Coordination of a two-level supply chain with contracts under complete or asymmetric information

Siao-Leu Phouratsamay, Safia Kedad-Sidhoum, Fanny Pascual

Sorbonne Universités, UPMC Univ Paris 06, CNRS, LIP6 UMR 7606, 4 place Jussieu 75005 Paris

Abstract

We consider the coordination of planning decisions of a single product in a supply chain composed of one supplier and one retailer by using contracts. We assume that the retailer has the market power to impose his optimal replenishment plan to the supplier. Our concern is on the minimization of the supplier's cost. In order to influence the retailer's planning decisions, the supplier can propose a side payment to the retailer by offering a contract to the retailer composed of a replenishment plan and a side payment. The focus of analysis is on evaluating how much the supplier can gain by designing contracts under complete or asymmetric information assumption. Different scenarios are identified and explored depending on the side payment coverage. We set the boundary between polynomially solvable and NP-complete cases. Under complete information, we show that for some scenarios the problem of designing contracts can be solved in polynomial time. Whereas if the side payment is on the retailer's holding cost, we prove that the problem of designing contracts is weakly NP-hard. Under asymmetric information, the problem without side payment is still polynomial. When side payment is allowed, we show that the supplier's cost can be arbitrarily large compared to the cost obtained under complete information assumption. We provide an experimental analysis that shows that side payment restricted to the retailer's holding cost is sufficient to decrease the cost of the supplier: the supplier's gain (resp. expected gain) can reach 30.8% (resp. 31%) under complete (resp. asymmetric) information assumption. We extend the experimental analysis in order to evaluate the supplier's gain when more than one retailer is considered.

Keywords: Two-level supply chain coordination, dynamic lot-sizing, contracts, mechanism design, asymmetric information, complexity.

1. Introduction

We address the problem of coordinating planning decisions in a supply chain for a single item. We consider a supply chain with two actors: one supplier and one retailer. The retailer has to satisfy a deterministic demand over a finite planning horizon of T periods. On the one hand, the retailer has to determine a replenishment plan over the horizon by setting the ordering periods and quantities to meet the demands. On the other hand, the supplier has to determine a production plan in order to satisfy the retailer's replenishment plan. Ordering units induces a fixed and a unit ordering cost for each actor. Carrying units in the inventory induces a unit holding cost for each actor.

The total cost of each actor is given by the sum of his total ordering and holding costs over the planning horizon. In this paper, we assume that the retailer has the market power [1, 2] and can impose his optimal replenishment plan to the supplier. If the supplier and the retailer act individually, the supplier will have to satisfy the retailer's optimal replenishment plan. In this case, the cost of the supplier for satisfying the replenishment plan can be very high. However, if the actors decide to collaborate, the supplier can sometimes decrease his cost by proposing another possible replenishment plan to the retailer. This plan has to be optimal for the retailer, since otherwise the retailer will reject it because of his leadership position. We also consider the case where the supplier propose a non-optimal plan for the retailer combined with a side payment to compensate the increase of the retailer's cost. In this paper, we consider the coordination of the supplier's and retailer's planning decisions with a focus on the minimization of the supplier's cost. The literature review reveals that few works have been addressed in the case of dynamic planning decisions over a discrete horizon. The state-of-the art on different aspects of the problem is provided in the following sections.

A natural question when designing contracts is the imbalance of power decision due to a lack of information. The following cases will be considered in our study: at first, we assume that the supplier knows the ordering and the holding costs of the retailer. Therefore, he can compute the value of an optimal plan for the retailer and then propose a take-it-or-leave-it contract to the retailer. The latter can decide to accept or to reject the contract which is composed of both a replenishment plan and a side payment. We analyze the following relevant scenarios:

- No side payment is allowed between the actors of the supply chain.
- The supplier can assume part of the retailer's cost through side payment. It may only concern the holding cost when the supplier is allowed to pay the retailer's holding cost or part of it over the horizon, or it may concern the retailer's total cost when the supplier is allowed to pay the retailer's ordering and holding costs or part of them.

Secondly, the retailer may have private information about his cost structure. In this case, the supplier does not possess full information about the retailer's cost parameters. The absence of information sharing between the actors reduces the supply chain performance [3, 4]. We use a screening game [5] to model this situation. Several types are considered regarding the retailers cost structure. The ordering and holding costs of each type are known as well as the probability that the retailer is of a certain type. The supplier proposes a set of contracts such that the retailer will accept one contract and such that his own expected cost is minimized.

Contributions

The purpose of our work is to evaluate how much the supplier can gain by designing contracts under complete or asymmetric information assumptions on the retailer's cost structure. In both cases, we consider different scenarios with regard to the side payment coverage on the cost parameters. When the supplier has complete information about the retailer's cost parameters, we show that designing contracts can be done in polynomial time either if no side payment is allowed or if side payment is used on the retailer's total cost. However, if the side payment is restricted to the retailer's holding cost, we prove that the problem is weakly NP-hard. Surprisingly, an experimental analysis shows that the side payment on the retailer's holding cost is sufficient to improve significantly the supplier's cost. In the asymmetric information setting, the problem of designing contracts is still polynomial when no side payment is allowed. We show that the supplier's cost can be arbitrarily high compared to the case where complete information is assumed. An experimental analysis shows that under asymmetric information assumption, the supplier's gain can be significant which indicates the usefulness of the cooperation between the actors. In addition, the analysis shows that side payments restricted to the retailer's holding cost provide significant gains for the supplier in this setting as well. Ultimately, we propose an experimental analysis for the multi-retailer problem to evaluate the supplier's gain if more that one retailer are involved in the supply chain.

This paper is organized as follows. In Section 2, we introduce useful notations and recall some results for the classical problem consisting in the minimization of the supply chain cost. The study of the problem when the supplier has complete information about the retailer's costs is addressed in Section 3. A literature review about the coordination of the supply chain with complete information is provided as well as an experimental analysis in order to evaluate the supplier's gain by considering different contracts. In Section 4, we address the problem of designing contracts assuming that the retailer has private information about his cost structure. We provide a literature review on coordination mechanisms under asymmetric information. An experimental analysis is performed to evaluate the supplier's gain when the actors collaborate and to compare the supplier's gain under complete and asymmetric information assumption. Finally, the analysis of the problem with more than one retailer is addressed in Section 5.

2. Preliminaries: minimizing the cost of the supply chain

The cost of the two-level supply chain is equal to the sum of the retailer's and the supplier's total costs. Determining the optimal cost of the supply chain is equivalent to solve a two-level Uncapacitated Lot-Sizing (2ULS) problem which is polynomially solvable [6]. Section 2.1 introduces the notations and gives the mathematical formulation for the 2ULS problem and Section 2.2 recalls the polynomial solving approach proposed by Zangwill [6].

2.1. Mathematical formulation

We denote by d_t the demand that the retailer has to satisfy at period t, d_{ij} represents the cumulative demand from period i to period j with $1 \leq i \leq j \leq T$, $d_{ij} = \sum_{k=i}^{j} d_k$. The set $\mathcal{T}$ stands for the set of all the periods $\{1, \ldots, T\}$. Ordering one unit at period t induces a fixed ordering cost f_t^R (resp. f_t^S) and a unit ordering cost p_t^R (resp. p_t^S) for the retailer (resp. supplier). Carrying one unit from period t to period $t+1$ induces a unit holding cost h_t^R (resp. h_t^S) for the retailer (resp. supplier).

We denote by x_t^R (resp. x_t^S) the retailer's (resp. supplier's) ordering quantity at period t, s_t^R (resp. s_t^S) the retailer's (resp. supplier's) inventory quantity at the end of period t and y_t^R (resp. y_t^S) the setup binary variable at period t at the retailer (resp. supplier) level which is equal to 1 if an order occurs at period t.

The total cost is given by $C^R(y^R, x^R, s^R) = \sum_{t\in\mathcal{T}}(f_t^R y_t^R + p_t^R x_t^R + h_t^R s_t^R)$ for the retailer and similarly by $C^S(y^S, x^S, s^S) = \sum_{t\in\mathcal{T}}(f_t^S y_t^S + p_t^S x_t^S + h_t^S s_t^S)$ for the supplier.

The mathematical formulation of the 2ULS problem is given by:

$$\min \quad C^R(y^R, x^R, s^R) + C^S(y^S, x^S, s^S)$$

$$\text{s.t. } s_{t-1}^R + x_t^R = d_t + s_t^R \qquad \forall t \in \mathcal{T}, \tag{1}$$

$$s_{t-1}^S + x_t^S = x_t^R + s_t^S \qquad \forall t \in \mathcal{T}, \tag{2}$$

$$x_t^R \leq d_{tT} y_t^R \qquad \forall t \in \mathcal{T}, \tag{3}$$

$$x_t^S \leq d_{tT} y_t^S, \qquad \forall t \in \mathcal{T}, \tag{4}$$

$$x_t^S, s_t^S, x_t^R, s_t^R \geq 0 \qquad \forall t \in \mathcal{T} \tag{5}$$

$$y_t^S, y_t^R \in \{0, 1\} \qquad \forall t \in \mathcal{T} \tag{6}$$

Constraints (1) are the balance constraints at the retailer level for satisfying the demand. Constraints (2) are the balance constraints at the supplier level where the demand is the amount ordered at the retailer level. Constraints (3) and (4) force the setup variables to be equal to 1 if an order occurs at period t, *i.e.* $x_t^R > 0$ and $x_t^S > 0$ respectively.

2.2. *Solving the 2ULS problem*

Zangwill [6] shows that the so-called Zero Inventory Ordering (ZIO) property holds at each level of the 2ULS problem, that is there exists an optimal solution such that at each period t an order occurs only if the incoming stock is null. Zangwill proposes a dynamic programming algorithm based on the ZIO property for solving the 2ULS problem in $O(T^3)$ [6]. More recently, Melo and Wolsey [7] provide a dynamic programming algorithm that improves the time complexity to $O(T^2 \log T)$. Even if the best time complexity is provided by the algorithm proposed by Melo and Wolsey, we present the dynamic programming scheme proposed by Zangwill since it is the one that we used to derive a polynomial algorithm for one of the problems addressed in this paper.

For sake of clarity, we assume that the retailer (resp. supplier) level corresponds to the first (resp. second) level. The subscripts and superscripts used for the retailer and the supplier will be replaced by 1 and 2 respectively in what follows.

