

# Substation Location and Transmission Network Expansion Problem in Power System

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In this paper, we propose a model for the generation and transmission network expansion planning problem that includes decisions related to substations' locations and sizes. In power system expansion planning problems, locations of generation units and/or transmission lines are determined. However, substation location decisions are not explicitly studied. Nevertheless, including the decisions of substations' locations and sizes to the problem can change the transmission network substantially. Hence, in this study, we first discuss the benefit of incorporating substation decisions to the transmission network design problem. We then propose a model that finds a minimum cost network, the optimal locations and sizes of substations and generation plants. To overcome the computational challenge of the proposed model, we present a heuristic solution approach, in which we decompose the multi-period problem into several single-period problems that are solved sequentially. Computational studies on 24-bus and 118-bus power systems show that the proposed solution approach yields near optimal solutions in minutes.

*Keywords:* location, network expansion, integrated planning, transmission switching.

## 1. Introduction

The goal of the transmission network expansion planning (TNEP) problem is to determine optimal dispatch of the power and find the optimal design of a network by expanding an existing network. Considering continuously increasing demand, expanding or reinforcing existing generation units (generation expansion planning-GEP) is also studied in the literature. For long-term planning, in practice, generation resources are planned first, and based on the solution, transmission network is planned [1]. In the literature, there are also studies which consider generation and transmission network expansion planning (GTNEP) problems simultaneously to find a less costly solution which may not be obtained in the sequential approach.

In GTNEP problems, although generation and transmission decisions are assessed simultaneously, substation components, which can be used to increase/decrease the voltage of the power at critical locations, are not explicitly included to the models. Investment cost of substations are either ignored or considered to be included in the investment costs of the generation units or transmission lines. However, including substations' locations and sizes to the GTNEP problem can change the transmission network substantially, and an integrated multi-period power system

planning that also includes substations could result in a different network configuration. To the best of our knowledge, there is no study that focuses on multi-period expansion of the GTNEP problem that includes substations explicitly. With the aim of filling this gap in the literature, in this work, we study an integrated generation, substation and transmission network expansion planning problem (GSTNEP).

In most of the large electricity networks, transmission is designed as a static system and the main aim is to determine power flows over a fixed network to minimize the total cost [2]. On the other hand, switching transmission lines (taking some lines out of operation) is a common practice in TNEP problems to increase transfer capacity, decrease power loss during the transmission or improve voltage profile of the nodes [3]. Transmission switching is important in TNEP problems because adding even a zero-cost transmission line may decrease the total capacity of the system and hence, may result with higher generator costs in order to satisfy the demand of each node due to Kirchoff's laws. Since considering substations in the power systems can change the network structure, taking out (or not using) some lines may also be beneficial in GSTNEP problem. Thus, in this paper, we integrated transmission switching to the proposed model of GSTNEP.

### ***1.1. Effect of including substations to GTNEP***

In the power investment/expansion planning problems, different types of substations are considered depending on the level of decomposition of the problem, such as *generation (step-up)*, *transmission* and *distribution (step-down)* substations [4]. Needless to say, when a new generation plant is built, it is also necessary to build a substation to connect the plant to the system. However, in GTNEP problems, the cost of these substations are not explicitly identified and it is considered within the investment cost of the generation plants. In the power systems, transmission substations are also required for routing power from generators to demand nodes [4]. Similar to step-up substations, investment cost of transmission substations could be added to the investment cost of transmission lines. However, these approaches for calculating investment cost of substations may not correctly estimate the total investment cost since generation amount at each plant (which also determines capacity of substations) and the number of transmission lines connected to each transmission substations are not known in advance.

Albert et al. [4] study North American power grid which includes generation plants, major substations and transmission lines. In their model, 14099 nodes are identified as substations for the North American power system. Substations that have a single high-voltage transmission line are identified as distribution substation, and substations that are located at a source of power are referred as step-up substations. The remaining ones are classified as transmission substations. In the paper, 1633 of 14099 nodes are distinguished as power plants (step-up substations), 2179 nodes are referred as distribution substations and the remaining ones are labeled as transmission

substations. In such a large power system, adding investment cost of substations to investment cost of generation units or to investment cost of transmission lines may result with different network structure, which may also have higher total investment cost.

In the following example, we discuss the value of explicitly modeling substations by comparing solutions of GTNEP and GSTNEP. We show that even in a small network (6-bus power system), including substation decisions affects the network structure and changes the substations' locations. For this example, demand of buses, and properties related to transmission lines are adapted from [5]. Load of buses is taken as 1/4 of the loads given in [5]. Bus 6 is also considered as a demand node with 10 MW load. In [5], there is only one transmission line type with 100 MW capacity. In this example, we include one more transmission line type with 50 MW capacity and investment cost of the line is considered to be 144000 \$/km. Three candidate generation capacities and five candidate substation capacities are considered and the investment costs of these units for each capacity type are given in Section 4. The model used to solve these problems will be explained in detail in Section 3.

Figure 1a and Figure 1b presents the networks resulting from GTNEP and GSTNEP problems, respectively. In the GTNEP output, one generation plant at node 2, and 5 transmission lines are built with the total cost of \$229.64M (Figure 1a). Thick lines have 100 MW capacity whereas the other lines have 50 MW capacity. Although it is not explicitly modeled in GTNEP problem, there should be 3 substations: one for step-up substation at node 2 and two for transmission substations at nodes 3 and 5, which are required to change the line type or to re-increase voltage of power in order to decrease power loss due to long distances between the nodes. We calculate the required capacities and cost of these substations with respect to the generation amount and power flows. When the costs of substation units are added to the cost of the GTNEP, the total cost turns out to be \$261.08M. However, in the GSTNEP output, with the same parameter setting, we have a star network solution (Figure 1b), i.e. power is directly sent to all the demand nodes from generation plant at node 2. In this case, we have only one substation at node 2 and 5 transmission lines. The total cost of GSTNEP is equal to \$247.82M. Hence, for this example, by integrating the subproblems, we decrease the total cost of the power system by \$13.26M.

One can question the effect of the investment cost of substations to the network structure and to the total cost. Hence, in order to analyze this effect, we come up with a parametric analysis by multiplying investment cost of substations with a factor,  $\alpha$ . Figure 1c and Figure 1d present the networks of GSTNEP problem when investment cost of substations are multiplied by  $\alpha=0.05, 0.3$ , respectively. In Table 1, we also show the results of all instances with the cost distribution in terms of investment costs of generation, substation, lines, and operation and maintenance (O&M) cost for both GTNEP and GSTNEP problems for  $\alpha =0.05, 0.3$  and 1.0.  $\alpha =1.0$  corresponds to the original investment cost of substations. Under the columns *GTNEP*,

the values given in bold show the calculated substation cost depending on  $\alpha$ .

When the investment cost of substations is small, (i.e.  $\alpha = 0.05$ ), the output of GSTNEP is the same as GTNEP (Figure 1a and 1c). That is, the output of the model where investment cost of substations are ignored (GTNEP) is the same with the one where they are explicitly considered (GSTNEP). When  $\alpha = 0.3$ , there are only two substations at nodes 2 and 5, and there exist 5 transmission lines with only one of them being 100 MW capacity (Figure 1d). Thus, even when  $\alpha = 0.3$ , beside the locations of substations, capacity of transmission lines also changes. For this case, the calculated cost of GTNEP after adding substations is 0.86% higher than the cost of GSTNEP. When  $\alpha = 1.0$ , value of integrating substation to the problem is more obvious; total cost of GTNEP is 5.35% higher than the total cost of GSTNEP.

