Modulation Design for MIMO-CoMP HARQ

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***Abstract*—Modulation diversity (MoDiv) is a simple and practical transmission enhancement technique that utilizes different modulation mappings to reduce packet loss rate and achieve higher link throughput. MoDiv is particularly meaningful and effective in hybrid-ARQ (HARQ) systems. We study the deployment and optimization of MoDiv for HARQ in a MIMO-coordinated multi-point (MIMO-CoMP) scenario under Rician fading channel to mitigate packet loss. We formulate the design optimization of MoDiv into a quadratic three-dimensional assignment problem (Q3AP), then solve it using a modified iterated local search (ILS) method. Numerical results demonstrate clear performance gain over simple retransmissions and over a heuristic design under fading channels.**

***Index Terms*—MoDiv, MIMO, CoMP, HARQ, Q3AP.**

I. Introduction

In wireless data communication systems, high rate transmission under poor channel conditions often leads to reception errors. To recover lost packets, Automatic Repeat reQuest (ARQ) or Hybrid ARQ (HARQ) are important mechanisms for improving reliability at both network layer [1] and PHY layer [2]. Constellation Rearrangement (CoRe) is a pragmatic technique that provides additional robustness via Modulation Diversity (MoDiv) to HARQ system [3]. As linear modulations of Q-ary constellations such as PSK, QAM are usually adopted in practical wireless communications systems, the same string of $\log_2 Q$ bits can be mapped to different symbols across the multiple HARQ (re)transmissions. This technique has been studied for point-to-point HARQ [4], relay networks [5], [6] and relay-HARQ [7], [8], [9] and is shown to greatly improve the BER performance and throughput.

Also, in recent years, coordinated multipoint (CoMP) transmission has become a promising technique which has been supported by LTE-Advanced [10]. By coordinating multiple base stations/remote radio heads (RRHs) from different cells/sectors, CoMP can effectively improve cell edge user data rate and spectral efficiency [11]. When applied to CoMP, MoDiv faces great opportunities and challenges at the same time which has not been fully understood. On the one hand, it is possible to adopt different constellation remapping schemes at different transmitters, which could potentially provide larger diversity gain. On the other hand, the optimization of more than one constellation mapping for each round of retransmission is non-trivial, especially when perfect channel state information (CSI) is not available. In this work, we propose a MoDiv design scheme for a CoMP system composed of one user equipment (UE) and two cooperative transmitters connected to a centralized control unit. We assume independent Rician fading MIMO channel between the transmitters and the UE, which is more general and practical than existing works on MIMO-CoRe [12], [13]. The CoRe is designed to minimize the expected bit error rate (BER) under statistical CSI assumption after each (re)transmission. The resulting optimization problem has a Successive Quadratic 3 Dimensional Assignment Problem (Q3AP) formulation [14]. Although Q3AP is generally NP-hard and cannot be solved exactly except for trivially small constellations, an efficient modified Iterated Local Search (ILS) approach is adopted. Our numerical results demonstrate significant gains achieved by the MoDiv techniques in various simulation settings and performance measurements.

This work was supported in part by the National Science Foundation under Grants CNS-1443870, ECCS-1307820, and CCF-1321143. The work of the second author was supported in part by AFOSR under grant FA 9550-15-1-0351.

II. System Model

Consider the constellation rearrangement (CoRe) in a HARQ-enabled MIMO-CoMP downlink channel composed of two transmitters and one UE. For the original transmission, bits are mapped to constellation symbols using conventional Gray mapping for both transmitters. However, in order to maximize the signal space diversity as an enhancement of beamforming, we allow different mappings of the same sequence of information bits at the two transmitters during the HARQ process. We assume a HARQ protocol based on Chase Combining (CC) and a maximum number of M retransmissions. For the m-th (re)transmission, $m = 0, 1, \ldots, M$ where $m = 0$ represents the original transmission, every consecutive $\log_2 Q$ bits are encoded into a label $p = 0, \ldots, Q-1$ and then mapped to two constellation symbols via a vector mapping function $\boldsymbol{\psi}^{(m)}[p] = [\psi_1^{(m)}[p], \psi_2^{(m)}[p]]^T$, where $\psi_a^{(m)}[p] \in \mathcal{C}$, $\psi_a^{(m)}[p] \neq \psi_a^{(m)}[q]$ if $p \neq q$ represents a permutation of the set of constellation symbols $\mathcal{C}$ (e.g. QAM, 16-QAM), $a = 1, 2$. The two symbols are then transmitted at the two transmitters, respectively. The received signal at the m-th (re)transmission of the bit sequence corresponding to label p is:

$$\begin{aligned}\mathbf{y}^{(m)} &= \mathbf{A}^{(m)}\boldsymbol{\psi}^{(m)}[p] + \mathbf{n}^{(m)} \\ &= \mathbf{H}_1^{(m)}\mathbf{p}_1\psi_1^{(m)}[p] + \mathbf{H}_2^{(m)}\mathbf{p}_2\psi_2^{(m)}[p] + \mathbf{n}^{(m)},\end{aligned} \quad (1)$$

where $\mathbf{H}_a^{(m)}$ is a N_R-by-$N_{T,a}$ MIMO channel between the a-th transmitter and the UE, $\mathbf{p}_a$ is a $N_{T,a}$-by-1 linear beamformer for the a-th transmitter, $a = 1, 2$, and the additive noise $\mathbf{n}^{(m)} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$. We assume the elements of $\mathbf{H}_a^{(m)}$ follow correlated Rician distributed channel [15] independent across retransmissions and between the two transmitters. Specifically,

$$\mathbf{H}_a^{(m)} = \sqrt{\frac{K_a}{K_a+1}}\bar{\mathbf{H}}_a + \sqrt{\frac{1}{K_a+1}}\mathbf{R}^{1/2}\mathbf{H}_{w,a}^{(m)}\mathbf{T}_a^{1/2}, \quad (2)$$

for $a = 1, 2$ where the N_R-by-N_R matrix $\mathbf{R}$ and the $N_{T,a}$-by-$N_{T,a}$ matrix $\mathbf{T}_a$ are the receive and transmit covariance matrices, respectively, while $\bar{\mathbf{H}}_a$ represents the line-of-sight component and $\mathbf{H}_{w,a}^{(m)}$ is a random matrix composed of independent entries of $\mathcal{CN}(0, 1)$. Here we denote $\mathbf{A}^{(m)} =$

$[\mathbf{H}_1^{(m)}\mathbf{p}_1, \mathbf{H}_2^{(m)}\mathbf{p}_2]$ as the equivalent MIMO channel from the two transmitters to the UE during the m-th retransmission.

After the m-th retransmission, the receiver attempts to demodulate p by combining all the symbols received so far with the ML detection:

$$p^* = \arg\min_p \sum_{n=0}^{m} \left\| \mathbf{y}^{(n)} - \sum_{a=1,2} \mathbf{H}_a^{(n)} \mathbf{p}_a \psi_a^{(n)}[p] \right\|^2. \quad (3)$$

III. Successive Constellation Mapping Design For Modulation Diversity

A. A BER Upper Bound

The BER of the ML demodulator after the m-th (re)transmission can be upper-bounded and approximated by

$$P_{BER}^{(m)} = \sum_{p=0}^{Q-1} \sum_{q=0}^{Q-1} \frac{B[p,q]}{Q} P_{PEP}^{(m)}(q|p) \quad (4)$$

where $B[p,q]$ is the Hamming distance between the binary representation of p and q divided by $\log_2 Q$ as in [4]. The computation of $P_{PEP}(q|p)$, the pairwise error probability (PEP) of mistakenly decoding p into q, generally follows the procedure in [16]:

$$P_{PEP}^{(m)}(q|p) = \mathbb{E}\left[Q\left(\sqrt{\sum_{n=0}^{m} \frac{\|\mathbf{A}^{(n)}\mathbf{e}^{(n)}[p,q]\|^2}{2\sigma^2}} \right) \right] \quad (5)$$

where $\mathbf{e}^{(n)}[p,q] = [e_1^{(n)}[p,q], e_2^{(n)}[p,q]]^T = \boldsymbol{\psi}^{(n)}[p] - \boldsymbol{\psi}^{(n)}[q]$. Since the Q-function can be bounded as $Q(x) \le e^{-x^2/2}/2$ [17], the PEP in (5) can be upper bounded by

$$\tilde{P}_{PEP}^{(m)}(q|p) = \frac{1}{2} \prod_{n=0}^{m} \mathbb{E}\left[\exp\left(-\frac{\|\mathbf{A}^{(n)}\mathbf{e}^{(n)}[p,q]\|^2}{4\sigma^2} \right) \right] \quad (6)$$

Denote $E_n[p,q]$ as the expectation in Eq. (6), which facilitates the recursion $\tilde{P}_{PEP}^{(m)}(q|p) = \tilde{P}_{PEP}^{(m-1)}(q|p) E_m[p,q]$, $\tilde{P}_{PEP}^{(-1)}(q|p) = 1/2$. $E_n[p,q]$ can be evaluated as follows:

Proposition 1.

$$E_n[p,q] = \frac{(4\sigma^2)^{N_R} \exp\left(-\boldsymbol{\mu}_n^H[p,q] \mathbf{S}_n^{-1}[p,q] \boldsymbol{\mu}_n[p,q] \right)}{\det(\mathbf{S}_n[p,q])} \quad (7)$$

$$\mathbf{S}_n[p,q] = 4\sigma^2 \mathbf{I} + \sum_{a=1,2} \frac{|e_a^{(n)}[p,q]|^2 \mathbf{p}_a^H \mathbf{T}_a \mathbf{p}_a}{K_a+1} \mathbf{R},$$

$$\boldsymbol{\mu}_n[p,q] = \sum_{a=1,2} \sqrt{\frac{K_a}{K_a+1}} \bar{\mathbf{H}}_a \mathbf{p}_a e_a^{(n)}[p,q]. \quad (8)$$

Proof. We have $\mathbf{A}^{(n)}\mathbf{e}^{(n)}[p,q] \sim \mathcal{CN}(\boldsymbol{\mu}_n[p,q], \mathbf{C}_n[p,q])$ where

$$\mathbf{C}_n[p,q] = \sum_{a=1,2} \frac{|e_a^{(n)}[p,q]|^2 \mathbf{p}_a^H \mathbf{T}_a \mathbf{p}_a}{K_a+1} \mathbf{R}. \quad (9)$$

For $\mathbf{v} \sim \mathcal{CN}(\boldsymbol{\mu}, \mathbf{C})$, using the technique of completing the square [18, Sec. 2.3.1], it is easy to show

$$\mathbb{E}\left[\exp(-\lambda \|\mathbf{v}\|^2) \right] = \frac{\exp\left(-\boldsymbol{\mu}^H (\lambda^{-1}\mathbf{I} + \mathbf{C})^{-1} \boldsymbol{\mu} \right)}{\det(\mathbf{I} + \lambda \mathbf{C})} \quad (10)$$

which in turn leads to Proposition 1 as $\lambda = 1/(4\lambda^2)$ and $\mathbf{v} = \mathbf{A}^{(n)}\mathbf{e}^{(n)}[p,q]$. □

B. Successive Quadratic 3-D Assignment Problem (S-Q3AP)

We adopt a successive optimization scheme as in [4], [9], in which $\boldsymbol{\psi}^{(m)}$ is sequentially optimized given $\boldsymbol{\psi}^{(n)}, n = 0, \ldots, m-1$, for $m = 1, \ldots, M$:

$$\min_{\boldsymbol{\psi}^{(m)} | \boldsymbol{\psi}^{(n)}, n=0,\ldots,m-1} \tilde{P}_{BER}^{(m)}, m = 1, \ldots, M, \quad (11)$$

where $\tilde{P}_{BER}^{(m)}$ is the approximated version of $P_{BER}^{(m)}$ by substituting $P_{PEP}^{(m)}(q|p)$ with $\tilde{P}_{PEP}^{(m)}(q|p)$. Similar to [9], denote the 3-D permutation matrix $\mathbf{x}^{(m)} = \{x_{pij}^{(m)} | p,i,j = 0,\ldots,Q-1\}$ as an equivalent representation of $\boldsymbol{\psi}^{(m)}$ by $x_{pij}^{(m)} = 1$ if $\psi_1^{(m)} = \psi_0[i], \psi_2^{(m)} = \psi_0[j]$ and otherwise $x_{pij}^{(m)} = 0$, where $\psi_0[\cdot] \in \mathcal{C}$ represents Gray mapping. We have $\mathbf{x}^{(m)} \in \mathcal{S}$ where

$$\mathcal{S} = \left\{ \mathbf{x} : \sum_{p=0}^{Q-1} x_{pij} = \sum_{i=0}^{Q-1} x_{pij} = \sum_{j=0}^{Q-1} x_{pij} = 1 \right\}. \quad (12)$$

In other words, given $\mathbf{x}^{(m)} \in \mathcal{S}$, $x_{pij}^{(m)} = 1$ means that in the m-th retransmission, $\psi_1^{(m)}$ maps label p to the same symbol as Gray mapping maps label i to, and $\psi_2^{(m)}$ maps label p to the same symbols as Gray mapping maps label j to.