Let $G_i(t,\alpha,\beta)$, with $1 \leq t \leq \alpha \leq \beta \leq T$ and $1 \leq i \leq 2$ ($G_i(t,\alpha,\beta)$ is equal to $+\infty$ in the other cases), be the minimum cost at level i for supplying $d_{\alpha\beta}$ units available at period t. Three cases have to be considered for computing $G_i(t,\alpha,\beta)$ at a given period t; either the quantity $d_{\alpha\beta}$ is stored until period $t+1$ at level i, or it is ordered at level $i-1$, or part of the quantity $d_{\alpha\beta}$ is stored until period $t+1$ at level i and the complementary part is ordered at level $i-1$. The recursion formula below consider these cases:

- if $d_\alpha > 0$,

$$G_i(t,\alpha,\beta) = \min\{ h_t^i d_{\alpha\beta} + G_i(t+1,\alpha,\beta), f_t^{i-1} + p_t^{i-1} d_{\alpha\beta} + G_{i-1}(t,\alpha,\beta),$$
$$\min_{\alpha+1\leq\gamma\leq\beta} (f_t^{i-1} + p_t^{i-1} d_{\alpha\gamma-1} + G_{i-1}(t,\alpha,\gamma-1) + h_t^i d_{\gamma\beta} + G_i(t+1,\gamma,\beta))\}$$

- if $d_\alpha = 0$, $\quad G_i(t,\alpha,\beta) = \begin{cases} G_i(t,\alpha+1,\beta), & \text{if } \alpha < \beta \\ 0, & \text{if } \alpha = \beta \end{cases}$

The initial conditions for the recursion are defined as:

$$G_1(t,\alpha,\beta) = \begin{cases} 0, & \text{if } t = \alpha = \beta \\ h_t^1 d_{\max(t+1,\alpha),\beta} + G_1(t+1, \max(t+1,\alpha),\beta), & \text{otherwise} \end{cases}$$

A dummy third level is considered in order to setup the total quantity d_{1T} that can be ordered at period 1 at the second level. The inventory cost parameter h_t^3 is null over the planning horizon. The optimal cost of the 2ULS problem is given by $G_3(1,1,T)$.

In Sections 3 and 4, we study some coordination mechanisms between the actors of the supply chain. We first assume in the following section that the supplier has a complete information about the retailer's cost parameters f^R, p^R and h^R.

3. Coordination with complete information

In this section, the aim of the supplier is to design a take-it-or-leave-it contract (x^R, z) where x^R is the replenishment plan proposed to the retailer and z is a side payment such that his total cost including the side payment is minimized. In order to ensure that the retailer will accept the contract, the retailer's cost must not exceed his optimal cost denoted by C^R_{opt} in the sequel. Therefore, the so-called Individual Rationality (IR) constraint must be satisfied. In the sequel, the supplier's total cost is equal to $C^S(y^S, x^S, s^S) + z$. The retailer's total cost is equal to $C^R(y^R, x^R, s^R) - z$.

3.1. Literature review

Most of the works about the coordination of a supply chain under complete information assumption involve the Economic Ordering Quantity (EOQ) problem which is a continuous time model with an infinite planning horizon and a stationary demand. The interested reader is referred to the comprehensive survey by Li and Wang [8]. Many works show the efficiency of coordination mechanism by providing contracts to achieve coordination. Cachon [9] proposes a survey of the works on designing contracts that coordinate multi-level supply chains. However, a few papers deal with the problem with dynamic demand. Recently, Geunes *et al.* [10] consider a two-level supply chain composed of a supplier and a retailer. They use a Stackelberg game where the supplier announces his ordering and/or holding costs to the retailer in order to persuade the retailer to change his replenishment plan. They show that price discount mechanisms can decrease the supplier's cost.

3.2. Designing contracts with side payment on the retailer's total cost

The problem of designing a contrat with side payment on the retailer's total cost is denoted by P_{TC}^{CI} where CI stands for complete information assumption. The supplier has to ensure that the contract is individually rational. The mathematical formulation of the problem P_{TC}^{CI} is given by:

$$\begin{aligned} \min \quad & C^S(y^S, x^S, s^S) + z \\ \text{s.t. } & (1), (2), (3), (4), (5), (6) \\ & C^R(y^R, x^R, s^R) - z \leq C_{opt}^R \qquad \text{(IR)} \\ & z \geq 0 \end{aligned}$$

Constraint (IR) represents the individual rationality constraint which ensures that the retailer's cost associated to the contract (x^R, z) will not exceed his optimal cost C_{opt}^R.

A key observation is that the supplier can decrease his cost by a factor arbitrarily large compared to the case where the side payment is on the retailer's holding cost as illustrated in the following example. The parameters are given by $T = 3$, $d = (0, 1, 1)$, $f^R = f^S = (0, 0, 0)$, $p^R = (1, 0, 1)$, $h^R = (1, 1, 1)$, $p^S = (0, M, M)$ and $h^S = (2M, 2M, 2M)$. The optimal replenishment plan is $\bar{x}^R = (0, 1, 1)$ of cost 1. The optimal production plan for the supplier to satisfy $\bar{x}^R$ is $\bar{x}^S = (0, 1, 1)$ of cost $2M$. If the actors collaborate with a side payment on the retailer's holding cost, the supplier can choose the replenishment plan $\tilde{x}^R = (1, 1, 0)$ of cost 3. The optimal production plan for the supplier to satisfy $\tilde{x}^R$ is $\tilde{x}^S = (1, 1, 0)$ of cost M. By proposing the contract $(\tilde{x}^R, 2)$ to the retailer, the supplier's cost is equal to $M + 2$. If the actors collaborate with a side payment on the retailer's total cost, the supplier can consider the replenishment plan $\bar{x}^R = (2, 0, 0)$ of cost 5. The production plan $\tilde{x}^S = (2, 0, 0)$ of cost 0 to satisfy $\bar{x}^R$ is optimal for the supplier. In that case, the supplier can propose the contract $(\bar{x}^R, 4)$ to the retailer, inducing a cost of 4 instead of $M + 2$ when the side payment is on the retailer's holding cost.

Since the supplier wants to minimize his total cost, the side payment z is exactly equal to the lower bound $C^R(y^R, x^R, s^R) - C_{opt}^R$ given by Constraint (IR). By replacing z by its lower bound, we obtain the mathematical formulation of the 2ULS problem. So, the problem P_{TC}^{CI} is a 2ULS problem which is solvable in polynomial time (see Section 2.2). The supplier's cost is then equal to $C^S(y^S, x^S, s^S) + C^R(y^R, x^R, s^R) - C_{opt}^R$. The retailer's cost is exactly equal to his optimal cost C_{opt}^R. In this case, the contract $(x_{opt}^R, C^R(y_{opt}^R, x_{opt}^R, s_{opt}^R) - C_{opt}^R)$ proposed by the supplier achieves the optimal cost of the supply chain, where x_{opt}^R is the optimal replenishment plan for P_{TC}^{CI}.

3.3. Designing contracts without side payment

When the retailer has several optimal replenishment plans, the supplier chooses among the retailer's replenishment plans of cost C_{opt}^R one that minimizes his cost. We denote by P_{WP}^{CI} the problem of designing a contract without side payment. Its mathematical formulation is given below:

$$\begin{aligned} \min \quad & C^S(y^S, x^S, s^S) \\ \text{s.t. } & (1), (2), (3), (4), (5), (6) \\ & y_t^R \leq x_t^R \qquad \forall t \in \mathcal{T}, & (7) \\ & C^R(y^R, x^R, s^R) \leq C_{opt}^R & \text{(IR)} \end{aligned}$$

Constraints (7) force the retailer's setup variables to be equal to 0 at period t if no order occurs at t. This problem is equivalent to minimizing the supply chain cost where the retailer's total cost cannot exceed C_{opt}^R.

Collaborating can lead to a significant decrease of the supplier's cost. The following example shows that the factor can be arbitrarily large: $T = 2$, $d = (0, 1)$, $f^R = p^R = h^R = f^S = (0, 0)$, $p^S = (0, M)$ and $h^S = (2M, 0)$. There are two optimal replenishment plans $\hat{x}^R = (0, 1)$ and $\bar{x}^R = (1, 0)$ of cost 0. The optimal production plan for the supplier to satisfy $\hat{x}^R$ is $\hat{x}^S = (0, 1)$. The optimal plan for the supplier to satisfy $\bar{x}^R$ is $\bar{x}^S = (1, 0)$: in the former case, the supplier's cost is M while it is 0 in the latter one. Therefore, from the supplier point of view, it is better to satisfy the replenishment plan $\bar{x}^R$. If the actors collaborate, the supplier can propose the contract $(\bar{x}^R, 0)$ to the retailer. The induced cost is 0 instead of M if no collaboration is assumed.

In the following, we propose a polynomial dynamic programming algorithm that solves P_{WP}^{CI}. The algorithm is based on the following decomposition of dominant solutions. There exists an optimal solution such that the ZIO property holds at both levels. Moreover, at the supplier level, if $d_{\alpha\beta}$ units are available at period t, where $t \leq \alpha \leq \beta$, either these units are stored at the supplier level until period $t+1$, or they are completely sent to the retailer at period t, or part of these units is stored at the supplier level until period $t+1$ and the other part is sent to the retailer at period t. At the retailer level, since the ZIO property holds, if $d_{\alpha\beta}$ units are ordered at period t then $s_{t-1}^R = s_\beta^R = 0$ and no order takes place at a period k such that $t < k \leq \beta$. Thus, knowing the retailer's cost for satisfying the demands $d_{\alpha\beta}$ from an order at period t, it is possible to compute the retailer's cost over the entire horizon by computing independently the retailer's minimum cost for satisfying the demands $d_{1,t-1}$ and $d_{\beta T}$.

We first compute the retailer's minimum cost $I(t, k)$ for satisfying the demands from period t to period k, with $1 \leq t \leq k \leq T$. We set the cost $I(t, k)$ to $+\infty$ if $k > t$. For a fixed value t, the cost $I(t, k)$ can be computed by solving an ULS problem in $O(T \log T)$ [11, 12, 13]. Therefore, the cost $I(t, k)$ can be computed in $O(T^2 \log T)$ for all t, k.

Let $H_i(t, \alpha, \beta)$ be the minimum cost of supplying a quantity $d_{\alpha\beta}$ of units available at level i at period t such that the total cost of the retailer does not exceed his optimal cost C_{opt}^R. $H_i(t, \alpha, \beta)$ is equal to $+\infty$ if $t > \alpha$. We recall that the retailer (resp. supplier) level corresponds to the first (resp. second) level. Similarly to Zangwill's algorithm, a dummy third level is added such that its inventory cost h_t^3 is null for all periods t. The computation of $H_i(t, \alpha, \beta)$ follows the one presented in Section 2.2 by setting an infinite cost to the solutions that violate the bound constraint on the retailer's total cost. The cost $H_i(t,\alpha,\beta)$ is computed by the same recursion formula provided in Section 2.2, except for the case where $d_\alpha > 0$ at the second level.