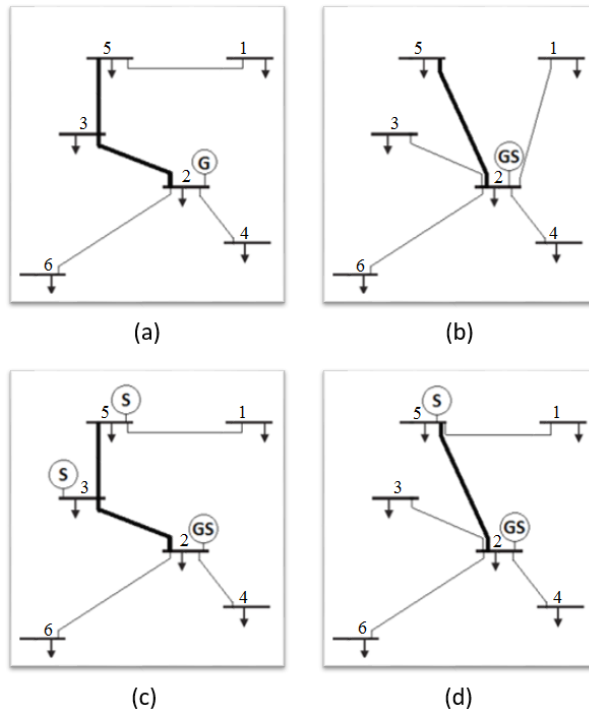


Figure 1: (a) Result of GTNEP (b) Result of GSTNEP (c) Result of GSTNEP for  $\alpha = 0.05$  (d) Result of GSTNEP for  $\alpha = 0.3$

A similar analysis is also made by multiplying investment cost of transmission lines by three and Figure 2 presents the networks resulting from GTNEP and GSTNEP problem for the original and adapted cost of transmission lines. GTNEP output remains the same when the investment cost of transmission lines is increased (Figure 2a). However, even in a small network, GSTNEP output changes significantly as the investment cost of the lines is increased. When cost of transmission lines are at original values, there are one generation unit, one step-up substation and 5 transmission lines with only one of them being 100 MW capacity (Figure 2b). But, as the cost increases, GSTNEP output changes and two generation units and two step-up substations

Table 1: Results of GTNEP and GSTNEP for different  $\alpha$  values

	$\alpha=0.05$		$\alpha=0.3$		$\alpha=1.0$	
	GTNEP	GSTNEP	GTNEP	GSTNEP	GTNEP	GSTNEP
total cost(\$10 <sup>6</sup> )	231.21	231.21	239.07	237.04	261.08	247.82
generation inv. cost (\$10 <sup>6</sup> )	180.00	180.00	180.00	180.00	180.00	180.00
line inv. cost (\$10 <sup>6</sup> )	22.56	22.56	22.56	23.28	22.56	26.16
O&M cost (\$10 <sup>6</sup> )	27.08	27.08	27.08	26.80	27.08	26.73
substation inv. cost (\$10 <sup>6</sup> )	<b>1.57</b>	1.57	<b>9.43</b>	6.95	<b>31.44</b>	14.92

are built at nodes 2 and 5 (Figure 2c). In this case, four transmission lines are built with 50 MW capacity and due to having two generation units, there is no need having a transmission line with 100 MW capacity.

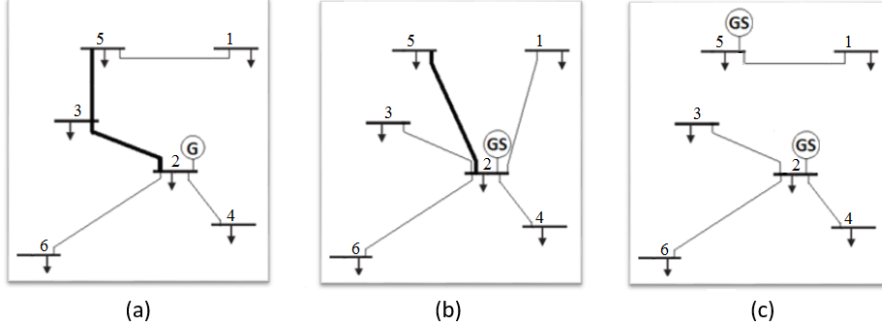


Figure 2: (a) Result of GTNEP (b) Result of GSTNEP with original transmission line costs (c) Result of GSTNEP with adapted transmission line costs

From this example, we can conclude that substation decisions should be included in the GTNEP problems since considering substations may change the network structure substantially (Figure 2). Hence, in this paper, we aim to provide an integrated model including all these components of the power system (generation, substation and transmission) to find a minimum cost network by determining the locations and sizes of these components simultaneously.

## 2. Literature Review

In the literature, the effect of co-optimization of generation and transmission components is discussed by Horne et al. in [6]. The authors first identify generation locations and then determine transmission capacities in a sequential manner. Samarakoon et al. [7] consider GEP and TNEP problems simultaneously and formulate GTNEP problem for a single period. Alizadeh and Jadid [8] also focus on the same problem by including additional environmental constraints

and fuel supply limitations. They analyze their method with 6-bus and 30-bus power systems. In [9], the same authors consider a multi-period GTNEP problem including time as a new dimension to their previous model given in [8] and solve the new dynamic model using Benders decomposition algorithm. Jenabi et al. [10] formulate a multi-period GTNEP problem considering demand side management technologies to reduce the peak demand. They also propose a Benders decomposition algorithm to solve the problem. Guerra et al. [11] provide an integrated model for GTNEP problem that includes renewable energy sources, demand side management and  $CO_2$  emission constraints. The model is applied to Colombian interconnected power system under different scenarios. Seddighi and Ahmadi-Javid [12] provide a stochastic programming model for multi-period GTNEP problem and the uncertainties are related to demand, generation amounts, cost and greenhouse gas emissions. They apply their model to 400-kV system of Iran's northwestern power grid which includes 17 nodes. Since long-term power investment/expansion planning problems are complex, some researchers work on improving computational quality of the problem. Munoz et al. [13] propose a new two-phase approach for GTNEP problem with Benders decomposition. They develop new bounds for the expected system cost using Jensen's inequality. Mínguez and García-Bertrand [14] propose an algorithm for solving robust TNEP problem. Grimm et al. [15] provide a model to analyze generation and transmission investment decisions in electricity market. By defining single and multiple price zones, they compare their solutions on 3-node and 6-node examples.

In the literature, there are a limited number of papers that study substation location and transmission network expansion problems (STNEP) and in those studies, generation decisions are included in the models. Cebeci et al. [16] work on a single period STNEP problem and determine the locations of substations, their connections to demand points as well as the HV lines between them. The authors also propose an algorithm that decomposes the problem into investment and feasibility-check subproblems. Akbari et al. [17] consider a scenario based stochastic STNEP problem for a single period. Their problem includes finding the generation amounts at prescribed generation plants, selection of new buses (can be considered as substations) and new transmission lines. However, authors do not consider building new generation plants or include some restrictions for the total incoming or outgoing power flow in the new buses. To solve the problem, a scenario reduction technique is applied to decrease number of scenarios.

In addition to co-optimization of subproblems, by switching transmission lines (taking some lines in and out of operation), total cost can be decreased. This transmission switching approach (TS), is applied to many problems in different areas. Fisher et al. [2] are the first authors that provide a mathematical programming model for TNEP problem considering TS. Hedman et al. [3] present a formulation for generation unit commitment by considering transmission switching. The authors decompose the problem into two subproblems by first solving their model without switching for a 24-hour problem. Then, they determine the status of lines by

fixing unit commitment decisions. Khodaei et al. [18] provide a Benders decomposition algorithm for solving GTNEP problem with transmission switching. By providing an existing network and limiting number of switchable lines, they compare the solutions for different cases. Villumsen and Philpott [19] add uncertainty to the TNEP problem with TS by a scenario approach. For a static (single period) problem, the authors apply Dantzig-Wolfe decomposition algorithm to the problem. Dehghan and Amjady [20] provide a new robust optimization model for TNEP including energy storage systems and transmission switching.

As it is discussed in the previous section, including substation decisions may change the topology of the network substantially. However, aforementioned studies show that in GTNEP problems, decisions related to substation locations are not included. Similarly, in the STNEP problems, power output from a substation is only dependent on the capacity of that substation and generation expansion decisions are not included to the problem. It is obvious that modeling substations in GTNEP problems may reduce the total cost. To the best of our knowledge, there is no study that includes substation decisions in GTNEP problem and in this paper, we present an integrated planning for generation, substation and transmission expansion planning problems (GSTNEP) considering transmission switching (TS).

