The Adaptive Robust Multi-Period Alternating Current Optimal Power Flow Problem

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Abstract

This paper jointly addresses two major challenges in power system operations: i) dealing with non-convexity in the power flow equations, and ii) systematically capturing uncertainty in renewable power availability and in active and reactive power consumption at load buses. To overcome these challenges, this paper proposes a two-stage adaptive robust optimization model for the multi-period AC optimal power flow problem (AC-OPF) with detailed modeling considerations such as reactive capability curves of conventional and renewable generators and transmission constraints. This paper then applies strong SOCP-based convex relaxations of AC-OPF combined with the use of an alternating direction method to identify worst-case uncertainty realizations, and also presents a speed-up technique based on screening transmission line constraints. Extensive computational experiments show that the solution method is efficient and that the robust AC OPF model has significant advantages both from the economic and reliability standpoints as compared to a deterministic AC-OPF model.

1 Introduction

A fundamental problem in power system operation is the AC optimal power flow (AC-OPF) problem, which consists of minimizing generation cost subject to satisfying load and several technical constraints on generation units and the transmission network [1]. Due to the non-convexity of the power flow equations, solving AC-OPF to global optimality is very difficult. The current industry practice typically solves a security-constrained DC approximation of AC-OPF, and then checks AC feasibility of the solution [2]. This practice has proven its effectiveness, however, there are significant potential economic and reliability advantages in trying to systematically identify better solutions to AC-OPF [3].

Various algorithms have been developed for solving AC-OPF to obtain stationary or locally optimal solutions [4], [5], [6], [7], [8]. Recently, another important line of work has explored convex relaxation methods. The idea is to develop tractable convex relaxations of this non-convex problem. This serves the purpose of providing a guarantee on the quality of the solutions obtained and also of finding good starting points for a local AC-OPF solver. The tightness and efficiency of solving such a relaxation will determine its usefulness. Several relaxations have been proposed and analyzed, including second-order cone programming (SOCP) relaxations [9], [10], [11], semidefinite programming (SDP) relaxations [12], [13], [14], and moment-based relaxations [15], for a few references. See [16] for a thorough survey.

Besides improving AC-OPF solutions, a challenge of practical significance in today's power systems is the economic and reliable operation under a deep adoption of intermittent renewable power, mainly wind and solar power [17]. The push for sustainability has led to an unprecedented increase in the levels of uncertainty under which power systems operate, and new operational and market methodologies are needed to effectively drive an economic and reliable power system under such large adoptions of wind and solar power. In this context, robust optimization has seen a wide reception by industry and academia as a key philosophy that could effectively help meet this challenge. An important example is given by the two-stage adaptive robust optimization models first developed for the day-ahead unit commitment problem in [18], [19], [20]. In these models, there is a set of first-stage decisions that are made before the uncertainty is revealed and encompass a schedule of on/off generator decisions for the next day,

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and a set of second-stage decisions composed of power dispatch actions that can adapt to the specific realization of uncertainty. After these works, a wide literature on robust optimization for power system operations has started to develop. Other important works on robust unit commitment have considered generator and transmission line contingencies [21], [22], uncertainty in wind power and demand response [23], the use of approximations of the AC-OPF equations [24], and multistage models with affine policies [25], [26], to name a few.

Robust optimization has also been applied to real-time operations. [27] integrated statistical methods for wind forecasting with a static robust look-ahead economic dispatch; [28], [29], and [30] employed affine policies for OPF; [31] developed a robust look-ahead economic dispatch model based on allowable intervals for wind power generation; [32] studied the problem of identifying the largest operating ranges of variable resources that maintain the feasibility of the system; and [33] proposed a two-stage economic dispatch model and the concept of dynamic uncertainty sets for wind speeds. However, all of the above mentioned works on robust optimization for power system operations have considered DC-OPF models, with the exception of [24], where AC-OPF equations are approximated using Taylor expansions and a "signomial transformation".

Other optimization methods incorporating uncertainty have also been applied for the OPF problem. [34], [35], and [36] present stochastic optimization methods for OPF, where [34] uses affine policies and considers storage units under a DC model; [35] considers an interesting two-stage stochastic AC-OPF model, using a sample-average approximation approach; [36] proposes a two-stage stochastic security-constrained DC-OPF model under uncertainty in transmission line contingencies; and [37] presents an interesting approach to manage preventive and corrective controls for security-constrained AC-OPF. Other related methods that have been employed for DC-OPF are distributionally robust optimization [38], [39] and chance constrained models [40], [41]. Chance constrained models have also been employed under AC-OPF formulations [42], [43], [44], [45].

The present paper addresses the challenge of jointly considering the inherent non-convexity in the AC-OPF equations and systematically modeling uncertainty through robust optimization. The main contributions of this paper are summarized as follows:

1. A robust multi-period AC-OPF model under uncertainty in active and reactive load and in renewable power is developed. The model includes detailed technical aspects such as transmission constraints and the reactive capability curves of conventional generators and renewable units.

2. We propose three Robust Conic-OPF models and an efficient algorithm for the approximate solution of the Robust AC-OPF model. We also implement a transmission line screening method as part of a rigorous constraint generation method to speed-up the algorithm.

3. In extensive computational experiments, we investigate the performance of Robust Conic-OPF models to find worst-case uncertainty realizations, show that the Robust AC-OPF model proposed can be efficiently solved, and analyze the potential practical benefits of the proposed approach using a realistic simulation platform.

The remainder of the paper is organized as follows. Section 2 reviews existing OPF models and formulates the Robust AC-OPF problem. Section 3 proposes solution methods for this problem. Section 4 presents extensive computational experiments to understand how the different approximations of AC-OPF can help in finding worst-case uncertainty realizations, the efficiency of the solution method, and the practical benefits of the proposed approach. Section 5 has concluding remarks.

2 Mathematical Models

2.1 Multi-period AC-OPF models

Consider a multi-period AC-OPF problem (denoted simply as AC-OPF in this paper):

$$\min_{\boldsymbol{p}^g,\, \boldsymbol{q}^g,\, \boldsymbol{c},\, \boldsymbol{s},\, \boldsymbol{\theta}} \quad \sum_{i\in\mathcal{G}} \sum_{t\in\mathcal{T}} C^g_{it}\, p^g_{it} \tag{1a}$$

$$\text{s.t.}\ \ \underline{p}^g_{it} \le p^g_{it} \le \overline{p}^g_{it} \quad \forall\, i \in \mathcal{G},\, t \in \mathcal{T} \tag{1b}$$

$$\underline{q}^g_{it} \le q^g_{it} \le \overline{q}^g_{it} \quad \forall\, i \in \mathcal{G},\, t \in \mathcal{T} \tag{1c}$$

$$-RD_{it} \le p^g_{it} - p^g_{i,t-1} \le RU_{it} \quad \forall\, i \in \mathcal{G},\, t \in \mathcal{T} \tag{1d}$$

$$a^g_{im}\, p^g_{it} + b^g_{im}\, q^g_{it} \le h^g_{im}\ \forall\, i \in \mathcal{G},\, t \in \mathcal{T},\, m \in \mathcal{M} \tag{1e}$$

$$\underline{V}_i^2 \le c_{iit} \le \overline{V}_i^2 \quad \forall\, i \in \mathcal{B},\, t \in \mathcal{T} \tag{1f}$$

$$\sum_{k \in \mathcal{G}(i)} p^g_{kt} - p^d_{it} = G_{ii}\, c_{iit} + \sum_{j \in \delta(i)} (G_{ij}\, c_{ijt} - B_{ij}\, s_{ijt}) \quad \forall i \in \mathcal{B}, t \in \mathcal{T} \tag{1g}$$

$$\sum_{k \in \mathcal{G}(i)} q^g_{kt} - q^d_{it} = -B_{ii}\, c_{iit} - \sum_{j \in \delta(i)} (B_{ij}\, c_{ijt} + G_{ij}\, s_{ijt}) \quad \forall i \in \mathcal{B}, t \in \mathcal{T} \tag{1h}$$

$$(f^{max}_{ij})^2 \geq (-G_{ij}\, c_{iit} + G_{ij}\, c_{ijt} - B_{ij}\, s_{ijt})^2 + (B_{ij}\, c_{iit} - B_{ij}\, c_{ijt} - G_{ij}\, s_{ijt})^2 \quad \forall (i,j) \in \mathcal{L},\, t \in \mathcal{T} \tag{1i}$$

$$c_{ijt} = c_{jit}, \quad s_{ijt} = -s_{jit} \quad \forall (i,j) \in \mathcal{L},\, t \in \mathcal{T} \tag{1j}$$

$$c^2_{ijt} + s^2_{ijt} = c_{iit}\, c_{jjt} \quad \forall (i,j) \in \mathcal{L},\, t \in \mathcal{T} \tag{1k}$$

$$\theta_{jt} - \theta_{it} = \arctan(s_{ijt}/c_{ijt}) \ \forall (i,j) \in \mathcal{L},\, t \in \mathcal{T}. \tag{1l}$$

