Airport Capacity Extension, Fleet Investment, and Optimal Aircraft Scheduling in a Multi-Level Market Model: On the Effects of Market Regulations

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Abstract. In this paper we present a four-level market model that accounts for airport capacity extension, fleet investment, aircraft scheduling, and ticket trade in a liberalized aviation market with independent decision makers. In particular, budget-constrained airports decide on the first level on their optimal runway capacity extension and on a corresponding airport charge. Airports anticipate optimal long-term fleet investment of airlines on the second level, optimal medium-term aircraft scheduling on the third level, and short-term outcomes of a competitive ticket market on the fourth level. We compare this market model with an integrated single-level benchmark model that simultaneously accounts for all investment, scheduling, and market clearing constraints. As we show, our multi-level model that resembles a liberalized aviation market may result in inefficient long-run investments in both runway capacity and new aircraft. We identify policy regulations that may have the potential to reduce inefficiencies of the market model relative to the integrated benchmark model.

1. Introduction

In the past decades, a lot of countries liberalized their air transportation sectors; see for instance Bowen 2002, Fu et al. 2010, or Burghouwt and Wit 2015. This liberalization yields markets, where different airlines compete with each other and invest in new aircraft on the basis of expected market developments and future profits. In contrast, regulated airports decide on their capacity extension based on an anticipation of the airlines' optimal fleet planning and on passenger growth. Obviously, both airport capacity extensions and optimal fleet planning highly depend on each other and directly affect future market outcomes including profits. Such a complex investment structure of independent agents challenges traditional planning processes that typically do not account for an interplay of different investment decisions, future market structures, and corresponding policy regulations; see also the literature review below. It is the purpose of this paper to provide a new model framework that adds valuable information on optimal investments in a complex market environment, both for policy markers and market participants, e.g., airlines or airport operators.

In particular, we propose a computational equilibrium model that allows to analyze optimal long-run investments of airlines in new aircraft as well as optimal capacity expansions of airports in a market environment. We elaborate on possible market regulations and their long-run impact on optimal fleet investment of imperfectly competitive airlines. As investment decisions base on expected future profits, we include scheduling decisions that directly affect the airlines' revenues on the market for flight tickets.

To be more precise, in our four-level market model airports choose their optimal long-run runway capacity investments on the first level in anticipation of future fleet investment of airlines on the second level. Observe that on the second level we directly account for different business models (see for instance Papatheodorou and Lei 2006, O'Connell 2011, or Nataraja and Al-Aali 2011) that airlines may pursue when investing in new aircraft, i.e., airlines may ex-ante decide on their airline-specific investment portfolio. Similar to airport infrastructure expansion, also new aircraft are only available in discrete amounts. Optimal fleet investment decisions of airlines on the second level depend on expected optimal aircraft scheduling and on flight planning decisions on the third level. In turn, optimal long-term fleet investment on the second level and optimal medium-term scheduling on the third level are dependent on expected short-term market outcomes on the fourth level, where flight tickets are sold by airlines to passengers on a ticket market. On the opposite, market outcomes on the fourth level determine the airlines' revenues that influence the described optimal long-term investments on the second level through corresponding returns on investment. Obviously, these interdependencies yield a complex market environment for long-run planning decisions.

Given an single-level benchmark model that plans the whole industry in an integrated and welfare-maximizing way, we demonstrate that our multi-level market model with its independent decision makers may yield an inefficient fleet investment and corresponding inefficient airport capacity extensions. Despite the fact that our multilevel model is in general worse than an integrated model, as a main advantage, it allows to quantitatively analyze – possibly inefficient – decision making structures of liberalized aviation markets around the world. In this context we identify policy regulations that may increase the efficiency of investments and move the economic multi-level system closer to an integrated benchmark regime, where the latter may equivalently be interpreted as some kind of social-planner problem. Amongst others, these regulations comprise the fields of competition policy and public subsidy programs. Against this backdrop, our work may shed new light on the efficiency of real-world market structures in the aviation industry.

Let us illustrate the relevance of our model by a current example. Given the lack of enough runway capacity of the two existing airports of Istanbul, the Turkish authorities decided to build a third airport in Istanbul; see for instance Saldiraner 2013, 2014. Obviously, the currently observed congestion directly limits operations of airlines arriving at or departing from Istanbul and highly influences optimal planning decisions of the major local airline Turkish Airlines, which in turn depend on the optimal demand behavior of passengers. On the other side, the expected fleet expansion strategy of Turkish Airlines and the anticipated future passenger demand are the main reasons for the construction of the new airport, which will have a capacity of 150 million passengers per year with estimated construction costs of more than 32 billion Euro. Quantitative models as the one proposed in this paper can provide a valuable tool to evaluate and assess such huge airport extension projects as well as their interdependent long-run effects on airlines' strategic decision making and optimal policy making.

Our paper directly contributes to different stands of literature. Given the typically large-sized and computationally challenging problems in the airline industry, the operations research related literature identified classes of real-word problems that deal with certain aspects of operational and strategic planning processes of airlines; for an overview see for instance Barnhart et al. 2003 or T. L. Jacobs et al. 2012. Amongst others, these problems comprise schedule design (see Teodorović and Stojković 1990, Lederer and Nambimadom 1998, or Burke et al. 2010), fleet assignment (see Abara

1989, Hane et al. 1995, Rushmeier and Kontogiorgis 1997, or Rexing et al. 2000), and fleet planning issues (see for instance List et al. 2003 or Clark 2007).

More recently, authors started to analyze integrated models that simultaneously account for several adjacent planning steps. For instance, Lohatepanont and Barnhart 2004 or Sherali et al. 2013 elaborate on an optimal schedule design and fleet assignment within a single planning model. Based on this work, Kölker and Lütjens 2015 combine network planning, scheduling, and aircraft rotation in an integrated planning approach, while Faust et al. 2017 focus on an integrated schedule design and aircraft maintenance routing. Cadarsoa and Marín 2011 and Cadarso and Marín 2013 propose a robust approach covering an integrated airline schedule development/design and fleet assignment. In our paper we extend these first integrated, multi-step models to the case of long-run investments that depend on expected airline profits in a multi-level market environment. In particular, to be best of our knowledge we are the first to analyze the effects of different market regulations on optimal decision making of airlines in a hierarchical airline market model that covers, amongst others, the above mentioned schedule design and fleet assignment steps. However, besides these short- and medium-run decisions of airlines, our model also covers long-run investment decisions that comprise both fleet planning of airlines and interdependent capacity extensions of airports. To this end, our work is also closely linked to the literature on airport capacity extensions, which, however, mainly built on very simplified models in the past, e.g., at most two airports were assumed and/or fleet investment of airlines was completely ignored; see for instance A. Zhang and Y. Zhang 2006, Basso 2008, Xiao et al. 2013, Santos and Antunes 2015, Sun and Schonfeld 2015, or Kidokoro et al. 2016.

This paper is organized as follows. Section 2 describes the model framework. In Sections 3 and 4 the reference benchmark model and the multi-level market model are presented, respectively. Our reformulation and our solution approach are described in Sections 5 and Section 6. Section 7 discusses the results of our case study. Finally, Section 8 summarizes main findings.

2. Model Framework

2.1. **Overview of the Relevant Market Players.** In this subsection we briefly sketch the main structure of the considered models and introduce the basic market participants. The following subsections will then present more detailed information on these players, their main characteristics, and key decision variables.

