35	1201.35	1201.35	1201.35	-	-	-	606.87
1	1147.68	(-4.46%)	0.1% (0.1%)	3750.68 (-1.43%)	1092.16	1092.16	1203.20	1203.20	9	0.16	0.16	603.42
2	1132.08	(-5.77%)	0.15% (0.16%)	3735.08 (-1.82%)	1062.20	1062.20	1204.34	1204.34	8	0.16	0.16	603.15
3	1128.94	(-6.03%)	0.26% (0.27%)	3731.94 (-1.90%)	1056.84	1056.84	1203.55	1203.55	6	0.18	0.18	604.08
4	1128.35	(-6.08%)	0.33% (0.34%)	3731.35 (-1.92%)	1055.84	1055.84	1203.62	1203.62	7	0.17	0.17	603.94