Figures 1, 2 and 3 represent the three cases described above for satisfying the demands from period α to period β where $d_{\alpha\beta}$ units are avaiable at a period t at the supplier level. The nodes correspond to the periods. The vertical inflows of each node correspond to the ordering quantities and the horizontal inflows correspond to the inventory quantities. The components of each term in the computation of $H_2(t,\alpha,\beta)$ when $d_\alpha > 0$ (described thereafter) are depicted in the figures.

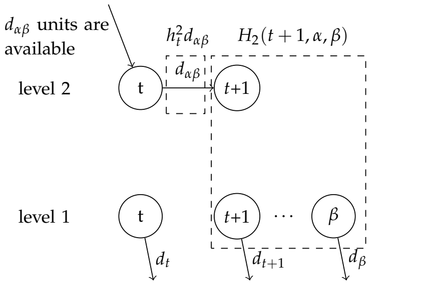

Figure 1: Illustration of the computation of $H_2(t,\alpha,\beta)$ when $d_{\alpha\beta}$ units are stored until period $t+1$ at the supplier level.

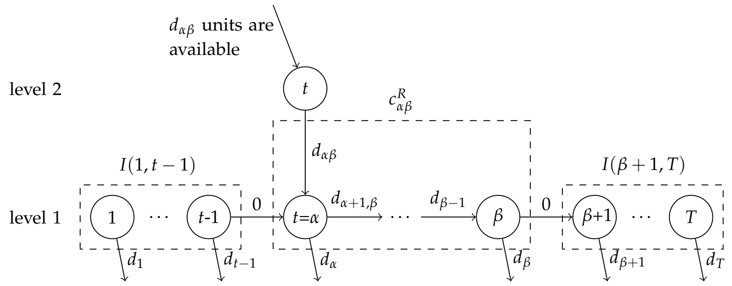

Figure 2: Illustration of the computation of $H_2(t,\alpha,\beta)$ when $t=\alpha$ and $d_{\alpha\beta}$ units are sent to the retailer at period t.

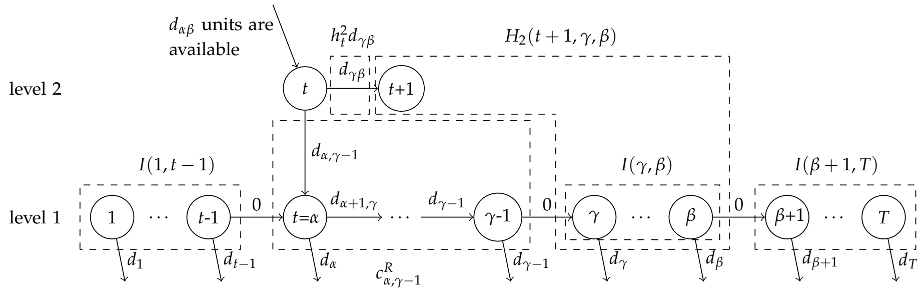

Figure 3: Illustration of the computation of $H_2(t,\alpha,\beta)$ when $t=\alpha$, $d_{\alpha,\gamma-1}$ units are sent to the retailer at period t and $d_{\gamma\beta}$ units are stored until period $t+1$ at the supplier level.

The recusion formula for computing $H_2(t,\alpha,\beta)$ are given by:

- if $d_\alpha = 0$, $\quad H_i(t,\alpha,\beta) = \begin{cases} H_i(t,\alpha+1,\beta), & \text{if } \alpha < \beta. \\ 0, & \text{if } \alpha = \beta. \end{cases}$

- if $d_\alpha > 0$,

$$H_3(t,\alpha,\beta) = \min\{ \quad H_3(t+1,\alpha,\beta), f_t^2 + p_t^2 d_{\alpha\beta} + H_2(t,\alpha,\beta),$$
$$\min_{\alpha+1\le\gamma\le\beta} (f_t^2 + p_t^2 d_{\alpha\gamma-1} + H_2(t,\alpha,\gamma-1) + H_3(t+1,\gamma,\beta)\}.$$

$$H_2(t,\alpha,\beta) = \min\{\, h^2_t d_{\alpha\beta} + H_2(t+1,\alpha,\beta), \underbrace{f^1_t + p^1_t d_{\alpha\beta} + H_1(t,\alpha,\beta)}_{c^R_{\alpha\beta}} + \mathbb{1}(I(1,t-1) + c^R_{\alpha\beta} + I(\beta+1,T)),$$

$$\min_{\alpha+1\leq\gamma\leq\beta} \underbrace{(f^1_t + p^1_t d_{\alpha\gamma-1} + H_1(t,\alpha,\gamma-1)}_{c^R_{\alpha,\gamma-1}} + h^2_t d_{\gamma\beta} + H_2(t+1,\gamma,\beta) + \mathbb{1}(I(1,t-1) + c^R_{\alpha,\gamma-1} + I(\gamma,\beta) + I(\beta+1,T)))\}. \tag{8}$$

where $\mathbb{1}(c)$ is equal to 0 if $c \leq C^R_{opt}$ and $+\infty$ otherwise. In Equation (8), the third term of the minimum $H_2(t+1,\gamma,\beta)$ is either not infinity, which implies that the retailer's cost for satisfying the demands $d_{\gamma\beta}$ computed by $H_2(t+1,\gamma,\beta)$ is equal to $I(\gamma,\beta)$, or it is equal to $+\infty$ which implies that the solution where the demands $d_\gamma,\ldots,d_\mu$ with $\gamma \leq \mu \leq \beta$ are satisfied from an order that occurs after period t is already pruned because the retailer's total cost exceeds C^R_{opt}.

The initialization of the recursion is given by the cost $H_1(t,\alpha,\beta) = G_1(t,\alpha,\beta)$. The optimal cost of the P^{CI}_{WP} problem is equal to $H_3(1,T,T)$. Since the precomputation of $I(t,k)$ is done in $O(T^2 \log T)$, the time complexity of the dynamic programming algorithm is $O(T^3)$.

3.4. Designing contracts with side payment on the retailer's holding cost

The problem studied in this section is denoted by P^{CI}_{HC}. In practice, this side payment is meaningful because the supplier can have consignment stocks [14], *i.e.* the supplier's inventory is located at the retailer level. The mathematical formulation of the problem P^{CI}_{HC} is given by:

$$\begin{aligned}
\min \quad & z + C^S(y^S, x^S, s^S) \\
\text{s.t.} \quad & (1),(2),(3),(4),(5),(6) \\
& C^R(y^R, x^R, s^R) - z \leq C^R_{opt} && \text{(IR)} \\
& z \leq \sum_{t\in\mathcal{T}} h^R_t s^R_t && (9) \\
& z \geq 0
\end{aligned}$$

Constraint (9) ensures that the side payment z covers at most the retailer's holding cost.

By summing up Constraints (IR) and (9), we obtain $\sum_{t\in\mathcal{T}} (f^R_t y^R_t + p^R_t x^R_t) \leq C^R_{opt}$. Since the supplier wants to minimize his total cost, the side payment z is exactly equal to the lower bound $C^R(y^R, x^R, s^R) - C^R_{opt}$ provided by (IR). Thus, the retailer's cost will be equal to C^R_{opt}. The formulation of the problem P^{CI}_{HC} reduces to:

$$\begin{aligned}
\min \quad & C^S(y^S, x^S, s^S) + C^R(y^R, x^R, s^R) - C^R_{opt} \\
\text{s.t.} \quad & (1),(2),(3),(4),(5),(6) \\
& \sum_{t\in\mathcal{T}} (f^R_t y^R_t + p^R_t x^R_t) \leq C^R_{opt} && (10)
\end{aligned}$$

The problem P_{HC}^{CI} comes down to the 2ULS problem with an additional constraint that imposes that the retailer's ordering cost cannot exceed C_{opt}^{R}.

Note that the supplier is be able to decrease his cost by a factor arbitrarily large compared to the case where no side payment is allowed. For example, consider the following instance: $T = 2$, $d = (0, 1)$, $f^R = p^R = f^S = (0, 0)$, $p^S = (0, M)$, $h^R = (1, 0)$ and $h^S = (2M, 0)$. The optimal replenishment plan is given by $\bar{x}^R = (0, 1)$: the retailer's optimal cost is 0. The optimal production plan for the supplier to satisfy $\bar{x}^R$ is given by $\bar{x}^S = (0, 1)$ of cost M. Assuming collaboration between the actors, the supplier can consider the replenishment plan $x^R = (1, 0)$ of cost 1. The optimal production plan for the supplier to satisfy x^R is $x^S = (1, 0)$ of cost 0. The supplier may propose the contract $(x^R, 1)$ to the retailer, inducing a decrease of his cost by M-1.

The following result settles the complexity of the P_{HC}^{CI} problem.

Theorem 1. *The P_{HC}^{CI} problem is NP-hard.*

Proof. We show that the P_{HC}^{CI} problem is NP-hard through a reduction from the knapsack problem which is NP-hard [15]. An instance of the knapsack problem is given by: a set $\mathcal{N} = \{1, \ldots, N\}$ of N items, a utility v_i for all $i \in \mathcal{N}$, a weight w_i for all $i \in \mathcal{N}$, a capacity $W \in \mathbb{N}$ such that $W < \sum_{i=1}^{N} w_i$ and a total value goal $V \in \mathbb{N}$. The recognition version of the knapsack problem requires a *yes/no* answer to the following question: is there a subset $\mathcal{S} \subset \mathcal{N}$ such that $\sum_{i \in \mathcal{S}} w_i \leq W$ and $\sum_{i \in \mathcal{S}} v_i \geq V$?

The recognition version of the P_{HC}^{CI} problem is defined by the following question: is there a production plan x^S and a replenishment plan x^R that satisfy the demands such that $C_O^R \leq C_{opt}^R$, where C_O^R is the retailer's ordering cost, and $C^R(y^R, x^R, s^R) + C^S(y^S, x^S, s^S) \leq K$ where K is a bound on the total cost?

