Two-stage stochastic programming model for routing multiple drones with fuel constraints

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Abstract

Uses of drones and unmanned vehicles (UAVs) in ground or aerial are increasing in both civil and military applications. This paper develops a two-stage stochastic optimization model with a recourse for a multiple drone-routing problem with fuel constraints under uncertainty for the travel between any pair of targets/refueling-sites/depot. We are given a set of n heterogeneous drones or UAVs, a set of targets, a set of refueling stations where any drone can refuel, and the objective is to minimize the overall fuel consumed or distance travelled by all the drones. To approximate the value function of the two-stage stochastic programming model, we use the sample average approximation method (SAA). Though the solution of SAA asymptotically converges to the optimal solution for a two-stage model, finding a lower bound for the original problem is computationally intensive. Therefore, we develop a heuristic to handle large scale instances. In the computational experiments, we have evaluated the robustness of solutions from a deterministic approach and the bounds obtained using the proposed methods.

Index terms— *vehicle routing, drones, fuel-constraints, unmanned aerial vehicles, two-stage stochastic model, stochastic programming, conditional value-at-risk*

1. Introduction

Advances in sensing, robotics and wireless networks have enabled the use of Unmanned Aerial Vehicles (UAVs) in surveillance applications such as environmental sensing including crop monitoring [33, 39, 41, 40], ocean bathymetry [8], forest fire monitoring [2], ecosystem management [3, 42], and civil security applications such as border surveillance [20, 14] and disaster management [21]. More recently, a variety of studies have indicated that UAVs/drones have a huge potential in package delivery, last-mile delivery, and emergency response [5]. The use of drones in these delivery and emergency response applications could aid in significant reduction of cost [4, 38] for several reasons. Firstly, when compared to a fleet of traditional delivery vehicles, maintaining a fleet of drones is less expensive. It can lead to a great reduction in labour costs as the drones can perform the deliveries autonomously. Furthermore, drone deliveries are faster than traditional ground deliveries as they are not limited by the ground transportation infrastructure. In the context of emergency response, they also have the potential to save lives by transporting and delivering the much-needed food and water supply over any terrain. Worldwide, corporate companies like Amazon and Google have started field-testing drone delivery capabilities through initiatives like Amazon Prime Air [38] and Project Wing [27], respectively. Drones have also inspired many start-ups that aim at using them in emergency response and last-mile delivery applications; a start-up by the name of "Matternet" is testing drone-delivery and emergency response capabilities by partnering with companies like DHL, Swiss post, and Mercedez [23].

Although significant efforts have been devoted to develop technologies required for drone-delivery applications, many challenges that are unique to drones such as limited flight time, variation in flight time, dependence of flight time on the weight of the drone and wind conditions etc. have been neglected in comparison. The main contribution of this paper is the development of a two-stage stochastic formulation for a *fuel-constrained multiple drone routing problem* (FCMDRP) where the fuel consumed to travel between any two targets/refueling-sites/depot is uncertain. The fuel consumed is analogous to the time of flight for a drone. The total fuel capacity of a drone then corresponds to the total flying time or distance of the vehicle starting with a fully charged battery (we present both

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interpretations because the drone can either be battery-operated or can run on fuel).

Formally, the FCMDRP is defined as follows: We are give a team of homogeneous drones stationed at a depot, a set of targets where deliveries have to be made and a set of refueling-sites where drones can be refueled (or batteries can be swapped). The fuel consumed by any drone between any pair or targets/depots is uncertain and we assume its description is known via numerous samples. The decisions on the routes of the drones are to be made under the uncertainty in the fuel consumption. The FCMDRP aims to compute a route for each drone such that a delivery is made at every target and each drone never runs out of fuel as it traverses its respective route. If a drone doesn't have sufficient fuel, it can visit a depot (also referred to as a refueling site) to refuel and continue its path. This refueling visit to a depot is a recourse action available for each drone as the fuel consumed is uncertain. The objective of the FCMDRP is to minimize the sum of the travel cost for all the routes and expected additional travel cost for every drone's recourse actions.

The stochastic version of the FCMDRP is formulated as a two state mixed integer linear program. In the first stage, a route for each drone is computed. In the second stage, recourse actions are added on the routes of the drones to any depot. To the best of authors' knowledge, stochastic variants of the FCMDRP have not been studied in the literature. In the next section, we present a detailed review of the literature related to the FCMDRP and its close variants.

2. Related work

The FCMDRP is NP-hard because it is a generalization of the Traveling Salesman Problem (TSP). The literature contains many variants of the vehicle routing problems (VRPs) for applications concerning delivery using ground vehicles. Both exact algorithms to obtain optimal solutions and heuristics have been developed for these variants [34]. The uncapacitated version of the FCMDRP is refered to as the Green Vehicle Routing problem (GVRP) and was introduced in [6]. Heuristics for solving the the GVRP was proposed in [6]. Since then, many algorithms have been developed to solve the deterministic variants of the GVRP; an excellent survey of this work is presented in [18].

The single vehicle deterministic variant of the FCMDRP for ground vehicles was first introduced in Khuller et al. [12]. An approximation algorithm is developed in [12] for the case when the travel costs are symmetric and satisfy the triangle inequality. They assume that the minimum fuel required to travel from any target to its nearest depot is at most equal to $F\alpha/2$ units, where α is a constant in the interval $[0, 1)$ and F is the fuel capacity of the vehicle. This is a reasonable assumption as, in any case, one cannot have a feasible tour if there is a target that cannot be visited from any of the depots. Using these assumptions, Khuller et al. [12] present a $(3(1+\alpha))/(2(1-\alpha))$ approximation algorithm for the problem. Variants of the deterministic FCMDRP in the context of routing UAVs are addressed in [28, 29, 16, 22, 30]. In particular, [28] was the first paper to address this problem in the context of drones. They formulate the single vehicle variant as a mixed-integer linear program (MILP) and present k-opt based exchange heuristics to obtain feasible solutions within 7% of the optimal, on an average. The MILP formulations are solved using off-the-shelf commercial solvers to obtain an optimal solution to the single vehicle variant; they also observe that the solvers are not able to handle their formulation for instances with more than 25 targets. Sundar et al. [29] extend the approximation algorithm in [12] to the asymmetric case and also present heuristics to solve the case where the costs can be asymmetric. Furthermore, variable neighborhood search heuristics for the deterministic FCMDRP with heterogeneous vehicles, *i.e.*, vehicles with different fuel capacities, are presented by Levy et al. [16]. More recently, an approximation algorithm and heuristics are developed for the FCMDRP in [22]. The authors [22] extend the MILP proposed in [28] to the multiple vehicle setting. Similar to [28], they also report that the off-the-shelf mixed-integer solvers had difficulty computing an optimal solution. More recently, Sundar et al. [31, 32] analytically and empirically analyze different MILP formulations for the deterministic FCMDRP and develop additional valid inequalities for the same.