Given a set of relevant planning periods $t \in T$, in this paper we consider an intertemporal problem where different airports N may decide to extend their current runway capacity by $x_n \in \mathbb{N}$. In an analogous way, airlines A can make a fleet expansion decision $y_{ap} \in \mathbb{N}$ based on a set of possible aircraft types $p \in P_a$ out of which an airline may choose from. Fleet expansion accounts for future returns on investments that are realized on a competitive ticket market. In turn, ticket trade depends on the aircraft scheduling $z_{apc} \in \mathbb{N}$ of the different airlines, i.e., on the decision which demand market or connection $c \in C_a$ with a connection-specific demand $d_c \in \mathbb{R}_{\geq 0}$ to serve by which aircraft $p \in P_a$. In particular, the number of offered or sold tickets $w_{ac} \in \mathbb{R}_{\geq 0}$ of airline a on a connection c will be constrained by the investment and scheduling decision of its aircraft. An overview of the relevant market players and their main decisions is given in Figure 1, while the tables in Appendix A summarize main sets, variables, and parameters.
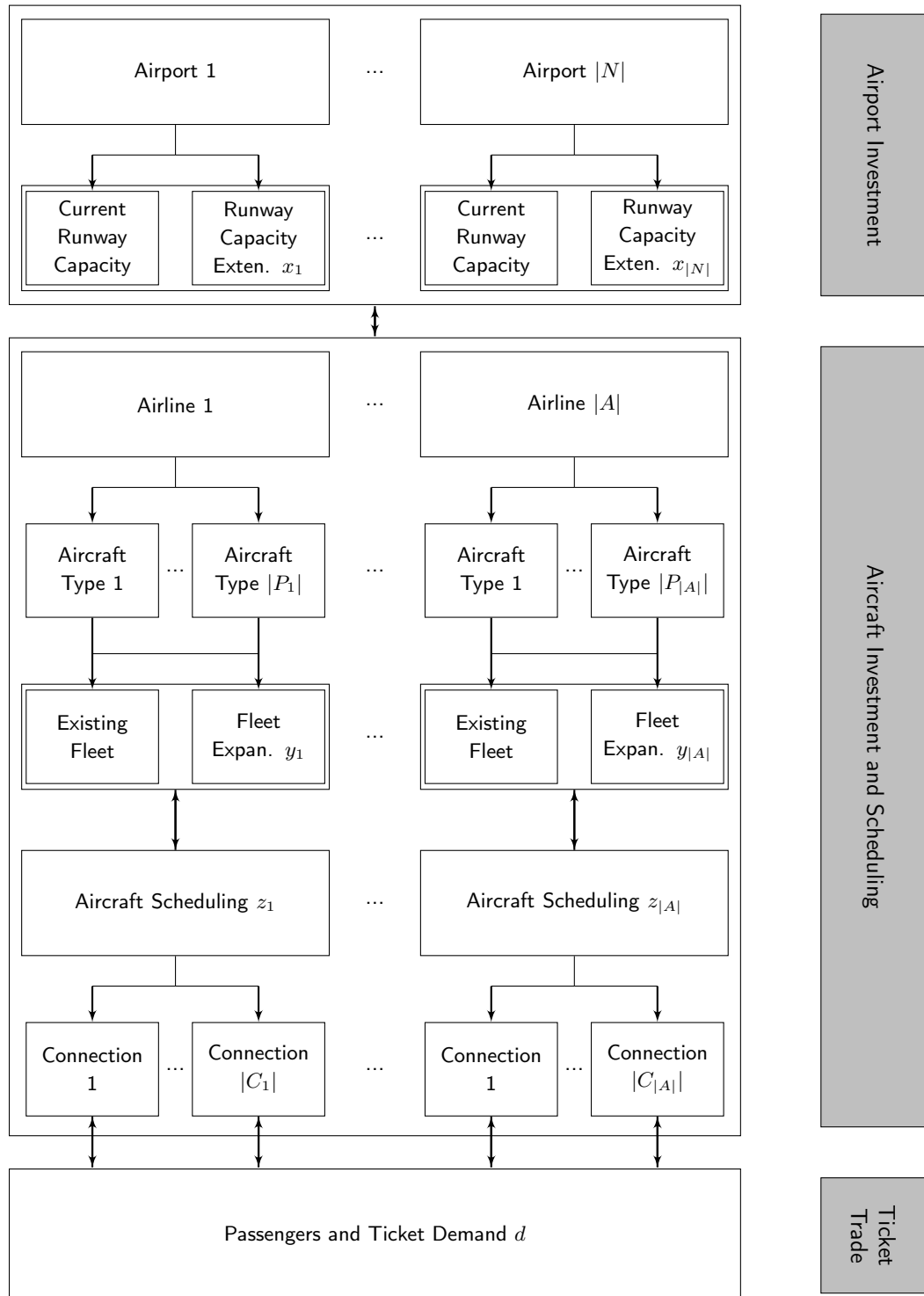
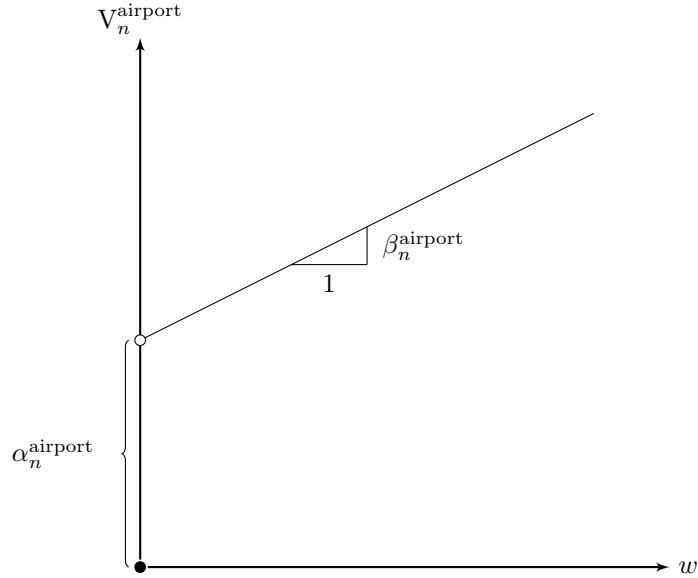FIGURE 1. Overview of the Relevant Market Players

FIGURE 2. Stepwise Variable Cost Function of an Airport

In order to simplify notation, in the following sections quantities x, y, z, w, and d without a subindex for airlines, connections, or aircraft types denote the corresponding vectors, e.g., $x = (x_n)_{n \in N}$ is the vector of all runway capacity expansion decisions. In analogy, an index for an airport or airline denotes the quantities that are controlled by the respective player. For instance the vector z_a describes the scheduling decisions related to airline $a \in A$.

2.2. Airports.

2.2.1. *Existing Runway Capacity and Airport Operation.* Given the set of relevant airports N, each airport $n \in N$ has a given runway capacity $\kappa_n^{\mathrm{airport}}$ that describes the maximum number of takeoffs and landings in one time period. Such a capacity choice implies the assumption of runways being operated in a so-called mixed mode, which is applied at various airports all around the world.

In order to operate an airport and to handle arriving and departing flights, airports face flight-dependent costs that are described by a variable cost function V_n^{airport}. In this paper we assume a piecewise linear variable cost function for each scheduled flight as depicted in Figure 2, which consists of two different cost components. First, for each aircraft arriving at or departing from that airport (given a chosen flight schedule) costs of $\alpha_n^{\mathrm{airport}}$ arise that are independent of the number of passengers aboard the respective aircraft. This cost component may for instance comprise taxi-in or -out, parking, or gate-usage expenses. In addition, we assume that for each passenger aboard the scheduled aircraft the airport faces additional costs of $\beta_n^{\mathrm{airport}}$ that include costs like baggage handling, passenger transportation, or security checking on a per-capita basis.

Let $\delta_n^{\mathrm{in}}(C_a)$ describe the set of all ingoing connections at airport $n \in N$ of airline $a \in A$. Analogously, let $\delta_n^{\mathrm{out}}(C_a)$ be the respective outgoing connections. If we set $\delta_n(C_a) := \delta_n^{\mathrm{in}}(C_a) \cup \delta_n^{\mathrm{out}}(C_a)$, then the variable cost function for an airport $n \in N$

writes as

$$V_n^{\text{airport}}(z, w) := \sum_{a \in A} \sum_{c \in \delta_n(C_a)} \left(\sum_{p \in P_a} \alpha_n^{\text{airport}} z_{apc} + \beta_n^{\text{airport}} w_{ac} \right). \qquad (1)$$

2.2.2. *Runway Capacity Investment.* As discussed above, airports may choose to invest in additional runway capacity $x_n \in \mathbb{N}$ at the different airports $n \in N$ with investment cost of i_n^{airport}. Note that besides the actual runway construction costs, runway capacity investment cost may also comprise gate or terminal extensions that might be necessary in order to handle the additional flights. In line with the existing literature, throughout this paper we will assume that all investment decisions are made (and realized) at the beginning of the planning horizon; see for instance Jenabi et al. 2013, Grimm, Martin, Schmidt, et al. 2016, Grimm, Martin, Weibelzahl, et al. 2016, or Weibelzahl 2016. In the case where at a given airport no investment is possible, we either set $x_n = 0$ or $i_n^{\text{airport}} = \infty$. By the term

$$I_n^{\text{airport}}(x) := i_n^{\text{airport}} x_n \qquad (2)$$

we will describe total investment cost of an airport $n \in N$.

2.3. Airlines.

2.3.1. *Existing Fleet and Fleet Operation.* Let P_a describe the set of different aircraft types, e.g., A330 or Boeing 747, which an airline $a \in A$ is willing to operate or to invest in according to its exogenous business model. Note that the importance of business models in the airline industry is recently highlighted by different authors. For instance Nataraja and Al-Aali 2011 discuss the strategy of Emirates that mainly build on the two aircraft types A380 and Boeing 777.

Let the number of aircraft of a specific type $p \in P_a$ that is owned by airline $a \in A$ at the beginning of the planing horizon, i.e., the existing fleet of airline $a \in A$, be given by e_{ap}^{airline}. Finally, the seat capacity of each aircraft type $p \in P_a$ is described by $\kappa_p^{\text{aircraft}}$.

Similar to airports, we assume that flight-dependent costs are described by a piecewise linear variable cost function V_a^{airline} that again depends on two different cost parameters. Independent of the number of passengers aboard the aircraft, constant costs of $\alpha_{apc}^{\text{airline}}$ arise for a completed flight that for instance refer to the fuel costs of the empty aircraft $p \in P_a$ on a connection $c \in C_a$, i.e., the fuel consumption of the aircraft with no passengers aboard. As fuel costs will increase with the number of passengers, we further assume a passenger-related cost coefficient[1] of $\beta_{ac}^{\text{airline}}$ that may vary between different connections $c \in C_a$. Note that this coefficient may additionally capture further passenger-related costs like for instance food or beverages. Using this notation, the variable cost function for an airline $a \in A$ is given by:

$$V_a^{\text{airline}}(z, w) := \sum_{c \in C_a} \left(\sum_{p \in P_a} \alpha_{apc}^{\text{airline}} z_{apc} + \beta_{ac}^{\text{airline}} w_{ac} \right). \qquad (3)$$

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[1]Observe that for the sake of simplicity and a reduced computational burden, the used cost coefficient is independent of the aircraft type. However, the model can easily be generalized to the case of aircraft type specific $\beta_{apc}^{\text{airline}}$ by extending the variable w_{ac} by an index p.

2.3.2. *Aircraft Investment and Fleet Expansion.* As discussed in Section 2.1, we allow airlines to invest in new aircraft, where aircraft investments may alternatively be interpreted as some kind of aircraft leasing (with a fixed leasing amount). Similar to runway capacity extensions, aircraft investments take place at the beginning of the planning horizon. The decision variable $y_{ap} \in \mathbb{N}$ describes the number of additional or new aircraft of type $p \in P_a$ that airline $a \in A$ purchases. Investment cost for airline $a \in A$ is denoted by the term

$$I_a^{\text{airline}}(y) := \sum_{p \in P_a} i_p^{\text{aircraft}} y_{ap}. \tag{4}$$

2.3.3. *Aircraft Scheduling.* Once investment in new aircraft has taken place, airlines schedule their whole fleet, which comprises existing and newly invested aircraft. In particular, a decision is made by each airline whether or not a certain connection is served by one of its aircraft. In this context a connection is defined as a tuple describing the origin where the flight starts from at a certain time and the corresponding destination that will be reached at another point in time. Note that our definition of a connection is independent of the assigned aircraft type and only depends on time-space characteristics, i.e., on the respective time and airport of departure and arrival. To give an example of such a connection, an aircraft may depart from Munich at 1 p.m. and reach Berlin at 2 p.m.. As before, C_a is the set of all possible connections that airline a can decide to serve. Observe that similar to fleet investment decisions, C_a may again be determined by an ex-ante given business model of airline $a \in A$. As described in Section 2.1, the variable $z_{apc} \in \mathbb{N}$ indicates for each airline $a \in A$ if an aircraft of a given type $p \in P_a$ operates on a connection $c \in C_a$.