We define an instance $\mathcal{I}$ of the P_{HC}^{CI} problem as follows (see Figure 4). We set $T = 3N$ and denote by $\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3$ the set of periods $\{1, 4, \ldots, 3i-2, \ldots, 3N-2\}$, $\{2, 5, \ldots, 3i-1, \ldots, 3N-1\}$, $\{3, 6, \ldots, 3i, \ldots, 3N\}$, respectively. The retailer setup costs are given by $f_t^R = 0$ for all $t \in \mathcal{T}_1$, $f_t^R = w_{(t+1)/3}$ for all $t \in \mathcal{T}_2$ and $f_t^R = NM$ for all $t \in \mathcal{T}_3$, where $M = \max_i\{W/N + \alpha_i, v_i + w_i + \alpha_i\}$ and $\alpha_i = \max\{0, W/N - w_i\}$. The retailer holding costs are set to $h_t^R = W/N$ for all $t \in \mathcal{T}_1$, $h_t^R = \alpha_{(t+1)/3}$ for all $t \in \mathcal{T}_2$ and $h_t^R = NM$ for all $t \in \mathcal{T}_3$. Finally, the retailer production cost p_t^R is set to zero for all periods t. The supplier costs are set to $f_t^S = M - W/N - \alpha_{(t+1)/3}$ for all $t \in \mathcal{T}_1$, $f_t^S = M - v_{(t+1)/3} - w_{(t+1)/3} - \alpha_{(t+1)/3}$ for all $t \in \mathcal{T}_2$ and $f_t^S = NM$ for all $t \in \mathcal{T}_3$. The supplier holding cost is $h_t^S = NM$ for all periods t. Finally, the supplier production cost p_t^S is set to zero for all periods t. The demand d_t is null for all $t \in \mathcal{T}_1 \cup \mathcal{T}_2$, and equal to 1 for all $t \in \mathcal{T}_3$. Finally, we set the bound K to $NM - V$.

An illustration of the instance is given in Figure 4. The nodes represent the periods at each level. We associate the fixed ordering cost to each vertical edge and the holding cost to each horizontal edge. In addition, the external demands at the retailer level are representing by outflow edges at each period.

We prove that the recognition version of the knapsack problems has a *yes* answer if and only if the above described

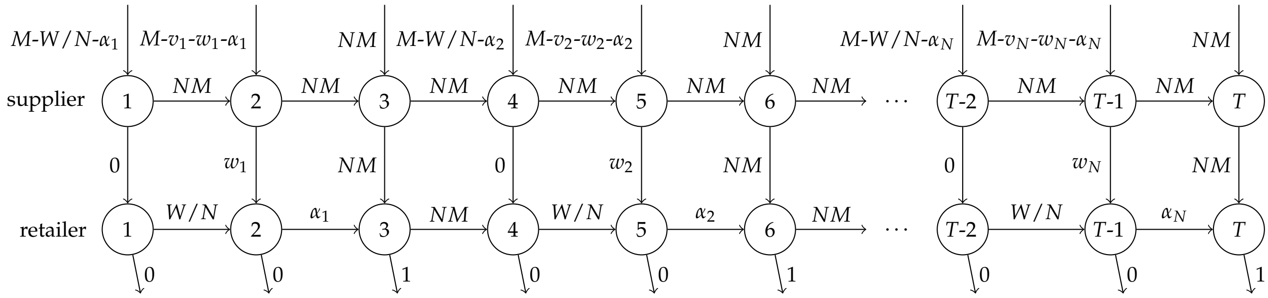

Figure 4: Instance $\mathcal{I}$ of the P^{CI}_{HC} problem where the fixed ordering cost is associated to a vertical edge and the holding cost to a horizontal edge.

instance of the recognition version of the P^{CI}_{HC} problem is a *yes* answer as well.

We first start by showing that the retailer's optimal cost C^R_{opt} is equal to W for instance $\mathcal{I}$. We show in what follows that the retailer can order only at a period $t \in \mathcal{T}_1 \cup \mathcal{T}_2$ and at most one unit per order. The cost for satisfying a demand d_t where $t \in \mathcal{T}_3$ from an order at period t is equal to $f^R_t = NM$. The one for satisfying d_t from an order at period $t-1 \in \mathcal{T}_2$ is equal to $f^R_{t-1} + h^R_{t-1} = w_{(t+1)/3} + \alpha_{(t+1)/3} < NM$. Finally, the retailer's cost for ordering the demand d_t at period $t-2 \in \mathcal{T}_1$ is equal to $h^R_{t-2} + h^R_{t-1} = W/N + \alpha_{(t+2)/3} < NM$. Then, in order to minimize his cost, the retailer will place an order either at period $t-1 \in \mathcal{T}_1$ or $t-2 \in \mathcal{T}_2$. Moreover, since $h^R_t = NM$ for $t \in \mathcal{T}_3$, the retailer can only store between a period $t \in \mathcal{T}_1$ and a period $k \in \mathcal{T}_3$, and he stores at most one unit.
If $w_i < W/N$, the retailer orders one unit at period $3i-1 \in \mathcal{T}_2$ which induces a cost of $w_i + \alpha_i = W/N$ (if the retailer orders at period $3i-2 \in \mathcal{T}_1$, his total total is equal to $f^R_{3i-2} + h^R_{3i-2} + h^R_{3i-1} = W/N + \alpha_i > W/N$ since $\alpha_i = W/N - w_i > 0$). In the case where $w_i \geq W/N$, the retailer orders one unit at period $3i-2 \in \mathcal{T}_1$ which induces a cost of W/N since $\alpha_i = 0$. Since there are N demands to satisfy, there are N ordering periods of cost W/N. Then, the retailer's optimal cost C^R_{opt} is equal to W.

Suppose that we have a solution $\mathcal{S}$ to the knapsack problem. Let us show that there is a solution to the corresponding instance of the P^{CI}_{HC} problem. Figure 5 represents the solution of problem P^{CI}_{HC} corresponding to an instance of the knapsck problem where $N = 3$, $w_1 + w_3 \leq W$ and $v_1 + v_3 \geq V$. Only the active edges are represented in the figure, *i.e.* the edges corresponding to an ordering period or a strictly positive inventory. For each item $i \in \mathcal{S}$, the supplier and the retailer order one unit at period $t = 3i-1 \in \mathcal{T}_2$ ($y^S_t = y^R_t = 1$, $x^S_t = x^R_t$=1). The retailer stores the unit until period $3i$ to satisfy the demand at period $3i$ ($s^R_t = 1$). For each item $i \notin \mathcal{S}$, the supplier and the retailer order one unit at period $t = 3i-2 \in \mathcal{T}_1$ in order to satisfy the demand at period $3i$ ($y^S_t = y^R_t = 1$ and $x^S_t = x^R_t = 1$). The retailer stores the unit until period $3i$ to satisfy the demand at period $3i$ ($s^R_t = s^R_{t+1} = 1$). Since $p^R_t = 0$ for $t \in \mathcal{T}$ and $f^R_t = 0$ for $t \in \mathcal{T}_1$, the retailer's ordering cost is equal to $C^R_O = \sum_{i\in\mathcal{S}} f^R_{3i-1} y^R_{3i-1} = \sum_{i\in\mathcal{S}} w_i$ which is less than or equal to $W = C^R_{opt}$. The retailer's total cost is equal to $C^R = C^R_O + \sum_{i\in\mathcal{S}}(w_i + \alpha_i) + \sum_{i\in\mathcal{S}} h^R_{3i-1} s^R_{3i-1} + \sum_{i\notin\mathcal{S}}(h^R_{3i-2} s^R_{3i-2} + h^R_{3i-1} s^R_{3i-1}) = \sum_{i\in\mathcal{S}}(w_i + \alpha_i) + \sum_{i\notin\mathcal{S}}(W/N + \alpha_i)$. Since $p^S_t = 0$ for $t \in \mathcal{T}$, the supplier's total cost is equal to $C^S = \sum_{i\in\mathcal{S}} f^S_{3i-1} y^S_{3i-1} + \sum_{i\notin\mathcal{S}} f^S_{3i-2} y^S_{3i-2} = \sum_{i\in\mathcal{S}}(M - v_i - w_i - \alpha_i) + \sum_{i\notin\mathcal{S}}(M - W/N - \alpha_i)$. Thus, the total cost of the supply chain is equal to $C^R + C^S = \sum_{i\in\mathcal{S}}(w_i + \alpha_i) + \sum_{i\notin\mathcal{S}}(W/N + \alpha_i) + \sum_{i\in\mathcal{S}}(M - v_i - w_i - \alpha_i) + \sum_{i\notin\mathcal{S}}(M - W/N - \alpha_i) = \sum_{i\in\mathcal{S}}(M - v_i) + \sum_{i\notin\mathcal{S}} M = \sum_{i\in N} M - \sum_{i\in\mathcal{S}} v_i \le NM - V$. Therefore, there exists a solution of the P^{CI}_{HC} problem for instance $\mathcal{I}$.

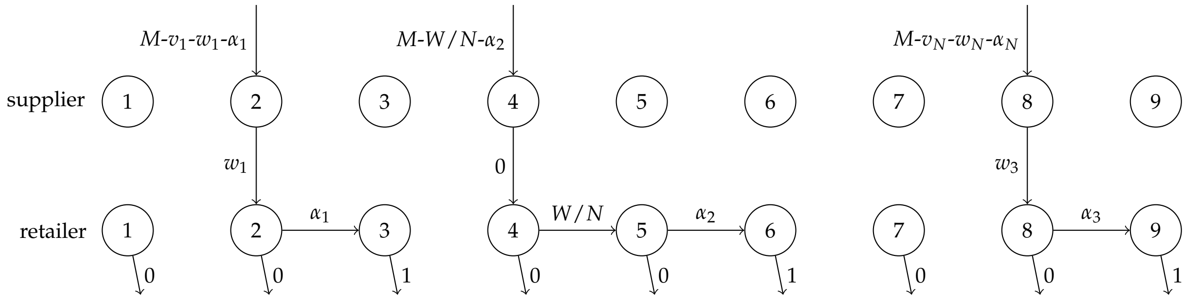

Figure 5: Solution of the P^{CI}_{HC} problem where $N = 3$, $w_1 + w_3 \le W$ and $v_1 + v_3 \ge V$

Conversely, suppose that there exists a solution of the P^{CI}_{HC} problem for instance $\mathcal{I}$. Since $h^S = NM$, if the supplier sets an order, then the retailer will order at the same period ($y^S_t = y^R_t$ and $x^S_t = x^R_t$ for all t), otherwise, there will be a period t where $s^S_t > 0$ and the cost will be greater than $NM - V$. Moreover, since the retailer can order at most one unit at a period $t \in \mathcal{T}_1 \cup \mathcal{T}_2$ (and thus store at most one unit), then in order to satisfy a demand at period $t \in \mathcal{T}_3$, the supplier and the retailer order at most one unit from period $t-2 \in \mathcal{T}_1$ or $t-1 \in \mathcal{T}_2$. Let us show that the subset of ordering periods $\mathcal{S} \subset \mathcal{T}_2$ gives a positive answer to the knapsask problem. In this solution, item i is selected if and only if there is an order at period $3i-1 \in \mathcal{S}$. Since $p^R_t = 0$ for $t \in \mathcal{T}$ and $f^R_t = 0$ for all $t \in \mathcal{T}_1$, we have $C^R_O = \sum_{t\notin\mathcal{S}} f^R_t y^R_t + \sum_{t\in\mathcal{S}} f^R_t y^R_t = \sum_{t\in\mathcal{S}} w_{(t+1)/3}$. By definition of the P^{CI}_{HC} problem, $C^R_O \le C^R_{opt}$. Therefore, $C^R_O = \sum_{t\in\mathcal{S}} w_{(t+1)/3} \le W$. Moreover, since $p^R_t = p^S_t = 0$ for $t \in \mathcal{T}$ and $f^R_t = 0$ for all $t \in \mathcal{T}_1$, $C^R(y^R, x^R, s^R) + C^S(y^S, x^S, s^S) = \sum_{t\notin\mathcal{S}}(f^S_t y^S_t + f^R_t y^R_t + h^R_t s^R_t + h^R_{t+1} s^R_{t+1}) + \sum_{t\in\mathcal{S}}(f^S_t y^S_t + f^R_t y^R_t + h^R_t s^R_t) = \sum_{t\notin\mathcal{S}} M + \sum_{t\in\mathcal{S}}(M - v_{(t+1)/3}) = NM - \sum_{t\in\mathcal{S}} v_{(t+1)/3}$. Similarly, by definition of the P^{CI}_{HC} problem, we have $C^R(y^R, x^R, s^R) + C^S(y^S, x^S, s^S) \le NM - V$. This implies that $\sum_{t\in\mathcal{S}} v_{(t+1)/3} \le V$. Thus, the subset $\mathcal{S}$ gives a positive answer to the knapsack problem.