Here, $\mathcal{B}, \mathcal{G}, \mathcal{L}, \mathcal{T}$ are the sets of buses, generators, transmission lines, and time periods, respectively. Decisions p^g_{it} and q^g_{it} are the active and reactive power output of generator i at time t, respectively, and θ_{it} is the voltage angle at bus i, time t. The other decisions correspond to $c_{ijt} = |V_{it}||V_{jt}|\cos(\theta_{it} - \theta_{jt})$ and $s_{ijt} = -|V_{it}||V_{jt}|\sin(\theta_{it} - \theta_{jt})$, for buses i, j at time t, where $|V_{it}|$ is the voltage magnitude at bus i, time t. For generator i at time t, parameters C^g_{it}, $\underline{p}^g_{it}$, $\overline{p}^g_{it}$, $\underline{q}^g_{it}$, $\overline{q}^g_{it}$, RD_{it}, RU_{it} correspond to the unit cost, lower and upper bounds of active power output, lower and upper bounds of reactive power output, and ramp-down and ramp-up capacities, respectively. Parameters a^g_{im}, b^g_{im}, h^g_{im} determine a linear constraint for active and reactive power output at generator i. For bus i, parameters $\underline{V}_i$, $\overline{V}_i$ are the lower and upper bounds on voltage magnitude, respectively, $\mathcal{G}(i)$ is the set of generators at this bus, and $\delta(i)$ is the set of buses connected through a transmission line. For buses i, j, parameters G_{ij} and B_{ij} are the real and imaginary parts of the corresponding admittance matrix element, respectively, and f^{max}_{ij} is the maximum apparent power flow allowed on line (i, j).

In multi-period AC-OPF (1), objective (1a) represents total cost minimization. Constraints (1b)-(1c) impose active and reactive power output bounds; (1d) enforces ramping capacities; and (1e) represents a polyhedral model for reactive capability curve limitations. Constraints (1f) are voltage magnitude bounds; (1g)-(1h) are the power flow equations that determine active and reactive power balance; and (1i) represents transmission line power flow capacities. Constraints (1j)-(1l) are required relations between the $\boldsymbol{c}, \boldsymbol{s}, \boldsymbol{\theta}$ variables that guarantee the consistency of the formulation (see [11] for details on formulating AC-OPF using the $\boldsymbol{c}, \boldsymbol{s}$ variables).

The AC-OPF problem can be equivalently formulated as a quadratically constrained optimization problem through the following "rectangular" formulation (denoted EF-AC-OPF):

$$\min_{\boldsymbol{p}^g, \boldsymbol{q}^g, \boldsymbol{c}, \boldsymbol{s}, \boldsymbol{e}, \boldsymbol{f}} \quad \sum_{i \in \mathcal{G}} \sum_{t \in \mathcal{T}} C^g_{it}\, p^g_{it} \tag{2a}$$

$$\text{s.t.} \quad \text{Eqs. (1b)-(1i) hold} \tag{2b}$$

$$c_{ijt} = e_{it}\, e_{jt} + f_{it}\, f_{jt} \quad \forall (i,j) \in \mathcal{L},\, t \in \mathcal{T} \tag{2c}$$

$$s_{ijt} = e_{it}\, f_{jt} - f_{it}\, e_{jt} \quad \forall (i,j) \in \mathcal{L},\, t \in \mathcal{T}. \tag{2d}$$

Here $\boldsymbol{e}, \boldsymbol{f}$ are such that the voltage phasor at bus i and time t is given by $V_{it} = e_{it} + \mathrm{i}\, f_{it}$.

As seen from formulations (1) and (2), AC-OPF is a non-convex optimization problem. Given the intrinsic difficulty this poses, it is useful to consider *convex relaxations* that can be solved to global optimality in polynomial time. They serve the purpose of providing an approximation for AC-OPF, a guaranteed lower bound on the optimal value, and a potentially good starting point for AC-OPF algorithms. In this paper, we will consider two such relaxations, described in what follows.

First, the SOCP relaxation of AC-OPF (1) (denoted SOCP-OPF) (see [46], [10], [11]) is given by:

$$\min_{\boldsymbol{p}^g, \boldsymbol{q}^g, \boldsymbol{c}, \boldsymbol{s}} \quad \sum_{i \in \mathcal{G}} \sum_{t \in \mathcal{T}} C^g_{it}\, p^g_{it} \tag{3a}$$

$$\text{s.t.} \quad \text{Eqs. (1b)-(1j) hold} \tag{3b}$$

$$c^2_{ijt} + s^2_{ijt} \leq c_{iit}\, c_{jjt} \quad \forall (i,j) \in \mathcal{L},\, t \in \mathcal{T}. \tag{3c}$$

The SOCP-OPF can also be strengthened through several techniques to provide a stronger relaxation [11].

Second, the SDP relaxation of EF-AC-OPF (2) (denoted SDP-OPF) (see [12], [11]) is given by:

$$\min_{\boldsymbol{p}^g, \boldsymbol{q}^g, \boldsymbol{c}, \boldsymbol{s}, \boldsymbol{X}} \quad \sum_{i \in \mathcal{G}} \sum_{t \in \mathcal{T}} C^g_{it}\, p^g_{it} \tag{4a}$$

$$\text{s.t.} \quad \text{Eqs. (1b)-(1i) hold} \tag{4b}$$

$$c_{ijt} = X_{ijt} + X_{i'j't} \quad \forall\,(i,j) \in \mathcal{L},\, t \in \mathcal{T} \tag{4c}$$
$$s_{ijt} = X_{ij't} - X_{i'jt} \quad \forall\,(i,j) \in \mathcal{L},\, t \in \mathcal{T} \tag{4d}$$
$$\boldsymbol{X}_t \succeq 0 \quad \forall\, t \in \mathcal{T}, \tag{4e}$$

where, $i' = i + |\mathcal{B}|$, $j' = j + |\mathcal{B}|$, and $\boldsymbol{X}_t$ is a symmetric postive-semidefinite $2|\mathcal{B}| \times 2|\mathcal{B}|$ matrix.

Besides relaxations of AC-OPF, we will also consider the widely used "DC approximation" of AC-OPF (denoted DC-OPF), given by:

$$\min_{\boldsymbol{p}^g,\,\boldsymbol{\theta}} \quad \sum_{i\in\mathcal{G}}\sum_{t\in\mathcal{T}} C^g_{it}\, p^g_{it} \tag{5a}$$
$$\text{s.t.}\ \ \text{Eqs. (1b), (1d) hold} \tag{5b}$$
$$\sum_{k\in\mathcal{G}(i)} p^g_{kt} - p^d_{it} = \sum_{j\in\delta(i)} B_{ij}(\theta_{it} - \theta_{jt}) \ \ \forall i \in \mathcal{B}, t \in \mathcal{T} \tag{5c}$$
$$B_{ij}\,(\theta_{it} - \theta_{jt}) \le f^{max}_{ij} \quad \forall\,(i,j) \in \mathcal{L},\, t \in \mathcal{T}. \tag{5d}$$

We would like to remark the advantage of a multi-period dispatch model versus a single-period dispatch model. The multi-period model is driven by the increasing level of uncertainty coming from renewable sources, which makes system ramping capability a significant concern. For example, in a recent paper [47], the authors showed that a multi-period dispatch model can substantially improve the system ramping capability versus a single-period dispatch model in a rolling horizon setting. ISOs also started adopting multi-period dispatch models. For example, the CAISO real-time market now solves a look-ahead economic dispatch in a rolling horizon fashion. MISO also has introduced ramping product through a multi-period look-ahead economic dispatch. From the wind forecast point of view, much research has pointed out the difficulty with even predicting hourly wind, for example, see [48] and references therein. So even for real-time dispatch, a multi-period look-ahead dispatch model will be more beneficial than a single-period model.

