Mixed integer formulations for a coupled lot-scheduling and vehicle routing problem in furniture settings

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Abstract

We propose and analyze two mathematical programming models for a production, inventory, distribution and routing problem considering real and relevant features from the furniture industry, such as production sequence-dependent setup times, heterogeneous fleet of vehicles, routes extending over one or more periods of the production planning horizon, multiple time windows and customers' deadlines, among others. These features are rarely jointly considered in the related literature, but commonly found in real-world applications. The models properly represent the problem in this industrial sector and can be used as a tool to support production and distribution planning in small companies. A large set of random and realistic instances is used to contrast the performance of the models in terms of both solution quality and computational effort. It is shown how much integrating production and distribution decisions in a single framework helps to reduce the total cost of the system, in comparison with a sequential approach that follows a common practice in this industry. This cost reduction comes at a higher computational effort, though.

Keywords: Production and distribution planning; production, inventory, distribution and routing problem; production routing problem; mixed integer programming; furniture industry.

1 Introduction

In typical supply chains, consisting of sequential activities of production, storage and distribution, each individual process is often planned and optimized using decisions made in preceding activities (Adulyasak et al. 2015b). In the case of production and distribution planning, for example, a common procedure is first to determine the lot sizes and the inventory levels over the planning horizon, and then use these decisions as input to decide how products should be delivered to customers. Since the distribution decisions are limited by the preceding production decisions, some benefits of a more global and integrated planning are lost. However, as pointed out by Díaz-Madroñero et al. (2015) and Chen (2004), companies should use their resources in a efficient manner in order to increase customers' service level and reduce lead times and inventory levels. In this sense, considering production and distribution activities in an integrated manner leads to increased efficiency and cost savings.

In this paper we integrate production and distribution planning activities in the industrial context of furniture companies trough mathematical optimization models, which might be used in other industries with minor modifications. Typically, production planning activities comprises decisions about lot sizing and scheduling, i.e., determine what to produce, how much to produce, when to produce, and how much to keep in stock, aiming to minimize the production, setup and inventory costs incurred in the production process (Karimi et al. 2003; Pochet and Wolsey 2006; Jans and Degraeve 2008). Distribution planning activities are concerned with making decisions about vehicle routing and scheduling, which requires to determine the optimal set of routes to be performed by a fleet of vehicles to serve a given set of customers at minimum cost (Toth and Vigo 2002).

In the literature, the problem integrating the aforementioned decisions is known as production routing problem (PRP) (Adulyasak et al. 2015b), production, inventory, distribution routing problem (PIDRP) (Lei et al. 2006), and integrated production-distribution problem (IPDP) (Armentano et al. 2011), and aims to jointly minimize the production, inventory, setup and routing costs at the system. In this paper, we prefer to term 'production, inventory, distribution routing problem (PIDRP)', as it clearly highlights the main decisions involved in the problem.

The seminal paper for the PIDRP is due to Chandra and Fisher (1994), who studied the economic value of integrating and coordinating production and distribution decisions. After this work, a number of papers have proposed mathematical models and solution approaches for some variants of the problem. The most commonly studied variant considers a single plant producing a single product and owning a homogeneous fleet of vehicles. Solution methods for this problem include branch-and-cut (Adulyasak et al. 2014a), Benders-based branch-and-cut (Adulyasak et al. 2015a), branch-and-price-based heuristics (Bard and Nananukul 2009a, 2010), mathematical programming-based heuristics (Bertazzi 2005; Archetti et al. 2011; Absi et al. 2014), greedy randomized adaptive search procedure (GRASP) (Boudia et al. 2007), genetic algorithm with population management (GAPM) (Boudia and Prins 2009), reactive tabu search (RTS) (Bard and Nananukul 2009b) and adaptive large neighborhood search (ALNS) (Adulyasak et al. 2014b).

The multi-product case is a natural extension that have been studied less intensively.

Once again, researchers have centered their efforts in developing effective solution strategies, such as decomposition heuristics (Chandra and Fisher 1994), Lagrangian heuristics (Fumero and Vercellis 1999), MIP-based heuristics (Brahimi and Aouam 2016) and tabu search-based algorithms (Shiguemoto and Armentano 2010; Armentano et al. 2011). The work of Amorim et al. (2013) is the only one in this category that includes parallel production lines and sequencing decisions in the production and distribution planning of perishable goods.

Lei et al. (2006) approach a problem with multiple plants that produce a single product and own a heterogeneous fleet of vehicles. To the best of our knowledge, this is the only paper considering such characteristics in the mathematical model. To solve the problem, the authors developed a two-stage decomposition heuristic that outperformed the CPLEX solver.

In the context of small furniture companies, Miranda et al. (2016) recently formulated a mixed integer program (MIP) and proposed relax-and-fix (RF) heuristics to solve a PIDRP. The authors considered a small furniture scenario whereby the manufacturer has one production line and only one vehicle that performs multiple trips over the planning horizon. It is important to notice the differences between this paper and the work of Miranda et al. (2016). First, the company considered in Miranda et al. (2016) involves a lot-sizing problem with no setup times. In this paper, we consider a production process scenario in which setup times are an important issue to take into account in the production planning, as they are highly sequence-dependent and not negligible, i.e., here we are concerned not only about a lot sizing problem, but with a lot sizing and scheduling problem. Second, Miranda et al. (2016) considered only one vehicle for making deliveries, while in the present paper we are assuming a small heterogeneous fleet of vehicles. It means that in this research we are concerned not only about the vehicle routes, but also with the proper allocation of customers to vehicles.

Tables 1 and 2 illustrate the main problem characteristics studied on the literature and compares with the problem studied in the present paper.

Notice that the PIDRP literature is scarce and most researches have studied the single product and homogeneous fleet problem. In the present paper, we propose non trivial mixed integer programming models for a typical scenario of small furniture companies, in which the manufacturer has one production line and a small fleet of heterogeneous vehicles to make deliveries. We model a number of real and common features in practice, such as producing and stocking multiple parts (instead of final products), sequence-dependent setup times, routes extending over one or more periods, multiple time windows, customers' deadlines, among others. Integrated problems with these characteristics are not usually addressed in the PIDRP literature, but commonly found in some industries, such as furniture, food, carrier services, electronic devices, among others. Results show that one of the models is capable of solving instances with up to twenty customers, whose size is considered as realistic in the context of small furniture companies. Finally, we analyze an alternative scenario in which all the vehicles have the same capacity, as opposite to the original situation where vehicles may have different ones. Results suggest that routing costs may increase considerably, affecting the total cost of the system. Still, further experimentation is required in order to validate this conclusion.

Tab. 1: Comparison of papers reported in the literature.

Reference	Production decisions		Number of products		Inventory location			Vehicle fleet		
	Lot-sizing	Lot-scheduling	One	Multiple	Plant	Plant/customers	None	One	Homogeneous	Heterogeneous
Chandra and Fisher (1994)	✓			✓		✓			✓	
Fumero and Vercellis (1999)	✓			✓		✓			✓	
Bertazzi (2005)	✓		✓			✓			✓	
Lei et al. (2006)	✓		✓			✓				✓
Boudia et al. (2007)	✓		✓			✓			✓	
Boudia and Prins (2009)	✓		✓			✓			✓	
Bard and Nananukul (2009a)	✓		✓			✓			✓	
Bard and Nananukul (2009b)	✓		✓			✓			✓	
Bard and Nananukul (2010)	✓		✓			✓			✓	
Shiguemoto and Armentano (2010)	✓			✓		✓			✓	
Archetti et al. (2011)	✓		✓			✓			✓	
Armentano et al. (2011)	✓			✓		✓			✓	
Amorim et al. (2013)		✓		✓			✓		✓	
Adulyasak et al. (2014a)	✓		✓			✓			✓	
Adulyasak et al. (2014b)	✓		✓			✓			✓	
Absi et al. (2014)	✓		✓			✓			✓	
Adulyasak et al. (2015a)	✓		✓			✓			✓	
Brahimi and Aouam (2016)	✓			✓		✓			✓	
Miranda et al. (2016)	✓			✓	✓			✓		
This paper		✓		✓	✓					✓

Tab. 2: Comparison of papers reported in the literature - *continuation.*

Reference	Time windows (TW)			Route duration		Routes per vehicle		Split deliveries	
	No TW	One TW	Multiple TW	< 1 period	> 1 period	1 per period	>1 per period	Yes	No
Chandra and Fisher (1994)	✓			✓		✓		✓	
Fumero and Vercellis (1999)	✓			✓		✓		✓	
Bertazzi (2005)	✓			✓		✓			✓
Lei et al. (2006)	✓			✓			✓		✓
Boudia et al. (2007)	✓			✓		✓			✓
Boudia and Prins (2009)	✓			✓		✓			✓
Bard and Nananukul (2009a)	✓			✓		✓			✓
Bard and Nananukul (2009b)	✓			✓		✓			✓
Bard and Nananukul (2010)	✓			✓		✓			✓
Shiguemoto and Armentano (2010)	✓			✓		✓			✓
Archetti et al. (2011)	✓			✓		✓			✓
Armentano et al. (2011)	✓			✓		✓			✓
Amorim et al. (2013)		✓		✓		✓			✓
Adulyasak et al. (2014a)	✓			✓		✓			✓
Adulyasak et al. (2014b)	✓			✓		✓			✓
Absi et al. (2014)	✓			✓		✓			✓
Adulyasak et al. (2015a)	✓			✓		✓			✓
Brahimi and Aouam (2016)	✓			✓					✓
Miranda et al. (2016)			✓		✓		✓		✓
This paper			✓		✓		✓		✓

Our study is motivated by the practice of Brazilian small furniture companies, in which the manufacturer is responsible for both producing and delivering the final products. We develop and apply mixed integer models that properly represent the common practice in those industries and that can be used to support the planning of production and distribution operations in small companies. Therefore, the contributions of this paper are twofold: (i) studying a challenging PIDRP in small furniture companies, considering the most relevant features on those industries; and (ii) proposing mathematical optimization models to represent and solve, through commercial solvers as CPLEX or Gurobi, moderate sized instances in an integrated manner.

The rest of this paper is organized as follows: Section 2 describes the production and distribution process in furniture companies, based on the practice of a typical Brazilian manufacturer, and presents the problem statement. Section 3 describes two mathematical models for the problem under study. Section 4 reports the computational results, and finally Section 5 give some concluding remarks and future research directions.

2 The furniture production and distribution process

In this section we describe the production and distribution process in furniture companies based on information provided by a Brazilian company. This company is a typical manufacturer of steel furniture, so that its production and distribution processes represent the common practice in this industry.

2.1 Furniture production process

Typical furniture companies use hardwood lumber, hardwood plywood, medium density fiberboard (MFD), particleboard, steel and other materials to produce a number of final products. Common production stages for furniture manufacturing are cutting, machining, painting and packing. However, some specific operations in each stage may change slightly depending on the specific raw material used in the process (i.e., wood, MDF, steel, etc.). These differences in the process do not affect our description as we have no interest on specific details of the process, but in a more global view for planning purposes.

The process starts at the cutting stage, wherein steel plates and coils are cut in smaller parts depending on the size of final products. At this stage it is common to have different cutting machines. Some machines are able to cut several parts without adjustments, while others require setup times to change and adjust the necessary tools to perform the operation.

