Integrated Generator Maintenance and Operations Scheduling under Uncertain Failure Times

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Abstract

Planning maintenances and operations is an important concern in power systems. Although optimization based joint maintenance and operations scheduling is studied in the literature, sudden disruptions due to random generator failures are not considered. In this paper we propose a stochastic mixed-integer programming approach for integrated condition-based maintenance and operations scheduling problem for a fleet of generators with explicit consideration of unexpected failures. The proposed stochastic program is based on failure scenarios derived from the remaining lifetime distributions of the generators, as well a chance constraint to ensure a reliable maintenance plan. We propose a deterministic safe approximation of the difficult chance constraint. The huge number of failure scenarios are handled by a combination of sample average approximation and an enhanced scenario decomposition algorithm in a distributed framework. We introduce a number of algorithmic improvements by exploiting the polyhedral structure of the problem, utilizing its time decomposability and an analysis of the transmission line capacities. Finally, we present a case study demonstrating the signficant computational benefits of our solution approach, and the importance of considering unexpected generator failures.

1 Introduction

Power systems operate under the premise of uncompromising generation capacity to satisfy the operational requirements of the network. Generator maintenance scheduling plays a pivotal role in this regard as it determines both the availability and reliability of the generation resources. Traditionally, utility companies have used different strategies to mitigate the effects of unexpected failures ranging from combinations of supplemental reserves to frequent maintenance intervals. Increased congestion coupled with an aging power grid infrastructure have generated

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new challenges that cannot be efficiently addressed by conventional solutions. Recent research efforts have focused on leveraging condition monitoring data to predict remaining life of generation assets and optimize maintenance schedules [16, 19]. Condition monitoring is the process of using sensor data to determine equipment health. Although condition monitoring based methods are very effective in preventing unexpected failures, they rely on proxy cost functions derived from remaining life prediction without explicit models of failure times. In this paper, we propose a stochastic mixed-integer program that uses condition monitoring data to identify the optimal maintenance and operational decisions yet considers the impact of extensive generator failure scenarios.

Generation maintenance scheduling has been studied extensively in the literature. The studies on this topic can be broadly classified into two groups, i) models that revolve mostly around maintenance optimization ([1,4,19]), and ii) models that integrate maintenance with operations decisions (cf. [7,8,10,12,13,15,16]). Recently, [16] proposed a framework that uses sensor data to schedule maintenance and operations. However, the underlying assumption of the optimization model is that generators do not experience sudden failures as long as they are being monitored. In contrast, this paper considers failure instances using a stochastic programming framework with failure scenarios and chance constraints.

Stochastic programming approaches have been widely used for optimizing operational problems, see [20] for a recent review. For instance, [6,17,18] consider uncertainty at the operational level and take into account a limited number of scenarios with a focus on demand uncertainty and different pricing strategies. However, there has been little to no emphasis on failure uncertainty or maintenance optimization much less on joint maintenance-operations planning under uncertainty. In this paper, we develop and solve a joint maintenance and operations planning problem with explicit consideration of unexpected generator failures. We model the condition of the generators using a degradation-based modeling framework similar to the one proposed in [9, 11], which uses condition monitoring sensor data to predict residual lifetime. We formulate the integrated problem as a stochastic mixed-integer program with failure scenarios, and introduce a chance constraint for ensuring a reliable maintenance plan using degradation-based residual life distributions (RLDs) to model the effects of random generator failures. The contributions of this paper are summarized as follows:

1. We propose a novel two-stage stochastic mixed-integer program to model the joint maintenance and operations scheduling problem with both reliability and cost perspectives. The proposed model differs from existing studies in that:

    (a) We consider the effects of unexpected generator failures on maintenance and operations planning through random failure scenarios that are generated by degradation-based generator RLDs.

    (b) We introduce a degradation-based chance constraint to restrict the number of corrective maintenance routines (CMs).

2. We adapt the scenario decomposition approach proposed by [2] to solve the stochastic program. We introduce a number of algorithmic improvements to this approach:

   (a) We develop a deterministic safe approximation for the proposed chance-constraint.

   (b) We exploit the polyhedral structure of the model to derive an alternative cut generation procedure.

   (c) We propose further enhancements to improve the computational performance of our algorithm, such as i) utilizing the time decomposability of the algorithm to minimize the number of resolves, ii) pre-screening transmission lines that never violate the transmission line constraints, and iii) implementing the proposed algorithm in a distributed fashion.

3. We conduct computational studies of the proposed approach using three IEEE test cases [3, 5, 21]. We demonstrate the computational performance of our enhanced algorithm relative to its generic version. Using sample average approximation method, we construct confidence intervals to the true objective and obtain solutions within 2% optimality gap. Moreover, our computational experiments highlight a 7-19% cost savings using the proposed approach relative to deterministic solutions.

The paper is organized as follows: Section 2 discusses the integrated condition-based generator maintenance and operations planning problem. It presents an overview of the degradation modeling framework, the scenario generation procedure, and the joint optimization model. Section 3 introduces the scenario decomposition algorithm along with our methodological enhancements. Section 4 presents the computational experiments and results. Section 5 is the conclusion.

2 Model Formulation

2.1 Degradation Modeling and Scenario Generation

Degradation is the progressive accumulation of damage due to natural wear and tear. Generators like many mechanical equipment and machinery undergo progressive degradation over time. In many cases, manifestations of physical degradation can be indirectly monitored through sensors (condition monitoring). We adopt the degradation modeling framework presented in [19] where sensor data is utilized to predict the RLD of each generator. Generator RLDs are then transformed into maintenance cost functions that are dynamically updated using a Bayesian framework as new sensor data is observed from the field. The maintenance cost function, expressed by equation (1), balances the costs of premature maintenance with the risk of unexpected generator failure and is given,

$$C_{i,t}^{t_i^o} = \frac{c_i^p \Pr(R_i^{t_i^o} > t) + c_i^c \Pr(R_i^{t_i^o} \leq t)}{\int_0^t \Pr(R_i^{t_i^o} > z)dz \ + t_i^o}, \tag{1}$$

where $C_{i,t}^{t_i^o}$ is the maintenance cost of generator i after t maintenance epochs from the observation time t_i^o, $R_i^{t_i^o}$ is a random variable denoting the residual life of generator i at the observation time t_i^0, $\Pr(R_i^{t_i^o} > t)$ is the degradation-based cumulative distribution function of $R_i^{t_i^o}$, namely F_i, c_i^p is the cost of preventive maintenance (PM) and c_i^c is the cost of CM.