2.4. **Passengers: Ticket Demand, Consumer Surplus, and Corresponding Revenues of Airlines.** We further assume (price-sensitive) passengers that can buy tickets for the different flight connections $c \in C_a$ of an airline $a \in A$, where each connection c departs and arrives within the assumed planning horizon T. For the sake of simplicity, we only consider a single, aggregated fare class as for instance in Lohatepanont and Barnhart 2004. As discussed above, we use the variable $d_c \in \mathbb{R}_{\geq 0}$ to describe the ticket demand on connection c, while we denote by $w_{ac} \in \mathbb{R}_{\geq 0}$ the number of tickets sold or offered by airline $a \in A$ on its flight connection $c \in C_a$. Throughout this paper we will make the (realistic) assumption that the ticket price P_c^{con} depends on the number of all passengers on each connection and that demand for two different connections is independent of each other; see again Lohatepanont and Barnhart 2004. In particular, we will assume that the demand or price functions are strictly decreasing and integral, which for instance is the case for a linear demand function.

Based on the above definitions, the integral

$$\int_0^{d_c} P_c^{\text{con}}(s) \mathrm{d}s \tag{5}$$

will be referred to as the gross consumer surplus.

Finally, airline revenues are generated by ticket sales. Therefore, the number of sold tickets multiplied by the above ticket price describes the corresponding revenues

$$R_a^{\text{airline}}(d, w) := \sum_{c \in C_a} P_c^{\text{con}}(d_c) w_{ac} \tag{6}$$

of an airline $a \in A$.

3. Integrated Planning Model

In our reference investment model, optimal prices are determined in a single-level welfare maximization problem that accounts for all airport expansion, fleet investment, aircraft scheduling, and ticket trade constraints. As all relevant planning constraints are simultaneously taken into account when investments are made, a first-best solution will always be achieved. Observe that our reference model may also be interpreted as an integrated-planning model, where a integrated planner makes welfare-maximizing investment and operational decisions, i.e., the whole industry is planned using a benevolent, single-level approach. Note that such an integrated benchmark implies that airports are not limited by budget restrictions, but in the case where an airport is in deficit, the planner can make a lump-sum transfer from the airlines to the respective loss-making airport operator (or vice versa). From an economic point of view, the integrated planner model is also equivalent to a perfect competition market and hence maximizes welfare.

In our framework, welfare ω can be expressed as the aggregated difference between gross consumer surplus and both investment and variable costs of airports/airlines:

$$
\begin{aligned}
\omega(d,x,y,z,w) \quad &:= \sum_{c \in C} \int_0^{d_c} \mathrm{P}_c^{\mathrm{con}}(s)\mathrm{d}s \\
&\quad - \sum_{n \in N} \left(\mathrm{I}_n^{\mathrm{airport}}(x) + \mathrm{V}_n^{\mathrm{airport}}(z,w) \right) \\
&\quad - \sum_{a \in A} \left(\mathrm{I}_a^{\mathrm{airline}}(y) + \mathrm{V}_a^{\mathrm{airline}}(z,w) \right).
\end{aligned}
\tag{7}
$$

In our integrated planning model we must guarantee that for any airline $a \in A$ the number of passengers on a connection $c \in C_a$ does not exceed the capacity of the aircraft scheduled:

$$
w_{ac} \leq \sum_{p \in P_a} \kappa_p^{\mathrm{aircraft}} z_{apc} \quad \forall\, a \in A, c \in C_a.
\tag{8}
$$

In addition, we ensure market clearing for each connection, i.e., for each connection the number of tickets bought by the passengers must be equal to the number of tickets sold by all airlines:

$$
d_c = \sum_{a \in A} w_{ac} \quad \forall\, c \in C.
\tag{9}
$$

Similar to these aircraft-related capacity constraints, we must also account for the respective runway capacity of airports. In particular, the sum of all in- and outgoing flights of an airport $n \in N$ must not exceed its runway capacity in any time period $t \in T$, where we explicitly account for possible runway capacity extensions:

$$
\sum_{a \in A} \sum_{c \in \delta_{nt}(C_a)} \sum_{p \in P_a} z_{apc} \leq \kappa_n^{\mathrm{airport}} + x_n \quad \forall\, n \in N, t \in T.
\tag{10}
$$

In Equation (10) we extended our delta-notation by a time index $t \in T$ with $\delta_{nt}(C_a)$ describing the set of all in- and outgoing connections at airport $n \in N$ of airline $a \in A$ in time period $t \in T$. Observe that airport congestion effects are not modeled by using delay cost functions, but in (10) each scheduled aircraft departs and arrives on time while satisfying the airport capacity. However, our model could easily be modified in order to incorporate explicit delay cost functions. In addition, we make the assumption that the scare airport infrastructures are always allocated globally
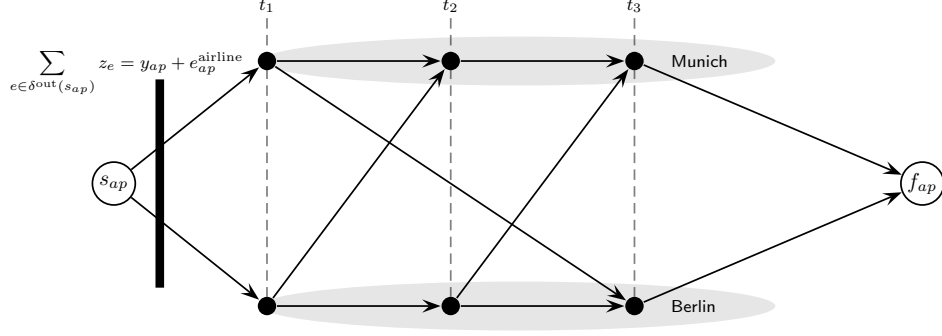
FIGURE 3. Graph Example for the Modeling of Scheduling Constraints

efficient, i.e., all scheduling decisions are made in a way that maximizes welfare of the whole industry.

In order to describe the relevant aircraft scheduling constraints, we introduce a graph for every $(a, p) \in (A, P_a)$, which models the trajectory of the aircraft $p \in P_a$ of an airline $a \in A$. We first define a start node s_{ap} and a final node f_{ap}. Moreover, we introduce nodes $\tilde{N}_{ap}$ for every airport and every time period to model the current location of the aircraft. To give a first example, let us assume two locations *Berlin* and *Munich* and three time periods t_1, t_2, and t_3 as depicted in Figure 3. Horizontal edges define holdover edges $e \in H_{ap}$, which enable the parking of an aircraft in a certain time period. Furthermore, we use the already introduced connections C_a to define the flight edges C_{ap} depending on the aircraft type $p \in P_a$. These edges only exist if a connection may be served by an airline given its ex-ante specified business model. In our example the aircraft may start in *Berlin* and arrive after one time period in *Munich*. The way back takes more time, which for instance may be due to bad wind or weather conditions.

Finally, we define start edges S_{ap} and final edges F_{ap} to connect the final and start node with the respective time- and location-specific nodes in time period t_1 and t_3. With this notation, the following constraints ensure a feasible trajectory of the aircraft:

$$\sum_{e \in S_{a,p}} z_e = y_{ap} + e_{ap}^{\mathrm{airline}} \quad \forall\, a \in A, p \in P_a, \tag{11a}$$

$$\sum_{e \in \delta_n^{\mathrm{out}}(H_{ap} \cup C_{ap})} z_e - \sum_{e \in \delta_n^{\mathrm{in}}(H_{ap} \cup C_{ap})} z_e = 0 \quad \forall\, a \in A, p \in P_a, n \in \tilde{N}_{ap}, \tag{11b}$$

$$\sum_{p \in P_a} z_{apc} \leq 1 \quad \forall\, a \in A, c \in C_a. \tag{11c}$$

Equation (11a) assures that the aircraft used are available, i.e., the aircraft either belong to the set of already existing aircraft or to the newly invested fleet. In addition, (11b) is the standard flow conservation, i.e., an aircraft can only serve connections if it is available at the respective airport. Furthermore, (11c) limits the number of scheduled aircraft on each connection for the different airlines. Note that for given flight edges e, z_e correspond to scheduling variables z_{apc}.

Finally, we assume the following variable restrictions:

$$x \in \mathbb{N}^N, \tag{12a}$$

$$y \in \mathbb{N}^{A \times P_a}, \tag{12b}$$

$$z \in \mathbb{N}^{A \times P_a \times C_a}, \tag{12c}$$

$$d \in \mathbb{R}_{\geq 0}^C, \tag{12d}$$

$$w \in \mathbb{R}_{\geq 0}^{A \times C_a}. \tag{12e}$$

Therefore, our mixed-integer integrated planning model writes as follows:

$$\max \quad \text{Welfare :} \ (7), \tag{13a}$$

$$\text{s.t.} \quad \text{Aircraft Capacity Limitations: } (8), \tag{13b}$$

$$\text{Market Clearing: } (9), \tag{13c}$$

$$\text{Runway Capacity Limitations: } (10), \tag{13d}$$

$$\text{Aircraft Scheduling Constraints: } (11), \tag{13e}$$

$$\text{Variable Restrictions: } (12). \tag{13f}$$

4. Four-Level Market Model

In liberalized real-world aviation markets, decisions are typically made by independent market players rather than by an integrated planner that controls all market facilities. In our paper these players comprise airport operators, airlines, and passengers.

