Algorithms and Software for the Golf Director Problem

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First draft: December 2016, This draft: October 21, 2017

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Abstract

The golf director problem introduced in Pavlikov et al. (2014) is a sports management problem that aims to find an allocation of golf players into fair teams for certain golf club competitions. The motivation for the problem is that club golf competitions are recreational events where the golf director wants to form teams that are competitive even though the players are of different skill levels, as measured by their USGA (http://www.usga.org) or R&A (http://www.randa.org) handicaps. We present an efficient simulation and optimization-based procedure that finds a near-optimal fair team allocation. A computer implementation of the proposed solution method is publicly available and located at http://www.fairgolfteams.com. The website provides a golf director with a variety of controls to manage and run club golf competitions in a fair way. Computational results of allocating real sets of golfers into teams are presented in a demonstration of the website.

Keywords: Simulation, Swap Heuristics, Golf Game Management, Golf Handicapping

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1. Introduction

1.1. Overview

This introduction provides a brief summary of mathematical approaches to golf team formulations and/or scoring with adjustments based on player skill level as measured by their handicap. A detailed example of the golf director problem then follows. The main sections of this paper describe our approach in which scenarios are generated and provide a basis for team allocation and adjustments in seeking a fair allocation.

1.2. Background and Related Literature

Determining a player's handicap is an involved calculation based on a partial history of the better scores recorded by the player. The final result is a regularly adjusted handicap index which is used to compute the player's course-dependent handicap. Studies of golf handicapping can be found in the works of Francis Scheid (see the references in Ragsdale et al. (2008)). An early reference that considers fairness in golf handicapping for individual games is Pollock (1974).

Most of the analytical literature for golf games takes the individual assigned handicaps as given, and we do the same in our study. The central issue is how to adjust, modify or combine individual handicaps for team play. For example, the game of fourball involves two 2-player teams where the team scores on any one hole are the two minima (best) of the scores of the two players on the teams after handicaps are taken into account. The paper by Hurley and Sauerbrei (2015) concludes, after considering many handicap combinations, that the authors had "...not been able to find a simple rule to apply to individual integer handicaps to make a net best-ball competition fair." See also Pollard and Pollard (2010) who state "...it is not strictly possible to rate individual players..." as fourball players. Similar conclusions were reached by Siegbahn and Hearn (2010) who also provide analysis, including a tiebreaker that induces fairness, and a simulation approach for the fourball game based on the player score distributions derived from 250,000 actual rounds of golf provided by GolfNet, Inc (http://golfnet.com/). This approach is embodied in the app on the web site http://www.siegbahn.com/golf/, which computes fair teams from the user input of four handicaps.

The papers by Grasman and Thomas (2013), Ball and Halper (2009), Ragsdale et al. (2008), Lewis (2005), Dear and Drezner (2000), and Tallis (1994) contain further references and each consider variants of a popular tournament game known as "scramble". In that game, all golfers on a team play an initial ("tee") shot on each hole and then all team members play subsequent shots from the best position achieved by any member on the prior shot. As shown below, for the problem we consider each golfer playing their own ball for every hole of play.

The golf director problem consists of finding an allocation of players into teams and is a challenging mathematical optimization problem. As part of the allocation of players to teams, the golf director controls the number of teams (and therefore the team sizes). We address the usual version of this net stroke play game in which individual gross scores are reduced on certain holes depending on handicap. The first challenge of the problem is that score performance of a player is stochastic by nature, and the model uses the estimated probability distributions of scores for every handicap, obtained in Siegbahn and Hearn (2010). In Pavlikov et al. (2014), an optimization model finding a fair team allocation was formulated where players of various handicaps were assumed to play a one-hole match and the result of a team is based on that of the best player score. Fairness of allocation is evaluated using the range of team probabilities to win a golf game and is formally defined in Section 2.1. This paper extends the golf director problem introduced in Pavlikov et al. (2014) in various directions. It considers a more realistic version of the golf director problem where the game is played over 18 holes and a team score can be based on the scores (hole by hole) of more than one player on a team. The main challenge of the 18-hole game is the fact that (gross) scores of different players are adjusted on certain holes according to player handicap and hole difficulty. Thus, all holes are not the same with respect to final scoring, and so the one-hole model can not generally solve the 18-hole game. Yet, one-hole games serve as a basis to obtain a set of good team allocation candidates for the real 18-hole game, and are central to the proposed solution procedure. One of the advantages of the solution method is flexibility in forming teams of same or different sizes, and ability to incorporate other real-life constraints, for example situations when certain players do not want to be on the same team.

1.3. The Golf Director Problem

Club golf competitions are regular events arranged by golf directors (or professionals) for club members at both public and private clubs. Player skill levels are measured by their USGA or R&A handicaps and it is the job of the director to use the handicaps to organize teams that are, in some sense, fair. In a real problem, the number of players could be large, say 40, and this would lead to 10 teams of 4 players each. The team score would be derived from one or more individual player scores for each hole and then totaled over 18 holes for the team score.

Example. This problem will be illustrated in the three tables below, which represent scorecards with two players on each of two teams. The team score on each hole is determined from the lower net score of the two players (known as the "team net best ball"), and the team total score is summed over the 18 holes. The player handicaps are known and each player receives a deduction from their gross score according to the course handicapping of holes by difficulty. Let the players be A, B, C, and D with handicaps 3, 8, 10 and 12, respectively. The first scorecard has team A and B versus team C and D. It is assumed that all holes are par 4 and the handicap holes are at the beginning of the 18 holes. Thus A gets a deduction on the first 3 holes, B on the first 8 holes, C on the first 10 holes and D on the first 12. These deductions are indicated with a minus sign (-) beside each gross score. So 4- means the score is counted as 3 in calculation of the team net score.