The variants that are closely related to the deterministic version of the FCMDRP are the distance constrained VRP [15, 17, 10, 11, 24] and the orienteering problem [9, 36]. All these variants are well suited for ground vehicles. The distance constrained VRP is a special case of FCMDRP with a single vehicle and single depot. The FCMDRP is also quite different and more general compared to the orienteering problem where one is interested in maximizing the number of targets visited by the vehicle subject to its fuel constraints.

2.1. Contributions of this Article

(i) The FCMDRP with uncertainities in the travel cost is formulated as a two stage stochastic program, (ii) the batch SAA technique is used to compute lower bounds to the optimal solution of the two-stage stochastic program, (iii) an optimization-based heuristic to compute feasible solutions to FCMDRP is developed, (iv) the performance of the heuristic and the quality of the solutions obtained using the SAA are corroborated using extensive computational experiments.

The rest of this paper is organized as follows: Section 3 presents notations for the formulations and the two-stage stochastic programming formulation is presented in section 4. SAA is presented in section 5 and heuristic is explained in 6. Extensive computational experiments are presented in 7, and finally, conclusions in 8.

3. Notations

Let $T = \{t_1, \dots, t_n\}$ denote the set of targets where deliveries have to be made by the drones, d_0 denote the depot where m homogeneous drones are initially stationed, and $\tilde{D} = \{d_1, \dots, d_k\}$ denote the set of additional depots or refueling-sites. Let $D = \tilde{D} \bigcup \{d_0\}$. All the m drones, initially stationed at the depot, d_0 are assumed to be fueled to capacity. The FCMDRP is defined on a directed graph $G = (V, E)$, where $V = T \cup D$ denotes the set of vertices and E denotes the set of edges joining any pair of vertices. We assume that G does not contain any self-loops. For each edge $(i, j) \in E$, c_{ij} and $\bar{f}_{ij}$ represents the travel cost and the nominal fuel consumed by any drone to traverse the edge (i, j). We remark that $\bar{f}_{ij}$ is the value of the fuel consumed that is computed directly using the length of the edge (i, j) and the fuel economy (discharge rate in case of battery-operated drone) of the drone. We assume that the fuel economy/discharge rate of the battery is a constant K. Furthermore, let F denote the fuel capacity for any drone. Additional notations that will be used in the mathematical formulation are as follows: for any set $S \subset V$, $\delta^+(S) = \{(i, j) \in E : i \in S, j \notin S\}$ and $\delta^-(S) = \{(i, j) \in E : i \notin S, j \in S\}$. When $S = \{i\}$, we shall simply write $\delta^+(i)$ and $\delta^-(i)$ instead of $\delta^+(\{i\})$ and $\delta^-(\{i\})$, respectively.

Let Ω denote the sample space of realizations for f_{ij}; the fuel consumed by a drone to travel between the pair of vertices (i, j). ω represents a random event or realization for the random variable $\tilde{\omega}$ with a probability of occurrence $p(\omega)$. We use $f_{ij}(\omega)$ to denote the fuel consumed by any drone to traverse the edge (i, j) in the realization ω. In a risk-neutral formulation, $\mathbb{E}_{\tilde{\omega}}$ is an expectation operator and for a random variable function ϕ, $\mathbb{E}_{\omega}(\phi) = \sum_{\omega \in \Omega} p(\omega)\phi$.

4. Two Stage Stochastic Program for FCMDRP

The first-stage decision variables in the stochastic program are used to compute the initial set of routes for each of the drones while satisfying the nominal fuel consumption values between every pair of targets. The second-stage decision variables are used to compute the refueling trips that must be added to the initial set of routes after a realization of the random variables is known.

Specifically, the first-stage decision variables are as follows: each edge $(i, j) \in E$ is associated with a variable x_{ij}, which equals 1 if the edge (i, j) is traversed by some drone, and 0 otherwise. We let $\boldsymbol{x} \in \{0, 1\}^{|E|}$ denote the vector of all decision variables x_{ij}. Also, associated with each edge (i, j) is a flow variable z_{ij} denoting the total nominal fuel consumed by any vehicle as it starts from a depot and reaches the vertex j, when the predecessor of j is i. Additionally, for any $A \subseteq E$ we let $x(A) = \sum_{(i,j) \in A} x_{ij}$.

All the variables that are used in the second-stage are functions of the realization $\omega \in \Omega$ of the fuel consumption of the drone to traverse any edge E. Analogous to the variable x_{ij} in the first stage, we define a binary variable $y_{ij}(\omega)$ for each edge $(i, j) \in E$. The purpose of these decision variables is to decide on the refueling trip for any drone when the route defined by $\boldsymbol{x}$ is not feasible for the realization ω. Similarly, analogous to the flow variable z_{ij} is a flow variable $v_{ij}(\omega)$ for every $(i, j) \in E$ and $\omega \in \Omega$. Additional variables $q_{id}(\omega)$ and $q_{di}(\omega)$ for every $\{(i, d) : i \in T, d \in D\}$ are used to compute the cost of a refueling trip. Finally, for every $(i, j) \in V$ and $\omega \in \Omega$, we let $\bar{d}$ is a refueling-site d that minimizes $f_{id}(\omega) + f_{dj}(\omega)$. We remark that the dependence of $\bar{d}$ on $(i, j) \in E$ and ω is not explicitly shown and ignored; we leave the onus on the reader to understand the dependence from the context.

