The Unmanned Aerial Vehicle Routing and Trajectory Optimisation Problem

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Abstract

Unmanned Aerial Vehicles (UAVs) are becoming increasingly popular over the past few years. The complexity of routing UAVs has not been fully investigated in the literature. In this survey, we aim to review recent contributions in UAV trajectory optimisation, UAV routing and contributions addressing these two problems simultaneously. A unified framework is introduced to describe UAV routing and trajectory optimisation problems. We conclude with the identification of future research opportunities.

Keywords: unmanned aerial vehicles, routing, trajectory optimisation

1. Introduction

Unmanned Aerial Vehicles (UAVs) are aircraft that do not need a human pilot on board. In general, these vehicles are either controlled by an embedded computer or by a pilot operating a remote control.

Drones, remote controlled helicopters and unmanned gliders are examples of UAVs. Gliders differ from the other types due to the lack of on-board propulsion (e.g., an electric or combustion engine). Modern UAVs have been first developed in the 1920s to support military operations in which the presence of human pilots was either impossible or too dangerous (Beard & McLain, 2012; Keane & Carr, 2013). However, UAVs have recently become very popular for logistics and surveillance applications (Tsourdos et al., 2010).

A report from the National Purchase Diary has shown that drones sales have increased 224% in 12 months from April 2015, reaching a total of 200 million dollars (NPD, 2016). Due to their ability of embedding several transmitters, sensors and photographing equipment, UAVs can be used in a large range of applications. Successful cases have been reported in, e.g., aerial reconnaissance (Ruzgienė et al., 2015), aerial forest fire detection (Yuan et al., 2015), target observation (Rysdyk, 2006), traffic monitoring and management (Kanistras et al., 2013), online commerce (Wang et al., 2017), geographic monitoring (Uysal et al., 2015), scientific data collection (Stöcker et al., 2015), meteorological sampling (Elston et al., 2014), three-dimensional mapping (Nex & Remondino, 2013), UAVs networks (Hayat et al., 2016), humanitarian relief (Bravo & Leiras, 2015), and disaster assessment and response (Quaritsch et al., 2010; Xu et al., 2014; Nedjati et al., 2016). More examples of the growing applications of UAVs are presented in Rao et al. (2016).

The academic routing community has acknowledged the interest of companies and organisations in adopting UAVs in their operations. A recent example is the approach of combining UAVs and trucks for distribution activities by dispatching drones from trucks for the last mile distribution within city centres (Ha et al., 2015; Murray & Chu, 2015; Wang et al., 2017). It can been shown that this solution could reduce

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the truck travel time (and the corresponding CO_2 emissions) of up to 50%. The UAV Task Assignment Problem (UAVTAP) is another name used in the literature for the problem of scheduling and routing a fleet of UAVs Khamis et al. (2015). A growing body of literature appeared on the UAVTAP in the last decade, e.g., Ramirez-Atencia et al. (2016), Wang et al. (2015), Hu et al. (2015a), Thi et al. (2012), Alidaee et al. (2010) and Edison & Shima (2011). However, the UAV routing literature has often neglected constraints due to the flight dynamics of UAVs. Finding feasible trajectories for UAVs in a routing problem is a complex task, but it is necessary to ensure the feasibility of the UAVs routes. Very few studies have attempted to solve simultaneously the routing and Trajectory Optimisation (TO) problem for a fleet of UAVs.

The computation of trajectories for UAVs has been widely studied in the aerospace engineering and optimal control literature (Yang et al., 2016). An optimal trajectory for a UAV must define a control manoeuvre while satisfying constraints on the kinematics (position, velocity and acceleration) and the dynamics (forces and moments) of the vehicle. A trajectory is generally associated to a set of Equations of Motion (EOMs) that describe the relationship between spatial and temporal changes of a system. The problem named Path Planning (PP), for example, consists of finding a feasible trajectory of a UAV visiting a given sequence of waypoints in a two-dimensional (2D) or three-dimensional (3D) space without considering the vehicle's dynamics. In this paper, we define a *waypoint* as a location that must be visited by the UAV during its mission (e.g., a target).

More complex variants of the PP problem including, for instance, wind and vehicle dynamics, etc. require substantial simplifications and assumptions to be solved heuristically (Kunchev et al., 2006; Rathinam & Sengupta, 2007). The books by Tsourdos et al. (2010) and Beard & McLain (2012) provide good overviews of PP algorithms for UAVs. On the other hand, high fidelity TO models (i.e., using more accurate physical models) have been developed for aircraft and spacecraft (Raivio et al., 1996; Conway, 2010; Fisch, 2011; García-Heras et al., 2014; Colasurdo et al., 2014). These models are currently solved by optimal control techniques. An overview of optimal control methods for TO is provided in Betts (1998, 2001).

UAV PP and TO are closely related problems. In general, a *path* is a curve traced in the vehicle's working space and a *trajectory* is a path that includes time information along the path (Goerzen et al., 2009). Several studies have been done in the field of UAV PP, e.g., Rathinam & Sengupta (2007), Goerzen et al. (2009) and Mac et al. (2016). Much of these were inspired by robot PP algorithms (Kunchev et al., 2006; Galceran & Carreras, 2013; Khamis et al., 2015; Yang et al., 2015). However, reviewing all UAV PP literature is considered out of the scope of this article.

The field of TO has however not included routing decisions: given a set of ordered waypoints, it is possible to find a feasible trajectory for a generic UAV, but it is not clear in the literature if the choice of waypoints is suboptimal. Given a fleet of UAVs, it is an open question how to combine routing and trajectory decisions in a single optimisation problem. As far as the authors are aware, there is not a survey summarising the literature about trajectory and routing optimisation for UAVs.

Research about integrated routing and TO problems seems, therefore, to be still very fragmented. As far as the authors are aware, there is no clear definition of the UAV Routing and Trajectory Optimisation Problem (UAVRTOP) and its variants. We believe that integrating TO and routing algorithms is a key research challenge in adopting UAVs for real world applications.

The purpose of this survey is to present the UAVRTOP, highlighting approaches already proposed in the literature and directions for further research. We introduce a taxonomy, able to identify the key components of routing and TO problems, as well as highlight assumptions and simplifications commonly adopted in the literature.