Hole #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
A	4-	5-	4-	3	4	4	4	5	4	4	4	5	4	5	3	4	4	5
B	4-	5-	5-	5-	4-	4-	4-	4-	4	4	5	4	6	4	4	5	4	5
net score	3	4	3	3	3	3	3	3	4	4	4	4	4	4	3	4	4	5
C	5-	4-	5-	4-	5-	4-	6-	5-	5-	4-	5	4	4	4	6	5	6	5
D	6-	6-	5-	4-	5-	5-	4-	4-	4-	5-	6-	5-	4	5	6	4	4	5
net score	4	3	4	3	4	3	3	3	3	3	5	4	4	4	6	4	4	5

Table 1: A scorecard for {A, B} team versus {C, D} team for a 18-hole game. A team score over an 18-hole game is defined as the sum of its net scores over 18 holes, where the team net score on an individual hole is determined on the basis of the score of its best player. Hence, in this example, the {A, B} team scores 65 and the {C, D} team scores 69.

On the scorecard in Table 1, team {A, B} wins by 4 strokes. But is there a fairer assignment of the four players to the two teams given this scorecard?

Consider the alternative pairings of {A, C} versus {B, D} and {A, D} versus {B, C}, with the individual player scores the same. The second and third scorecards show the results. Team {A, C} defeats {B, D} by a score of 66 to 67 and {A, D} versus {B, C} allocation results in a tied score of 67. Thus, it might be argued that either of these pairings is more fair than the original one since, in one case, the difference of team scores is just one stroke, and, in the other, the scores are identical.

Hole #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
A	4-	5-	4-	3	4	4	4	5	4	4	4	5	4	5	3	4	4	5
C	5-	4-	5-	4-	5-	4-	6-	5-	5-	4-	5	4	4	4	6	5	6	5
net score	3	3	3	3	4	3	4	4	4	3	4	4	4	4	3	4	4	5
B	4-	5-	5-	5-	4-	4-	4-	4-	4	4	5	4	6	4	4	5	4	5
D	6-	6-	5-	4-	5-	5-	4-	4-	4-	5-	6-	5-	4	5	6	4	4	5
net score	3	4	4	3	3	3	3	3	3	4	5	4	4	4	4	4	4	5

Table 2: A scorecard for {A, C} team versus {B, D} team for a 18-hole game. A team score over an 18-hole game is defined as the sum of its net scores over 18 holes, where the team net score on an individual hole is determined on the basis of the score of its best player. Hence, in this example, the {A, C} team scores 66 and the {B, D} team scores 67.

Hole #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
A	4-	5-	4-	3	4	4	4	5	4	4	4	5	4	5	3	4	4	5
D	6-	6-	5-	4-	5-	5-	4-	4-	4-	5-	6-	5-	4	5	6	4	4	5
net score	3	4	3	3	4	4	3	3	3	4	4	4	4	5	3	4	4	5
B	4-	5-	5-	5-	4-	4-	4-	4-	4	4	5	4	6	4	4	5	4	5
C	5-	4-	5-	4-	5-	4-	6-	5-	5-	4-	5	4	4	4	6	5	6	5
net score	3	3	4	3	3	3	3	3	4	3	5	4	4	4	4	5	4	5

Table 3: A scorecard for {A, D} team versus {B, C} team for a 18-hole game. A team score over an 18-hole game is defined as the sum of its net scores over 18 holes, where the team net score on an individual hole is determined on the basis of the score of its best player. Hence, in this example, the {A, B} team ties with the {C, D} team, both scoring 67.

As mentioned, a mathematical approach to a one-hole version of the golf director problem can be found in the paper Pavlikov et al. (2014). However, the size of real problems makes a precise solution impossible even on the fastest computers. For example, 24 players can generate up to 4^{24} scoring scenarios if there are 4 possible scores per player per hole and, if they are to be assigned to six teams of size four, there are $O(10^{12})$ possible teams.

The approach discussed in the following sections generates a large number of scenarios using the scoring distributions for players with handicaps 0 to 36 developed in Siegbahn and Hearn (2010) and combinatorial techniques then search the possible team combinations for an assignment where the teams have similar probabilities to win, i.e., the teams determined are, in that sense, fair. An online (beta) implementation is located at https://www.fairgolfteams.com.

2. Technical Description

Our model for the golf director problem is based on the following set of assumptions:

- each player's hole score is a random variable, for which the probability distribution solely depends on player's handicap,
- games have multiple players and it is assumed that their scores are independent of each other, and
- the game involves playing 18 holes of different difficulty, with the score of every player being adjusted according to some predefined rule that depends on the hole number and player's handicap.