GSTNEP problem that we present is a multi-period power system planning problem. In this paper, we propose a mixed integer linear programming model (MILP) with the aim of finding the optimal siting, sizing and timing of power system's generation, transmission and substation components simultaneously. We also determine power flow paths and generation amounts for each time period. In the literature, to overcome the computational challenge of the power expansion planning, problem is decomposed into subproblems and the solution is obtained by iteratively solving these subproblems [1]. In this study, we present a heuristic solution approach, in which we decompose our integrated multi-period problem into a set of single period problems as many as the number of expansion periods. Computational studies on 24-bus and 118-bus power systems show that the proposed solution approach yields better results than the results of traditional method which is based on decomposing the problem and solving the subproblems sequentially.

The rest of the paper is organized as follows: In the next section, we provide the mathematical model and solution approaches for the problem. We show and discuss the results of the proposed model in Section 4. We also compare the results of our proposed solution approach with the solutions that are obtained by sequentially solving the subproblems. The paper concludes with Section 5.

### 3. GSTNEP Problem

Given the demand of nodes and possible generation amounts from different source types, we aim to find optimal siting, sizing and timing of system components (generation plants, substation units and lines) in order to fulfill power demand in a multi-period planning horizon. The problem includes optimization of power generation amounts, power flows and flow directions on the transmission lines considering transmission switching option. In the electrical power systems, when a new generation plant is built, it is also necessary to build a substation to connect the plant to the system in order to adjust or transform the voltage to the required level. So, if a generation unit is located at a node, we need to guarantee that there is also a step-up substation at the same node. Moreover, a node which connects other demand nodes or substations to a generation unit (or to a step-up substation) also has a transmission substation in order to transform/re-increase the voltage level.

Direct current (DC) approximation of Kirchhoff's first and second laws, and a single load scenario for one time period are considered in this study as in [7]-[11]. However, similar to [18, 21], to have a better representation of the reality, we divide each time period into a set of demand blocks to consider seasonality and variability of demand within year. In the model, as customarily done in the literature [11], we also assume that a predetermined ratio of the transferred power is lost due to line resistance and the ratio depends on line types. For brevity, throughout the paper, we refer a node that has just substations as *substation node*, a node that has substation and generation units as *generation node* and the remaining nodes (which have no generation and substation units) as *demand node*.

#### 3.1. Mathematical programming model of GSTNEP

We use the following notation for the mathematical programming model. Let  $G = (N, E)$  be a graph where  $N$  is the node set for demand nodes and candidate nodes for locating generation and substation units; and  $E$  is the candidate corridors for building transmission lines. The sets and corresponding indices, parameters and decision variables used in the mathematical model are shown in Table 2.



Table 2: Nomenclature

<b>Sets</b>	<b>Explanation and corresponding indices</b>
$N$	set of all nodes ( $i$ and $j$ )
$E$	set of corridors ( $e = \{i, j\}$ )
$\Omega^+(e)$	sending-end bus of corridor $e$
$\Omega^-(e)$	receiving-end bus of corridor $e$
$A$	set of line types ( $a$ )
$C$	set of generation technologies ( $c$ )
$S$	set of substations ( $s$ )
$B$	set of load blocks ( $b$ )
$T$	set of periods ( $t$ )
<b>Param.</b>	<b>Explanation of parameters</b>
$d_{ibt}$	demand of node $i$ at load block $b$ at period $t$ (MW)
$CapG_{ict}$	generation capacity in node $i$ from source type $c$ at period $t$ (MW)
$CapL_a$	capacity of transmission line type $a$ (MW)
$CapS_s$	capacity of substation type $s$ (MVA)
$c-inv_c$	investment cost of source type $c$ (\$)
$c-om_c$	operation&maintenance cost of source type $c$ (\$/MWh)
$c-sub_s$	investment cost of substation type $s$ (\$)
$c-line_a$	investment cost of transmission line type $a$ (\$/km)
$l_e$	distance of corridor $e$ (km)
$b_{ea}$	susceptance of line type $a$ of corridor $e$ (p.u.)
$r$	discount rate
$dur_{bt}$	duration of load block $b$ at period $t$ (h)
$ef_a$	transmission loss on the line type $a$ per unit flow
$TI$	number of years in each period
<b>Dec.Var.</b>	<b>Explanation of decision variables</b>
$X_{ict}$	1 if source type $c$ is located at node $i$ at period $t$ , 0 o.w.
$Y_{icbt}$	generation amount in node $i$ from source type $c$ at load block $b$ at period $t$
$K_{ist}$	1 if substation type $s$ is located at node $i$ at period $t$ , 0 o.w.
$\bar{K}_{it}$	1 if at least one substation is located at node $i$ until period $t$ , 0 o.w.
$\theta_{ibt}$	voltage angle of node $i$ at load block $b$ at period $t$ (rad)
$L_{eat}$	1 if line type $a$ is built at corridor $e$ at period $t$ , 0 o.w.
$L'_{eabt}$	1 if line type $a$ at corridor $e$ is used at load block $b$ at period $t$ , 0 o.w.
$f_{eabt}$	flow on line type $a$ at corridor $e$ at load block $b$ at period $t$

The proposed model for GSTNEP (*GSTNEP-M*) is as follows:

$$\min \quad z_{inv} + z_{sub} + z_{line} + z_{om} \quad (1)$$

$$z_{inv} = \sum_{t \in T} (1+r)^{-\Pi(t-1)} \sum_{i \in N} \sum_{c \in C} c_{-inv_c} X_{ict}$$

$$z_{sub} = \sum_{t \in T} (1+r)^{-\Pi(t-1)} \sum_{i \in N} \sum_{s \in S} c_{-sub_s} K_{ist}$$

$$z_{line} = \sum_{t \in T} (1+r)^{-\Pi(t-1)} \sum_{e \in E} \sum_{a \in A} c_{-line_{e_a}} l_e (L_{eat} - L_{eat-1})$$

$$z_{om} = \sum_{t \in T} (1+r)^{-\Pi(t-1)} \kappa(r, \Pi) \sum_{i \in N} \sum_{c \in C} \sum_{b \in B} c_{-om_c} dur_{bt} Y_{icbt}$$

$$\begin{aligned} s.t \quad & \sum_{c \in C} Y_{icbt} + \sum_{e \in E | \Omega^-(e)=i} \sum_{a \in A} f_{eabt} - \\ & \sum_{e \in E | \Omega^+(e)=i} \sum_{a \in A} (1 + e f_a) f_{eabt} = d_{ibt} \quad \forall i \in N, b \in B, t \in T \quad (2) \end{aligned}$$

$$Y_{icbt} \leq \sum_{t' \in T: t' \leq t} CapG_{ict'} X_{ict'} \quad \forall i \in N, c \in C, b \in B, t \in T \quad (3)$$

$$\sum_{e \in E | \Omega^+(e)=i} \sum_{a \in A} (1 + e f_a) |f_{eabt}| \leq \sum_{s \in S} \sum_{t' \in T: t' \leq t} CapS_s K_{ist'} \quad \forall i \in N, b \in B, t \in T \quad (4)$$

$$\sum_{e \in E | \Omega^-(e)=i} \sum_{a \in A} |f_{eabt}| \leq \sum_{s \in S} \sum_{t' \in T: t' \leq t} CapS_s K_{ist'} +$$

$$d_{ibt}(1 - \bar{K}_{it}) \quad \forall i \in N, b \in B, t \in T \quad (5)$$

$$- CapL_a L'_{eabt} \leq (1 + e f_a) f_{eabt} \leq CapL_a L'_{eabt} \quad \forall e \in E, a \in A, b \in B, t \in T \quad (6)$$

$$f_{eabt} = b_{ea} L'_{eabt} (\theta_{ibt} - \theta_{jbt}) \quad \forall e \in E, a \in A, b \in B, t \in T \quad (7)$$

$$L'_{eabt} \leq L_{eat} \quad \forall e \in E, a \in A, b \in B, t \in T \quad (8)$$

$$X_{ict} \leq \bar{K}_{it} \quad \forall i \in N, c \in C, t \in T \quad (9)$$

$$\bar{K}_{it} \leq \sum_{s \in S} \sum_{t' \in T: t' \leq t} K_{ist'} \quad \forall i \in N, t \in T \quad (10)$$

$$K_{ist} \leq \bar{K}_{it} \quad \forall i \in N, s \in S, t \in T \quad (11)$$

$$\bar{K}_{it-1} \leq \bar{K}_{it} \quad \forall i \in N, t \in T \quad (12)$$

$$L_{eat-1} \leq L_{eat} \quad \forall e \in E, a \in A, b \in B, t \in T \quad (13)$$

