Optimal threshold classification characteristics

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Abstract.
This study looks at the application of mathematical concepts of entropy and Fibonacci sequence in creating optimal dimensional relations of classification character. The paper is devoted to optimization of some numerical relations and integers as unified threshold characteristics of classification type, aimed for example at systemic optimizing the measuring information of various processes. The paper is devoted to optimization of some numerical relations and integers as unified threshold characteristics of classification type. The dimensional concepts of information optimality, harmony and balance in the complex are major criteria underlying the present study. Based on the criteria, the maximum deviations from the optimal values and the way to optimize the redundancy of information are determined and justified. As the model in determining informational optimality has used the principle of equivalence, based on the equality of total entropy of system and the entropy of its informative components. The application of suggested theory is discussed on examples of characteristics relating to binary classification from the sphere of practical metrology, such as confidence levels, tolerance uncertainty ratios, and test uncertainty ratios. As accompanying results of the research the optimality of system of decimal numbering and the information nature of "magic number seven" are proved.

Keywords: optimization, informative components, information redundancy, binary classification.

1. Introduction

Unified numerical estimates of a systemic nature are in use in various fields of human activity to solve practical problems of classification type. Depending on the application and purpose, such standard estimates are expressing by means of numbers (e.g. the number of periods in classification of chemical elements), ratios of numbers (e.g. the ratio of a tolerance of parameter to an uncertainty of measurement), percentage or decimals (e.g. the level of confidence, or the level of probability). They may be neutral, for example, the base of numeral system, relatively independent, for example, in determining the normal biochemical and physical parameters of blood, or systemically interconnected as, for example, in ensuring the accuracy and uniformity of measurements in metrology and of quality of metrological support in industrial measuring practice.

Generalized numerical standards, used in human practice, are essentially a product of evolution, based on experience, observation and an agreement to systemic unification. Despite the versatility and practicality, their direct mathematical substantiation is not feasible, which creates a confidence problem to the process of developing and implementing of unified rules for different objects. It requires the use of complex quality criteria, combining mathematical optimization and evolutionary sound principles of general preferences. In fact, the goal is to create a new concept of optimization of generalized heuristic quantitative estimates.

Dimensional concept of harmony, balance and informational optimality as of dimensional perfection ideas, expressed in the form of concrete numerical ratios are applicable for this purpose. Generalized permissible deviations from these ratios is the significant necessity of their practical usage and solution on the optimization base also.

The paper deals with the criteria of optimization for two-component systems. Regardless of the type and number of components, if consider the components in terms of their grouping onto the informative and non-informative part of full quality, the system represents the simple classification according to dichotomy (binary classification) for which the above criteria also suitable.

Some principles of qualimetry [1] and of information theory [2] are the basis of the proposed optimization method. Their combination allowed using weights of components as arguments of optimization, representing relative characteristics of their individual contribution to the quality of measurements or other object of evaluation [3].

2. Criteria of informational optimization

Optimal separation of two components or the two parts of a whole onto informative and non-informative can consist of one or two main stages. First, according to the formal criteria of necessity and sufficiency, and second on the criterion of optimal information redundancy to achieve a reliable estimate. The first stage is implemented by a substantiation of optimum classification index (section 2.1), and the second through the identification and proof of the optimal redundancy of information (section 2.3). The transition to the second stage requires a preliminary discussion the concept of dimensional perfection, and the optimization of tolerances of mathematical constants expressing these concepts that is carrying out in section 2.2.

2.1. Optimal classification index

In generalized form, it is possible to describe a system of two components or two parts of a whole using their weights K_m and K_ψ, as estimates of contributions into the system's quality. Relative values of weights can be expressed as $K_m(\rho) = 1/(1 + \rho)$ and $K_\psi(\rho) = \rho/(1 + \rho)$, where the ratio $\rho = K_\psi/K_m$ one can identify as *classification index* that is the initial object of optimization.

