

Optimal Installation for Electric Vehicle Wireless Charging Lanes

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Abstract

RANGE anxiety, the persistent worry about not having enough battery power to complete a trip, remains one of the major obstacles to widespread electric-vehicle adoption. As cities look to attract more users to adopt electric vehicles, the emergence of wireless in-motion car charging technology presents itself as a solution to range anxiety. For a limited budget, cities could face the decision problem of where to install these wireless charging units. With a heavy price tag, an installation without a careful study can lead to inefficient use of limited resources. In this work, we model the installation of wireless charging units as an integer programming problem. We use our basic formulation as a building block for different realistic scenarios, carry out experiments using real geospatial data, and compare our results to different heuristics. Reproducibility: all datasets, algorithm implementations and mathematical programming formulation presented in this work are available at <https://github.com/hmwesigwa/smartcities.git>

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1 Introduction

The transportation sector is the largest consumer in fossil fuel worldwide. As cities move towards reducing their carbon footprint, electric vehicles (EV) offer the potential to reduce both petroleum imports and greenhouse gas emissions. The batteries of these vehicles however have a limited travel distance per charge. Moreover, the batteries require significantly more time to recharge compared to refueling a conventional gasoline vehicle. An increase in the size of the battery would proportionally increase the driving range. However, since the battery is the single most expensive unit in an EV, increasing its size would greatly increase the price discouraging widespread adaptation. Given the limitations of on-board energy storage, concepts such as battery swapping (Pan et al., 2010) have been proposed as possible approaches to mitigate these limitations. In the case of battery swapping, the battery is exchanged at a location that stores the equivalent replacement battery. This concept, however, leads to issues such as battery ownership in addition to significant swapping infrastructure costs. Another approach to increase the battery range of the EV is to enable power

exchange between the vehicle and the grid while the vehicle is in motion. This method is sometimes referred to as *dynamic charging* (Li et al., 2015; Vilathgamuwa and Sampath, 2015) or *charging-while-driving* (Chen et al., 2016). In this approach, the roads can be electrified and turned into charging infrastructure (He et al., 2013b). Ko and Jang (2013) demonstrated that dynamic charging can significantly reduce the high initial cost of EV by allowing the battery size to be downsized. This method could be used to complement other concepts such as battery swapping to reduce driver range anxiety.

There have been many studies on the design, application and future prospects of wireless power transfer for electric vehicles (see e.g., Bi et al. (2016); Cirimele et al. (2014); Fuller (2016); Li and Mi (2015); Lukic and Pantic (2013); Ning et al. (2013); Qiu et al. (2013); Vilathgamuwa and Sampath (2015)). Some energy companies are teaming up with automobile companies to incorporate wireless charging capabilities in EVs. Examples of such partnerships include Tesla-Plugless and Mercedes-Qualcomm. Universities, research laboratories and companies have invested in research for developing efficient wireless charging systems for electric vehicles and testing them in a dynamic charging scheme. Notable institutions include Auckland University (Cho and Kim, 2011), HaloIPT (Qualcomm) (Lee et al., 2010), Oak Ridge National laboratory (ORNL) (Kim et al., 2012), MIT (WiTricity) and Delphi (Jung et al., 2012). However, there is still a long way to go for a full commercial implementation, since it requires significant changes to be made in the current transportation infrastructure.

A few studies focus on the financial aspect of the implementation of a dynamic charging system. A smart charge scheduling model is presented by Li et al. (2015) that maximizes the net profit to each EV participant while simultaneously satisfying energy demands for their trips. An analysis of the costs associated with the implementation of a dynamic wireless power transfer infrastructure and a business model for the development of a new EV infrastructure are presented by Gill et al. (2014). He et al. (2013b) explores the integrated pricing of electricity in a power network and usage of electrified roads in order to maximize the social welfare.

In regards to the planning infrastructure, a number of studies have focused on the implications of dynamic charging to overall transportation network. Mohrehkesh and Nadeem (2011) analyze the effectiveness of placing wireless charging units at traffic intersections in order to take advantage of the frequent stops at these locations. Sarker et al. (2016) show how to effectively distribute power to the different charging coils along a wireless charging lane in a *vehicle-to-infrastructure* (V2I) communication system. Johnson et al. (2013) carry out simulations over a traffic network to show how connected vehicle technology, such as *vehicle-to-vehicle* (V2V) or V2I communications can be utilized in order effectively facilitate the EV charging process at fast-charging stations. Routing algorithms that take dynamic charging into account have also be developed. Li et al. (2016) develop an ant colony optimization based multi-objective routing algorithm that utilizes V2V and V2I communications systems to determine the best route considering the current battery charge.

Given the effectiveness and advances in dynamic charging technology, cities face the challenge of budgeting and deciding on what locations to install these *wireless charging lanes* (WCL) within a transportation network. In this article, we seek to optimize the installation locations of WCLs.

1.1 Related Work

Owing to advances in technology, there have been recent studies related to the optimal placement of wireless charging lanes. The basic difference in these studies arise in the objective function and/or

the type of routes, between the origin and destination, that are considered.

Recently, [Chen et al. \(2016\)](#) considered the optimal placement of wireless charging lanes when the charging infrastructure is considered to affect the EV driver’s route choice. They developed a mathematical model with an objective to minimize the total system travel times which they defined as the total social cost. There have also been studies devoted optimal locations of refueling or recharging stations of EVs when the EV driver route choice is not fixed (see, e.g, [He et al. \(2013a, 2015\)](#); [Jung et al. \(2014\)](#); [Kang and Recker \(2014\)](#)).

One prominent study where the charging infrastructure does not affect the route choice is presented by [Ko and Jang \(2013\)](#). In this study, they focus on a single route and seek the optimal system design of the *online electric vehicle* (OLEV) that utilizes wireless charging technology. They apply a *particle swarm optimization* (PSO) method to find a minimum cost solution considering the battery size, total number of WCLs (power transmitters) and their optimal placement as decision variables. The model is calibrated to the actual OLEV system and the algorithm generates reliable solutions. However, the formulation contains a non-linear objective function making it computationally challenging for multi-route networks. Moreover, speed variation is not considered in this model, which is typical in a normal traffic environment. The OLEV and its wireless charging units were developed in Korea Advanced Institute of Science and Technology (KAIST) ([Jang et al., 2012b](#)). At Expo 2012, an OLEV bus system was demonstrated, which was able to transfer 100KW (5×20 KW pick-up coils) through 20 cm air gap with an average efficiency of 75%. The battery package was successfully reduced to 1/5 of its size due to this implementation ([Jang et al., 2012a](#)). This study was recently extended ([Mouhrim et al., 2016](#)) to take multiple routes into account in which they carried out experiments on example with five routes.

