Decision support for strategic energy planning: a complete robust optimization framework

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This paper presents a complete robust optimization framework to deal with a large range of uncertainties in optimization-based energy models. Robust formulations are proposed to address specific features of long-term energy models - such as multiplied uncertain parameters in the objective and many uncertainties in the constraints. Then, we introduce an original approach to make use of the obtained robust formulations for decision support and provide a real case study of a national energy system for validation. It is demonstrated that, by first ensuring feasibility and then evaluating cost-optimality, the proposed method offers solutions which are both reliable and cost-effective, while limiting the computational requirements.

Key words: energy; uncertainty; robust optimization; strategic planning; decision support

1. Introduction

Strategic energy planning for large-scale energy systems defines investment roadmaps for energy conversion technologies. In other words, making a strategic energy plan means deciding which resources and technologies will supply our future energy needs. Due to the lifetime of these technologies, energy plans at urban and national scale have a time horizon of 20 to 50 years.

Optimization-based energy models can support this decision-making process (Zeng et al. 2011). Most of the commonly used models are in origin *deterministic* (Powell et al. 2012), i.e. they do not consider uncertainty and rely on long-term forecasts for important parameters. As these forecasts are often inaccurate (Moret et al. 2017), various authors recommend the integration of uncertainty in the energy planning practice, making it an emerging topic in the energy modeling literature (Mirakyan and De Guio 2015, DeCarolis et al. 2017).

When dealing with optimization under uncertainty, "*a major modeling decision is whether one should rely on robust optimization (RO), or whether one should use stochastic programming (SP)*" (Grossmann et al. 2015). Historically, SP is the "traditional" approach (Babonneau et al. 2010). In a nutshell, SP

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optimizes the expected value of the objective over all the possible realizations of uncertainty (modeled as a scenario tree), under the fundamental assumption that the probability density functions (PDFs) of the uncertain parameters are known (Birge and Louveaux 2011, Gorissen et al. 2015). First proposed by Dantzig (1955), it has found multiple applications in the contexts of multistage decision-making and long-term planning (Grossmann et al. 2015). The main limitations of SP are the difficulties in defining PDFs for the uncertain parameters (Gorissen et al. 2015), and thus the scenarios, and the fact that it quickly leads to intractable model sizes (Babonneau et al. 2010). Strategic energy planning models often have a large number of uncertain parameters along with scarce quantity and quality of data to characterize their uncertainty. Thus, in this context, SP applications are often limited to a handful of uncertain inputs (Usher and Strachan 2012), and the associated PDFs seldom have a solid empirical basis (Siddiqui and Marnay 2006).

Compared to SP, the idea behind RO is to ensure protection against worst-case realizations of uncertainty. As a consequence, "*the main feature of this approach is that it is does not resort to the calculus of probability, which makes it immune against the curse of dimensionality and computational intractability*" (Babonneau et al. 2010). The roots of the method are found in the work by Soyster (1973) for linear programming (LP) models. In his robust formulation, all the uncertain parameters are considered at their worst case value. On the one hand, this ensures full protection against infeasibility, i.e. the obtained solution is feasible for every possible value of the uncertain parameters; on the other hand, it has the disadvantage of producing over-conservative solutions. This issue was first addressed by Ben-Tal and Nemirovski (1999), whose approach offers less conservative solutions, but with the drawback of formulating a non-linear robust counterpart for a LP problem. A few years later, Bertsimas and Sim (2004) also addressed this issue by proposing an alternative approach, resulting in a linear formulation of the robust counterpart for mixed-integer linear programming (MILP) problems. Their fundamental idea is that "*nature will be restricted in its behavior, in that only a subset of the coefficients will change in order to adversely affect the solution*". The authors introduce in their formulation a parameter (Γ_i) that controls the number of parameters taking their worst-case value simultaneously in the i-th constraint (containing $|J_i|$ uncertain parameters). If $\Gamma_i = 0$ the deterministic solution is obtained, whereas if $\Gamma_i = |J_i|$ then Soyster's solution is obtained. The interest is then to evaluate how the solution of the optimization problem changes when varying Γ_i between these two extreme cases.

The literature review presented in Table 1 reveals that, despite an increasing trend in the last years, the use of RO methods is still rather limited in the energy field. The approach proposed by Bertsimas and Sim (2004) appears to be the most diffused one, probably due to the linearity of their formulation. In terms of considered uncertain parameters, most applications aim at ensuring feasibility against uncertain demand, or consider market uncertainty for costs and prices in the objective function. Applications range from small scale (buildings, specific systems) to districts and industrial processes, and up to urban and national energy systems, although in the last case the focus is often on the operation of the electricity

Table 1 Review of application of RO methods to energy models. Abbreviations for methods: Ben-Tal and Nemirovski (1999) (BT&N), Bertsimas and Sim (2004) (B&S), linear decision rules (LDR), adjustable robust optimization (ARO). "Own" indicates that the paper also introduces a RO framework. Abbreviations for model types: linear programming (LP), mixed-integer linear programming (MILP), mixed-integer non linear programming (MINLP), fuzzy radial interval linear programming (FRILP).

	Method(s)	Uncertain parameters	Application & Model type
Mulvey et al. (1995)	Own (scenarios)	energy demand	power capacity expansion (LP)
Janak et al. (2007)	Own	processing time, demand, prices	chemical plant scheduling (MILP)
Babonneau et al. (2010)	BT&N, LDR, own	pollutant transfer, demand	environmental and energy planning (LP)
Ribas et al. (2010)	Scenarios (Kouvelis and Yu 1997)	oil production, demand, prices	oil supply chain strategic planning (LP)
Hajimiragha et al. (2011)	B&S	electricity prices	Plug-in Hybrid Electric Vehicles (MILP)
Koo et al. (2011)	Scenarios (Yu and Li 2000)	fuel prices, emission targets	sustainable energy planning (LP)
Jiang et al. (2012)	B&S, own	wind power production	wind & hydro unit commitment (MILP)
Parisio et al. (2012)	B&S	conversion efficiencies	energy hub management (MILP)
Zhao and Zeng (2012)	Own	wind production	wind unit commitment (MILP)
Dong et al. (2013)	B&S, own	prices, cost, efficiencies	energy management planning (FRILP)
Street et al. (2014)	ARO	generation/transmission outages	electricity market scheduling (MILP)
Akbari et al. (2014)	B&S	demand, fuel costs	building energy system (MILP)
Rager (2015)	B&S, own	cost, demand, efficiencies	urban energy system (MILP)
Grossmann et al. (2015)	LDR	reserve demand	air separation unit scheduling (MILP)
Moret et al. (2016)	B&S	fuel prices	household supply, conceptual example (MILP)
Sy et al. (2016)	Own (Ng and Sy 2014)	selling prices, demand	polygeneration system (MILP)
Nicolas (2016)	B&S	fuel prices, inv. cost; climate	national energy system; climate policy (LP)
Gong et al. (2016)	Own	feedstock price, biofuel demand	optimal biomass conversion pathways (MINLP)
Majewski et al. (2016)	Soyster (1973), own	demand, fuel prices, emissions	decentralized energy supply system (MILP)
Majewski et al. (2017)	Soyster (1973), own	energy demand, fuel prices	decentralized energy supply system (MILP)