Such assumptions appear reasonable and allow us to create a useful model. Suppose there are n players playing an 18-hole game; that game can be fully described by an $18 \times n$ matrix of gross player scores:

$$G' = \begin{bmatrix} s_{11} & s_{12} & s_{13} & \dots & s_{1n} \\ \dots & \dots & \dots & \dots & \dots \\ s_{18,1} & s_{18,2} & s_{18,3} & \dots & s_{18,n} \end{bmatrix}.$$

In order to account for different player handicaps, the gross scores are typically adjusted according to player handicaps as explained in the example in the Introduction (Section 1.3). However, there can be variations on these adjustments which we designate by a (golf director chosen) function $f(h_i, j)$ defined for every player handicap h_i and every hole number j, so that the matrix of net scores is as follows:

$$G = \begin{bmatrix} s_{11} - f(h_1, 1) & s_{12} - f(h_2, 1) & s_{13} - f(h_3, 1) & \dots & s_{1n} - f(h_n, 1) \\ \dots & \dots & \dots & \dots & \dots \\ s_{18,1} - f(h_1, 18) & s_{18,2} - f(h_2, 18) & s_{18,3} - f(h_3, 18) & \dots & s_{18,n} - f(h_n, 18) \end{bmatrix}.$$

In our simulations, the holes are assumed to be in order of decreasing difficulty. When a player handicap is no greater than 18, the usual adjustment function is:

$$f(h, j) = \begin{cases} 1, & \text{if } h > 0 \text{ and } j \leq h\,, \\ 0, & \text{otherwise.} \end{cases} \tag{2.1}$$

A player with handicap above 18 receives the score adjustment in all holes, however, in some cases, some golf directors allow further adjustments (e.g., 2 strokes on the most difficult holes) for such players. Hence, the strokes adjustment function may look as follows:

$$f(h,j) = \begin{cases} 1, & \text{if } j \le h \text{ and } 0 < h \le 18\,, \\ 2, & \text{if } j \le h - 18 \text{ and } h > 18\,, \\ 1, & \text{if } j > h - 18 \text{ and } h > 18\,, \\ 0, & \text{otherwise.} \end{cases} \tag{2.2}$$

Given the net score matrix G, and an allocation of n players into m teams, team scores on any hole are determined by summing the best scores of some defined number s as shown in the earlier example.

Suppose $T_1 \subset \{1, \ldots, n\}$ defines the indices of players on Team 1. Then, the s-best player score of Team 1 on hole h can be defined by the optimization problem:

$$\min_{q^1_{hi}} \sum_{i \in T_1} q^1_{hi} g_{hi} \tag{2.3}$$

subject to

$$\sum_{i \in T_1} q^1_{hi} = s\,, \tag{2.4}$$

$$q^1_{hi} \in [0,1]\,, \qquad i \in T_1\,. \tag{2.5}$$

The above linear program represents the score of a team on one particular hole. Since the game score is based on 18 holes, the overall games score will be the sum of scores:

$$S(1) = \sum_{h=1}^{18} \min_{q^1_{hi}} \sum_{i \in T_1} q^1_{hi} g_{hi} \tag{2.6}$$

subject to

$$\sum_{i \in T_1} q^1_{hi} = s\,, \qquad h = 1, \ldots, 18\,, \tag{2.7}$$

$$q^1_{hi} \in [0,1]\,, \qquad i \in T_1\,,\ h = 1, \ldots, 18\,. \tag{2.8}$$

Note that the matrix of game scores G is not known in advance. However, using the independence assumption of player scores and the estimated probability distributions of handicap scores calculated in Siegbahn and Hearn (2010), we can sample as many scenarios of equally probable future game scores as we want:

$$\begin{bmatrix} G_1 & G_2 & G_3 & \ldots & G_Q \end{bmatrix}.$$

From now on, the sample of Q scenarios becomes the universe, the space of possible game outcomes, which is used to define and solve the golf director problem of how to divide n players into m teams in a fair way, in other words in a way that the teams obtained have similar chances to win the game.

2.1. Formal Statement of the Golf Director Problem

When there exists an allocation of players to teams, the question arises as to how fair it is. Club golf competitors would like to play a fair game, i.e., no team wants to appear in the position when it has no real opportunity to win and be dominated by some other team, made up of good players. There are multiple ways to define fairness and measure it, we suggest and focus on the following definitions of team's power to win and the overall fairness.

Definition 1. *The measure of each team's ability to win is defined by the probability of the* *team to winning the 18-hole game given the universe of game scenarios:*

$$\mathbb{P}(\textit{Team } j \textit{ wins}) = \frac{1}{Q}\sum_{r=1}^{Q} \prod_{t\in\{1,\ldots,m\}\setminus\{j\}} \mathbb{1}\{S(r,j) < S(r,t)\} +$$
$$\frac{1}{Q}\sum_{r=1\,,\,\tau(r)\geq 2}^{Q} \frac{1}{\tau(r)}\mathbb{1}\{S(r,j) = \min_{t\in\{1,\ldots,m\}} S(r,t)\}\,, \tag{2.9}$$

where $S(r,j)$ denotes the score of team j in game scenario r and $\mathbb{1}\{\}$ stands for the standard indicator function, with $\mathbb{1}\{true\} = 1$ and $\mathbb{1}\{false\} = 0$. The quantity $\tau(r)$ denotes the degree of tie (number of tied teams) in scenario r:

$$\tau(r) = \sum_{j=1}^{m} \mathbb{1}\{S(r,j) = \min_{t\in\{1,\ldots,m\}} S(r,t)\}\,. \tag{2.10}$$

Definition 2. *The measure of team allocation fairness is the range of team's probabilities to win the game:*

$$\max_{j\in\{1,\ldots,m\}} \mathbb{P}(\textit{Team } j \textit{ wins}) - \min_{j\in\{1,\ldots,m\}} \mathbb{P}(\textit{Team } j \textit{ wins}). \tag{2.11}$$

Hence, using the above definitions, the golf director problem can formally be defined as follows:

$$\min_{T_1,\ldots,T_m} \max_{j\in\{1,\ldots,m\}} \mathbb{P}(\text{Team } j \text{ wins}) - \min_{j\in\{1,\ldots,m\}} \mathbb{P}(\text{Team } j \text{ wins})\,, \tag{2.12}$$

where $T_1 \ldots, T_m$ is an allocation of n players into m teams.