The remaining of this paper is organised as follows. In Section 2, we formally define the UAVRTOP. In Section 3, a background on TO problems is provided. The same is done in Section 4 for vehicle routing problems. In Section 5, a taxonomy of UAV routing and TO problems is provided. An application of the proposed taxonomy to a selected number of papers is demonstrated in Section 6. This section continues with an analysis of the results obtained from the taxonomic review. In Section 7, we discuss future research opportunities.

2. The UAV Routing and Trajectory Optimisation Problem

In this section, we formally define the UAV Routing and Trajectory Optimisation Problem (UAVRTOP), the optimisation problem in which a fleet of UAVs has to visit a set of waypoints assuming generic kinematics and dynamics constraints. Wind conditions, collision avoidance between UAVs and obstacles can be incorporated in the model as well.

In the following, we assume a fleet $\mathcal{C}$ of UAVs is available at the launching site 0. Let $G = (V, A)$ be a graph, where the set V represent all the waypoints that need to be visited by the UAVs and A represent the set of arcs between waypoints. In addition, let $0'$ represent the landing site. The cost of using a vehicle $k \in \mathcal{C}$ is F_k. The parameters (e.g., mass, wing area, aerodynamics coefficients) of the UAV k travelling between i and j are stored in the vector $\mathbf{p}_{ijk}$. Note that these parameters may change during the mission due, for example, to a change in the flight mode (if hybrid UAVs are used). The state of a UAV is a vector fully defining the position, orientation and velocity of the vehicle in some coordinate system (alternative state representations will be described in Section 3).

For simplicity, we recall $\mathbf{y}_{ijk}(t_{ijk}) \in \mathbb{R}^{n_y^k}$, $n_y^k \in \mathbb{Z}$, the state variable of the UAV k travelling between waypoints i and j at time $t_{ijk} \in \mathbb{R}$. Similarly, the control variables model the inputs that are given to the physical systems in order to achieve a desired trajectory. Typical control variables for UAVs are the thrust (the impulse given by the UAV engine, if any), the roll angle, a.k.a. bank angle (which "banks" the aircraft to change its horizontal flight direction), and the angle-of-attack (which is related to how much lift the aircraft's wing generate). We define $\mathbf{u}_{ijk}(t_{ijk}) \in \mathbb{R}^{n_u^k}$, $n_u^k \in \mathbb{Z}$, the control variables for a UAV k flying on arc (i, j) at time $t_{ijk} \in \mathbb{R}$.

The physical laws governing the UAV k travelling between the waypoints i and j at time t_{ijk} are called the *system dynamics*. In general terms, the system dynamics can be expressed by a set of EOMs in the form of a system of Ordinary Differential Equations (ODEs) as follows:

$$\dot{\mathbf{y}}_{ijk} = \mathbf{f}_k(\mathbf{y}_{ijk}(t_{ijk}), \mathbf{u}_{ijk}(t_{ijk}), \mathbf{p}_{ijk}, t_{ijk}) \forall i, j \in V, \forall k \in C \tag{1}$$

The functions $\mathbf{f}_k, \forall k \in \mathcal{C}$, in the right hand side of the EOMs 1, represent the relationship between the variables and parameters with the derivatives over time of the state variables (here denoted by " $\dot{}$ ").

State and control variables have to be specified for a time instant to initialise the ODEs. In what follows we assume that the initial conditions need to be specified at time $t = 0$. It is also reasonable to assume that only the control variables need to be optimised since the values of the states can be determined, provided an initial condition and the evolution of the controls over time.

The routing cost for a UAV k to travel between waypoints i and j can be computed as:

$$\int_{t_{ijk}^o}^{t_{ijk}^f} w_k(\mathbf{y}_{ijk}(t_{ijk}), \mathbf{u}_{ijk}(t_{ijk}), \mathbf{p}_{ijk}, t_{ijk}) \mathrm{d}t_{ijk}. \tag{2}$$

The variables t_{ijk}^o and t_{ijk}^f represent the initial and final flight times of the UAV k travelling between waypoints i and j such that $t_{ijk} \in [t_{ijk}^o, t_{ijk}^f]$.

Bounds on the state and control variables are usually imposed by a given UAV technology. We denote $\mathbf{y}_{ijk}^{lb}$ and $\mathbf{y}_{ijk}^{ub}$ the lower and upper bounds on the state variables $\mathbf{y}_{ijk}(t_{ijk})$ of the UAV k travelling on arc (i, j) for all $t_{ijk} \in \mathbb{R}$, respectively. Similarly, $\mathbf{u}_{ijk}^{lb}$ and $\mathbf{u}_{ijk}^{ub}$ represent the lower and upper bounds on the control variables $\mathbf{u}_{ijk}(t_{ijk})$ of the UAV k travelling on arc (i, j) for all $t_{ijk} \in \mathbb{R}$. We also assume lower and upper bounds on the operational constraints, here denoted as $\mathbf{g}_{ijk}^{lb}$ and $\mathbf{g}_{ijk}^{ub}$.

According to our assumption on the initial conditions, the initial flight time from the launching point must be defined as $t_{0jk}^o = 0, \forall j \in V, \forall k \in \mathcal{C}$. Let $\bar{\mathbf{y}}_o$ and $\bar{\mathbf{u}}_o$ represent predetermined initial conditions. Thus, the initial state and control variables can be defined as $\mathbf{y}_{0jk}(t_{0jk}^o) = \bar{\mathbf{y}}_o$ and $\mathbf{u}_{0jk}(t_{0jk}^o) = \bar{\mathbf{u}}_o$, respectively, if UAV k departs from the launching point.