Next, we consider the generation of failure scenarios. We use the degradation-based RLDs of the generators to determine the probability π_k associated with each failure scenario k. To arrive at a finite set of scenarios, a planning horizon of length H is discretized into d time intervals each corresponding to a possible failure period. The probability that generator i fails in the range j is calculated for every generator i and range $j = 1, \cdots, d+1$, using F_i. An additional period is added to account for generator non-failures, i.e., a generator does not fail within the planning horizon. The probability that generator i does not fail is given by $\Pr(R_i^{t_i^o} > H)$, which corresponds to the probability that the residual lifetime of generator i is at least H periods.

A scenario consists of failure times corresponding to each generator. In the extensive scenario generation case, all possible scenarios are considered resulting in $(d+1)^{|\mathcal{G}|}$ many scenarios. Hence, every generator has an assigned failure period for any given scenario k. We denote the failure time of generator i in scenario k by τ_i^k, and assume that failure occurs in the middle of the selected failure period. For non-failures, τ_i^k is set to $H+1$.

2.2 Optimization Model

In this section, we formulate the integrated generator maintenance and operations scheduling problem as a stochastic mixed integer program by considering unexpected generator failure scenarios. The model aims to minimize the degradation-based expected maintenance and operational costs while determining maintenance schedules, and operational decisions related with commitment and dispatch amounts specific to each scenario. We consider this problem over a set of generators with a finite planning horizon. The planning horizon consists of maintenance epochs for determining the maintenance schedule, and each maintenance epoch contains operational subperiods to determine commitment decisions, dispatch and demand curtailment amounts. We consider a one-year planning horizon with 12 maintenance epochs corresponding to monthly maintenance decisions where each maintenance epoch consists of 30 days corresponding for operational level decisions. We assume that each generator undergoes a single maintenance routine, preventive or corrective, within the planning horizon. Generators cannot produce electricity while under maintenance. A CM is conducted on the spot if a generator fails before its scheduled maintenance time. In that case, the maintenance takes Y_c periods and costs C_c.

Otherwise, PM is carried out at its scheduled time, which lasts Y_p periods and costs $C_{i,t}$. The cost $C_{i,t}$ is calculated using equation (1). Here, we note that $Y_c > Y_p$ since CM is conducted under emergency conditions that typically require unplanned dispatch of maintenance resources.

The decision variables of the optimization model are z, x, y, ψ, γ. The binary variable, $z_{i,t}$, is 1 if the maintenance of the generator i starts at maintenance epoch t, and 0 otherwise. The variables, $x^k_{i,t,s}$ and $y^k_{i,t,s}$, correspond to the commitment and dispatch decisions, respectively, for generator i in operational period s within maintenance epoch t in scenario k. The variable $\psi^k_{b,t,s}$ is the demand curtailment amount in operational period s within maintenance epoch t at demand bus b in scenario k, and the variable γ^k_t is the additional labor required for PM at maintenance epoch t in scenario k.

The sets $\mathcal{G}$, $\mathcal{B}$, $\mathcal{L}$, $\mathcal{T}$, $\mathcal{S}$, $\mathcal{K}$ denote sets of generators, buses, lines, maintenance epochs, operational time periods within a maintenance epoch, and scenarios respectively. The model parameters are as follows: $V_{i,t,s}$ and $F_{i,t,s}$ are no-load cost and per unit dispatch cost of generator i in operational period s within maintenance epoch t; P_{DC} is demand curtailment cost per unit demand; C_{add} is the cost of additional labor for conducting preventive maintenance; L is the maintenance capacity on the number of ongoing PMs; p_i^{max} and p_i^{min} are maximum and minimum production levels of generator i; $D_{b,t,s}$ is the demand of bus b within maintenance epoch t in operational period s; a_l is the shift factor vector for line l, and $M_{b,i}$ is the generation location matrix, which is 1 if generator i is on bus b, and 0 otherwise. $\zeta_{i,t}$ is a random variable that is 1 if $t \geq \tau_i$ and 0 otherwise, where the distribution of τ_i is given by the RLD of generator i. As mentioned earlier (in Section II-A), the parameters $C_{i,t}$ and π_k are the dynamic maintenance cost and scenario probability. The formulation of our optimization problem is outlined below:

$$\begin{aligned}
\min \;& \sum_{k\in\mathcal{K}} \pi_k \left(\sum_{i\in\mathcal{G}} \sum_{t=1}^{\tau_i^k-1} C_{i,t} z_{i,t} + \sum_{i\in\mathcal{G}} \sum_{t=\tau_i^k}^{H} C_c z_{i,t} \right) \\
& + \sum_{k\in\mathcal{K}} \sum_{i\in\mathcal{G}} \sum_{t\in\mathcal{T}} \sum_{s\in\mathcal{S}} \pi_k \left(V_{i,t,s} x^k_{i,t,s} + F_{i,t,s} y^k_{i,t,s} \right) \\
& + \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} \sum_{s\in\mathcal{S}} \sum_{b\in\mathcal{B}} \pi_k P_{DC} \psi^k_{b,t,s} + \sum_{k\in\mathcal{K}} \sum_{t\in\mathcal{T}} \pi_k C_{add} \gamma^k_t && \text{(2a)}
\end{aligned}$$

$$\text{s.t. } \Pr\left(\sum_{i\in\mathcal{G}} \sum_{t\in\mathcal{T}} \zeta_{i,t} z_{i,t} \leq \rho \right) \geq 1 - \epsilon \qquad \text{(2b)}$$

$$\sum_{\substack{i\in\mathcal{G}: \\ t\leq\tau_i^k-1}} \sum_{e=0}^{Y_p-1} z_{i,t-e} \leq L + \gamma^k_t \qquad t\in\mathcal{T}, k\in\mathcal{K} \qquad \text{(2c)}$$

$$\sum_{t\in\mathcal{T}} z_{i,t} = 1 \qquad i\in\mathcal{G} \qquad \text{(2d)}$$

$$x^k_{i,t,s} \leq 1 - \sum_{e=0}^{Y_p-1} z_{i,t-e}$$

$$i \in \mathcal{G}, t \in \{1, \ldots, \tau_i^k + Y_p - 1\}, s \in \mathcal{S}, k \in \mathcal{K} \tag{2e}$$