2.2. Fairness Evaluation of a Given Team Allocation

Suppose we are given $T_1, \ldots, T_m$ index sets for m teams. In this section we describe a formal procedure how to evaluate this allocation, i.e., how to find the range of probabilities for a given team to win the game.

3. Approximate Solution of the Golf Director Problem

The solution of the problem (2.12) to provable optimality is a computationally challenging task. First of all, note that it involves the range of probability values, each of which is defined through a set of indicator variables – hence leading to a mixed integer problem. Moreover, the indicator variables designate when the score of one team is smaller than that of another team, but the expressions for the teams scores are not well-defined. Indeed, the team score expression, (2.6) – (2.8), is in fact an optimization problem even if that team's members are known, and is an even more challenging problem if the team members are not known in advance and need to be determined. Therefore, in this section we focus on finding an approximate (heuristic) solution to the golf director problem. The quality of an approximate solution is easy to evaluate: if the range of probabilities to win of the obtained teams is acceptably small (say, up to 5% or up to 10%), then it is reasonable to accept such an allocation for practical purposes.

The overall approximate solution scheme can be summarized in the following three steps:

- Generate a set of candidate team allocations.
- Evaluate each of the candidate allocations in terms of team probabilities to win the game using Algorithm 1, and select an allocation with the smallest range of probabilities.
- If the obtained allocation appears acceptably fair, then stop. Otherwise, we perform a set of swaps of players between teams, in order to obtain a more efficient allocation.

Algorithm 1 Finding the range of team probabilities to win an 18-hole game

for $j = 1$ to m **do**
  $\mathbb{P}(\text{team } j \text{ wins}) := 0$
end for
for $r = 1$ to Q **do**
  for $j = 1$ to m **do**
    $S_j := 0$
    for $h = 1$ to 18 **do**
      $S_j := S_j + \min_{i \in T_j\,,\, q_i \in [0,1]\,,\, \sum_{i \in T_j} q_i = s} q_i g^r_{hi}$
    end for
  end for
  for $j = 1$ to m **do**
    if $\prod_{t \in \{1,\ldots,m\} \setminus j} \mathbb{1}\{S_j < S_t\} = 1$ **then**
      $\mathbb{P}(\text{team } j \text{ wins}) := \mathbb{P}(\text{team } j \text{ wins}) + \dfrac{1}{Q}$
    else if $S_j = \min_{t \in \{1,\ldots,m\}} S_t$ **then**
      $\mathbb{P}(\text{team } j \text{ wins}) := \mathbb{P}(\text{team } j \text{ wins}) + \dfrac{1}{Q \sum_{t \in \{1,\ldots,m\}\,,\, S_t = \min_{l \in \{1,\ldots,m\}} S_l} 1}$
    end if
  end for
end for
return $\max_{j \in \{1,\ldots,m\}} \mathbb{P}(\text{team } j \text{ wins}) - \min_{j \in \{1,\ldots,m\}} \mathbb{P}(\text{team } j \text{ wins})$

The procedure only looks at a finite number of candidate allocations and hence the optimal (most fair) one might be missing from consideration. However, we find such allocations by optimally solving simpler and more tractable problems: 18 one-hole golf director problems. Therefore it is possible that an optimal solution to one of these can turn out to be optimal to the 18-hole game, which makes our candidate set of solutions to be attractive. Further, the candidate solutions can be either used directly, or serve as starting points in search for the globally optimal solution.

3.1. Generating a Set of Candidate Allocations Using One-Hole Games

Finding a set of candidate solutions is based on the more simple one-hole game considered in Pavlikov et al. (2014). Suppose that the game consists of one hole only and the goal of the golf director is the same as before: form a set of teams with similar probabilities to win that one hole. Because of the score adjustment function $f(h_i, j)$, i.e., all holes are generally different with respect to the scoring players. Thus an optimal solution to hole 1 is likely to be different from an optimal solution for hole 2, and so on. Hence, if we consider the scoring scenario data for an 18-hole game as the data for 18 separate one-hole games, we obtain up to 18 different candidate solutions. Further, we will mention two heuristic solution approaches which also can be used to generate candidates for solving the true 18-hole game. Thus the pool of solutions can consist of up to 54 different allocations.

3.1.1. Solving 18 one-hole games optimally

Suppose we are given Q scenarios of game scores over 18 holes after adjustments applied according to the function $f(h, j)$ descried above, or any other function:

$$\begin{bmatrix} G_1 & G_2 & G_3 & \ldots & G_Q \end{bmatrix}\,, \quad G_s = \{g^s_{ji}\}\,,\ j = 1, \ldots, 18\,,\ i = 1\,\ldots, n\,.$$

Consider the matrix of the first rows of the matrices G_s

$$\begin{bmatrix} g_{11}^1 & g_{12}^1 & g_{13}^1 & \cdots & g_{1n}^1 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ g_{11}^Q & g_{12}^Q & g_{13}^Q & \cdots & g_{1n}^Q \end{bmatrix}, \tag{3.1}$$

they clearly form a set of scenarios for n players to score on that first hole. Suppose we want to form a set of teams that minimizes the range of team probabilities to win this particular hole. We will be solving this problem under the assumption that the team score equals the lowest score obtained by the players on the team. In golfing parlance, this is a "one best-ball" game.