Let us define the following binary variable:

$$x_{ijk} = \begin{cases} 1, & \text{if UAV } k \text{ flies directly from waypoint } i \text{ to } j \\ 0, & \text{otherwise.} \end{cases} \tag{3}$$

Hereafter, we will describe the optimisation problem defined by Equations (4-19). The objective function (4) minimises the sum of the fixed cost of using a UAV, the routing cost of flying between waypoints i and j and a measure of the quality of the trajectories at the end points of each arc (i,j). Non desirable features at the end points of the UAVs' trajectories can be penalised in the objective function by means of the functions $\phi_k(\mathbf{y}_{ijk}(t^f_{ijk}), \mathbf{u}_{ijk}(t^f_{ijk}), \mathbf{p}^{ijk}, t^f_{ijk})$. Such undesirable characteristics may include, e.g., sharp flight angles, prohibited flight speeds and noise levels (Vanderbei, 2001; Zhang et al., 2012). Constraints (5) and (6) ensure that every waypoint is visited exactly once and that, if a UAV arrives at a waypoint $l \in V$, it must also depart from l. Constraints (7) make sure that each UAV departs from the launching point 0 and lands in $0'$, if the UAV k is used. Constraints (8) ensure that the UAVs' dynamics are preserved if arc (i,j) is used in a solution. In a similar way, Constraints (9-11) make sure the bounds on the state variables, control variables and *operational constraints* $(\mathbf{g}_{ijk}(\mathbf{y}_{ijk}(t_{ijk}), \mathbf{u}_{ijk}(t_{ijk}), \mathbf{p}_{ijk}, t_{ijk}))$ are respected for every arc (i,j) and for every UAV k if these are travelled in the optimal solution. These constraints can model, for example, obstacles in the space, collisions between UAVs, non desirable flying dynamics, etc. Constraints (12) and (13) ensure that the final state and control variables at every arc (i,j) visited by UAV k is linked to the state and control variables of its subsequent arc (j,l) if waypoints i, j and l are visited by UAV k in this order. Constraints (14) preserve the continuity of the time variable $t_{ijk}, \forall i,j \in V$, along the UAV's k trajectory for all $k \in \mathcal{C}$. Constraints (15) and (16) provide the initial states and controls for every UAV departing from the launching point. Finally, Constraints (17-19) define the domain of the variables.

The UAVRTOP can be modelled as follows:

$$\begin{aligned}
\min \quad & \sum_{k\in C}\sum_{i\in V} F_k x_{0ik} \\
& + \sum_{k\in C}\sum_{(i,j)\in A} \left\{ \int_{t^o_{ijk}}^{t^f_{ijk}} w_k(\mathbf{y}_{ijk}(t_{ijk}), \mathbf{u}_{ijk}(t_{ijk}), \mathbf{p}_{ijk}, t_{ijk}) \mathrm{d}t_{ijk} \right\} x_{ijk} \\
& + \sum_{k\in C}\sum_{(i,j)\in A} \phi_k(\mathbf{y}_{ijk}(t^f_{ijk}), \mathbf{u}_{ijk}(t^f_{ijk}), \mathbf{p}^{ijk}, t^f_{ijk}) x_{ijk} && (4) \\
\text{s.t.} \quad & \sum_{k\in C}\sum_{i\in V} x_{ijk} = 1, \forall j \in V && (5) \\
& \sum_{i\in V} x_{ijk} - \sum_{i\in V} x_{jik} = 0, \forall j \in V, \forall k \in C && (6) \\
& \sum_{i\in V} x_{0ik} = \sum_{i\in V} x_{i0'k} \le 1, \forall k \in C && (7) \\
& \dot{\mathbf{y}}_{ijk} = \mathbf{f}_k(\mathbf{y}_{ijk}(t_{ijk}), \mathbf{u}_{ijk}(t_{ijk}), \mathbf{p}_{ijk}, t_{ijk}) x_{ijk}, \forall i,j \in V, \forall k \in C && (8) \\
& \mathbf{g}^{lb}_{ijk} x_{ijk} \le \mathbf{g}_{ijk}(\mathbf{y}_{ijk}(t_{ijk}), \mathbf{u}_{ijk}(t_{ijk}), \mathbf{p}_{ijk}, t_{ijk}) \le \mathbf{g}^{ub}_{ijk} x_{ijk}, \\
& \forall i,j \in V, \forall k \in C && (9) \\
& \mathbf{y}^{lb}_{ijk} x_{ijk} \le \mathbf{y}_{ijk}(t_{ijk}) \le \mathbf{y}^{ub}_{ijk} x_{ijk}, \forall i,j \in V, \forall k \in C && (10) \\
& \mathbf{u}^{lb}_{ijk} x_{ijk} \le \mathbf{u}_{ijk}(t_{ijk}) \le \mathbf{u}^{ub}_{ijk} x_{ijk}, \forall i,j \in V, \forall k \in C && (11) \\
& \mathbf{y}_{jlk}(t^o_{jlk}) = \mathbf{y}_{ijk}(t^f_{ijk}) x_{ijk} x_{jlk}, \forall i,j,l \in V, \forall k \in C && (12) \\
& \mathbf{u}_{jlk}(t^o_{jlk}) = \mathbf{u}_{ijk}(t^f_{ijk}) x_{ijk} x_{jlk}, \forall i,j,l \in V, \forall k \in C && (13) \\
& t^o_{jlk} = t^f_{ijk} x_{ijk} x_{jlk}, \forall i,j,l \in V, \forall k \in C && (14) \\
& \mathbf{y}_{0jk}(t^o_{0jk}) = \bar{\mathbf{y}}_o x_{0jk}, \forall j \in V, \forall k \in C && (15) \\
& \mathbf{u}_{0jk}(t^o_{0jk}) = \bar{\mathbf{u}}_o x_{0jk}, \forall j \in V, \forall k \in C && (16) \\
& x_{ijk} \in \{0,1\}, \forall i,j \in V, \forall k \in C && (17) \\
& \mathbf{u}_{ijk}(t_{ijk}) \in \mathbb{R}^{n^k_u}, \forall i,j \in V, \forall k \in \mathcal{C} && (18)
\end{aligned}$$

$$t_{ijk}, t^o_{ijk}, t^f_{ijk} \in \mathbb{R}. \forall i, j \in V, \forall k \in \mathcal{C} \tag{19}$$

3. The Trajectory Optimisation Problem

Trajectory Optimisation Problems (TOPs) are Optimal Control Problems (OCPs) determining the trajectory of a vehicle (e.g. a spacecraft, an aircraft, a rover) while minimising a measure of performance and satisfying a set of boundary (initial and final) conditions and path constraints.