□

3.5. Experimental analysis

In order to evaluate the decrease of the supplier's cost, we have carried out an experimental analysis. The tests were performed using concert technology of CPLEX 12.6 on a machine running under Debian x86_64 GNU/Linux with Intel Xeon(R) E5-1630 v3 3.70GHz processor and 8Gb of RAM memory.

The instance generation scheme follows the one proposed in [16]. Unless otherwise mentioned, the cost parameters used in our experiments are not stationary and generated using a uniform distribution. We recall that a cost parameter is stationary if it is constant over the planning horizon. For our experiments, we define four classes of instances A, B, C, D described in Table 1. The class A (resp. B) represents the case where h^S is small (resp. large) and h^R is small. The class C (resp. D) represents the case where h^S is small (resp. large) and h^R is large. In order to compare the results under both the complete and the asymmetric information assumptions (see Section 4.4), parameter p^R is generated in $[1, 20] \cup [70, 90]$. Different cost structures have been considered for p^S in a preliminary work. The results show that: for $p^S \in [1, 10]$ or $p^S \in [1, 50]$, the supplier's gain when p^S is stationary is significantly smaller than the one obtained when p^S is not stationary. For $p^S \in [40, 50]$, the supplier's gain does not depend on the stationarity assumption of p^S. Moreover, when $p^S \in [1, 10]$ or $p^S \in [40, 50]$, the supplier's gain is smaller than the one observed when $p^S \in [1, 50]$. We consider T periods with $T \in \{5, 10, 20, 30\}$, the demands are randomly generated from the uniform distribution $[0, 300]$ and the cost parameters are randomly chosen in the discrete intervals described in Table 1 where $\delta = 500$. This value has been set following preliminary results which show that other values do not change the qualitative results of the analysis. The supplier's fixed ordering costs are given by $f_t^S = \delta p_t^S$ for all $t = 1, \ldots, T$. For each class of instances, we randomly generate 100 quintuplets (p^S, h^S, f^R, h^R, d) and for each quintuplet, we randomly generate two costs p^R such that the first one is in $[1, 20]$ and the second one in $[70, 90]$. Thus, in total, we have 200 instances for each class of instances. For example, an instance of the class A is generated such that: at each period t, d_t is randomly chosen in $[0, 300]$, h_t^S is a random integer in $[0, 6]$, h_t^R is a random integer in $[0, 6]$, $f_t^R = \delta c$ where c is randomly chosen in $[1, 100]$, p^S is randomly chosen in $[1, 50]$, p_t^R is a random integer in $[1, 20]$.

Class	h^S	p^S	h^R	p^R	f^R
A	[0,6]	[1,50]	[0,6]	[1,20] ∪ [70,90]	$\delta\times$[1,100]
B	[12,18]				
C	[0,6]	[1,50]	[12,18]	[1,20] ∪ [70,90]	$\delta\times$[1,100]
D	[12,18]				

Table 1: Description of the classes of instances

To asses the supplier's gain in percentage, we compute $(1 - c_P^S/c^S) \times 100$ where c_P^S is the supplier's cost for each of the addressed problems (P_{WP}^{CI}, P_{HC}^{CI} or P_{TC}^{CI}), and c^S is the supplier's cost when the retailer imposes his optimal replenishment plan. If several optimal plans exist, we choose the one that is the less profitable for the supplier. The results are provided in Tables 2 and 3 where the column *Mean* (resp. *Max*) represents the average (resp. maximum) value of the supplier's gain over the 200 instances for each class of instances. Some preliminary experiments show that the supplier's gain in average for the problem without side payment P_{WP}^{CI} is smaller than 1% for every class of instances. Thus, the results for this problem are not reported in Tables 2 and 3.

Table 2 presents the supplier's gain for the problems P_{HC}^{CI} and P_{TC}^{CI} where $T = 5$. The results show that the difference of the supplier's gain between P_{HC}^{CI} and P_{TC}^{CI} is at most 2.2 percentage points. Similar results for a larger

	P^{CI}_{HC}		P^{CI}_{TC}	
Class	Mean	Max	Mean	Max
A	1.2	37.3	2	37.3
B	3.5	81.8	5.7	81.8
C	0.6	23.6	0.6	23.6
D	7.2	75.5	7.4	75.5

Table 2: Comparison of the supplier's gain in average (%) between the problems P^{CI}_{HC} and P^{CI}_{TC} where $T = 5$

horizon allow us to say that a side payment on the retailer's holding cost is sufficient in order to decrease the supplier's cost.

	$T = 10$		$T = 20$		$T = 30$	
Class	Mean	Max	Mean	Max	Mean	Max
A	5.9	54.6	12.2	63.3	12.8	49.9
B	15.7	74.7	27.3	75.4	30.8	66.4
C	2.2	31.9	3	27.1	3.3	24
D	11.5	56.9	14	40.5	14.1	53.8

Table 3: Supplier's gain in average (%) for the problem P^{CI}_{HC}

Table 3 presents the supplier's gain in average for each class of instances when side payment is allowed on the retailer's holding cost for larger values of T. The results show that for most of the class of instances, the supplier's can decrease his cost. In the case where h^S is small and h^R is large (class C), the results show that the supplier's cannot decrease his cost significantly. For $T = 30$, the supplier's gain is 3.3% in average for the class C, whereas it is at least 12.8% for the other classes. This can be explained by the fact that the supplier has no incentive to put more units in the retailer's inventory and to cover part of his cost because $h^S < h^R$. Finally, we can also notice that the supplier's gain observed for the class B is higher than the other classes (for $T = 30$ the supplier's gain is equal to 30.8% whereas the second highest gain is equal to 14.1% for the class D). Since $h^R < h^S$, the supplier will have an incentive to propose a contract where the units are stored at the retailer's level and pay part of the retailer's holding cost.

Figure 6 gives a representation of the results as boxplots. A boxplot can be analyzed in the following way: the middle line represents the median, the borders of the boxplot represent the 25th and the 75th quantile denoted by Q_1 and Q_3 respectively, the "whiskers" represent the maximum and the minimum value such that it is less than Q_3 plus 1.5 times the length of the boxplot and greater than Q_1 minus 1.5 times the length of the boxplot respectively, the outliers represented by points correspond to the values that are greater than Q_3 plus 1.5 times the length of the boxplot or smaller than Q_1 minus 1.5 times the length of the boxplot. We can observe that for the class B, the supplier's gain is

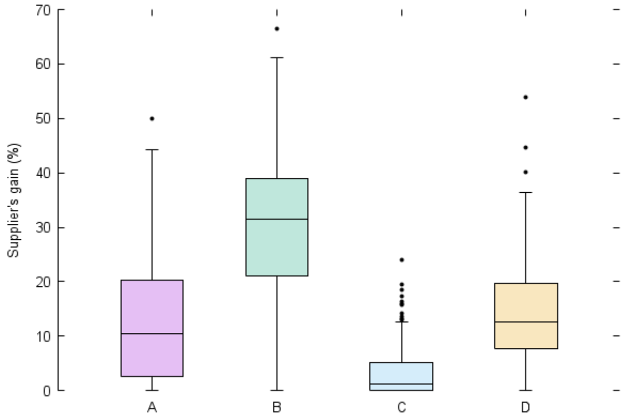

Figure 6: The supplier's gain of the problem P^{CI}_{HC} for each class of instances when $T = 30$

at least equal to 32% for half of the instances and 38% for 25% of the instances.

4. Coordination with asymmetric information

In this section, we consider that the retailer has private information about his cost structure. The supplier assumes that a retailer has N possible types. These types represent different retailer's profiles that the supplier may supply. We denote by $\mathcal{N} = \{1, \ldots, N\}$ the set of types. The real type of the retailer is necessarily in the set $\mathcal{N}$. Moreover, the supplier knows the probability P_i for a retailer to be of type $i \in \mathcal{N}$ ($\sum_{i\in\mathcal{N}} P_i = 1$). Contrary to the case where the supplier has a complete information about the retailer's cost parameters, a set of contracts will be proposed to the retailer, each one is associated to a type. Then, the retailer may choose one contract in this set or reject all the contracts. This process is called a screening game [5].

Under the asymmetric information assumption, it is possible that the retailer does not report his type truthfully. Indeed, a retailer of type i may have an incentive to choose a contract designed for a retailer of type j that minimizes his cost. We say that there is no partial verification when for each type, the set of types that the retailer of type i can report is the entire set $\mathcal{N}$. When there is no partial verification, the revelation principle [17] states that there always exists a set of optimal contracts for which the retailer will truthfully choose the contract that corresponds to his real type. Therefore, we can focus our study on the set of contracts which ensures that the retailer will report his true type. We say that these contracts are Incentive Compatible (IC).

In order to differentiate the types, we add a subscript to the notations defined in Section 2. The ordering (resp. replenishment) quantity at period t for the type i is given by x^S_{it} (resp. x^R_{it}). The inventory quantity at the end of period t at the supplier (resp. retailer) level is denoted by s^S_{it} (resp. s^R_{it}) for the type i. The binary variables associated to the setup at period t for the supplier (resp. retailer) is given by y^S_{it} (resp. y^R_{it}) for the type i. The side payment for the retailer of type i is z_i. The supplier wants to minimize his expected cost $\sum_{i\in\mathcal{N}} P_i(C^S_i + z_i)$ where $C^S_i = \sum_{t\in\mathcal{T}}(f^S_t y^S_{it} + p^S_t x^S_{it} + h^S_t s^S_{it})$

is the supplier's total cost associated to the retailer of type i. We denote by $C^R_{ij} = \sum_{t\in\mathcal{T}}(f^R_{it}y^R_{jt} + p^R_{it}x^S_{jt} + h^R_{it}s^R_{jt})$ the retailer's total cost when the retailer's real type is i and the type selected by the retailer is j. The optimal cost of the retailer of type i is denoted by $C^{R_i}_{opt}$.