sector. In general, applications are typically limited to specific cases or sectors - often the electricity sector - and consider only a subset of the uncertain parameters. The latter is often due to the difficulty in incorporating uncertainties in complex deterministic model formulations, initially developed without accounting for uncertainty. As an example, the presence of multiplied uncertain parameters (Kwon et al. 2013) and of uncertainty in the constraints (Hajimiragha et al. 2011, Rager et al. 2016), which are common features in existing energy models, make it difficult to develop robust formulations. Overall, the literature shows that a gap still exists between fields of mathematical programming and of energy systems. In the first one, energy applications are often simply used as illustrative numerical examples for fundamental methodological contributions, while energy researchers and professionals are interested in applying the developed methods to support the actual decision-making process.

As a contribution towards bridging this gap, we address these limiting features to provide a complete RO framework for MILP strategic energy planning models that offers the possibility of considering *all* uncertain parameters, both in the objective function and in the other constraints. First, we extend, in Section 2, the formulation of Kwon et al. (2013) for the case of multiplied uncertain parameters in Bertsimas and Sim (2004) to account for the uncertainty in the annualization factor of the investment cost in the objective function. Second, in Section 3, the methodology proposed by Babonneau et al. (2012) is integrated in the framework to systematically consider uncertainty in the constraints. Third, Section 4 introduces a decision support method to link the provided methodological framework to the energy planning practice. By first ensuring feasibility and then evaluating cost-optimality, the proposed method offers solutions which are both reliable and cost-effective, while limiting the computational requirements. In all sections, we systematically validate our developments on the real case study of the national Swiss energy system that has been presented in (Moret 2017)[1]. Given the end-use energy demand, the efficiency

and cost of energy conversion technologies, the availability and cost of energy resources, the model identifies the optimal investment and operation strategies to meet the demand and minimize the total annual cost of the energy system. It is representative model of an energy system, including electricity, heating and mobility, with a multiperiod formulation accounting for seasonality and energy storage.

2. Uncertainty in the Objective Function

In our MILP cost minimization problem, the uncertain parameters in the objective function (1) are the discount rate (i_{rate}), the technology lifetime (n), the costs of investment (c_{inv}), operation and maintenance (O&M) (c_{maint}), and resources (c_{op}). The uncertainty of these parameters is characterized in Table 2; a sensitivity analysis study in (Moret 2017) reveals that these parameters are the most impacting on the model output.

Table 2   Ranges of uncertainty (relative to the nominal values) for the uncertain parameters in the objective function as defined in (Moret et al. 2017).

Parameter	Range of variation	
	min	max
i_{rate}	-46.2%	46.2%
$c_{inv,mature}$	-21.6%	21.6%
$c_{inv,new}$	-39.6%	39.6%
c_{inv}(NUCLEAR)	-21.6%	119.3%
c_{inv}(HYDRO DAM)	-21.6%	73.8%
c_{inv}(*Thermal plants*)	-21.6%	25.0%
c_{inv}(WIND)	-21.6%	22.9%
c_{inv}(District Heating Network)	-39.3%	39.3%
c_{inv}(*Geothermal*)	-39.7%	62.1%
c_{maint}	-48.2%	35.7%
n	-26.5%	26.5%
$c_{op,local}$	-2.9%	2.9%
$c_{op,import}$	-47.3%	89.9%

The objective function minimizes the total annual cost ($\mathbf{C_{tot}}$) of the energy system, defined as the sum of the total investment cost ($\mathbf{C_{inv}}$, reported to one year with the annualization factor τ), the total O&M cost ($\mathbf{C_{maint}}$) of technologies (*TECH*), and the operating cost ($\mathbf{C_{op}}$) of resources (*RES*). The decision variables are the installed capacity of technologies ($\mathbf{F}$) and the use of resources over different periods ($\mathbf{F_t}$).

$$\mathbf{C_{tot}} = \sum_{j \in TECH} \tau(j)\mathbf{C_{inv}}(j) + \sum_{j \in TECH} \mathbf{C_{maint}}(j) + \sum_{i \in RES} \sum_{t \in T} \mathbf{C_{op}}(j,t) = \tag{1}$$

$$= \underbrace{\sum_{j \in TECH} \frac{i_{rate}(i_{rate}+1)^{n(j)}}{(i_{rate}+1)^{n(j)}-1} c_{inv}(j)\mathbf{F}(j)}_{\mathbf{Obj_1}: \text{ Novel formulation (Section 2.1)}} + \underbrace{\sum_{j \in TECH} c_{maint}(j)\mathbf{F}(j) + \sum_{i \in RES}\sum_{t \in T} c_{op}(i,t)\mathbf{F_t}(i,t)t_{op}(t)}_{\mathbf{Obj_2}: \text{ Direct application of Bertsimas and Sim (2004)}}$$

Here, t_{op} is the duration of the time periods and the uncertain parameters are highlighted in red.

As indicated in (1), the formulation by Bertsimas and Sim (2004) can be directly applied to obtain the robust counterpart of the second half of the objective function ($\mathbf{Obj_2}$), as there is only one uncertain

parameter multiplying each decision variable. However, their approach does not allow to individually consider the uncertainty of the multiplied parameters in the first half of the equation ($\mathbf{Obj_1}$). Thus, novel developments are presented, extending the formulation by Bertsimas and Sim (2004) to this special case, commonly found in energy models.

2.1. Novel robust formulation

A simple option to deal with multiplied uncertain parameters would consist in defining a new parameter as the product of independent uncertain parameters and then applying Bertsimas and Sim (2004). The worst case value for this additional parameter is thus simply equal to the multiplication of worst case values for all parameters. This option though is known to lead to excessively conservative solutions. This issue of multiplied uncertain parameters was first addressed by Kwon et al. (2013), who considered a robust shortest path problem in which the cost coefficient is the product of two uncertain factors. In particular, they derived the robust counterpart for a problem of the type min $\sum_j a_j c_j \mathbf{x}_j$, with $a_j = [a_j, a_j + \delta_{a,j}]$ and $c_j = [c_j, c_j + \delta_{c,j}]$. Their formulation extends Bertsimas and Sim (2004) as it allows to separately consider the uncertainty of the multiplied uncertain parameters a_j and c_j.