The origin of OCPs dates to as early as the 17th century when Johann Bernoulli proposed the Brachistochrone problem (Ross, 2009), one of the first problems in calculus of variations. One of the first applications of the calculus of variations to the control of flying vehicles was presented by Robert Goddard in "A method of reaching extreme altitudes" (Goddard, 1919), where the objective was to determine the minimum initial mass of a ground-based rocket necessary to achieve a given altitude. Optimal control methods are a classical tool in the computation of spacecraft trajectories, e.g., for interplanetary travels and satellite transfer orbits around Earth (Conway, 2010; Colasurdo et al., 2014).

Usually, the system dynamics is modelled by a set of equations of motion that can be nonlinear and discontinuous. *Six degree of freedom* (6DOF) EOMs are composed by translational equations (containing forces, position, velocity, acceleration, etc.) and rotational equations (containing moments, angular velocities, angular acceleration, etc.). Under the assumption that the translational mechanics is much faster than the rotational mechanics, 6DOF EOMs can be decoupled. In this case, the translational EOMs are refered as *three degree of freedom* (3DOF) EOMs. Usual state variables in 6DOF EOMs are the position vector, velocity, pitch angle, pitch rate, weight and flight path angle. In the 3DOF case, the state vector can represent, for instance, the position, velocity, flight path angle and yaw angle of the vehicle.

Other difficulties can be added to the problem if one considers that the boundary conditions depend on unknown variables or if the dynamics of the vehicles change over time. In this cases, TOPs can be divided into two or more *phases* in order to properly model the changes in the operational or physical characteristics of the vehicles. A phase can be defined as a segment of a trajectory in which the dynamical system remains unchanged. Phases can be described by their own boundary conditions, system of differential equations, operational constraints and time events. Finally, all phases can be linked or not depending on the behaviour of the dynamical system.

Aircraft TO models have also gained much popularity over the last decades. For instance, Schultz & Zagalsky (1972) present solutions for several fixed end point aircraft TOPs using calculus of variations. In Raivio et al. (1996), a nonlinear programming-based method is proposed to compute optimal trajectories for a descending aircraft. Fisch (2011) presents a high fidelity optimisation framework for the computation of air race trajectories under safety requirements. García-Heras et al. (2014) compare several optimal control methods for the TO of cruise flight with fixed arrival time. Finally, Delahaye et al. (2014) present a survey of mathematical models for the computation of aircraft trajectories.

Optimal control methods for UAVs are similar to those of full size aircraft, and therefore similar dynamical systems can be used for both types of planes. On the other hand, new challenges are introduced when specific mission's demands are considered. Moreover, due to their limited capacity, extra effort must be put on determining successful flight plans. Therefore, algorithms that are capable of tackling the UAVs' particularities while developing flight plans must be developed.

3.1. Direct and Indirect methods for Trajectory Optimisation Problems

Two main classes of numerical methods became very popular for solving TOPs, namely, *direct* and *indirect* methods. The so-called *direct* methods rely on the discretisation of a infinite-dimensional OCP into a finite-dimensional optimisation problem. This strategy is commonly known as "discretise, then optimise". In a *direct single shooting* method, for example, the controls are discretised on a fixed grid using an arbitrary parametrisation scheme. The next step of this method consists of solving a non-linear programming problem in order to find an optimal vector of parameters. The *indirect* methods consist of determining necessary optimality conditions for an OCP and then using a discretisation method to solve the resulting equations. Indirect methods generally apply an "optimise, then discretise" strategy. In an *indirect single shooting* method, for example, the resulting optimality conditions consist of a boundary value problem, which can be solved by means of a simple single shooting algorithm (Betts, 2001).

There are other more sophisticated algorithms in the literature that are able to tackle TOPs. Reviewing such works is considered out of the scope of this papers. More information about algorithms for optimal control and TO can be found, e.g., in the papers by Stryk & Bulirsch (1992), Betts (1998), Ross (2009), Wang (2009) and Rao (2014); and the books by Bryson (1975), Bertsekas (1979), Betts (2001), Bryson (2002) and Kirk (2012).

3.2. The UAV Path Planning Problem

Using the notation defined by Latombe (1991), the basic PP problem can be defined as follows. Let $\mathcal{A}$ be an object (a robot) moving in a workspace $\mathcal{S}$ (e.g., in an Euclidean space $\mathcal{S} = \mathbb{R}^n, n = 2$ or 3). A set of obstacles $\mathcal{B}_1, \ldots, \mathcal{B}_m$ is assumed to be distributed over $\mathcal{S}$. The problem consists in, given initial and final *configurations* (position and orientation) for $\mathcal{A}$, find a path in $\mathcal{S}$ that avoids collisions with the objects $\mathcal{B}_1, \ldots, \mathcal{B}_m$. It has been shown that this problem is $\mathcal{NP}$-hard if the velocity of the object $\mathcal{A}$ is unbounded and no rotation is considered (Reif & Sharir, 1994).

In the literature, the terms PP and *motion planning* are used almost interchangeably (Barraquand & Latombe, 1991). Both problems have gained much popularity over the years. Figure 1 shows the number of publications by year about UAV PP problems.

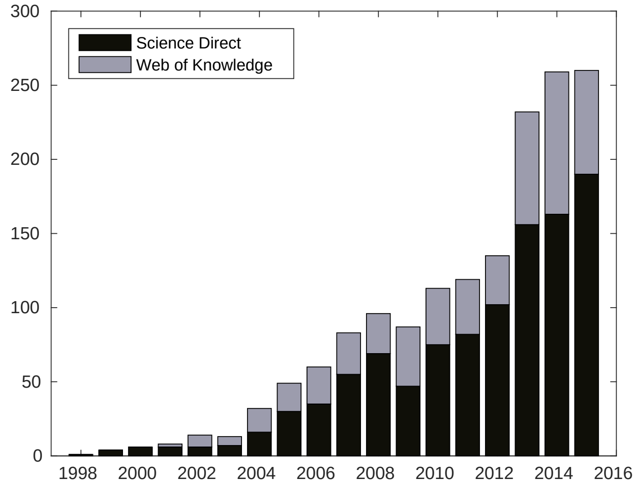

Figure 1: Number of published papers by year on PP problems

Problems integrating UAV routing and PP have already been studied before (Manyam et al., 2015; Ho & Ouaknine, 2015; Enright et al., 2015; Sundar & Rathinam, 2014; Levy et al., 2014). Under simplifying assumptions, a PP problem can be modelled as a network problem and standard VRP techniques can be used. A common assumption is that the UAV can be modelled as a Dubin's vehicle (Medeiros & Urrutia, 2010). A Dubin's vehicle has a limited turning angle and is restricted to move forward, therefore it can be a good representation for some types of UAVs. This simplification is very popular specially for modelling rotary-wing aircraft such as quadcopters. However, for most of the fixed-wing UAVs, the Dubin's assumption might not be suitable due to their complicated dynamics. The reader is referred to Tsourdos et al. (2010) for more details on UAVs PP methods.