In considering of relative weights by probabilities, as an observer relative version of probabilities [4] and using information entropy, the optimization is achievable by applying the principle of equivalence. This principle is based on the equality of total entropy of system and the entropy of ψ_o informatively necessary components in the assumption that by the fact of grouping all ψ_o components are equivalent by weights. Then the value of optimum classification index ($\rho_o = K_{\psi o}/K_m$) can define as follows:

$$\rho_o = arg\ min\ [\psi(\rho) - \psi_o(\rho) = 0] \approx 1/2\pi, \qquad (1)$$

where $\psi(\rho) = exp\ [- K_m(\rho) ln\ K_m(\rho) - K_{\psi o}(\rho) ln\ K_{\psi o}(\rho)];$ (2)

$\psi_o(\rho) = 1.5$ corresponds to the most uncertain situation (confidence level 50%) on the ignoring or accepting of the component of less weight.

Difference $(1 - \rho_o)$ indicates a necessity and sufficiency in relation to the component of a higher weight, while ρ_o reflects the boundary of lack of information redundancy for the second component.

The equality $\rho_o = 1/2\pi$ matches the approximation as three significant digits. More exact approximation by four significant digits, leads to negligible information redundancy inherent $\rho_o = 1/2\pi$ comparing with calculated $\rho_o = 0.163$. The relative value of the redundancy is $|(0.163 - 1/2\pi)/(1 - 1/2\pi)|*100\% = 0.48\%$. This allows considering $\rho_o \approx 1/2\pi = 0.159$ as the parameter suitable for determining information periodicity in classification systems, since in such systems classification period is always expressing by integers, and the rounded up to integer $\|2\pi\| = \|1/0.163\| = 6$.

2.2. Perfect dimensional ratios

On purely mathematical grounds and of the relevance in nature the concepts of perfection can express via Perfect Dimensional Relations (PDR). Each PDR represents certain ratio of two parts of two-dimensional system. In this sense, in addition to the optimal classification index ρ_o = 0159, there are two PDRs: the golden proportion $\Phi = 0.618$ represents the mathematical harmony objectively prevailing in nature and often used in human practice, as well as the concept of balance, expressed as the ratio $\lambda = 0.5$.

Deviations (D_{PDR}) from rated PDR values, ideally from their mathematical constants (C_{PDR}), always characterize their manifestations in nature and human practice. The absence of substantiated permissible ranges of D_{PDR} poses problems of PDR practical application, which are solving by analogy with ρ_o determination. Thus, if $\pm 0.5\delta$ is the variation of estimation error in determining C_{PDR}, the permissible range of deflection from C_{PDR} is equal to the optimal error $\delta_o = K_{\psi o}(\delta)/K_m(\delta)$, which is easy to define as follows:

$$\delta_o = arg\ min[\psi(\delta) - \psi_o(\delta) = 0] = \rho_o C_{CPO} \qquad (3)$$

where $\psi(\delta) = exp[- K_m(\delta) ln\ K_m(\delta) - K_{\psi o}(\delta) ln\ K_{\psi o}(\delta)]$;
$K_m(\delta) = (C_{CPO} - 0.5\delta)/(C_{CPO} + 0.5\delta)$;
$K_{\psi o}(\delta) = \delta/(C_{CPO} + 0.5\delta)$;
$\psi_o(\delta) = 1.5$

Resulting PDR values with deviations $D_{PDR} = \pm 0.5\rho_o C_{PDR}$ (approximately ±8% from a constant) are listed in table 1. These values one can consider as tolerances by the criterion of practical acceptability.