$$-\pi \leq \theta_{ibt} \leq \pi \quad \forall i \in N, b \in B, t \in T \quad (14)$$

$$\theta_{1bt} = 0 \quad \forall b \in B, t \in T \quad (15)$$

$$L_{ea0} = 0 \quad \forall e \in E, a \in A, b \in B \quad (16)$$

$$\bar{K}_{i0} = 0 \quad \forall i \in N \quad (17)$$

$$X_{ict} \in \{0, 1\}, Y_{icbt} \geq 0, \theta_{ibt} \text{ unrestricted} \quad \forall i \in N, c \in C, b \in B, t \in T \quad (18)$$

$$K_{ist} \in \{0, 1\} \quad \forall i \in N, s \in S, t \in T \quad (19)$$

$$\bar{K}_{it} \in \{0, 1\}, \quad \forall i \in N, t \in T \quad (20)$$

$$L_{eat}, L'_{eabt} \in \{0, 1\}, f_{eabt} \text{ unrestricted} \quad \forall e \in E, a \in A, b \in B, t \in T \quad (21)$$

The objective function (1) minimizes net present value of total cost; the first three terms are

for the investment costs of generation plants, substation units and transmission lines, respectively. The last term is for the O&M cost. The function  $\kappa(r, TI) = (1+r)(1 - (1+r)^{-TI})/r$  is to calculate the present value of annual cost that has  $TI$  years in each time period with interest rate  $r$  [22]. We remark here that, the cost of losses is not explicitly modeled in the objective function since it is inherently included within the O&M cost. Constraint (2) provides the power balance, that is equivalent to Kirchhoff's first law and implies conservation of the power at each node (after adding losses on the transmission lines for the transferred power) for each time period. Constraint (3) states that generation amount in node  $i$ , at load block  $b$ , at period  $t$  cannot exceed the possible generation amount that could be produced from all of the generation units located at node  $i$  until time period  $t$ . Constraints (4) and (5) limit the total flow leaving from and entering to the substation node or generation node  $i$  with the capacity of the substations located at that node. Constraint (6) is the capacity constraint for the transmission lines. Constraint (7) represents the power flow constraints based upon the Kirchhoff's second law and it determines that the power flow on each line (if used) is equal to the susceptance of the line multiplied by the difference of the phase angles of the nodes. Constraint (8) enforces that a line can be used at period  $t$ , only if the line is built. Constraints (9) and (10) satisfy that if a generation plant is built at node  $i$ , at least one step-up substation should also be built at the same node. Constraint (11) guarantees that  $\bar{K}_{it}$  should be 1 if there is at least one substation unit at node  $i$  at period  $t$ . Constraints (12) and (13) couple time units. Constraint (14) sets limits on the voltage angles at every node. Constraint (15) is the reference point for voltage profile of nodes, and without loss of generality, node 1 is selected as the reference point. Constraints (16) and (17) are the initial points that should be given due to constraints (12) and (13). Constraints (18)-(21) are the domain constraints.

We remark here that, constraints (4), (5) and (7) are nonlinear. Similar to the linearization techniques used in [23], we provide a linear formulation for the problem. In the linear model, the flow amount on each line is expressed as the difference of two nonnegative flow variables  $f_{eabt}^+$  and  $f_{eabt}^-$ :

$$f_{eabt} = f_{eabt}^+ - f_{eabt}^- \quad \forall e \in E, a \in A, b \in B, t \in T \quad (22)$$

and the difference of voltage angles are expressed as the difference of two nonnegative variables  $\Delta\theta_{ebt}^+$  and  $\Delta\theta_{ebt}^-$  as follows:

$$\theta_{ibt} - \theta_{jbt} = \Delta\theta_{ebt}^+ - \Delta\theta_{ebt}^- \quad \forall e = \{i, j\} \in E, b \in B, t \in T \quad (23)$$

By using the above substitutions, constraint (7) is linearized and replaced with the following

ones:

$$f_{eabt}^+ \leq b_{ed}\Delta\theta_{ebt}^+ \quad \forall e \in E, a \in A, b \in B, t \in T \quad (24)$$

$$f_{eabt}^- \leq b_{ed}\Delta\theta_{ebt}^- \quad \forall e \in E, a \in A, b \in B, t \in T \quad (25)$$

$$f_{eabt}^+ \geq b_{ed}\Delta\theta_{ebt}^+ - M(1 - L'_{eabt}) \quad \forall e \in E, a \in A, b \in B, t \in T \quad (26)$$

$$f_{eabt}^- \geq b_{ed}\Delta\theta_{ebt}^- - M(1 - L'_{eabt}) \quad \forall e \in E, a \in A, b \in B, t \in T \quad (27)$$

where  $M$  in (26) and (27) represents a sufficiently large number so that the new constraints does not cut any feasible solution if the line  $e$  is not in operation.

In order to linearize  $|f_{eabt}|$ , we utilize equation (22) and a linear expression of the absolute value in constraints (4) and (5) can be derived as follows:

$$|f_{eabt}| = f_{eabt}^+ + f_{eabt}^- \quad \forall e \in E, a \in A, b \in B, t \in T \quad (28)$$

Thus, a mixed integer linear model for GSTNEP (*GSTNEP-L*) is as follows:

$$\min \quad z_{inv} + z_{sub} + z_{line} + z_{om} \quad (1)$$

s.t (3), (8)-(20), (23)-(27)

$$\sum_{c \in C} Y_{icbt} + \sum_{e \in E | \Omega^-(e)=i} \sum_{a \in A} (f_{eabt}^+ - f_{eabt}^-) - \sum_{e \in E | \Omega^+(e)=i} \sum_{a \in A} (1 + ef_a)(f_{eabt}^+ - f_{eabt}^-) = d_{ibt} \quad \forall i \in N, b \in B, t \in T \quad (2')$$

$$\sum_{e \in E | \Omega^+(e)=i} \sum_{a \in A} (1 + ef_a)(f_{eabt}^+ + f_{eabt}^-) \leq$$

$$\sum_{s \in S} \sum_{t' \in T: t' \leq t} CapS_s K_{ist'} \quad \forall i \in N, b \in B, t \in T \quad (4')$$

$$\sum_{e \in E | \Omega^-(e)=i} \sum_{a \in A} (f_{eabt}^+ + f_{eabt}^-) \leq \sum_{s \in S} \sum_{t' \in T: t' \leq t} CapS_s K_{ist'} +$$

$$d_{ibt}(1 - \bar{K}_{it}) \quad \forall i \in N, b \in B, t \in T \quad (5')$$

$$(1 + ef_a)f_{eabt}^+ \leq CapL_a L'_{eabt} \quad \forall e \in E, a \in A, b \in B, t \in T \quad (6')$$

$$(1 + ef_a)f_{eabt}^- \leq CapL_a L'_{eabt} \quad \forall e \in E, a \in A, b \in B, t \in T \quad (6'')$$

$$L_{eabt}, L'_{eabt} \in \{0, 1\}, f_{eabt}^+ \geq 0, f_{eabt}^- \geq 0 \quad \forall e \in E, a \in A, b \in B, t \in T \quad (21')$$

$$\Delta\theta_{ebt}^+ \geq 0, \Delta\theta_{ebt}^- \geq 0 \quad \forall e \in E, b \in B, t \in T \quad (29)$$

### 3.2. Strengthening the formulation

We also derive valid inequalities to strengthen the model.

**Proposition 1.** *Inequality*

$$\sum_{i \in N} d_{ibt} \leq \sum_{i \in N} \sum_{c \in C} \sum_{t' \in T: t' \leq t} CapG_{ict'} X_{ict'} \quad \forall b \in B, t \in T \quad (30)$$

is valid for *GSTNEP-L*.

The left hand side of the constraint (30) is the total demand in the power system for each load block, at each period and the right hand side of the constraint is the total generation that can be produced from all generation plants. Constraint (30) guarantees that total production amount should meet the total demand in the system for each load block.