Most algorithms for UAV PP have been originated from adaptations of existing algorithms for robot PP. However, we do not intend to survey all PP algorithms as it has already been done in other articles (Kunchev et al., 2006; Goerzen et al., 2009; Galceran & Carreras, 2013; Yang et al., 2016).

4. The Vehicle Routing Problem

The Vehicle Routing Problem (VRP) is a very well known problem in Operational Research and Combinatorial Optimisation. In the VRP, routes must be assigned to a set of vehicles that must serve a set of customers such that the total cost of the operation is minimised. Its classical variant is called Capacitated Vehicle Routing Problem (CVRP), where a load capacity is assigned to each vehicle.

The CVRP can be formally defined as follows. A set of vertices $V = \{0, \ldots, n\}$ and a set of arcs A connecting these vertices are given. Each vertex represents a customer with demand d_i, $i \in V \setminus \{0\}$. A value $c_{i,j}$ is assigned to each arc $(i, j) \in A$ representing the travel cost between two customers. Let $C = \{1, \ldots, m\}$ be a set of homogeneous vehicles with capacity Q. Here we denote the vertex $i = 0$ representing the depot. The CVRP consists of finding a minimum cost set of m routes starting and ending at the depot such that all customers are visited exactly once, all customers' demands are satisfied and the capacity of the vehicles are respected. The CVRP is well known to be $\mathcal{NP}$-hard. More information about the VRP and its variants can be found, e.g., in Golden & Assad (1988), Cordeau et al. (2007), Golden et al. (2008) and Toth & Vigo (2002).

The m-TSP is closely related to the VRP. In the m-TSP, m minimum cost tours starting at the depot must be found such that every vertex in $V \setminus \{0\}$ is visited exactly once. The m-TSP can be reduced to the CVRP if all vehicles are considered to have infinite capacity. An extensive literature review on models and algorithms for the m-TSP is presented by Bektas (2006).

The VRP has been widely studied for terrestrial applications, but with the development of new technologies, e.g. unmanned vehicles, new variants of this problem are gaining interest among the scientific community. The problem of routing an aerial vehicle is more complex than the VRP, because its intrinsic TO problem is hard on its own.

4.1. UAV Task Assignment Problem

The UAV Task Assignment Problem (UAVTAP) consists of finding an optimal assignment of UAVs to a set of tasks, subject to a set of constraints. Often, the UAVs have different characteristics and the tasks present particular requirements. It has been shown that this problem is $\mathcal{NP}$-hard (Alidaee et al., 2010). Due to the quick development of UAV technology, new challenging assignment problems arise every day and many algorithms have been developed to address the new challenges. Figure 2 shows the number of publications by year in UAVTAPs. One can observe that this field of research has gained attention by the scientific community. A detailed literature review about algorithms for multi-robot Task Assignment (TA) problems can be found in Khamis et al. (2015).

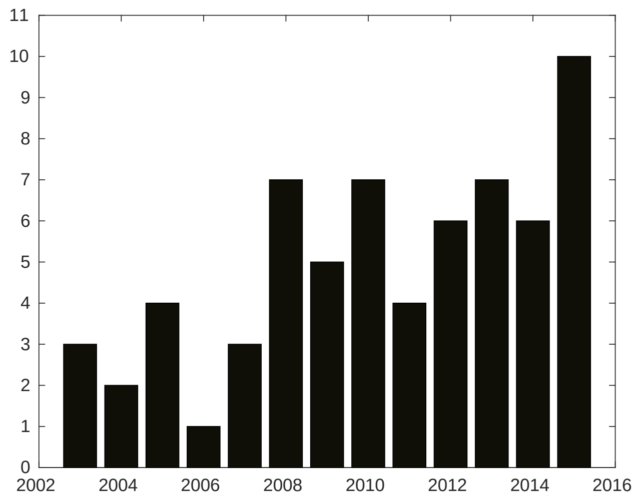

Figure 2: Number of published papers by year on TA problems

The UAVTAP shares many similarities with the VRP. Many examples in the literature support this claim. One can cite, for instance: TA with time windows (Karaman & Inalhan, 2008), multi-depot (launching points) (Darrah et al., 2012), task allocation with resource constraints (Kim et al., 2015), TA with flexible demand (Alidaee et al., 2011), real-time and dynamic assignment (Kim et al., 2007; Lin et al., 2013), time-dependent TA (Kingston & Schumacher, 2005), and finally, TA under uncertainty (Alighanbari & How, 2008; Hu et al., 2015a). Nonetheless, new features have been also introduced, e.g., the possibility of heterogeneous UAVs to perform multiple operations at the same time (Shima & Schumacher, 2009).

On the other hand, the kinematics and dynamics of the UAVs are usually not considered, as opposed to the formulation of path/motion planning and TO problems.

5. UAVRTOP taxonomy

In this section, a taxonomy is proposed in order to help the readers to identify the key differences among various UAV routing problems and guide their research towards the development of new algorithms.

We have identified 20 attributes that are common in the UAV routing and UAV TO literatures. They define common features of UAVs routing problems, such as the kind of fleet, the mission characteristics, the flying dynamic assumptions, etc. Attributes are further grouped into five classes. The first class collects the characteristics of the UAVs, the second class the characteristics of the waypoints, the third class the characteristics of the environment, the fourth class the characteristics of the launching point(s) and the last class the flight duration. These are listed in Table 1. The last two lines of this table are not part of the taxonomy, but they describe the methods and applications described in the reviewed papers.