4.1. Literature review

Many authors study the EOQ problem in a two-player supply chain context with asymmetric information. Corbett and De Groote [18] consider an EOQ problem where the supplier does not know the retailer's holding cost. In order to influence the retailer's behavior, the supplier proposes a set of optimal contracts with quantity discounts to the retailer. However, for the EOQ problem, if the supplier knows the retailer's ordering cost and his optimal ordering quantity, he can deduce the retailer's holding cost. Thus, the information asymmetry in this setting is not meaningful [19]. Sucky [19] study the EOQ problem where the supplier does not know the retailer's ordering and holding costs. He proposes to use a screening model: the retailer has two types associated to the unknown cost parameters in order to improve the supply chain cost. The supplier offers a set of contracts composed of a side payment and a replenishment plan in order to influence the retailer's decision. His model leads to a non-convex problem for which he characterizes sets of contracts that are candidate to be an optimal solution. Pishchulov and Richter [20] consider the same model and complete Sucky's analysis of the optimal sets of contracts. More recently, Kerkkamp *et al.* [21] consider a general number of retailer's types for the EOQ problem contrary to the works of [19, 20]. They consider that the retailer has only one private cost parameter that corresponds to his holding cost. The purpose of their work is to design a menu of contracts for each type of retailer by using a screening game. The authors propose structural properties of the optimal solutions by looking at a directed graph associated to the individual rational and the incentive compatible constraints. They give a sufficient condition to guarantee that the menu of contracts in an optimal solution is composed of a unique contract for each retailer's type. In the specific case of two retailer's types, they give a complete characterization of the unique optimal menu of contracts.

The following works address the problem with asymmetric information and dynamic demand. Mobini *et al.* [22] consider the two-level supply chain problem for minimizing the supplier's cost. They use a screening model by considering several types contrary to most of the papers in the literature which only consider two types. In the mathematical formulation of the problem, the number of incentive compatible constraints which ensure that the retailer will always reveal his real type is quadratic. The purpose of their paper is to reduce the number of incentive compatible constraints. In the case where the private costs are stationary, they show that only a linear number of incentive compatible constraints should be considered in the model. Dudek and Stadtler [23] propose to coordinate a two-level supply chain with one supplier, one retailer and several items using a negotiation based model. Each actor only knows his own cost parameters. They consider a centralized method where the planning decisions are synchronized by a coordinator in order to minimize the supplier cost. The synchronization is made from information exchange about ordering proposals and impacts on local costs. A side payment is allowed between the actors in order to compensate the inscrease of the retailer's cost. They extend their work by considering one supplier and several retailers [24].

Lee and Kumara [25] study the problem with one supplier and several retailers. They coordinate the actor's planning decisions by using auction markets. The actors do not have to reveal their ordering costs, their ordering capacity and the external demand during the auction process. At each step of the auction, the revealed information for each actor concern the inventory (for example the inventory costs).

4.2. Designing contracts without side payment

The problem of minimizing the supplier's cost without side payment under the asymmetric information assumption is referred to as P^{AI}_{WP} where AI stands for asymmetric information. Since side payment is not allowed, the contracts proposed by the supplier are necessarily incentive compatible. Indeed, the supplier wants to minimize his expected cost by determining a contract $(x_{R_i}, 0)$ for each type i that minimizes his cost such that the cost of the replenishment plan x_{R_i} is equal to $C^{R_i}_{opt}$. Thus, a retailer of type i has no incentive to choose a contract that is designed for another type. The problem is then decomposed into N independent subproblems. Each subproblem is a coordination problem with complete information P^{CI}_{WP} addressed in Section 3.3 which gives the optimal replenishment plans $x^{R_i}_{opt}$ of cost $C^{R_i}_{opt}$ that minimizes the supplier's cost. The supplier will then propose N contracts $(x^{R_i}_{opt}, 0)$ for each $i \in \mathcal{N}$ to the retailer.

4.3. Designing contracts with side payment

The problem of minimizing the supplier's cost with side payment on the retailer's holding (resp. total) cost under the asymmetric information assumption is referred to as P^{AI}_{HC} (resp. P^{AI}_{TC}) where AI stands for asymmetric information. When side payment is allowed under the asymmetric information assumption, we have to ensure that the set of contracts are individually rational and incentive compatible.

The mathematical formulation of the problem P^{AI}_{HC} is given by:

$$\min \sum_{i\in\mathcal{N}} P_i(z_i + C^S_i)$$

$$\text{s.t. } s^R_{i,t-1} + x^R_{it} = d_t + s^R_{it} \qquad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}, \tag{11}$$

$$s^S_{i,t-1} + x^S_{it} = x^R_{it} + s^S_{it} \qquad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}, \tag{12}$$

$$x^R_{it} \leq M_t y^R_{it} \qquad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}, \tag{13}$$

$$x^S_{it} \leq M_t y^S_{it} \qquad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}, \tag{14}$$

$$y^R_{it} \leq x^R_{it} \qquad \forall i \in \mathcal{N}, \forall t \in \mathcal{T}, \tag{15}$$

$$C^R_{ii} - z_i \leq C^{R_i}_{opt} \qquad \forall i \in \mathcal{N}, \tag{IR}$$

$$C^R_{ii} - z_i \leq C^R_{ij} - z_j \qquad \forall i \in \mathcal{N}, \forall j \in \mathcal{N}, \tag{IC}$$

$$z_i \leq \sum_{t\in\mathcal{T}} h^R_{it} s^R_{it} \qquad \forall i \in \mathcal{N}, \tag{16}$$

$$x^S_{it}, x^R_{it}, s^S_{it}, s^R_{it} \geq 0 \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N},$$

$$y^S_{it}, y^R_{it} \in \{0, 1\} \qquad \forall t \in \mathcal{T}, \forall i \in \mathcal{N}$$

$$z_i \geq 0 \qquad \forall i \in \mathcal{N}$$

Constraints (11) and (12) are the inventory balance constraints for each type $i \in \mathcal{N}$ at each level of the supply chain, at each period $t \in \mathcal{T}$. Constraints (13) and (14) ensure that the setup variable is equal to 1 if there is an order for each type $i \in \mathcal{N}$, at each period $t \in \mathcal{T}$. Since we do not minimize the retailer's cost, Constraints (15) ensure that the setup variable is equal to 0 if there is no order at period $t \in \mathcal{T}$ for a type $i \in \mathcal{N}$. Constraints (IR) represent the individual rationality constraint for each type $i \in \mathcal{N}$. Constraints (IC) represent the incentive compatible constraints which ensure that a retailer of type i will not have an incentive to choose a contract designed for another retailer's type j. Constraints (16) ensure that for each retailer's type $i \in \mathcal{N}$, the side payment is only on the retailer's holding cost. The mathematical formulation of the problem P^{AI}_{TC} can be derived from the above described model by relaxing Constraint (16).

As shown when complete information is assumed (see Sections 3.4 and 3.2), the supplier's cost can be arbitrarily large. This is true even if the side payment is only assumed on the retailer's holding cost.

Since the problem P^{CI}_{HC} is NP-hard, the problem P^{AI}_{HC} is at least NP-hard. The complexity of the problem P^{AI}_{TC} is an open question. However, if the incentive compatible constraint is relaxed, this problem can be decomposed into N problems for which complete information can be assumed and then the problem is solvable in polynomial time (Sections 3.2).

We show that when the incentive compatible constraint is assumed, the cost of the supplier can be arbitrarily high compared to the case with complete information.

Property 1. *For any $c > 1$, there is no c-approximate truthful mechanism for P^{AI}_{HC} and P^{AI}_{TC}.*

Proof. Let $c > 1$. We consider the following instance where the retailer's holding cost is private: $T = 2$, $N = 2$, $d = (0,1)$, $f^R_i = p^R_i = (0,0)$ for $i \in \{1,2\}$, $h^R_1 = (1,0)$, $h^R_2 = (c,0)$, $p^S = (0,0), f^S = (0,c^2)$ and $h^S = (c^2,0)$. The probability that the retailer is of type 1 is $P_1 = 1 - 1/c$, and thus the probability that the retailer is of type 2 is $P_2 = 1 - P_1 = 1/c$. The optimal replenishment plan for each retailer's type is $x^{R_1}_{opt} = x^{R_2}_{opt} = (0,1)$ of cost $C^{R_1}_{opt} = C^{R_2}_{opt} = 0$. The supplier's cost for satisfying this replenishment plan is equal to c^2. When the supplier knows the retailer's cost, the supplier will propose contracts to the retailer such that the retailer's cost is his optimal cost and the supplier's cost is minimized. If the retailer's type is 1, the supplier will propose a contract (x^R_1, z_1) such that $x^R_1 = (1,0)$ and $z_1 = 1$. The retailer's cost including the side payment is equal to 0, and the supplier's cost is equal to 1, i.e. the cost of his optimal plan $x^S = [1,0]$ to satisfy x^R_1 which is equal to 0, plus the side payment z_1. Likewise, if the retailer's type is 2, the supplier will propose the following contract (x^R_2, z_2) where $x^R_2 = (1,0)$ and $z_2 = c$. The retailer's cost is then equal to 0 and the supplier's cost is equal to c. Therefore, assuming complete information, the supplier's expected cost is equal to $P_1 + P_2 c = (1 - 1/c)1 + 1 < 2$.

When asymmetric information on the retailer's cost is assumed, the supplier should consider the following contracts. We only consider the contracts that are interesting for the supplier with respect to his cost:

- Contract 1: the supplier proposes the replenishment plan $x^R = (0,1)$ which is the optimal replenishment plan for each type of retailer. Thus, no side payment is needed. The cost of the supplier is then equal to c^2.
- Contract 2: the supplier proposes the replenishment plan $x^R = (1,0)$ with a side payment equal to c. Both types of retailer have an incentive to choose this contract; the retailer of type 1 will have a negative cost and the retailer of type 2 will have a cost equal to 0. The supplier's cost related to this contract is equal to c.
- Contract 3: the supplier proposes the replenishment plan $x^R = (1,0)$ with the side payment equal to 1. In that case, the only retailer that will have an incentive to choose this contract is the retailer of type 1, his cost will be equal to 0. The supplier's cost is then equal to 1.