Then Eq. (11) can be formulated into a S-Q3AP as follows:

$$\min_{\mathbf{x}^{(m)} \in \mathcal{S}} \sum_{p=0}^{Q-1} \sum_{i=0}^{Q-1} \sum_{j=0}^{Q-1} \sum_{q=0}^{Q-1} \sum_{k=0}^{Q-1} \sum_{l=0}^{Q-1} f_{pq}^{(m)} d_{ikjl} x_{pij}^{(m)} x_{qkl}^{(m)}, \quad (13)$$

$$f_{pq}^{(m)} = \frac{B[p,q]}{Q} \tilde{P}_{PEP}^{(m-1)}(q|p) \quad (14a)$$

$$d_{ikjl} = \frac{(4\sigma^2)^{N_R} \exp(-\boldsymbol{\mu}_{ikjl}^H \mathbf{S}_{ikjl}^{-1} \boldsymbol{\mu}_{ikjl})}{\det(\mathbf{S}_{ikjl})} \quad (14b)$$

Here d_{ikjl} is derived from Eq. (7)(8) with

$$\mathbf{S}_{ikjl} = 4\sigma^2 \mathbf{I} + \left(\frac{|e_{ik}|^2 \mathbf{p}_1^H \mathbf{T}_1 \mathbf{p}_1}{K_1+1} + \frac{|e_{jl}|^2 \mathbf{p}_2^H \mathbf{T}_2 \mathbf{p}_2}{K_2+1} \right) \mathbf{R},$$

$$\boldsymbol{\mu}_{ikjl} = \sqrt{\frac{K_1}{K_1+1}} \bar{\mathbf{H}}_1 \mathbf{p}_1 e_{ik} + \sqrt{\frac{K_2}{K_2+1}} \bar{\mathbf{H}}_2 \mathbf{p}_2 e_{jl}. \quad (15)$$

and $e_{ik} = \psi_0[i] - \psi_0[k]$, $e_{jl} = \psi_0[j] - \psi_0[l]$. $f_{pq}^{(m)}$ can be updated recursively while solving the S-Q3AP, since:

$$\tilde{P}_{PEP}^{(m)}(q|p) = \sum_{i,k,j,l=0}^{Q-1} \tilde{P}_{PEP}^{(m-1)}(q|p) d_{ikjl} \hat{x}_{pij}^{(m)} \hat{x}_{qkl}^{(m)} \quad (16)$$

and $\tilde{P}_{PEP}^{(-1)}(q|p) = 1/2$. Due to the geometry of the QAM constellation, the evaluation of d_{ikjl} is actually $\mathcal{O}(Q^2)$ instead of $\mathcal{O}(Q^4)$, and it only needs to be evaluated once. Consequently, the main computational complexity lies in solving the Q3AP problems instead of evaluating their coefficients.

The size of the search space for Eq. (11) or (13) is $(Q!)^2$. For typical constellations such as 16-QAM and 64-QAM, it is impractical to apply the exact branch-and-bound algorithm [14]. Also, our tests show that they do not have enough symmetry to exploit for faster solution as does the 16-PSK constellation [19]. Consequently, the MoDiv problem is solved with the ILS method [14, Sec. 5.5] extended from its QAP version [20]. The only difference of our implementation from the one in these references is that, as in Eq. (13) a Q3AP is defined with a 4-D matrix and a 2-D matrix instead of the general Q3AP defined with a 6-D matrix [14, Eq.(2)]. The outline of ILS-Q3AP is as follows: starting from a random initial mappings $\boldsymbol{\psi} = [\psi_1, \psi_2]$, the algorithm repeatedly executes a local search by exchanging the mappings of 2 labels whenever the objective function is reduced so as to lower the objective function and update the mapping locally. When the process hits a local minimum, it executes a perturbation step to break from it and explore new solutions by exchanging the mappings of k_p labels, where integer k_p is adaptively adjusted within a range $[k_{p,min}, k_{p,max}]$. The perturbation is accepted with a probability defined as in simulated annealing, after which the local search is restarted from the new mappings until the stopping criterion is satisfied. The algorithm is implemented in the centralized control unit connected to the two transmitters.