Table 1: The list and expressions of perfect numerical ratios

Dimensional concept	PDR with permissible deviations	Numerical values
Optimality	$\rho_o \pm D_{\rho o}$	0.158 ± 0.013
Harmony	$\Phi \pm D_\Phi$	0.618 ± 0.049
Balance	$\lambda \pm D_\lambda$	0.5 ± 0.04

To note, while dimensional harmony is the widely known concept, more rare manifestations of balance and of optimality have not yet attracted noticeable attention. At the same time the harmony and balance are associated via the division ($0 < F_n/F_{n+1} < 1$) of adjacent numbers in Fibonacci sequence, which fractions start with 0.5 (the balance), and the next number 0.667 represents the upper boundary of permissible deviation from Φ. The comparison of this property with result of calculation of limiting value $[1 - \Phi(1 + 0.5\rho_o)]/\Phi(1 + 0.5\rho_o) = \lambda$ is one of proofs of its informational nature also in terms of optimization.

2.3. Key indexes of information redundancy

Usually practical reliability when optimizing the information in information processing systems associates with some redundancy proportional to the decrease of classification index.

Obtained ρ_o value is the basic characteristic in determining indexes of maximum (ρ_{rm}) and optimum (ρ_{ro}) redundancy of information. Index ρ_{rm} can be determined via the entropy of the complete system of PDR constants. This is realizable using the difference between 3 (the number of perfect dimensional ratios) and the numeric value to which matches informational optimality, and is determined by means of relative weights $K_{\Phi(1)} = \Phi^{-1}/(\Phi^{-1} + \lambda^{-1} + \rho_o^{-1})$, $K_{\lambda(1)} + \lambda^{-1}/(\Phi^{-1} + \lambda^{-1} + \rho_o^{-1})$ and $K_{\rho o(1)} + \rho_o^{-1}/(\Phi^{-1} + \lambda^{-1} + \rho_o^{-1})$ as arguments of entropy function as follows:

$$\rho_{rm} = \rho_o - \Delta_{r(1)} = 0.08, \qquad (4)$$

where $\Delta_{r(1)} = \rho_o(3 - t_{o(1)})$;
$t_{o(1)} = exp\ (-K_{\Phi(1)}\ lnK_{\Phi(1)} - K_{\lambda(1)}\ lnK_{\lambda(1)} - K_{\rho o(1)}\ ln\ K_{\rho o(1)}) = 2.48$

Index ρ_{ro}, located in two components system between ρ_o and ρ_{rm}, is calculated with using their relative weights $K_{\rho o(2)} = \rho_o/(\rho_o + \rho_{rm})$ and $K_{\rho rm} = \rho_{rm}/(\rho_o + \rho_{rm})$ as follows:

$$\rho_{ro} = \rho_{rm} + \Delta_{r(2)} = 0.10, \qquad (5)$$

where $\Delta_{r(2)} = \rho_o(2 - t_{o(1)})$;
$t_{o(1)} = exp\ (-K_{\rho o(2)}\ ln\ K_{\rho o(2)} - K_{\rho rm}\ ln\ K_{\rho rm}) = 1.89$

On the other hand, the same results for the indexes relevant to optimal and maximal information redundancy is obtained when calculating directly using PDR constants: $\rho_{o\Phi} = \rho_o\Phi = 0.10$, and $\rho_{o\lambda} = \rho_o\lambda = 0.08$. This reveals the optimization nature of mathematical harmony in applying to information redundancy in boundaries of permissible redundancy, which correspond to ρ_o and λ.

3. Optimization of confidence level

The optimization of the level of confidence (LC) one can consider as highly representative model of applying optimal classification indexes. For the consideration of confidence level in terms of PDR it is necessary to pass from probabilistic characteristics to dimensional. As known, an interval representing LC is convenient to characterize by the value ($\mathcal{E}_{LC}$), which one can name *dimensional factor of confidence*:

$$\mathcal{E}_{LC} = (1 - 0.01\text{LC})^{-1} \quad (6)$$

For example, in measurement, there is one chance from the number $\mathcal{E}_{LC}$ that the measured value lies outside the interval. In case of LC = 50%, factor $\mathcal{E}_{50} = (1 - 0.01*50)^{-1} = 2$ corresponds to the most uncertain classification situation about allowing or ignoring of the lesser component in a two-component system. Then, considering ratio $\mathcal{E}_{50}/\mathcal{E}_{LC\rho o} = \rho_o$, factor $\mathcal{E}_{LC\rho o} = 12.6$ relates to confidence level $\text{LC}_{\rho o} = (1 - 0.5\rho_o)*100\% = 92\%$. Correspondingly, LC_x as function of any ρ_x is determined as follows:

$$\text{LC}_x = (1 - 0.5\rho_x)*100\% \quad (7)$$

Other required evaluations, found according to expression (7), are: $\text{LC}_\Phi = (1 - 0.5\rho_{o\Phi})*100\% = 95\%$ and $\text{LC}_\lambda = (1 - 0.5\rho_{o\lambda})*100\% = 96\%$.

The adequate result for optimal informational redundancy: $\rho_{or} = \rho_{o\Phi} = 0.10$ substantiates the wide usage of 95% level of confidence, and indicates the informatively prevailing value of dimensional harmony for the generalized estimation.

In some practical cases, when determining the minimum level of confidence (LC_{min}) it is reasonable to use the upper permissible value ρ_o. Then the practically admissible minimum level of confidence is determined as follows:

$$\text{LC}_{\text{min}} = [1 - 0.5\rho_o(1 + 0.5\rho_o)]*100\% = 91\% \quad (8)$$

that is somewhat less than $\text{LC}_{\rho o}$. Then, in practice the maximum limitation for the classification criteria can consider admissible in the range from $\rho_{min} = 2(1 - 0.01\text{LC}_\lambda) = 0.08$ to $\rho_{max} = 2(1 - 0.01\text{LC}_{\text{min}}) = 0.18$.

4. Dimensionally perfect integers as parameters binary classification

Among PDR the optimality represents the most concrete concept of informativeness. This, for instance, can be exemplified by the direct usage of optimal classification index in determining the optimal base of numeral system. It is achievable with applying the model of Benford's probabilities. According to Benford's law [5], the probability $P(d)$ of any number d in the system of different integers from 1 to $(z - 1)$ is equal to $P(d) = log_z (1 + 1/d)$. These probabilities form a logarithmic sequence of complete group of independent events. Boundary values $P_{min} = P\,[d = (z - 1)]$ and $P_{max} = P\,(d = 1)$ represent parameters of binary classification, their necessary and sufficient ratio $(P_{min}/P_{max})_o = \rho_o$. Thus, after rounding off the optimal value of base for numeral system z_o can calculate as follows:

$$z_o = arg\ min\ \| log_z\,[1 + 1/(z - 1)]/log_z\,(1 + 1) - \rho_o \| = \|9.6\| = 10, \quad (9)$$

which is a proof of optimality of the decimal numeral system.

When considering the harmony and balance in terms of optimality, one may find that:

a) two classification integers $X_{\rho o(max)}$ and $X_{\rho o(min)}$ exist according to permissible limitations regarding the optimality which as rounded values of $x_{\rho o(max)}$ and $x_{\rho o(min)}$ respectively are determined as follows:

$$X_{\rho o(max)} = \|x_{\rho o(max)}\| = \|arg\ min\ [|1/x - \rho_o(1 - 0.5\rho_o)| = 0]\| = \|6.9\| = 7; \quad (10)$$

$$X_{\rho o(min)} = \|x_{\rho o(min)}\| = \|arg\ min\ [|1/x - \rho_o(1 + 0.5\rho_o)| = 0]\| = \|5.8\| = 6, \quad (11)$$

b) irrespective of $x_{\rho o(max)}$ or $x_{\rho o(min)}$ as the base of calculation, the classification integers associated with concepts of harmony and balance are single-valued and leads to the following results:

$$X_{o(\Phi)} = \|x_{\Phi o(max)}\| = \|\Phi * x_{\rho o(max)}\| = \|4.2\| = \|x_{\Phi o(min)}\| = \|\Phi * x_{\rho o(min)}\| = \|3,6\| = 4; \quad (12)$$

$$X_{o(\lambda)} = \|x_{\lambda o(max)}\| = \|\lambda * x_{\rho o(max)}\| = \|3.4\| = \|x_{\lambda o(min)}\| = \|\lambda * x_{\rho o(min)}\| = \|2,9\| = 3 \quad (13)$$