At the machining stage, a group of operations, such as folding, drilling and welding, gives the final shape to parts, which then are joined into sub-assemblies required for final products. At the painting stage, parts and sub-assemblies are coated using powder paint. The process starts by cleaning parts and sub-assemblies in order to remove oils and greases on the surface. Afterward, parts and sub-assemblies pass through a phosphating process that increases corrosion resistance properties and prepares the surfaces to be painted.

Once prepared, parts and sub-assemblies go into the painting line, where they are coated with powder paint. Afterwards, parts and sub-assemblies are heated at temperatures ranging between 150 °C–200 °C in order to melt the powder. In general, color changeover operations are complex and time-consuming, since the painting line has to be cleaned to avoid contamination of parts and sub-assemblies. Since cleaning time depends on the color sequence, setup times are highly sequence-dependent.

After the painting stage, parts and sub-assemblies are packaged and stored. At this stage, parts that do not require painting, such as handles, locks and screws, are also packaged to guarantee that all parts needed to assemble the final product are included. Note that only parts and sub-assemblies are stocked and not final products. This practice is common in furniture companies, since the same part or sub-assembly can be used to make up different final products.

In this paper, we focus on the painting line planning, since this stage is considered the production bottleneck. Furthermore, planning of the remaining stages can be made based on the production plan of the painting stage.

2.2 Furniture distribution process

The furniture distribution process starts by picking packages containing parts and sub-assemblies that together make up a final product. Customer orders may include different products, each one comprising several parts and sub-assemblies that should be collected from stock. Afterward, packages are loaded into the vehicles and delivered to customers, which are usually retail stores. The final assembly of the products is made at the retail store, or by the final customer (a kind of production postponement strategy). In this process, the vehicle routing is considered the main decision, since picking and loading plans can be obtained based on the delivery plan.

In furniture companies is possible to find particularities that make vehicle routing a more difficult task to be performed effectively. For example, vehicles have different capacities, so that the fleet is heterogeneous. Therefore, besides the order in which customers should be visited, it is also necessary to define which vehicles should be used and how to allocate customers to vehicles. Routes are long in duration and can extend over several days, which means that vehicles may leave the depot in one period (i.e., one day) and return in a different one (i.e., another day). Each vehicle may perform multiple trips over the planning horizon in order to meet the customers demand. Hence, when a vehicle returns to the depot it can either remain idle or be reloaded in order to perform a new route. Notice that the last two characteristics are not commonly approached in the related literature, where is often supposed that routes are short enough to be performed within a single period and that each vehicle performs at most one trip per day.

Other important characteristic is that customers have multiple time windows, but they must be visited only once over the planning horizon. Moreover, deadlines can be agreed with customers and deliveries should take place before this deadline. In consequence, each customer can be visited at any period, provided that it is visited only once and its deadline is met. This situation is common in long-haul transportation, where distances traveled during a journey can be considerably long and locations are scattered over a wide region, so that routes can extend over several days (Bodin et al. 2003). A recent paper approaching a vehicle

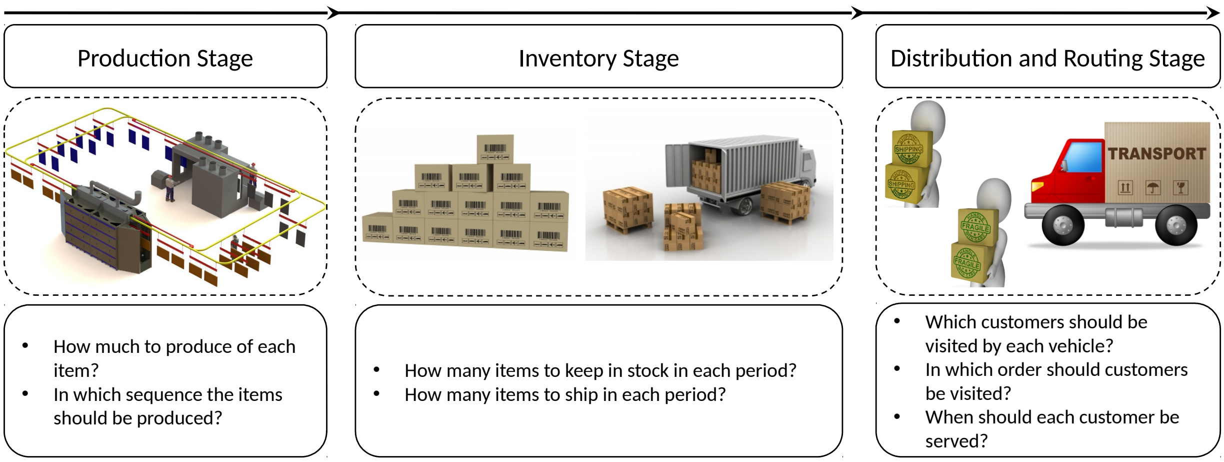

Fig. 1: Decision involved in the PIDRP considered in this paper.

routing problem involving long travel distances and multiple time windows is presented by Rancourt et al. (2013).

As described in Sections 2.1 and 2.2, we consider painting and delivering stages as production and distribution bottlenecks, respectively. Thus, we study a PIDRP in which lot-sizing, scheduling, routing decisions are considered in an integrated manner with the goal of finding production and delivery plans that achieve a compromise between setup, inventory and routing costs. Figure 1 depicts the decisions we are considering in the problem studied in this paper.

2.3 Problem statement

The PIDRP considered in this work consists of one production line that produces a set $\mathcal{C}$ of items, $a \in \mathcal{C}$, required to meet the demand of a set $\mathcal{P}$ of final products, $p \in \mathcal{P}$. To assembly one unit of product p it is necessary to produce η_{ap} units of item a. We assume that $\eta_{ap} = 0$ if $c \notin \mathcal{F}_p$, where $\mathcal{F}_p \subseteq \mathcal{C}$ represents the subset of items required for assembling product p. Switching from item a to item b requires ς_{ab} time units to prepare the line in order to avoid contamination of items. The line capacity in time units is K_t, $t \in \mathcal{T}$, and the time required to produce one unit of item a is ρ_a. Back-orders are not allowed, the unit inventory holding cost of item a is h_a, and the initial and minimum inventory of item a are denoted by I_{a0} and $I_a^{\min}$, respectively.

Let $\mathcal{G} = (\mathcal{N}, \mathcal{A})$ define a complete directed graph, where $\mathcal{N} = \{0, 1, \ldots, n+1\}$ is the set of nodes and $\mathcal{A} = \{(i,j) : i,j \in \mathcal{N}, i \neq j\}$ is the set of arcs. The depot is denoted by both 0 and $n+1$ and $\bar{\mathcal{C}} = \mathcal{N} \backslash \{0, n+1\}$ denotes the set of customers. The travel time (cost) for going from node i to node j is denoted by τ_{ij} (c_{ij}), $i,j \in \mathcal{N}$. For each customer i, $i \in \bar{\mathcal{C}}$, let d_{pi} be the demand of product p at customer i and $[\delta_{it}, \bar{\delta}_{it}]$ denote the time window of customer i in period t. These time windows represent the time intervals in which a delivery is allowed. In furniture companies is common the time windows to be the regular business hours, so δ_{it} and $\bar{\delta}_{it}$ represent the beginning and end of a customer regular working day. When a vehicle arrives to the node before δ_{it}, then it has to wait until δ_{it} to start the service.

Similarly, when a vehicle arrives after $\bar{\delta}_{it}$, then it has to wait until δ_{it+1} to start the service. Each customer i must be visited only once over the planning horizon (i.e., split deliveries are not allowed) and before a preset deadline Δ_i.

Deliveries are performed by a set $\mathcal{V}$ of heterogeneous vehicles. The capacity, in weight units, of vehicle $v \in \mathcal{V}$ is θ_v and each unit of product p on board of vehicle v consumes φ_p units of capacity. Each vehicle v can perform several routes $r = 1, \ldots, R$ over the planning horizon, and each route must departure from node 0 and arrive to node $n+1$. R denotes an upper bound on the number of routes performed over the planning horizon (for example, $R = n$). Before starting a new route, vehicles must be reloaded at the depot. We assume that the loading time is proportional to the service time of the customers served by the route, and so it is variable. The loading/unloading rate is denoted by λ. We do not impose a limit in the length of the route, so that a route can depart from node 0 in period $t \in \mathcal{T}$ and return to node $n+1$ at period $t' \in \mathcal{T} : t' \geq t$. It means that routes can extend over several periods in the planning horizon.

Let M_{0j} and M_{ij} denote two sufficiently large numbers, defined as:

$$
\begin{aligned}
M_{0j} &= \bar{\delta}_{0|\mathcal{T}|} + \lambda \sum_{i \in \bar{\mathcal{C}}} \sum_{p \in \mathcal{P}} \varphi_p d_{pi} + \tau_{0j}, && j \in \bar{\mathcal{C}}. \\
M_{ij} &= \Delta_i + \lambda \sum_{p \in \mathcal{P}} \varphi_p d_{pi} + \tau_{ij}, && i \in \bar{\mathcal{C}}, j \in \{\bar{\mathcal{C}}, n+1\}.
\end{aligned}
$$

The problem consists in determining how much to produce each item, the sequence in which the items should be produced, how much to keep in stock each item, the vehicle routes and the timing in which each customer should be served over a finite multi-period planning horizon. The objective is to minimize the sum of setup, inventory and routing costs. Note that we do not consider inventory holding costs at customers' sites, since we do not assume that the manufacturer manages the customers' inventory.

This scenario is very common in the practice of small Brazilian manufacturers, which usually have a single painting line and a small number of (different) vehicles to perform deliveries.

3 Mathematical formulations

In this section we present two mathematical models to represent the main scenario studied in this paper. These models, named LSMVRP-F1 and LSMVRP-F2, respectively, are different from each other by the way in which routes are assigned to vehicles, and both of them use a hybrid time framework.

The lot sizing and scheduling part uses a discrete time structure that divides the planning horizon on one day periods. In each period we allow the production of several items. In order to get a production sequence, we implement a polynomial sub-tours elimination constraints proposed by Miller et al. (1960) and extensively used in the recent literature. The vehicle routing part relies on a continuous time structure that allow us to determine the time in which each route starts and finishes, as well as the time in which customers are served. Then, the lot-scheduling part and the vehicle routing part are coupled through the inventory balance constraints, which take into account the production and delivery quantities of each item in each period.

In general, model LSMVRP-F1 has more constraints and binary variables than model LSMVRP-F2 and, consequently, solving it might be more difficult than solving LSMVRP-F2. Moreover, model LSMVRP-F2 provides solutions with lower total cost and slightly better lower bounds than those obtained by model LSMVRP-F1. More details on the performance of the models will be given in Section 4.