The solution approach from Pavlikov et al. (2014) is based on first finding P_i, $i = 1, \dots, n$ values, the probability for every player i to score strictly less than any other player. Finding these is a relatively simple task given the matrix of score scenarios (3.1). Proceeding iteratively, let all $P_i := 0$ for all players. Consider the first row of that matrix:

$$[g_{11}^1 \; g_{12}^1 \; g_{13}^1 \; \cdots \; g_{1n}^1],$$

the first scenario of player scores on the hole, which occurs with probability $1/Q$. In this scenario, there may be a single winner, say the player with index l, then, the probability of player l winning the game increases from 0 to $1/Q$:

$$P_l := 0 \implies P_l := P_l + \frac{1}{Q}.$$

When there are more players in a scenario that score the lowest, there is a slight change in how this scenario is classified. For simplicity, suppose there are two players, with indices l_1 and l_2 who scored the best, then, intuitively speaking, we split the probability of this scenario in two and add corresponding parts towards P_{l_1} and P_{l_2}:

$$P_{l_1} := 0\,,\; P_{l_2} := 0 \implies P_{l_1} := P_{l_1} + \frac{1}{2Q}\,,\; P_{l_2} := P_{l_2} + \frac{1}{2Q}.$$

Formally speaking, when there is a tie, i.e., more than one player having the lowest score, we still want to determine the winner among them and for that purpose we draw an outcome of a uniform random variable that determines which player actually wins. Certainly, there may be more advanced methods to determine the winner in case of tie, but that does not change the idea of scenario splitting. When this procedure is done for all Q scenarios of scores, we obtain a vector of probabilities for every player to win the one hole:

$$[P_1\,, P_2\,, P_3\,, \dots, P_n]. \tag{3.2}$$

With these values known, then taking into account the above assumption on how to score a team, the following simple relation holds:

$$\mathbb{P}(\text{Team } j \text{ wins}) = \sum_{i \in T_j} P_i\,. \tag{3.3}$$

In other words, the probability of a team to win a game is just a linear function of probabilities to win the game of its members. Since team members of the team j are not known in advance and we need to determine them, we introduce the set of binary variables

$$x_i^j = \begin{cases} 1, & \text{if player } i \text{ is in team } j, \\ 0, & \text{otherwise.} \end{cases} \tag{3.4}$$

Then the probability of the team j to win is expressed as follows:

$$\mathbb{P}(\text{Team } j \text{ wins}) = \sum_{i=1}^{n} P_i x_i^j \,, \tag{3.5}$$

and the even-sized (when $n = m \times k$) golf director problem is formulated as the following combinatorial optimization problem:

$$\begin{aligned}
&\min \;\; b - a && && (3.6)\\
&\text{subject to} && && \\
&b \geq \sum_{i=1}^{n} P_i x_i^j \,, && j = 1, \ldots, m \,, && (3.7)\\
&a \leq \sum_{i=1}^{n} P_i x_i^j \,, && j = 1, \ldots, m \,, && (3.8)\\
&\sum_{j=1}^{m} x_i^j = 1 \,, && i = 1, \ldots, n \,, && (3.9)\\
&\sum_{i=1}^{n} x_i^j = k \,, && j = 1, \ldots, m \,, && (3.10)\\
&x_i^j \in \{0, 1\} \,, && i = 1, \ldots, n \,,\; j = 1, \ldots, m \,. && (3.11)
\end{aligned}$$

More details about this optimization problem can be found in Pavlikov et al. (2014).

3.1.2. Heuristic solutions for the one-hole game

In addition to this optimization approach, two heuristics in Pavlikov et al. (2014) were used to solve the one-hole game. One was based on the familiar ABCD method for determining teams that appears in the USGA supported software for golf professionals, but using the above probabilities (3.2) rather than player handicaps. The other is called the "Zigzag" method, and based on the same vector (3.2). First, this vector is sorted in descending order of probability to win:

$$[P_{(1)} \,, P_{(2)} \,, P_{(3)} \,, \ldots, P_{(n)}] \,. \tag{3.12}$$

Recall that the goal is to distribute players into teams so that the corresponding sums of probabilities are close to each other. Hence, the player with highest win probability goes to the first team; the second best goes to the second team, etc. If we only split players into teams of three, then the player with the fourth best probability has to be assigned after the first three. Since teams one and two have the two best players, the fourth player is assigned to team three, etc. Therefore, the team allocation pattern for nine players into three teams of three, will have a zigzag pattern as follows:

$P_{(1)}$	$P_{(2)}$	$P_{(3)}$	$P_{(4)}$	$P_{(5)}$	$P_{(6)}$	$P_{(7)}$	$P_{(8)}$	$P_{(9)}$	
1	0	0	0	0	1	1	0	0	Team 1
0	1	0	0	1	0	0	1	0	Team 2
0	0	1	1	0	0	0	0	1	Team 3

The main point is that the one-hole game leads to three assignment methods for the 18 holes and thereby provides up to 54 candidate allocations. As mentioned above, our software selects the one for which the range of team win probabilities is smallest. If this range is acceptable, the process stops. Otherwise, heuristic swapping of the assignments is done (similar to local search techniques). The swapping techniques are explained below.

3.2. Swapping Players Between Teams

The swapping heuristic described below applies to an allocation with a smallest range of win probabilities, obtained after examining a set of allocations obtained by solving a number of one-hole games. It is best to describe the procedure based on an example with three teams. Let us have the following allocation of players in three teams with corresponding probabilities to win:

Team A	{4, 4, 8, 11, 13}	44.0
Team B	{5, 6, 9, 15, 19}	30.0
Team C	{6, 6, 7, 12, 18}	26.0

Clearly, this allocation is not perfect for practical purposes since the difference in probabilities between teams A and C is 18%. The procedure attempts to rectify uneven team allocations by weakening any of the stronger teams while strengthening the weakest one. The heuristic first enumerates a subset of all the possible swaps that can be performed between the strongest team and the remaining weaker teams. The following rules are used to produce the the list of swaps:

- a handicap of a player to swap in the stronger team is strictly less than that of a player in a weaker team,
- a swap can be applied to players with handicaps of at least 7.