Our contribution here is twofold: we first show in Proposition 1 that $\mathbf{Obj_1}$ can be approximated by a simpler formulation; then, we present in Theorem 1 a novel robust formulation, extending Kwon et al. (2013) to obtain the robust counterpart of the studied problem.

PROPOSITION 1. *The annualization factor in the first half of the objective function can be approximated as follows*

$$\mathbf{Obj_1} = \sum_{j \in TECH} \frac{i_{rate}(i_{rate} + 1)^{n(j)}}{(i_{rate} + 1)^{n(j)} - 1} c_{inv}(j) \mathbf{F}(j) \approx \sum_{j \in TECH} \left(\alpha \frac{i_{rate}}{2} + \frac{1}{n(j)} \right) c_{inv}(j) \mathbf{F}(j), \quad (2)$$

in which α is a correction factor. This formulation is equivalent to

$$\min \sum_j (a_j + a'_j) c_j \mathbf{x}_j \quad (3)$$

in which $a_j = [a_j, a_j + \delta_{a,j}]$, $a'_j = [a'_j, a'_j + \delta_{a',j}]$ and $c_j = [c_j, c_j + \delta_{c,j}]$, $j \in J$.

Proof. The proof is reported in the Appendix.

Unfortunately (3) still cannot be treated with the robust formulation proposed by Kwon et al. (2013). Thus, building on their developments, we derive the following Theorem.

THEOREM 1. *The robust counterpart of problem (3) is given by*

$$\min \sum_j (a_j + a'_j) c_j \mathbf{x}_j + \mathbf{z_u} \Gamma_u + \mathbf{z_{u'}} \Gamma_{u'} + \mathbf{z_v} \Gamma_v + \sum_j (\mathbf{p}_j + \mathbf{p}'_j + \mathbf{q}_j) \quad (4)$$

$$s.t. \ \mathbf{z_u} - \boldsymbol{\eta}_j + \mathbf{p}_j \geq \delta_{a,j} c_j \mathbf{x}_j \qquad \forall j$$

$$\mathbf{z_{u'}} - \boldsymbol{\eta}'_j + \mathbf{p}'_j \geq \delta_{a',j} c_j \mathbf{x}_j \qquad \forall j$$

$$\mathbf{z_v} - \boldsymbol{\pi}_j - \boldsymbol{\pi}'_j + \mathbf{q}_j \geq \delta_{c,j} (a_j + a'_j) \mathbf{x}_j \qquad \forall j$$

$$\boldsymbol{\eta}_j + \boldsymbol{\pi}_j \geq \delta_{a,j}\delta_{c,j}\mathbf{x}_j \qquad\qquad \forall j$$

$$\boldsymbol{\eta}'_j + \boldsymbol{\pi}'_j \geq \delta_{a',j}\delta_{c,j}\mathbf{x}_j \qquad\qquad \forall j$$

$$\mathbf{x}_j, \mathbf{p}_j, \mathbf{p}'_j, \mathbf{q}_j, \boldsymbol{\pi}_j, \boldsymbol{\pi}'_j, \boldsymbol{\eta}_j, \boldsymbol{\eta}'_j, \mathbf{z_u}, \mathbf{z_{u'}}, \mathbf{z_v} \in \mathbb{R}^+$$

in which Γ are protection parameters and $\mathbf{p}$, $\mathbf{p}'$, $\mathbf{q}$, $\boldsymbol{\pi}$, $\boldsymbol{\pi}'$, $\boldsymbol{\eta}$, $\boldsymbol{\eta}'$, $\mathbf{z_u}$, $\mathbf{z_{u'}}$ and $\mathbf{z_v}$ are additional variables.

Proof. Let's consider the robust optimization problem in (5), where the control parameters Γ_u, $\Gamma_{u'}$ and Γ_v, associated to the uncertain parameters a, a' and c, respectively, are positive integers.

$$\min_{\mathbf{x}} \max_{\mathbf{u},\mathbf{u}',\mathbf{v}} \sum_j \left(a_j + \delta_{a,j}\mathbf{u}_j + a'_j + \delta_{a',j}\mathbf{u}'_j\right)\left(c_j + \delta_{c,j}\mathbf{v}_j\right)\mathbf{x}_j \qquad\qquad (5)$$

$$\mathbf{u}_j, \mathbf{u}'_j, \mathbf{v}_j \in [0,1] \qquad\qquad \forall j$$

$$\sum_j \mathbf{u}_j \leq \Gamma_u, \quad \sum_j \mathbf{u}'_j \leq \Gamma_{u'}, \quad \sum_j \mathbf{v}_j \leq \Gamma_v$$

Assuming $\mathbf{w}_j = \mathbf{u}_j\mathbf{v}_j, \forall j$ and $\mathbf{w}'_j = \mathbf{u}'_j\mathbf{v}_j, \forall j$, the maximization problem in $\mathbf{u}$, $\mathbf{u}$' and $\mathbf{v}$ can be rewritten as in (6), in which the dual variables are indicated in parentheses.

$$\max_{\mathbf{u},\mathbf{u}',\mathbf{v},\mathbf{w},\mathbf{w}'} \sum_j \left(\delta_{a,j}c_j\mathbf{u}_j + \delta_{a',j}c_j\mathbf{u}'_j + a_j\delta_{c,j}\mathbf{v}_j + \delta_{a,j}\delta_{c,j}\mathbf{w}_j + a'_j\delta_{c,j}\mathbf{v}_j + \delta_{a',j}\delta_{c,j}\mathbf{w}'_j\right)\mathbf{x}_j \qquad (6)$$

$$\text{s.t. } \mathbf{u}_j - 1 \leq 0 \qquad (\mathbf{p}_j) \qquad\qquad \forall j$$

$$\mathbf{u}'_j - 1 \leq 0 \qquad (\mathbf{p}'_j) \qquad\qquad \forall j$$

$$\mathbf{v}_j - 1 \leq 0 \qquad (\mathbf{q}_j) \qquad\qquad \forall j$$

$$-\mathbf{u}_j + \mathbf{w}_j \leq 0 \qquad (\boldsymbol{\eta}_j) \qquad\qquad \forall j$$

$$-\mathbf{v}_j + \mathbf{w}_j \leq 0 \qquad (\boldsymbol{\pi}_j) \qquad\qquad \forall j$$

$$-\mathbf{u}'_j + \mathbf{w}'_j \leq 0 \qquad (\boldsymbol{\eta}'_j) \qquad\qquad \forall j$$

$$-\mathbf{v}_j + \mathbf{w}'_j \leq 0 \qquad (\boldsymbol{\pi}'_j) \qquad\qquad \forall j$$

$$\sum_j \mathbf{u}_j - \Gamma_u \leq 0 \qquad (\mathbf{z_u})$$

$$\sum_j \mathbf{u}'_j - \Gamma_{u'} \leq 0 \qquad (\mathbf{z_{u'}})$$

$$\sum_j \mathbf{v}_j - \Gamma_v \leq 0 \qquad (\mathbf{z_v})$$

$$\mathbf{u}, \mathbf{u}', \mathbf{v}, \mathbf{w}, \mathbf{w}' \in \mathbb{R}^+$$