In the literature, studies on optimal locations of plug-in charging facilities are often related to the maximal covering location problem (MCLP), in which each node has a demand and the goal is to maximize the demand coverage by locating a fixed number of charging facilities. For a more comprehensive study on the MCLP, one can refer to the work in [Church and Velle \(1974\)](#); [Daskin \(2008\)](#); [Farahani et al. \(2012\)](#); [Hale and Moberg \(2003\)](#). [Hodgson \(1990\)](#) builds on the MCLP and defines the flow-capturing location problem (FCLP) which seeks to maximize the *captured flow* between all origin-destination pairs. He defines a flow along a path as being captured if there exists at least one facility on the path. The definition of a flow being captured however does not carry over to the case of vehicle refueling, as a vehicle may need to refuel more than once to successfully complete the entire path. As a result, [Kuby and Lim \(2005\)](#) formulate the flow-refueling location model (FRLM). Subsequently, extensions of the FRLM have been formulated (see, e.g., [Huang et al. \(2015\)](#); [Kim and Kuby \(2012, 2013\)](#); [Kuby and Lim \(2007\)](#); [Upchurch et al. \(2009\)](#); [Wang and Lin \(2013\)](#)). In the FRLM and its extensions, the assumption that the vehicle is fully refueled at a facility does not carry over to the case of in-motion wireless charging as a EV may not be fully charged after passing over a wireless charging unit. [Riemann et al. \(2015\)](#) formulate an extension FRLM where the routes are not fixed and apply their formulation to wireless charging facilities. Similarly to other FRLM extensions, they assume that an EV is fully charged once it passes over a link containing a wireless charging facility. [Wang and Lin \(2009\)](#) proposed a model a flow-based set covering model for fast-refueling station, such as battery exchange or hydrogen refueling stations. In particular, their approach does not assume that the fuel or charge after passing through refueling or recharging facility to be full. Their work was subsequently extended ([Wang, 2011](#); [Wang and Wang, 2010](#)) while keeping a similar objective to minimize the locating cost.

[Dong et al. \(2014\)](#) took a different approach in order to optimize the locations of public charging

facilities for EVs. They take into account the long charging times of these charging stations which increases the preference for a charging facility to be located at a user activity destination.

1.2 Contribution

In this work, we seek to address the optimal location problem of wireless charging lanes with a limited budget. Our objective is to maximize the number of origin-destination routes that benefit, to a given threshold, from a deployment. This objective function is different from the one considered by [Chen et al. \(2016\)](#). Given that the battery charge may not significantly increase when an EV drives over a single wireless charging lane, the minimum budget to cover an entire network may be significantly higher than the available budget. In order to best utilize the available budget, we define a feasible path, as an origin-destination path, in relation to the final battery charge an EV would have at the end of its trip along this path. We then seek to maximize the number of feasible paths over the network. We formulate the WCL installation problem as an integer programming model that is built upon taking into account different realistic scenarios. We compare the computational results for the proposed model to faster heuristics and demonstrate that our approach provides significantly better results for fixed budget models. Using a standard optimization solver with parallelization, we provide solutions for networks of different sizes including the Manhattan road network, whose size is significantly larger than the ones considered in previous studies.

2 Optimization Model Development

The purpose of developing a mathematical model of the WCL installation problem is to construct an optimization problem that maximizes the battery range per charge within a given budget and road network. This, in turn, will minimize the driver range anxiety within the road network. In this section, we first define the road segment graph model with notation, then we discuss the modeling assumptions.

2.1 Road Segment Graph

Consider a physical network of roads within a given location. We define a road segment as the one-way portion of a road between two intersections. Let $G = (V, E)$ be a directed graph with node set V such that $v \in V$ if and only if v is a road segment. Two road segments u and v are connected with a directed edge (u, v) if and only if the end point of road segment u is adjacent to the start point of road segment v . We refer to G as the *road segment graph*. For a given road segment graph and budget constraint, our goal is to find a set of nodes that would minimize driver range anxiety within the network.

2.2 State of Charge of an EV

State of charge (SOC) is the equivalent of a fuel gauge for the battery pack in a battery electric vehicle and hybrid electric vehicle. The SOC determination is a complex non-linear problem and there are various techniques to address it (see e.g., [Chang \(2013\)](#); [Koirala et al. \(2015\)](#); [Wang et al. \(2016\)](#)). As discussed in the literature, the SOC of an EV battery can be determined in real time using different methods, such as terminal voltage method, impedance method, coulomb counting method, neural network, support vector machines, and Kalman filtering. The input to the models

are physical battery parameters, such as terminal voltage, impedance, and discharging current. However, the SOC related input to our optimization model is the change in SOC of the EV battery to traverse a road segment rather than the absolute value of the real time SOC of the EV battery. So, we formulate a function that approximates the change in SOC of an EV to traverse a road segment using several assumptions, as mentioned in the following. The units of SOC are assumed to be percentage points (0% = empty; 100% = full). The change in SOC is assumed to be proportional to the change in battery energy. This is a valid assumption for very small road segments that form a large real road network, which is the case in this analysis (range of 0.1 to 0.5 mile).

We compute the change in SOC of an EV as a function of the time t spent traversing a road segment by

$$\Delta\text{SOC}_t = \frac{E_{\text{end}} - E_{\text{start}}}{E_{\text{cap}}}, \quad (1)$$

where E_{start} and E_{end} is the energy of the battery (KWh) before and after traversing the road segment respectively and E_{cap} is the battery energy capacity. We follow the computation of the battery energy given by [Sarker et al. \(2016\)](#). We, however, assume that the velocity of an EV is constant while traversing the road segment. This gives us

$$E_{\text{end}} - E_{\text{start}} = (P_{2t} \cdot \eta)t - P_{1t}t, \quad (2)$$

where P_{1t} is the power consumption (KW) while it traverses the road segment, and P_{2t} is the power delivered to the EV in case a WCL is installed on the road segment, otherwise P_{2t} is zero. In order to take into account the inefficiency of charging due to factors such as misalignment between the primary (WCL) and secondary (on EV) charging coils and air gap, an inefficiency constant η is assumed.

The power consumption P_{1t} varies from EV to EV. In this work, we take an average power consumption calculated by taking the average mpge (miles per gallon equivalent) and battery energy capacity rating from a selected number of EV. We took the average of over 50 EVs manufactured in 2015 or later. For each EV, its fuel economy data was obtained from [US \(2017\)](#).

For P_{2t} and η , the power rating of the WCL, and the efficiency factor, we average the values from [Bi et al. \(2016\)](#), Table 2, where the authors make a comparison of prototype dynamic wireless charging units for electric vehicles.

2.3 Modeling Assumptions

For a road segment graph G , we assume that each node has attributes such as average speed and distance that are used to compute the average traversal time of the road segment. Note that since nodes represent road segments, an edge represents part of an intersection, thus, the weight of an edge does not have a typical general purpose weighting scheme associated to it (such as a length). For a given pair of nodes s and t , where s represents the origin and t the destination of a user within the road network, respectively, we assume that the user will take the fastest route from s to t . As a result, we assign the weight of each edge (u, v) in the road segment graph with a value equal to the average traversal time of road segment u . Then we can find a shortest path with road segment graph that represents the fastest route from the start point of road segment s to the end point of t .

We assume that SOC of any EV whose journey starts at the beginning of a given road segment is fixed. For example, we may assume that if a journey starts at a residential area, then any EV at this starting location will be fully charged or follows a charge determined by a given probability

distribution which would not significantly change the construction of our model. For example, in real applications, one could choose the average SOC of EV's that start at that given location. In our empirical studies, for simplicity, we first assume that all EVs start fully charged. We later give results for studies where we take the initial SOC to be chosen uniformly at random. We also assume that SOC takes on real values such that $0 \leq \text{SOC} \leq 1$ at any instance where $\text{SOC} = 1$ implies that the battery is fully charged and $\text{SOC} = 0$ implies that the battery is empty.