The second rule is imposed for the reason that changes of players with lower handicaps (less than 7), have a greater impact on the fairness of the allocation, and tend to lead to unpredictable results that often times decrease the fairness of the allocation. The enumeration process yields a list of tuples, each containing a player of the weakest team, and a player of another teams, which represent the candidate swaps.

$$(A, 11 \longleftrightarrow C, 12),\ (A, 11 \longleftrightarrow C, 18),\ (A, 8 \longleftrightarrow C, 12),\ (A, 8 \longleftrightarrow C, 18),\ (A, 13 \longleftrightarrow C, 18),\ (B, 9 \longleftrightarrow C, 12),\ (B, 9 \longleftrightarrow C, 18)\,.$$

The returned tuples are sorted in ascending order of the difference of the handicaps of their elements, and in case of ties, sorted in descending order of the values of handicaps. Once sorted, the swaps are iteratively applied to the current team allocation. If a swap improves the fairness of the allocation, the new configuration is kept in memory, and the process continues until the fairness of the team allocation reaches the desired range, or until all the possible swaps are exhausted. After a successful swap, the weakest team may change. Whenever this happens, the remaining swaps are discarded and recomputed by enumerating the swaps between the players of new weakest team the players of the remaining ones.

If a satisfying team allocation is not found, an analogous player swap heuristic, with similar rules, attempting to strengthen any of the weaker teams while weakening the strongest one is also considered.

4. The FairGolfTeams.com Website

This section provides details on the input and output for the website www.fairgolfteams.com, a tool for golf directors who assign players to teams in golf club competitions. Individual player scores are generated using the distributions derived in Siegbahn and Hearn (2010) and the solver determines teams that have probabilities of winning that are close to the ideal. That is, for a competition with 4 teams, the aim is to assign players so that each team has an approximate win probability of 25%; for ten teams the aim would be for each team to have a 10% probability to win.

Player List Input. The primary input is a text file of csv (comma separated values) of players as shown in Table 4. The header line is required and the commas are also required. The last

name,lastname,handicap
a,alast, 5
b,blast, 7
c,clast, 7
d,dlast, 8
e,elast, 9
f,flast, 9
g,glast, 10
h,hlast, 12
i,ilast, 1
j,jlast, 5
k,klast, 11
l,llast, 10
m,mlast, 21
n,nlast, 10
o,olast, 9
p,plast, 26
q,qlast, 12
r,rlast, 1
s,slast, 20
t,tlast, 8
u,ulast, 12
v,vlast,20
w,wlast, 17
x,xlast, 13

Table 4: Player list input.

name is optional, so the first entry could be “a,,5” and only the first name is used. The list can be input from a file by clicking on the button. This populates the player list as shown in the figure below. Once entered, the list may be edited manually by adding and deleting players.

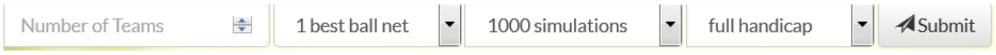

Players

	NAME	LAST NAME	USGA Handicap™
	a	alast	5
	b	blast	7
	c	clast	7
	d	last	8
	e	elast	9
	f	flast	9
	g	glast	10
	h	hlast	12
	i	ilast	1
	j	jlast	5
	k	klast	11
	l	llast	10
	m	mlast	21
	n	nlast	10
	o	olast	9
	p	plast	26
	q	qlast	12
	r	rlast	1
	s	slast	20
	t	tlast	8
	u	ulast	12
	v	vlast	20
	w	wlast	17
	x	xlast	13

Importing the csv data file to the website.

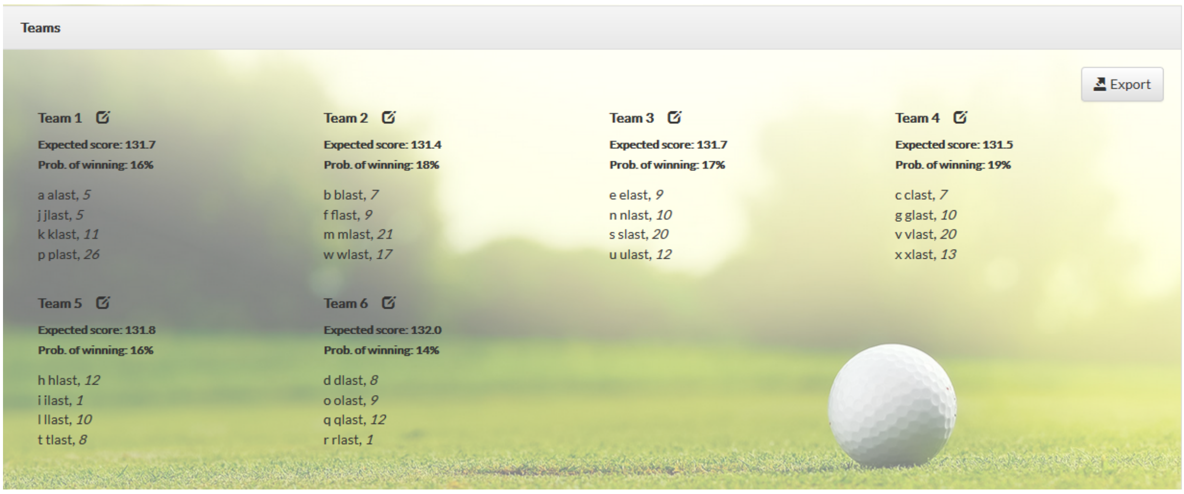
Deletes are done by clicking the box to the left of a player’s name and then clicking the button that appears at the bottom of the list after one or more players is selected for deletion. Additions are made using the button.