As a main planning challenge of such a liberalized market structure, an optimal decision of a player will highly depend on the optimal decisions of all other market players. Therefore, in this section we present a market model, where the optimal behavior of the considered players is influenced by the expectations of the optimal decision-response of the others. As we will see below, such a sequential decision structure may yield investments that quite differ from an integrated planning solution.

The timing of the considered decision situation is depicted in Figure 4, where long-term airport capacity investments and airline fleet investment decisions are followed by medium-term aircraft scheduling and short-term market trade. Let us note that similar planning and operation time structures are commonly highlighted in the literature, e.g., in Smith and T. Jacobs 1997 or T. L. Jacobs et al. 2012. We translate our multi-stage game into the following four-level market model, which is depicted in Figure 5: On the first level we assume airports that cooperatively decide on their runway capacity extension. Airports anticipate optimal fleet expansion and scheduling on the second and third level. In turn, optimal aircraft investment and scheduling of airlines on the second and third level account for expected returns on investment that will be realized at several ticket markets on the fourth level. Note that in the case where the seat capacity of an aircraft scheduled on a specific connection on the third level is much too low to serve the respective demand on the ticket market on the fourth level, revenues may be lost on that connection. On the opposite, if the seat capacity of the scheduled aircraft will be much too high with a lot of empty seats, revenues may be lost on another (more profitable) connection with a high demand; see also Lohatepanont and Barnhart 2004. Such interdependencies underline the complex interaction between the different market players and the respective problem levels.
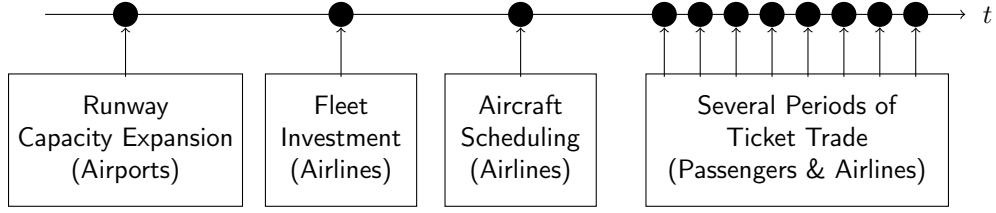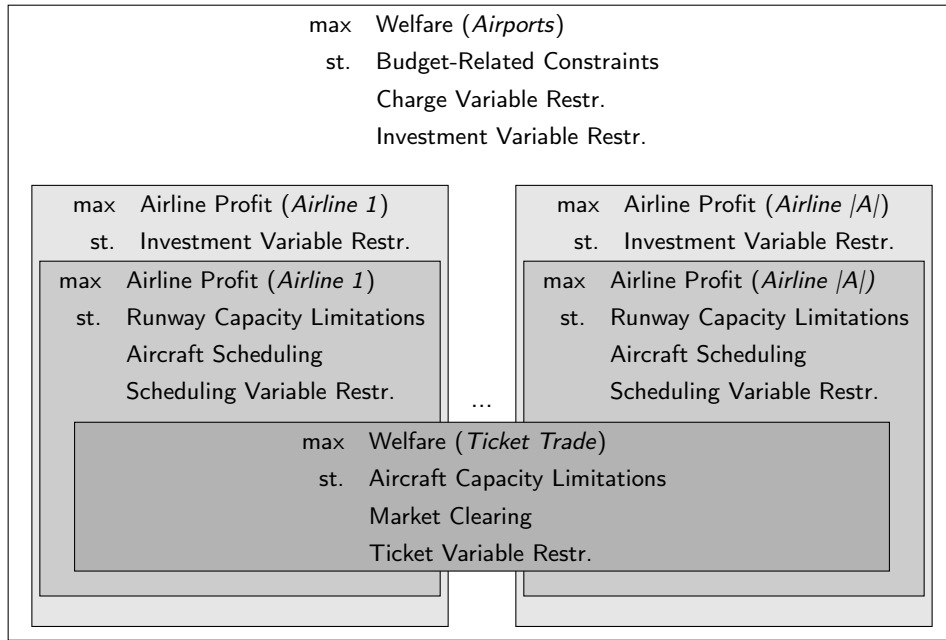
FIGURE 4. Timing of the Multi-Stage Game

FIGURE 5. The Four-Level Problem

For the sake of an intuitive presentation of the multi-level model, the following sections will describe step-by-step the four problem levels, before we present an equivalent single-level problem reformulation in Section 5 that circumvents an hierarchical problem solving.

4.1. **First-Level Problem: Airport Capacity Investment.** On the first level airports provide the necessary infrastructure that is used by both airlines and passengers. In particular, airports decide on an optimal runway capacity extension. We assume that airports and their charges are regulated [2] such that it is in an airport's own interest to maximize welfare (see also Basso 2008), where welfare is again given by Equation (7).

Airports impose charges in order to generate revenues. In particular, in this paper we consider airport charges as measures to recover investment and/or operational airport cost. Even though, in practice different types of charges may be used, we

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[2]In a lot of countries all around the world airports are regulated by governmental authorities. For instance the German Aviation Law (§19b) requires airports in Germany to get an official permission for their charges, where amongst other (public interest) criteria objectivity, transparency, and non-discrimination must be guaranteed.

assume a passenger-based charge

$$\phi_n \in \mathbb{N}^{|N|} \tag{14}$$

imposed on each passenger aboard an aircraft that arrives at or departs from an airport. Observe that we restricted ϕ_n to integer values mainly in order to reduce numerical instabilities in our decomposition algorithm in Section 6. However, this will not affect the general statement of our analysis, as all monetary quantities can for instance be measured in Cent instead of Dollar.

Revenues of an airport $n \in N$ paid by an airline $a \in A$ are given by:

$$R_{na}^{\text{airport}}(\phi, w) := \sum_{c \in \delta_n(C_a)} \phi_n w_{ac}. \tag{15}$$

For similar charges that are used at different airports all around the world see, e.g., Frankfurt Airport 2016, Heathrow Airport 2017, or John F. Kennedy International Airport 2017. We also note that our model can easily be adapted in order to model other types of airport charges.

Finally, observe that such charges may either be treated as endogenous variables or exogenous parameters. For both variants examples exist in the literature; see for instance Jenabi et al. 2013 or Grimm, Martin, Schmidt, et al. 2016. As in our paper purely exogenous charges would limit and bias the investment behavior of the different airports, we consider the decision on optimal airport charges as endogenous variables.

In addition, we make the (realistic) assumption that airports are budget-constrained, i.e., their profits must always be nonnegative. Profits of an airport equal its income from airport charges minus runway capacity investments and operational costs. Therefore, profits can be expressed as

$$\rho_n^{\text{first}}(x, \phi_n) := \sum_{a \in A} R_{na}^{\text{airport}}(\phi, w) - \left(V_n^{\text{airport}}(z, w) + I_n^{\text{airport}}(x) \right) \tag{16}$$

for all airports $n \in N$. The corresponding budget constraints for each airport are given by:

$$\rho_n^{\text{first}}(x, \phi_n) \geq 0 \quad \forall \, n \in N. \tag{17}$$

Finally, we require runway capacity extension variables to take integer values, which measure the maximal number of takeoffs/landings at an airport within one time period. Thus, the complete first-level problem can be stated as:

$$\max \quad \text{Welfare}: \ (7), \tag{18a}$$

$$\text{s.t.} \quad \text{Budget-Related Constraints}: \ (16), (17), \tag{18b}$$

$$\text{Charge Variable Restrictions}: \ (14), \tag{18c}$$

$$\text{Investment Variable Restrictions}: \ (12a). \tag{18d}$$

4.2. Second-Level Problem: Fleet Expansion and Aircraft Investment.

On the second level imperfectly competitive airlines decide on their optimal fleet expansion by investing in new aircraft; for a discussion of imperfect competition in the airline industry see, e.g., Adler 2001. We assume that each individual airline $a \in A$ maximizes its profits, where profits ρ_a^{second} can be expressed as the difference between revenues from ticket trade on the fourth level and the corresponding aircraft

investment cost, operational aircraft cost, and airport charges

$$
\begin{aligned}
\rho_a^{\text{second}}(y) := \quad & R_a^{\text{airline}}(d, w) \\
& - \left(V_a^{\text{airline}}(z, w) + I_a^{\text{airline}}(y) + \sum_{n \in N} R_{na}^{\text{airport}}(\phi, w) \right)
\end{aligned}
\tag{19}
$$

for all airlines $a \in A$. Note that on the second level the imperfectly competitive airlines may decide to limit investment in order to increase their profits through higher prices on the ticket market on the fourth level; see below.

Taking the integrality condition of its aircraft investments into account, the second-level investment problem for each airline $a \in A$ can be stated as:

$$
\max \quad \text{Airline Profit : (19)}, \tag{20a}
$$

$$
\text{s.t.} \quad \text{Investment Variable Restrictions: (12b)}. \tag{20b}
$$

4.3. **Third-Level Problem: Aircraft Scheduling.** On the third level imperfectly competitive airlines $a \in A$ schedule their existing and newly invested aircraft for given fleet and runway capacities, i.e., airlines decide, in which time period $t \in T$, which aircraft of their respective fleet P_a will serve which connection $c \in C_a$. In particular, given imperfect competition on the third level airlines need to compete for the possibly scare runway capacities that are available in discrete amounts; see above.

Again, let us consider an arbitrary airline $a \in A$ that maximizes its individual profits. As optimal aircraft scheduling on the third level takes place after optimal fleet expansion on the second level, profits of an airline $a \in A$ on the third level only account for relevant operational aircraft cost and corresponding airport charges. In other words, sunk investment costs of the second level are irrelevant for the scheduling and operational decisions of the airlines on the third level. Therefore, third-level profits of each airline $a \in A$ are given by

$$
\rho_a^{\text{third}}(z) := \quad R_a^{\text{airline}}(d, w) - \left(V_a^{\text{airline}}(z, w) + \sum_{n \in N} R_{na}^{\text{airport}}(\phi, w) \right). \tag{21}
$$

Maximizing their profits, on the third level the scheduling-related constraints of the integrated planning model are taken into account by airlines. In addition, airlines consider the runway capacity restrictions of airports, which are known by airlines on the third level. However, in contrast to the global runway capacity constraint (10) in our integrated planner model, in the multi-level model runway capacity limitations become individual airline constraints, where the decision of an airline on the use of the runways influences the set of decisions of the other airlines. Therefore, airlines explicitly compete at the third level for the scare runway capacities, which does not necessarily imply an efficient use of airport infrastructures as in the integrated planner model anymore.