Using these decision variables, the two-stage formulation is formulated as follows:

$$\text{TS: } \min \sum_{(i,j) \in E} c_{ij} x_{ij} + \mathbb{E}_{\tilde{\omega}} \phi(\boldsymbol{x}, \omega) \tag{1a}$$

subject to:

$$x(\delta^+(d)) = x(\delta^-(d)) \quad \forall d \in D \setminus \{d_0\}, \tag{1b}$$

$$x(\delta^+(d_0)) = m \text{ and } x(\delta^-(d_0)) = m, \tag{1c}$$

$$x(\delta^+(S)) \geqslant 1 \quad \forall S \subset V \setminus \{d_0\} : S \cap T \cap D \neq \emptyset, \tag{1d}$$

$$x(\delta^+(i)) = 1 \text{ and } x(\delta^-(i)) = 1 \quad \forall i \in T, \tag{1e}$$

$$\sum_{j \in V} z_{ij} - \sum_{j \in V} z_{ji} = \sum_{j \in V} \bar{f}_{ij} x_{ij} \quad \forall i \in T, \tag{1f}$$

$$z_{di} = \bar{f}_{di} x_{di} \quad \forall i \in V,\, d \in D, \tag{1g}$$

$$0 \leqslant z_{ij} \leqslant F x_{ij} \quad \forall (i,j) \in E, \tag{1h}$$

$$x_{ij} \in \{0,1\} \quad \forall (i,j) \in E. \tag{1i}$$

In the formulation (1), $\phi(\boldsymbol{x}, \omega)$ is the recourse function which is defined as follows:

$$\phi(\boldsymbol{x}, \omega) = \min \sum_{i \in T, d \in D} (c_{id} q_{id}(\omega) + c_{di} q_{di}(\omega)) \tag{2a}$$

subject to:

$$\sum_{j \in V} v_{ij}(\omega) - \sum_{j \in V} v_{ji}(\omega) = \sum_{j \in V} f_{ij}(\omega) y_{ij}(\omega) \;\; \forall i \in T, \tag{2b}$$

$$v_{di}(\omega) = f_{di}(\omega) y_{ij}(\omega) \quad \forall i \in V,\, d \in D, \tag{2c}$$

$$0 \leqslant v_{ij}(\omega) \leqslant F y_{ij}(\omega) \quad \forall (i,j) \in E, j \in T, \tag{2d}$$

$$y_{i\bar{d}}(\omega) + y_{\bar{d}j}(\omega) \geqslant 2(x_{ij} - y_{ij}(\omega)) \quad \forall (i,j) \in E, \tag{2e}$$

$$y_{ij}(\omega) \leqslant x_{ij} \quad \forall (i,j) \in E, \tag{2f}$$

$$q_{id}(\omega) \geqslant y_{id}(\omega) - x_{id} \quad \forall i \in T, d \in D, \tag{2g}$$

$$q_{di}(\omega) \geqslant y_{di}(\omega) - x_{di} \quad \forall i \in T, d \in D, \tag{2h}$$

$$0 \leqslant q_{ij}(\omega) \leqslant 1, y_{ij}(\omega) \in \{0,1\}, \quad \forall\, (i,j) \in E. \tag{2i}$$

The constraints (1b) state that the in-degree of each refueling station is equal to its out-degree. Constraint (1c) ensures that all the drones leave and return to the depot d_0. The constraints (1d) ensure that a feasible solution is connected and does not contain any sub-tours. For each target i, the pair of constraints in (1e) require that some drone makes a delivery at i. The constraints (1f) eliminate sub-tours of the targets and also defines the flow variables z_{ij} for each edge $(i,j) \in E$ using the nominal fuel consumption values, $\bar{f}_{ij}$. The constraints (1f)–(1h) together impose the fuel constraints on the routes for all the drones. Finally, for the formulation (1), the constraints (1i) impose the binary restrictions on the decision variables x_{ij}. In the formulation (2), the constraints (2b)–(2d) are analogous to the constraints (1f)–(1h); they prevent sub-tours of the targets and impose the fuel constraints when the realization of the fuel consumption is given by ω. The constraints (2f) ensure that sequence of target visits by each drone is not altered by the recourse action. The constraints (2e) defines the actual recourse action for any drone *i.e.*, given a realization ω, if the route defined by $\boldsymbol{x}$ is not feasible for ω, it can add a refueling visit to its route without altering the sequence in which it is making deliveries to the targets. Furthermore, if route chosen for any drone through the first-stage variables $\boldsymbol{x}$ includes an edge (i,j), we force the refueling-site visit to be only at depot $\bar{d}$, corresponding to (i,j), if the drone needs a refueling-site visit after making a delivery at target i. Finally, the constraints (2g) and (2h) ensure that the recourse cost of additional visits to refueling stations is captured.

We now present a proposition to strengthen the inequalities (1h) and (2d). Since the proposition is similar for (1h) and (2d), we present the proposition only for (1h)

Proposition 1. *For any $i,j,k \in V$ and, suppose that $\bar{f}_{ij} + \bar{f}_{jk} \geqslant \bar{f}_{ki}$, then the inequalities in* (1h) *can be strengthened as follows:*

$$z_{ij} \leqslant (F - t_j) x_{ij} \quad \forall j \in T,\, (i,j) \in E, \tag{3}$$

$$z_{id} \leqslant F x_{id} \quad \forall i \in V \text{ and } d \in D, \tag{4}$$

$$z_{ij} \geqslant (s_i + \bar{f}_{ij}) x_{ij} \quad \forall i \in T,\, (i,j) \in E, \tag{5}$$

where, $t_i = \min_{d \in D} \bar{f}_{id}$ and $s_i = \min_{d \in D} \bar{f}_{di}$.