2.2 Robust AC-OPF model

We can now describe the multi-period Robust AC-OPF model. Consider a time horizon encompassing time periods $\{0,\ldots,T\}$. We will assume that in this horizon, all parameters are exactly known at $t = 0$, while uncertain parameters in $t \in \{1,\ldots,T\}$ are uncertain. We formulate a robust optimization problem where the dispatch decision at $t = 0$ is selected in such a way that the cost of the current time period at $t = 0$ plus the worst-case cost over $t \in \{1,\ldots,T\}$ is minimized, where the worst-case cost is subject to uncertain parameters in some *uncertainty set*. The Robust AC-OPF problem is formulated as follows:

$$\min_{\boldsymbol{x}\in X_{AC}} \left(\boldsymbol{c}^\top \boldsymbol{x} + \max_{\boldsymbol{\xi}\in\Xi} \ \min_{\boldsymbol{y}\in Y_{AC}(\boldsymbol{x},\boldsymbol{\xi})} \boldsymbol{b}^\top \boldsymbol{y} \right). \tag{6}$$

In this problem, $\boldsymbol{x} = (\boldsymbol{p}^g_0, \boldsymbol{q}^g_0)$ and X_{AC} represents all AC-OPF constraints (1b)-(1l) at $t = 0$. Also, $\boldsymbol{\xi} = (\boldsymbol{p}^d, \boldsymbol{q}^d, \overline{\boldsymbol{p}}^r)$, and $\Xi = \mathcal{D} \times \overline{\mathcal{P}}^r$, where $\mathcal{D}$ is an uncertainty set for active ($\boldsymbol{p}^d$) and reactive ($\boldsymbol{q}^d$) load over $t \in \{1,\ldots,T\}$, and $\overline{\mathcal{P}}^r$ is an uncertainty set for available active power ($\overline{\boldsymbol{p}}^r$) at all renewable units over $t \in \{1,\ldots,T\}$. Finally, $Y_{AC}(\boldsymbol{x},\boldsymbol{\xi})$ represents all AC-OPF constraints (1b)-(1l) over $t \in \{1,\ldots,T\}$, under given dispatch decisions at $t = 0$ as well as given active and reactive loads and available active power at renewable units over $t \in \{1,\ldots,T\}$.

In all OPF models presented, we assume that there is a set $\mathcal{R}$ of renewable units such that $\mathcal{R} \subset \mathcal{G}$, with which the vector of available active power at all renewable units over $t \in \{1,\ldots,T\}$ is given by $\overline{\boldsymbol{p}}^r = (\overline{p}^g_{it} : i \in \mathcal{R}, t \in \{1,\ldots,T\})$. With this notation, we do not need to write separate equations for conventional generators and for wind and solar farms in the formulation of the problem.

Problem (6) is a two-stage adaptive robust optimization problem. We refer to $\boldsymbol{x}$ as first-stage variables, since they are decided before uncertainty $\boldsymbol{\xi}$ is revealed, and we refer to $\boldsymbol{y}$ as second-stage variables, since they are decided after a specific realization of uncertain parameters is revealed, and can thus adapt to this realization. See [17] for more discussion on two-stage adaptive robust optimization models.

2.3 Robust Conic-OPF models

The Robust AC-OPF model (6) has complicated non-convex AC-OPF constraints in both the first and second stages. We want to find a computationally efficient way to approximately solve it. To do this, we replace the non-convex

AC-OPF constraints by tractable relaxation or approximation models, such as the SOCP-, SDP- and DC-OPF. We construct three Robust *Conic*-OPF models, each of which uses a type of convex conic relaxation (or approximation) of the AC-OPF constraints. In particular, if problem (6) replaces AC-OPF constraints with the SOCP relaxation defined in (3), we obtain a Robust SOCP-OPF; if the AC-OPF constraints are replaced with the SDP relaxation defined in (4), we obtain a Robust SDP-OPF; and if these non-convex constraints are replaced with the DC-OPF (5), we obtain a Robust DC-OPF.

Using the framework of conic optimization, we can write the Robust SOCP-, SDP-, and DC-OPF models in the following unified form:

$$\min_{\boldsymbol{x}\in X_M} \left(\boldsymbol{c}^\top \boldsymbol{x} + \max_{\boldsymbol{\xi}\in\Xi} \min_{\boldsymbol{y}\in Y_M(\boldsymbol{x},\boldsymbol{\xi})} \boldsymbol{b}^\top \boldsymbol{y} \right), \tag{7}$$

where $Y_M(\boldsymbol{x},\boldsymbol{\xi}) = \{\boldsymbol{y} : \boldsymbol{A}\boldsymbol{y} \geq_K \boldsymbol{f} + \boldsymbol{G}\boldsymbol{x} + \boldsymbol{H}\boldsymbol{\xi}\}$, where K is a closed, pointed, and self-dual cone with non-empty interior, and X_M has an analogous representation. In particular, Robust SOCP-, SDP-, and DC-OPF has K as the Lorentz cone (more precisely, a cross product of Lorentz cones), the SDP cone, and the non-negative orthant, respectively. To specifically denote each of these three cases in (7), we will write $M = \text{SOCP}$, $M = \text{SDP}$, and $M = \text{DC}$, respectively.

For readers not familiar with convex optimization, the notation $\boldsymbol{u} \geq_K \boldsymbol{v}$ means $\boldsymbol{u} - \boldsymbol{v} \in K$ for some cone K. As an example, if $\boldsymbol{z} \in \mathbb{R}^n$ and K is the Lorentz cone (i.e. in the SOCP case), then $\boldsymbol{z} \geq_K \boldsymbol{0}$ means $||(z_1, \ldots, z_{n-1})|| = \sqrt{z_1^2 + \cdots + z_{n-1}^2} \leq z_n$. See [49] and [50] for the basic theory of convex optimization.

Given the structure above, we can exploit conic duality to reformulate problem (7) in a convenient form. Assuming $Y_M(\boldsymbol{x},\boldsymbol{\xi})$ is bounded with non-empty interior, strong duality of the inner min problem holds [49], and taking the dual of this problem leads to

$$\max_{\boldsymbol{\xi}\in\Xi} \min_{\boldsymbol{y}} \left\{ \boldsymbol{b}^\top \boldsymbol{y} : \boldsymbol{A}\,\boldsymbol{y} \geq_K \boldsymbol{f} + \boldsymbol{G}\,\boldsymbol{x} + \boldsymbol{H}\boldsymbol{\xi} \right\} \tag{8a}$$

$$= \max_{\boldsymbol{\xi}\in\Xi} \max_{\boldsymbol{\pi}} \left\{ \boldsymbol{\pi}^\top (\boldsymbol{f} + \boldsymbol{G}\boldsymbol{x} + \boldsymbol{H}\boldsymbol{\xi}) : \boldsymbol{\pi}^\top \boldsymbol{A} = \boldsymbol{b}^\top, \boldsymbol{\pi} \geq_{K^*} 0 \right\} \tag{8b}$$

$$= \max_{\boldsymbol{\xi},\boldsymbol{\pi}} \left\{ \boldsymbol{\pi}^\top (\boldsymbol{f} + \boldsymbol{G}\boldsymbol{x}) + \boldsymbol{\pi}^\top \boldsymbol{H}\boldsymbol{\xi} : \boldsymbol{\xi} \in \Xi, \boldsymbol{\pi}^\top \boldsymbol{A} = \boldsymbol{b}^\top, \boldsymbol{\pi} \geq_K 0 \right\}, \tag{8c}$$

where K^* is the dual cone of cone K, and due to self-duality of K we have $K^* = K$.