Thus, the third-level problem of an arbitrary airline $a \in A$ is given by the following optimization problem:

$$
\max \quad \text{Airline Profit : (21)}, \tag{22a}
$$

$$
\text{s.t.} \quad \text{Runway Capacity Limitations: (10)}, \tag{22b}
$$

$$
\text{Aircraft Scheduling: (11)}, \tag{22c}
$$

$$
\text{Scheduling Variable Restrictions: (12c)}. \tag{22d}
$$

4.4. **Fourth-Level Problem: Ticket Trade.** On the last level ticket trade takes place at a competitive market for flight tickets. Despite the fact that optimal demand behavior of passengers is completely captured by a corresponding price function, under the assumption of perfect competition on the ticket market, the amount of sold tickets is only determined by market forces and not by price-setting airlines. In other words, given their optimal flight schedules, no airline can affect ticket prices by strategic supply decisions w_{ac}, i.e., no airline can systematically increase profits by artificially reducing ticket supply on their scheduled flights. This price-taking behavior of airlines keeps and nests the ticket-equilibrium problem as a true level in our market model.

As it is well known in the literature (see for instance Grimm and Zoettl 2013), the assumption of competitive markets allows to formulate the fourth level as a single welfare maximization problem, where welfare is given by

$$
\begin{aligned}
\omega^{\text{fourth}}(d, w) \quad &:= \sum_{c \in C} \int_0^{d_c} \text{P}_c^{\text{con}}(s)\text{d}s \\
&\quad - \sum_{a \in A} \left(\text{V}_a^{\text{airline}}(z, w) + \sum_{n \in N} \text{R}_{na}^{\text{airport}}(\phi, w) \right).
\end{aligned}
\tag{23}
$$

Observe that in contrast to the welfare objective that is used in the integrated planning problem and in the first-level airport capacity investment problem, on the ticket market on the fourth level welfare does not account for (sunk) investment costs, but only considers variable flight-dependent costs and the relevant airport charges.

Accounting for (i) the aircraft capacity limitations determined on the second and third level as well as for (ii) the ticket variable restrictions (including market clearing), the fourth-level is given by:

$$
\begin{aligned}
\max \quad &\text{Welfare}: \ (23), &\text{(24a)} \\
\text{s.t.} \quad &\text{Aircraft Capacity Limitations}: (8), &\text{(24b)} \\
&\text{Market Clearing}: (9), &\text{(24c)} \\
&\text{Ticket Variable Restrictions}: (12d), (12e). &\text{(24d)}
\end{aligned}
$$

5. Problem Reformulation

Our proposed four-level market model can be seen as an instance of a general multilevel optimization problem. Multilevel problems are known to be computationally very challenging, see for instance Dempe 2002, Colson et al. 2007, or Dempe et al. 2014 that also discuss the NP-hardness of much easier linear bilevel programs. Therefore, in this paper we developed a reformulation strategy that builds on a detailed three-step problem analysis as depicted in Figure 6.

In a first step, we replace the fourth-level problem by its Karush-Kuhn-Tucker (KKT) equilibrium system. In a second step, we show that the second-level and third-level problems have affine-equivalent objective functions, which implies that they can be replaced by an "aggregated" second level resulting from a concatenation of levels 2 and 3. Finally, in step 3 we reformulate the aggregated second-level problem by its Nash equilibrium representation, which yields a single-level problem, with all airport, Nash, and KKT constraints being placed on the same level. In other words, we arrive at a combination of the airport runway optimization, the equilibrium problem of competing airlines that buy and schedule their fleets, and the KKT conditions of the ticket market.
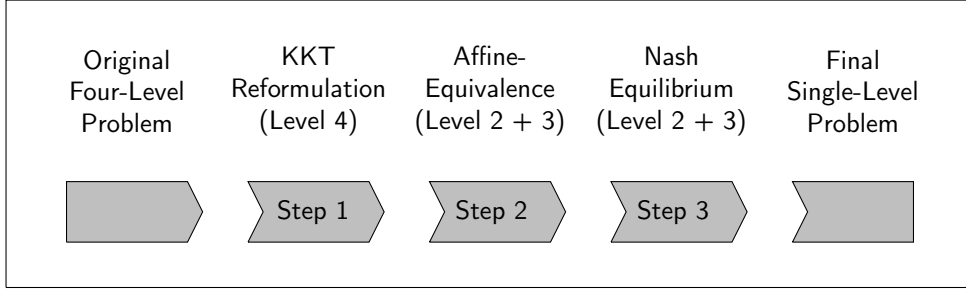
FIGURE 6. Solution Approach

5.1. **Step 1: KKT Reformulation of the Fourth Level.** The fourth-level problem is a maximization problem with a quadratic objective and linear constraints, which are all differentiable. Therefore, we can replace the last problem level by its KKT formulation; see for instance Dempe 2002 or Boyd and Vandenberghe 2004. These conditions comprise primal feasibility, dual feasibility, and complementary slackness. In particular, the KKT conditions equivalent to Problem (24) are given by

$$-\beta_{ac}^{\text{airline}} - \sum_{n \in N} \frac{\partial R_{na}^{\text{airport}}(\phi, w)}{\partial w_{ac}} - \gamma_{ac}^{\text{dual}} + \lambda_{ac}^{\text{dual}} + \mu_c^{\text{dual}} = 0 \quad \forall a \in A, c \in C_a, \quad \text{(25a)}$$

$$P_c^{\text{con}}(d_c) - \mu_c^{\text{dual}} + \nu_c^{\text{dual}} = 0 \quad \forall c \in C, \quad \text{(25b)}$$

$$w_{ac} - \sum_{p \in P_a} \kappa_p^{\text{aircraft}} z_{apc} \leq 0 \quad \forall a \in A, c \in C_a, \quad \text{(25c)}$$

$$-w_{ac} \leq 0 \quad \forall a \in A, c \in C_a, \quad \text{(25d)}$$

$$-d_c \leq 0 \quad \forall c \in C, \quad \text{(25e)}$$

$$d_c - \sum_{a \in A} w_{ac} = 0 \quad \forall c \in C, \quad \text{(25f)}$$

$$\gamma_{ac}^{\text{dual}} \geq 0 \quad \forall a \in A, c \in C_a, \quad \text{(25g)}$$

$$\lambda_{ac}^{\text{dual}} \geq 0 \quad \forall a \in A, c \in C_a, \quad \text{(25h)}$$

$$\nu_c^{\text{dual}} \geq 0 \quad \forall c \in C, \quad \text{(25i)}$$

$$\gamma_{ac}^{\text{dual}} \perp w_{ac} - \sum_{p \in P_a} \kappa_p^{\text{aircraft}} z_{apc} = 0 \quad \forall a \in A, c \in C_a, \quad \text{(25j)}$$

$$\lambda_{ac}^{\text{dual}} \perp -w_{ac} = 0 \quad \forall a \in A, c \in C_a, \quad \text{(25k)}$$

$$\nu_c^{\text{dual}} \perp -d_c = 0 \quad \forall c \in C, \quad \text{(25l)}$$

where the derivative of the airport revenues is described by:

$$\frac{\partial R_{na}^{\text{airport}}(\phi, w)}{\partial w_{ac}} = \begin{cases} \phi_n, & \text{if } n \text{ is origin/destination of } c \\ 0, & \text{otherwise} \end{cases} \quad \text{(26)}$$

Given that the fourth level is always feasible, the above KKT reformulation is valid as the fourth level problem has a unique solution in w_{ac} and d_c under quite reasonable assumptions:

Theorem 1. *The fourth level problem has a unique optimal solution if for any given connection $c \in C$ the two following conditions are satisfied:*

(1) *For all airlines $a \in A$, variable cost coefficients $\beta_{ac}^{airline}$ on connection c are pairwise distinct.*
(2) *Each airline $a \in A$ schedules at most one aircraft on connection c.*

Proof. As can directly be seen, the welfare maximization problem on the fourth level is strictly concave in d_c. Under the two above assumptions, by using Equation (11c) we can uniquely determine on each connection c the order in which the available aircraft of the different airlines will be used on a connection c for any optimal demand level d_c. Note that in the case where at least one of the two conditions does not hold, we can not guarantee a unique order of the used aircraft. On the opposite, under the two assumptions, the fourth level problem reduces to a cost-minimization problem with a unique solution in w_{ac} for any fixed demand level d_c. Thus, the fourth level problem has a unique global optimal solution. $\square$

5.2. **Step 2: Affine-Equivalence of the Second- and Third-Level Objectives.** Next, we show that for each airline $a \in A$ its third-level objective is an affine transformation of its second-level objective.

Proposition 1. *Consider an arbitrary airline $a \in A$ and let ρ_a^{second} and ρ_a^{third} be the corresponding objective functions of the second and third level. Then, $\rho_a^{second} = \rho_a^{third} + b$ holds, where*

$$b := -I_a^{airline}(y)$$

only depends on second-level aircraft investment variables.

We now prove that given affine equivalent objective functions for each airline $a \in A$, we can reformulate the second level and the third level as a single level for all airlines.