Consider the following common variables for the two models:

Lot-Scheduling Variables

x_{at} : Production quantity of item a in period t;

I_{at} : Inventory of item a at the end of period t;

y_{at} : Equal to 1 if the line is set up for item a at the beginning of period t, 0 otherwise;

z_{abt} : Equal to 1 if there is a changeover from item a to item b in period t, 0 otherwise;

π_{at} : Auxiliary variable for sequencing item a in period t;

3.1 Lot-scheduling multi-vehicle routing problem (LSMVRP-F1)

In the first model the relationship between routes and vehicles is established using a single set of binary variables, w_{ijvr}. The routing variables used in the LSMVRP-F1 model are the following:

Routing Variables

w_{ijvr} : Equal to 1 if arc (i, j) is traveled by route r of vehicle v, 0 otherwise;

Q_{pvrt} : Quantity of product p shipped on route r of vehicle v in period t;

ϕ_{ivrt} : Equal to 1 if node i is visited by route r of vehicle v in period t, 0 otherwise;

μ_{ivr} : Starting time at which node i is serviced by route r of vehicle v;

The LSMVRP-F1 model is formulated as follows:

The objective function (1) minimizes the sequence-dependent setups, inventory and routing costs.

$$\min \sum_{a\in\mathcal{C}} \sum_{t\in\mathcal{T}} h_a I_{at} + \sum_{a\in\mathcal{C}} \sum_{b\in\mathcal{C}} \sum_{t\in\mathcal{T}} c_{ab} z_{abt} + \sum_{v\in\mathcal{V}} \sum_{r=1}^{R} \sum_{i=0}^{n} \sum_{j=1}^{n+1} c_{ij} w_{ijvr}. \tag{1}$$

Inequalities (2) are the inventory balance constraints of items a in period t. Constraints (3) ensure that the quantity of item a shipped in period t (i.e., $\sum_{v\in\mathcal{V}} \sum_{r=1}^{R} \sum_{p\in\mathcal{P}} \eta_{ap} Q_{pvrt}$) is not greater than the inventory at the end of period $t-1$.

$$I_{at-1} + x_{at} = \sum_{v\in\mathcal{V}} \sum_{r=1}^{R} \sum_{p\in\mathcal{P}} \eta_{ap} Q_{pvrt} + I_{at}, \qquad a \in \mathcal{C}, t \in \mathcal{T}. \tag{2}$$

$$I_{at-1} \geq \sum_{v \in \mathcal{V}} \sum_{r=1}^{R} \sum_{p \in \mathcal{P}} \eta_{ap} Q_{pvrt}, \qquad a \in \mathcal{C}, t \in \mathcal{T}. \tag{3}$$

Constraints (4) impose a minimum inventory level at the end of each period and (5) represent the production capacity constraints.

$$I_{at} \geq I_a^{\min}, \qquad a \in \mathcal{C}, t \in \mathcal{T}. \tag{4}$$

$$\sum_{a \in \mathcal{C}} \rho_a x_{at} + \sum_{a \in \mathcal{C}} \sum_{b \in \mathcal{C}} \varsigma_{ab} z_{abt} \leq K_t, \qquad t \in \mathcal{T}. \tag{5}$$

Constraints (6) guarantee that there is production of item a in period t only if the line is set up for item a at the beginning of period t, or if there is a changeover from some item b to item a in period t. Note that these constraints also give an upper bound on the production quantity of item a in period t, $M_{at} = \min\left(\lfloor \frac{K_t}{\rho_a} \rfloor, \sum_{i \in \bar{\mathcal{C}}} \sum_{p \in \mathcal{P}} \eta_{ap} d_{pi} \right)$.

$$x_{at} \leq M_{at} \left(y_{at} + \sum_{\substack{b \in \mathcal{C} \\ b \neq a}} z_{bat} \right), \qquad a \in \mathcal{C}, t \in \mathcal{T}. \tag{6}$$

Constraints (7) establish that the line is set up for exactly one item a at the beginning of each period t, even if the period is idle (i.e., there is no production on it).

$$\sum_{a \in \mathcal{C}} y_{at} = 1, \qquad t \in \mathcal{T}. \tag{7}$$

Equalities (8) are flow constraints and establish that if the line is set up for item a at the beginning of period t, then we have either a changeover from item a to some different item b or the line is still set up for item a at the beginning of period $t+1$. Similarly, if there is a changeover from some item b to item a in period t, then we have either a changeover from item a to some different item b or the line setup is the same at the beginning of period $t+1$. They relate the line initial setup variables, y_{at}, with changeover variables, z_{abt}, in each period of the planning horizon.

$$y_{at} + \sum_{\substack{b \in \mathcal{C} \\ b \neq a}} z_{bat} = y_{a,t+1} + \sum_{\substack{b \in \mathcal{C} \\ b \neq a}} z_{abt}, \qquad a \in \mathcal{C}, t \in \mathcal{T}. \tag{8}$$

Constraints (9) determine a coherent production sequence in each period. For a fixed period t, whenever a changeover from item b to item a takes place ($z_{bat} = 1$), the expression $|\mathcal{C}|(1 - z_{bat})$ is equal to zero, and thus $\pi_{at} \geq \pi_{bt} + 1$ must hold, implying that item a is produced after item b.

$$\pi_{at} \geq \pi_{bt} + 1 - |\mathcal{C}|\left(1 - z_{bat}\right), \qquad a \in \mathcal{C}, b \in \mathcal{C}, t \in \mathcal{T}. \tag{9}$$

The depot is represented by nodes 0 and $n+1$, so that a route starts leaving node 0 and finishes returning to node $n+1$, as stated by constraints (10)–(11), respectively. We assume that a route traveling directly from node 0 to node $n+1$ is an empty route.

$$\sum_{j \in \{\bar{\mathcal{C}}, n+1\}} w_{0jvr} = 1, \qquad v \in \mathcal{V}, r = 1, \dots, R. \tag{10}$$

$$\sum_{i \in \{0, \bar{\mathcal{C}}\}} w_{i(n+1)vr} = 1, \qquad v \in \mathcal{V}, r = 1, \dots, R. \tag{11}$$

Flow conservation of routing decisions is guaranteed by the set of constraints (12), while constraints (13) establish that each customer must be visited exactly once over the planning horizon (i.e., split deliveries are not allowed).

$$\sum_{\substack{j \in \{\bar{\mathcal{C}}, n+1\} \\ j \neq i}} w_{ijvr} = \sum_{\substack{j \in \{0, \bar{\mathcal{C}}\} \\ j \neq i}} w_{jivr}, \qquad i \in \bar{\mathcal{C}}, v \in \mathcal{V}, r = 1, \dots, R. \tag{12}$$

$$\sum_{\substack{i \in \{0, \bar{\mathcal{C}}\} \\ i \neq j}} \sum_{v \in \mathcal{V}} \sum_{r=1}^{R} w_{ijvr} = 1, \qquad j \in \bar{\mathcal{C}}. \tag{13}$$

Constraints (14) ensure that the total number of non-empty routes does not exceed the maximum allowed number, R.

$$\sum_{v \in \mathcal{V}} \sum_{r=1}^{R} \sum_{j \in \bar{\mathcal{C}}} w_{0jvr} \leq R, \tag{14}$$

The set of constraints (15) establish precedences between routes in order to reduce symmetry on the solution space. These constraints indicate that route $r+1$ of vehicle v can be used if route r of the same vehicle is used. Some routes may be empty, so these constraints force those routes to be the last ones.

$$\sum_{i \in \bar{\mathcal{C}}} w_{i(n+1)vr} \geq \sum_{i \in \bar{\mathcal{C}}} w_{0iv(r+1)}, \qquad v \in \mathcal{V}, r = 1, \dots, R-1. \tag{15}$$

Constraints (16) set the quantity of product p shipped on route r in period t (Q_{pvrt}) to zero if route r does not start in period t ($\phi_{0vrt} = 0$). Constraints (17) determine the quantity of product p shipped on route r. Since any non-empty route must satisfy $\sum_{t \in \mathcal{T}} \phi_{0vrt} = 1$, and taking into account constraints (16), it follows that just one variable Q_{pvrt} in the left-side summation of (17) is positive.

$$Q_{pvrt} \leq \min \left\{ \left\lfloor \frac{\theta_v}{\varphi_p} \right\rfloor, \sum_{i \in \bar{\mathcal{C}}} d_{pi} \right\} \phi_{0vrt}, \qquad p \in \mathcal{P}, v \in \mathcal{V}, r = 1, \dots, R, t \in \mathcal{T}. \tag{16}$$

$$\sum_{t \in \mathcal{T}} Q_{pvrt} = \sum_{i \in \bar{\mathcal{C}}} d_{pi} \Bigg(\sum_{\substack{j \in \{\bar{\mathcal{C}}, n+1\} \\ j \neq i}} w_{ijvr} \Bigg), \qquad p \in \mathcal{P}, v \in \mathcal{V}, r = 1, \dots, R. \tag{17}$$

Constraints (18) guarantee that vehicle v on route r must visit customer i in just one period t only if customer i belongs to route r of vehicle v, while constraints (19)–(20) ensure that each non-empty route starts and finishes in just one period (not necessarily the same), respectively. Note that $w_{0(n+1)vr} = 1$ implies that route r of vehicle v was not used, so $\phi_{0vrt} = \phi_{(n+1)vrt} = 0$ for all $t \in \mathcal{T}$.

$$\sum_{t\in\mathcal{T}} \phi_{ivrt} = \sum_{\substack{j\in\{\bar{\mathcal{C}},n+1\}\\ j\neq i}} w_{ijvr}, \qquad i \in \bar{\mathcal{C}}, v \in \mathcal{V}, r = 1,\ldots,R. \tag{18}$$

$$\sum_{t\in\mathcal{T}} \phi_{0vrt} = 1 - w_{0(n+1)vr}, \qquad v \in \mathcal{V}, r = 1,\ldots,R. \tag{19}$$

$$\sum_{t\in\mathcal{T}} \phi_{0vrt} = \sum_{t\in\mathcal{T}} \phi_{(n+1)vrt}, \qquad v \in \mathcal{V}, r = 1,\ldots,R. \tag{20}$$

Inequalities (21) model time windows. If node i is not visited by route r of vehicle v (i.e., $\sum_{t\in\mathcal{T}} \phi_{ivrt} = 0$), then $\mu_{ivr} = 0$. Conversely, if node i is visited by route r of vehicle v, then $\phi_{ivrt} = 1$ for some $t \in \mathcal{T}$ and the time window is $\delta_{it} \leq \mu_{ivr} \leq \bar{\delta}_{it}$.

$$\sum_{t\in\mathcal{T}} \delta_{it}\phi_{ivrt} \leq \mu_{ivr} \leq \sum_{t\in\mathcal{T}} \bar{\delta}_{it}\phi_{ivrt}, \qquad i \in \mathcal{N}, v \in \mathcal{V}, r = 1,\ldots,R. \tag{21}$$

Constraints (22)–(23) impose lower bounds on the time at which node j is serviced by route r of vehicle v. Let j be the first customer on route r of vehicle v, then (22) establishes that the time when customer j starts being served is at least the start time of the route plus the loading time plus the travel time between the depot and the customer location. Similarly, for the remaining nodes, $j' \in \{\bar{\mathcal{C}}, n+1\}$, (23) ensures that the time when the route starts serving customer j' is at least the time when the previous customer i is served plus the service time at i plus the travel time between i and j'.