If the set Ξ is a conic set (i.e. $\Xi = \{\boldsymbol{\xi} : W\boldsymbol{\xi} \geq_{\hat{K}} \boldsymbol{w}\}$ for some closed, pointed cone with non-empty interior $\hat{K}$, probably different from K), the max-min problem (8a) can be reformulated as a *bi-conic* optimization problem (8c). For instance, if the uncertainty set Ξ is a polytope and the cone K is the non-negative orthant (which leads to the Robust DC-OPF case), then the max-min problem reduces to a bilinear optimization problem.

2.4 Uncertainty sets for load and renewables

In this paper, we consider the following *budget* uncertainty set for active and reactive load:

$$\mathcal{D} = \left\{ (\boldsymbol{p}^d, \boldsymbol{q}^d) : \sum_{i\in\mathcal{B}^d} \frac{|p_{it}^d - \tilde{p}_{it}^d|}{\hat{p}_{it}^d} \leq \Gamma\sqrt{|\mathcal{B}^d|} \right. \tag{9a}$$

$$\sum_{i\in\mathcal{B}^d} \frac{|q_{it}^d - \tilde{q}_{it}^d|}{\hat{q}_{it}^d} \leq \Gamma\sqrt{|\mathcal{B}^d|} \tag{9b}$$

$$p_{it}^d \in [\tilde{p}_{it}^d - \Gamma\hat{p}_{it}^d, \tilde{p}_{it}^d + \Gamma\hat{p}_{it}^d] \;\; \forall i \in \mathcal{B}, t \in \mathcal{T} \tag{9c}$$

$$\left. q_{it}^d \in [\tilde{q}_{it}^d - \Gamma\hat{q}_{it}^d, \tilde{q}_{it}^d + \Gamma\hat{q}_{it}^d] \;\; \forall i \in \mathcal{B}, t \in \mathcal{T} \right\}, \tag{9d}$$

where $\mathcal{B}^d \subset \mathcal{B}$ is the set of load buses, $\tilde{p}_{it}^d$, $\tilde{q}_{it}^d$ are the nominal values for active and reactive load at bus i, time t, respectively, and $\hat{p}_{it}^d$, $\hat{q}_{it}^d$ are variability factors for active and reactive load, respectively (e.g. standard deviation from historic data). Parameter Γ determines the size of the uncertainty set.

For renewable power, we consider the following dynamic uncertainty set, recently proposed in [26]:

$$\overline{\mathcal{P}}^r = \left\{ \overline{\boldsymbol{p}}^r = (\overline{p}_{it}^r)_{it} \;:\; \exists\, \boldsymbol{u},\, \boldsymbol{v} \;\text{ s.t.} \right. \tag{10a}$$

$$\overline{p}_{it}^r = f_{it} + g_{it}\, u_{it} \quad \forall\, i \in \mathcal{R},\, t \in \mathcal{T} \tag{10b}$$

$$\boldsymbol{u}_t = \sum_{l=1}^{L} \boldsymbol{A}^l \, \boldsymbol{u}_{t-l} + \boldsymbol{B} \, \boldsymbol{v}_t \quad \forall \, t \in \mathcal{T} \tag{10c}$$

$$\|\boldsymbol{v}_t\| \leq \Gamma \quad \forall \, t \in \mathcal{T} \tag{10d}$$

$$\sum_{t \in \mathcal{T}} \|\boldsymbol{v}_t\| \leq \rho \, \Gamma \, |\mathcal{T}| \tag{10e}$$

$$\left. 0 \leq \overline{p}^r_{it} \leq \overline{p}^{r,max}_{it} \quad \forall \, i \in \mathcal{R}, \, t \in \mathcal{T} \right\}. \tag{10f}$$

Here, $\overline{p}^r_{it}$ is the available renewable real power at renewable unit i at time t, parameters f_{it} and g_{it} are deterministic seasonality components, matrices $\boldsymbol{A}^l$ and $\boldsymbol{B}$ determine spatial and temporal correlations (of up to L time periods), $\| \cdot \|$ is a norm, Γ is an uncertainty set size parameter, ρ determines a budget of uncertainty over time periods, and $\overline{p}^{r,max}_{it}$ is an upper bound on $\overline{p}^r_{it}$. Component $\boldsymbol{u_t} \in \mathbb{R}^{|\mathcal{R}|}$ can be seen as a residual uncertainty after seasonal components are filtered out, and $\boldsymbol{v_t} \in \mathbb{R}^{N_v}$ as a primitive uncertainty for which temporal and spatial correlations are further filtered out as well, where $N_v \in \{1, \ldots, |\mathcal{R}|\}$.

3 Solution Methods for Robust OPF

In this section, we propose solution strategies for the Robust Conic-OPF models (7) and for the Robust AC-OPF (6). Section 3.1 reviews a constraint-generation framework. Section 3.2 studies a method to solve the max-min problem under Conic-OPF models. Section 3.3 develops a speed-up technique.

3.1 Overall solution framework

Let us first observe that the Robust AC-OPF (6) and the Robust Conic-OPF (7) models share the following general Robust OPF structure:

$$\min_{\boldsymbol{x} \in X} \left(\boldsymbol{c}^\top \boldsymbol{x} + \max_{\boldsymbol{\xi} \in \Xi} \; \min_{\boldsymbol{y} \in Y(\boldsymbol{x}, \boldsymbol{\xi})} \boldsymbol{b}^\top \boldsymbol{y} \right). \tag{11}$$

Now, with one additional variable, this problem can be reformulated as:

$$\min_{\boldsymbol{x} \in X, \eta} \; \boldsymbol{c}^\top \boldsymbol{x} + \eta \tag{12a}$$

$$\text{s.t.} \quad \eta \geq \min_{y \in Y(\boldsymbol{x}, \boldsymbol{\xi})} \boldsymbol{b}^\top \boldsymbol{y} \quad \forall \boldsymbol{\xi} \in \Xi, \tag{12b}$$

which can in turn be reformulated as

$$\min_{\boldsymbol{x} \in X, \eta, \boldsymbol{y}(\cdot)} \; \boldsymbol{C}^\top \boldsymbol{x} + \eta \tag{13a}$$

$$\text{s.t.} \quad \eta \geq \boldsymbol{b}^\top \boldsymbol{y}(\boldsymbol{\xi}) \quad \forall \boldsymbol{\xi} \in \Xi \tag{13b}$$

$$\boldsymbol{y}(\boldsymbol{\xi}) \in Y(\boldsymbol{x}, \boldsymbol{\xi}) \quad \forall \boldsymbol{\xi} \in \Xi. \tag{13c}$$

This problem structure suggests a separation-based strategy. The idea is as follows. In the k-th iteration, consider set $S = \{\boldsymbol{\xi}^*_1, \ldots, \boldsymbol{\xi}^*_k\}$, where $\boldsymbol{\xi}^*_j \in \Xi$ for $j = 1, \ldots, k$, and solve the following master problem:

$$\min_{\boldsymbol{x} \in X, \eta \geq 0, \boldsymbol{y}(\cdot)} \; \boldsymbol{c}^\top \boldsymbol{x} + \eta \tag{14a}$$

$$\text{s.t.} \quad \eta \geq \boldsymbol{b}^\top \boldsymbol{y}(\boldsymbol{\xi}) \quad \forall \boldsymbol{\xi} \in S \tag{14b}$$

$$\boldsymbol{y}(\boldsymbol{\xi}) \in Y(\boldsymbol{x}, \boldsymbol{\xi}) \quad \forall \boldsymbol{\xi} \in S, \tag{14c}$$

to obtain a first-stage solution $\boldsymbol{x} \in X$ and $\eta \geq 0$. Then, solve

$$Q(\boldsymbol{x}) = \max_{\boldsymbol{\xi} \in \Xi} \; \min_{\boldsymbol{y} \in Y(\boldsymbol{x}, \boldsymbol{\xi})} \boldsymbol{b}^\top \boldsymbol{y}, \tag{15}$$

to obtain the corresponding worst-case realization of uncertainty, $\boldsymbol{\xi}^*_{k+1} \in \Xi$. Then, if $Q(\boldsymbol{x}) > \eta$ holds then it means that more scenarios from Ξ are needed in S, so we add $\boldsymbol{\xi}^*_{k+1}$ to S and continue iterating, whereas if $Q(\boldsymbol{x}) \leq \eta$ holds then the algorithm terminates. This solution method is formally presented in Algorithm 1.