**Proposition 2.** *The following inequality is valid for GSTNEP-L:*

$$\sum_{c \in C} \sum_{t' \in T: t' \leq t} X_{ict'} + \sum_{e \in E | \Omega^-(e) = i} \sum_{a \in A} L'_{eabt} \geq 1 \quad \forall i \in N, b \in B, t \in T \quad (31)$$

If there is no generation plants at node  $i$  at period  $t$  ( $X_{ict'} = 0 \forall c \in C$  and  $t' \leq t$ ), from constraint (2') there should be power entering to node  $i$ . Thus, due to constraints (6') and (6''), there should also be at least one transmission line in operation entering to node  $i$  to transmit the power entering to the same node.

### 3.3. Solution approaches

As explained in Section 1, power system expansion planning problems are highly complex and nonlinear problems. Thus, to overcome the computational burden of the problems, dividing the original problem into subproblems and solving these subproblems sequentially based on the output of previous subproblem is a widely used approach in the literature. In this study, we also propose a heuristic approach for the integrated GSTNEP problem to be able to solve larger instances within reasonable solution times. In the computational study, the solutions of the proposed model and heuristic are compared with the solutions obtained using the traditional method in which the subproblems are solved sequentially. We remark here that, solving the model in a sequential manner is also actually solving the GSTNEP problem heuristically since this approach also requires decomposing and optimizing the subproblems of GSTNEP. The sequential solution approach and the proposed time-based solution approach are explained below.

#### 3.3.1. Sequential approach

In this approach, first, the GTNEP problem is solved and decisions related to power flows, transmission lines, generation units and generation amounts are determined. Then, based on the output of GTNEP, we determine the location and capacities of substation units, and calculate the cost of these substations. In order to obtain GTNEP problem, we remove the constraints that are related to substations from GSTNEP-L; that is, constraints (4') and (5') which are related to capacity limitation of substations, constraints (9)-(12) which are the relationship constraints between generation units and substations, and domain constraints ((17), (19) and (20)). The remaining constraints of the GSTNEP-L constitutes a model for the GTNEP problem and referred as *GTNEP-L*:

$$\begin{aligned}
\min \quad & z_{inv} + z_{line} + z_{om} \\
s.t \quad & (2'), (3), (6'), (6'')(8), (13)-(16), (18), (21'), (23)-(27), (29)
\end{aligned} \tag{32}$$

$\hat{X}$ ,  $\hat{G}$ ,  $\hat{f}^+$ ,  $\hat{f}^-$ ,  $\hat{L}$  and  $\hat{L}'$  are the outputs of GTNEP-L and optimum solution value is denoted by  $\hat{z}^{GTNEP}$ . Then, based on the values of these decision variables,  $\hat{K}$  and  $\hat{\bar{K}}$  are determined. Total investment cost of substations ( $z_{sub}$ ) is also calculated and denoted by  $\hat{z}^{SUB}$ . Hence, at the end of this procedure, we obtain a solution for the original problem (GSTNEP) using the sequential solution approach. The corresponding objective value for the solution is equal to  $\hat{z}^{GTNEP} + \hat{z}^{SUB}$ .

### 3.3.2. Time-based approach

We develop a time-based heuristic solution approach for the GSTNEP problem. In the proposed heuristic, instead of dividing the problem into subproblems with respect to system components, we decompose our multi-period problem into a set of single period problems as many as the number of expansion periods ( $|T|$ ) considered.

In this approach, we first define a problem for each time period  $t$  and refer it *Single Period Expansion Problem (SPEP<sub>t</sub>)*. In SPEP<sub>t</sub>, we consider only the first  $t$  periods and ignore the remaining ones. For period  $t$ , we know all the investment decisions (generation units, substations and lines), generation amounts and power flows up to period  $t$ , which are obtained from the solution of SPEP<sub>t-1</sub>, and these variables are fixed in SPEP<sub>t</sub>. Then, we obtain the investment decisions, generation amounts and power flows at period  $t$  by solving SPEP<sub>t</sub>. All the one-period expansion problems (SPEP<sub>1</sub>, SPEP<sub>2</sub>, ... SPEP<sub>|T|</sub>) are solved iteratively by feeding the result of a period as an existing infrastructure for the next time period in a nested manner. Hence, after solving SPEP<sub>|T|</sub>, we obtain a solution for the original GSTNEP problem. For time period  $t$ , (or when  $|T|=t$ ), the model of SPEP<sub>t</sub> is given below.

$$\begin{aligned}
\min \quad & z_{inv} + z_{sub} + z_{line} + z_{om} \\
s.t \quad & (2'), (3), (4'), (5'), (6'), (6''), (8)-(20), (21'), (23)-(27), (29) \\
& X_{ic1} \dots X_{ict-1}, G_{icb1} \dots G_{icbt-1} \text{ are fixed} & \forall i \in N, b \in B, c \in C \\
& K_{is1} \dots K_{ist-1} \text{ are fixed} & \forall i \in N, s \in S \\
& L_{ea1} \dots L_{eat-1} \text{ are fixed} & \forall e \in E, a \in A \\
& f_{eab1}^+ \dots f_{eat-1}^+, f_{eab1}^- \dots f_{eat-1}^- \text{ are fixed} & \forall e \in E, a \in A, b \in B
\end{aligned} \tag{1}$$

## 4. Computational Study

In this section, we introduce the data sets and parameters that are used in the computational study. We test the combination of valid inequalities for the proposed model (GSTNEP-L) and present the solution of the GSTNEP-L for 24-bus power system. In order to discuss the value

of integrating subproblems, we compare the results of GSTNEP-L along with the results of the sequential and time-based solution approaches. We, then, discuss the effect of transmission switching on the results. Lastly, we solve a large instance and present results of the sequential and time-based solution approaches for 118-bus power system. For all the instances, we stopped computations once the reported gap by the solver is less than 1%. All computation results are obtained on a Linux environment with 2.4GHz Intel Xeon E5-2630 v3 CPU server with 64GB RAM. All the experiments of the model and heuristics are implemented using Java Platform, Standard Edition 8 Update 91 (Java SE 8u91) and Gurobi 6.5.1 in parallel mode using up to 32 threads.

#### 4.1. 24-bus power system

The proposed model has been applied to a modified version of IEEE Reliability test system. IEEE Reliability test system includes 24 nodes, 32 generation plants and 35 corridors for building transmission lines [24]. A 15-year planning horizon is considered and divided into three time periods of equal length. Demand of nodes at the initial time period ( $t=0$ ) are the same as the original demand values given in [24] for 24 bus. Annual demand growth rate is assumed to be 6% and the discount rate that includes inflation,  $r$ , is 5% [25]. In the computational study, four load blocks are considered for one year and load-duration curve is given in Figure 3. Percentage of the peak demand ( $p_b$ ) at each load block  $b$  and duration of each load block are obtained from [18]. Demand in each load block ( $d_{ibt} = d_{it}p_b$ ) is calculated with respect to given percentages.

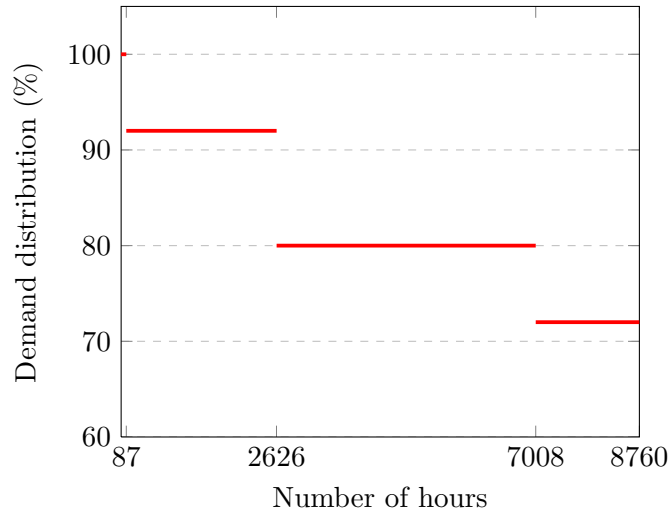


Figure 3: Load-duration curve

For 24-bus, specifications of the transmission lines (capacity, investment cost and reactance of the lines) are obtained from [26]. Maximum three lines are allowed for each corridor and the lines in any corridor have the same characteristics. To allow investment decisions, the maximum

capacity of each line is set to the half of the capacity given in [26]. Moreover, we add seven new corridors for expansion [27]. In [26], instead of providing the distance between the nodes, investment costs of the transmission lines ( $c\text{-}line_{al_e}$ ) are given. The parameters used for 24-bus power system are shown in Table 3. Transmission loss parameter is equal to 2.5% of flow amounts on all the lines [28].