For any two road segments s and t in G , we assume that there is a unique shortest path between them. Since we are using traversal time as a weighting scheme of the road segment graph, this is a reasonable assumption. If there exists two road segments such that this assumption is not realistic, then one can treat these paths using distinct routes and include them in the model since we will define a model based on distinct routes.

We call a route *infeasible* within a network if any EV that starts its journey at the beginning of this route (starts fully charged in our empirical studies), will end with a final $\text{SOC} \leq \alpha$, where $0 \leq \alpha \leq 1$. The constant α is a global parameter of our model called a global SOC threshold. The value of α could be chosen in relation to the minimum SOC an EV driver is comfortable driving with (Franke et al., 2012). Introducing different types of EV and more than one type of α would not significantly change the construction of the model.

Given the total length of all road segments in the network, T , we define the budget, $0 \leq \beta \leq 1$, as a part of T for which funds available for WCL installation exist. For example, if $\beta = 0.5$, the city planners have enough funds to install WCL's across half the road network. We use our model and its variations to answer the following problems that the city planners are interested in.

1. For a given α , determine the *minimum budget*, β , together with the corresponding locations, needed such that the number of infeasible routes is zero.
2. For a given α and β , determine the optimal installation locations to *minimize the number of infeasible routes*.

We assume that minimizing the number of infeasible routes would reduce the driver range anxiety within the network.

2.4 Single Route Model Formulation

Let *Routes* be the set of all possible fastest routes between each pair of nodes $i, j \in V$. In our model, we assume that the fastest route between a pair of nodes is represented by a unique route. For each route $r \in \text{Routes}$, assume that each EV whose journey is identical to this route has a fixed initial SOC, and a variable final SOC, termed $i\text{SOC}_r$, and $f\text{SOC}_r$, respectively, depending on whether or not WCL's were installed on any of the road segments along the route. The goal of the optimization model is to guarantee that either $f\text{SOC}_r \geq \alpha$ where α is a global threshold or $f\text{SOC}_r$ is as close as possible to α for a given budget. The complete optimization model takes all routes into account. Given that realistic road segment graphs have a large number of nodes, taking all routes into account may overwhelm the computational resources, thus, the model is designed to give the best solution for any number of routes considered.

We describe the model by first defining it for a single route and then generalizing it to multiple routes. For simplicity, we will assume that the initial SOC, $i\text{SOC}_r = 1$, for each route r , i.e., all EV's start their journey fully charged. This assumption can easily be adjusted with no significant changes to the model. Since we assume that SOC takes discrete values, we define a unit of increase

or decrease of SOC as the next or previous discrete SOC value respectively. For simplicity, we will also assume a simple SOC function in this section. We will assume that SOC of an EV traversing a given road segment increases by one unit if a WCL is installed, otherwise it decreases by one unit. A more realistic SOC function can be incorporated into the following model without any major adjustments, as presented in the following.

For a single route $r \in Routes$, with $iSOC_r = 1$, consider the problem of determining the optimal road segments to install WCL's in order to maximize $fSOC_r$ within a limited budget constraint. Define a SOC-state graph, $socG_r$, for route r , as an acyclic directed graph whose vertices describe the varying SOC an EV on a road segment would have depending on whether or not the previously visited road segment had a WCL installed. More precisely, let $r = (u_1, u_2, \dots, u_k)$, for $u_i \in V$ with $i = 1, \dots, k$ and $k > 0$. Let $nLayers \in \mathbb{N}$ represent the number of discrete values that the SOC can take. For each $u_i \in r$, let $u_{i,j}$ be node in $socG_r$ for $j = 1, \dots, nLayers$ representing the $nLayers$ discrete values that the SOC can take at road segment u_i . Let each node $u_{i,j}$ have out-degree at most 2, representing the two different scenarios of whether or not a WCL is installed at road segment u_i . The edge (u_{i,j_1}, u_{i+1,j_2}) has weight 1 represents the scenario if a WCL is installed at u_i and has weight 0 if a WCL is not installed. An extra nodes are added accordingly to capture the output from the final road segment u_k , we can think of this as adding an artificial road segment u_{k+1} . Two dummy nodes, s and t , are also added to the graph $socG_r$ to represent the initial and final SOC respectively. There is one edge of weight 0 between s and u_{1,j^*} where u_{1,j^*} represents the initial SOC of an EV on this route. Each node $u_{k,j}$ for each j is connected to node t with weight 0. Figures 1 shows an example for a SOC graph constructed from a route with three road segments.

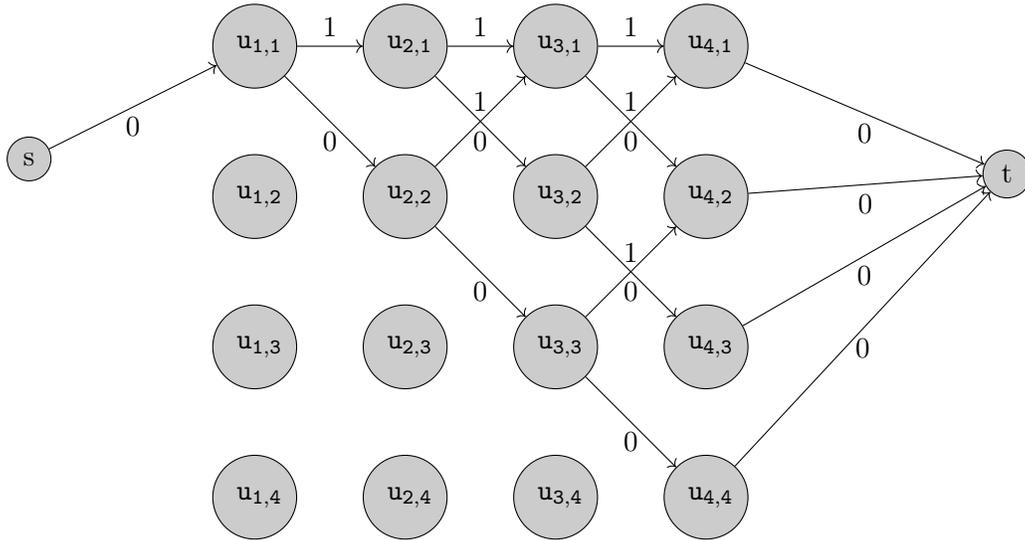


Figure 1: Example of $socG_r$ with $r = (u_1, u_2, u_3)$ and $nLayers = 4$. u_4 is an artificial road segment added to capture the final SOC from u_3 . The nodes in the set $\mathcal{B} = \{u_{i,j} | i = 4 \text{ or } j = 4\}$ are referred to as the boundary nodes. The out going edges of each node $u_{i,j}$ are determined by an SOC function. Each node represents a discretized SOC value. For example, the nodes $u_{i,1}$ and $u_{i,4}$ represent an SOC value of 1 and 0, respectively.