In the class **UAVs**, the fleet and the way the UAVs' kinematics and dynamics are modelled are considered in the following way:

1. *Multiple* - Defines whether a fleet is available or there is only one vehicle.
2. *Heterogeneous* - Considers whether a problem dealt with identical (homogeneous) UAVs or with a fleet of different types of vehicles (regarding capabilities and technologies, for example).
3. *Fleet Size* - Considers whether the size of the fleet is given or not.
4. *Capacity* - Defines whether vehicles are capacitated or not. A capacitated UAV might have, for example, a maximum flight endurance or maximum payload capacity.
5. *Path Planning* - A PP algorithm is used to construct UAV paths or vehicles' kinematics are ignored (e.g., as for most of the papers on UAV task assignment).
6. *Dubin's* - Dubin's vehicles are used to model the UAVs or not.
7. *EOM* - A set of differential equations has been used to model the vehicles' kinematics and dynamics or not.

The class **Waypoints** presents the attributes of the waypoints (vertices):

8. *Multiple* - Considers whether there are multiple or a single waypoint.
9. *Unordered* - Determines if the waypoints' visiting order should be defined or not.
10. *Visits* - Defines whether the waypoints can be visited multiple times or not.
11. *Constraints* - Is concerned with the presence of special mission constraints. For instance, time-windows, precedence and customer-specific vehicles are common Vehicle Routing Problem (VRP) constraints that often apply to UAV routing problems.
12. *Covering Region* - Specifies whether or not a continuous, but not necessarily convex, region (or airspace) is defined over the waypoint. We believe this characteristic is important to the UAVRTOP since most UAVs' sensors require at least a minimum radius of action to be effective.

The **Environment** class collects the attributes about the environment where the UAVs operate:

13. *3D* - Considers whether the UAVs fly in a 3D or 2D space.
14. *Obstacles* - Is concerned about the presence of fixed or moving obstacles.
15. *Wind* - Determines whether the effects of wind were considered or not.

16. *Real-time* - Defines whether a real-time application has been considered or not. For example, waypoints and tasks arriving at random time and location.

The class **Launching** groups the attributes about the number of launching points (depots):

17. *Multiple* - Specifies whether there exists a single or multiple launching points.
18. *Inter-depot* - Considers the presence of inter-depots (e.g., for refuelling, battery replenishment or maintenance of the UAVs).

Papers are classified in class **Time** according to the way the flight time is considered in their models:

19. *Fixed* - If the UAVs' flight time has been precomputed beforehand. This is a common characteristic of some PP methods (e.g., the Dubin's model).
20. *Variable* - If the UAVs' velocities and flight times have been considered as optimisation variables.

In order to provide a survey of the most relevant papers, we adopted the following procedure. Papers published since 2010 were collected from the following databases: *The Web of Science, Google Scholar* and *ScienceDirect*. We have limited our search to papers published in English. In order to cover the most common types of UAVs, we considered Unmanned Combat Aerial Vehicle (UCAV), Unmanned Aerial Systems (UAS) and *aerial gliders* in our search. The following keywords were used:

- UAV/UCAV/UAS/aerial glider trajectory optimization
- UAV/UCAV/UAS/aerial glider PP
- UAV/UCAV/UAS/aerial glider motion planning
- UAV/UCAV/UAS/aerial glider task assignment
- UAV/UCAV/UAS/aerial glider routing

Papers that apply *Control Theory* to UAVs were not reviewed.

6. Critical review of the recent literature

In this section, we apply our taxonomy to 70 articles published between 2010 and 2016. We have balanced our analysis by considering articles dedicated to UAV TO/PP and UAV routing/TA. Papers devoted to technical and theoretical aspects of UAV flight dynamics were excluded from our analysis. Articles published in journals and conferences have been included in a number that we consider to be representative. Nonetheless, we apologise for any inadvertent omission of relevant papers.

The selected papers have been organised into Table 3. Each line of this table contains one article. Columns correspond to the taxonomy specified in Table 1. Each time an attribute is present in a paper the respective column is marked with "✕". Therefore, empty cells indicate that its corresponding paper has not addressed the attribute indicated by that column. A table with a detailed description of methods and applications for each article can be found in the appendices. Statistics about the Table 3 are provided in Table 2.

Three types of articles can be identified in Table 3. Papers focusing on UAV routing and TA can be identified by the presence of attributes 8 and 9. The second type, which involves papers on UAV PP and TO, correspond to the ones where attribute 9 is absent. The third type consists of articles that integrate UAV routing and TO, which can be identified by the presence of attributes 7 and 9 together.

In Table 3, one can notice that 70% of the articles considered a fixed flight time. This indicates that most of the UAV literature is concerned with routing and PP algorithms, where constant velocity along the trajectories is a common assumption. The EOMs of the vehicles where employed in 17.1% of the articles. In 60% of the papers on PP that applied a Dubin's model (which consist of only 17.1% of the total number of papers), the flight time has been considered as a variable.

Table 1: Characteristics of the problems considered in this literature review.

	UAVs	
1	Multiple	If a fleet of UAVs is available
2	Heterogeneous	If the fleet has different types of UAVs
3	Fleet size	If the size of the fleet is to be defined
4	Capacity	If capacity constraints (e.g., maximum flight endurance, limited battery or fuel, maximum payload) have been considered
5	Path planning	If path planning models have been used or the dynamics of the UAVs has been neglected
6	Dubin's	If a Dubin's vehicle model has been used
7	EOM	If a set of EOMs is used to model the UAVs
	Waypoints	
8	Multiple	Whether there are multiple waypoints or not
9	Unordered	Whether the visiting order is to be defined or not
10	Visits	If waypoints can be visited more than once
11	Constraints	If special VRP-like constraints (e.g., time-windows, precedence) or boundary conditions on waypoints have been considered
12	Covering Region	If there is a continuous covering region around the waypoints
	Environment	
13	3D	If the UAV flies in a 3D environment
14	Obstacles	If there are obstacles
15	Wind	Whether the effects of the wind have been considered or not
16	Real-time	If the problem needs to be solved in real-time
	Launching	
17	Multiple	If there are multiple launching points
18	Inter-depot	If the vehicles can stop at intermediary points for refueling, battery replenishment or maintenance
	Time	
19	Fixed	If the flight time between waypoints can be computed *a priori*
20	Variable	If the velocity of the aerial vehicle is a decision variable
	Approach	The type of algorithm used to solve the path planning or trajectory optimisation problem
	Application	The real-world motivation to solve the problem