Golf Director Choices. The main web site page has a bar of buttons for these choices:



1. Number of Teams has a drop down menu (or the number can be entered directly). If the number of players is a multiple of the number of teams, then the teams will be of the same size. However, the simulator can handle teams of unequal sizes. For example if there are 13 players and 4 teams, there will be three teams of 3 players and one with 4.
2. The next drop down menu allows the choices of 1, 2 or 3 best ball net scores of players to be counted toward the team score on each hole.
3. The third drop down menu is for the number of simulations. The menu allows up to 10,000 scenarios, but 1,000 is usually sufficient. A greater number of scenarios should generally lead more accurate probability estimates.
4. The final menu button is the handicap allowance for each player, from full handicap, 90% of handicap, 80% of handicap etc., down to No handicap allowance. (The USGA recommends 100% (full) handicaps for fourball match paly and 90% for fourball stroke play.)

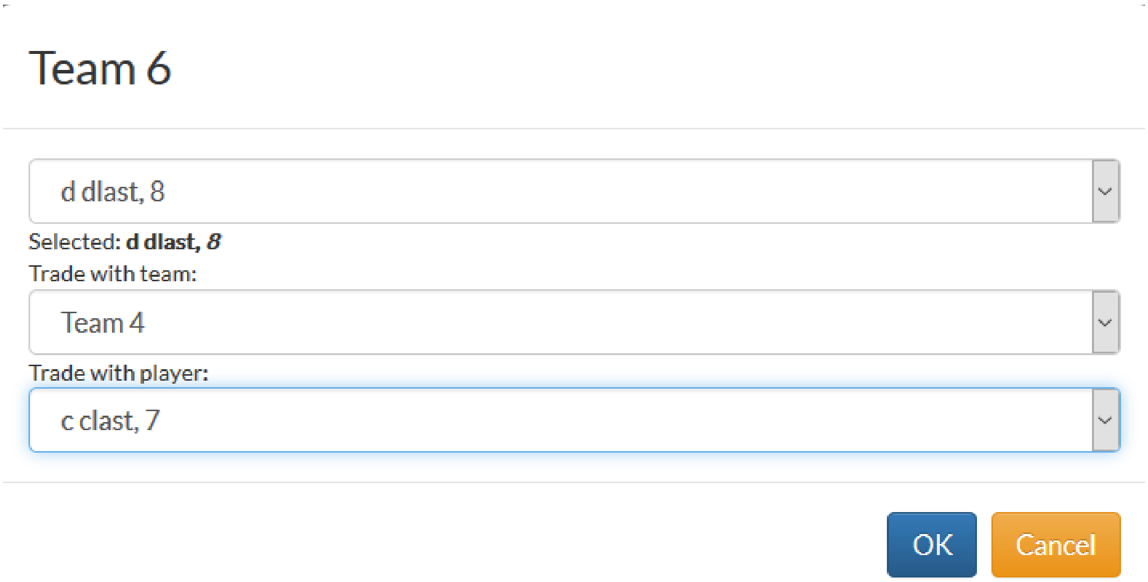
Once these choices are entered, clicking on Submit starts the simulator. While running, changes cannot be made and the screen is grayed out. The final team score is the sum of the best ball totals for 18 holes. For the 24-player example above, a typical choice would be 6 teams, 2 best balls net, 2,000 simulations and full handicap allowance. Ideally, the team probabilities to win would be one-sixth (= 16.7%) for each team. Below is the simulator output with these choices for the data file above. The simulator has determined an assignment of players to teams with a

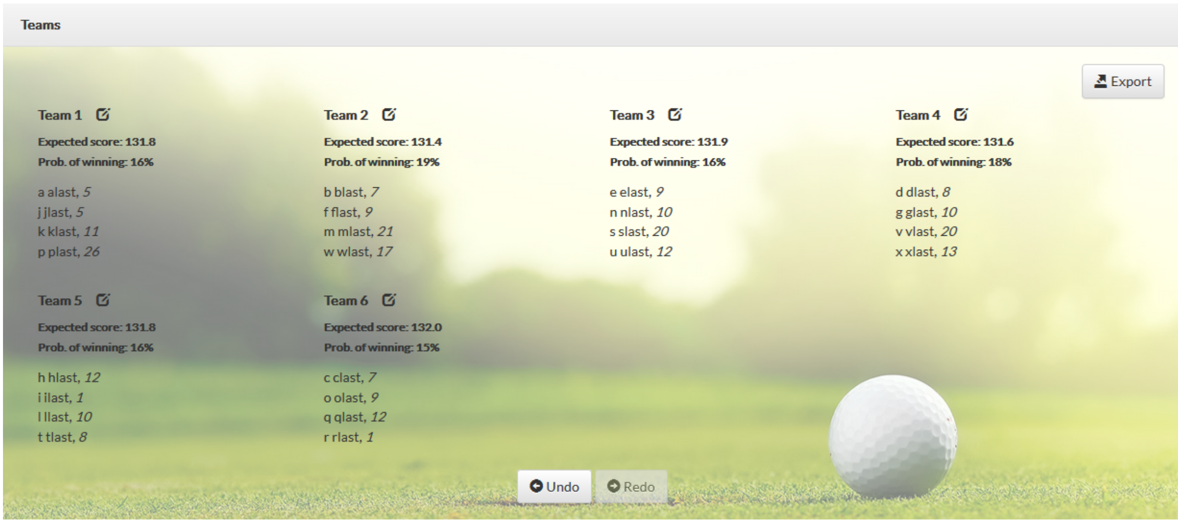


probability to win range of 14% – 19%. The expected score range is 131.5 – 132, showing that the teams are fairly assigned.