Algorithm 1 Solution method for Robust OPF (11)

```
1: k ← 0, S ← ∅
2: repeat
3:    (x, η) ← optimal solution of the master problem (14)
4:    Evaluate Q(x): ξ*_{k+1} ← optimal solution of (15)
5:    S ← S ∪ {ξ*_{k+1}}
6:    k ← k + 1
7: until Q(x) ≤ η
```

Algorithm 1 is called column-and-constraint generation in [51], and it is also independently developed in [18] as a way to speed up the usual Benders' decomposition. In [51], the authors show that Algorithm 1 converges in a finite number of iterations to the optimal solution of (11) when both Ξ and $Y(\boldsymbol{x}, \boldsymbol{\xi})$ are polyhedral. A natural generalization is the following result.

Proposition 1. *For the two-stage robust problem* (11)*, assume Ξ is a non-empty polytope, $Y(\boldsymbol{x}, \boldsymbol{\xi})$ is a non-empty and bounded convex conic set for any $\boldsymbol{x} \in X$ and $\boldsymbol{\xi} \in \Xi$, and the optimal solution $\boldsymbol{\xi}^*_{k+1}$ of* (15) *in Step 4 of Algorithm 1 is an extreme point of Ξ for any $\boldsymbol{x}$, then the solution $\boldsymbol{x}$ produced in Step 3 of Algorithm 1 converges in a finite number of iterations to an optimal solution of* (11)*.*

Proof. Assuming $\boldsymbol{\xi}^*_{k+1}$ in Step 4 of Algorithm 1 is an extreme point of Ξ for any $\boldsymbol{x}$, we have that

$$Q(\boldsymbol{x}) = \max_{\boldsymbol{\xi} \in \mathrm{Ext}[\Xi]} \min_{\boldsymbol{y} \in Y(\boldsymbol{x}, \boldsymbol{\xi})} \boldsymbol{b}^\top \boldsymbol{y},$$

where Ext $[\Xi]$ is the set of extreme points of Ξ, which is a finite set, since we assume that Ξ is a non-empty polytope. Let us also observe that every time the master problem is solved in Step 3 of Algorithm 1 we must have

$$\eta = \max_{\boldsymbol{\xi} \in S} \min_{\boldsymbol{y} \in Y(\boldsymbol{x}, \boldsymbol{\xi})} \boldsymbol{b}^\top \boldsymbol{y},$$

and since $S \subset \Xi$, we must have $Q(\boldsymbol{x}) \geq \eta$ in every iteration. From this we can see that whenever $Q(\boldsymbol{x}) \leq \eta$ it means that $\boldsymbol{x}$ must be an optimal solution of (11), which is the condition under which Algorithm 1 terminates. Now, if Algorithm 1 reaches an iteration where $S = \mathrm{Ext}[\Xi]$ (that is, after adding all extreme points to S), we can see from the above that at that point we will have $\eta = Q(\boldsymbol{x})$ and the Algorithm will terminate. Thus, the number of iterations of Algorithm 1 is at most the same as the number of extreme points of Ξ, and the Algorithm terminates with an optimal solution of (11). □

For brevity, the description of Algorithm 1 does not include cases where infeasibility occurs, however, these situations can be addressed as follows. First, if in any iteration of the method it occurs that the master problem (14) is infeasible, then this means that the Robust OPF (11) is infeasible, and thus the algorithm terminates. This follows directly from the fact that the master problem (14) is a relaxation of the Robust OPF (13), since $S \subset \Xi$ in all iterations. Second, when evaluating $Q(\boldsymbol{x})$ it could occur that the inner min problem in Eq. (15) is infeasible for some $\boldsymbol{\xi}^* \in \Xi$, that is, it could occur that $Y(\boldsymbol{x}, \boldsymbol{\xi}^*) = \emptyset$. In this case, we can define $Q(\boldsymbol{x}) = \infty$ and continue Algorithm 1 with $\boldsymbol{\xi}^*_{k+1} = \boldsymbol{\xi}^*$. This does not necessarily mean that the Robust OPF (11) is infeasible, because there could be another first-stage solution $\boldsymbol{x}'$ such that $Y(\boldsymbol{x}', \boldsymbol{\xi}^*) \neq \emptyset$, and such solution could be identified when solving the master problem (14).

We can see that there are two key steps in Algorithm 1: solving the master problem (14) and evaluating $Q(\boldsymbol{x})$. Under Conic-OPF models the master problem can be solved to optimality in polynomial time, whereas under AC-OPF known solution methods can only guarantee finding a locally optimal solution. Regarding the evaluation of $Q(\boldsymbol{x})$, under any OPF model this accounts to solving a difficult max-min problem (15). We will study this in what follows.

3.2 An alternating direction method for Conic-OPF

As shown in Eq. (8), under conic structures given by $Y_M(\boldsymbol{x}, \boldsymbol{\xi}) = \{\boldsymbol{y} : \boldsymbol{A}\boldsymbol{y} \geq_K \boldsymbol{f} + \boldsymbol{G}\boldsymbol{x} + \boldsymbol{H}\boldsymbol{\xi}\}$ and $\Xi = \{\boldsymbol{\xi} : W\boldsymbol{\xi} \geq_{\hat{K}} w\}$ evaluating $Q(\boldsymbol{x})$ amounts to solving a bi-conic optimization problem (8c). This means that, if $\boldsymbol{\xi}$ is fixed, problem (8c) becomes a conic problem:

$$\max_{\boldsymbol{\pi}} \left\{ \boldsymbol{\pi}^\top (\boldsymbol{f} + \boldsymbol{G}\boldsymbol{x}) + \boldsymbol{\pi}^\top \boldsymbol{H}\boldsymbol{\xi} : \boldsymbol{\pi}^\top \boldsymbol{A} = \boldsymbol{b}^\top, \boldsymbol{\pi} \geq_K \boldsymbol{0} \right\} \tag{16}$$

and if $\boldsymbol{\pi}$ is fixed problem (8c) also becomes a conic problem:

$$\max_{\boldsymbol{\xi}} \left\{ \boldsymbol{\pi}^\top (\boldsymbol{f} + \boldsymbol{G}\boldsymbol{x}) + \boldsymbol{\pi}^\top \boldsymbol{H}\boldsymbol{\xi} : W\boldsymbol{\xi} \geq_{\hat{K}} w \right\}. \tag{17}$$

This suggests an iterative method, first proposed in [52] for the bilinear case. Here we extend it to the bi-conic case, which we call an alternating direction method. The idea is to fix some starting $\boldsymbol{\xi}$ and optimize over $\boldsymbol{\pi}$, to then fix the obtained $\boldsymbol{\pi}$ and optimize over $\boldsymbol{\xi}$, and so on until convergence. This method is formally presented in Algorithm 2.