$$x_{i,t,s}^k \leq \sum_{t'=1}^{\tau_i^k - 1} z_{i,t'}$$

$$i \in \mathcal{G}, t \in \{\tau_i^k, \ldots, \tau_i^k + Y_c - 1\}, s \in \mathcal{S}, k \in \mathcal{K} \tag{2f}$$

$$\sum_{i \in \mathcal{G}} y_{i,t,s}^k + \sum_{b \in \mathcal{B}} \psi_{b,t,s}^k = \sum_{b \in \mathcal{B}} D_{b,t,s}$$

$$t \in \mathcal{T}, s \in \mathcal{S}, k \in \mathcal{K} \tag{2g}$$

$$p_i^{min} x_{i,t,s}^k \leq y_{i,t,s}^k \leq p_i^{max} x_{i,t,s}^k$$

$$i \in \mathcal{G}, t \in \mathcal{T}, s \in \mathcal{S}, k \in \mathcal{K} \tag{2h}$$

$$\left| \sum_{b \in \mathcal{B}} a_{l,b} \left(\sum_{i \in \mathcal{G}} M_{b,i} y_{i,t,s}^k + \psi_{b,t,s}^k - D_{b,t,s} \right) \right| \leq f_l^{max}$$

$$t \in \mathcal{T}, s \in \mathcal{S}, k \in \mathcal{K}, l \in \mathcal{L} \tag{2i}$$

$$z_{i,t}, x_{i,t,s}^k \in \{0, 1\}, \gamma_t^k, y_{i,t,s}^k \geq 0, D_{b,t,s} \geq \psi_{b,t,s}^k \geq 0$$

$$i \in \mathcal{G}, t \in \mathcal{T}, s \in \mathcal{S}, k \in \mathcal{K}, b \in \mathcal{B}. \tag{2j}$$

The objective function (2a) consists of four parts. The first part corresponds to the expected maintenance cost. If the maintenance of generator i is scheduled before its failure time in scenario k, then "a PM cost derived from" the dynamic cost function is incurred, otherwise the cost of CM is incurred. The second and third parts of the objective function represent the expected operational cost where commitment, dispatch, and demand curtailment costs are taken into account. The last part corresponds to the expected cost of additional PMs. Normally, L many PMs are allowed in each period t, and additional PMs are penalized with the cost, C_{add}.

Constraint (2b) is a chance constraint which ensures that the number of generators that enter CM should be less than a predefined threshold ρ with high probability $1 - \epsilon$. We derive safe approximations of this intractable constraint in Section 2.2.1. Constraint (2c) assures that there can be at most $L + lb_t^k$ ongoing PMs for each period t and scenario k, and constraint (2d) ensures that one maintenance must be scheduled within the planning horizon for every generator.

Constraints (2e) and (2f) impose the logical relationship between a generator's failure at a specific scenario and the maintenance decision. If a maintenance $z_{i,t}$ is scheduled before the time of failure τ_i^k of generator i, then the generator enters PM at its scheduled time. In that case, constraint (2e) guarantees that generator i is under PM, and hence unavailable during the time interval $[z_{i,t}, z_{i,t} + Y_p - 1]$ of scenario k. However, if there is no maintenance scheduled for generator i before its failure time τ_i^k, then the generator fails at τ_i^k as determined by the scenario k. In this case, constraint (2f) guarantees that generator i is under a CM, and is unavailable during the time interval $[\tau_i^k, \tau_i^k + Y_c - 1]$ of scenario k.

Constraint (2g) ensures that demand is satisfied by the total generation output and demand curtailment during each operational period, and constraint (2h) restricts the dispatch of each generator to its corresponding minimum and maximum production amounts. Constraint (2i) corresponds to the linearized power flow equations using shift factors for restricting the maximum flow allowed in each transmission line.

2.2.1 Safe approximations of the chance constraint (2b)

Here we propose a deterministic safe approximation of the chance constraint (2b). Note that we can equivalently write (2b) as $\Pr\left(\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}}\zeta_{i,t}z_{i,t}\geq\rho\right)\leq\epsilon$. Therefore, it suffices to find upper bounds on the probability expression on left hand side of this inequality to obtain a safe approximation. We use Markov and generalized Bernstein inequalities towards this.

Proposition 1. *The deterministic linear constraint*

$$\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}}\mathbf{E}[\zeta_{i,t}]\,z_{i,t}\leq\max\left(\rho\,\epsilon,\max_{\alpha>0}\left[\frac{((\epsilon\,e^{\alpha\rho})^{1/|\mathcal{G}|}-1)|\mathcal{G}|}{e^{\alpha}-1}\right]\right)$$

is a safe approximation of (2b)*, i.e. any* $z\in\{0,1\}^{|\mathcal{T}|\times|\mathcal{G}|}$ *satisfying the above inequality, satisfies* (2b).

Proof. We prove the result in two parts. First, using Markov's inequality and linearity of expectation, we have:

$$\begin{aligned}\Pr(\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}}\zeta_{i,t}z_{i,t}\geq\rho)&\leq\mathbf{E}[\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}}\zeta_{i,t}z_{i,t}]/\rho\\&=\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}}\mathbf{E}[\zeta_{i,t}]z_{i,t}/\rho.\end{aligned}$$

Hence, $\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}}\mathbf{E}[\zeta_{i,t}]\,z_{i,t}/\rho\leq\ \epsilon$ provides a safe approximation of the constraint (2b).

Second, let $G_i(z)=\sum_{t\in\mathcal{T}}\zeta_{i,t}z_{i,t}$ be a random variable depending on generator i and a maintenance decision z. Since $\sum_{t\in\mathcal{T}}z_{i,t}=1$ due to constraint (2d), $G_i(z)$ is a Bernoulli random variable, which equals to 1 with probability $p_i(z)$, and 0 otherwise. Here, $p_i(z)$ represents the probability that the generator i fails under the solution z. Thus, $p_i(z)=\sum_{t\in\mathcal{T}}\Pr(t\geq\tau_i)\,z_{i,t}$ and $\Pr\left(\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}}\zeta_{i,t}z_{i,t}\geq\rho\right)$ can be rewritten as $\Pr\left(\sum_{i\in\mathcal{G}}G_i(z)\geq\rho\right)$. Note that given a maintenance decision z, $G_i(z)$ for each generator i are independently distributed.