$$\mu_{jvr} \geq \mu_{0vr} + \lambda\left(\sum_{p\in\mathcal{P}} \varphi_p \sum_{t\in\mathcal{T}} Q_{pvrt}\right) + \tau_{0j} - M_j\big(1 - w_{0jvr}\big), \qquad j \in \bar{\mathcal{C}}, v \in \mathcal{V}, r = 1,\ldots,R. \tag{22}$$

$$\mu_{jvr} \geq \mu_{ivr} + \lambda\left(\sum_{p\in\mathcal{P}} \varphi_p d_{pi}\right) + \tau_{ij} - M_{ij}\big(1 - w_{ijvr}\big), \qquad \begin{array}{c} i \in \bar{\mathcal{C}}, j \in \{\bar{\mathcal{C}}, n+1\}, \\ v \in \mathcal{V}, r = 1,\ldots,R : i \neq j. \end{array} \tag{23}$$

Constraints (24) guarantee that each customer i is visited before its deadline. Constraints (25) states that route $r+1$ of vehicle v can start only after vehicle v finishes the previous route, r. Notice that these constraints are active only if route $r+1$ of vehicle v is non-empty.

$$\mu_{ivr} \leq \Delta_i \sum_{\substack{j\in\{\bar{\mathcal{C}},n+1\}\\ j\neq i}} w_{ijvr}, \qquad i \in \bar{\mathcal{C}}, v \in \mathcal{V}, r = 1,\ldots,R. \tag{24}$$

$$\mu_{0v(r+1)} \geq \mu_{(n+1)vr} - \bar{\delta}_{(n+1)|\mathcal{T}|}\left(1 - \sum_{j \in \bar{\mathcal{C}}} w_{0jv(r+1)}\right), \qquad r = 1, \dots, R-1, v \in \mathcal{V}. \tag{25}$$

Constraints (26) avoid that the vehicle capacity is exceeded.

$$\sum_{p \in \mathcal{P}} \varphi_p \left(\sum_{t \in \mathcal{T}} Q_{pvrt}\right) \leq \theta_v, \qquad v \in \mathcal{V}, r = 1, \dots, R. \tag{26}$$

Finally, the variables domain is given by (27)–(32).

$$x_{at}, I_{at}, \pi_{at} \geq 0, \qquad a \in \mathcal{C}, t \in \mathcal{T}. \tag{27}$$
$$\mu_{ivr} \geq 0, \qquad i \in \mathcal{N}, v \in \mathcal{V}, r = 1, \dots, R. \tag{28}$$
$$Q_{pvrt} \geq 0, \qquad p \in \mathcal{P}, v \in \mathcal{V}, r = 1, \dots, R, t \in \mathcal{T}. \tag{29}$$
$$y_{at}, z_{abt} \in \{0,1\}, \qquad a \in \mathcal{C}, b \in \mathcal{C}, t \in \mathcal{T}. \tag{30}$$
$$\phi_{ivrt} \in \{0,1\}, \qquad i \in \mathcal{N}, v \in \mathcal{V}, r = 1, \dots, R, t \in \mathcal{T}. \tag{31}$$
$$w_{ijvr} \in \{0,1\}, \qquad i, j \in \mathcal{N}, v \in \mathcal{V}, r = 1, \dots, R. \tag{32}$$

3.2 Lot-scheduling multi-vehicle routing problem (LSMVRP-F2)

In the second model, LSMVRP-F2, we separate routing and vehicle allocation decisions in order to reduce the number of binary variables of the problem. Consider the following sets of routing variables for this model:

Routing Variables

w_{ijr} : Equal to 1 if arc (i, j) is traveled by route r, 0 otherwise;
α_{rv} : Equal to 1 if route r is performed by vehicle v, 0 otherwise;
Q_{prt} : Quantity of product p shipped on route r in period t;
ϕ_{irt} : Equal to 1 if node i is visited by route r in period t, 0 otherwise;
μ_{ir} : Starting time at which node i is serviced by route r;

The objective function (33) minimizes the production setups, inventory and routing costs.

$$\min \sum_{a \in \mathcal{C}} \sum_{t \in \mathcal{T}} h_a I_{at} + \sum_{a \in \mathcal{C}} \sum_{b \in \mathcal{C}} \sum_{t \in \mathcal{T}} c_{ab} z_{abt} + \sum_{r=1}^{R} \sum_{i=0}^{n} \sum_{j=1}^{n+1} c_{ij} w_{ijr}. \tag{33}$$

Constraints (2)–(9) related to lot-sizing and scheduling decisions are the same as the ones in the LSMVRP-F1 model, except for constraints (2) and (3), which are rewritten using the new variables Q_{prt} as (34) and (35), respectively.

$$I_{at-1} + x_{at} = \sum_{r=1}^{R} \sum_{p \in \mathcal{P}} \eta_{ap} Q_{prt} + I_{at}, \qquad a \in \mathcal{C}, t \in \mathcal{T}. \tag{34}$$

$$I_{at-1} \geq \sum_{r=1}^{R} \sum_{p \in \mathcal{P}} \eta_{ap} Q_{prt}, \qquad a \in \mathcal{C}, t \in \mathcal{T}. \tag{35}$$

In the vehicle routing part, we need to drop the vehicle index from all variables and constraints. Besides that, consider the following changes:

Constraints (16) are replaced by constraints (36). Notice that it was necessary to change the upper bound on the right hand side of (36) since we have no information about the vehicle that will perform route r.

$$Q_{prt} \leq \min \left\{ \left\lfloor \frac{\max_{v \in \mathcal{V}} \theta_v}{\varphi_p} \right\rfloor, \sum_{i \in \bar{\mathcal{C}}} d_{pi} \right\} \phi_{0rt}, \qquad p \in \mathcal{P}, r = 1, \ldots, R, t \in \mathcal{T}. \tag{36}$$

Similarly, constraints (25) are replaced by constraints (37), which ensure that route s starts after route r finishes, $r < s$, if both of them are performed by the same vehicle v.

$$\mu_{0s} \geq \mu_{(n+1)r} - \bar{\delta}_{(n+1)|\mathcal{T}|} \Big(2 - \alpha_{rv} - \alpha_{sv} \Big), \qquad v \in \mathcal{V}, r, s = 1, \ldots, R : r < s. \tag{37}$$

The vehicle capacity constraints (26) must be replaced by the inequalities (38).

$$\sum_{p \in \mathcal{P}} \varphi_p \left(\sum_{t \in \mathcal{T}} Q_{prt} \right) \leq \sum_{v \in \mathcal{V}} \theta_v \alpha_{rv}, \qquad r = 1, \ldots, R. \tag{38}$$

The new set of constraints (39) it necessary to ensure that each non-empty route is performed by only one vehicle.

$$\sum_{v \in \mathcal{V}} \alpha_{rv} = \sum_{t \in \mathcal{T}} \phi_{0rt}, \qquad r = 1, \ldots, R. \tag{39}$$

Finally, the domain of α_{rv} is given by (40).

$$\alpha_{rv} \in \{0, 1\}, \qquad r = 1, \ldots, R, v \in \mathcal{V}. \tag{40}$$

Therefore, model LSMVRP-F2 reads as follows: minimize (33), subject to (34)–(35), (4)–(9), (36)–(40), and all the following constraints accordingly modified by dropping the vehicle index v: (10)–(13), (15), (17)–(24), (27)–(32).

4 Computational Experiments

In this section, we first present the computational results of the proposed models LSMVRP-F1 and LSMVRP-F2 on a set of randomly generated instances described in Section 4.1. All models were coded in C++ language using Concert Technology and solved on a workstation with an Intel Xeon 3.07 GHz processor and 96 GB of RAM using the default setting of CPLEX 12.6.1. A computational time limit of three hours was arbitrarily set for each

Tab. 3: Parameters used to generate random instances.

Demand of product p at customer i	$d_{pi} \in U[10, 100]$
Units of item a per unit of product p	$\eta_{ap} \in U[0, 5]$
Width (W_a) and length (L_a) of item a	$W_a, L_a \in U[5, 10]$
Coordinates of node i	$(X_i, Y_i) \in [0, 500]$
Initial (Minimum) stock of item c	$I_{c0} = I_c^{\min} = 0$
Unit processing time of item a	$\rho_a = 1$
Unit weight of product p	$\varphi_p = 1$
Production capacity in period t	$K_t = \left\lceil \frac{\sum_{a\in\mathcal{C}} \rho_a \left(\sum_{p\in\mathcal{P}} \sum_{i\in\mathcal{N}} d_{pi}\eta_{ap}\right)}{0.6\cdot\lvert\mathcal{T}\rvert} \right\rceil$
Setup time between items a and b	$\varsigma_{ab} \in U[6, 10]$ if $a \neq b$, 0 otherwise
Travel time from nodes i to node j	$\tau_{ij} = \left\lceil \frac{60}{80} \left(\sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2} \right) \right\rceil$
Capacity of vehicle v	$\theta_v = \left\lceil U[0.25, 0.50] \cdot \sum_{p\in\mathcal{P}} \sum_{i\in\mathcal{N}} d_{pi} \cdot \varphi_p \right\rceil$
Time window of node $i \in \{0, \bar{\mathcal{C}}\}$ in period t	$[\underline{\delta}_{it}, \bar{\delta}_{it}] = [480 + 1440\,(t-1)\,, 1080 + 1440\,(t-1)]$
Time window of node $n+1$ in period t	$[\underline{\delta}_{(n+1)t}, \bar{\delta}_{(n+1)t}] = [1440\,(t-1)\,, 1440t]$
Loading and unloading rates	$\lambda = 1/50$
Due date of customer $i \in \bar{\mathcal{C}}$	$\Delta_i = \bar{\delta}_{i\lvert\mathcal{T}\rvert}$
Maximum number of routes over the planning horizon	$R = n$
Unit inventory holding cost of item a	$h_a = 0.001 \cdot W_a \cdot L_a$
Setup cost of a changeover from item a to item b	$c_{ab} = 25 \cdot \varsigma_{ab}$
Travel cost from node i to node j	$c_{ij} = \left\lceil \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2} \right\rceil$

instance. This limit seems to be acceptable for supporting decisions in practical settings of small furniture companies.

4.1 Instances Generation

We generated a set of random instances based on information provided by a Brazilian furniture company. We notice that the size of the instances is considered realistic in the context of small furniture companies, which produce a few final products to meet the demand of a set of customers over a short planning horizon (usually a week or a bit longer).

We follow a similar approach to the ones proposed by Gramani et al. (2009), for a cutting stock problem in furniture companies, and Armentano et al. (2011), for a multi-product PIDRP. The number of final products ($|\mathcal{P}|$), items ($|\mathcal{C}|$), customers (n) and vehicles ($|\mathcal{V}|$) are {3,5}, {3,5}, {10,15,20,30,40,50} and {2,3}, respectively. The planning horizon is fixed to 8 periods for all the instances. We generate five instances for each combination of final products, items, customers and vehicles, resulting in a total of 240 instances. Table 3 presents how the remaining parameters were generated.

Observe that we defined the width and length for each item a in order to estimate inventory holding costs as a function of the size of the items. It means that the larger a given item is, the more expensive is to hold it in stock. The capacity usage for each time period is defined to be around 60%, following the work of Amorim et al. (2013). For the time windows generation, we used the common practice in the Brazilian furniture industry: customers may be visited at any time within regular business hours, vehicles should also leave the depot within regular working hours, but they may return at any time. Furthermore, since we considered a relatively short planning horizon (8 days), we supposed that the customers' due dates match the end of the last time windows. The loading/unloading rates were estimated as 1/50, which means that fifty kilograms are loaded/unloaded per minute. Finally, we set the upper bound on the number of routes as the number of customer on the problem.