Algorithm 2 Alternating direction method for (8c)

1: Choose an initial $\boldsymbol{\xi} \in \Xi$
2: **repeat**
3: $\quad \boldsymbol{\pi} \leftarrow$ optimal solution of (16) with objective θ
4: $\quad \boldsymbol{\xi} \leftarrow$ optimal solution of (17) with objective θ'
5: **until** $\theta' = \theta$
6: Output: θ as estimate of $Q(\boldsymbol{x})$ with solution $\boldsymbol{\xi}^*$

In the bilinear case, Algorithm 2 converges monotonically to a KKT point of problem (8c) in a finite number of iterations [52]. This also holds for the bi-conic case when the uncertainty set is polyhedral (i.e. when $\hat{K} = \mathbb{R}^n_+$).

Based on the alternating direction method, for the Robust Conic-OPF models (7) we approximate $Q(\boldsymbol{x})$ as

$$Q_M(\boldsymbol{x}) = \min_{\boldsymbol{y} \in Y_M(\boldsymbol{x}, \boldsymbol{\xi}^*_M)} \boldsymbol{b}^\top \boldsymbol{y}, \tag{18}$$

where $\boldsymbol{\xi}^*_M \in \Xi$ is the best solution identified by the alternating direction method, and $M =$ SOCP, $M =$ SDP or $M =$ DC. This motivates us to use any Conic-OPF model to approximate $Q(\boldsymbol{x})$ for the Robust AC-OPF (6) as

$$Q_{AC}(\boldsymbol{x}) = \min_{\boldsymbol{y} \in Y_{AC}(\boldsymbol{x}, \boldsymbol{\xi}^*_M)} \boldsymbol{b}^\top \boldsymbol{y}. \tag{19}$$

That is, we propose to run the alternating direction method under a Conic-OPF model to obtain a scenario $\boldsymbol{\xi}^*_M \in \Xi$ under which we solve the second-stage AC-OPF problem (19). We will study the performance of the different Conic-OPF models for this purpose in the next Section.

3.3 Screening transmission line constraints

We describe here a critical enhancement for accelerating Algorithm 1 (under any OPF formulation). Large-scale power systems have many transmission lines, among which typically only a small subset is close to the power flow capacity. Therefore, we propose to augment Algorithm 1 by iteratively adding transmission line constraints on an as-needed basis. The idea is to start with a subset of these constraints in both the master problem (14) and the max-min problem (15) and in every iteration check if the constraints not included are satisfied or not. If they are not satisfied, they are incorporated to the problem in explicit form. The resulting enhanced solution method is presented in Algorithm 3. The transmission line screening in Algorithm 3 is similar to the Simultaneous Feasibility Test (SFT) used in the ISO markets. However, the key difference is that SFT usually relies on DC power flows rather than AC, and it is usually manually terminated after a few rounds. The screening algorithm we proposed uses AC power flow and checks all possible transmission violations.

4 Computational Experiments

We conduct extensive computational experiments to study the performance of the algorithms developed as well as the potential advantages in practice of the Robust AC-OPF. To carry out these experiments, we work with adapted versions of the IEEE 14- 118-, and 2736-bus systems [8]. The main features of the test cases employed are summarized in Table 1. All experiments have been performed using Python 2.7 in a PC with an Intel Core i5 processor at 2.4 GHz with 4GB memory, using GUROBI as linear and SOCP solver, MOSEK as SDP solver and IPOPT as local solver for AC-OPF master- and sub-problems. In the Robust OPF, we consider 4 time periods (one corresponding to first-stage decisions and three corresponding to second-stage decisions), and other modeling aspects as described in what follows.

Algorithm 3 Enhanced solution method for Robust OPF (11)

Choose initial sets $\mathcal{L}_M$, $\mathcal{L}_S$ of transmission constraints (1i) to consider in the master (14) and max-min (15) problems
$k \leftarrow 0$, $S \leftarrow \emptyset$
repeat
    $(\boldsymbol{x}, \eta) \leftarrow$ optimal solution of (14) under $\mathcal{L}_M$
    for all $(i, j) \in \mathcal{L}$ **do**
        If the (i, j)-th transmission constraints are not satisfied in (14), do $\mathcal{L}_M \leftarrow \mathcal{L}_M \cup \{(i, j)\}$
    end for
    repeat
        Evaluate $Q(\boldsymbol{x})$: $\boldsymbol{\xi}^*_{k+1} \leftarrow$ optimal solution of (15) under $\mathcal{L}_S$
        for all $(i, j) \in \mathcal{L}$ **do**
            If the (i, j)-th transmission constraints are not satisfied in (15), do $\mathcal{L}_S \leftarrow \mathcal{L}_S \cup \{(i, j)\}$
        end for
    until All transmission constraints in (15) are satisfied
    $S \leftarrow S \cup \{\boldsymbol{\xi}^*_{k+1}\}$
    $k \leftarrow k + 1$
until $Q(\boldsymbol{x}) \leq \eta$ and all transmission constraints are satisfied

Table 1: Main features of the test cases employed

Buses	14	118	2,736
Conventional generators	5	54	253
Wind farms	4	40	60
Solar farms	1	20	30
Loads	12	99	2,011
Lines	20	186	3,504
Conventional generation capacity (MW)	400	9,966	28,868
Installed wind power capacity (MW)	240	2,394	8,400
Installed solar power capacity (MW)	29	1,462	4,363
Average total nominal demand (MW)	254	6,256	14,264
Max total nominal demand (MW)	320	7,878	18,075

Reactive power capability curves of conventional generators, wind farms, and solar farms are adapted from [53], using two linear cuts in Eq. (1e) (that is, $|\mathcal{M}| = 2$), in such a way that the model obtained is equivalent to having bounds on reactive power output that depend linearly on active power output. That is, reactive capability curves are represented through constraints of the following form:

$$- h^g_{i1} + a^g_{i1}\, p^g_{it} \leq q^g_{it} \leq h^g_{i2} - a^g_{i2}\, p^g_{it}.$$

For the dynamic uncertainty set (10) we estimate $\boldsymbol{f}, \boldsymbol{g}, \boldsymbol{A}_l, \boldsymbol{B}$ from data and use $L = 1$, $\rho = 0.2$, several values for the uncertainty set size parameter Γ, and $\|\cdot\| = \max\{\|\cdot\|_1/\sqrt{N_v},\ \|\cdot\|_\infty\}$, with which the uncertainty set becomes polyhedral.

To avoid infeasibility, all experiments employ OPF models augmented with penalty variables that allow to shed load at any load bus, with a unit cost of \$5000/MWh.

Section 4.1 analyzes the solutions obtained for the max-min component of the Robust OPF. Section 4.2 studies the efficiency of the solution method developed for Robust AC-OPF. Section 4.3 studies the performance of the Robust AC-OPF in a simulation platform.

4.1 Solving the max-min problem

4.1.1 Robust Conic-OPF models

We study the alternating direction method proposed in Section 3.2 to solve the second-stage max-min problem of the Robust Conic-OPF (7) models, i.e. using DC-, SOCP- and SDP-OPF for $Y(\boldsymbol{x}, \boldsymbol{\xi})$ in (15). For SOCP-OPF, we strengthen the formulation using the technique proposed in [11, Section 4.4] to obtain tight bounds for the c_{ijt} and s_{ijt} variables. We test 120 max-min problems for each model and solve them over a 5-day horizon, using "small" ($\Gamma = 0.5$) and "large" ($\Gamma = 2$) uncertainty set sizes. In order to try to understand how stable the alternating

Table 2: Max-min problem performance for Robust Conic-OPF

14-bus system						
Conic-OPF model	DC		SOCP		SDP	
Γ	0.5	2	0.5	2	0.5	2
Running time (s)	0.1	0.1	0.3	0.3	5.1	4.0
Iterations	3.0	3.1	3.0	3.1	2.9	3.2
Impr. mul. ini. sols. (%)	0.00	0.07	0.00	0.00	0.00	0.10
118-bus system						
Conic-OPF model	DC		SOCP		SDP	
Γ	0.5	2	0.5	2	0.5	2
Running time (s)	0.8	0.8	1.3	1.7	160.2	151.9
Iterations	3.7	4.1	3.0	4.2	2.4	2.9
Impr. mul. ini. sols. (%)	0.64	0.67	0.00	0.10	0.00	0.01
2736-bus system						
Conic-OPF model	DC		SOCP		SDP	
Γ	0.5	2	0.5	2	0.5	2
Running time (s)	0.8	1.0	75.4	79.6	–	–
Iterations	3.0	3.4	2.9	3.0	–	–
Impr. mul. ini. sols. (%)	0.00	0.02	0.00	0.06	–	–

direction method is to changes in the initial solution, we test starting the algorithm from three different initial solutions corresponding to high, nominal and low net load trajectories in Ξ.