Obtained values $X_{o(\Phi)} = 4$ and $X_{o(\lambda)} = 3$ are the proof of practical reasonability of these integers in terms of dimensional balance and harmony, explaining the popularity of the usage of 4:1 and 3:1 for generalized tolerance uncertainty ratios and test uncertainty ratios (TUR) as a kind of classification standards. The current trend in metrology requires applying 4:1 TUR [6] that definitely is preferable because of the evolutionary value of harmony, compared with balance.

Number 6, being the integer approximation of $\rho_o = 1/2\pi$, in terms of classification periodicity can be identified as is the *informatively optimal classification period*. Such an approximation results in 0.9% information redundancy in comparing $(1 - 1/2\pi)$ and $(1 - 1/6)$ that leads to some increasing of information reliability.

The classification manifestation of the "magical number 7" in nature and in many spheres of science and practice is well known. The consistency of integers 6 and 7 is intrinsic to informatively optimal classification scheme when at a certain hierarchy level classification items represent 6 categories with their 7 boundaries. The joint application of integers 7 and 6 is rather rational, for example when forming optimal systems of accuracy classification [7].

5. Optimization peculiarities relevant to decimal system and integer 7

The unique feature of a decimal numeral system is that of its optimality is provable using both the model of two-component system in accordance with expression (9) and the complete model of calculating informative components. Indeed by the alternative to expression (9) is the proof of optimality of decimal numeral system in calculating z_o when using $z = \|\mathcal{E}_{LC\rho o}\| = 13$ as follows:

$$z_o = exp \sum_{d=1}^{12} -log_{13}(1+1/d)\ ln\ [log_{13}(1+1/d))] = \|9.7\| = 10 \quad (14)$$

As for integer 7, its informational uniqueness is inherent not just in binary classification, but is true for the variety of its other known manifestations. Such a generalization relies on the fact that mathematically integer 7 represents the most informatively optimal value $X_{inf(o)}$ as the optimization of information within decimal numeral system itself. This can prove with the principle of entropy equivalence applied to the complete group of nine $(z_o - 1) = 9$ Benford's probabilities as follows:

$$X_{inf(o)} = exp\ [\sum_{d=1}^{9} - log_{10}(1+1/d)\ ln\ [log_{10}(1+1/d))] = \|7.3\| = 7, \quad (15)$$

To note, the inverse estimate by formula (16), using arguments z and $X_{inf(o)}$ of Benford's function, also results in conclusion on the optimality of decimal system:

$$z_o = arg\ min\{\|exp \sum_{d=1}^{z-1} -log_z(1+1/d)*ln\ [log_z(1+1/d)]\| - X_{inf(o)}\} = 10 \quad (16)$$

Clearly in the informatively optimal decimal system the maximal value of optimal integer $X_{inf(max)} = 9$. Correspondingly the minimum value $X_{inf(min)}$ one can calculate when $z = X_{inf(o)} - 1 = 6$ as follows:

$$X_{inf(min)} = exp \sum_{d=1}^{6} -log_7(1+1/d)\ ln\ [log_7(1+1/d))] = \|5.2\| = 5 \quad (17)$$

Expressions (15) and (16), as well as the same result ($z_o = 10$), obtained with two different models of information optimization using expressions (9) and (16), indicate onto the unique information bond between decimal system and integer seven.