Multiple UAVs were considered in 25.7% of the papers dealing with TO and PP. An interesting fact arises counting the number of papers dealing with multiple UAVs and their EOMs. There seems to be a preference for using PP methods and the Dubin's model when a fleet of UAVs is taken into account. One can notice that the preferred strategy is to simplify the physical models of the UAVs so as to make the problem of designing multiple flyable routes more tractable. This happens in 44.4% of the articles on UAV PP and TO and in 93.1% of the articles on UAV routing and TA.

Around 43% of the papers on TO and PP problems dealt with visiting multiple waypoints. Among those, only 8.6% (3 articles) included routing decisions. This gives an indication that integrated routing and TO is yet to be fully investigated in the literature.

Regarding environmental conditions, 40% of the papers have studied three dimensional problems. Obstacle avoidance was tackled in 22.8% of the articles. Only a few studies (10%) included the effects of the wind in the UAVs' trajectories. In addition, only 7.1% of the papers studied real-time applications.

In 76.3% of the papers articles focusing on UAV routing and TA, a fleet of UAVs was considered. A large amount (73.7%) of the articles on routing and TA have either neglected or simplified the dynamics of the UAVs. Only about 8% of the articles have employed the UAVs' EOMs and approximately 21% have modelled the UAVs as Dubin's vehicles. This suggests the preference for simplified vehicle models when dealing with UAV routing.

Table 2 illustrates other differences between the literature on UAV TO/PP and UAV routing/TA. Each row of Table 2 shows four classes that were defined in the proposed taxonomy and their respective frequencies (defined as the number of non-empty cells divided by the total number of cells in that class). One can notice that while the routing/TA literature is able to include more VRP-like attributes (like multiple UAVs and waypoints), the literature on TO/PP is more concerned about modelling environmental aspects. Including environmental attributes (such as obstacles and wind) is usually possible when the UAVs physical models are integrated to the optimisation problem.

In addition, the number of articles using approximations for the flight time between waypoints is higher in the routing literature (81.6%) than in the TO literature (54.3%). This is also related to the preference of simplified physical models in the UAV routing research.

Table 2: Densities per class for each classification

Class	TO/PP	routing/TA
UAVs	23.2%	36.1%
Waypoints	18.8%	58.9%
Environment	32%	13.2%
Launching	7.8%	30.3%

6.1. Integrating routing and trajectory optimisation

Hereafter we highlight the contribution of articles that studied UAV routing and TO in an integrated framework. These papers present alternative frameworks to the UAVRTOP formulation presented in Section 2.

Zhang et al. (2012) investigated the problem of routing a combat UCAV in a 3D environment through stationary ground targets while avoiding no-fly threat areas. In order to succeed on the attacks, the UCAV must fly within the targets' allowable attack region (which consisted of a hollow-cone-like airspace around the target) and respect projectile release attitude and velocity constraints. The UCAV was modelled by a high fidelity 3DOF EOMs which take the wind velocities into account.

In order to solve this problem, Zhang et al. (2012) propose a hierarchical heuristic with two levels. In the first level, the vehicle's state space is discretised into a set of feasible points that intersects the targets' allowable attack region by using a modified probabilistic road map method. Then, for every pair of sampled points not in the same target a TO problem was solved to obtain feasible trajectories (with respect to the vehicle's dynamics and operational constraints) and their respective costs. The second decision level consists of solving a Generalised Traveling Salesman Problem (GTSP) over the network produced in the first level. This is accomplished by transforming the GTSP into an instance of the Asymmetric TSP by means of the noon-bean transformation method. The Lin–Kernighan heuristic was then employed to solve the ATSP. In addition, the authors embedded this algorithm into a real-time framework in order to make this approach more flexible in practical applications. Numerical experiments showed that this approach is computationally intensive. The authors reported that roughly 50 minutes were necessary to solve a test case with three targets and one no-fly zone.

Fügenschuh & Müllenstedt (2015) studied the problem of designing and routing a fleet of heterogeneous UAVs over a set of waypoints. The waypoints have to be selected from a list, where a score was associated to each waypoint. The objective was to maximise the total score (defined as the sum of the individual scores) while minimising the total flight time. The UAVs' motion was modelled by a piece-wise linear dynamics based on Newton's law of motion. The advantage of using this model lies on its simplicity, since the discretised version of these EOMs is also linear. On the other hand, the accuracy of such a model regarding UAVs flight dynamics is limited. In order to represent the range of the UAVs' sensors, the waypoints were considered to rest inside a sphere. A waypoint would be considered visited if a UAV passes through its covering sphere. No-fly zones and collision avoidance among the UAVs were also considered. Finally, different locations could be chosen to launch each UAV.

The authors proposed a Mixed-Integer Non-linear Programming (MINLP) formulation to this problem, which was linearised and could be solved by a commercial Mixed-Integer Linear Programming (MILP) optimisation software. The authors created 8 instances by varying the number of waypoints between 3 and 15, the number of no-fly zones between 0 and 3 and the number of UAVs between 1 and 2. Computational experiments showed that bigger instances with 10 and 15 waypoints could not be solved within one hour. The computation time required to solve smaller problems to optimality varied between 57 and 3400 seconds.

A similar approach was presented by Forsmo (2012). The author applied Newton's second law in order to model the motion of the UAVs. However, constraints on the magnitudes of forces, velocities and yaw rates were also imposed, which increased the complexity of the physical representation of the UAVs. The authors have considered several operational constraints, such as obstacles and collision avoidance. Scenarios with up to two UAVs and multiple waypoints were generated. A MILP formulation was proposed in order to find minimum flight time trajectories visiting all waypoints subject to mission and operational constraints.