4.1. Manual Override

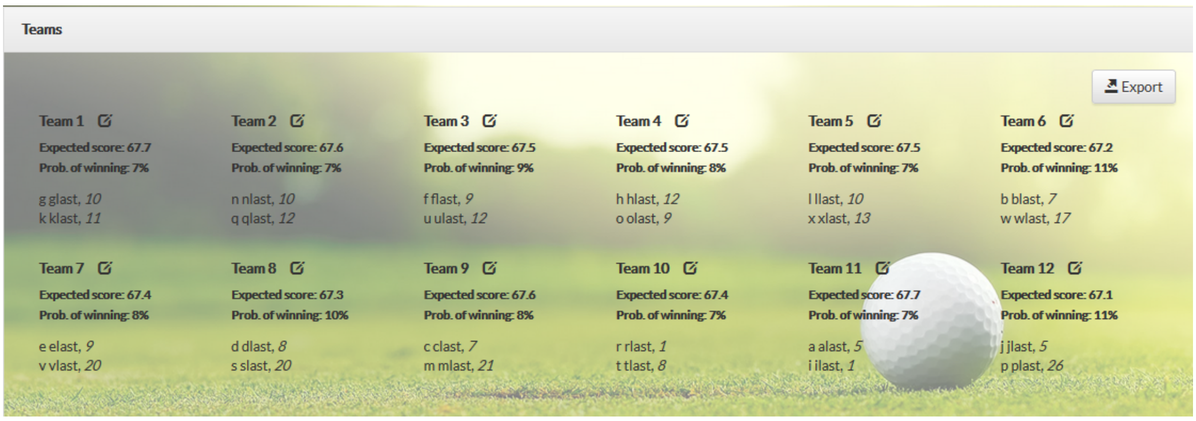
The golf director may choose to change the assignments. For example, consider exchanging the 8-handicap player on Team 6 with the 9-handicap player on Team 4 in an attempt to improve the chances for Team 6. This done by clicking on the small edit box beside Team 6 which opens a window for making the exchange. Below is the window with the entries made. To complete the exchange, click OK. The assignments are then as follows. This action has resulted in a slightly better set of teams because Team 6 now has a 15% chance of winning while the strongest team still wins with probability 19%. The Undo button takes the assignments back to where they were and the Redo button provides toggling between the two assignments.



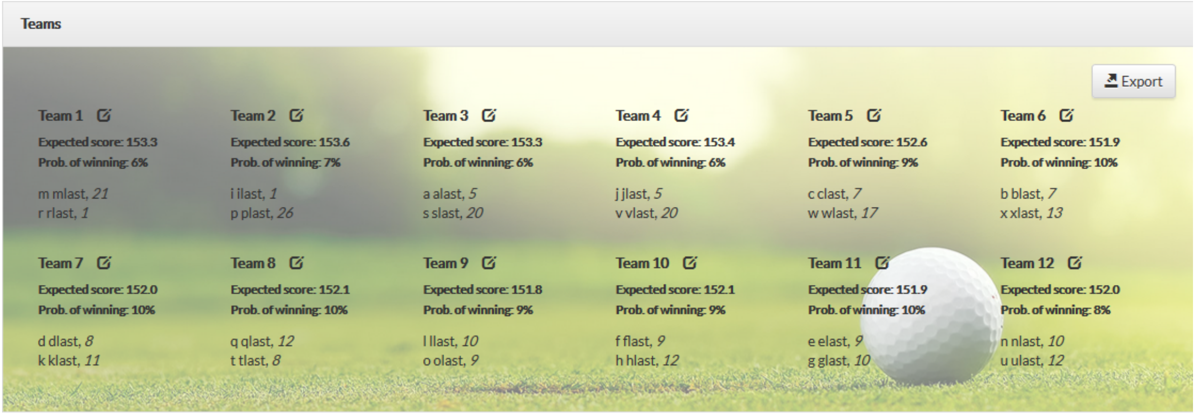


4.2. Two player teams

A natural question is how the simulator works with 2–player teams. In the case of this example, the Number of Teams would be changed to 12 since there are 24 players. The next figure shows what happens with 1–best ball for each team and using full handicaps.



If there are 2–best balls per team the probability range is similar as is shown in the output below.



5. Conclusion

This paper has described an approach to a common problem of golf directors who arrange club golf competitions with the goal of forming fair teams of players with varying handicaps. Player handicap distributions derived in earlier work form the primary database, therefore the *variability* of player scores in form of probability distributions is taken into account besides their handicaps.

The results of a prior optimization model for the one-hole problem form a basis for a scenario based simulation approach to the more realistic 18-hole problem. A website http://www.fairgolfteams.com has been developed and gives the golf director control over team sizes, the number of best balls used in scoring, and handicap allowances. The golf director also has the possibility of manually changing the assignments from the website output. Computational results show that, with these controls, the director can arrange teams for which the win probabilities are close to equal.

6. Acknowledgments

Partick Siegbahn's Master's thesis first addressed fairness in the fourball game and he developed the score distributions by player handicap using a data base provided by Golfnet.com. Also, Patrick Fieldson developed a desktop system for an earlier version of the golf director problem.

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