Proposition 2. *Consider a general bilevel problem with n players on each of the two problem levels*

$$\min_{x_1,y_1} \quad o_1(x_1, y_1, \ldots, x_n, y_n), \qquad \ldots, \qquad \min_{x_n,y_n} \quad o_n(x_1, y_1, \ldots, x_n, y_n)$$
$$\text{st.} \quad g_1(x_1, \ldots, x_n) \geq 0, \qquad \ldots, \qquad \text{st.} \quad g_n(x_1, \ldots, x_n) \geq 0$$
$$y_1 = \arg\min_{z_1}\{o_1 : \qquad \ldots, \qquad y_n = \arg\min_{z_n}\{o_n :$$
$$h_1(x_1, y_1, \ldots, x_n, y_n) \geq 0\}, \qquad \ldots, \qquad h_n(x_1, y_1, \ldots, x_n, y_n) \geq 0\}.$$

and with equivalent objective functions. We denote the set of optimal solutions by $\mathcal{S}^{bl}$. Let

$$\min_{x_1,y_1} \quad o_1(x_1, y_1, \ldots, x_n, y_n), \qquad \ldots, \qquad \min_{x_n,y_n} \quad o_n(x_1, y_1, \ldots, x_n, y_n)$$
$$\text{st.} \quad g_1(x_1, \ldots, x_n) \geq 0, \qquad \ldots, \qquad \text{st.} \quad g_n(x_1, \ldots, x_n) \geq 0$$
$$h_1(x_1, y_1, \ldots, x_n, y_n) \geq 0, \qquad \ldots, \qquad h_n(x_1, y_1, \ldots, x_n, y_n) \geq 0.$$

be the corresponding single-level problem with n players. Its solution set will be denoted by $\mathcal{S}^{sl}$. Then, we have: $\mathcal{S}^{sl} = \mathcal{S}^{bl}$.

Proof. Note that a solution of the above bilevel problem is feasible for the described single-level problem and vice versa. As additionally the upper and lower level share the same objective function, we arrive at the proposed result. $\square$

5.3. **Step 3: Generalized Nash Equilibrium.** In the following, we will denote by $-a$ the set of all airlines except from airline a, i.e., we set $-a := A \setminus \{a\}$. In this paper we assume that the decision of an airline on its fleet expansion and flight scheduling depends on the optimal decision-making of the remaining airlines. In analogy to Barroso et al. 2006 or Pozo et al. 2013, we use the concept of a (generalized) Nash equilibrium to determine the investment and scheduling decisions of all airlines. A Nash strategy (y_a^*, z_a^*) for airline $a \in A$ is characterized by rendering the maximum profit as compared to any other of its strategies, where we assume that all other airlines $-a$ stick to their equilibrium strategy (y_{-a}^*, z_{-a}^*). More formally, in a Nash equilibrium the strategy (y_a^*, z_a^*) of an arbitrary airline $a \in A$ maximizes its profits given that the strategies of all other airlines $-a$ are fixed to their equilibrium decision; see also the definition of a Nash equilibrium in the seminal paper Nash 1951. This directly implies that a unilateral deviation of the Nash equilibrium only reduces profits of an airline a, i.e.,

$$
\begin{aligned}
&\rho_a^{\text{second}}(y^*, z^*, w^*(y^*, z^*), d^*(y^*, z^*)) \\
&\geq \max_{y_a, z_a} \rho_a^{\text{second}}(y_a, z_a, y_{-a}^*, z_{-a}^*, w^*(y_a, z_a, y_{-a}^*, z_{-a}^*), d^*(y_a, z_a, y_{-a}^*, z_{-a}^*)).
\end{aligned}
\tag{27}
$$

Equivalently, the maximum operator in Equation (27) may be replaced by corresponding inequalities for every feasible airline strategy (y_a, z_a):

$$
\begin{aligned}
&\rho_a^{\text{second}}(y^*, z^*, w^*(y^*, z^*), d^*(y^*, z^*)) \\
&\geq \rho_a^{\text{second}}(y_a, z_a, y_{-a}^*, z_{-a}^*, w^*(y_a, z_a, y_{-a}^*, z_{-a}^*), d^*(y_a, z_a, y_{-a}^*, z_{-a}^*)).
\end{aligned}
\tag{28}
$$

Note that even though the actual ticket trade is competitive, in our setting a unilateral deviation of the equilibrium investment and scheduling decision of an airline implicitly affects sales on the market for flight tickets. Therefore, Equations (27) and (28) account for supply effects on the ticket market caused by changes in the investment and scheduling decisions, which yields a restricted Nash equilibrium. In particular, optimal ticket demand and supply variables (w^*, d^*) are determined by a corresponding KKT system analogous to (25). These dependencies are also captured in the expressions $w^*(y^*, z^*)$ and $d^*(y^*, z^*)$.

5.4. **Final Single-Level Problem.** Applying the above reformulation steps, we arrive at a single-level problem that consists of

- the original first-level problem and
- the (generalized) Nash equilibrium formulation for each airline (including the corresponding KKT systems).

Note that we assume the optimistic multilevel case, where for multiple optimal Nash equilibria the optimal solution that is welfare-maximizing will be realized. From our point of view such an assumption is appropriate, as in general airlines will have a corporate social responsibility that will not lead them to choose a solution out of their set of optimal solutions with a welfare-diminishing effect. For instance, a responsible business practice is part of the corporate strategy of the Lufthansa Group that aims at "meeting (its) responsibilities towards the environment and society"; see Group 2000.

6. Solution Strategy

An obvious drawback of the above problem reformulation is the high number of Nash constraints and corresponding KKT systems. Therefore, in this section we present a solution strategy that iteratively solves master and subproblems with a

---

Algorithm 1: Decomposition approach for the reformulated market model

---

    Input : Parameters for the market model
    Output: Optimal solution $\hat{x}, \hat{\phi}, \hat{y}, \hat{z}, \hat{w}, \hat{d}$ of the problem

1 Solve the integrated planner model (13) with the KKT system (25) and the
    budget constraints (17). Denote the solution by $(\hat{x}^{\mathrm{m}}, \hat{\phi}^{\mathrm{m}}, \hat{y}^{\mathrm{m}}, \hat{z}^{\mathrm{m}}, \hat{w}^{\mathrm{m}}, \hat{d}^{\mathrm{m}})$.

2 for *all airlines* $a \in A$ **do**

3   Fix $\hat{x}^{\mathrm{m}}, \hat{\phi}^{\mathrm{m}}, \hat{y}^{\mathrm{m}}_{-a}, \hat{z}^{\mathrm{m}}_{-a}$ and solve the subproblem with a solution
        $(\hat{y}^{\mathrm{s}}_a, \hat{z}^{\mathrm{s}}_a, \hat{w}^{\mathrm{s}}, \hat{d}^{\mathrm{s}})$

4   **if** $\rho^{second}_a(\hat{y}^m_{-a}, \hat{z}^m_{-a}, \hat{y}^s_a, \hat{z}^s_a, \hat{w}^s, \hat{d}^s) - \rho^{second}_a(\hat{y}^m, \hat{z}^m, \hat{w}^m, \hat{d}^m) \geq \epsilon$ **then** Add a
        corresponding Nash inequality (28) and KKT system (25) to the new
        master problem.

5 if *violated Nash inequalities* (28) *and corresponding KKT systems* (25) *are
    identified for at least one* $a \in A$ **then** solve new master problem and **go to**
    Line 2.

6 else

7   **return** optimal solution $(\hat{x}, \hat{\phi}, \hat{y}, \hat{z}, \hat{w}, \hat{d}) = (\hat{x}^{\mathrm{m}}, \hat{\phi}^{\mathrm{m}}, \hat{y}^{\mathrm{m}}, \hat{z}^{\mathrm{m}}, \hat{w}^{\mathrm{m}}, \hat{d}^{\mathrm{m}})$.

---

reduced number of Nash-related constraints. In particular, as a main advantage, not all Nash equilibrium constraints have to be considered a-priori.

6.1. **Description of the Algorithm.** The main idea of our Algorithm is to relax all Nash equilibrium constraints (28) in a first step. The resulting master problem consists of only the integrated planner constraints (13), a single set of KKT ticket trade restrictions (25), and the budget constraints (17). Suppose we solved this augmented planner problem and got the optimal solution for all variables. In a second step, we then check in $|A|$ subproblems whether at least one airline $a \in A$ has an incentive to deviate from its investment and scheduling solution identified in the master problem. In particular, in every subproblem each given airline $a \in A$ maximizes its profits subject to all scheduling restrictions (10), (11), (12c), fleet investment constraints (12b), the KKT system (25), and fixed scheduling and investment values for all other airlines $-a$, which were determined in the current master problem. If we identify a profit-enhancing strategy (y_a, z_a) for an airline a that unilaterally deviates from the identified master problem solution, we add a corresponding Nash inequality (28) and KKT system (25) to the new master problem, which cuts-off the current infeasible solution. Then we add these constraints to the augmented planner problem and find a solution to this further augmented planner problem. An overview of the resulting iterative solution strategy can be found in Algorithm 1, where the master problem iteratively generates solution candidates, whose feasibility is verified in a collection of subproblems.

6.2. **Correctness of the Algorithm.** In this section we will prove correctness of our Algorithm. We first note that in each iteration of Algorithm 1 the added Nash inequalities (28) and corresponding KKT systems (25) do not cut-off a feasible solution of the four-level market model:

Theorem 2. *The Nash inequalities* (28) *and KKT systems* (25) *that are added in Step 5 of Algorithm 1 do not cut-off a feasible solution of the four-level market model.*

Proof. By definition of a Nash equilibrium, the Nash inequalities (28) and KKT systems (25) must be met for all available airline strategies. In Step 5 of Algorithm 1

we only add to the master problem a subset of these constraints, which every feasible solution to the Nash equilibrium and in direct consequence to the four-level market model must satisfy. This implies that in each step 5 of Algorithm 1 no feasible solution will be cut-off. □

We next show that in each iteration the current infeasible solution is indeed cut-off:

Theorem 3. *In each iteration of Algorithm 1 the current infeasible solution is cut-off.*

Proof. Let us be given a solution to the master problem $(\tilde{x}, \tilde{\phi}, \tilde{y}, \tilde{z}, \tilde{w}, \tilde{d})$ and an airline a with a strategy (y_a^*, z_a^*) that yields by unilaterally deviating from this master problem solution an increased profit, i.e.,

$$\begin{aligned} &\rho_a^{\text{second}}(\tilde{y}, \tilde{z}, \tilde{w}(\tilde{y}, \tilde{z}), \tilde{d}(\tilde{y}, \tilde{z})) \\ &< \rho_a^{\text{second}}(y_a^*, z_a^*, \tilde{y}_{-a}, \tilde{z}_{-a}, \tilde{w}(y_a^*, z_a^*, \tilde{y}_{-a}, \tilde{z}_{-a}), \tilde{d}(y_a^*, z_a^*, \tilde{y}_{-a}, \tilde{z}_{-a})). \end{aligned} \tag{29}$$

Obviously, the current master problem solution $(\tilde{x}, \tilde{\phi}, \tilde{y}, \tilde{z}, \tilde{w}, \tilde{d})$ violates the identified Nash inequality.