For any $\alpha>0$, we have

$$\begin{aligned}\Pr\left(\sum_{i\in\mathcal{G}}G_i(z)\geq\rho\right)&=\Pr\left(e^{\alpha\,\sum_{i\in\mathcal{G}}G_i(z)}\geq e^{\alpha\rho}\right)\\&\leq\frac{\mathbf{E}[e^{\alpha\,\sum_{i\in\mathcal{G}}G_i(z)}]}{e^{\alpha\rho}}\end{aligned}$$

$$
\begin{aligned}
&= \frac{\mathbf{E}[\prod_{i\in\mathcal{G}} e^{\alpha\, G_i(z)}]}{e^{\alpha\rho}} \\
&= \frac{\prod_{i\in\mathcal{G}} \mathbf{E}[e^{\alpha\, G_i(z)}]}{e^{\alpha\rho}} \\
&= \frac{\prod_{i\in\mathcal{G}}[p_i(z)\, e^{\alpha} + (1-p_i(z))]}{e^{\alpha\rho}} \\
&\leq \frac{[p(z)\, e^{\alpha} + (1-p(z))]^{|\mathcal{G}|}}{e^{\alpha\rho}},
\end{aligned}
$$

where $p(z) = \frac{\sum_{i\in\mathcal{G}} p_i(z)}{|\mathcal{G}|}$.

The first inequality follows from Markov's inequality, and the last inequality follows from the geometric-arithmetic means inequality. By upper bounding the resulting expression by ϵ, we obtain a safe approximation $\frac{[p(z)\, e^{\alpha}+(1-p(z))]^{|\mathcal{G}|}}{e^{\alpha\rho}} \leq \epsilon$ for the constraint (2b). By plugging in the value of $p(z) = \frac{\sum_{i\in\mathcal{G}} p_i(z)}{|\mathcal{G}|} = \frac{\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}} \Pr(t\geq\tau_i)\, z_{i,t}}{|\mathcal{G}|}$ and after some algebra, we have

$$
\sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}} \Pr(t\geq\tau_i)\, z_{i,t} \leq \frac{((\epsilon\, e^{\alpha\rho})^{1/|\mathcal{G}|}-1)|\mathcal{G}|}{e^{\alpha}-1}.
$$

Since $\mathbf{E}[\zeta_{i,t}] = \Pr(t \geq \tau_i)$, we obtain the desired bound. The proposed inequality provides a safe approximation to the chance-constraint (2b) for any α positive. To achieve a least conservative approximation, we select the α value that maximizes the right hand side of the constraint. □

2.2.2 Decomposition Structure

The stochastic program (2) can be compactly represented as:

$$
\begin{aligned}
\min_{z,x,y} \sum_{k\in\mathcal{K}} \pi_k(\quad & c_k^\top z + v^\top x^k && +b^\top \tilde{y}^k) && \text{(3a)} \\
\text{s.t.} \quad & Az && \leq l && \text{(3b)} \\
& B_k z + E x^k && \leq m \;\; k\in\mathcal{K} && \text{(3c)} \\
& F x^k && +H\tilde{y}^k \leq n \;\; k\in\mathcal{K} && \text{(3d)}
\end{aligned}
$$

$$
z \in \{0,1\}^{|\mathcal{T}|\times|\mathcal{G}|},\; x^k \in \{0,1\}^{|\mathcal{T}|\times|\mathcal{G}|\times|\mathcal{S}|},\; \tilde{y}^k \geq 0\; k\in\mathcal{K}.
$$

Here, $\tilde{y}$ is a vector consisting of the dispatch variable y, demand curtailment variable ψ and additional labor variable γ. Constraint (3b) refer to the safe approximation of the chance-constraint (2b), and maintenance constraints ((2c), (2d)). Constraints (3c) and (3d) represent the coupling constraints between maintenance and operations ((2e), (2f)), and operational constraints ((2g), (2h)), respectively.

This formulation is a two-stage stochastic program with first-stage variables z corresponding to maintenance decisions and the second-stage variables x and $\tilde{y}$ corresponding to the operational decisions, and can be rewritten as

$$\min_{z}\{\sum_{k\in\mathcal{K}}\pi_k f_k(z) : Az \le l, z \in \{0,1\}^{|\mathcal{T}|\times|\mathcal{G}|}\}, \tag{4}$$

where

$$\begin{aligned} f_k(z) = c_k^\top z + \min_{x,\tilde{y}}\{v^\top x^k + b^\top \tilde{y}^k : Ex^k \le m - B_k z, \\ Fx^k + H\tilde{y}^k \le n, x^k \in \{0,1\}^{|\mathcal{T}|\times|\mathcal{K}|\times|\mathcal{S}|}, \tilde{y}^k \ge 0\}. \end{aligned} \tag{5}$$

Given a maintenance decision z, we can observe that the operational decisions corresponding to each maintenance epoch t, namely $x^{t,k}$ and $\tilde{y}^{t,k}$, are independent from each other as there are no coupling constraints between the maintenance epochs in the operational problem. Thus, given a maintenance decision z, the scenario subproblems $f_k(z)$ can be decomposed with respect to the maintenance epochs such that $f_k(z) = c_k^\top z + \sum_{t\in\mathcal{T}} f_k^t(z)$ where $f_k^t(z)$ is defined as:

$$\begin{aligned} f_k^t(z) = \min_{x^t,\tilde{y}^t}\{{v^t}^\top x^{t,k} + {b^t}^\top \tilde{y}^{t,k} : E^t x^{t,k} \le m^t - B_k^t z, \\ F^t x^{t,k} + H^t \tilde{y}^{t,k} \le n^t, x^{t,k} \in \{0,1\}^{|\mathcal{G}|\times|\mathcal{S}|}, \tilde{y}^{t,k} \ge 0\}. \end{aligned} \tag{6}$$

Using this formulation we can solve smaller subproblems corresponding to each maintenance epoch t and scenario k rather than solving the operational problem over the entire planning horizon, which results in significant computational advantage over (5).

3 Solution Methodology

In this section, we discuss the proposed solution methodology for solving the resulting two-stage stochastic program (4). To do this, we first adopt a sample average approximation approach and solve an approximation of the original problem by optimizing the stochastic program over a random scenario subset K. The resulting problem is still challenging to solve and requires efficient solution techniques, which we address by using a scenario decomposition algorithm and proposing various algorithmic enhancements.