Problem (6) can be written in matrix notation as in (7), in which $I_{|J|}$ is the $|J| \times |J|$ identity matrix, $1_{|J|}$ is a vector of size $|J| \times 1$ with all elements equal to 1, $0_{|J|}$ is a vector of size $|J| \times 1$ with all elements

equal to 0, $\underline{\mathbf{u}} = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_{|J|}]$ (the same applies to $\underline{\mathbf{u}}$', $\underline{\mathbf{v}}$, $\underline{\mathbf{w}}$, $\underline{\mathbf{w}}$'), and empty fields are all zeros.

$$
\begin{array}{l}
\text{(Row Group 1)} \\
\text{(Row Group 2)} \\
\text{(Row Group 3)} \\
\text{(Row Group 4)} \\
\text{(Row Group 5)} \\
\text{(Row Group 6)} \\
\text{(Row Group 7)} \\
\text{(Row 8)} \\
\text{(Row 9)} \\
\text{(Row 10)}
\end{array}
\begin{bmatrix}
I_{|J|} & & & & \\
& I_{|J|} & & & \\
& & I_{|J|} & & \\
-I_{|J|} & & & I_{|J|} & \\
& & -I_{|J|} & I_{|J|} & \\
& -I_{|J|} & & & I_{|J|} \\
& & -I_{|J|} & & I_{|J|} \\
1^T_{|J|} & & & & \\
& 1^T_{|J|} & & & \\
& & 1^T_{|J|} & &
\end{bmatrix}
\begin{bmatrix}
\underline{\mathbf{u}} \\
\underline{\mathbf{u}}' \\
\underline{\mathbf{v}} \\
\underline{\mathbf{w}} \\
\underline{\mathbf{w}}'
\end{bmatrix}
\leq
\begin{bmatrix}
1_{|J|} \\
1_{|J|} \\
1_{|J|} \\
0_{|J|} \\
0_{|J|} \\
0_{|J|} \\
0_{|J|} \\
\Gamma_u \\
\Gamma_{u'} \\
\Gamma_v
\end{bmatrix}
\tag{7}
$$

The constraint matrix $\mathcal{A}$ is totally unimodular, i.e. each subdeterminant of $\mathcal{A}$ is 0, $+1$, or -1. In fact, total unimodularity is preserved under the following operations: *i*) taking the transpose; *ii*) multiplying a row or column by -1; *iii*) adding an all-zero row or column, or adding a row or column with one non-zero, being ± 1 (Schrijver 1998). Based on *iii*), row groups 1 to 3 can be eliminated from the analysis. Focusing on rows 4 to 10, Ghouila-Houri (1962) shows that a necessary and sufficient condition for total unimodularity is that each collection of columns (or rows, based on *i*)) of $\mathcal{A}$ can be split into two parts so that the sum of the columns (rows) in one part minus the sum of the columns (rows) in the other part is a vector with entries only 0, $+1$, and -1. Equivalently, for any collection of rows, if each row is multiplied by $+1$ or -1 (based on *ii*)), then the sum of rows must have only $-1, 0, +1$ in each column (Kwon et al. 2013). For any collection of rows 4-10 of $\mathcal{A}$, this is obtained by multiplying rows 5, 6 and 9 by -1, and by appropriately multiplying row 10 by ± 1. This proves the total unimodularity of $\mathcal{A}$.

Given an optimization problem of the type $\min \sum_j c_j \mathbf{x}_j$ s.t. $\sum_j a_{ij} \mathbf{x}_j \leq b_i \ \forall i$, if the constraint matrix $\mathcal{A}$ is totally unimodular and the coefficients b_j are all integers, a solution $\mathbf{x}_j$ of the problem is integral. Hence, in problem (6) the decision variables $\mathbf{u}_j, \mathbf{u}'_j, \mathbf{v}_j, \mathbf{w}_j, \mathbf{w}'_j$ are all binaries at optimality. As a consequence, at optimality, $\mathbf{w}_j = \mathbf{u}_j \mathbf{v}_j, \forall j$ and $\mathbf{w}'_j = \mathbf{u}'_j \mathbf{v}_j, \forall j$. This proves the previously assumed equalities, and thus the equivalence between formulation (6) and the inner problem of (5).

The dual problem of formulation (6) is expressed by (8).

$$
\min \ \mathbf{z_u}\Gamma_u + \mathbf{z_{u'}}\Gamma_{u'} + \mathbf{z_v}\Gamma_v + \sum_j \left(\mathbf{p}_j + \mathbf{p}'_j + \mathbf{q}_j \right) \tag{8}
$$

$$
\text{s.t.} \ \mathbf{z_u} - \boldsymbol{\eta}_j + \mathbf{p}_j \geq \delta_{a,j} c_j \mathbf{x}_j \qquad \forall j
$$

$$
\mathbf{z_{u'}} - \boldsymbol{\eta}'_j + \mathbf{p}'_j \geq \delta_{a',j} c_j \mathbf{x}_j \qquad \forall j
$$

$$
\mathbf{z_v} - \boldsymbol{\pi}_j - \boldsymbol{\pi}'_j + \mathbf{q}_j \geq \delta_{c,j}(a_j + a'_j)\mathbf{x}_j \qquad \forall j
$$

$$
\boldsymbol{\eta}_j + \boldsymbol{\pi}_j \geq \delta_{a,j}\delta_{c,j}\mathbf{x}_j \qquad \forall j
$$

$$
\boldsymbol{\eta}'_j + \boldsymbol{\pi}'_j \geq \delta_{a',j}\delta_{c,j}\mathbf{x}_j \qquad \forall j
$$

$$
\mathbf{x}_j, \mathbf{p}_j, \mathbf{p}'_j, \mathbf{q}_j, \boldsymbol{\pi}_j, \boldsymbol{\pi}'_j, \boldsymbol{\eta}_j, \boldsymbol{\eta}'_j, \mathbf{z_u}, \mathbf{z_{u'}}, \mathbf{z_v} \in \mathbb{R}^+
$$

Hence, using strong duality, the robust counterpart of problem (3) is expressed by (4). $\quad\square$

2.1.1. Complete formulation for the objective function. The obtained robust formulation for the first half of the objective function ($\mathbf{Obj_1}$) is then combined with the application of Bertsimas and Sim (2004) to $\mathbf{Obj_2}$. This gives the complete formulation of the robust counterpart of (1).