Proof. When $j \in D$ or $i \in D$, the constraints (4) and (1h) specify the values of z_{id} and z_{di}, respectively. When both $i,j \in D$, the constraint (1h) bounds the value of z_{ij}. Hence, we need only to discuss the case when $i,j \in T$. When $x_{ij} = 1$, the total fuel consumed by any vehicle that traverses the edge (i,j) cannot be greater that $(F - t_j)$, where t_j is the minimum amount of nominal fuel required by any vehicle to reach a refueling station or the depot from target j. Therefore, the constraint in (3) strengthens the upper bound of z_{ij} in (1h). Similarly, any vehicle that traverses the edge (i,j) consumes at least $(s_i + \bar{f}_{ij})$ amount of fuel. As a result, the constraint in (5) strengthens the lower bound of z_{ij} in (1h). □

5. Sample Average Approximation

SAA is a well-known solution technique to tackle two-stage or multi-stage stochastic programs through the use of Monte Carlo simulations [25, 19]. In the SAA, the expectation in the objective function of the first-stage problem is approximated by a sample average estimated from a random finite set of samples. The two-stage problem with this approximated objective can then be solved directly or using decomposition techniques [1, 37]. The approximated optimization problem, when solved repeatedly for different set of samples, results in multiple candidate solutions which can be combined to obtain statistical estimates of their optimality gaps. We shall now detail how SAA can be used to solve the FCMDRP using the formulation (1)–(2), with strengthened constraints from Prop. 1. To that end, we let S^{ω} denote a finite set of realizations of the fuel consumption. We can then estimate $\mathbb{E}_{\tilde{\omega}}\phi(\boldsymbol{x},\omega)$ over $S^{\tilde{\omega}}$ as

$$\frac{1}{|S^{\tilde{\omega}}|}\sum_{\omega\in S^{\tilde{\omega}}}\phi(\boldsymbol{x},\omega). \tag{6}$$

The resulting optimization problem that uses (6) to approximate the expectation in (1a) is a large-scale mixed integer linear program (MILP). It can be computationally expensive to solve this MILP due to the presence of binary variables in the second-stage formulation, unless the cardinality of $S^{\tilde{\omega}}$ is sufficiently small. To counter this difficulty, we use the technique of batch-SAA, suggested in [25] and implemented in [19]; convergence properties for the batch-SAA technique were studied in [13]. The batch-SAA prescribes a method to this sampling process in order to obtain statistical estimates of upper and lower bounds on the original objective value in Eq. (1a), as well as estimates of the variances of these bounds.

5.1. Statistical estimate for the lower bound

This section details the procedure to obtain a statistical estimate for the lower bound of the objective function in (1a) using the batch SAA technique. The procedure is as follows: generate N independent sets of samples and solve the the associated N separate SAA problems. Let the objective value of the N SAA problems be denoted by $\alpha_1,\alpha_2,\ldots,\alpha_N$ and the candidate solutions be denoted by $\boldsymbol{x}_1,\boldsymbol{x}_2,,\ldots,\boldsymbol{x}_N$. The average of the optimal objective values of the N SAA problems is a statistical estimate for the lower bound of the objective value for the FCMDRP. The statistical estimate of this lower bound and the variance of this estimator are given by the following expressions:

$$\underline{\alpha}=\frac{1}{N}\sum_{i=1}^{N}\alpha_i \text{ and } \sigma^2_{\underline{\alpha}}=\frac{1}{N(N-1)}\sum_{i=1}^{N}(\underline{\alpha}-\alpha_i)^2. \tag{7}$$

5.2. Statistical estimate for the upper bound

A statistical estimate for the upper bound is obtained by observing that for any candidate solution $\hat{\boldsymbol{x}}$, the value of the objective function in Eq. (1a) is an upper bound of the optimal objective for the two-stage model. Using such a feasible solution $\hat{\boldsymbol{x}}$, the upper bound bound can therefore be estimated using a set $\hat{S}^{\tilde{\omega}}$ of newly generated realizations, independently of the previously created realization used to solve the SAA problem. Thus, the statistical estimate of the upper bound is given by

$$\bar{\alpha}(\hat{\boldsymbol{x}})=\sum_{(i,j)\in E}c_{ij}\hat{x}_{ij}+\frac{1}{|\hat{S}^{\tilde{\omega}}|}\sum_{\omega\in\hat{S}^{\tilde{\omega}}}\phi(\hat{\boldsymbol{x}},\omega). \tag{8}$$

We then utilize each candidate solution $\boldsymbol{x}_1,\boldsymbol{x}_2,,\ldots,\boldsymbol{x}_N$, obtain the estimate for its corresponding upper bound, and compute the best upper bound as

$$\boldsymbol{x}^*=\operatorname{argmin}\{\bar{\alpha}(\hat{\boldsymbol{x}}):\hat{\boldsymbol{x}}\in\boldsymbol{x}_1,\boldsymbol{x}_2,,\ldots,\boldsymbol{x}_N\}. \tag{9}$$

Typically $|\hat{S}^{\tilde{\omega}}|\gg|S^{\tilde{\omega}}|$ since the scenarios in $|\hat{S}^{\tilde{\omega}}|$ are used only to evaluate a candidate solution, as opposed to the set $S^{\tilde{\omega}}$ which is used to optimize a large-scale MILP. The variance of the estimate $\bar{\alpha}$ is computed using an expression similar to the one used for computing the variance of the lower bound estimate in Eq. (7).

6. Heuristic

As the solution approach for the SAA problem described in the previous section relies on solving a large MILP, this section presents an optimization-based heuristic that is based on solving smaller MILPs to construct a good feasible solution to the FCMDRP. The motivation for developing such an optimization-based heuristic is that solving a deterministic variant of the FCMDRP is computationally not intensive and hence, we perform multiple solves of the deterministic variant of the problem and combine all of these solutions in a particular way to construct another solution.

As the objective in Eq. (1a) is to minimize the total cost of travel while making deliveries to all the targets, the emphasis of the heuristic is to determine the likeliness an edge (i,j) being present in the optimal solution of the two-stage problem. Hence, the heuristic proceeds by solving the deterministic version of FCMDRP using each of the scenarios in the set S^{ω} (or a subset of scenarios) independently in the decreasing order of the probability of the scenarios. Suppose that we have scenario $\omega\in S^{\omega}$ with probability p_{ω} and the solution of the deterministic problem with the scenario ω is given by $\boldsymbol{x}^{\omega}$. This procedure is repeated for every scenario $\omega\in S^{\omega}$. The solution obtained from all these scenarios is used to construct a cumulative weight for each edge $w_{ij}=1-\sum_{\omega\in S^{\omega}}p_{\omega}x^{\omega}_{ij}$.