Consider a path p from s to t , namely, $p = (s, u_{1,j_1}, u_{2,j_2}, \dots, u_{k,j_k}, t)$, then each node in p represents the SOC of an EV along the route. We use this as the basis of our model. Any feasible

$s - t$ path will correspond to an arrival at a destination with an SOC above a given threshold. A minimum cost path in this network would represent the minimal number of WCL installations in order to arrive at the destination. Let $socG_r = (V', E')$ with weighted edges w_{ij} , then the minimum cost path can be formulated as follows:

$$\begin{aligned} & \text{minimize} && \sum_{ij \in E'} w_{ij} x_{ij} \\ & \text{subject to} && \sum_j x_{ij} - \sum_j x_{ji} = \begin{cases} 1, & \text{if } i = s; \\ -1, & \text{if } i = t; \\ 0, & \text{otherwise} \end{cases} \\ & && x_i \in \{0, 1\} \end{aligned}$$

where $\sum_j x_{ij} - \sum_j x_{ji} = 0$ ensures that we have a path i.e., number of incoming edges is equal to number of out going edges.

Decision Variable for Installation Let R_i be the decision variable for installation of a WCL at road segment i . Then, for a single route, we have $R_i = \sum_k w_{ik} \in \{0, 1\}$. For multiple routes we

have

$$R_i = \begin{cases} 1, & \text{if } \sum_k w_{ik} \geq 1 \\ 0, & \text{if } \sum_k w_{ik} = 0. \end{cases}$$

In other words, install a WCL if at least one route requires it. Under these constraints for a single route, an optimal solution to the minimum cost path from s to t would be a solution for the minimum number of WCL's that need to be installed, in order for EV to arrive at the destination with its final SOC greater than a specified threshold.

2.5 General Model Description and Notations

In this section, we describe the model when multiple routes are taken into consideration. Similar to the model for a single route, we define a different graph $G_r = (V, E_r)$ for each route, r , together with one global constraint. Note that each graph G_r is defined over the same node set V however, the edge set E_r is dependent on the route. Consider all fastest routes in the network. Construct a SOC-state graph for each of these routes. We can think of this network as $nRoutes$ distinct graphs interconnected by at least one constraint.

Decision Variables:

We first describe the different notation necessary for describing the model.

$nRoadSegs$	number of road segments
$nLayers$	number of discrete values SOC can take
$nRoutes$	number of routes taken into account
$nNodes$	number of nodes in soc-state graph

For a given route, r , define a SOC graph $G_r = (V, E_r)$ where the edges E_r are defined according to the SOC function. The weights for edges in E_r are given by

$$w_{r,i,j} = \begin{cases} 1, & \text{if WCL is installed in respective road segment for } i \\ 0, & \text{otherwise} \end{cases}$$

Then the decision variables of the model are given by

$$R_s = \begin{cases} 1, & \text{if at least one route requires a WCL installation} \\ 0, & \text{otherwise} \end{cases}$$

for $s = 1, \dots, nRoadSegs$

$$x_{r,i,j} = \begin{cases} 1, & \text{if edge } (r, i, j) \text{ is in an } s-t \text{ path in } G_r \\ 0, & \text{otherwise} \end{cases}$$

for $r = 1, \dots, nRoutes$

For the decision variable R_s on the installation of a WCL at road segment s , we install a WCL if at least one route requires an installation within the different $s-t$ paths for each route. For road segment s , and for any set of feasible $s-t$ paths, let $p(s)$ be the number of routes that require a WCL installation, then $p(s)$ is given by

$$p(s) = \sum_{r=1}^{\overbrace{nRoutes}^{\text{all routes}}} \sum_{u=\overbrace{nLayers(s-1)+1}^{\text{road segment } s}}^{\overbrace{s \cdot nLayers}^{\text{road segment } s}} \sum_{(r,u,v) \in E} w_{(r,u,v)} \cdot x_{(r,u,v)}$$

Then,

$$R_s = \begin{cases} 1, & \text{if } p(s) \geq 1 \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

models the installation decision.

Objective function:

For the problem of minimizing the budget, the objective function is simply given by

$$\sum_{s=1}^{nRoads} c_s \cdot R_s. \quad (4)$$

where c_s is the cost of installing a WCL at road segment s .

For the problem of minimizing the number of infeasible routes for any fixed budget, we modify the SOC graph such that there exists an $s - t$ path for any budget. We accomplish this by adding an edge of weight 0 between the nodes $u_{i,nLayers}$ to t for all $i = 1, \dots, k + 1$, in each route, where k is the number of road segments in the route. We then define the *boundary nodes* of the SOC graph to be all the nodes that are adjacent to node t . Let \mathcal{B} be the set of all boundary nodes and \mathcal{B}_r be the boundary nodes with respect to route r . Assign each node in $u_{i,j} \in \mathcal{B}_r$ weights according to the function:

$$w(u_{i,j}) = \begin{cases} 1, & \text{if } \mathbf{s}(u_{i,j}) \geq \alpha \\ 0, & \text{otherwise} \end{cases}$$

where $\mathbf{s}(u_{i,j})$ represents the discretized SOC value that node $u_{i,j}$ represents and $|r|$ is the number of road segments in route r . In the weighting scheme above, we make no distinctions between two infeasible routes. However, a route in which an EV completes, say, 90% of the trip would be preferable to one in which an EV completes, say, 10% of the trip. One can easily take this preference into account and weight the boundary nodes by the function

$$w(u_{i,j}) = \begin{cases} 1, & \text{if } \mathbf{s}(u_{i,j}) \geq \alpha \\ \frac{i-|r|}{|r|}, & \text{otherwise,} \end{cases} \quad (5)$$

where the term $\frac{i-|r|}{|r|}$ measures how close an EV comes to completing a given route, i.e, if $\frac{i-|r|}{|r|} = -1$, then the EVs SOC falls below α after traversing the first road segment.

For simplicity, we relabel the nodes in the SOC graph in a canonical way indexed by the set \mathbb{N} . For a node i in the SOC graph, let $w(r, i, i)$ represent the weight of i . Then the objective is maximize charge of each route, which is given by

$$\sum_{r=1}^{nRoutes} \sum_{(u,t) \in \mathcal{B}_r} w(r,u,u) \cdot x(r,u,t) \quad (6)$$

Budget Constraint:

Cost of installation cannot exceed a budget B . Since this technology is not yet widely commercialized, we can discuss only the estimates of the budget for WCL installation. Currently, the price of installation per kilometer ranges between a quarter million to several millions dollars (Gill et al., 2014). For simplicity, in our model the cost of installation at a road segment is assumed to be proportional to the length of the road segment which is likely to be a real case. Thus, a budget would represent a fraction of the total length of all road segments.

$$\sum_{s=1}^{nRoads} c_s \cdot R_s \leq B \quad (7)$$

Route Constraints:

The constraints defining an s - t path

$$\sum_j x_{r,i,j} - \sum_j x_{r,j,i} = \begin{cases} 1, & \text{if } i = s; \\ -1, & \text{if } i = t; \\ 0, & \text{otherwise} \end{cases}$$

2.5.1 Model

The complete model formulation for minimizing the number of infeasible routes, for a fixed budget is given by:

$$\text{maximize} \quad \sum_{r=1}^{nRoutes} \sum_{(u,t) \in \mathcal{B}_r} w_{(r,u,u)} \cdot x_{(r,u,t)}$$

$$\text{subject to} \quad \sum_{s=1}^{nRoads} c_s \cdot R_s \leq B$$

$$\sum_j x_{(r,i,j)} - \sum_j x_{(r,j,i)} = \begin{cases} 1, & \text{if } (r,i) = (r,s) \\ -1, & \text{if } (r,i) = (r,t) \\ 0, & \text{otherwise} \end{cases} \quad r = 1, \dots, nRoutes$$

$$\begin{aligned} R_s &\leq p(s) && s = 1, \dots, nRoadSegs \\ \mathbf{M}R_s &\geq p(s) - \epsilon && s = 1, \dots, nRoadSegs \\ R_s &\leq 1 && s = 1, \dots, nRoadSegs \\ R_s &\in \mathbb{N} && s = 1, \dots, nRoadSegs \end{aligned}$$

where

$$p(s) = \underbrace{\sum_{r=1}^{nRoutes}}_{\text{all routes}} \underbrace{\sum_{u=nLayers(s-1)+1}^{s \cdot nLayers}}_{\text{road segment } s} \sum_{(r,u,v) \in E} w_{(r,u,v)} \cdot x_{(r,u,v)}$$

and \mathbf{M} , a large constant and $0 < \epsilon < 1$ are used to model the logic constraints given in equation (3).