Table 3: Specifications of transmission lines on 24-bus power system

corridor	reactance (per unit)	investment cost ( $10^5\$$ )	capacity (MW)	corridor	reactance (per unit)	investment cost ( $10^5\$$ )	capacity (MW)
existing corridors							
1 - 2	0.0139	7.04	87.5	11 - 13	0.0476	24.1	250
1 - 3	0.2112	106.92	87.5	11 - 14	0.0418	21.16	250
1 - 5	0.0845	42.78	87.5	12 - 13	0.0476	24.1	250
2 - 4	0.1267	64.14	87.5	12 - 23	0.0966	48.9	250
2 - 6	0.192	97.2	87.5	13 - 23	0.0865	43.79	250
3 - 9	0.119	60.24	87.5	14 - 16	0.0389	19.7	250
3 - 24	0.0839	42.47	200	15 - 16	0.0173	8.76	250
4 - 9	0.1037	52.5	87.5	15 - 21	0.049	24.81	250
5 - 10	0.0883	44.7	87.5	15 - 24	0.0519	26.27	250
6 - 8	0.0614	31.08	87.5	16 - 17	0.0259	13.11	250
6 - 10	0.0605	30.63	87.5	16 - 19	0.0231	11.7	250
7 - 8	0.0614	31.08	87.5	17 - 18	0.0144	7.29	250
8 - 9	0.1651	83.58	87.5	17 - 22	0.1053	53.31	250
8 - 10	0.1651	83.58	87.5	18 - 21	0.0259	13.11	250
9 - 11	0.0839	42.47	200	19 - 20	0.0396	20.05	250
9 - 12	0.0839	42.47	200	20 - 23	0.0216	10.93	250
10 - 11	0.0839	42.47	200	21 - 22	0.0678	34.32	250
10 - 12	0.0839	42.47	200				
new corridors							
1 - 8	0.1344	35	87.5	14 - 23	0.062	86	250
2 - 8	0.1267	33	87.5	16 - 23	0.0822	114	250
6 - 7	0.192	50	87.5	19 - 23	0.0606	84	250
13 - 14	0.0447	62	250				

The specifications of the new generation units are given in Table 4 for 24-bus power system. Permitted nodes for expanding generation capacities are 2, 3, 9, 13, 15 and 23. Existing source types and the capacities are obtained from [24]. Table 5 shows different alternatives for the substations and their corresponding investment costs. The costs are estimated using various sources [22, 29].



Table 4: Available generation types

type	1	2	3
capacity (MW)	200	400	600
inv. cost( $10^6$ \$)	100	180	260
O&M cost (\$/MWh)	20	15	12

Table 5: Specifications of available substations types

type	1	2	3	4	5
capacity (MVA)	100	200	300	400	500
inv. cost ( $10^6$ \$)	8.26	14.92	19.98	23.44	25.3

For 24-bus power system, we first compare the combinations of valid inequalities given in Section 3.2 for GSTNEP-L. In each row in Table 6, the second column lists the best bound at the root node, the third column presents the best integer solution found and the last column shows the gap reported by the solver at the end of the two-hour time limit. Our preliminary analyses show that the best bound at the root node is achieved when inequality (30) is added to GSTNEP-L (second row and fourth row in Table 6). The lowest gap at the end of the time limit is achieved when only inequality (30) is added to the model. Although the lowest solution at the end of time limit is obtained when inequalities are not added to the model, we decided to use inequality (30) in the computational experiments for GSTNEP-L since the difference in the best solutions is less than 0.2%.

Table 6: Effect of valid inequalities on the GSTNEP-L for 24-bus power system

	best bound at root node( $10^6$ \$)	best solution ( $10^6$ \$)	gap(%)
no valid ineq	4061.86	4270.18	2.02
(30)	4091.16	4277.04	1.44
(31)	4061.86	4367.82	4.55
(30) & (31)	4091.16	4320.70	2.58

Figure 4 illustrates the optimal expansion of the network for 24-bus power system for the GSTNEP problem. In Figure 4a, black lines and black circles represent existing transmission

lines and generation units, respectively. Candidate lines for expansion are shown in Table 3. Total cost of the optimum solution is represented in Table 7 under the columns titled with “GSTNEP-TS” and it is equal to \$4266.30M. The total expanded capacity of generation, substation and transmission lines at the end of planning horizon are 3200 MV, 6900 MVA and 3075 MW, respectively. At time period 1, at least one substation is built at 16 nodes and 11 of them are the step-up substations. The remaining ones at nodes 9, 10, 11, 12 and 17 are the transmission substations.

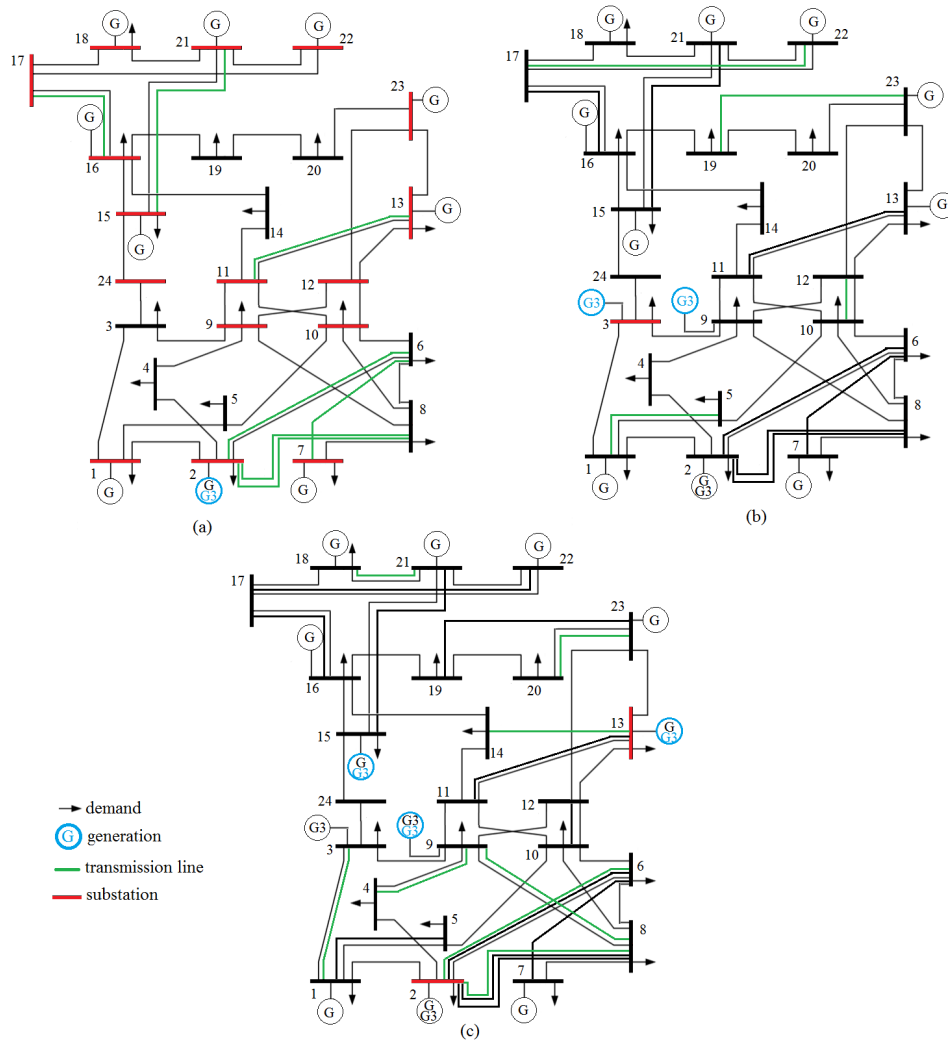


Figure 4: Optimum solution of the problem for 24-bus power system (a) Time period 1 (b) Time period 2 (c) Time period 3

#### 4.1.1. Computational analysis of the solution approaches

This section compares the results of our integrated model (GSTNEP-L) with the sequential and time-based solution approaches. In Table 7, the columns titled with “GSTNEP-TS” show

the results of these three approaches for GSTNEP problem. As well as providing total cost, the cost distribution in terms of investment costs of generation, substation and lines, and operation and maintenance (O&M) cost are reported. We again note that, both approaches solve GSTNEP problem heuristically.