In the experiment, the DC and SOCP cases did not present numeric issues, however, for the SDP case numeric problems appeared for 32% of the runs under the 118-bus system and for all of the runs under the 2736-bus system (in these cases, the algorithm terminates without any solution). For the successful runs, Table 2 summarizes the average running time, number of iterations, and improvement in the objective function when starting the algorithm from multiple initial solutions ("Impr. mul. ini. sols."). From the results obtained, we can observe that the method runs extremely fast for the DC-OPF case, very fast for the SOCP-OPF case, and significantly slower for the SDP-OPF case (for the cases that could be solved); the number of iterations is similar for the three cases; and testing different initial solutions does not significantly help (especially for the SOCP and SDP cases).

4.1.2 Robust AC-OPF

To approximately solve the Robust AC-OPF model (11), we propose to estimate $Q(\boldsymbol{x})$ by first using Conic-OPF (SOCP-, SDP- or DC-OPF) in the second-stage problem to obtain a solution $\boldsymbol{\xi}_M \in \Xi$, and then solve AC-OPF, taking the cost obtained as approximation of $Q(\boldsymbol{x})$, as described in Eq. (19).

We test 120 max-min problems. For each of them, we run the alternating direction method under each of the Conic-OPF models to obtain a corresponding solution $\boldsymbol{\xi}_M$, with $M \in \{\mathrm{DC}, \mathrm{SOCP}, \mathrm{SDP}\}$, respectively. Then, we solve AC-OPF under each of these three $\boldsymbol{\xi}_M$'s, and rank the $\boldsymbol{\xi}_M$'s according to their AC-OPF cost, given by

$$v_{AC}(\boldsymbol{\xi}_M) = \min_{\boldsymbol{y} \in Y_{AC}(\boldsymbol{x}, \boldsymbol{\xi}_M)} \boldsymbol{b}^\top \boldsymbol{y},$$

with which we denote as $\boldsymbol{\xi}^*$ the one that achieves the highest AC-OPF cost $v_{AC}(\boldsymbol{\xi}^*) = \max\{v_{AC}(\boldsymbol{\xi}_M) : M \in \{\mathrm{DC}, \mathrm{SOCP}, \mathrm{SDP}\}\}$. With this, $\boldsymbol{\xi}^*$ is the best solution identified for the max-min problem (15) under AC-OPF. Finally, under $\boldsymbol{\xi}^*$ we also calculate the cost obtained by each Conic-OPF model, given by

$$v_M(\boldsymbol{\xi}^*) = \min_{\boldsymbol{y} \in Y_M(\boldsymbol{x}, \boldsymbol{\xi}^*)} \boldsymbol{b}^\top \boldsymbol{y},$$

for $M \in \{\mathrm{DC}, \mathrm{SOCP}, \mathrm{SDP}\}$, respectively.

Table 3 summarizes the average results obtained for: i) the relative difference between $v_{AC}(\boldsymbol{\xi}_M)$ and $v_{AC}(\boldsymbol{\xi}^*)$, which measures the quality under AC-OPF of the solution identified for the max-min problem (15) using each of the Conic-OPF models; and ii) the relative difference between $v_M(\boldsymbol{\xi}^*)$ and $v_{AC}(\boldsymbol{\xi}^*)$, which measures how close each Conic-OPF model is to AC-OPF in terms of costs. The SDP case under the 2736-bus system is excluded due to numeric instability.

From the results obtained, we can observe that: i) SDP-OPF is closest as approximation to AC-OPF and the best solutions identified for the max-min problem (15) are naturally obtained by using this approximation (for the cases that can be solved); ii) using SOCP-OPF has a better performance than DC-OPF and leads to good solutions

Table 3: Max-min problem performance for Robust AC-OPF (in %)

14-bus system						
Conic-OPF model (M)	DC		SOCP		SDP	
Γ	0.5	2	0.5	2	0.5	2
$v_{AC}(\boldsymbol{\xi}_M)$ vs $v_{AC}(\boldsymbol{\xi}^*)$	-0.84	-2.91	-0.13	-2.65	0.00	-0.02
$v_M(\boldsymbol{\xi}^*)$ vs $v_{AC}(\boldsymbol{\xi}^*)$	370.17	317.69	-3.15	-7.20	0.00	0.00
118-bus system						
Conic-OPF model (M)	DC		SOCP		SDP	
Γ	0.5	2	0.5	2	0.5	2
$v_{AC}(\boldsymbol{\xi}_M)$ vs $v_{AC}(\boldsymbol{\xi}^*)$	-0.26	-6.85	-0.12	-2.42	0.00	-1.58
$v_M(\boldsymbol{\xi}^*)$ vs $v_{AC}(\boldsymbol{\xi}^*)$	188.72	261.17	-1.98	-8.00	-0.54	-4.62
2736-bus system						
Conic-OPF model (M)	DC		SOCP		SDP	
Γ	0.5	2	0.5	2	0.5	2
$v_{AC}(\boldsymbol{\xi}_M)$ vs $v_{AC}(\boldsymbol{\xi}^*)$	-0.23	-1.16	0.00	0.00	–	–
$v_M(\boldsymbol{\xi}^*)$ vs $v_{AC}(\boldsymbol{\xi}^*)$	8.09	4.32	-1.00	-1.13	–	–

Table 4: Running time (s) for Robust AC-OPF (average over 120 min-max-min problems)

14-bus system						
Γ	0.5	1	1.5	2	2.5	3
Without TS	1.0	1.1	1.2	1.2	1.2	1.3
With TS	0.8	0.9	1.0	1.0	1.0	1.0
118-bus system						
Γ	0.5	1	1.5	2	2.5	3
Without TS	10.4	11.6	15.3	25.7	23.1	26.9
With TS	7.7	8.2	11.2	14.9	17.8	20.7
2736-bus system						
Γ	0.5	1	1.5	2	2.5	3
Without TS	798	904	820	936	1144	1343
With TS	342	356	353	394	414	462

under $\Gamma = 0.5$ but can lose quality under $\Gamma = 2$; and iii) DC-OPF is not a good approximation to AC-OPF under the cases studied, as it can find a significantly different cost (which we find to be a result of high penalty costs that occur under DC-OPF but not under AC-OPF). From these results, it seems that it would be most convenient to use SDP-OPF in the alternating direction method if it could be solved reliably and efficiently, however, as mentioned in Section 4.1.1 this choice is not stable and also very slow. Therefore, we conclude that SOCP-OPF provides a good overall choice as approximation to AC-OPF, for the purpose of approximately solving the max-min problem (15).

4.2 Overall algorithmic performance

We now turn our attention to the efficiency of the overall algorithm proposed in Section 3.3. We solve 120 Robust AC-OPF problems under different conditions corresponding to a simulation over 5 days, using the SOCP-OPF approximation of AC-OPF in the alternating direction method (a choice we justify from the analysis in Section 4.1). Tables 4 and 5 respectively summarize the average running time and number of iterations of the algorithm with ("With TS") and without ("Without TS") the transmission screening technique described in Section 3.3. From these Tables we can observe that the transmission screening technique can significantly speed-up the solution method and does not significantly affect the number of iterations. In particular, we obtain a 42% average time reduction for the 118-bus system with $\Gamma = 2$ and a 66% average time reduction for the 2736-bus system with $\Gamma = 3$. Further, we believe that the average running time below 21 seconds obtained for the 118-bus system is very promising for real-time applications of the Robust AC-OPF in medium-sized power systems with a significant adoption of intermittent renewable power.