$$
\min \sum_{j \in TECH} \big(\tau_r + \tau_n(j)\big) c_{inv}(j)\mathbf{F}(j) + \sum_{j \in TECH} c_{maint}(j)\mathbf{F}(j) + \sum_{i \in RES}\sum_{t \in T} c_{op}(i,t)\mathbf{F_t}(i,t)t_{op}(t) \tag{9}
$$

$$
+ \mathbf{z_r}\Gamma_r + \mathbf{z_n}\Gamma_n + \mathbf{z_{inv}}\Gamma_{inv} + \mathbf{z_0}\Gamma_0 + \sum_{j \in TECH}\big(\mathbf{p_r}(j) + \mathbf{p_n}(j) + \mathbf{p_{inv}}(j) + \mathbf{p_{maint}}(j)\big) + \sum_{i \in RES}\sum_{t \in T}\mathbf{p_{op}}(j)(j,t)
$$

$$
\text{s.t.} \quad \mathbf{z_r} + \sum_{j \in TECH}\big(-\boldsymbol{\eta_r}(j) + \mathbf{p_r}(j)\big) \geq \delta_{\tau_r}\sum_{j \in TECH}\big(c_{inv}(j)\mathbf{F}(j)\big)
$$

$$
\mathbf{z_n} - \boldsymbol{\eta_n}(j) + \mathbf{p_n}(j) \geq \delta_{\tau_n}(j)c_{inv}(j)\mathbf{F}(j) \qquad\qquad \forall j \in TECH
$$

$$
\mathbf{z_{inv}} - \boldsymbol{\pi_{inv}}(j) - \boldsymbol{\pi'_{inv}}(j) + \mathbf{p_{inv}}(j) \geq \delta_{inv}(j)\big(\tau_r + \tau_n(j)\big)\mathbf{F}(j) \qquad\qquad \forall j \in TECH
$$

$$
\boldsymbol{\eta_r}(j) + \boldsymbol{\pi_{inv}}(j) \geq \delta_{\tau_r}\delta_{inv}(j)\mathbf{F}(j) \qquad\qquad \forall j \in TECH
$$

$$
\boldsymbol{\eta_n}(j) + \boldsymbol{\pi'_{inv}}(j) \geq \delta_{\tau_n}(j)\delta_{inv}(j)\mathbf{F}(j) \qquad\qquad \forall j \in TECH
$$

$$
\mathbf{z_0} + \mathbf{p_{maint}}(j) \geq \delta_{maint}(j)\mathbf{y_{maint}}(j) \qquad\qquad \forall j \in TECH
$$

$$
\mathbf{F}(j) \leq \mathbf{y_{maint}}(j) \qquad\qquad \forall j \in TECH
$$

$$
\mathbf{z_0} + \sum_{t \in T}\mathbf{p_{op}}(i,t) \geq \delta_{op}(i)\sum_{t \in T}\mathbf{y_{op}}(i,t) \qquad\qquad \forall i \in RES
$$

$$
\mathbf{F_t}(i,t)t_{op}(t) \leq \mathbf{y_{op}}(i,t) \qquad\qquad \forall i \in RES, \forall t \in T
$$

$$
\mathbf{z_r}, \mathbf{z_n}, \mathbf{z_{inv}}, \mathbf{z_0}, \mathbf{p_r}, \mathbf{p_n}, \mathbf{p_{inv}}, \mathbf{p_{maint}}, \mathbf{p_{op}}, \boldsymbol{\eta_r}, \boldsymbol{\eta_n}, \boldsymbol{\pi_{inv}}, \boldsymbol{\pi'_{inv}}, \mathbf{y_{maint}}, \mathbf{y_{op}} \in \mathbb{R}^+
$$

In which $\tau_r = \alpha i_{rate}/2$, $\tau_n(j) = 1/n(j), \forall j \in TECH$, and δ is the difference between the upper bound and the nominal value of the uncertain parameters. In particular, $\delta_{\tau_r} = \frac{1}{2}\big(i_{rate}(\alpha' - \alpha) + \alpha'\delta_{i_{rate}}\big)$, with $\delta_{i_{rate}}$ being the maximum worst case deviation from the nominal value of the interest rate, α being the correction factor for the nominal value of the interest rate, and α' being the correction factor for its worst case value; $\delta_{\tau_n}(j) = \frac{\delta_n(j)}{n(j)(n(j)-\delta_n(j))}$, with $\delta_n(j)$ being the maximum worst case deviation from the nominal value of the technologies' lifetime.

2.2. Application to the case study

In this section we apply formulation (9) to our MILP energy model with the uncertainty ranges reported in Table 2.

In total, the MILP has 417 uncertain parameters, out of which 160 are in the objective function. These 160 uncertain inputs are controlled by four different protection parameters: $\Gamma_r \in [0,1]$ for i_{rate}, $\Gamma_n \in [0,51]$ for n, $\Gamma_{inv} \in [0,52]$ for c_{inv}, $\Gamma_0 \in [0,56]$ for c_{maint} and c_{op}. We define $\mathbf{z_{obj}} = \mathbf{z}_k, \forall k \in K = \{r, n, inv, 0\}$, thus replacing $\mathbf{z_{obj}}\Gamma_{obj} = \mathbf{z_r}\Gamma_r + \mathbf{z_n}\Gamma_n + \mathbf{z_{inv}}\Gamma_{inv} + \mathbf{z_0}\Gamma_0$ in formulation (9). This means that we directly use Γ_{obj} as the control parameter of the objective function. For the studied problem this is equivalent from a practical standpoint to having four different control parameters.

Figures 1 and 2 illustrate how the optimal configuration of the energy system changes when gradually increasing (at integral steps) the protection parameter Γ_{obj} from its lower bound ($\Gamma_{obj} = 0$, deterministic solution) to its upper bound ($\Gamma_{obj} = |J_{obj}|$, "Soyster's" solution).

The energy strategy changes dramatically when uncertainty is accounted for. In terms of energy resources (Figure 1), the deterministic solution is dominated by natural gas (NG). For medium levels of Γ_{obj}, there is a "risk pooling" effect, i.e. the dependency on only one resource is replaced by a mix of alternatives (coal, renewables and electricity imports) to offer more protection against worst case. For high levels of Γ_{obj}, the solution stabilizes towards a mix of NG (used in more efficient technologies) and renewables. This tendency, i.e. a diversification for medium uncertainty budgets and a stabilization on a restricted set of alternatives for very low and very high protection levels, is rather common in RO. In fact, Bertsimas and Sim (2004) and Nicolas (2016) observe similar results in their numerical experiments.