In direct consequence, if we add the following inequality and a corresponding KKT system to the next master problem, the current infeasible solution $(\tilde{x}, \tilde{\phi}, \tilde{y}, \tilde{z}, \tilde{w}, \tilde{d})$ will be cut-off:

$$\begin{aligned} &\rho_a^{\text{second}}(y, z, w(y, z), d(y, z)) \\ &\geq \rho_a^{\text{second}}(y_a^*, z_a^*, y_{-a}, z_{-a}, w(y_a^*, z_a^*, y_{-a}, z_{-a}), d(y_a^*, z_a^*, y_{-a}, z_{-a})). \end{aligned} \tag{30}$$

□

With these results we can now prove that Algorithm 1 is correct, i.e., it terminates after finitely many iterations with a solution that is optimal. We have the following theorem:

Theorem 4. *Let $|P_a| < \infty$ and let all investment variables x, y as well as airport charges ϕ have a finite upper bound. Then, Algorithm 1 terminates after a finite number of iterations with an optimal solution to the four-level market model.*

Proof. We first note that (i) under the above assumptions only finitely many discrete strategies exist and that (ii) according to Theorem 3 in each iteration the current infeasible strategy is cut-off. If we keep adding cuts as indicated by the respective subproblem, we further restrict the constraint set of the master problem and Algorithm 1 eventually converges to a feasible solution after a maximum number of sub- and master problems equal to the number of discrete strategies.

In order to show optimality, we consider the solution of the final master problem $(x^*, \phi^*, y^*, z^*, w^*, d^*)$. As Algorithm 1 terminates with no unilateral, profit-enhancing deviation strategy for all airlines, the found solution $(y^*, z^*, w^*(y^*, z^*), d^*(y^*, z^*))$ directly forms a feasible Nash equilibrium according to Theorem 2.d On the other hand, the solution maximizes the objective at the first level, which proves optimality. □

7. Case Study

7.1. **Test Network.** In this section we present a six-airport network that is linked by different flight connections. The test network was originally introduced by Dobson and Lederer 1993 and is used to illustrate the relevant economic effects in an intuitive and simple—but also rich enough—example.
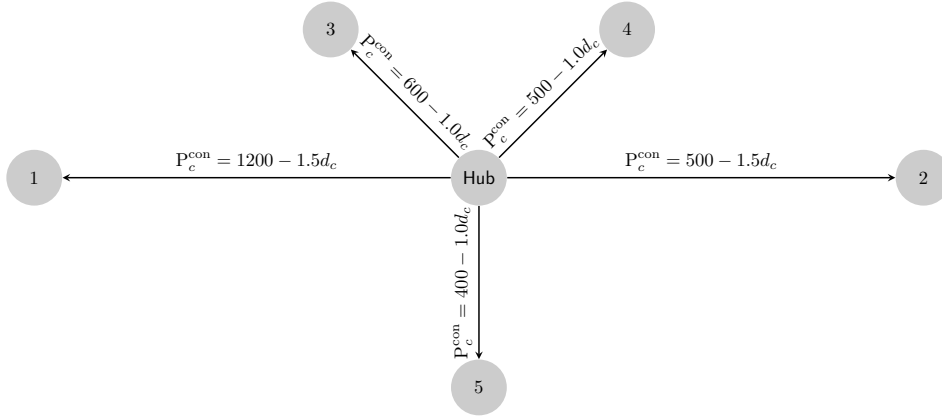
FIGURE 7. Hub-and-Spoke Topology with Demand Functions

As can be seen in Figure 7, the six airports are divided into the hub airport H and the destination airports 1 to 5. Let us assume that there are no existing runway capacities at the beginning of the planning horizon, which may alternatively be interpreted as an analysis of the residual flight demand that can not be covered by existing runway capacities. We consider two testsets $\mathcal{A}$ and $\mathcal{B}$, which only differ in the investment cost i^{airport} of an additional runway. In particular, we will use the analysis of airport investment cost in order to model and evaluate some kind of public subsidy program; see also Section 7.3. For $\mathcal{A}$ the corresponding values for the hub H and the airports 1–5 amount to \$20,000 and \$10,000, respectively. For $\mathcal{B}$ these values are halved, i.e., we consider a subsidy that yields investment cost that decline by 50 percent. Furthermore, the intercept $\alpha^{\mathrm{airport}}$ and the slope β^{airport} of the variable cost function for all airports amount to \$5000 and \$5, respectively. In order to keep our example (and the corresponding scheduling decisions) as simple as possible, we only consider five connections with the corresponding takeoffs taking place in subsequent periods. In particular, the connections from the hub H to airports 1 and 2 are rather long-haul flights with three flight periods, while the connections with destination 3, 4, and 5 are short-haul flights with only one flight period. Therefore, in our test example we consider a total planning horizon of six periods. In addition, we assume that connections $(H, 1)$ and $(H, 3)$ are the two high-demand connections of the network. Note that the five considered connections may equivalently be interpreted as return-trips without affecting our main analysis. As can be seen in Figure 7, all demand functions used are linear, which implies that consumer surplus is a concave function.

In analogy to runway capacities, we assume that there are no existing aircraft available. We consider two possible candidate aircraft with a seat capacity $\kappa^{\mathrm{aircraft}}$ of 300 and 600, respectively. Investment cost i^{aircraft} of the "small" and "large" aircraft type amount to \$10,000 and \$20,000. All five connections may possibly be served by each of the two aircraft. Moreover, the intercept $\alpha^{\mathrm{airline}}$ of the variable cost function of the "small" aircraft is assumed to be \$22,500 for the long-haul flights and \$7200 for the short-haul flights. For the "large" aircraft these values are given by \$45,000 and \$22,500, respectively. The slope β^{airline} of the variable cost function for the long-haul flights and the short-haul flights amounts to \$25 and \$8, respectively. [3]

---

[3]In order to ensure a unique solution, we slightly petabyte these cost coefficients in the case of several airlines.

TABLE 1. Results for airports N. Passenger-based charge (ϕ, in \$), runway capacity extension (x), airport investment cost (I, in \$), and airport variable cost (V, in \$) for different models (integrated planner, monopoly, duopoly) and testsets ($\mathcal{A}, \mathcal{B}$)

		integrated planner				monopoly				duopoly			
	N	ϕ	x	I	V	ϕ	x	I	V	ϕ	x	I	V
$\mathcal{A}$	H	—	1	20,000	35,410	44	1	20,000	19,500	35	1	20,000	21,000
	1	—	1	10,000	8000	55	1	10,000	6500	30	1	10,000	8000
	2	—	1	10,000	6500	0	0	0	0	0	0	0	0
	3	—	1	10,000	7910	55	1	10,000	6500	55	1	10,000	6500
	4	—	1	10,000	6500	55	1	10,000	6500	55	1	10,000	6500
	5	—	1	10,000	6500	0	0	0	0	0	0	0	0
$\mathcal{B}$	H	—	1	10,000	35,410	32	1	10,000	25,595	25	1	10,000	27,690
	1	—	1	5000	8000	39	1	5000	6500	22	1	5000	8000
	2	—	1	5000	6500	0	0	0	0	0	0	0	0
	3	—	1	5000	7910	39	1	5000	6500	29	1	5000	7095
	4	—	1	5000	6500	39	1	5000	6500	39	1	5000	6500
	5	—	1	5000	6500	51	1	5000	6095	51	1	5000	6096

For the discussed test network we will present the welfare optimum of the integrated planner problem as well as the market outcomes of our four-level market model. In particular, for our four-level model we analyze the effects of different degrees of competition among airlines, where we consider both a monopoly airline and a duopoly.

Note that for the case of linear demand functions, our integrated planner model reduces to a mixed-integer problem with linear constraints and a concave objective, while the four-level market model is inherent nonconvex. Therefore, we modeled and solved our problem instances with SCIP 3.2.1; see Gamrath et al. 2016. Using our decomposition Algorithm 1, all instances of our four-level market model were solved in only a few iterations and seconds. All computations have been performed on an Intel© Core™i5-3360M CPU with 4 cores and 2.8 GHz each and 4 GB RAM.

7.2. **Testset $\mathcal{A}$.** An integrated planner extends the capacity of all airports; see Table 1. Moreover, investment in five aircraft that serve all available connections takes place; see Table 2 and Table 3. Two large aircraft are scheduled on the high-demand connections (H, 1) and (H, 3). In addition, on the three remaining connections only small aircraft are used. Overall welfare amounts to \$553,262, which can be used as a planning benchmark in order to assess possible market inefficiencies; see Table 4.

We now discuss the results of our four-level market model, which are summarized in Tables 1, 2, 3, and 4. We first assume a single firm, i.e., a monopolist, that invests in its fleet and schedules its aircraft on the second and third level. As expected, the monopolist reduces ticket supply in order to increase its profits. In particular, the monopolist only invests in three small aircraft that are scheduled on connections (H, 1), (H, 3), and (H, 4). Given this supply shortage, welfare reduces to \$364,300 as compared to the reference planning solution. Profits of the monopolist amount to \$206,700.