The usage in expression (7) the classification index $\rho_x = 1/z_o = 0.10$ results in $LC_x = LC_\Phi = 95\%$, which points to very important information interrelation between above two models of optimization. The model based on binary classification leads to the optimization by necessity and sufficiency, and the model based on complete group of probabilities results in determining optimal redundancy of information. Clearly this property peculiar to the decimal numeration system only and being its informational merit.

Evolutionary nature of integer 7 is predetermining the existence of some permissible range ($7 \pm \Delta_o$) as regards of its practical value in various aspects, including for a classification. On practical deviations of this number possibly first wrote George A. Miller in his famous article [8], naming them as assuming "a variety of disguises sometimes a little larger and sometimes a little smaller than usual, but never changing so much as to be unrecognizable".

The permissible deviation Δ_o one may define by two ways, applying either the model of binary classification and PDR, or the modeling by complete groups of Benford's probabilities within ($z_o - 1$), as following. In the first case Δ_o is being defined based on $x_{\rho o(max)}$ and in correspondence with permissible deviations regarding the balance ($\Delta_{o\lambda}$) and harmony ($\Delta_{o\Phi}$) as follows:

$$\Delta_{o\lambda} = arg\ [(x_{\rho o(max)} - \Delta)/(x_{\rho o(max)} + \Delta) = \lambda(1 \pm 0.05)]; \quad (18)$$

$$\Delta_{o\Phi} = arg\ [(x_{\rho o(max)} - \Delta)/(x_{\rho o(max)} + \Delta) = \Phi(1 \pm 0.05)] \quad (19)$$

In the second case the deviation $\Delta_o = \Delta_{min(o)} = \Delta_{max(o)}$, where the module of components is calculated by following expressions:

$$\Delta_{min(o)} = X_{inf(o)} - exp[\sum_{d=1}^{7-1} -P(d)\ ln\ P(d)]; \quad (20)$$

$$\Delta_{max(o)} = (z_o - 1) - X_{inf(o)} \quad (21)$$

The calculation by expressions (18), (19), (20), (21) after rounding off, results in the same value: $\Delta_o = \Delta_{o\lambda} = \Delta_{o\Phi} = \Delta_{min(o)} = \Delta_{max(o)} = 2$.

The range ($7 \pm \Delta_o$) points to the estimation stability of Δ_o in terms of dimensional perfection and informational optimality. The obtained result perfectly conforms with determined experimentally Miller's "magical number 7 ± 2" [8], and can serve by its additional proof. Moreover, relying on core principles dimensional perfection and optimal information redundancy, obtained result is well agree with the idea of "a tradeoff between consistency and redundancy" [9].

It is remarkable, for instance, that in two most traditional measurement fields, measurement of length [10] and of mass [11], the number of accuracy classes was specified as 5 and 9 correspondingly that justified by perennial practice.

6. Optimization LC and TUR in multicomponent systems

In spite of simplicity and universality, the usage of generalized TUR and LC in ensuring of confidence to measurements in many ways contradicts to the situation for m-component systems ($m > 2$), characterized by distinctions of components weights. This largely concerns various manufacturing and other processes and may lead to the overstating of measurement accuracy for some components and/or understating for others. As a result, there is on the one hand the risk of economic loss when selecting measuring instruments and in their calibration, and of quality loss on the other.

The solution of problem is possible when determining TUR for each component individually that inevitably leads to changing the confidence in obtained measurement result in the valuation of system's quality. In so doing, we should be convinced that the total confidence of estimation (LC_Σ) of the system satisfies the following boundary conditions:

$$(LC_{min} = 91\%) \leq LC_\Sigma \leq (LC_\lambda = 96\%). \quad (22)$$

This condition one can check using the linear diagram of weights: $K_j = K_1 (m + 1 - j)/m$ (when $K_j > K_{j+1}$) as the optimal unified model for generalizing estimates [12]. As shown in Appendix, it is the model that applicable to the selection of informative components in multicomponent systems by the principle of equivalence in the binary classification mode. Considered all this, the total confidence, being function of average dimensional factor $\mathcal{E}_{\psi av}$ for the number ψ of informative components, is determined as follows:

$$LC_\Sigma = (1 - \mathcal{E}_{\psi av}^{-1})*100\%, \quad (23)$$

where $\mathcal{E}_{\psi av} = \sum_{j=1}^{\psi} \mathcal{E}_j/\psi$;