The best solution that satisfies Constraints (IC) for the supplier is to propose Contract 2 to the retailer. The supplier's expected cost is then equal to $P_1 c + P_2 c = c$. Indeed, the supplier could also propose another set of contracts such as Contract 1 (to retailers of type 2) and Contract 3 (to retailers of type 1). But, in this case, the supplier's expected cost will be equal to $P_1 1 + P_2 c^2 = (1 - 1/c) + c > c$. Therefore, assuming asymmetric information, the supplier's expected cost is equal to c whereas it is less than 2 under complete information assumption. The supplier increases his cost by a factor larger than $c/2$ under the asymmetric information assumption. Then, when c tends to the infinity, this ratio can be as large as wanted. □

Note that both the fixed ordering cost and the unit ordering cost are null in the instance considered in the proof. Then, the above proposition holds even if the side payment is allowed only on the retailer's holding cost.

4.4. Experimental analysis

An experimental analysis is provided in order to evaluate the supplier's expected gain with regard to the different scenarios. We consider that either f^R, p^R or h^R is private and use an extended mathematical formulation based on the facility location model for the two-level lot-sizing problem proposed by Wolsey and Pochet [26] to solve the addressed problems.

We generate the instances based on the classes described in Table 1. For every class of instances, we create 100 instances with $T \in \{10, 20, 30\}$, $N \in \{2, 10\}$. The probability P_i for $i \in \mathcal{N}$ is randomly generated from the uniform distribution $[0, 1]$, and then normalized in order to have $\sum_{i \in \mathcal{N}} P_i = 1$. The demands are randomly generated from the uniform distribution $[0, 300]$.
The supplier's fixed ordering cost is generated such that $f_t^S = \delta p_t^S$ where $\delta = 500$. When the retailer's costs f^R, h^R are not private, they are randomly generated (uniform distribution) in the discrete interval described in Table 1. If the cost p^R is not private, it is randomly generated from the uniform distribution $[1, 100]$. We generate the private cost such that the retailer has several profiles. If f^R is the retailer's private cost, we generate the fixed ordering cost for the different types such that for type 1, $f_1^R = \delta c$ where c is randomly generated from the uniform distribution $[c_1^{R\min}, c_1^{R\max}] = [1, 20]$. Type 1 represents a retailer with a fixed ordering cost that is low. The subsequent types are generated such that the fixed ordering cost is increasing. For each type $i \in \{2, \ldots, N\}$, f_i^R is randomly generated from the uniform distribution $[c_{i-1}^{R\max} + 100/N, c_{i-1}^{R\max} + 20 + 100/N]$: 20 represents the length of the interval and $100/N$ enable us to generate different profiles of retailers. For example, we consider an instance where $N = 2$, f^R is the private cost and the instance is in the class A. The costs are generated in the following way: h^S, p^S, h^R are randomly chosen in the interval described in Table 1, p^R is randomly chosen in $[1, 100]$, f_1^R is generated such that $f_{1t}^R = \delta c$ where c is randomly chosen in $[1, 20]$ and f_2^R is generated such that $f_{2t}^R = \delta c$ where c is randomly chosen in $[70, 90]$. Similarly, if p^R (resp. h^R) is private, p_1^R (resp. h_1^R) is randomly generated from the uniform distribution $[p_1^{R\min}, p_1^{R\max}] = [1, 20]$ (resp. $[h_1^{R\min}, h_1^{R\max}] = [0, 6]$), and p_i^R is randomly generated from the uniform distribution $[p_{i-1}^{R\max} + 100/N, p_{i-1}^{R\max} + 20 + 100/N]$ (resp. $[h_{i-1}^{R\max} + 5, h_{i-1}^{R\max} + 7 + 5]$) for each type $i \in \{2, \ldots, N\}$. In order to compare the results when complete information is assumed and the ones obtained assuming asymmetric information, when p^R is private and $N = 2$, the instances are generated from the instances used for the experimental analysis with complete information in Section 3.5: from one pair of instances where $p^R \in [1, 20]$ or $p^R \in [70, 90]$ under complete information, we create an instance with asymmetric information where the retailer of type 1 has a cost $p^R \in [1, 20]$ and the retailer of type 2 has a cost $p^R \in [70, 90]$. Observe that in the case where h^R is private, we do not have to differentiate the case where h^R is high or not because of the retailer's profiles. Thus, we can only consider the classes of instances A and B.

The supplier's expected gain, in percentage, is defined by $(1 - \mathrm{E}[c_P^S]/\mathrm{E}[c^S]) * 100$ where $\mathrm{E}[c_P^S]$ is the supplier's expected cost where P stands for problem P_{WP}^{AI}, P_{HC}^{AI} or P_{TC}^{AI}, and $\mathrm{E}[c^S]$ is the supplier's expected cost when the retailer imposes his optimal replenishment plan that is the less profitable for the supplier. The results are summarized in Tables 4, 5 and 6. The column *Mean* (resp. *Max*) represents the average (resp. maximum) value of the supplier's expected gain over the 100 instances for each class of instances.

When side payment is not allowed, the supplier's gain is null for most of the classes. Moreover, whenever a supplier's gain is observed, the expected gain is less than 0.5%. Therefore, the results for the problem P^{AI}_{WP} are not reported in Table 4, 5 and 6. Similarly to the results obtained under the complete information assumption, the supplier's expected gain when side payment is only allowed on the retailer's holding cost is close to the case where side payment is allowed on the retailer's total cost. Thus, we only provide the results for the problem P^{AI}_{HC} in Tables 4, 5 and 6.

		$T = 10$		$T = 20$		$T = 30$	
N	Class	Mean	Max	Mean	Max	Mean	Max
2	A	5.6	43.9	12.1	60.6	13.1	39.4
2	B	15	63.9	27.2	63.9	31	61
2	C	2.2	29.5	2.6	15.6	3.2	21.8
2	D	10.8	41.3	13.8	34	13.8	52.6
10	A	5.8	47.5	9.5	31.7	12.4	29.8
10	B	11	51.6	22	54.8	29.2	55.8
10	C	2.2	20.4	2.5	12.2	2.8	11.8
10	D	7	33.2	11.8	37	13.5	32.8

Table 4: Supplier's expected gain in average (%) when p^R is private for the problem P^{AI}_{HC}

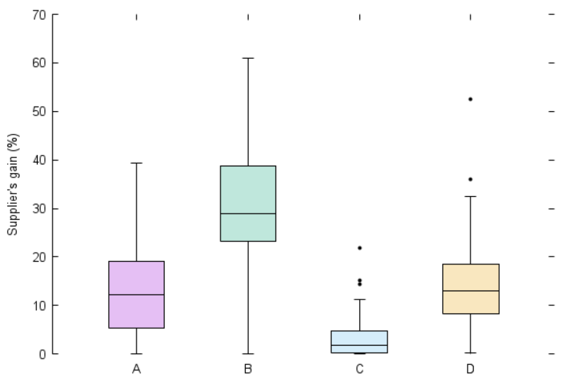

Figure 7: The supplier's expected gain of the problem P^{AI}_{HC} for each class of instances where p^R is private, $N = 2$ and $T = 30$

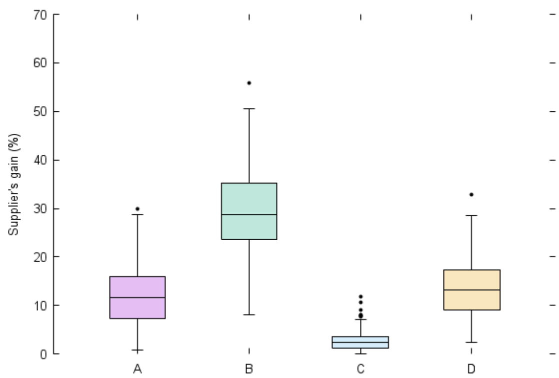

Figure 8: The supplier's expected gain of the problem P^{AI}_{HC} for each class of instances where p^R is private, $N = 10$ and $T = 30$

Table 4 and Figures 7, 8 report the supplier's expected gain in average when p^R is private. We compare the results obtained for 2-type instances with the ones observed assuming complete information. We can notice that, the asymmetric information assumption does not deteriorate significantly the supplier's gains compared to the complete information case: the difference is at most 1 percentage point. Therefore, although Proposition 1 states that the supplier's cost with asymmetric information can increase by a factor arbitrarily large compared to the case with complete information, these results show that, in practice, the supplier's cost with asymmetric information does not increase significantly. Similarly to the case with complete information, the supplier's expected gain in average is the lowest for the class C. It is equal to 3.2% for the class C when $T = 30$ and $N = 2$ whereas it is at least equal to 13.1% for the other classes. Moreover, the supplier has the highest expected gain in average with the class B (31% when $T = 30$ and $N = 2$). The number of types seems to have a little impact on the supplier's expected gain. There is at most a difference of 5.2 percentage points between the expected supplier's gain in average when $N = 2$ and $N = 10$: for the class B and $T = 20$, it is equal to 27.2% and 22% when $N = 2$ and $N = 10$ respectively. Moreover, the boxplots show that there is no instance where the supplier's expected gain is equal to 0 for the classes B and D when $N = 10$ contrary to the case $N = 2$. We observe the same trend when f^R is private (see Table 5).

		$T = 10$		$T = 20$		$T = 30$	
N	Class	Mean	Max	Mean	Max	Mean	Max
2	A	2.5	42.3	9.5	63.2	9.7	52.8
2	B	7	74.6	16.9	59.3	20.4	65.1
2	C	2.9	31.1	3.8	29.2	4	25.4
2	D	8.4	56	11.7	52.3	14.1	37.4
10	A	1.1	10.2	4.8	37.4	7.9	39.8
10	B	2.3	22.5	11.3	69.1	15.8	47.1
10	C	2	18.8	3.1	17.5	3.7	17.6
10	D	5.3	33.7	10.8	34.7	14.1	42.4

Table 5: Supplier's expected gain in average (%) when f^R is private for the problem P_{HC}^{AI}

		$T = 10$		$T = 20$		$T = 30$	
N	Class	Mean	Max	Mean	Max	Mean	Max
2	A	2.5	22.3	4.1	30.1	5	32
2	B	6.7	42.9	10.7	50.4	13.4	30.6
10	A	0.6	8.3	1	11.1	1.9	8.2
10	B	2	14.2	3.7	15.5	4.4	9.4

Table 6: Supplier's expected gain in average (%) when h^R is private for the problem P_{HC}^{AI}

When h^R is private, the supplier's expected gain is not as high as for the other cost parameters. For $T = 30$ and $N = 2$, the best expected gain in average is equal to 13.4% whereas it is equal to 31% when p^R is private and 20.4% when f^R is private. Moreover, the number of types seems to have an important impact on the supplier's expected gain. For $N = 10$, the supplier's expected gain in average is not significant. For $T = 30$, the best result is 4.4%, when $N = 10$ and it is 13.4% when $N = 2$. The retailer's holding cost increases with the number of types since a type represents a retailer's profile. However, when the retailer's holding cost is high, the supplier does not improve his cost significantly. Thus, if there are lot of types with high holding costs, the supplier's expected gain will not be significant.