This cumulative weight is then used to transform the cost co-efficients c_{ij} as $\bar{c}_{ij} \leftarrow c_{ij}w_{ij}$ and one final solve of the deterministic problem using the coefficients $\bar{c}_{ij}$ is performed to obtain the final heuristic solution. We remark that this approach is massively parallelizable.

Heuristic

Step 0. Initialize. Let $TS_f()$ denotes a model with first-stage constraints and objective function, i.e, constraints (1b)-(1i) of TS model with an objective function $\sum_{(i,j)\in E} c_{ij}x_{ij}$.

Step 1. Solve scenario problem. For each $\omega \in \Omega$, set $\bar{f}_{ij} \leftarrow f_{ij}(\omega)$ and solve $TS_f(\omega)$, and let $x_{ij}(\omega) \leftarrow x_{ij}$.

Step 2. Determine edge weight. Based on the solutions $x_{ij}(\omega)$ from Step 1, determine the weight for each edge, $\bar{x}_{ij} = 1 - \sum_{\omega\in\Omega} p_\omega x_{ij}(\omega)$, $\forall(i,j) \in E$. Also, let $\hat{f}_{ij} = \sum_{\omega\in\Omega} p_\omega \bar{f}_{ij}(\omega)$, $\forall(i,j) \in E$.

Step 3. Adjust objective costs. Modify the objective co-efficients as $c_{ij} \leftarrow c_{ij}\bar{x}_{ij}$ and let $\bar{f}_{ij} \leftarrow \hat{f}_{ij}$ and solve TS_f, and let the solution be x^*.

Figure 1: *Pseudo-code for heuristic*

7. Computational Experiments

In this section, we discuss the computational performance of the heuristics for SAA. All the formulations and algorithms presented in this paper were implemented using Java programming language and CPLEX 12.6.2 was used as the underlying MILP solver. All the simulations were performed using Intel(R) Core(TM) i7 processor @2.70 GHz and 16 GB RAM. The performance of the algorithm was tested on two classes of instances: the first class comprised of a set of randomly generated test instances and the second class is from the standard orienteering problem instances that are modified for the FCMDRP.

7.1. Instance generation

The problem data for the first class of instances were randomly generated. The coordinates of all the targets were uniformly generated from a square grid of size 100 × 100. The number of refueling stations was set to 4 and the locations of the depot and all the refueling-sites were fixed for all the test instances. The number of targets was varied from 10 to 30 in steps of ten; for each $|T| \in \{10, 20, 30\}$, we generated five random instances. For each of the above generated instances, the number of vehicles in the depot was varied from 3 to 5, and the fuel capacity of the vehicles, F, was varied linearly with a parameter λ, which is the maximum distance between the depot and any target. The fuel capacity F was assigned a value from the set $\{2.25\lambda, 2.5\lambda, 2.75\lambda, 3\lambda\}$. The travel costs and the nominal fuel consumed to travel between any pair of vertices were assumed to be directly proportional to the Euclidean distances between the pair and rounded down to the nearest integer. In total, the class of randomly generated test bed consisted of 180 instances.

We now describe the procedure used for defining uncertainty on the fuel consumed by any drone to traverse an edge (i, j). We let the mean fuel consumed to traverse the edge (i, j) to be the Euclidean distance between the vertices i and j [7]. As for the distributions, we resort to the literature in [7], which suggests a normal distribution or a gamma distribution to model the fuel consumption. Hence, we use generate scenarios from both these distributions to test our algorithm. Although, authors in [7] perform experiments with both the normal and gamma distribution to model fuel consumption, they have argued that a gamma distribution is more realistic to represent the uncertainty in fuel consumption. In general, the fuel consumption is observed to have the following two properties [7]: (i) the fuel consumed by a drone to traverse any edge (i, j) has a higher deviation from the expected value due to external factors like wind and (ii) the deviation from its mean that are higher than its mean are much higher when compared to the deviations which are lower than its mean. A gamma distribution is an ideal candidate to exhibit such a behaviour. For the normal distribution case, we chose $f_{ij} \sim \mathcal{N}(f_{ij}, (0.25 \cdot f_{ij})^2)$, truncated with zero. The scenarios generated using the gamma distribution had a scale parameter of $0.25 \cdot f_{ij}$ and a shape parameter value of 4. We also remark that, since SAA and heuristic are immune to the distributions used, the algorithm would still work with scenarios generated from any other fitting distribution. To make the scenarios more realistic, we divided the co-ordinate space in all test instances into four equal quadrants, and randomly designated one of the quadrants as *'congested'*, one as *'sparse'*, and the other two as *'mean'*. All the edges connected with a node in a congested quadrant will get a $f_{ij}(\omega)$ higher than its mean f_{ij} for a scenario ω based on the distributions described above. Similarly, all the edges connected with a node in a sparse quadrant will get a $f_{ij}(\omega)$ lower than its mean f_{ij}. For other two quadrants, $f_{ij}(\omega)$ is set to f_{ij}. Instead of randomly assigning $f_{ij}(\omega)$ for the edges based on the prescribed distributions, this design creates correlation among the edges which is more practical.

The second class of instance is a modified version of the "Set 21" instance from the standard orienteering problem library [35]. For this class of instance, we perform experiments only with the gamma distribution.

7.2. Experiments

In this section, we provide the details of the settings used for batch-SAA procedure and the heuristics.

7.2.1 SAA settings

The estimate of the lower bound for the FCMDRP is obtained using the batch SAA technique detailed in Sec. 5.1. To that end, we used $|S^{\tilde{\omega}}| = 8$ scenarios for each batch of the run. This was repeated in $N = 10$ separate SAA problems (run) in order to derive an estimated lower bound on the true expected cost. Each scenario ω has a realization for each edge $(i, j) \in E$ based on the distribution and quadrant explained in the earlier section. Hence, for each ω, $|E|$ random realizations were created. To estimate the upper bound for SAA, we consider each of the solutions from the batch M and evaluate them using $|\hat{S}^{\tilde{\omega}}| = 1000$ scenarios, independent of the scenarios constructed for the initial optimal runs of the N batches. The estimate of the upper bound using all these runs is obtained using the procedure detailed in Sec. 5.2. As noted earlier, though $|S^{\tilde{\omega}}| = 8 \ll |\hat{S}^{\tilde{\omega}}| = 1000$, obtaining lower bound is an optimization problem and the run-time complexity generally grows with the increase in nodes, however obtaining upper bound is an evaluation problem and is computationally very efficient.