$\mathcal{E}_j = \mathcal{E}_{LC\Phi}(m + 1 - j)/m = 20.4(m + 1 - j)/m$;

ψ = the number of informative components ($\psi \leq m$) with weights K_j which, comparing with maximal weight K_{max}, satisfy the condition $K_j \geq \rho_\Phi K_{max}$, where $\rho_\Phi = 1/Z_o \approx 0.1$.

The calculations by formula (23) allows concluding that, depending of components number, the total confidence of estimation LC_Σ is located between 93.5% ($m = 2$), and 91% ($m \to \infty$), i.e. satisfies to the boundary conditions (22).

7. Conclusions

Difficulties of systemic optimization for the strict substantiation of commonly applicable characteristics of binary classification can overcome, using dimensional concepts of information optimality, harmony and balance as objective criteria for achieving the goal.

The application of perfect dimensional ratios enabled fully confirm the expectation of optimality of 95% LC and 4:1 TUR. Both characteristics meet also the requirement of dimensional harmony, as the prevailing concept of perfectness, which objectively exists in nature and human practice. A practical value one may expect from the discovery that dimensional harmony can serve by reliable criterion in the optimization of information redundancy for solving various problems of system character.

The interrelation of classification integers 6 and 7 and their information optimality is proven by the criteria of dimensional perfection that corresponds with their classification manifestations in Nature and in human practice.

The proof of optimality of decimal numeral system serves by example of applying information optimality individually, for the optimization in neutral systems of binary classification. On this basis the nature of common importance of number 7 is revealed as a necessary and sufficient value by informativity in a full group of numbers that form a decimal numeral system.

The outcome of the study is also the revealing and proof of permissible deflections from the optimal value concerning the most known classification integer 7, and their coincidence with the famous "magical 7±2". Compliance with these boundaries for accuracy levels in the most traditional areas of measurement of weight and length illustrates the significance of obtained result to improve classification systems of accuracy in metrology.

The acceptability of optimizing LC and TUR individually for each component in a multicomponent system by the condition of optimality requirements for the system as a whole was proven.

Now then, the correspondence of objectively existing in nature mathematical constants expressing concepts of perfection and some significant empirical metrological estimates has been demonstrated. As a whole, the obtained results once more acknowledge the great value of trust to human experience.

The author hopes on an interest to the proposed ideas and of their further development, for example, the determination of informative components in the case of interdependent components and their correlations. One can also expect that in the broadest sense, the inventive concept of perfect dimensional relationship will be a versatile tool for the improvement and optimization of many classifying quantitative evaluations.

Appendix: On the selection informative components

Formula (2) originates from the general expression $\psi_{eo} = exp[-\sum_{j=1}^{m} K_j \ln K_j]$ for a multicomponent system. This approach differs from the known principle of proportionality: $\psi_{po} = [m/ln(m)]*exp[-\sum_{j=1}^{m} K_j lnK_j]$, according to which the ratio ψ_{po}/m equates to the ratio between the entropy of real system and the entropy of m components in the assumption that all m components are equivalent by weights. Results of comparing these approaches for two-component system are in table 2, where $\delta_\psi = [1 - \psi(\rho)/1.5]*100\%$ is the error in determining of informatively necessary components by the condition of boundary (50%) confidence.