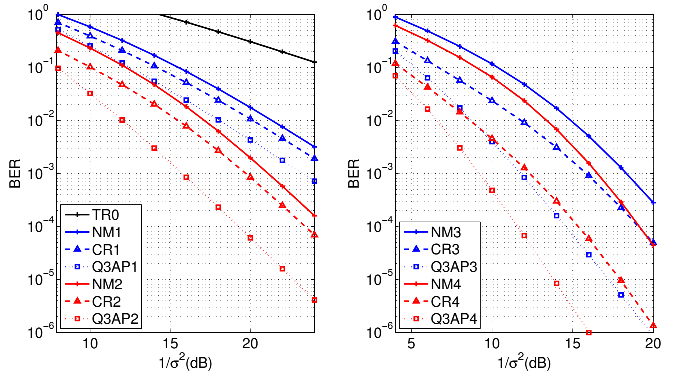
IV. Numerical Results

Unless otherwise noted, we adopt the following settings throughout our simulation. For the MIMO-CoMP channel, we have $N_R = 1$ and $N_{T,1} = N_{T,2} = 2$. For the correlated Rician fading channels, we have $\mathbf{T}_1 = \mathbf{T}_2 = [1, 0.7; 0, 7, 1]$, $\bar{\mathbf{H}}_1 = [0.2540; 0.2457]$ and $\bar{\mathbf{H}}_2 = [-0.1027; -0.2320]$. The Rician coefficients are $K_1 = K_2 = 4$ by default. A simple maximum SNR beamformer (MSNRB) [21] generalized for MIMO channel is used at the two transmitters. The maximum number of HARQ retransmissions is set to $M = 4$. Three MoDiv schemes are compared, namely the simple retransmission with no MoDiv, a heuristic CoRe scheme proposed for HSDPA [22] generalized to our CoMP scenario by fixing $\psi_1 = \psi_2$ and our Q3AP-based MoDiv design, denoted as NMm, CRm and Q3APm, respectively for m retransmissions. The original transmission using Gray mapping is labeled as TR0. 64-QAM is considered in our simulations, for which each Q3AP is solved with 5 random initializations and several minutes of ILS iterations.

First we compare the upper bound and Monte-Carlo average of the uncoded BER for different σ^2 in Fig. 1 and Fig. 2. Apparently, the BER upper bound in (4) appears to be a descent indicator of the actual BER when comparing different MoDiv schemes, and the Q3AP solution offers a substantial performance gain over the other 2 schemes. For instance, at low SNR regime Q3AP1 achieves almost the same uncoded BER as NM2, and Q3AP3 is comparable to CR4.

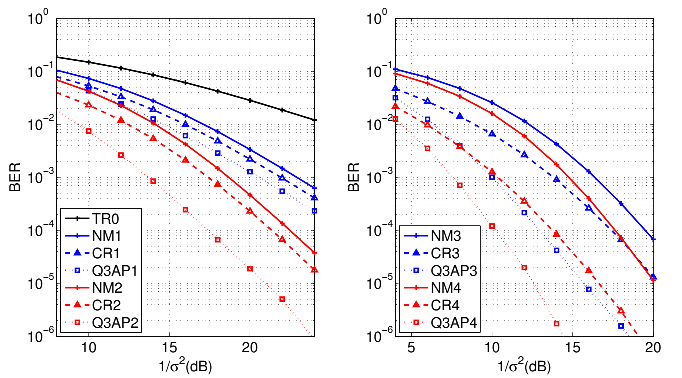
In Fig. 3 and Fig. 4, the uncoded BER is plotted against varying K under fixed $\sigma^2 = 18$dB for $m = 1, 2$ and $\sigma^2 = 8$dB for $m = 3, 4$. A significant performance gain is observed for all channel conditions ranging from heavily Rayleigh fading (small K) to very light fading (large K).

As another practical performance measurement, the coded BER performances of the three MoDiv schemes are compared in a LDPC-coded system. A LDPC code of length $L = 2400$ and code rate $r = 3/4$ is used. To demonstrate the robustness of our Q3AP-based MoDiv scheme against design parameter mismatch, we optimize the remapping only for $\sigma^2 = 5$dB and test the coded BER for a wide range of σ^2. As shown in Fig. 5, despite of the mismatch in the design parameter σ^2, our Q3AP-based MoDiv is still able to outmatches the other two MoDiv schemes, especially in the more likely cases of a smaller number of retransmissions. Specifically, the waterfall curve of coded BER for Q3AP1 almost overlaps with that of NM2, and Q3AP2 outperforms the other 2 schemes by around 1.7dB and 4dB, respectively. The average HARQ throughput defined in [22] for the same LDPC coded system is also shown in Fig. 6, which further verifies the performance gain of the Q3AP-based MoDiv design approach.