Since we are interested in reducing the number of routes with a final SOC less than α , we can take the set of routes in the above model to be the all the routes that have a final SOC below the given threshold. We evaluate this computationally and compare it with several fast heuristics.

2.6 Heuristics

Integer programming is NP-hard in general and since the status of the above optimization model is unknown, we have little evidence to suggest that it can be solved efficiently. For large road networks, it may be desirable to use heuristics instead of forming the above integer program. In particular, since we know the structure of the network, one natural approach may be to apply concepts from

network science to capture the features of the best candidates for a WCL installation. In this section, we outline different heuristics for deciding on the a set of road segments. We then compare these structural based solutions to the optimization model solution, and demonstrate the superiority of proposed model.

Different centrality indexes is one of the most studied concepts in network science (Newman, 2010). Among them, the most suitable to our application are betweenness and vertex closeness centralities. In (Freeman, 1978), a node closeness centrality is defined as the sum of the distances to all other nodes where the distance from one node to another is defined as the shortest path (fastest route) from one to another. Similar to interpretations from (Borgatti, 1995), one can interpret closeness as an index of the expected time until the arrival of something "flowing" within the network. Nodes with a low closeness index will have short distances from others, and will tend to receive flows sooner. In the context of traffic flowing within a network, one can think of the nodes with low closeness scores as being well-positioned or most used, thus ideal candidates to install WCL.

The betweenness centrality (Freeman, 1978) of a node k is defined as the fraction of times that a node i , needs a node k in order to reach a node j via the shortest path. Specifically, if g_{ij} is the number of shortest paths from i to j , and g_{ikj} is the number of i - j shortest paths that use k , then the betweenness centrality of node k is given by

$$\sum_i \sum_j \frac{g_{ikj}}{g_{ij}}, \quad i \neq j \neq k,$$

which essentially counts the number of shortest paths that path through a node k since we assume that $g_{ij} = 1$ in our road network because edges are weighted according to time. For a given road segment in the road segment graph, the betweenness would basically be the road segments share of all shortest-paths that utilize that the given road segment. Intuitively, if we are given a road network containing two cities separated by a bridge, the bridge will likely have high betweenness centrality. It also seems like a good installation location because of the importance it plays in the network. Thus, for a small budget, we can expect the solution based on the betweenness centrality to give to be reasonable in such scenarios. There is however an obvious downfall to this heuristic, consider a road network where the betweenness centrality of all the nodes are identical. For example, take a the cycle on n nodes. Then using this heuristic would be equivalent to choosing installation locations at random. A cycle on n vertices can represent a route taken by a bus, thus, a very practical example. Figure 2 shows an optimal solution from our model to minimize the number of infeasible routes with a budget of at most four units.

The eigenvector centrality (Newman, 2008) of a network is also considered. As an extension of the degree centrality, a centrality measure based on the degree of the node, the concept behind the eigenvector centrality is that the importance of a node is increased if it connected to other important nodes. In terms of a road segment graph, this would translate into the importance of a road segment increasing if its adjacent road segments are themselves important. For example, if a road segment is adjacent to a bridge. One drawback of using this centrality measure is that degree of nodes in road segment graphs is typically small across the graph. However, it still helps to find regions of potentially heavy traffic.

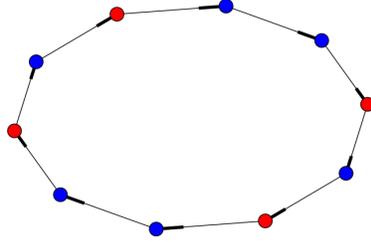


Figure 2: Optimal solution with a four unit installation budget. The thick ends of the edges are used to indicate the direction of the edge. Taking $\alpha = 0$ without any installation, there are 70 number of infeasible routes. An optimal installation of 5 WCLs would ensure zero infeasible routes. With an optimal installation of 4 WCLs, the nodes colored in red, there would have 12 infeasible routes.

2.7 Data collection and post-processing

The geospatial data is collected from OpenStreetMap (OSM). It is a free collaborative project to generate editable maps of any location on earth (Haklay and Weber, 2008). A region of interest can be selected on OpenStreetMap user interface, and all available data can be generated for the selected region. OpenStreetMap offers different formats for the user. In this case, the selected format is XML format. Also, there is a website named planet.osm that already contains captured OSM XML files for different cities in different parts of the world. The contents of an OSM XML file are described in Table 1.

A script is developed to process the raw XML data and extract meaningful information. First, the road network is filtered using the tag 'highway' that specifies the characteristics of a road. The 'highway' tag must also contain sub-tags such as 'motorway', 'trunk', 'primary', 'secondary', 'tertiary', 'road', 'residential', 'living_street' etc. This eliminates the unnecessary parts of the network, as the model only deals with the road network. Then, the filtered road network is segmented into small road segments having two end points (points with specific latitude and longitude). In the segmented graph, each node represents a road, and connectivity between nodes represent connection of roads. Two nodes in a graph are connected if they share a point with the same latitude and longitude. Then, the adjacency matrix of the segmented graph is extracted representing the network's connectivity (i.e. if two road segments have one point in common, they are connected). This is a sparse matrix with 1 as its only non-zero entry.

Finally, the length of each road segment is calculated using the haversine formula. The haversine formula gives great-circle distances between two points on a sphere from their longitudes and latitudes. It is a special case of a more general formula in spherical trigonometry, the law of haversines, relating the sides and angles of spherical triangles. For any two points on a sphere, the distance (d) between the points is calculated using the following equations (Veness, 2011).

$$a = \sin^2(\Delta\phi/2) + \cos \phi_1 \cdot \cos \phi_2 \cdot \sin^2(\Delta\lambda/2) \quad (8)$$

$$c = 2 \arctan 2(\sqrt{a}, \sqrt{1-a}) \quad (9)$$

$$d = R \cdot c \quad (10)$$

where, ϕ_1, ϕ_2 are latitudes, $\Delta\phi$ is a difference of latitudes, λ_1 and λ_2 are longitudes, $\Delta\lambda$ is a difference of longitudes, and R is the Earth’s radius (mean radius = 3959 miles).