We first compare the objective function values of solutions obtained with sequential and time-based approaches. The proposed time-based approach finds a very good solution since the gap from optimum solution is less than 1%. The solution of the sequential approach is larger than the solution obtained with time-based approach and the difference between the result of sequential approach and optimum solution is 3.86%. The optimum solution of GSTNEP problem is found in approximately 12 hours. Time-based and sequential approaches find the solutions in 2.3 minutes and 1.8 hours, respectively. Thus, in a very short time, the proposed time-based approach finds a better solution than the solution obtained with sequential approach.

#### 4.1.2. Analysis of transmission switching

In this section, we discuss the additional benefit of allowing transmission switching (TS). We again compare the solutions of the three approaches; namely, the optimization model, the sequential approach and the time-based approach.

In the original problem, GSTNEP, all the lines are considered as switchable. In order to guarantee that line switching is not allowed, we added the following constraint to the model:

$$L_{eat} \leq L'_{eabt} \quad \forall e \in E, a \in A, b \in B, t \in T \quad (33)$$

Constraint (33) guarantees that, if a line is built at  $t$ , then the line should be used in the same period. With constraint (13), this requirement holds for all time periods.

In Table 7, we present the results for no TS case under the columns ‘‘GSTNEP-noTS’’. In sequential approach, in order not to allow switching lines, constraint (33) is also added to the GTNEP-L. When TS is not allowed, we obtain similar results for the three solution methods; GSTNEP-L has the lowest objective function value and the sequential solution approach has the largest objective value. The difference between the results of GSTNEP-L and sequential approach is 2.30%, whereas the difference between the solutions of GSTNEP-L and time-based approach is 1.22%. For this case, we report the best solution that is obtained within 12 hours for the GSTNEP-L and the gap reported by the solver is 1.48% at the end of 12 hours. Interestingly, when TS is not allowed, the solution time of the sequential approach (20 seconds) is shorter than the solution time of the proposed approach (13 minutes). We remark here that, in both cases (with and without TS) time-based solution approach finds a better solution than the sequential approach.

In sequential approach, when TS is allowed, total cost decreases by \$10.47M and the main reason of this decrease is the difference in the investment costs of generation units ( $z_{inv}$ ) (Table

Table 7: Comparison of cases GSTNEP-TS and GSTNEP-noTS for three approaches on 24-bus power system

	GSTNEP-TS			GSTNEP-noTS		
	GSTNEP-L	Sequential	Time-based	GSTNEP-L	Sequential	Time-based
total cost ( $10^6$ \$)	4266.30	4430.98	4306.14	4341.42	4441.45	4394.49
$z_{inv}$ ( $10^6$ \$)	1048.06	1017.17	1017.17	1048.06	1048.06	1017.17
$z_{sub}$ ( $10^6$ \$)	395.53	583.87	412.73	448.93	580.16	469.44
$z_{line}$ ( $10^6$ \$)	73.15	61.05	82.64	73.61	49.71	112.64
$z_{om}$ ( $10^6$ \$)	2749.54	2768.88	2793.59	2770.81	2763.53	2795.23
(gap%)		3.86%	0.93%		2.30%	1.22%

7). However, the effect of TS is more obvious in GSTNEP problem and when TS is allowed, total cost decreases by \$75.12M. Although investment costs of generation units and transmission lines are almost the same in GSTNEP-L, investment cost of substations is decreased significantly by allowing transmission switching. Hence, by allowing TS in the GSTNEP problem total cost decreases. Thus, the effect of TS is more pronounced for GSTNEP-L problem (0.17% vs. %0.02) and this amplifies optimizing the substation location decisions.

In Table 8 and Table 9, we show the details of the optimum solutions of GSTNEP problem for the cases when TS is allowed and not allowed. Although the total added generation capacity is the same for both cases, the locations of new generation units at each time period are different. As noted above, the main difference in the solutions of GSTNEP-TS and GSTNEP-noTS is due to investment cost of substations. When TS is not allowed (GSTNEP-noTS), two and four more substations are built at  $t = 1$  and at  $t = 2$ , respectively and total capacities at the end of time horizon are 6900 and 8100 MVA, for the cases TS and no-TS, respectively. As explained above, investment costs of transmission lines in GSTNEP-L are almost the same for both cases. However, the networks at the end of planning horizon are significantly different from each other (Table 9). When TS is not allowed, total number of transmission lines is also increased; one and two more transmission lines are built at  $t = 1$ ,  $t = 2$ , respectively.

#### 4.2. 118-bus power system

The proposed solution approach is also tested with a large network which includes 118 nodes, 54 generation units and 186 corridors for building transmission lines [30]. The total load at the initial time period ( $t=0$ ) is equal to peak load of the power system and the load ratio of each demand node is given in [31]. Maximum generation in the current network is 7220 MW. Demand growth rate is assumed to be 3% per year. Investment cost of transmission lines is 144000 \$/km. The same 61 corridors in [14] are considered for expanding transmission network and 32 nodes

Table 8: Installed generation and substation units for GSTNEP-TS and GSTNEP-noTS on 24-bus power system

	GSTNEP-TS		GSTNEP-noTS	
$t$	bus number (generation type)	# of added subs. units	bus number (generation type)	# of added subs. units
1	2(3)	16	9(3)	18
2	3(3), 9(3)	1	2(3), 3(3)	5
3	9(3), 13(3), 15(1)	2	9(1), 13(3), 15(3)	2

Table 9: Installed number of lines and corridors for GSTNEP-TS and GSTNEP-noTS on 24-bus power system

	GSTNEP-TS		GSTNEP-noTS	
$t$	# of added lines	transmission lines	# of added lines	transmission lines
1	7	(2,6), (2,8), (2,8), (6,7) (11,13), (15,21), (16,17)	8	(7,8), (8,9), (10,12), (12,23), (15,21), (15,24), (16,17), (17,22)
2	4	(1,5), (10,12) (17,22), (19,23)	7	(1,2), (1,5), (2,6), (2,8), (2,8), (14,16), (20,23)
3	8	(1,3), (2,6), (2,8), (4,9) (8,9), (13,14), (18,21), (20,23)	8	(2,6),(2,8), (4,9), (8,9), (13,14), (15,16), (18,21), (20,23)

whose current generation capacities are more than 100 MW are allowed for expanding generation capacities. The remaining parameters are the same with the parameters used in 24-bus power system.

Table 10 shows the results of the three approaches for the GSTNEP problem. We first note that the solution times in 118-bus power system are less than the solution times in 24-bus power system. For the choice of parameters, the density of the transmission network in 24-bus power system is higher than the density of the transmission network in 118-bus power system. Moreover, the ratio of the number of corridors for expanding the network to the total number of corridors is also larger in 24-bus power system which increases number of feasible solutions.

In Table 10, we compare total costs of three solution approaches. We also report cost distributions in terms of investment costs of generation, substation, transmission lines and O&M costs. The value of adding substation location decision to the GTNEP problem can be easily

observed from Table 10. The investment cost of generation units in GSTNEP-L is \$197.31M higher than the investment cost of generation units in sequential approach whereas the investment cost of substations in GSTNEP-L is \$565.49M less than the investment cost of substations in sequential approach. Hence, by adding substation location decisions to the problem, the objective value is decreased by 4.28%. For 118-bus power system, GSTNEP-L finds the optimum solution in approximately 9.3 hours whereas sequential and time-based solution approaches find the solution in 65 seconds and 8 minutes, respectively. Although the solution time of the sequential approach is less than the solution time of the proposed time-based solution approach, the proposed method finds a solution that has smaller objective value than solution of the sequential approach. The gap between the solution of the time-based approach and optimal solution is 2.77%.