Computational experiments were performed over 5 test cases, constructed by varying the number of UAVs (1 or 2), waypoints (6 or 8) and by imposing or not a visiting order. The authors showed that by CPU times could be reduced by decreasing the flight time horizon.

Table 3: Characteristics of the articles on UAV Trajectory Optimisation and Path/Motion Planning Problems

Author(s)	**UAVs**							**Waypoints**					**Env.**				**Depot**		**Time**	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Al-Sabban et al. (2012)							×								×					×
Babel (2011)					×									×					×	
Babel (2012)				×	×								×							×
Bae et al. (2015)				×	×			×											×	
Baiocchi (2014)					×			×					×						×	
Bandeira et al. (2015)					×			×	×			×							×	
Bednowitz et al. (2012)	×				×			×	×		×								×	
Besada-Portas et al. (2010)	×				×			×					×						×	
Besada-Portas et al. (2013)	×				×			×					×						×	
Casbeer & Holsapple (2011)	×	×		×	×			×	×		×						×		×	
Chakrabarty & Langelaan (2011)					×			×					×		×					×
Chen et al. (2016)	×						×	×			×		×	×						×
Choe et al. (2016)	×				×			×					×				×		×	
Cobano et al. (2013)	×				×			×	×		×		×				×		×	
Cons et al. (2014)						×		×	×										×	
Crispin (2016)	×						×						×						×	
Dilão & Fonseca (2013)							×						×							×
Edison & Shima (2011)	×	×				×		×	×										×	
Enright et al. (2015)	×				×	×		×	×		×	×					×		×	
Evers et al. (2014)				×	×			×	×		×					×			×	
Faied et al. (2010)	×				×			×	×		×						×		×	
Filippis et al. (2011)					×									×					×	
Forsmo (2012)	×						×	×	×								×		×	
Fügenschuh & Müllenstedt (2015)	×	×	×	×			×	×	×		×	×	×		×		×			×
Furini et al. (2016)					×			×	×		×			×					×	
Gottlieb & Shima (2015)	×					×		×	×		×			×			×			×
Guerriero et al. (2014)	×		×	×	×			×	×		×						×		×	
Han et al. (2014)				×	×									×					×	
Henchey et al. (2016)						×		×			×	×		×	×	×				×
Huang et al. (2016)	×				×			×	×					×					×	
Hu et al. (2015b)	×			×	×			×	×								×		×	
Jaishankar & Pralhad (2011)					×						×		×	×					×	
Jiang & Ng (2011)	×				×			×	×		×								×	
Kagabo (2010)						×		×					×						×	
Kivelevitch et al. (2016)	×				×			×	×								×		×	
Kumar & Padhi (2013)							×	×												×
Kwak et al. (2013)	×	×			×			×	×	×	×		×						×	
Levy et al. (2014)	×	×		×	×			×	×	×							×	×	×	
Liu et al. (2013)					×									×						×
Liu et al. (2016)					×			×					×						×	
Manyam et al. (2015)	×					×		×	×		×						×			×
Mersheeva (2015)	×			×	×			×	×	×	×						×	×	×	
Mufalli et al. (2012)	×			×	×			×	×										×	
Murray & Karwan (2010)	×	×		×	×			×	×		×						×		×	
Murray & Karwan (2013)	×	×		×	×			×	×	×							×		×	
Myers et al. (2016)						×		×	×					×		×	×			×
Nguyen et al. (2015)					×			×	×	×		×	×							×
Niccolini et al. (2010)	×	×			×			×	×		×		×						×	
Park et al. (2012)				×	×			×	×	×	×								×	
Pepy & Hérissé (2014)							×													×
Pharpatara et al. (2015)						×							×						×	
Rogowski & Maroński (2011)							×													×
Shanmugavel (2013)					×														×	
Silva et al. (2015)							×						×		×					×
Song et al. (2016)	×	×		×	×			×			×						×	×	×	
Stump & Michael (2011)	×			×	×			×	×	×	×						×		×	
Sundar & Rathinam (2014)				×		×		×	×		×		×					×	×	
Techy et al. (2010)						×									×					×
Thi et al. (2012)	×			×	×			×	×										×	
Vilar & Shin (2013)	×				×			×	×		×			×					×	
Wang et al. (2015)	×	×		×	×			×	×		×		×	×			×		×	
Wang et al. (2016)					×								×						×	
Wu et al. (2011)					×						×		×	×					×	
Xu et al. (2017)	×						×				×		×	×						×
Yakıcı (2016)	×			×	×			×	×										×	
Yang et al. (2015)	×		×		×			×	×	×			×				×		×	
Yomchinda et al. (2016)						×					×		×							×
Zhang et al. (2011)					×						×		×	×		×			×	
Zhang et al. (2012)							×	×	×			×	×		×	×				×
Zhang et al. (2014)	×					×		×	×		×	×					×			×

7. Conclusions and Directions for Further Research

The UAVRTOP is a routing problem that takes into account the flight dynamics of UAVs, therefore it is a more realistic approach. This problem arises from the current development of UAV technology and the vast number of applications that these vehicles can be used for. In this paper, we first formalise the UAVRTOP. Next, an introduction to TOPs, VRPs and their variants is provided. In addition, we introduce a taxonomy capable of classifying UAV routing/TA and UAV TO/PP problems according to their most relevant features. This taxonomy includes 20 attributes that are common in the literature. Finally, we apply the proposed taxonomy to 70 recent papers.

The literature on UAVs routing problems has been surveyed and a lack of articles integrating UAV routing and TO has been identified. In particular, the UAVs' flight dynamics is often simplified or neglected. In many cases the behaviour of UAVs cannot be satisfactorily approximated only by their kinematics, like in the case of terrestrial robots (Forsmo, 2012). We believe that integrating the UAVs dynamical systems into routing problems is a key concept for complex operations.

Mathematical formulations and algorithms capable of tackling complex unmanned aerial systems in a routing framework are recently appearing in the literature. A first step in this direction has been made by Zhang et al. (2012), Forsmo (2012) and Fügenschuh & Müllenstedt (2015). However, only problems with limited size have been solved. Therefore, the development of efficient frameworks for solving UAVRTOPs still raises research questions that need to be answered.

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