In this section, we discuss the results of the proposed model used to solve the WCL installation problem. We use Pyomo, a collection of Python packages by (Hart et al., 2012, 2011) to model the integer program. As a solver, we used CPLEX 12.7 (ILOG, 2014) with all our results attained with an optimality gap of at most 10%. Designing a fast customized solver is not the central goal of this paper. However, it is clear that introducing customized parallelization and using advanced solvers will make the proposed model solvable for the size of a large city in urban area.

The measurement of WCL installation effectiveness on a particular road segment depends on the SOC function used. However, the SOC function varies from EV to EV and is dependent on such factors as vehicle and battery type and size together with the effectiveness of the charging technology used. However, the purpose of this paper is to propose a model that is able to accommodate any SOC function.

2.8 Small networks

In order to demonstrate the effectiveness of our model, we begin with presenting the results on two small toy graphs in which all road segments are identical. We incorporate a simple toy SOC function, one in which the SOC increases and decreases by one unit if a wireless charging lane is installed or not installed, respectively. For the first toy graph (see Figure 3(a)), we are interested in determining the minimum budget such that all routes are feasible. For this we assume that a fully charged battery has four different levels of charge 0, 1, 2, and 3, where a fully charged battery contains three units. This would imply that the SOC-layered graph would contain four layers. The parameter α is fixed to be 0. For the second toy graph (see Figure 3(b)), we are interested in minimizing the number of infeasible routes with a varying budget. For this example, we take the number of layers for the SOC-layered graph to be five while also taking $\alpha = 0$.

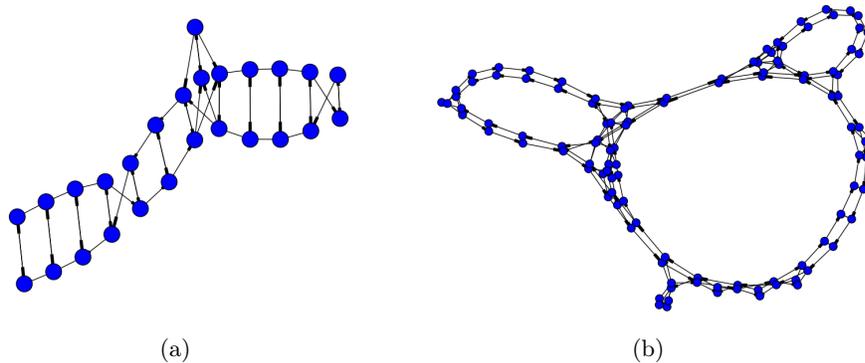


Figure 3: Directed toy graphs of 26 and 110 vertices used for problems 1 and 2, respectively. The bold end points on the edges of (a) represent edge directions. The graphs are subgraphs of the California road network taken from the dataset SNAP in (Leskovec et al., 2009)

In experiments with the first graph, we take **all** routes into consideration and compute an optimal solution which is compared with the betweenness and eigenvector centralities. We rank the nodes

based on their centrality indexes, and take the smallest number of top k central nodes that ensure that all routes are feasible. The installation locations for each method are shown in Figure 4 of which the solution to our model uses the smallest budget. We observe a significant difference in the required budget to ensure feasibility of the routes (see values B in the figure).

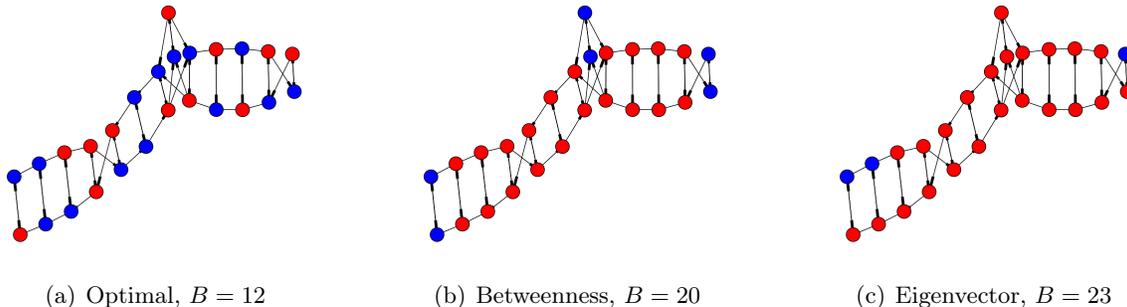


Figure 4: Comparison of the different methods. The minimum number of WCL installation needed to eliminate all infeasible routes is B . The nodes colored red indicate location of WCL installation. In (a), we demonstrate the result given by our model requiring a budget of 12 WCL's in order to have zero infeasible routes. In (b) and (c), we demonstrate solutions from the betweenness and eigenvector heuristics that give budgets of 20 and 23 WCL's, respectively.

For the second graph, we vary the available budget β . The results are shown in Figure 5. The plots also indicate how the optimal solution affects the final SOC of all other routes. The solutions to our model were based on 100 routes, with length at least 2, that were sampled uniformly without repetitions. We observe that our solution gives a very small number of infeasible routes for all budgets. We notice that for a smaller budget, taking a solution based on betweenness centrality gives a similar but slightly better solution than that produced by our model. However, this insignificant difference is eliminated as we increase the number of routes considered in our model. Note that if our budget was limited to one WCL, then the node chosen using the betweenness centrality would likely be a good solution because this would be the node that has the highest number of shortest paths traversed through it compared to other nodes. As we increase the budget, the quality of our solution is considerably better than the other techniques. For budgets close to 50% in Figure 5 (d) and (e), our model gives a solution with approximately 90% less infeasible routes compared to that of the betweenness centrality heuristic. This is in spite of only considering about 1% of all routes as compared to betweenness centrality that takes all routes into account.

2.9 Experiments with Manhattan network

In the above example, the input to our model is a road segment graph with identical nodes, and a simple SOC function. We next test our model with real data and the SOC function defined in Section 2.2. We extract data of lower Manhattan using Openstreet maps. The data is preprocessed by dividing each road into road segments. Each road in Openstreet maps is categorized into one of eight categories presented in Table 2, together with the corresponding speed limit for a rural or urban setting. For this work, we consider roads from categories 1 to 5 as potential candidates for installing

wireless charging lanes due to their massive exploitation. Thus, we remove any intersections that branch off to road categories 6 to 8. The resulting road segment network contains 5792 nodes for lower Manhattan. We also study a neighborhood of lower Manhattan that forms a graph of 914 nodes. The graphs are shown in Figure 6.

Similar to experiments on the second toy example, we carry out experiments on the Manhattan network using 200 routes. We sample routes that have a final SOC less than the threshold α uniformly at random without repetitions. Due to relatively small driving radius within the Manhattan neighborhood graph shown in Figure 6 (b), we increase the length of each road segment by a constant factor in order to have a wider range of a final SOC within each route. We take $\alpha = 0.8$ and 0.85 with a corresponding budget of $\beta = 0.1$ and 0.2 respectively for the Manhattan neighborhood graph while $\alpha = 0.7$ and $\beta = 0.1$ for the lower Manhattan graph. We compare our results with the heuristic of choosing installation locations based on their betweenness centrality. *In our experiments, the betweenness centrality produces significantly better results than other heuristics, so it is used as our main comparison.*

For a threshold $\alpha = 0.8$ in the Manhattan neighborhood graph, there are 42,001 infeasible routes with no WCL installation. With a budget $\beta = 0.1$, our model was able to reduce this number to 4,957. Using the heuristic based on betweenness centrality, the solution found contained 21,562 infeasible routes. For a budget of $\beta = 0.2$ with threshold $\alpha = 0.85$, there were 170,393 infeasible routes without a WCL installation, 57,564 using the betweenness centrality heuristic and only 14,993 using our model. Histograms that demonstrate the distributions of SOC are shown in Figure 7. The green bars represent a SOC distribution without any WCL installation. The red and blue bars represent SOC distributions after WCL installations based on the betweenness centrality heuristic and our proposed model, respectively.