We also look at the closeness between the worst-case cost of Robust AC-OPF estimated using the solution method proposed and a guaranteed lower bound to the optimal worst-case Robust AC-OPF cost. This lower bound comes from solving the last master problem (14) in Algorithm 3 using SOCP-OPF (3), which gives a relaxation of Robust AC-OPF (6). This follows from two facts: i) the master problem (14) is a relaxation of the Robust OPF problem (11) (under any OPF formulation); and ii) Robust SOCP-OPF is a relaxation of Robust AC-OPF, which in turn follows from the fact that deterministic SOCP-OPF (3) is a relaxation of deterministic AC-OPF (1). Table 6 shows the

Table 5: Number of iterations for Robust AC-OPF (average over 120 min-max-min problems)

14-bus system						
Γ	0.5	1	1.5	2	2.5	3
Without TS	2.0	2.1	2.2	2.2	2.2	2.2
With TS	2.0	2.1	2.2	2.2	2.2	2.3
118-bus system						
Γ	0.5	1	1.5	2	2.5	3
Without TS	2.0	2.0	2.3	2.4	2.6	2.8
With TS	2.0	2.0	2.3	2.4	2.8	2.9
2736-bus system						
Γ	0.5	1	1.5	2	2.5	3
Without TS	2.0	2.0	2.0	2.1	2.1	2.1
With TS	2.0	2.0	2.0	2.0	2.0	2.1

Table 6: Estimated optimality gap for Robust AC-OPF (average over 120 min-max-min problems) (in %)

Γ	0.5	1	1.5	2	2.5	3
14-bus system	3.48	3.48	4.30	5.31	9.02	16.49
118-bus system	2.50	2.53	6.69	7.87	8.54	13.64
2736-bus system	0.91	0.92	0.95	1.08	1.33	1.65

corresponding average estimated optimality gaps obtained. From this Table, we can observe that as the uncertainty set size parameter Γ increases, this gap also increases.

4.3 Model comparison through rolling-horizon simulations

This part compares the practical performance that the Robust AC-OPF can offer, using a simulation platform that mimics the hour to hour operation of the power system under the true full AC-OPF power flow equations. The simulation consists of a rolling-horizon process, where Robust AC-OPF is repeatedly solved as time moves forward. The simulation starts by observing uncertain parameters at the first hour and solving the Robust AC-OPF under a horizon that spans from this hour (first-stage decision with known parameters) to three hours ahead (second-stage decisions under uncertain parameters). Then, the decision for this first hour is implemented and we move on to observe the parameters at the second hour, solving now a Robust AC-OPF that spans this second hour (first-stage decision) and three hours ahead (second-stage decisions). This process is repeated moving forward until the last simulated hour, after which we compute several cost and reliability performance metrics achieved by the implemented AC-OPF decisions. We consider a simulation horizon of 100 contiguous days (i.e. 2400 hours). To simulate uncertain parameters, we use a normal distribution for active and reactive power demand at each node (assuming temporal and spatial independence), and a vector autoregressive process for wind and solar power [26] (that captures temporal and spatial correlations), for which we use data from NREL's Western Wind and Solar Integration Datasets [54] for parameter estimation. The average renewable penetration under these simulated trajectories is 21.6% for the 14-bus system and 18.8% for the 118-bus system.

The results from these simulations are summarized in Table 7, under several values for the uncertainty set size parameter Γ. In this Table, "Cost Avg" is the average hourly cost over the simulated horizon, "Cost Std" is the standard deviation of hourly cost, "Penalty Avg" is the average hourly penalty cost, "Penalty Freq" is the proportion of time periods where penalty occurred, "T Congestion" is the average transmission congestion, based on a metric corresponding to the proportion of transmission line capacities that have an utilization of 90% or more, "Ramp Capacity" is the average system-level ramp-up capacity as a proportion to the sum of all ramp-up rates, and "Renewable Util" is the average utilization of renewable power, corresponding to the proportion of dispatched renewable power with respect to its availability.

We can first observe that the tradeoff between economic effectiveness and system reliability can be controlled through the uncertainty set size parameter Γ. As Γ increases we can see that: average costs tend first to decrease (until $\Gamma = 1$ for both systems) and then to increase, cost standard deviation decreases sharply first and then slowly increases, penalty costs and frequency decrease until being completely eliminated, transmission congestion decreases, ramping capacities increase, and renewable power can be slightly curtailed. These results show that the value of Γ can be used to "tune" the conservativeness with which the power system is operated. A higher Γ provides a more reliable operation, but potentially at a significant cost if Γ is too large.

Table 7: Rolling-horizon simulation results under Robust AC-OPF

14-bus system						
Γ	0	0.5	1	1.5	2	2.5
Cost Avg ($)	9162	8123	7841	8116	8677	9405
Cost Std ($)	10569	5700	2984	2625	2796	3056
Penalty Avg ($)	1642	540	78	6	0	0
Penalty Freq	4.88%	2.29%	0.50%	0.04%	0.00%	0.00%
T Congestion	4.97%	4.66%	4.31%	3.78%	2.71%	1.71%
Ramp Capacity	57.51%	60.72%	66.50%	74.73%	84.71%	92.09%
Renewable Util	99.98%	99.93%	99.75%	99.00%	96.85%	91.45%
118-bus system						
Γ	0	0.5	1	1.5	2	2.5
Cost Avg ($)	181625	155358	149778	153822	163126	181540
Cost Std ($)	206116	102140	59992	63461	68016	68415
Penalty Avg ($)	37032	9717	181	0	0	0
Penalty Freq	7.62%	3.33%	0.36%	0.00%	0.00%	0.00%
T Congestion	2.83%	2.72%	2.43%	2.05%	1.61%	1.07%
Ramp Capacity	33.32%	34.97%	37.89%	41.41%	46.45%	52.17%
Renewable Util	99.99%	99.99%	99.99%	99.99%	99.98%	99.95%

We can also observe that the Robust AC-OPF provides a tremendous advantage when compared to the Deterministic AC-OPF, which corresponds to the case with $\Gamma = 0$. For the 14-bus system the average cost savings can be up to 14.4% (with $\Gamma = 1$) and cost standard deviation can be reduced up to 75.2% ($\Gamma = 1.5$), while for the 118-bus system the average cost savings can be up to 17.5% ($\Gamma = 1$) and cost standard deviation can be reduced up to 70.9% ($\Gamma = 1$). Also, for both systems a sufficiently large Γ results in null penalty, while Deterministic AC-OPF presents a significant level of penalty.

In summary: i) a convex relaxation of AC-OPF, such as SOCP-OPF, combined with the alternating direction method provides a good way for approximately solving the difficult max-min problem of finding a worst-case uncertainty realization; ii) the Robust AC-OPF model can be efficiently solved in approximate form; and iii) Robust AC-OPF provides an effective way to manage the uncertainty in active and reactive power demands, as well as in wind and solar power, offering an explicit way to control the desired conservativeness in the operation of the power system.

5 Conclusion

We have developed an adaptive robust optimization model for the multi-period AC-OPF problem, proposed an effective solution method using convex relaxations of AC-OPF, and carried out extensive experiments to understand the benefits and limitations of this approach. The modeling framework developed is innovative in offering a systematic treatment of the non-convexities presented by the power flow equations and the inherent uncertainty in power demand and renewable power, considering technical details such as transmission constraints and reactive capability curves of conventional generators and renewable units. The Robust AC-OPF proposed can be efficiently solved and transmission line screening techniques can significantly speed-up the algorithm. Finally, using a realistic simulation platform that mimics the hour to hour operation of the power system, we showed that the Robust AC-OPF can offer significant advantages in terms of cost effectiveness and system reliability, as compared to a Deterministic AC-OPF model, further offering a simple mechanism for balancing the operational conservativeness.

Regarding future work, an interesting challenge raised by this paper is to find theoretical guarantees for Robust AC-OPF. For example, is there an efficient solution method that guarantees convergence to a local optimum of this problem? Another important direction for future work is to extend the proposed approach to incorporate security constraints.

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