4.2 Computational results of models LSMVRP-F1 and LSMVRP-F2

Tables 4–6 show the results for models LSMVRP-F1 and LSMVRP-F2 for instances with up to $n = 20$ customers. For $n \geq 30$, the models are not able to return feasible solutions within the preset computational time limit of 3 hours. Columns labeled n, $|\mathcal{P}|$, $|\mathcal{C}|$, $|\mathcal{T}|$ and $|\mathcal{V}|$ show information related to the size of the instances, namely, number of customers, final products, items, periods and vehicles, respectively. Then, for each formulation we report the value of the linear relaxation at the root node ('Root'), the cost of the best feasible solution found by CPLEX ('Total Cost'), the optimility gap ('Gap(%)'), the computing time in seconds ('CPU(s)') and the number of nodes explored in the branch-and-bound tree.

In general, proving optimality is difficult with either of the two formulations, but better solutions are more likely to be found with model LSMVRP-F2, which also provides slightly better lower bounds. Both models are capable of finding feasible solutions for instances of moderate size, with model LSMVRP-F2 providing feasible solutions for most of the instances with $n \leq 20$ customers, and model LSMVRP-F1 limited to solve instances with up to $n = 15$ customers. In general, model LSMVRP-F1 has more constraints and binary variables than model LSMVRP-F2 and, consequently, solving LSMVRP-F1 might be more difficult than solving LSMVRP-F2. Indeed, the lower number of explored nodes in the branch-and-bound tree of model LSMVRP-F1, in comparison with model LSMVRP-F2, highlights the difficulty of solving this model.

For instances with $n = 10$ customers, model LSMVRP-F2 found a better solution than model LSMVRP-F1 in 28 out of 40 instances, model LSMVRP-F1 provided a better solution for other 11 instances, while for the remaining one both models found the same solution. We also note that over the 11 instances whereby model LSMVRP-F1 provided a better solution, in only one (32th instance) it achieved a lower optimality gap. This is in line with our claim that lower bounds from model LSMVRP-F2 are slightly better than those from model LSMVRP-F1. However, computational results suggest that lower bounds from model LSMVRP-F2 are also weak and, therefore, CPLEX struggled to prove optimality of some instances. To show that, we solved again all the instances using model LSMVRP-F2 and setting a much longer CPU time limit of one day per instance. Results showed that CPLEX only proved optimality of one instance more (6th instance) in comparison with the results of Table 4. Figure 2 shows the behavior of the lower and upper bounds for this instance. The optimal solution was found in less than one hour of CPU time, but CPLEX took more than 10 hours to prove its optimality.

For instances with $n = 15$, model LSMVRP-F1 found feasible solutions for 24 out 40 instances and, from those, only in four instances it provided a better solution than model LSMVRP-F2. The average optimality gap was 46.75%, which shows that CPLEX was far from proving any optimal solution after three hours of CPU time. On the other hand, model

Tab. 4: Models results for instances with $n = 10$ customers.

						LSMVRP-F1					LSMVRP-F2				
Instance	n	$\|\mathcal{P}\|$	$\|\mathcal{C}\|$	$\|\mathcal{T}\|$	$\|\mathcal{V}\|$	Root	Total Cost	Gap(%)	CPU(s)	Nodes	Root	Total Cost	Gap(%)	CPU(s)	Nodes
1	10	3	3	8	2	1,983.05	4,013.31	24.91	10,800.00	96,025	2,077.43	4,045.03	20.43	10,800.00	209,851
2	10	3	3	8	2	1,430.10	3,904.91	22.52	10,800.00	177,340	1,866.56	3,873.99	10.69	10,800.00	657,112
3	10	3	3	8	2	1,465.19	3,572.92	29.81	10,800.00	215,436	1,612.51	3,573.19	21.73	10,800.00	596,060
4	10	3	3	8	2	1,928.56	3,819.34	8.87	10,800.00	107,832	2,155.90	3,803.46	0.00	10,507.70	209,729
5	10	3	3	8	2	1,823.55	3,758.01	20.26	10,800.00	199,671	1,912.67	3,797.04	15.88	10,800.00	664,842
6	10	3	3	8	3	2,016.46	4,126.04	23.72	10,800.00	190,955	2,241.56	4,074.22	4.48	10,800.00	227,288
7	10	3	3	8	3	1,790.79	3,842.32	25.18	10,800.00	147,310	1,957.19	3,817.56	12.72	10,800.00	422,615
8	10	3	3	8	3	1,817.66	4,643.83	27.92	10,800.00	86,806	2,071.12	4,668.97	17.34	10,800.00	721,265
9	10	3	3	8	3	1,705.47	3,786.86	20.40	10,800.00	113,400	1,926.63	3,786.86	11.48	10,800.00	678,848
10	10	3	3	8	3	2,325.14	4,369.69	26.02	10,800.00	72,213	2,533.60	4,237.05	13.05	10,800.00	280,493
11	10	3	5	8	2	2,570.45	5,413.06	34.76	10,800.00	113,968	2,621.88	5,250.37	34.32	10,800.00	175,476
12	10	3	5	8	2	2,609.69	6,966.45	41.56	10,800.00	112,670	2,888.37	6,988.09	40.45	10,800.00	183,098
13	10	3	5	8	2	3,061.44	6,887.40	38.68	10,800.00	161,304	3,212.91	6,695.40	35.81	10,800.00	238,188
14	10	3	5	8	2	2,238.67	5,983.55	45.54	10,800.00	59,852	2,647.55	6,105.33	41.43	10,800.00	234,870
15	10	3	5	8	2	1,912.77	4,288.12	24.88	10,800.00	151,136	2,206.63	4,251.06	21.43	10,800.00	223,723
16	10	3	5	8	3	2,255.50	6,784.48	40.42	10,800.00	74,565	3,068.79	6,744.75	34.31	10,800.00	395,217
17	10	3	5	8	3	2,529.27	5,345.39	36.44	10,800.00	109,642	2,618.85	5,214.48	28.92	10,800.00	195,310
18	10	3	5	8	3	2,633.59	5,703.54	36.14	10,800.00	28,830	2,910.47	5,465.00	30.10	10,800.00	460,508
19	10	3	5	8	3	3,665.00	7,834.99	37.85	10,800.00	60,554	3,970.80	7,493.86	29.18	10,800.00	258,073
20	10	3	5	8	3	2,398.35	5,348.56	37.13	10,800.00	60,326	2,483.96	5,261.29	35.92	10,800.00	291,710
21	10	5	3	8	2	2,775.91	5,410.53	27.01	10,800.00	151,592	2,803.51	5,425.21	22.79	10,800.00	298,856
22	10	5	3	8	2	2,069.86	3,983.37	27.69	10,800.00	58,063	2,240.59	3,972.29	20.05	10,800.00	128,931
23	10	5	3	8	2	2,433.71	4,446.00	22.64	10,800.00	81,967	2,613.53	4,414.16	15.49	10,800.00	95,870
24	10	5	3	8	2	2,306.00	4,852.70	22.34	10,800.00	142,892	2,659.07	4,919.61	16.62	10,800.00	245,912
25	10	5	3	8	2	2,051.20	4,106.12	26.92	10,800.00	76,121	2,183.93	4,064.24	20.87	10,800.00	466,300
26	10	5	3	8	3	2,265.94	4,699.06	23.20	10,800.00	137,024	2,774.79	4,696.54	9.49	10,800.00	184,863
27	10	5	3	8	3	2,059.30	4,356.66	32.01	10,800.00	108,726	2,112.32	4,333.78	25.14	10,800.00	205,573
28	10	5	3	8	3	2,699.78	5,639.80	28.20	10,800.00	82,524	3,370.43	5,641.26	16.15	10,800.00	119,126
29	10	5	3	8	3	2,229.84	4,471.96	25.73	10,800.00	62,773	2,344.04	4,514.89	15.50	10,800.00	121,510
30	10	5	3	8	3	2,328.65	4,961.75	36.51	10,800.00	18,291	2,543.25	4,872.25	21.87	10,800.00	175,492
31	10	5	5	8	2	3,746.60	–	–	10,800.00	9,003	3,862.69	7,090.63	36.41	10,800.00	245,393
32	10	5	5	8	2	3,052.55	7,282.79	42.29	10,800.00	69,739	3,275.49	7,669.47	42.38	10,800.00	169,482
33	10	5	5	8	2	3,225.63	8,163.11	44.04	10,800.00	25,562	4,006.55	7,919.50	38.57	10,800.00	153,257
34	10	5	5	8	2	2,714.76	6,236.01	46.25	10,800.00	19,159	2,990.73	6,058.22	35.86	10,800.00	404,461
35	10	5	5	8	2	3,688.11	7,860.27	35.81	10,800.00	36,277	4,019.43	7,447.01	34.85	10,800.00	100,860
36	10	5	5	8	3	3,412.44	7,868.78	47.48	10,800.00	13,573	3,741.29	7,256.83	34.94	10,800.00	139,331
37	10	5	5	8	3	3,176.21	7,631.78	47.24	10,800.00	27,855	3,438.16	7,252.94	39.22	10,800.00	73,187
38	10	5	5	8	3	3,719.02	7,396.04	39.35	10,800.00	30,803	3,886.42	7,148.21	35.46	10,800.00	75,108
39	10	5	5	8	3	2,412.25	–	–	10,800.00	8,636	2,697.53	5,738.86	33.76	10,800.00	100,384
40	10	5	5	8	3	3,274.00	7,682.54	49.00	10,800.00	7,001	3,397.50	6,591.28	33.61	10,800.00	86,096
Mean						2,495.06	5,459.00	32.28	10,800.00	87,685.40	2,748.67	5,404.35	25.22	10,792.69	272,856.70

Tab. 5: Models results for instances with $n = 15$ customers.