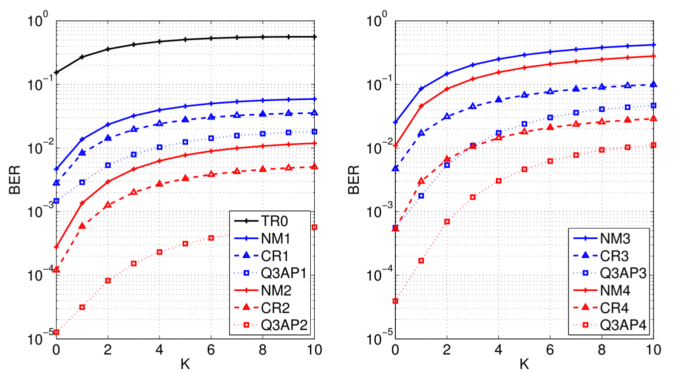
(a) $m = 1, 2$ (b) $m = 3, 4$

Fig. 1. Analytical approximation results of uncoded BER.



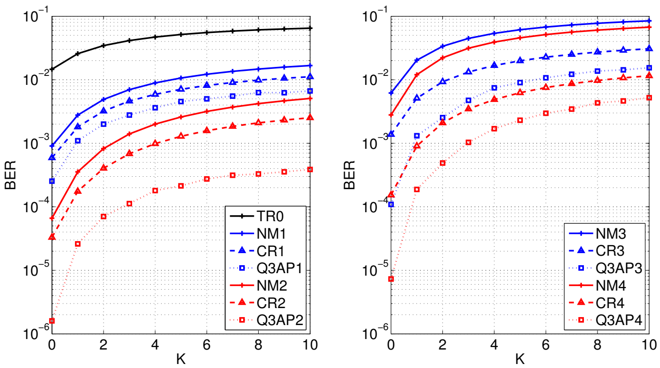
(a) $m = 1, 2$ (b) $m = 3, 4$

Fig. 2. Monte-Carlo simulation results of uncoded BER.



(a) $m = 1, 2$ (b) $m = 3, 4$

Fig. 3. Analytical approximation results of uncoded BER.



(a) $m = 1, 2$ (b) $m = 3, 4$

Fig. 4. Monte-Carlo simulation results of uncoded BER.

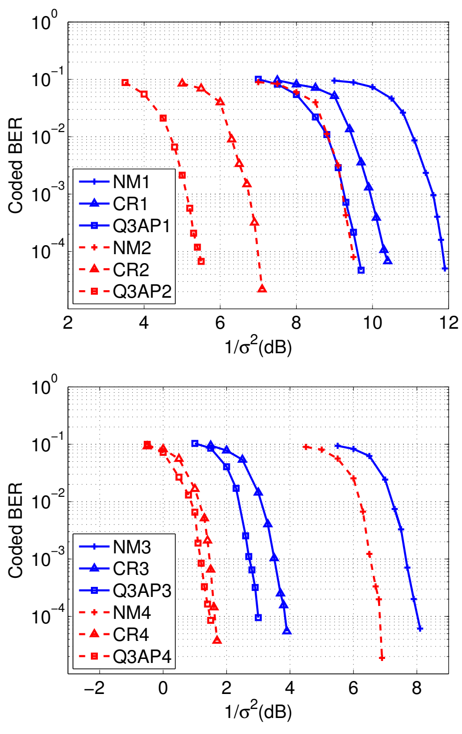


Fig. 5. Coded BER. $m = 1, 2$ (top) and $m = 3, 4$ (bottom).

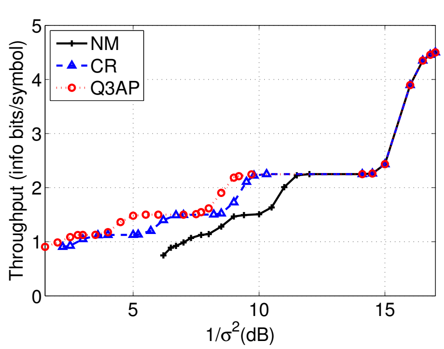


Fig. 6. Average throughput.

V. Conclusion

In this work, we investigated the modulation diversity (MoDiv) design problem for HARQ in a CoMP-MIMO system. Aiming to minimize the bit error rate (BER) upper bound, we formulated the MoDiv design into a quadratic three-dimensional assignment problem (Q3AP), and presented an efficient modified iterative local search (ILS) solution. Our numerical tests demonstrate the performance advantage and robustness of our MoDiv design over simply repeated use of Gray mapping and an existing heuristic scheme.

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