5. Extension to the multi-retailer case

In this section, we extend the analysis by considering a two-level supply chain with one supplier and multiple retailers. Each retailer has to satisfy a demand for a single product. The problem of satisfying these demands in order to minimize the cost of the supply chain is the so-called the One-Warehouse Multi-Retailers (OWMR) problem that is known to be NP-hard [27]. The problems of designing contracts under complete and asymmetric information are refereed to as OWMR-Supp-CI and OWMR-Supp-AI respectively in order to minimize the supplier's cost.

5.1. Problem definition

As described for the single-retailer case, We consider the following contracts: the first one assumes no side payment between the supplier and the retailers, the second (resp. third) one allows a side payment on the retailer's holding (resp total) cost. We extend the screening game to multi-retailer case. We consider M retailers and denote by $\mathcal{M}$ the set of retailers $\{1, \dots, M\}$. Each retailer m has N_m possible types. We denote by $\mathcal{N}_m$ the set of types of the retailer m. The supplier knows that the retailer m is of type $i \in \mathcal{N}_m$ with a probability P_i^m ($\sum_{i \in \mathcal{N}_m} P_i^m = 1$). Similarly to the single-retailer case, the supplier proposes to each retailer m a set of contracts (one contract for each type $i \in \mathcal{N}_m$). We have to consider every combination of types among the retailers. We denote by C the set of all the combinations. To illustrate these possible combinations, let us consider the following example for $M = 2$. The retailer 1 has $N_1 = 2$ types and the retailer 2 has $N_2 = 3$ types. Then, the set of combinations is $C = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\}$ where (i, j) means that the retailer 1 is of type i and the retailer 2 is of type j. The probability to have a combination c is given by $P_c = \prod_{m \in \mathcal{M}} P_{c_m}^m$, where c_m is the type of the retailer m in the combination $c \in C$. Moreover, the contracts still have to be individual rational and incentive compatible to ensure that each retailer m will not have an incentive to lie.

The mathematical formulations of the problems OWMR-Supp-CI and OWMR-Supp-AI are derived from the multi-commodity formulation proposed by Cunha and Melo [28] for the OWMR problem. The mathematical formulations are not detailed for sake of conciseness. However, we provide some useful information. Under the complete information assumption, the mathematical formulation of the problem OWMR-Supp-CI is the same than the one for the OWMR problem with in addition the IR constraint for each retailer. Under the asymmetric information assumption, the mathematical formulation of the problem OWMR-Supp-AI is an extension of the one proposed for the OWMR problem where the IR constraint for each retailer and the IC constraint for each combination of retailer's types have to be taken into account.

5.2. Experimental analysis

The first part of the experiments deals with the complete information assumption. Since the OWMR variants are NP-hard, we restrict the analysis to instances with $M = 3$ and $T = \{10, 20, 30\}$ in order to solve them optimally. The instances are generated using the same generation scheme than the one used for one retailer (see Section 3.5). The supplier's costs and each retailer's cost are generated according to the classes of instances described in Table 1.
The experimental results for the OWMR-Supp-CI problem show that using contracts without side payment do not induce the decrease of the supplier's cost. Moreover, similarly to the case with one retailer, allowing a side payment on the retailer's holding cost is sufficient to decrease the supplier's cost.
Table 7 and Figure 9 report the supplier's gain for the OWMR-Supp-CI problem when side payment is allowed on the retailer's holding cost. The observations provided for the single-retailer case still hold for the multi-retailer setting. The highest gain in average for the supplier is observed for the class B (33.3% when $T = 30$) and the lowest one is observed for the class C (4.4% when $T = 30$). Moreover, for the class B and $T = 30$, the boxplot shows that the supplier's gain is at least equal to 26% for 75% of the instances.

	$T = 10$		$T = 20$		$T = 30$	
Class	Mean	Max	Mean	Max	Mean	Max
A	9.3	61.3	13.9	58.1	16.2	55.7
B	17.7	65.2	30.2	69.9	33.3	65.7
C	4.4	24.7	4.1	24.5	4.4	19.2
D	13.9	48.8	12.9	35.6	14.8	33.1

Table 7: Supplier's gain in average (%) with complete information, side payment on the retailer's holding cost, for $M = 3$

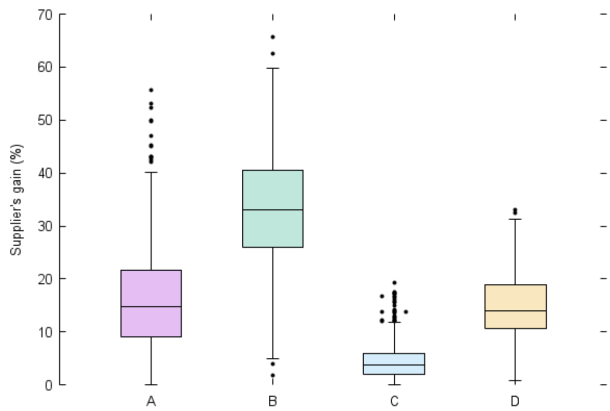

Figure 9: Supplier's gain for each class of instances with complete information, side payment on the retailer's holding cost, for $M = 3$ and $T = 30$

In the following part, we assume that a cost parameter is private. Similarly, we only run the tests with $M = 3$,

$N_i = 2$ for all $i = 1, 2, 3$ and $T = \{10, 20, 30\}$ to get the optimal expected gains. We generate the instances following the single-retailer generation scheme. The supplier's costs are generated according to the class of instances described in Table 1. The retailer's costs are generated following the procedure described for the single retailer case under asymmetric information assumption (Section 4.4). Similarly to the complete information case, the supplier cannot decrease his cost without side payment. Moreover, side payment on the retailer's holding cost is sufficient to decrease the supplier's cost. The results are given in Tables 8, 9, 10 and Figure 10.

	$T = 10$		$T = 20$		$T = 30$	
Class	Mean	Max	Mean	Max	Mean	Max
A	8.6	52.9	13.4	49.2	15.8	49.2
B	16.3	61.4	28.9	65.2	32.8	55.3
C	3.8	20.3	3.7	18.6	4.3	15
D	12.7	35	12	31.4	14.4	30.1

Table 8: Supplier's expected gain in average (%) with asymmetric information, side payment on the retailer's holding cost, p^R is private, for $M = 3$ and $N_i = 2$ for all i

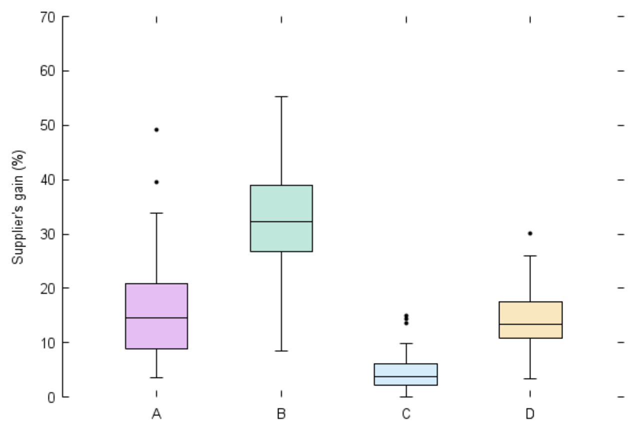

Figure 10: Supplier's expected gain for each class of instances with asymmetric information side payment on the retailer's holding cost, p^R is private, for $M = 3$, $N_i = 2$ for all i and $T = 30$

	$T = 10$		$T = 20$		$T = 30$	
Class	Mean	Max	Mean	Max	Mean	Max
A	3.4	30.4	9.9	53.7	11.0	30.9
B	9.2	50.0	20.6	55.1	22.3	42.3
C	3.9	26.8	4.3	16.7	4.7	19.0
D	10.3	36.8	12.9	30.0	13.6	25.6

Table 9: Supplier's expected gain in average (%) with asymmetric information where f^R is private, $M = 3$ and $N_i = 2$ for all i

	$T = 10$		$T = 20$		$T = 30$	
Class	Mean	Max	Mean	Max	Mean	Max
A	3.2	20.6	4.8	22.4	5.2	23.2
B	6.9	26.5	11.8	31.9	14.1	37.8

Table 10: Supplier's expected gain in average (%) with asymmetric information where h^R is private, $M = 3$ and $N_i = 2$ for all i

The same trend is observed regarding the impact of the asymmetric information that does not have a significant impact on the supplier's expected gain. Besides, for the classes A, B and D, when p^R is private, the boxplots show that the supplier's expected gain is always positive. It is at least equal to 9% for the class B and 2% for the classes A, D.

6. Conclusion and future research

In this paper, we study the coordination of planning decisions in a two-level supply chain by using contracts. Different variants of the problem have been discussed regarding complete or asymmetric information assumption. We design contracts where side payment can be allowed or not between the actors, a retailer that has the power market and a supplier in order to decrease the supplier's cost. We show that for the single-retailer problem with complete information, designing optimal contracts that minimize the supplier's cost can be done in polynomial time when side payment is not allowed or side payment is allowed on the retailer's total cost. When side payment is only allowed on the retailer's holding cost, we prove that the problem is NP-hard. Under the asymmetric information assumption, we show that designing optimal contracts without side payment is still polynomial. However, in the case where side payment is allowed on the retailer's total cost, the complexity issue is open. We prove that under the asymmetric information assumption, there is no constant factor approximation algorithm which is truthful for the problems with side payment on the retailer's cost. An experimental analysis shows that under complete or asymmetric information, contracts without side payment are not effective in decreasing the supplier's cost. On the contrary, when side payment is allowed, the benefit of the supplier can be substantial. Moreover, we have seen that contracts with side payment on the retailer's holding cost are sufficient to decrease significantly the supplier's cost. We extended the study for a multi-retailer setting. The analysis shows that the same trends are observed.

Certain open research questions remain. The complexity of the problem under the asymmetric information assumption when side payment is allowed on the holding cost is a challenging question. In the multi-retailer case, it is also possible to consider the problem where side payment can be allowed between the retailers. In addition, our study concerns a single product. It can be interesting to study a supply chain with several products where side payments are shared between all the products or a subset of products. A strong assumption in our study is that the retailer has the market power. However, we can also consider the problem of designing contracts when the supplier is the leader.

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