3.1 Sample Average Approximation Methodology

Stochastic program (4) is computationally expensive to solve for large instances as the number of scenarios increases exponentially in the number of generators. Instead, we solve (4) over an independently and identically distributed (i.i.d.) scenario subset corresponding to generator failure times. To assess the quality of the solutions that we obtain, it is necessary to derive confidence intervals of the true optimal value. Sample average approximation (SAA) approach provides a guideline for constructing these intervals [14]. The main steps of this algorithm are presented in Algorithm 1.

To obtain the confidence intervals, we first generate M batches of N scenarios where each scenario indicates the failure times for every generator. The resulting SAA problems are solved (Algorithm 1, step 4), and the average of the objectives of the corresponding programs provides a lower bound estimate of the true optimal value. Then, we evaluate the resulting M feasible solutions over N' scenarios where $N' \gg N$, and the solution coming from each batch with the smallest upper bound is selected for evaluating the upper bound estimate of the true optimal value. After obtaining the confidence intervals for the lower and upper bounds, a $(1-\alpha)\%$ confidence interval can be constructed as follows:

$$(\mu_L - t_{\alpha/2,M-1}\sigma_L/\sqrt{M}, \mu_U + z_{\alpha/2}\sigma_U/\sqrt{N'}) \tag{7}$$

Algorithm 1 SAA method

1: Generate an i.i.d. scenario sample of size N'.
2: **for all** $k = 1, \ldots, M$ **do**
3: Generate an i.i.d. sample of size N.
4: Solve $v_N^k = \min\{\frac{1}{N}\sum_{j=1}^{N} f_j(z) : Az \leq l,\ z \in \{0,1\}^{|\mathcal{T}|\times|\mathcal{G}|}\}$ using Algorithm 2 and obtain z_N^k.
5: Evaluate the solution z_N^k over N' scenarios: $\mu_{N'}^k = \frac{1}{N'}\sum_{k\in N'}\left(c_k^\top z_N^k + \sum_{t\in\mathcal{T}} f_k^t(z_N^k)\right)$.
6: Select the best upper bound estimate: $\mu_U = \min\limits_{k\in\{1,\ldots,M\}} \mu_{N'}^k$, and the corresponding solution $\bar{z}$.
7: Compute variance of the true upper bound estimate, σ_U^2 :

$$\sigma_U^2 = \frac{1}{N'-1}\sum_{k=1}^{N'}\left(\left(c_k^\top \bar{z} + \sum_{t\in\mathcal{T}} f_k^t(\bar{z})\right) - \mu_U\right)^2.$$

8: Construct the $(1-\alpha)$ level confidence interval for the upper bound estimate as $\mu_U \pm z_{\alpha/2}\sigma_U/\sqrt{N'}$.
9: Compute mean and variance of the true lower bound estimate, μ_L and σ_L^2 as

$$\mu_L = \frac{1}{M}\sum_{k=1}^{M} v_N^k \text{ and } \sigma_L^2 = \frac{1}{M-1}\sum_{k=1}^{M}(v_N^k - \mu_L)^2.$$

10: Construct the $(1-\alpha)$ level confidence interval for the lower bound estimate as $\mu_L \pm t_{\alpha/2,M-1}\sigma_L/\sqrt{M}$.

3.2 Scenario Decomposition Algorithm

To solve model (4) over a scenario sample, we introduce Algorithm 2, an enhanced version of a scenario decomposition algorithm proposed in [2]. Algorithm 2 has two components, a lower and an upper bounding step. In the lower bounding step, each scenario subproblem is solved separately to obtain lower bounds and candidate feasible first stage maintenance decisions z. In the upper bounding step, the candidate binary feasible solutions are evaluated under each scenario setting, and the best upper bound of that iteration is found. The algorithm proceeds by using integer cuts to eliminate the solutions that are already explored in order to obtain new candidate solutions. This process is repeated until the lower bound is close enough to the upper bound.

Algorithm 2 Scenario Decomposition

```
LB = −∞, UB = ∞, 𝒵 = ∅, z* = ∅.
Set ϒ_t = ∅, Φ_t = ∅ for all t ∈ 𝒯.
while UB > LB do
  Lower Bounding:
  for all k ∈ 𝒦 (in parallel) do
    z^k = argmin{f_k(z) : Az ≤ l, z ∈ {0,1}^{|𝒯|×|𝒢|} \ 𝒵}.
  Compute LB = Σ_{k∈𝒦} π_k f_k(z^k).
  Ẑ = ⋃_{k∈𝒦} {z^k}, 𝒵 = 𝒵 ∪ Ẑ {Add the set of explored solutions in this iteration, namely
  Ẑ, to the set 𝒵.}
  Upper Bounding:
  Identify generator statuses υ_t, over every pair (ẑ, k) ∈ Ẑ × 𝒦 for all t ∈ 𝒯, and set
  Φ_t = ⋃_{(ẑ,k,t)∈Ẑ×𝒦×𝒯} {υ_t, (ẑ, k, t)}. {Store solution, scenario, time triple (ẑ, k, t) with its
  status υ_t.}.
  Set ϒ̂_t = (⋃_{t∈𝒯} {υ_t}) \ ϒ_t, ϒ_t = ϒ_t ∪ ϒ̂_t for all t ∈ 𝒯.
  for all t ∈ 𝒯 do
    for all υ_t ∈ ϒ̂_t (in parallel) do
      Obtain a (ẑ, k, t) triple that υ_t maps in the set Φ_t.
      Solve (6) to obtain f_k^t(ẑ) and set f_k^t(ẑ) for all (ẑ, k, t) triple that υ_t maps in the set
      Φ_t.
  for all ẑ ∈ Ẑ do
    Compute u_ẑ = Σ_{k∈K} π_k (c_k^⊤ ẑ + Σ_{t∈𝒯} f_k^t(ẑ)).
  if UB > min_{ẑ∈Ẑ} u_ẑ then
    UB = min_{ẑ∈Ẑ} u_ẑ, z* = argmin_{ẑ∈Ẑ} u_ẑ.
```

Utilizing the time decomposability of the second stage problem, we propose an algorithmic improvement that identifies generator statuses for each time period t as a binary vector of size $|\mathcal{G}|$ in the upper bounding step. This vector, namely υ_t, shows whether generators are available for production in period t based on their maintenance status in $\hat{z}$ and failure times τ_i^k in scenario k. A generator is not available for production if it is under a PM or CM routine. Consequently, the availability information is sufficient to solve the second stage operational problem for each time t. As a preprocessing step (before the upper bounding part), the unique set of generator statuses for each time period t are identified in the set Υ_t. It becomes sufficient to solve the second stage subproblems only for the unique set of generator statuses found in that iteration. This provides significant computational gains by storing the values corresponding to the previously explored generator statuses.