7.2.2 Heuristic settings

For the heuristic, the first-stage MILP models with $f_{ij} = f_{ij}(\omega)$ were constructed and $|S^{\tilde{\omega}}| = 8$ such models were run for each iteration of N. Since each model depends on its realization of ω and is independent to other, the models were run in parallel. The models were developed using Java programming language and concurrent libraries in Java were used for parallelization of models. The MILPs were solved using CPLEX 12.6 libraries.

7.3. Results

The tables 1–3 show the results of the computation experiments for the randomly generated instances and the table 4 shows the results for the experiments on the modified orienteering instance. The column headings are defined as follows: 'I' denotes instance number, 'λ' is a factor to denote the maximum distance before refueling, 'EV-SAA' and 'Heuristic' denote the objective function values using the FCMDRP model and heuristics, respectively. Similarly, 'TS-SAA-LB' denotes the optimal solution of the FCMDRP model which is a lower bound for SAA model, and 'EV-SAA', 'TS-SAA' and 'H-SAA' represents the upper bound for SAA using solutions from deterministic, FCMDRP model, and heuristic approaches. Also, since TS model had longer runtime, FCMDRP model was run to optimal only 10 targets.

As observed from tables, the estimates of the upper bound obtained using the formulation is very close to the lower bound. But, the heuristic takes a computation time much less than the lower bound indicating its effectiveness. The solution quality obtained using the heuristic is also good indicating the heuristic can be used as an alternate way of computing good quality solutions to the two-stage FCMDRP quickly. Furthermore, when compared the deterministic version of the problem (EVP), the solution provided by the heuristic is consistently better. Similar results are also observed for the modified orienteering instance.

In terms of runtime, the average runtime for heuristics for 10, 20, and 30 targets instances were 370, 670, and 860 seconds, respectively. However, the runtime for optimal lower bound for TS model for 10 targets is 850 seconds with a high standard deviation of 940 seconds. Whereas heuristic's standard deviation was 95 seconds.

Additionally, we noted that for some of the instances, deterministic solution was slightly better or very close to solution from heuristic, and there were two major reasons: targets were too close to each other in these instances, so there was no difference in terms of targets visited before refueling a drone from a deterministic or heuristic approach, and the number of targets in a 'congested' or 'sparse' quadrant were too low, and most of the targets were in 'mean' quadrant.

#I	λ	Expected cost(standard deviation)								
		EV-SAA		TS-SAA		H-SAA		EV	TS-SAA-LB	Heuristic
		Normal distribution								
1	2.25	389.50	(0.27)	345.35	(1.22)	349.55	(4.80)	325.00	344.69	364.40
1	2.50	421.54	(0.26)	419.15	(1.23)	420.55	(3.23)	361.00	409.95	410.80
1	2.75	335.25	(0.20)	325.00	(1.03)	328.00	(3.36)	323.00	324.80	337.70
1	3.00	388.89	(0.10)	361.71	(0.10)	365.71	(0.10)	344.00	361.00	365.00
2	2.25	404.00	(0.10)	404.00	(0.39)	383.23	(2.74)	404.00	404.00	382.60
2	2.50	475.25	(0.54)	475.34	(0.52)	438.72	(4.32)	427.00	430.31	443.40
2	2.75	415.48	(0.16)	402.65	(1.54)	413.09	(3.75)	378.00	399.63	413.00
2	3.00	392.29	(0.13)	389.48	(0.37)	388.44	(2.22)	378.00	378.94	382.60
3	2.25	461.58	(0.17)	402.28	(1.70)	397.04	(5.95)	393.00	399.02	395.70
3	2.50	437.26	(0.10)	437.26	(0.10)	430.78	(0.10)	429.00	429.00	430.00
3	2.75	513.59	(0.10)	429.16	(0.10)	430.00	(0.10)	406.00	429.00	430.00
3	3.00	456.77	(0.10)	429.00	(0.10)	434.47	(0.10)	394.10	429.00	406.00
4	2.25	344.92	(0.10)	336.00	(0.99)	329.05	(3.65)	335.00	335.90	329.20
4	2.50	352.28	(0.23)	326.00	(0.12)	324.03	(2.48)	316.00	326.00	331.70
4	2.75	334.26	(0.14)	319.85	(0.29)	320.63	(2.58)	309.00	318.40	321.60
4	3.00	376.25	(0.12)	361.41	(0.14)	365.11	(0.10)	343.00	356.00	360.00
5	2.25	354.16	(0.17)	353.00	(0.31)	349.07	(3.41)	342.00	348.06	352.60
5	2.50	367.17	(0.34)	342.06	(0.18)	341.50	(2.34)	329.00	342.20	334.20
5	2.75	344.32	(0.26)	323.75	(0.06)	323.75	(0.92)	322.00	323.00	322.70
5	3.00	322.76	(0.22)	320.11	(0.56)	320.05	(4.32)	319.00	319.50	326.50
		Gamma distribution								
1	2.25	403.48	(0.12)	403.51	(0.12)	354.72	(4.91)	325.00	330.48	362.10
1	2.50	362.60	(0.60)	345.56	(2.28)	350.42	(2.64)	325.00	343.40	342.80
1	2.75	350.81	(0.25)	331.02	(2.64)	332.17	(2.18)	323.00	324.70	330.30
1	3.00	332.88	(0.29)	325.60	(0.08)	325.60	(3.59)	323.00	325.00	330.70
2	2.25	404.09	(0.08)	404.09	(0.08)	387.34	(1.18)	404.00	404.00	379.60
2	2.50	410.48	(0.19)	406.04	(2.00)	407.74	(4.87)	378.00	402.99	410.80
2	2.75	491.86	(0.12)	448.29	(0.14)	448.29	(0.12)	419.00	441.63	432.00
2	3.00	426.15	(0.09)	403.40	(1.16)	415.00	(3.69)	378.00	396.52	419.10
3	2.25	469.07	(0.11)	425.99	(1.59)	393.21	(5.26)	393.00	396.28	393.50
3	2.50	402.29	(0.35)	390.00	(1.74)	377.37	(3.00)	354.00	387.24	361.60
3	2.75	368.06	(0.30)	362.91	(1.34)	363.04	(2.90)	354.00	362.52	362.30
3	3.00	461.61	(0.14)	432.87	(0.12)	431.43	(0.14)	394.00	429.00	391.00
4	2.25	350.11	(0.15)	336.31	(1.45)	329.21	(2.58)	335.00	335.97	329.30
4	2.50	357.93	(0.14)	357.93	(0.14)	324.00	(5.15)	316.00	319.06	329.80
4	2.75	333.52	(0.29)	324.00	(0.94)	329.97	(1.90)	309.00	321.73	320.50
4	3.00	346.99	(0.20)	316.16	(0.32)	331.76	(1.61)	309.00	316.30	338.90
5	2.25	363.60	(0.27)	363.60	(0.27)	353.00	(2.82)	342.00	343.99	348.20
5	2.50	388.12	(0.19)	388.12	(0.19)	342.12	(1.27)	329.00	333.74	324.90
5	2.75	357.15	(0.30)	325.74	(0.13)	326.12	(2.91)	322.00	324.27	324.40
5	3.00	333.21	(0.21)	321.32	(1.85)	321.32	(1.71)	319.00	320.00	325.40