Table 10: Comparison of cases GSTNEP-TS for three approaches on 118-bus power system

	GSTNEP-TS		
	GSTNEP-L	Sequential	Time-based
total cost ( $10^6$ \$)	9143.83	9535.15	9397.45
$z_{inv}$ ( $10^6$ \$)	921.03	723.72	662.70
$z_{sub}$ ( $10^6$ \$)	999.36	1564.85	1044.50
$z_{line}$ ( $10^6$ \$)	2.56	0.43	7.81
$z_{om}$ ( $10^6$ \$)	7220.85	7246.16	7682.44
gap(%)		4.28%	2.77%

## 5. Conclusion

In this paper, we study a multi-period long-term power expansion planning problem that includes decisions related to substations' locations and sizes. In the literature, substation decisions are not explicitly considered in the transmission network design problems. In this paper, we propose a mathematical programming model for the integrated problem that finds a minimum cost network and locations of substations and generation units. In the computational study, we discuss the value of adding substation decisions to the problem on 24-bus power system. To overcome computational burden of the problem, we also propose a time-based solution approach in which we decompose the multi-period problem into single-period problems and proceed by fixing the output of the one period in the next time periods. We conclude that this approach provides better results than the results of sequential approach both in terms of solution quality and solution time.

We also analyze the effect of TS to the GSTNEP problem and discuss the results of the model, sequential and time-based approaches when TS is not allowed. In this case, the proposed solution approach also provides better results than the sequential approach in terms of solution quality. We then apply and compare the solution approaches to a larger network, 118-bus power system. Similar to the 24-bus power system, the proposed solution approach provides better solutions compared to solutions obtained with the sequential approach.

## References

- [1] V. Krishnan, J. Ho, B.F. Hobbs, A.L. Liu, J.D. McCalley, M. Shahidehpour, and Q.P. Zheng. Co-optimization of electricity transmission and generation resources for planning and policy analysis: review of concepts and modeling approaches. *Energy Systems*, 7(2): 297–332, 2016.
- [2] E.B. Fisher, R.P. O’Neill, and M.C. Ferris. Optimal transmission switching. *IEEE Transactions on Power Systems*, 23(3):1346–1355, 2008.
- [3] K.W. Hedman, M.C. Ferris, R.P. O’Neill, E.B. Fisher, and S.S. Oren. Co-optimization of generation unit commitment and transmission switching with N-1 reliability. *IEEE Transactions on Power Systems*, 25(2):1052–1063, 2010.
- [4] R. Albert, I. Albert, and G.L. Nakarado. Structural vulnerability of the North American power grid. *Physical Review E*, 69(2):025103, 2004.
- [5] M.S. Sepasian, H. Seifi, A.A. Foroud, and A.R. Hatami. A multiyear security constrained hybrid generation-transmission expansion planning algorithm including fuel supply costs. *IEEE Transactions on Power Systems*, 24(3):1609–1618, 2009.
- [6] P.R.V. Horne, L.L. Garver, and A.E. Miscally. Transmission plans impacted by generation siting a new study method. *IEEE Trans. Power Appa. Syst.*, (5):2563–2567, 1981.
- [7] H.M.D.R.H. Samarakoon, R.M. Shrestha, and O. Fujiwara. A mixed integer linear programming model for transmission expansion planning with generation location selection. *Int. J. Electr. Power Energy Syst.*, 23(4):285–293, 2001.
- [8] B. Alizadeh and S. Jadid. Reliability constrained coordination of generation and transmission expansion planning in power systems using mixed integer programming. *IET Gener. Transm. Distrib.*, 5(9):948–960, 2011.
- [9] B. Alizadeh and S. Jadid. A dynamic model for coordination of generation and transmission expansion planning in power systems. *Int. J. Electr. Power Energy Syst.*, 65:408–418, 2015.
- [10] M. Jenabi, S.M.T.F. Ghomi, S.A. Torabi, and S.H. Hosseinian. A Benders decomposition algorithm for a multi-area, multi-stage integrated resource planning in power systems. *J. Oper. Res. Society*, 64(8):1118–1136, 2012.
- [11] O.J. Guerra, D.A. Tejada, and G.V. Reklaitis. An optimization framework for the integrated planning of generation and transmission expansion in interconnected power systems. *Applied Energy*, 170:1–21, 2016.



- [12] A.H. Seddighi and A. Ahmadi-Javid. Integrated multiperiod power generation and transmission expansion planning with sustainability aspects in a stochastic environment. *Energy*, 86:9–18, 2015.
- [13] F.D. Munoz, B.F. Hobbs, and J.-P. Watson. New bounding and decomposition approaches for MILP investment problems: Multi-area transmission and generation planning under policy constraints. *European Journal of Operational Research*, 248(3):888–898, 2016.
- [14] R. Mínguez and R. García-Bertrand. Robust transmission network expansion planning in energy systems: Improving computational performance. *European Journal of Operational Research*, 248(1):21–32, 2016.
- [15] V. Grimm, A. Martin, M. Schmidt, M. Weibelzahl, and G. Zöttl. Transmission and generation investment in electricity markets: The effects of market splitting and network fee regimes. *European Journal of Operational Research*, 254(2):493–509, 2016.
- [16] M.E. Cebeci, S. Eren, O.B. Tor, and N. Güven. Transmission and substation expansion planning using mixed integer programming. In *North American Power Symposium (NAPS), 2011*, pages 1–5. IEEE, 2011.
- [17] T. Akbari, M. Heidarizadeh, M.A. Siab, and M. Abroshan. Towards integrated planning: Simultaneous transmission and substation expansion planning. *Electric Power Syst. Res.*, 86:131–139, 2012.
- [18] A. Khodaei, M. Shahidehpour, and S. Kamalinia. Transmission switching in expansion planning. *IEEE Transactions on Power Systems*, 25(3):1722–1733, 2010.
- [19] J.C. Villumsen and A.B. Philpott. Investment in electricity networks with transmission switching. *European Journal of Operational Research*, 222(2):377–385, 2012.
- [20] S. Dehghan and N. Amjady. Robust transmission and energy storage expansion planning in wind farm-integrated power systems considering transmission switching. *IEEE Transactions on Sustainable Energy*, 7(2):765–774, 2016.
- [21] L. Baringo and A.J. Conejo. Correlated wind-power production and electric load scenarios for investment decisions. *Applied energy*, 101:475–482, 2013.
- [22] A. Tabares, J.F. Franco, M. Lavorato, and M.J. Rider. Multistage long-term expansion planning of electrical distribution systems considering multiple alternatives. *IEEE Transactions on Power Systems*, 31(3):1900–1914, 2016.
- [23] L. Bahiense, G.C. Oliveira, M. Pereira, and S. Granville. A mixed integer disjunctive model for transmission network expansion. *IEEE Trans. Power Syst.*, 16(3):560–565, 2001.

- [24] IEEE reliability test system. *IEEE Trans Power Appar Syst*, 98(6):2047–2054, 1979.
- [25] A.S. Kocaman, C. Abad, T.J. Troy, W.T. Huh, and V. Modi. A stochastic model for a macroscale hybrid renewable energy system. *Renewable and Sustainable Energy Rev.*, 54: 688–703, 2016.
- [26] N. Alguacil, A.L. Motto, and A.J. Conejo. Transmission expansion planning: a mixed-integer LP approach. *IEEE Transactions on Power Systems*, 18(3):1070–1077, 2003.
- [27] Risheng Fang and David J Hill. A new strategy for transmission expansion in competitive electricity markets. *IEEE Transactions on power systems*, 18(1):374–380, 2003.
- [28] International Electrotechnical Commission. Efficient electrical energy transmission and distribution. *Report, Switzerland*, 2007.
- [29] TEİAŞ. 2015 yılı yatırım programı, 2015. URL <http://www.teias.gov.tr/Yatirimlar.aspx>. [Accessed 7 April 2017].
- [30] IEEE 118 bus system, . URL [http://motor.ece.iit.edu/data/Data\\_118\\_Bus.pdf](http://motor.ece.iit.edu/data/Data_118_Bus.pdf). [Accessed 7 April 2017].
- [31] IEEE 118 bus system, . URL [http://motor.ece.iit.edu/data/118\\_UMP.xls](http://motor.ece.iit.edu/data/118_UMP.xls). [Accessed 7 April 2017].