An analysis of the output shows that the first parameter which is "activated" (at $\Gamma_{obj} = 1$) is i_{rate}, followed by fixed costs (hydro dams, grids, efficiency) and by the operating cost of resources; investment-related parameters (c_{inv}, c_{maint}, n) are activated for higher values of Γ_{obj}. This trade-off between operating and investment cost uncertainties pushes the solution in opposite directions, as showed by the performance indicators in Figure 2. The highest reduction of fossil fuel consumption (-42%) in the system and the lowest global warming potential (GWP) emissions (-23%) compared to the deterministic case are obtained at medium uncertainty budgets ($\Gamma_{obj} = 31$ and $\Gamma_{obj} = 62$, respectively), while at higher protection levels there is again an increase of both indicators. However, the cost of resources emerges as the most impacting parameter on the energy strategy, and thus robust solutions are in general less dependent on volatile imported resources. As an example, the fully robust solution ($\Gamma_{obj} = |J_{obj}|$), features lower emissions (-20%), a lower consumption of fossils (-23%) and more installed capacity (+8%) compared to the deterministic case.

With increasing protection levels, there is a tendency to shift towards renewables and more efficient technologies, i.e. towards higher investment-to-operating cost ratios. Efficiency thus acts as an *uncertainty damper* as, by optimizing the conversion of resources, it reduces the exposure of the energy system to the volatility of fuel prices. A similar trend is observed for electricity production and heat supply.

Overall, the results show the interest of the solutions obtained at medium protection levels, and thus of the approach by Bertsimas and Sim (2004), which allows to identify them. In fact, various technologies and system configurations appear only at medium uncertainty budgets. In the standard RO approach by Soyster (1973), assuming all parameters at worst case, these solutions do not emerge.

2.3. Evaluation of the robust solutions

When RO aims at ensuring feasibility against constraint violations (e.g. inability to meet the demand), robust solutions are normally compared based on the value of the objective, i.e. by measuring the difference in the cost of the robust solutions compared to the nominal case. In fact, by increasing protection against
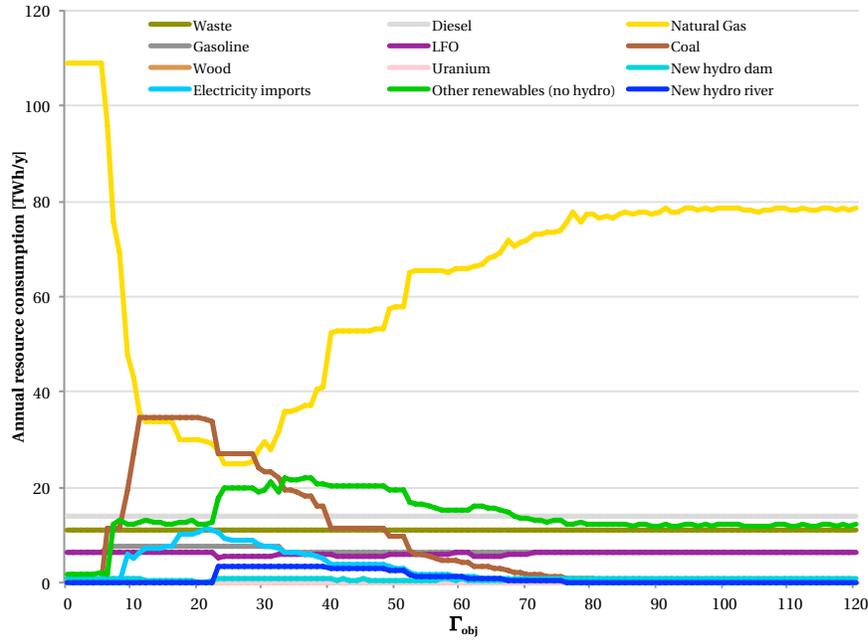
Figure 1   Evolution of the energy system configuration vs. protection level Γ_{obj}: annual consumption of resources.
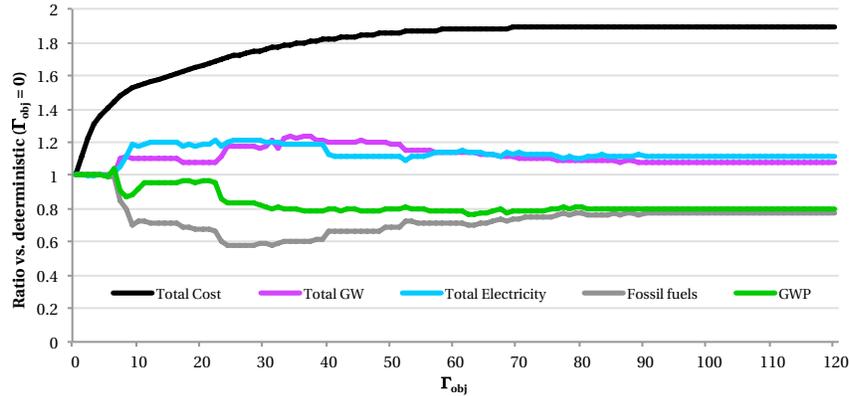

Figure 2   Performance indicators (cost, GWP emissions, installed capacity, electricity in the system, consumption of fossil fuels.) vs. protection level Γ_{obj}: ratio vs. deterministic solution ($\Gamma_{obj} = 0$).

worst case, constraint violations are reduced at the price of a higher objective value. Bertsimas and Sim (2004) define this trade-off as the *price of robustness (PoR)*. However, when uncertainty is considered for the cost coefficients in the objective, the use of this metric is discouraged. In fact, as discussed by Gorissen et al. (2015), each value of Γ_{obj} corresponds to a different "scenario" for the uncertain parameters. Thus, the objectives at different values of Γ_{obj} are not directly comparable. In this case, they recommend comparing solutions via *simulation* studies.

To do this, 15 representative solutions are selected using the *k-medoids* algorithm (Kaufman and Rousseeuw 1987), which clusters the obtained energy system configurations based on the total annual output of technologies and resources. These representative energy system configurations are characterized in Table 3. A simulation study is carried out to evaluate the performance of these configurations. In the

simulation, all the decision variables of the MILP problem are fixed to the values obtained in output of the RO runs. This means that both the investment and the operation strategy of the system are fixed for all the simulations. The MILP model is run $n_{sample} = 10000$ times sampling the values of the uncertain parameters (i_{rate}, c_{inv}, c_{maint}, c_{op}, n) from uniform distributions over the entire uncertainty range. For all the simulations, the fixed costs which are constant for all the solutions and which have a high impact on the objective - such as the investment cost of existing hydroelectric power plants, of the electricity grid and of energy efficiency measures - are set to zero. The simulation results are displayed in Figure 3. For all the representative solutions in Table 3, the mean ($\overline{x}$), standard deviation (σ) and the maximum value over the different runs are compared to the same statistics for the deterministic case ($\Gamma_{obj} = 0$).