Table 1: *Computational experiments with 10 Targets.*

Furthermore, we provide details of the results for larger instances using expected value problem, optimization and heuristic procedures. The lower bounds are calculated using the appropriate realizations and methods as explained

#I	λ	Expected cost(standard deviation)				EV	Heuristic
		EV-SAA		H-SAA			
		Normal distribution					
1	2.25	586.56	(0.16)	555.19	(0.10)	523.00	536.00
1	2.50	528.10	(0.19)	507.89	(2.92)	473.00	505.90
1	2.75	519.54	(0.08)	476.00	(5.96)	452.00	488.50
1	3.00	517.54	(0.05)	480.13	(3.20)	452.00	477.40
2	2.25	521.12	(0.19)	507.34	(2.85)	465.00	500.00
2	2.50	498.75	(0.34)	487.45	(2.96)	465.00	491.20
2	2.75	520.30	(0.24)	478.55	(3.98)	446.00	486.90
2	3.00	476.37	(0.13)	460.80	(3.42)	439.00	461.20
3	2.25	470.63	(0.27)	446.64	(2.14)	435.00	444.80
3	2.50	484.79	(0.11)	440.68	(2.96)	433.00	440.20
3	2.75	494.54	(0.13)	457.11	(3.09)	442.00	451.90
3	3.00	486.78	(0.10)	446.77	(0.80)	435.00	446.80
4	2.25	436.91	(0.19)	433.27	(6.13)	411.00	445.10
4	2.50	440.11	(0.08)	414.14	(2.21)	402.00	414.60
4	2.75	433.58	(0.13)	412.43	(5.87)	402.00	434.80
4	3.00	416.71	(0.20)	407.19	(4.35)	402.00	434.00
5	2.25	407.13	(0.02)	402.76	(2.20)	407.00	410.50
5	2.50	404.14	(0.08)	399.37	(2.34)	403.00	407.90
5	2.75	395.57	(0.05)	391.09	(3.33)	394.00	405.00
5	3.00	394.04	(0.01)	389.38	(3.81)	394.00	401.60
		Gamma distribution					
1	2.25	578.09	(0.23)	534.28	(2.79)	523.00	545.40
1	2.50	516.67	(0.25)	505.41	(5.17)	473.00	507.70
1	2.75	514.24	(0.07)	480.00	(4.76)	452.00	492.00
1	3.00	514.48	(0.04)	464.71	(1.88)	452.00	459.20
2	2.25	542.92	(0.27)	501.22	(7.54)	485.00	520.70
2	2.50	465.26	(0.22)	461.45	(3.79)	446.00	475.60
2	2.75	464.11	(0.28)	460.11	(4.24)	446.00	459.70
2	3.00	532.97	(0.07)	519.00	(5.34)	476.00	527.40
3	2.25	447.71	(0.14)	445.00	(4.08)	435.00	448.20
3	2.50	476.42	(0.02)	433.08	(2.03)	433.00	432.10
3	2.75	412.06	(0.02)	412.06	(2.39)	412.00	418.10
3	3.00	412.00	(0.10)	412.00	(2.22)	412.00	419.10
4	2.25	426.91	(0.17)	418.59	(8.26)	411.00	434.50
4	2.50	454.89	(0.01)	412.18	(1.33)	402.00	408.00
4	2.75	424.15	(0.32)	411.22	(1.91)	402.00	406.20
4	3.00	402.00	(0.01)	402.00	(3.53)	402.00	410.70
5	2.25	407.00	(0.10)	419.00	(4.01)	407.00	428.10
5	2.50	403.00	(0.20)	404.27	(2.60)	403.00	412.80
5	2.75	394.03	(0.01)	398.48	(2.06)	394.00	401.10
5	3.00	394.00	(0.16)	398.00	(2.35)	394.00	404.30

Table 2: *Computational experiments with 20 Targets.*

in the earlier sections. Based on the lan obtained from each of the methods, the appropriate upper bounds are evaluated using large number of scenarios. The upper bounds indicate the robustness of solutions in the presence of