In Figure 8, we demonstrate the results for the lower Manhattan graph, with $\alpha = 0.7$ and $\beta = 0.1$. Due to the large number of routes, we sample about 16 million routes. From this sample, our model gives a solution with 10% more infeasible routes compared to the heuristic based on betweenness centrality. Note that in this graph, there are about 13 million infeasible routes. From these routes, we randomly chose less than 1000 routes for our model without any sophisticated technique for choosing these routes, while the heuristic based on betweenness centrality takes all routes into account. The plot in Figure 8, shows the number of routes whose final SOC falls below a given SOC value. Similar to Figure 5 (a), the results demonstrate that for a relatively small budget, our model gives a similar result compared to the betweenness centrality heuristic.

2.10 Experiments with Random Initial SOC

In the preceding experiments, EVs were assumed to start their journey fully charged. However, the assumption in our model was that the initial SOC be any fixed value. Thus, as an alternative scenario, one can take the initial SOC to follow a given distribution selected either by past empirical data or known geographic information about a specific area. For example, one could assume higher values in residential areas compared to non-residential areas. In this work, we carried out experiments where the initial SOC was chosen uniformly at random in the interval $(a, 1)$. The left endpoint of the interval was chosen such that the final SOC associated to any route would be positive. In order to give preference to longer routes, we define Ω_k as the set of all routes with distance greater than $\mu + k\sigma$, where μ is the average distance of a route with standard deviation σ , for some real number k . We then study the average of the final SOC of all routes in Ω_k which we denote as \bar{x}_k .

In the Manhattan neighborhood graph, Figure 6 (b), we chose 1200 routes as an input to the

model. These routes were chosen uniformly at random from Ω_k . In our solution analysis, we took $a = 0.4$ and computed \bar{x}_k . Without any installation, we had $\bar{x}_k \approx 0.39$ for $k \geq 2$ while $0.5 \leq \bar{x}_k \leq 0.51$ based the betweenness heuristic with a installation budget of 20% of the entire network. However, our model gives us $0.59 \leq \bar{x}_k \leq 0.71$ given the same 20% installation budget. The value of \bar{x}_k in this case significantly increases for an increase in k or in the number of routes sampled from Ω_k .

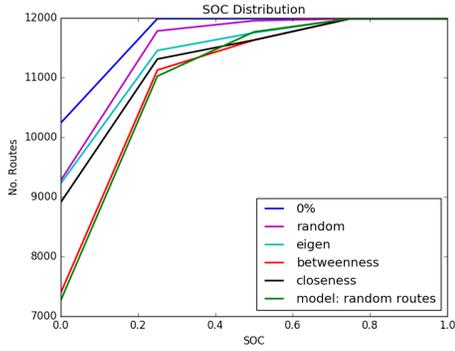
In this work, we have presented an integer programming formulation for modeling the WCL installation problem. With a modification to the WCL installation optimization model, we present a formulation that can be used to answer two types of questions. First, determining a minimum budget to reduce the number of infeasible routes to zero, thus, assuring EV drivers of arrival at their destinations with a battery charge above a certain threshold. Second, for a fixed budget, minimizing the number of infeasible routes and thus reducing drivers' range anxiety. Our experiments have shown that our model gives a high quality solution that typically improves various centrality based heuristics. The best reasonable candidate (among many heuristics we tested) that sometimes not significantly outperforms our model is the betweenness centrality. In our model, the routes were chosen randomly based on whether their final SOC is below α or not. We notice that a smarter way of choosing the routes leads to a better solution, for example choosing the longest routes generally provided better solutions. In future research, a careful study on the choice of routes to include in the model will give more insight into the problem. For a more comprehensive study of this model and its desired modifications, we suggest to evaluate their performance using a large number of artificially generated networks using (Gutfraind et al., 2015; Staudt et al., 2016).

Acknowledgment

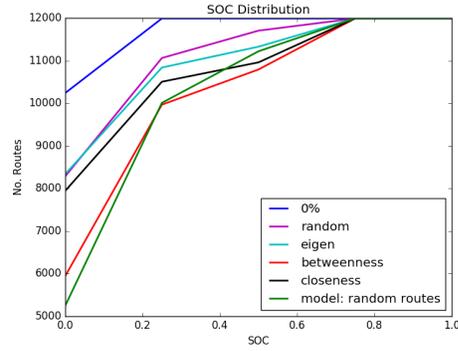
This research is supported by the National Science Foundation under Award #1647361. Any opinions, findings, conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the of the National Science Foundation.

Table 1: Contents of an OSM XML file.

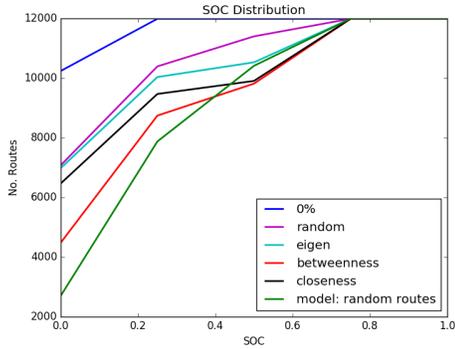
Item	Sub-item	Features
XML Suffix		<ul style="list-style-type: none"> • Introduces the UTF-8 character encoding for the file
OSM Elements	Node, Way and Relation	<ul style="list-style-type: none"> • Contains the version of the API (features used) • Contains the generator that distilled this file (the editor tool)
	Node	<ul style="list-style-type: none"> • Set of single points in space defined by unique latitude, longitude and node id according to the World Geodetic System (WGS84) • WGS84 is the reference coordinate system (for latitude, longitude) used by GPS (Global Positioning System) • Data contains tags of each node
	Way	<ul style="list-style-type: none"> • An ordered list of nodes which normally also has at least one tag or is included within a relation • A way can have between 2 and 2,000 nodes, unless there is some error in data • A way can be open or closed, a closed way is one whose last node is also the first • Data contains the references to its nodes and tags of each way
	Relation	<ul style="list-style-type: none"> • One or more tags and an ordered list of one or more nodes, ways and/or relations as members which is used to define logical or geographic relationships between other elements • Data contains the references to its members for each relation and tags of each relation.