Algorithm 2 can be implemented in a parallel fashion. In the lower bounding step, scenario subproblems can be solved in parallel as the problems are independent, and it suffices to collect the candidate solutions at the end. Similarly, for the upper bounding step, given a maintenance schedule each scenario subproblem is time decomposable with respect to the maintenance epochs. The decomposed subproblems for each time period with respect to the each generator status can be solved in a distributed framework.

3.3 Alternative cuts

Starting from the second iteration of Algorithm 2, the previously explored solutions are discarded from the set of feasible solutions in step 6 by utilizing the binary nature of the first stage variables. Given a solution $\hat{z}$, we can remove it from the solution set by adding 'no good' cut of the form,

$$\sum_{(i,t):\hat{z}_{i,t}=1} (1 - z_{i,t}) + \sum_{(i,t):\hat{z}_{i,t}=0} z_{i,t} \geq 1, \tag{8}$$

to the original set of constraints of the scenario subproblems. In our problem, we have a special structure that every generator can enter maintenance only once during the planning horizon, which is ensured by the constraint (2d). We can exploit this structure to improve (8) to the following cut

$$\sum_{(i,t):\hat{z}_{i,t}=0} z_{i,t} \geq 1. \tag{9}$$

Proposition 2. *Inequality* (9) *is valid and dominates the standard 'no good' cut (8).*

Proof. Let

$$\bar{S_1}(\hat{z}) := \{z : \sum_{(i,t):\hat{z}_{i,t}=1} (1 - z_{i,t}) + \sum_{(i,t):\hat{z}_{i,t}=0} z_{i,t} \geq 1, \sum_{t\in\mathcal{T}} z_{i,t} = 1\, \forall i \in \mathcal{G}, z_{i,t} \geq 0\, \forall i \in \mathcal{G}, t \in \mathcal{T}\}$$

be the set of all vectors excluding satisfying (8) for the solution $\hat{z}$, and the single maintenance constraint (2d). Let

$$\bar{S_2}(\hat{z}) := \{z : \sum_{(i,t):\hat{z}_{i,t}=0} z_{i,t} \geq 1, \sum_{t\in\mathcal{T}} z_{i,t} = 1\, \forall i \in \mathcal{G}, z_{i,t} \geq 0\, \forall i \in \mathcal{G}, t \in \mathcal{T}\}.$$

be the set with (9), and (2d). Let $S_1(\hat{z}) = \bar{S_1}(\hat{z}) \cap \{0,1\}^{|T|\times|G|}$ and $S_2(\hat{z}) = \bar{S_2}(\hat{z}) \cap \{0,1\}^{|T|\times|G|}$. We first show that the linear programming (LP) relaxation of $S_1(\hat{z})$, namely $\bar{S_1}(\hat{z})$, is weaker than the LP relaxation of $S_2(\hat{z})$, namely $\bar{S_2}(\hat{z})$, i.e., $\bar{S_1}(\hat{z}) \supseteq \bar{S_2}(\hat{z})$. Since $z_{i,t} \leq 1$ is implied by $\sum_{t\in\mathcal{T}} z_{i,t} = 1$, we have $\sum_{(i,t):\hat{z}_{i,t}=1}(1 - z_{i,t}) \geq 0$. Thus, $\sum_{(i,t):\hat{z}_{i,t}=1}(1 - z_{i,t}) + \sum_{(i,t):\hat{z}_{i,t}=0} z_{i,t} \geq \sum_{(i,t):\hat{z}_{i,t}=0} z_{i,t}$, which implies that $\bar{S_1}(\hat{z}) \supseteq \bar{S_2}(\hat{z})$.

Next, we show that $S_1(\hat{z}) = S_2(\hat{z})$. Since $S_1(\hat{z}) \supseteq S_2(\hat{z})$ follows from the previous claim, it suffices to show $S_1(\hat{z}) \subseteq S_2(\hat{z})$. For this purpose, we show that only $\hat{z}$ is removed from the solution space and not the other possible feasible solutions. Suppose there exists z such that $\sum_{(i,t):\hat{z}_{i,t}=0} z_{i,t} \leq 0$, hence $z_{i,t} = 0$ for all $\hat{z}_{i,t} = 0$. As $\sum_{t\in\mathcal{T}} z_{i,t} = 1 = \sum_{t:\hat{z}_{i,t}=1} z_{i,t} + \sum_{t:\hat{z}_{i,t}=0} z_{i,t}$ for all $i \in \mathcal{G}$, this implies that $z_{i,t} = 1$ for all $\hat{z}_{i,t} = 1$. Thus, $z = \hat{z}$. This result shows that only $\hat{z}$ is removed from the feasible solution space by using the new cut. Combining the above, we proved that (9) is stronger than (8) due to the special structure of the problem. □

3.4 Transmission line analysis

We propose an enhancement by identifying the lines that never violate the transmission line constraint (2i) by solving auxiliary linear programs. For this purpose, we solve a relaxation of the operational problem in (10) by maximizing the amount of flow on each line l subject to the worst case nodal demands and operational decisions. We then check whether the resulting flow is larger than the corresponding line's flow capacity, namely f_l^{max}.

$$g_1(l) = \max_{x,y,\psi,d} \sum_{b\in\mathcal{B}} a_{l,b} \left(\sum_{i\in\mathcal{G}} M_{b,i} y_i + \psi_b - d_b \right) \tag{10a}$$

$$\text{s.t. } \sum_{i\in\mathcal{G}} y_i + \sum_{b\in\mathcal{B}} \psi_b = \sum_{b\in\mathcal{B}} d_b \tag{10b}$$

$$p_i^{min} x_i \leq y_i \leq p_i^{max} x_i \quad \forall i \in \mathcal{G} \tag{10c}$$

$$\psi_b \leq d_b \leq D_{max,b} \qquad \forall b \in \mathcal{B} \tag{10d}$$

$$x \in [0,1]^{|\mathcal{G}|}, y, \psi, d \geq 0. \tag{10e}$$

Here, the first set of decision variables are x, y, ψ which represents commitment decision, dispatch and demand curtailment amounts, respectively. Additionally, we consider every possible demand scenario by representing demand at bus b as a decision variable d_b with an upper bound $D_{max,b}$, which denotes the maximum demand at bus b over all the operational time periods of the planning horizon. As (10) is a relaxation of the original operational problem by relaxing the commitment variable x and removing constraints (2e) and (2f); if $g_1(l)$ is smaller than f_l^{max}, then we assure that the maximum possible flow on line l will be less than the right hand side of constraint (2i), and hence omit this constraint. We can do a similar analysis by maximizing the magnitude of the negative flow and solve (10) as a minimization problem. Omitting such provably redundant constraints significantly reduces the total number of constraints in the operational problem.