In a second step, we assume a duopoly, i.e., we consider two competing airlines on level two and level three. In particular, we assume (i) a "large" airline that—according to its assumed airline-specific fleet portfolio—can invest in the large aircraft type as well as (ii) a "small" airline that can only invest in the small aircraft type,

TABLE 2. Results for airline(s) A. New aircraft ordered (y^{300}, y^{600}), airline investment cost (I, in \$), and airline variable cost (V, in \$) for different models (above: integrated planner, monopoly; below: duopoly) and testsets $(\mathcal{A}, \mathcal{B})$

	A	y^{300}	y^{600}	I	V	y^{300}	y^{600}	I	V
			integrated planner				monopoly		
$\mathcal{A}$	—	3	2	70,000	128,256	3	—	30,000	49,200
$\mathcal{B}$	—	3	2	70,000	128,256	4	—	40,000	58,152

	A	y^{300}	y^{600}	I	V
			duopoly		
$\mathcal{A}$	small	2	—	20,000	19,200
	large	—	1	20,000	60,000
$\mathcal{B}$	small	2	—	20,000	18,552
	large	—	2	40,000	77,752

TABLE 3. Results for connections C. Number of passengers (w), scheduled aircraft capacity (κ), and price (P, in \$) for different models (integrated planner, monopoly, duopoly) and testsets $(\mathcal{A}, \mathcal{B})$

		integrated planner		monopoly		duopoly		
						Airline large	Airline small	
	C	$w(\kappa)$	P	$w(\kappa)$	P	$w(\kappa)$	$w(\kappa)$	P
$\mathcal{A}$	(H, 1)	600(600)	300	300(300)	750	600(600)	0(0)	300
	(H, 2)	300(300)	50	0(0)	500	0(0)	0(0)	500
	(H, 3)	582(600)	18	300(300)	300	0(0)	300(300)	300
	(H, 4)	300(300)	200	300(300)	200	0(0)	300(300)	200
	(H, 5)	300(300)	100	0(0)	400	0(0)	0(0)	400
$\mathcal{B}$	(H, 1)	600(600)	300	300(300)	750	600(600)	0(0)	300
	(H, 2)	300(300)	50	0(0)	500	0(0)	0(0)	500
	(H, 3)	582(600)	18	300(300)	300	419(600)	0(0)	181
	(H, 4)	300(300)	200	300(300)	200	0(0)	300(300)	200
	(H, 5)	300(300)	100	219(300)	181	0(0)	219(300)	181

TABLE 4. Welfare (in \$), (net) consumer surplus (in \$), and the sum of airline/airport profits (in \$) for different models (integrated planner, monopoly, duopoly) and testsets $(\mathcal{A}, \mathcal{B})$

		integrated planner	monopoly	duopoly
$\mathcal{A}$	welfare ω	553,262	364,300	478,800
	(net) consumer surplus	—	157,500	360,000
	$\sum$ airport profits	—	100	1000
	$\sum$ airline profits	—	206,700	117,800
$\mathcal{B}$	welfare ω	588,262	416,778	540,555
	(net) consumer surplus	—	365,891	426,761
	$\sum$ airport profits	—	887	1290
	$\sum$ airline profits	—	234,410	112,504

respectively. As expected, the increased competition yields a welfare increase to $478,800 as compared to the monopoly solution, which underlines the importance of adequate competition policy regulations; see also the vast literature on competition and competition policy in the airline industry, e.g., Borenstein and Rose 1994, Mazzeo 2003, or Silva et al. 2014. The observed welfare gain is mainly driven by the fact that the large airline schedules one large aircraft on connection $(H, 1)$, which increases the seat capacity on this connection by 300. In addition, we can observe a reduction of airline profits to $117,800 relative to the monopoly, whereas (net) consumer rents move in the opposite direction.

We also note that in line with the above observations, airport charges at the hub and at airport 1 decrease in the case of a duopoly as compared to the monopolistic case, since corresponding airport investment and operational costs can be carried by an increased number of passengers on the respective connections. In addition, given efficient ticket prices under the integrated planner problem, our hierarchical market model will in general distort the optimal price structure. In particular, in our example ticket prices under both the duopoly and the monopoly will be inefficiently high, which directly imply an underinvestment in aircraft and airport capacity; see Tables 1 and 2. In line with the current policy debate on the bankruptcy of the German carrier AirBerlin, prices tend to increase if we reduce the number of airlines, i.e., if we move from a duopoly to a monopoly market environment.

We finally note that our model inherently allows airlines to identify an optimal strategy for a given investment and scheduling decision of its competitor(s). Obviously, if we fix the decision of the small airline to its discussed equilibrium strategy, by definition we will arrive at the same equilibrium decision for the large airline as described above. However, we may also fix the strategy of the small airline to any other (suboptimal) strategy and evaluate an optimal reaction of the large airline. Therefore, our model may not only be used for policy analyses, but can also serve as a valuable tool for business support.

7.3. **Testset $\mathcal{B}$.** Let us consider the case, where the regulator decides to implement some kind of subsidy program that lowers overall investment costs of airports. In particular, in this section we assume that all investment costs of airports are halved by the public subsidy. Such a program may aim at enhancing airport extension activities in order to being able to increase the degree to which growing flight demand can be satisfied. In other words, public subsidies try to heal inefficiently low investments of welfare-maximizing airports in a deregulated market environment; for a discussion see also A. Zhang and Y. Zhang 2006.

As expected, the lowered investment costs directly increase welfare under all model variants. Since under the integrated planner solution all five connections are served already in the standard case (see testset $\mathcal{A}$), we arrive at identical scheduling and investment decisions. However, the monopolistic airline now also serves connection $(H, 5)$ by investing in an additional small aircraft.

In the duopoly case, it is now profitable for the large airline to invest in two large aircraft that are scheduled on connections $(H, 1)$ and $(H, 3)$. Accordingly, the small airline invests in two small aircraft that serve connections $(H, 4)$ and $(H, 5)$. As the number of sold tickets increases for both the monopolist and the duopoly as compared to testset $\mathcal{A}$, we also observe a rise in overall consumer surplus.

Like in the original testset $\mathcal{A}$, the integrated planner solution yields a benchmark with the highest welfare of $588,262. Again, the monopolist has the lowest welfare level with a value of $416,778, with the duopolist solution ($540,555) lying between the two latter cases.

As discussed above, in the current example the implemented subsidy program yields an increased welfare for all three model variants as compared to the no-subsidy case, i.e., the subsidy program enhances efficiency from an overall welfare perspective. Since this may not always be true in a general context, our model may therefore provide a valuable tool for a quantitative assessment of such subsidy programs.

8. Conclusion

In this paper we present a four-level market model that accounts for different economic agents including airport operators, airlines, and passengers. In particular, we assume airports that choose their optimal runway capacity expansion as well as an optimal airport charge on the first level. On the second level investment in new aircraft takes place by airlines. Optimal investment behavior of airlines on the second level accounts for their expected optimal aircraft scheduling and on corresponding revenues from ticket trade on the third and fourth level, respectively.

We demonstrate that our market model may result in inefficient investments as compared to an integrated single-level benchmark model. We also identify policy regulations that may reduce such investment inefficiencies. In this context, our results call for a careful economic assessment of (future) market structures and regulations, as otherwise investment incentives for airports or airlines may be aligned in a way that yields severe market inefficiencies in the long-run. To this end, our model may be seen as a valuable tool to evaluate and assess different policy options on a quantitative basis.

Acknowledgments

Appendix A. Parameters, Sets, and Variables

In this appendix we summarize main parameters, sets, and variables.

Table 5. Sets

Symbol	Description
T	Set of all time periods
N	Set of all airports
A	Set of all airlines
P	Set of all aircraft or plane types
$P_a \subset P$	Set of aircraft types of airline a
C	Set of all connections
$C_a \subset C$	Set of connections of airline a

TABLE 6. Variables and derived quantities

Symbol	Description	Unit
x_n	Runway capacity extension at airport n	$\mathbb{N}$
ϕ_n	Passenger-based charge of airport n	\$
y_{ap}	Number of new aircraft of type p ordered by airline a	$\mathbb{N}$
z_{apc}	Number of aircraft of type p scheduled on connection c by a	$\mathbb{N}$
d_c	Ticket demand on connection c	$\mathbb{R}_{\geq 0}$
$\mathrm{P}_c^{\mathrm{con}}$	Ticket price or inverse demand function for connection c	\$
w_{ac}	Ticket supply of airline a on connection c	$\mathbb{R}_{\geq 0}$
$\mathrm{V}_n^{\mathrm{airport}}$	Variable cost function of airport n	\$
$\mathrm{I}_n^{\mathrm{airport}}$	Investment cost function of airport n	\$
$\mathrm{R}_{na}^{\mathrm{airport}}$	Revenues of airport n paid by a	\$
$\mathrm{V}_a^{\mathrm{airline}}$	Variable cost function of airline a	\$
$\mathrm{I}_a^{\mathrm{airline}}$	Investment cost function of airline n	\$
$\mathrm{R}_a^{\mathrm{airline}}$	Revenues of airline a	\$

TABLE 7. Parameters

Symbol	Description	Unit
$\kappa_n^{\mathrm{airport}}$	Existing runway capacity of airport n	$\mathbb{N}$
i_n^{airport}	Slope of investment cost function of airport n	\$
$\alpha_n^{\mathrm{airport}}$	Intercept of variable cost function of airport n	\$
$\beta_n^{\mathrm{airport}}$	Slope of variable cost function of airport n	\$
$e_{ap}^{\mathrm{airline}}$	Number of existing aircafts of type p owned by airline a	$\mathbb{N}$
$\alpha_{apc}^{\mathrm{airline}}$	Intercept of variable cost function of airline a	\$
$\beta_{ap}^{\mathrm{airline}}$	Slope of variable cost function of airline a	\$
$\kappa_p^{\mathrm{aircraft}}$	Seat capacity of aircraft type p	$\mathbb{N}$
i_p^{aircraft}	Slope of investment cost function of aircraft p	\$

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