						LSMVRP-F1					LSMVRP-F2				
Instance	n	$\lvert\mathcal{P}\rvert$	$\lvert\mathcal{C}\rvert$	$\lvert\mathcal{T}\rvert$	$\lvert\mathcal{V}\rvert$	Root	Total Cost	Gap(%)	CPU(s)	Nodes	Root	Total Cost	Gap(%)	CPU(s)	Nodes
41	15	3	3	8	2	1,513.58	3,977.65	41.36	10,800.00	55,547	1,759.03	3,966.34	26.49	10,800.00	401,780
42	15	3	3	8	2	2,565.14	5,987.54	44.74	10,800.00	12,002	2,734.98	5,625.78	25.74	10,800.00	276,861
43	15	3	3	8	2	2,291.98	5,074.26	42.78	10,800.00	12,012	2,440.57	4,718.21	23.45	10,800.00	89,380
44	15	3	3	8	2	2,854.02	5,855.40	34.60	10,800.00	21,057	3,075.04	5,618.68	24.11	10,800.00	71,080
45	15	3	3	8	2	2,547.18	–	–	10,800.00	16,587	2,686.77	5,298.50	34.78	10,800.00	62,279
46	15	3	3	8	3	2,263.23	5,611.07	49.88	10,800.00	18,333	2,349.75	5,465.94	35.49	10,800.00	297,782
47	15	3	3	8	3	2,608.21	6,346.25	49.40	10,800.00	7,538	2,750.31	5,344.93	24.78	10,800.00	125,961
48	15	3	3	8	3	2,968.55	6,271.81	41.91	10,800.00	3,783	3,342.17	5,821.80	21.90	10,800.00	113,485
49	15	3	3	8	3	2,742.98	6,929.88	43.90	10,800.00	41,208	3,104.22	6,088.72	20.74	10,800.00	103,179
50	15	3	3	8	3	2,227.05	5,276.76	41.44	10,800.00	13,361	2,575.13	4,761.04	18.93	10,800.00	113,400
51	15	3	5	8	2	2,929.63	8,753.77	53.02	10,800.00	42,277	3,679.65	–	–	10,800.00	24,248
52	15	3	5	8	2	2,102.73	–	–	10,800.00	7,107	2,166.02	5,761.02	40.93	10,800.00	213,950
53	15	3	5	8	2	1,915.72	5,484.08	53.07	10,800.00	21,155	2,104.19	5,667.40	54.67	10,800.00	10,711
54	15	3	5	8	2	2,524.27	6,964.86	57.53	10,800.00	6,452	2,653.08	6,246.09	46.97	10,800.00	21,420
55	15	3	5	8	2	2,758.40	7,272.88	54.98	10,800.00	4,440	2,885.33	7,315.13	54.95	10,800.00	7,248
56	15	3	5	8	3	3,566.72	–	–	10,800.00	4,079	3,861.85	8,677.79	49.88	10,800.00	12,035
57	15	3	5	8	3	3,368.12	–	–	10,800.00	7,225	3,555.83	7,425.28	44.74	10,800.00	22,860
58	15	3	5	8	3	3,076.12	–	–	10,800.00	6,001	3,336.69	7,208.09	41.06	10,800.00	36,860
59	15	3	5	8	3	2,851.33	–	–	10,800.00	4,159	2,943.36	6,918.86	49.97	10,800.00	12,333
60	15	3	5	8	3	2,272.73	–	–	10,800.00	8,144	2,720.41	6,418.21	41.31	10,800.00	78,380
61	15	5	3	8	2	3,039.91	6,416.36	41.97	10,800.00	27,135	3,148.88	6,428.42	33.22	10,800.00	71,700
62	15	5	3	8	2	2,637.22	5,680.32	39.95	10,800.00	16,292	2,906.87	5,572.08	33.11	10,800.00	112,940
63	15	5	3	8	2	2,986.00	6,362.16	41.63	10,800.00	12,693	3,133.58	5,847.36	35.14	10,800.00	16,965
64	15	5	3	8	2	3,734.66	7,738.15	44.29	10,800.00	3,393	3,828.55	7,025.78	30.05	10,800.00	37,492
65	15	5	3	8	2	3,306.70	7,235.02	44.17	10,800.00	10,069	3,571.17	6,408.19	26.13	10,800.00	171,096
66	15	5	3	8	3	2,765.02	8,218.68	56.51	10,800.00	8,661	2,834.41	6,382.74	33.91	10,800.00	242,889
67	15	5	3	8	3	2,865.40	6,598.18	45.40	10,800.00	4,178	3,210.67	5,769.22	25.14	10,800.00	56,209
68	15	5	3	8	3	2,795.92	6,213.96	46.09	10,800.00	13,383	3,068.52	5,721.24	32.49	10,800.00	31,835
69	15	5	3	8	3	2,047.48	–	–	10,800.00	4,323	2,317.34	4,115.19	12.13	10,800.00	107,394
70	15	5	3	8	3	2,549.86	6,358.26	49.67	10,800.00	7,326	2,764.00	4,895.53	23.35	10,800.00	51,716
71	15	5	5	8	2	4,040.90	9,184.84	50.11	10,800.00	6,071	4,260.73	8,725.19	46.58	10,800.00	13,771
72	15	5	5	8	2	3,916.01	–	–	10,800.00	8,283	4,138.42	8,436.51	42.40	10,800.00	17,658
73	15	5	5	8	2	3,718.36	–	–	10,800.00	7,872	3,951.78	8,543.94	45.38	10,800.00	15,233
74	15	5	5	8	2	4,204.57	–	–	10,800.00	8,444	4,427.07	10,975.40	51.32	10,800.00	9,703
75	15	5	5	8	2	3,772.43	–	–	10,800.00	5,115	3,859.83	–	–	10,800.00	13,783
76	15	5	5	8	3	2,701.33	–	–	10,800.00	1,275	2,871.03	6,834.79	48.16	10,800.00	14,947
77	15	5	5	8	3	4,101.34	–	–	10,800.00	4,382	4,432.72	–	–	10,800.00	6,354
78	15	5	5	8	3	4,183.82	–	–	10,800.00	4,329	4,403.57	10,206.70	48.18	10,800.00	14,519
79	15	5	5	8	3	3,872.96	–	–	10,800.00	3,581	4,127.53	8,775.01	43.90	10,800.00	19,637
80	15	5	5	8	3	3,409.27	8,827.61	53.72	10,800.00	1,833	3,557.44	8,267.41	48.67	10,800.00	13,805
Mean						2,964.92	6,609.99	46.75	10,800.00	11,777.63	3,188.46	6,575.09	36.22	10,800.00	78,371.95

Tab. 6: Models results for instances with $n = 20$ customers.

						LSMVRP-F1					LSMVRP-F2				
Instance	n	$\|\mathcal{P}\|$	$\|\mathcal{C}\|$	$\|\mathcal{T}\|$	$\|\mathcal{V}\|$	Root	Total Cost	Gap(%)	CPU(s)	Nodes	Root	Total Cost	Gap(%)	CPU(s)	Nodes
81	20	3	3	8	2	3,458.56	–	–	10,800.00	3,999	3,734.94	6,693.56	31.73	10,800.00	79,830
82	20	3	3	8	2	3,299.97	–	–	10,800.00	2,567	3,480.67	7,670.04	48.89	10,800.00	5,739
83	20	3	3	8	2	2,670.63	–	–	10,800.00	1,613	2,795.89	6,559.22	50.65	10,800.00	8,860
84	20	3	3	8	2	2,564.48	–	–	10,800.00	2,221	2,695.48	–	–	10,800.00	3,971
85	20	3	3	8	2	3,037.86	–	–	10,800.00	1,951	3,115.50	–	–	10,800.00	5,013
86	20	3	3	8	3	3,446.00	11,518.00	64.44	10,800.00	7,137	3,654.88	6,112.95	23.23	10,800.00	36,780
87	20	3	3	8	3	3,197.65	–	–	10,800.00	2,573	3,272.90	6,422.40	31.60	10,800.00	91,112
88	20	3	3	8	3	3,024.74	11,411.50	68.54	10,800.00	1,968	3,205.41	–	–	10,800.00	11,244
89	20	3	3	8	3	2,929.10	–	–	10,800.00	645	2,973.25	6,216.14	43.94	10,800.00	5,315
90	20	3	3	8	3	2,976.17	–	–	10,800.00	1,765	3,206.19	6,262.44	35.78	10,800.00	196,438
91	20	3	5	8	2	4,036.65	–	–	10,800.00	2,238	4,184.05	–	–	10,800.00	6,843
92	20	3	5	8	2	3,476.55	–	–	10,800.00	1,573	3,559.01	–	–	10,800.00	6,514
93	20	3	5	8	2	3,312.15	–	–	10,800.00	1,239	3,754.98	–	–	10,800.00	9,619
94	20	3	5	8	2	3,259.73	–	–	10,800.00	3,388	3,464.40	–	–	10,800.00	17,666
95	20	3	5	8	2	4,159.15	–	–	10,800.00	2,009	4,376.46	9,576.09	48.30	10,800.00	8,426
96	20	3	5	8	3	5,131.78	–	–	10,800.00	2,622	5,460.74	–	–	10,800.00	2,270
97	20	3	5	8	3	3,704.72	–	–	10,800.00	1,399	3,794.15	8,480.65	48.87	10,800.00	19,517
98	20	3	5	8	3	4,524.38	14,583.60	65.45	10,800.00	1,566	4,888.21	10,968.50	49.13	10,800.00	4,776
99	20	3	5	8	3	3,687.45	–	–	10,800.00	1,223	3,941.72	8,797.52	49.77	10,800.00	7,126
100	20	3	5	8	3	3,907.46	–	–	10,800.00	1,525	4,019.82	–	–	10,800.00	4,287
101	20	5	3	8	2	3,747.03	–	–	10,800.00	1,413	3,850.13	–	–	10,800.00	3,509
102	20	5	3	8	2	4,365.74	–	–	10,800.00	1,781	4,488.75	8,285.40	38.61	10,800.00	47,819
103	20	5	3	8	2	3,482.27	–	–	10,800.00	2,245	3,790.95	8,174.80	49.08	10,800.00	4,892
104	20	5	3	8	2	3,558.59	–	–	10,800.00	2,243	3,674.59	9,069.97	55.05	10,800.00	3,248
105	20	5	3	8	2	3,337.34	–	–	10,800.00	2,108	3,518.11	7,076.40	44.76	10,800.00	5,361
106	20	5	3	8	3	4,201.02	–	–	10,800.00	486	4,315.80	–	–	10,800.00	5,032
107	20	5	3	8	3	2,621.02	–	–	10,800.00	489	2,736.48	7,222.34	56.96	10,800.00	3,459
108	20	5	3	8	3	3,778.36	–	–	10,800.00	956	3,917.24	8,538.12	47.65	10,800.00	4,224
109	20	5	3	8	3	3,337.80	–	–	10,800.00	984	3,525.39	–	–	10,800.00	11,614
110	20	5	3	8	3	3,958.90	–	–	10,800.00	956	4,088.95	7,990.06	41.46	10,800.00	11,722
111	20	5	5	8	2	5,045.44	–	–	10,800.00	929	5,122.76	10,578.90	48.53	10,800.00	3,825
112	20	5	5	8	2	6,169.67	–	–	10,800.00	1,061	6,431.45	–	–	10,800.00	3,554
113	20	5	5	8	2	6,826.87	–	–	10,800.00	2,466	7,092.44	–	–	10,800.00	3,946
114	20	5	5	8	2	3,943.73	–	–	10,800.00	1,840	4,174.88	–	–	10,800.00	2,438
115	20	5	5	8	2	4,629.55	–	–	10,800.00	1,489	4,896.83	–	–	10,800.00	2,619
116	20	5	5	8	3	6,478.13	–	–	10,800.00	2,004	6,542.24	12,824.40	43.34	10,800.00	8,723
117	20	5	5	8	3	5,720.74	–	–	10,800.00	1,019	5,842.08	–	–	10,800.00	2,447
118	20	5	5	8	3	6,073.86	–	–	10,800.00	2,505	6,419.24	–	–	10,800.00	3,577
119	20	5	5	8	3	5,124.53	–	–	10,800.00	1,029	5,544.10	9,951.43	31.71	10,800.00	9,836
120	20	5	5	8	3	6,444.60	–	–	10,800.00	2,004	6,516.04	–	–	10,800.00	1,971
Mean						4,066.26	12,504.37	66.14	10,800.00	1,880.70	4,251.68	8,260.54	43.76	10,800.00	16,879.05