4 Computational Results

In this section, we provide our computational results on WSCC 9-bus instance [21], 39-bus New-England Power System [3], and 118-bus instance [5]. The algorithm is implemented using Python 2.7 and Gurobi 6.5 is used as the solver. We study a one-year maintenance plan with monthly maintenance decisions corresponding to 12 maintenance epochs. Additionally, we have daily operational decisions with respect to the demand in the peak hour, which implies 30 operational time periods within a maintenance epoch. Thus, $|\mathcal{T}| = H = 12$ months and $|\mathcal{S}| = 30$ days. We assume that $Y_p = 1$, and $Y_c = 2$ months and $L = 2$. For the chance constraint (2b), experiments are conducted by setting ρ as $\lfloor |\mathcal{G}|/3 \rfloor$ with $\epsilon = 0.05$ and 0.10. This means that with a probability of at least $1 - \epsilon$, at most one third of the generators can enter corrective maintenance due to a failure. The proposed safe approximation in Section 2.2.1 is used for modeling the chance-constraint.

A database of historical degradation signals generated by a rotating machinery application is used to reproduce generator degradation. This setup is explained in detail in [11]. These degradation signals are used to estimate dynamic maintenance cost and failure scenarios as outlined in Section II-A. To generate failure scenarios, the RLD of each generator is discretized in monthly periods ($d = 12$).

Our goal is to present the computational results and demonstrate the advantages of the proposed solution methodology along three dimensions:

1. We demonstrate the computational efficiency of the proposed algorithm by comparing the solution times with the generic scenario decomposition algorithm [2].

2. We provide SAA analysis for constructing confidence intervals on the true optimal value and determining the sufficient sample sizes for the presented test cases.

3. To demonstrate the relevance of a stochastic modeling approach, we compare the proposed stochastic program with the failure scenarios and the chance-constraint (2), which we refer to as the chance-constrained model with failure scenarios (CCMFS) with two other simplified models: i) a deterministic model (DM), and ii) a chance-constrained model (CCM). The DM formulation ignores the unexpected failures by not considering scenarios or the chance constraint. CCM is a simplified stochastic formulation that only uses the chance constraint without incorporating the failure scenarios.

4.1 Computational efficiency

We demonstrate the computational efficiency of the proposed solution methodology by comparing its performance with the generic scenario decomposition algorithm [2]. Figure 1 presents the computational performance of a single processor for the 9-bus case with samples of 50, 100, 150, 200 scenarios when $\rho = 1$ and $\epsilon = 0.10$. Run time increases sublinearly with respect to the sample size for the proposed algorithm, which is at a much higher rate in the generic algorithm. We observe speedups of 5.77, 6.32, 6.52, 9.41 times compared to the original computation time corresponding to 50, 100, 150, 200 scenarios. As sample size increases, the proposed algorithm becomes more advantageous.

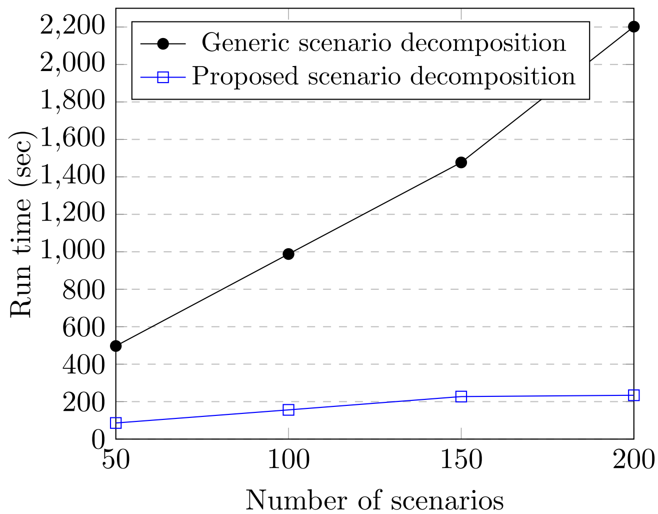

Figure 1: Effect of the proposed enhancements.

In our experiments, we use a distributed framework in which the proposed scenario decomposition algorithm is parallelized. To evaluate the impact of parallelization, the speedup of the proposed algorithm is evaluated using different numbers of processors for solving the 9-bus instance with a sample of 500 scenarios when $\rho = 1$ and $\epsilon = 0.10$ (see Figure 2). The speed up ratio increases almost linearly until 8 processors. After 16 processors, the effect of parallelism begins to decrease. Nevertheless, the speedup is 134.83 times compared to the original algorithm when 32 processors are used. Note that the run time comparison is only presented for the 9-bus instance as the results are analogous for other test cases.

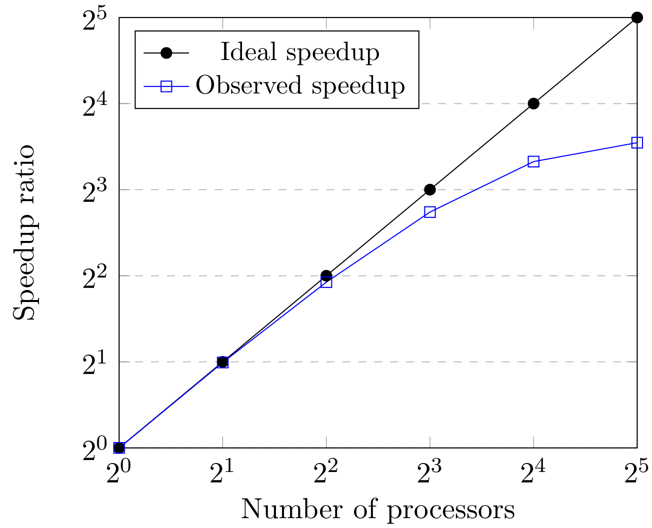

Figure 2: Effect of parallellism.