Table 3   Characterization of the representative energy system configurations selected via *k-medoids* clustering. %$_{Dhn}$ is the share of heat demand supplied by district heating. Renewables include photovoltaics, wind, new hydro dam and run-of-river plants, deep geothermal, solar thermal. [†]integrated gasification combined cycle (IGCC).

				Electricity production							Other			
Γ_{obj}	CCGT	Coal	PV	Wind	New dams	New river	Imports	Boilers	CHP	Elec HPs	Solar Th.	Deep Geo	%$_{Dhn}$	Renewables
	[GW$_e$]	[GW$_e$]	[GW$_e$]	[GW$_e$]	[GW$_e$]	[GW$_e$]	[TWh$_e$/y]	[GW$_{th}$]	[GW$_{th}$]	[GW$_{th}$]	[GW$_{th}$]	[GW$_{th}$]	[-]	[TWh/y]
0	3	0	0	0	0.428	0	0	22.26	0.02	0.75	0	0.25	0.100	2.76
6	3	0	0	0	0.440	0	0	22.29	0.08	0.73	0	0.25	0.100	2.79
7	2	0	0	5.10	0.440	0	0	19.49	0.04	3.57	0	0.28	0.100	13.2
8	2.5	0	0	5.30	0.440	0	0.01	15.34	0.1	7.57	0	0.35	0.100	13.9
9	1.5	0.5	0	5.30	0.440	0	6.20	11.29	0.08	11.73	0	0.25	0.100	13.5
14	0.5	1.5	0	5.30	0.352	0	7.51	11.27	0.06	11.75	0	0.28	0.100	13.4
19	0	1.5	0	5.25	0.208	0	10.29	11.27	0.08	11.63	0	0.39	0.100	13.6
27	0	1	2.30	5.30	0.440	0.850	9.09	13.18	0.12	10.54	0	1.17	0.221	24.5
30	0	1[†]	1.32	5.30	0.440	0.850	7.78	12.46	0.38	10.10	1.63	1.01	0.189	24.2
37	1	0.5[†]	2.91	5.30	0.440	0.850	6.12	12.02	0.08	9.69	3.64	0.78	0.149	26.4
44	1.5	0	1.85	5.30	0.367	0.726	3.89	15.03	0.06	7.70	2.68	0.81	0.195	24.2
56	1.5	0	0.85	5.30	0.349	0.334	1.79	13.40	0.86	8.71	1.59	0.37	0.142	18.0
68	1	0	0.36	5.30	0.427	0.157	0.75	12.78	1.56	9.22	0.67	0.32	0.148	15.6
91	1	0	0.02	5.30	0.440	0.006	0.03	14.12	1.56	7.67	0	0.21	0.100	13.2
200	1	0	0	5.30	0.440	0	0	13.53	1.64	8.18	0	0.18	0.100	12.9
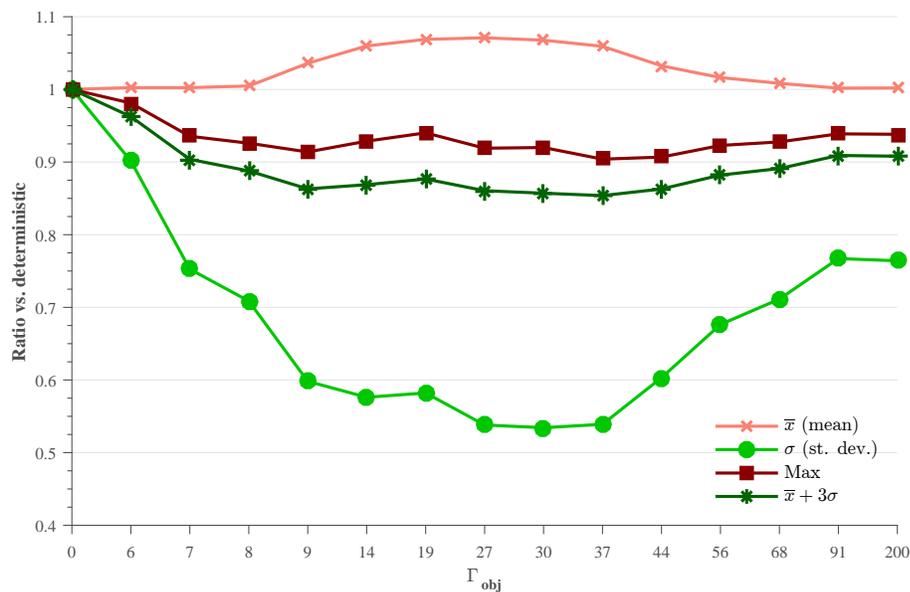
Figure 3   Simulation results for the robust solutions in Table 3: mean ($\overline{x}$), standard deviation (σ), maximum value and $\overline{x} + 3\sigma$ for the objective value. In the simulations, all decision variables fixed, and uncertain parameters (i_{rate}, c_{inv}, c_{maint}, c_{op}, n) are sampled from entire uncertainty range.

On the one hand, robust solutions have a higher average total cost than the nominal case ($\overline{x}_0 = 7786$ MCHF/y). On the other hand, the standard deviation of robust solutions is significantly lower, reaching almost half of the deterministic case standard deviation ($\sigma_0 = 1694$ MCHF/y) for $\Gamma_{obj} \in \{27; 30; 37\}$. Looking at the worst-case performance, the maximum objective value for all robust solutions is lower than in the deterministic case; also, evaluating the likelihood of being 3σ higher than the mean, it emerges that solutions obtained at medium uncertainty budgets (with a higher penetration of renewables) offer more stability and protection against unfavorable realizations of uncertainty. Similarly to the results presented in the previous section, the indicators in Figure 3 denote a sharp deviation away from the deterministic solution for low and medium protection levels, and then a convergence back in the direction of the deterministic solution for higher values of Γ_{obj}. Interestingly, a similar trend for the standard deviation is observed in the portfolio example application in Bertsimas and Sim (2004).

Overall, the simulations further confirm the interest of the presented RO approach, i.e. of generating solutions at low and medium protection levels.

3. Uncertainty in the other constraints

Robust optimization works *constraint-wise*, i.e. the robust formulations are applied individually to the different constraints of an optimization model. In energy system cost minimization problems, most impacting uncertain parameters are often found in a "special" constraint, i.e. the objective function. This justifies the focus on uncertainty in the objective in most of the reviewed literature. However, as revealed by a sensitivity analysis study on our MILP example, various impacting parameters - such as the availability of resources ($avail$), the capacity factors ($c_{p,t}$), the demand ($endUses_{year}$) and the conversion efficiencies (η) - are also found in the other constraints.