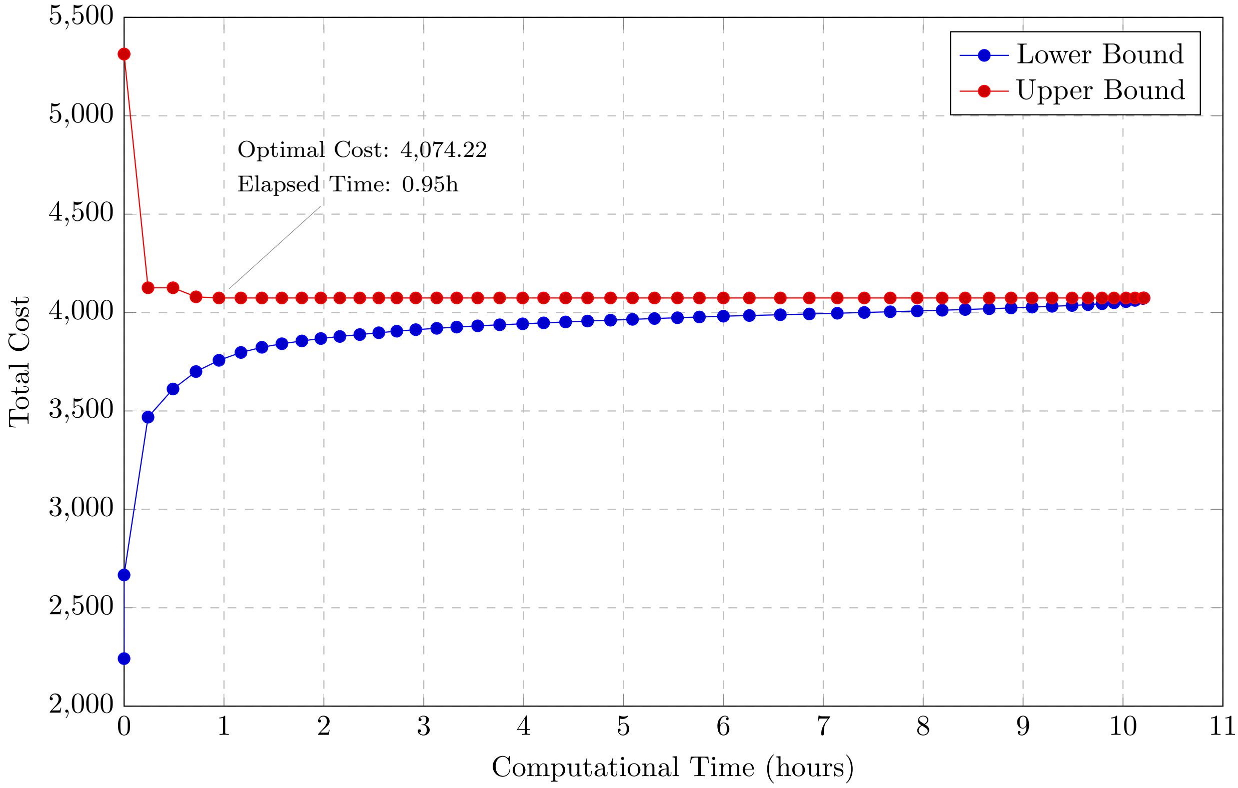

Fig. 2: Comparison of lower bound and upper bound for the 6th instance.

LSMVRP-F2 found feasible solutions for 37 out 40 instances, providing a better solution than model LSMVRP-F1 in 33 cases. Similarly to LSMVRP-F1, model LSMVRP-F2 could not prove the optimality of any instance, reporting large optimality gap's for most of them (36.22% on average).

Solving instances with more than fifteen customers appears to be very difficult, especially for model LSMVRP-F1. Table 6 shows that for instances with $n = 20$ customers, model LSMVRP-F1 just found feasible solution for three instances and model LSMVRP-F2 only solved 21 out of 40 instances. The small number of explored nodes for both formulations suggest that solving the linear relaxation of models LSMVRP-F1 and LSMVRP-F2 becomes hard when the size of the problem increase and, therefore, CPLEX is not able to explore so many nodes in the branch-and-bound tree.

4.3 Benefits of integrating production and distribution decisions

In order to show the benefits of integrating production and distribution decisions, we compare the results of model LSMVRP-F2 with those obtained by a sequential approach based on the common practice of furniture companies. The routing decisions (routes and timing) are made first with the objective of minimizing only the routing costs. Then, the amount of final products carried in each route is used by the production planner to calculate the demand of each item in each period. Those demands are then used as input to determine a production plan which minimizes the setups and inventory holding costs over the planning horizon.

Notice that this approach corresponds to solve first a multi-trip vehicle routing problem with time windows (MTVRPTW) in order to obtain the routes schedules, and then to solve a multi-item lot-sizing and scheduling problem (LSSP) to determine lot-sizes, setup operations and inventory levels. Therefore, solving the problem with this two-step procedure may still be difficult and time-consuming. In fact, our preliminary test showed that solve to optimality instances with $n \geq 15$ customers using this approach is time-consuming, since finding the optimal solution of the MTVRPTW is very hard.

To overcome this issue, we only solve to optimality instances with $n = 10$ customers. This allows a fair comparison between the optimal solution found by the sequential procedure and the best feasible solution found by the model LSMVRP-F2. It is worth mentioning that by 'optimal' solution of the sequential procedure, we mean the solution found when both the MTVRPTW and LSSP are solved optimally.

Table 7 compares the total cost and CPU time of the integrated model LSMVRP-F2 and the sequential strategy described above. All the columns are self-explanatory, except for column 'Reduction(%)', which shows the cost reduction, in percentage, that model LSMVRP-F2 achieved in comparison with the sequential procedure. In general, solving model LSMVRP-F2 requires more effort than the sequential procedure, at least for those instances, but it also provides lower cost solutions for most of the instances (5.80% on average). Only in four cases (those with a negative Reduction(%)) the sequential procedure found better solutions than those reported by CPLEX after three hours of computing time. However, according to Table 4, the optimality gap for those instances is still large and so CPLEX might still find better feasible solutions by solving model LSMVRP-F2. In any case, these results strongly suggest that integrating production and distribution decisions may lead to significant savings in the system.

Figures 3–4 show how by integrating decisions CPLEX is able to find a better solutions than the sequential procedure for two different instances (4th and 35th, respectively). In both cases, considering production and distribution decisions simultaneously allow us to reduce the production costs (i.e., inventory plus setup costs) without increasing the routing costs. The sequential procedure fails to achieve those solutions as it is myopic: it does not consider the impact that routing decisions may have on the production side. The integrated model, however, is able to achieve a better trade-off between the different components of the total cost, since it has a holistic view of the problem.

4.4 Results on real-based instances

In this section we solve a small set of instances generated using real data provided by a small Brazilian furniture company. The data included information such as processing times, painting line capacity, initial inventory levels, product structure, customers' demand, customers' location, vehicle capacities, etc. All instances have the same size and each one corresponds to a problem with $n = 12$ customers, $|\mathcal{P}| = 5$ final products, $|\mathcal{C}| = 6$ items or sub-assemblies, $|\mathcal{V}| = 2$ vehicles and a planning horizon of $|\mathcal{T}| = 8$ days. We note that these instances are considered realistic in the context of a small company, which usually produce a few final products to meet the demand of a small group of customers in a short planning

Tab. 7: Economic benefits of integrating production and distribution decisions.

						Sequential Approach		Integrated Approach		
Instance	n	$\|\mathcal{P}\|$	$\|\mathcal{C}\|$	$\|\mathcal{T}\|$	$\|\mathcal{V}\|$	Total Cost	CPU(s)	Total Cost	CPU(s)	Reduction(%)
1	10	3	3	8	2	4,171.88	307.59	4,045.03	10,800.00	3.04
2	10	3	3	8	2	3,963.09	69.04	3,873.99	10,800.00	2.25
3	10	3	3	8	2	3,537.38	404.08	3,573.19	10,800.00	−1.01
4	10	3	3	8	2	3,819.34	28.42	3,803.46	10,507.70	0.42
5	10	3	3	8	2	3,888.43	285.46	3,797.04	10,800.00	2.35
6	10	3	3	8	3	4,232.14	21.08	4,074.22	10,800.00	3.73
7	10	3	3	8	3	3,965.76	163.59	3,817.56	10,800.00	3.74
8	10	3	3	8	3	4,939.41	142.45	4,668.97	10,800.00	5.48
9	10	3	3	8	3	3,952.91	212.87	3,786.86	10,800.00	4.20
10	10	3	3	8	3	4,441.59	151.67	4,237.05	10,800.00	4.61
11	10	3	5	8	2	5,220.70	57.45	5,250.37	10,800.00	−0.57
12	10	3	5	8	2	7,359.16	238.96	6,988.09	10,800.00	5.04
13	10	3	5	8	2	6,968.84	182.10	6,695.40	10,800.00	3.92
14	10	3	5	8	2	5,781.36	131.74	6,105.33	10,800.00	−5.60
15	10	3	5	8	2	4,260.97	39.89	4,251.06	10,800.00	0.23
16	10	3	5	8	3	7,267.31	520.15	6,744.75	10,800.00	7.19
17	10	3	5	8	3	5,386.77	70.65	5,214.48	10,800.00	3.20
18	10	3	5	8	3	5,629.61	123.54	5,465.00	10,800.00	2.92
19	10	3	5	8	3	7,724.88	46.86	7,493.86	10,800.00	2.99
20	10	3	5	8	3	5,725.96	150.83	5,261.29	10,800.00	8.12
21	10	5	3	8	2	5,793.68	246.07	5,425.21	10,800.00	6.36
22	10	5	3	8	2	4,343.06	147.08	3,972.29	10,800.00	8.54
23	10	5	3	8	2	4,608.93	68.08	4,414.16	10,800.00	4.23
24	10	5	3	8	2	5,139.10	293.95	4,919.61	10,800.00	4.27
25	10	5	3	8	2	4,106.02	134.02	4,064.24	10,800.00	1.02
26	10	5	3	8	3	4,976.28	37.97	4,696.54	10,800.00	5.62
27	10	5	3	8	3	4,469.94	927.21	4,333.78	10,800.00	3.05
28	10	5	3	8	3	6,794.30	444.74	5,641.26	10,800.00	16.97
29	10	5	3	8	3	4,810.93	49.62	4,514.89	10,800.00	6.15
30	10	5	3	8	3	5,683.28	308.62	4,872.25	10,800.00	14.27
31	10	5	5	8	2	8,128.01	165.82	7,090.63	10,800.00	12.76
32	10	5	5	8	2	7,483.61	3,079.27	7,669.47	10,800.00	−2.48
33	10	5	5	8	2	9,602.79	479.81	7,919.50	10,800.00	17.53
34	10	5	5	8	2	7,232.59	90.34	6,058.22	10,800.00	16.24
35	10	5	5	8	2	9,043.95	161.78	7,447.01	10,800.00	17.66
36	10	5	5	8	3	8,043.94	333.46	7,256.83	10,800.00	9.79
37	10	5	5	8	3	7,677.66	145.37	7,252.94	10,800.00	5.53
38	10	5	5	8	3	8,072.74	58.59	7,148.21	10,800.00	11.45
39	10	5	5	8	3	5,861.19	343.00	5,738.86	10,800.00	2.09
40	10	5	5	8	3	7,730.39	77.61	6,591.28	10,800.00	14.74
Mean						5,796.00	273.52	5,404.35	10,792.69	5.80