4.2 Sample Average Approximation Results

In this section, we present the confidence intervals for upper and lower bounds of the true optimal objective values to the proposed stochastic optimization models. To do so, we apply SAA method to the presented instances with $M = 5, N = 50, N' = 500$ when $\epsilon = 0.05, 0.10$. The resulting 95% confidence intervals for the lower bound and upper bound for 9-bus, 39-bus and 118-bus instances are presented in Table 1. The first column "rhs" gives the right hand side obtained by Proposition 1. In the last column of the tables, we report the percentage gap between the lower and upper ends of the confidence interval to the true optimal in (7).

For different ϵ values, the resulting stochastic programs give the same solutions in 9-bus instance as the chance constraint restricts the feasible space considerably for small-scale instances. Our computational studies suggest that $N = 50$ provides sufficient accuracy for solving the stochastic program for the 9-bus instance, which has an extensive scenario size of $13^3 = 2197$. We also note that, σ_U is notably large since a maintenance schedule can perform considerably different under different failure scenarios as a failure of a single generator affects the objective value significantly in small-scale instances.

To study the performance in larger instances, we extend our results to 39-bus and 118-bus cases. Percentage gap indicates that $N = 50$ is sufficient for solving these instances to within 2% optimality as well for both ϵ values; instead of solving the extensive case with 13^{10} and 13^{19} many scenarios, respectively. We observe that as the decision maker becomes less conservative regarding the chance constraint, that is as ϵ value gets larger, we obtain a less expensive maintenance plan.

Table 1: SAA results (Intervals are in million \$).

	rhs	ϵ	CI for LB	CI for UB	Gap (%)
9-bus	0.05 0.10	0.05 0.10	(2.43 , 2.49)	(2.41 , 2.48)	2.15
39-bus	0.55	0.05	(81.88, 82.03)	(81.92, 82.12)	0.30
	0.72	0.10	(81.74, 81.98)	(81.83, 82.03)	0.36
118-bus	2.05	0.05	(57.00, 57.87)	(57.63, 57.99)	1.74
	2.41	0.10	(57.02, 57.95)	(57.63, 58.04)	1.79

4.3 Model comparison

In this section, we highlight the importance of using the stochastic formulation by comparing the quality of the solutions of the deterministic model, and the proposed chance-constrained models in Table 2. The column "Evaluated solution estimate" presents the performance of the solutions using 500 failure scenarios. The solutions are obtained by the following procedure: i) the optimal solutions to the DM, CCM and CCMFS for $N = 50$ with $\epsilon = 0.05, 0.10$ are obtained, ii) the resulting maintenance schedules are evaluated for the operational problem by fixing the first stage decision z and solving the second stage problem (5) for each scenario, and iii) the mean of the objective values of each scenario subproblem is calculated. CCMFS is solved over $M = 5$ batches and the solution with the best evaluated estimate is reported. The "Cost Improvement" column provides the percentage improvements gained by solving the stochastic programs. DM-CCM and DM-CCMFS columns give the percentage improvement gained by solving CCM instead of DM and CCMFS instead of DM, and reporting the percentage gap between the corresponding evaluated solution estimates, respectively. The "p-value" column represents the results of the paired t-tests for determining whether CCMFS outperforms CCM. In the paired t-test, the objectives of the 500 failure scenarios from the solutions of CCM and CCMFS are compared in a pairwise manner. The null hypothesis of the test is that CCM outperforms CCMFS, which corresponds to smaller objective values. The p-values are reported for a one-sided test. Note that p-value is not applicable for 9-bus instance with $\epsilon = 0.05$ since CCM and CCMFS arrive at the same solution.

Table 2: Model comparison for all instances (Estimates are in million \$).

		Evaluated Solution Estimates ($\hat{\mu}, \hat{\sigma}$)			Cost Improvement (%)		p-value
		DM	CCM	CCMFS	DM-CCM	DM-CCMFS	CCM-CCMFS
9-bus	$\epsilon = 0.05$	(2.62, 0.87)	(2.45, 0.44)	(2.45, 0.44)	7.05	7.05	N/A
	$\epsilon = 0.10$		(2.53, 0.64)	(2.45, 0.44)	3.28	7.05	2.29e-06
39-bus	$\epsilon = 0.05$	(93.82, 5.96)	(82.51, 2.33)	(82.02 1.16)	13.67	14.39	9.33e-13
	$\epsilon = 0.10$		(82.20, 1.48)	(81.93, 1.15)	14.13	14.50	2.61e-06
118-bus	$\epsilon = 0.05$	(69.36, 4.79)	(58.65, 3.16)	(57.81, 2.12)	18.26	19.98	0
	$\epsilon = 0.10$		(58.27, 3.07)	(57.83, 2.36)	19.04	19.94	7.45e-06

Since CCMFS captures more uncertainty using the chance constraint and the scenarios, the resulting solutions have less variance when evaluated under 500 scenarios, compared to the solutions obtained by DM and CCM. Thus, CCMFS results in more robust solutions that take into account various failure cases with less disruption to the maintenance plan. The improvement percentages in Table 2 indicate that the stochastic programs, CCM and CCMFS, are necessary for all instances since the solutions found by the DM gives significantly higher objective values when it is evaluated under different failure scenarios. This demonstrates that unexpected failures should be considered explicitly when scheduling maintenance routines and determining operational level decisions. Finally, the p-values over all instances indicate the we have significant evidence to reject the null hypothesis. Thus, the proposed CCMFS approach performs better than CCM. This also demonstrates that the proposed stochastic programming approach with failure scenarios and chance constraint is required for solving this problem effectively.

5 Conclusion

This paper presents a comprehensive framework for solving the joint maintenance and operations problem under unexpected generator failures. We consider the conditions of the generators through degradation-based predictions of their RLDs which are then used to compute maintenance costs and failure probabilities. The problem is formulated as a two-stage stochastic mixed-integer program that determines maintenance schedules and operational decisions while considering unexpected failure scenarios. We present a chance-constraint that adapts to the generator RLDs in order to restrict the number of generators that enter maintenance due to a failure with high probability. We also provide a safe approximation for its representation. We adopt the SAA method to solve the program, and present a scenario decomposition algorithm by introducing various enhancements that leverage the structural properties of our model. Our experiments show significant computational gains over generic scenario decomposition and serial implementations. We also demonstrate the solution quality of the proposed approach relative to deterministic and alternative stochastic solutions.

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