Table 2: Comparison approaches in determining $\psi(\rho_o)$ for a two-component system

Used approach	Expression for $\psi(\rho)$	ρ_o	$\psi(\rho_o)$	δ_ψ
Equivalence	$\psi_e(\rho) = exp\{-(1+\rho)^{-1}ln(1+\rho)^{-1} - \rho(1+\rho)^{-1}ln[\rho(1+\rho)^{-1}]\}$	0.159	1.49	0.7%
Proportionality	$\psi_p(\rho) = (2/ln2)\{-(1+\rho)^{-1}ln[(1+\rho)^{-1} - \rho(1+\rho)^{-1}ln[\rho(1+\rho)^{-1}]\}$	0.274	1.15	23%

Thus for two-component system the principle of equivalence has significant advantage in estimating the accuracy against the principle of proportionality.

In passing to multicomponent system, we can avail using linear diagram of weights. The comparison of the principles is carried out by with functions $\rho_{o(l)e} = (1 - \psi_{o(l)e}/m)$ and $\rho_{o(l)p} = (1 - \psi_{o(l)p}/m)$. Graphically figure 1, convincingly demonstrates the complete fulfillment with requirements of permissible criteria regarding the principle of equivalence ($\rho_{min} < \rho_{o(l)e} < \rho_{max}$), and on the other hand, the complete disparity to the requirements regarding the principle of proportionality ($\rho_{o(l)e} < \rho_{min}$). Thus, the analysis points onto the essential advantage of the principle of equivalence also for a multicomponent system.

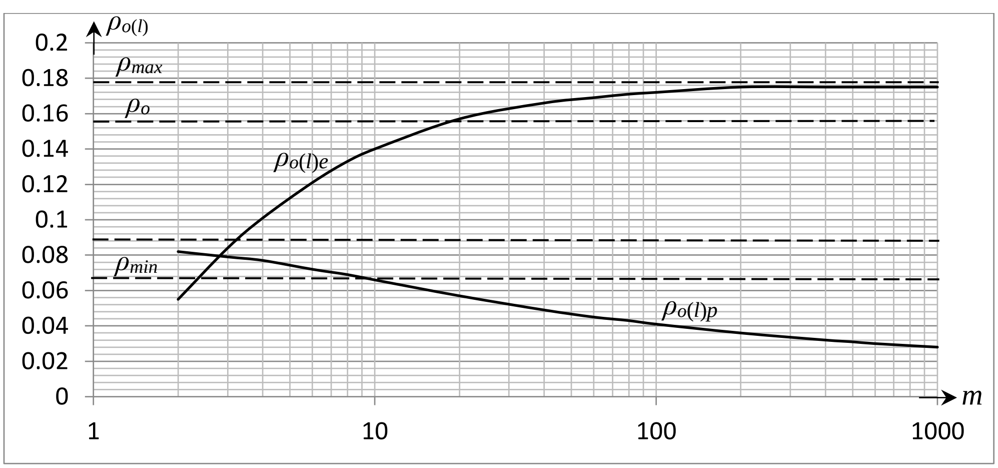

Figure 1: Graphical illustration of comparing two principles of selecting informative components for multicomponent system with a linear distribution of weights

Relying on above considerations, in frame of boundaries denoted in Fig. 1, generalized selection criteria one may classify by accuracy coefficients and confidences of estimation as demonstrated in table 3.

Table 3: Possible ways of selecting informative components by generalized characteristics

Most pessimistic		Most optimistic		Moderately pessimistic		Moderately optimistic	
ρ_{min}	LC_{max}	ρ_{max}	LC_{min}	ρ_{or}	LC_{mp}	ρ_{o}	LC_{mo}
0.08	96%	0.18	91%	0.1	95%	0.159	92%

In practice, the usage of one of moderate estimates is preferred. The application of moderately optimistic estimates, e.g. one can find in [13] and [3] devoted to the interpolation errors in measurement, and to improving the evaluation of proficiency tests data respectively. Evidently, in a number of cases for refining the estimation, the full number of analyzed components one can also take in account for calculation.

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