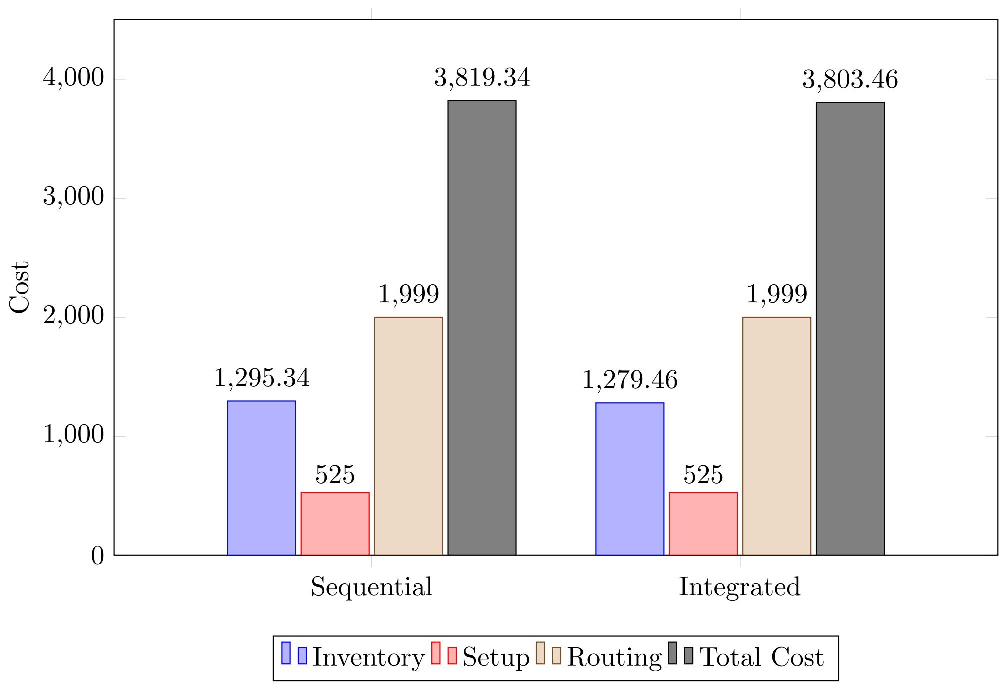

Fig. 3: Total cost distribution of the solution found by the sequential procedure and the integrated model for the 4th instance.

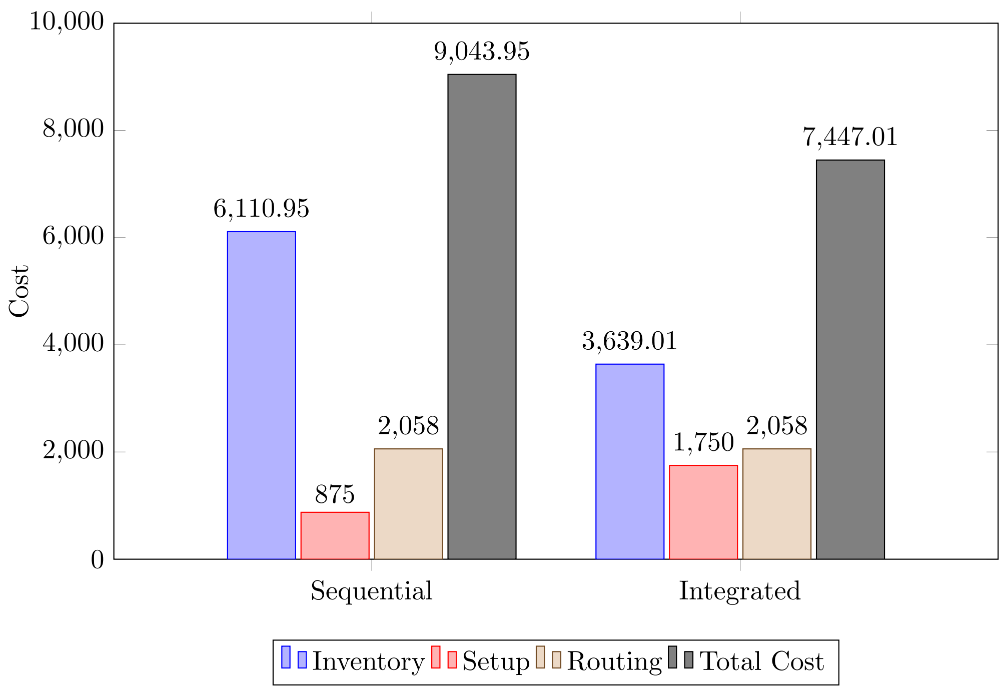

Fig. 4: Total cost distribution of the solution found by the sequential procedure and the integrated model for the 35th instance.

Tab. 8: Computational results on a set of real-based instances.

	Sequential Approach				Integrated Approach				Variation(%)		
Instance	Total Cost	Inv.+Setup	Routing	CPU(s)	Total Cost	Inv.+Setup	Routing	CPU(s)	Inv.+Setup	Routing	Total Cost
1	5,208.95	460.15	4,748.80	927.30	5,173.30	424.50	4,748.80	10,800.00	−7.75	0.00	−0.68
2	4,604.23	431.23	4,173.00	638.27	4,545.51	372.51	4,173.00	10,800.00	−13.62	0.00	−1.28
3	5,153.49	466.69	4,686.80	706.03	5,061.53	374.73	4,686.80	10,800.00	−19.71	0.00	−1.78
4	5,000.78	478.38	4,522.40	3,483.98	4,993.12	414.72	4,578.40	10,800.00	−13.31	1.24	−0.15
5	4,824.04	434.04	4,390.00	1,503.84	4,793.35	372.65	4,420.70	10,800.00	−14.15	0.70	−0.64
6	4,287.24	414.44	3,872.80	79.07	4,204.33	303.93	3,900.40	10,800.00	−26.66	0.71	−1.93
7	4,765.56	389.76	4,375.80	381.72	4,721.27	345.47	4,375.80	10,800.00	−11.36	0.00	−0.93
8	4,786.59	427.89	4,358.70	435.64	4,733.80	375.10	4,358.70	10,800.00	−12.34	0.00	−1.10
9	4,940.46	376.26	4,564.20	986.42	4,934.66	370.46	4,564.20	10,800.00	−1.54	0.00	−0.12
10	3,988.61	344.41	3,644.20	839.92	3,962.21	318.01	3,644.20	10,800.00	−7.66	0.00	−0.66
Mean	4,756.00	422.33	4,333.67	998.22	4,712.31	367.21	4,345.10	10,800.00	−12.81	0.27	−0.93

horizon.

We solved all the instances with both the integrated model LSMVRP-F2 and the sequential procedure described in Section 4.3. Table 8 presents cost details of the solutions provided by the aforementioned approaches.

In general, the integrated model provides better solutions than the sequential approach, although in no case it could prove their optimality after three hours of computing time. Similarly to the results of Section 4.3, the integrated model achieved a better compromise between all the costs involved on the problem. Indeed, it reduced the production costs (i.e., inventory plus setup cost) by 12.81%, on average, with a small increase in the routing costs (0.27%, on average). The overall cost reduction is 0.93%, on average.

The reason behind such a small cost reduction is that, for this set of real-based instances, the routing costs represent about 92% of the total cost and, therefore, substantial reductions in the inventory and setup costs have a negligible impact on the total cost. This is also the reason why the sequential procedure succeeds to find high quality solutions for these instances (it first minimizes routing costs and then the inventory and setup costs). We believe that greater reductions in the overall cost may be achieved in scenarios where both, production and routing costs, have more or less the same order of magnitude.

4.5 Alternative scenario: homogeneous fleet

We also considered an alternative scenario in which the fleet is composed by homogeneous vehicles. Our goal is to evaluate how the use of a homogeneous fleet would impact the total cost, in comparison with the original situation where the vehicles have different capacities. For our experiments, we only consider random instances 4 and 6, for which the optimal solutions are known. In order to account for a homogeneous fleet, for each instance we simply set the vehicles capacity as $\theta = \sum_{v \in \mathcal{V}} \theta_v / |\mathcal{V}|$. In this way, the overall capacity remains the same as in the original instances. To solve the instances, we run CPLEX during a maximum CPU time of one day. Table 9 compares the results of solving instances 4 and 6 with both homogeneous and heterogeneous fleet.

Although neither instance 4 nor 6 were solved optimally after one day of computing time, the lower bounds provided by CPLEX allow us to draw some conclusions. In both cases, the value of the lower bound is higher than the corresponding optimal solution when

Tab. 9: Comparison of results for instances 4 and 6 considering heterogeneous and homogeneous fleet, respectively.

	Instance 4		Instance 6	
	Heterogeneous	Homogeneous	Heterogeneous	Homogeneous
Total Cost	3,803.46	4,017.60	4,074.22	4,335.14
Inv. Cost	1,279.46	1,271.60	1,269.22	1,273.14
Setup Cost	525.00	525.00	725.00	725.00
Routing Cost	1,999.00	2,221.00	2,080.00	2,337.00
LB	3,803.46	3,912.09	4,074.22	4,076.32
Gap(%)	0.00	2.63	0.00	5.97
CPU(s)	10,507.70	86,400.00	36,744.30	86,400.00

a heterogeneous fleet is considered. It means that, for these two instances, the optimal solution with homogeneous fleet will be more expensive than the optimal solution we found with heterogeneous fleet. Indeed, taking as a reference the current lower and upper bounds provided by CPLEX, we estimate that using a homogeneous fleet will increase the total cost between 2.86%–5.63% and 0.05%–6.40% for instances 4 and 6, respectively.

We also note that the main impact on the total cost comes from the increase on the routing cost, which is 11.70%, on average, for the two instances. These results suggest that, although the overall vehicle capacity remains the same, changing to a homogeneous fleet of vehicles may lead to considerable increments on the routing cost and, in consequence, on the overall cost of the system for these problem instances. Further experimentation is required to validated these conclusions, though.

5 Concluding remarks and research directions

We proposed and analyzed two mixed integer models, LSMVRP-F1 and LSMVRP-F2, for a PIDRP considering real features from the furniture industry. The models properly represent the scenario considered and can support the planning of production and distribution activities in small furniture companies. Extensive computational experimentation on a set of random and realistic instances showed that LSMVRP-F2 performs better, as it can usually provide better solutions. Moreover, it can solve instances with up to twenty customers and five products, whose size is considered realistic for small companies.

Our computational results also pointed out that integrating production and distribution decisions can lead to significant savings in the system, when compared with a sequential procedure that mimics the common practice in the industry. The extent of the savings, however, depends on the relative weight of each component of the total cost. Exploratory experiments considering two instances with homogeneous fleet suggested that the value of the total cost of the optimal solution can be negatively affected by considerable increments on the routing costs. However, a more thorough experimentation is required in order to draw more solid conclusions on this subject.

As a research direction, it is important to develop specific procedures to solve the problem on hands. Given its complexity, an interesting perspective is to develop solution strategies that combine mathematical programming techniques and heuristic approaches in order to achieve a good trade-off between solution quality and computational time. These kind of strategies, whereby mathematical models are used in a heuristic fashion, may allow us to solve larger problem instances, which are currently beyond the capabilities of state-of-the-art commercial solvers.

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