#I	λ	Expected cost(standard deviation)				EV	Heuristic
		EV-SAA		H-SAA			
		Normal distribution					
1	2.25	590.12	(0.22)	587.00	(4.49)	565.00	592.40
1	2.50	577.69	(0.12)	568.00	(8.21)	539.00	592.60
1	2.75	567.67	(0.10)	532.31	(6.33)	528.00	542.80
1	3.00	535.24	(0.02)	516.12	(3.24)	499.00	523.70
2	2.25	561.01	(0.05)	536.42	(10.83)	513.00	549.50
2	2.50	515.36	(0.04)	499.18	(4.80)	475.00	503.10
2	2.75	553.73	(0.09)	522.07	(1.79)	513.00	524.90
2	3.00	515.36	(0.04)	499.18	(4.80)	475.00	503.10
3	2.25	525.36	(0.04)	512.00	(5.28)	505.00	526.50
3	2.50	572.16	(0.04)	546.00	(6.62)	515.00	559.10
3	2.75	544.00	(0.16)	505.09	(5.26)	500.00	517.30
3	3.00	569.21	(0.03)	564.38	(2.24)	512.00	566.30
4	2.25	636.73	(0.02)	550.91	(0.20)	539.00	549.50
4	2.50	554.99	(0.14)	537.00	(3.89)	504.00	545.40
4	2.75	529.79	(0.04)	504.00	(0.16)	494.00	504.60
4	3.00	540.05	(0.12)	512.00	(2.54)	504.00	512.60
5	2.25	545.66	(0.16)	535.09	(5.32)	534.00	540.00
5	2.50	545.85	(0.21)	535.13	(2.15)	534.00	548.00
5	2.75	546.10	(0.12)	535.09	(3.23)	534.00	546.30
5	3.00	527.01	(0.27)	529.47	(2.86)	488.00	522.30
		Gamma distribution					
1	2.25	592.95	(0.23)	576.17	(6.14)	539.00	584.50
1	2.50	570.75	(0.13)	550.32	(2.63)	528.00	547.10
1	2.75	536.66	(0.03)	507.27	(4.79)	499.00	515.90
1	3.00	542.75	(0.05)	519.05	(4.40)	499.00	520.50
2	2.25	563.12	(0.06)	550.64	(1.44)	513.00	530.90
2	2.50	519.81	(0.09)	505.83	(4.83)	475.00	508.80
2	2.75	519.81	(0.09)	505.83	(4.83)	475.00	508.80
2	3.00	544.09	(0.17)	526.75	(1.28)	494.00	528.20
3	2.25	605.42	(0.22)	558.78	(2.10)	556.00	550.90
3	2.50	516.83	(0.09)	514.99	(2.27)	505.00	517.20
3	2.75	576.07	(0.08)	546.12	(1.54)	515.00	551.30
3	3.00	544.00	(0.36)	506.10	(2.01)	500.00	509.80
4	2.25	640.45	(0.10)	556.12	(3.54)	539.00	563.50
4	2.50	553.81	(0.05)	520.22	(0.95)	504.00	513.90
4	2.75	558.53	(0.09)	547.86	(2.20)	504.00	541.70
4	3.00	535.91	(0.06)	504.00	(4.45)	494.00	511.50
5	2.25	488.07	(0.03)	488.12	(2.31)	488.00	501.00
5	2.50	574.07	(0.28)	551.09	(3.83)	534.00	548.80
5	2.75	487.05	(0.20)	488.00	(3.58)	472.00	505.10
5	3.00	540.77	(0.21)	554.47	(4.64)	534.00	558.20

Table 3: *Computational experiments with 30 Targets.*

uncertainty.

#I	Expected cost(standard deviation)				EV	Heuristic
	EV-SAA		H-SAA			
	Gamma distribution					
21	47.00	(2.03)	43.29	(2.03)	46.67	46.96
21	47.00	(0.34)	43.26	(0.33)	43.67	43.94
21	47.00	(2.27)	44.25	(2.00)	46.00	46.53
21	47.04	(1.79)	44.31	(1.42)	44.00	44.31
21	47.52	(1.91)	45.71	(1.41)	45.00	45.71
21	47.48	(2.05)	44.73	(2.00)	48.00	48.82
21	48.67	(0.58)	44.46	(0.33)	44.33	45.47
21	56.99	(3.46)	54.85	(3.61)	58.00	59.53
21	56.48	(4.23)	49.64	(4.73)	56.00	57.51
21	62.62	(7.42)	58.17	(7.09)	62.68	68.31

Table 4: *Computational experiments with 21 Targets.*

8. Conclusions and future works

In this paper we introduced a new approach to deal with route planning for multiple drones or UAVs with fuel constraints under uncertainty (FCMDRP) in distance or travel time between two targets. The new approach is a two-stage stochastic model with first-stage costs to minimize the route costs and a second-stage recourse costs to additional visits to refueling depots when the fuel is inadequate to complete the tour proposed by the first-stage. In addition, we provide constraints to strengthen our formulation.

We adopted sample average approximation (SAA) method to solve the two-stage FCMDRP model where the second-stage expectation function is approximated using Monte Carlo sampling. The SAA objective value will asymptotically converge to the optimal expected cost of the two-stage model, but it is still computationally expensive for larger instances. Therefore, for larger instances we developed an optimization based heuristic which is very conducive for parallelization. Our heuristic provides good quality solution and is very computationally efficient.

For our computational experiments, we used instances from literature and created uncertainty which reflects more practical environments. The results indicate that the solution from heuristic are far superior than solutions from deterministic model, and also very robust in the face of uncertainty. Future extensions could possibly focus on considering risk-averse model where additional risk-measure weights are considered in the objective functions. One such example for quantile based CVaR risk-measure is presented below. Mean-risk two-stage stochastic programming models are introduced in the literature where a deviation is added along with mean in the second-stage objective function. We present a formulation for a well known quantile based risk-measure conditional-value-at-risk (CVaR). CVar measures the expectation of worst outcomes for a given α-quantile. With the user parameters $\lambda > 0$ and α denoting the risk-averseness of the user and quantile deviation, respectively, a deterministic equivalent problem formulation with CVar risk-measure is given as follows:

$$\text{Minimize} \quad \text{TS-CVaR:} \sum_{(i,j)\in E} c_{ij}x_{ij} + \mathbb{E}_\omega \phi^{ec}(x,\omega) + \lambda\eta + \frac{\lambda}{1-\alpha}\sum_{\omega\in\Omega} p_\omega v'(\omega)$$

subject to: Constraints (1)-(20),

$$\sum_{(i,j)\in E} c_{ij}x_{ij} + \sum_{i\in T, d\in D}(c_{id}q_{id}() + c_{di}q_{di}()) - \eta \le v'(\omega) \quad \forall\,\omega\in\Omega, \tag{10}$$

$$\eta \in \mathbb{R}, v'(\omega) \ge 0, \forall\omega\in\Omega. \tag{11}$$

The above formulation is based on the work in [26]. It should be noted that the proposition 1 is applicable to the above mean-risk formulation too. Another potential extension is to consider developing an optimization based decomposition algorithm for the problem.

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