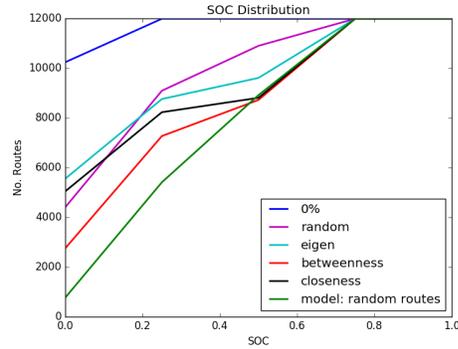
(a) $\beta = \frac{10}{110}$



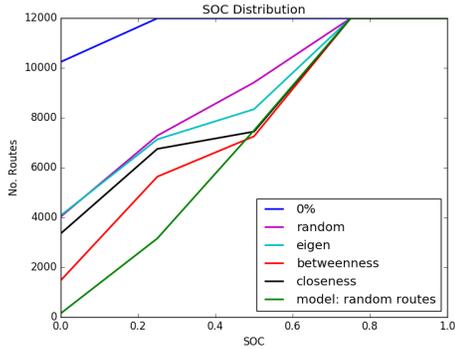
(b) $\beta = \frac{20}{110}$



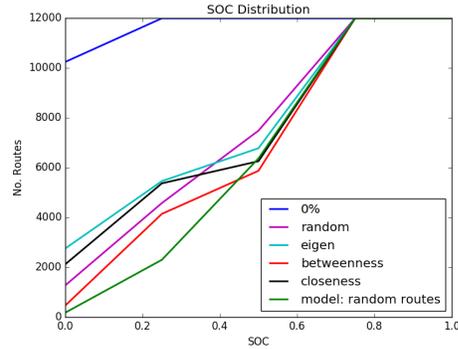
(c) $\beta = \frac{30}{110}$



(d) $\beta = \frac{40}{110}$



(e) $\beta = \frac{50}{110}$

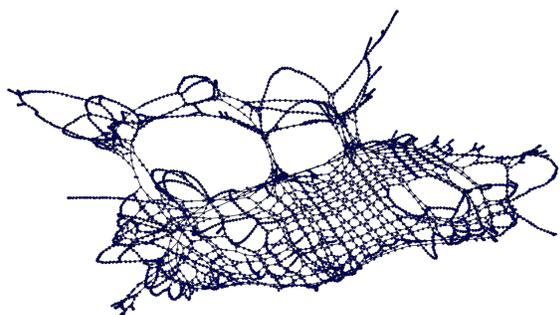


(f) $\beta = \frac{60}{110}$

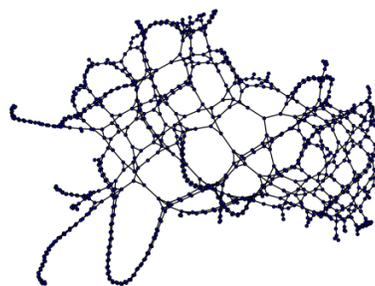
Figure 5: In each figure from (a) to (f) we show plots of number of routes ending with final SOC below a given value. The model was solved by optimizing 100 routes chosen uniformly at random with $\alpha = 0$ with a varying budget. The y -intercept of the different lines shows the number of infeasible routes for the different methods. Our model gives a smaller number in all cases. The plots go further and show how a specific solution affects the SOC of all routes. As the budget approaches 50%, we demonstrate that our model gives a significant reduction to the number of infeasible routes while also improving the SOC in general of the feasible routes

Table 2: Road category with corresponding speed in Miles/Hr

Category	Road Type	Urban Speed	Rural Speed
1	Motorway	60	70
2	Trunk	45	55
3	Primary	30	50
4	Secondary	20	45
5	Tertiary	15	35
6	Residential/Unclassified	8	25
7	Service	5	10
8	Living street	5	10

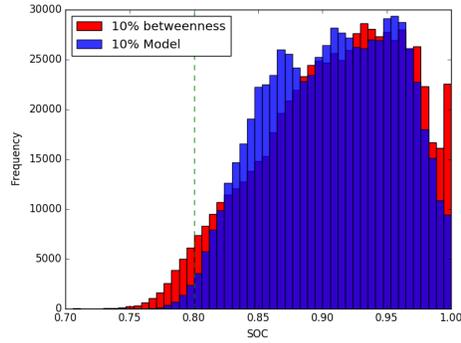


(a) Lower Manhattan

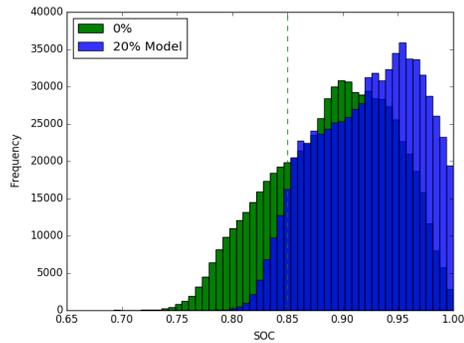


(b) Manhattan Neighborhood

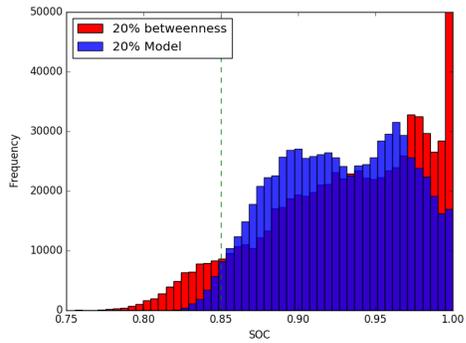
Figure 6: Road segment graphs from real geospatial data: a node, drawn in blue, represents a road segment. Two road segments u and v are connected by a directed edge (u, v) if and only if the end point of u is that start point of v



(a)



(b)



(c)

Figure 7: Histograms showing the number of infeasible routes for different values of α and β for the Manhattan neighborhood graph. The vertical line indicates the value of α . In (a) with a budget of 10%, our model gives a solution with at least 50% less infeasible routes compared to the betweenness heuristic. In (b), we demonstrate how the effects of a 20% budget on the SOC distribution within the network. In (c), our model gives a solution with at least 25% less infeasible routes.

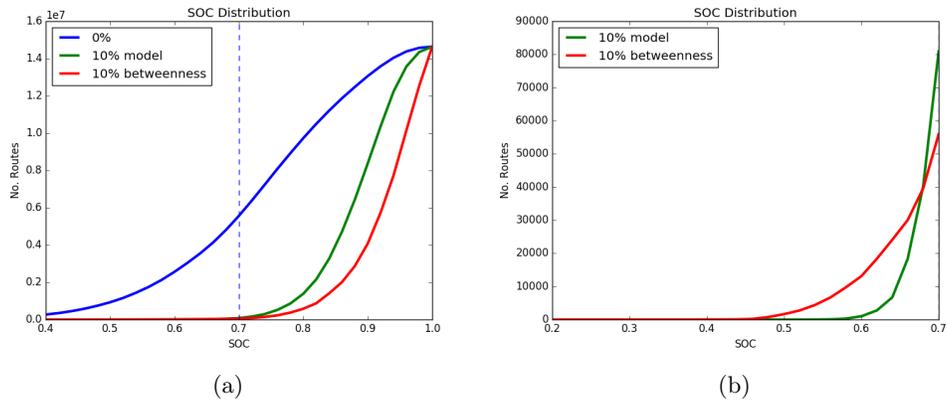


Figure 8: The number of routes ending with final SOC below a given value in the lower Manhattan graph. The solution was obtained with $\alpha = 0.7$ and $\beta = 0.1$. The blue curve shows the SOC distribution when no WCL are installed. Green and red curves show the SOC distribution after an installation using the proposed model and the betweenness heuristic, respectively. Plot (b) a gives closer look into (a) for the SOC values below 0.7.

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