In the reviewed literature, uncertainty in the constraints is seldom addressed. When it is addressed, uncertainty is normally considered only for an *a priori* selected subset of parameters, most often the demand (to ensure that demand is met for any realization of the uncertain parameters). This is mainly due to two reasons: *i*) the fact that the same uncertain parameters appear in multiple constraints; *ii*) the fact that most constraints contain very few - and often only one - uncertain parameters.

The first issue, highlighted by Hajimiragha et al. (2011) ("*given the limitation of the RO techniques [...], uncertain parameters [...] which simultaneously appear in multiple constraints and the objective function cannot be handled by this methodology*"), can often be addressed by improving the model structure, aiming at obtaining concise and compact formulations.

The second issue, discussed by Rager et al. (2016), can be exemplified by a model with N constraints, each of them with only one uncertain parameter. Hence, if applying Bertsimas and Sim (2004), $\Gamma_i \in [0,1], \forall i = 1, \ldots, N$. In this case, on the one hand the problem becomes trivial for the i-th constraint; on the other hand, the problem becomes combinatorial when considering all the N constraints, as there are N binary protection parameters Γ_i. Although this second issue can also be addressed (at least partly) by

working on the model formulation, it constitutes nonetheless a rather typical and intrinsic characteristic of energy system models.

Babonneau et al. (2012) propose a method to deal with this problem. The idea behind their approach is to ensure global protection for a set of constraints: instead of protecting each constraint individually, a new "global" redundant constraint is added to the model formulation by summing over the constraint indices. The robust formulation is then applied to this new constraint. This leads to having only one protection parameter $\Gamma \in [0, N]$, instead of N protection parameters $\Gamma_i \in [0, 1], \forall i = 1, \ldots, N$, which has the advantage of replacing an intractable combinatorial problem with an easily manageable uncertainty set.

3.1. Application to the case study

The method proposed by Babonneau et al. (2012) is applied to the MILP case study for the following parameters: *avail*, $c_{p,t}$, $endUses_{year}$ and η. The application of the method is not automatic as, although mathematically possible, in a real situation not all elements of a set can share the same uncertainty budget. Thus, the implementation is separately discussed for the uncertain parameters of interest, which offer different representative examples. As an example, we show the application to the most representative constraint, i.e. those having efficiency and energy demand as uncertain parameters.

3.1.1. Efficiencies and energy demand. The conversion efficiency of technologies (η) and the yearly energy demand ($endUses_{year}$) are accounted for in the layer[2] balance equation (10). The equation expresses the balance for each layer: all outputs from resources and technologies (including storage) are used to satisfy the demand or as inputs to other resources and technologies. As an example, the electricity which is imported or produced in the system is used to satisfy the electricity demand; additionally, it can be stored in hydroelectric dams or used as input to other energy conversion technologies (such as electric vehicles). In total, 52 uncertain instances of η and 15 uncertain instances of $endUses_{year}$ are considered.

$$\underbrace{\sum_{i \in RES \cup TECH \setminus STO} f(i,l)\mathbf{F_t}(i,t) + \sum_{j \in STO}(\mathbf{Sto_{out}}(j,l,t) - \mathbf{Sto_{in}}(j,l,t)) - \mathbf{EndUses}(l,t)}_{\mathbf{LB}} = 0 \qquad \forall l \in L, \forall t \in T \quad (10)$$

A first problem is that these parameters are present in different constraints: η is linked to the parameter f, while $endUses_{year}$ is linked to the decision variables **EndUses** (see Moret (2017)). However, as these are all equality constraints, (10) can be rewritten in an extended form as in (11). The equation takes as an example the electricity layer, which is representative as it includes resources, technologies (producers and consumers), demand and losses.

$$\sum_{i \in RES \cup TECH \setminus STO} f(i,l)\mathbf{F_t}(i,t) + \sum_{j \in STO}(\mathbf{Sto_{out}}(j,l,t) - \mathbf{Sto_{in}}(j,l,t)) - \frac{\sum_{s \in S} endUses_{year}(l,s)}{\sum_{t \in T} t_{op}(t)}$$

$$- \frac{\%_{lighting}(t)}{t_{op}(t)} \sum_{s \in S} endUses_{year}(Lighting, s) - \Big(\sum_{i \in RES \cup TECH \setminus STO | f(i,l) > 0} f(i,l)\mathbf{F_t}(i,t) \Big) \%_{loss}(l) = 0 \qquad l = Elec., \forall t \in T \quad (11)$$

In the equation, $\%_{lighting}$ is the yearly share (adding up to 1) of lighting end-uses; the decision variables $\mathbf{Sto_{in}}$ and $\mathbf{Sto_{out}}$ are inputs to and outputs from storage units, respectively; $\%_{loss}$ are the losses in the electricity grid.

The possibility of including all the uncertain parameters in a single set of constraints simplifies - and, in some cases, makes possible - the development of the robust counterpart. To obtain the robust counterpart, the first step is the relaxation of the equality constraint (10) to an inequality constraint, i.e. $\mathbf{LB} \geq 0$. This does not pose problems as it simply implies allowing an excess of production in the energy system. As a matter of fact, transforming equalities into inequalities is quite common in RO, *"since often [equality] constraints restrict the feasibility region drastically or even lead to infeasibility"* (Gorissen et al. 2015), as in this case. Second, the application of the method by Babonneau et al. (2012) is considered by summing over the set L.

Summing over the set of layers is possible only if the summed layers are "compatible", e.g. if an additional demand in one layer can be compensated by an additional consumption in the other one. This is clearly not always possible, e.g. an additional transportation demand cannot be satisfied by the import of more uranium. In particular, resource layers need to remain separate. As a consequence, for resource layers the method by Babonneau et al. (2012) cannot be meaningfully adopted, and thus the classical robust formulation by Bertsimas and Sim (2004) applies, as in (12).

$$\mathbf{LB} - \mathbf{z_{lb}}(l,t)\Gamma_{lb} - \sum_{i \in TECH \backslash STO} \mathbf{p_f}(i,l,t) \geq 0 \qquad\qquad \forall l \in L \cap RES, \forall t \in T \quad (12)$$

$$\mathbf{z_{lb}}(l,t) + \mathbf{p_f}(i,l,t) \geq \delta_f(i,l)\mathbf{y_{lb}}(i,t)$$

$$\forall i \in TECH \backslash STO, \forall l \in L \cap RES, \forall t \in T | f(i,l) < 0 \vee f(i,l) > 0 \wedge l \neq Elec.$$

$$\mathbf{z_{lb}}(l,t) + \mathbf{p_f}(i,l,t) \geq \delta_f(i,l)\mathbf{y_{lb}}(i,t)(1 - \%_{loss}(l))$$

$$\forall i \in TECH \backslash STO, \forall l \in L \cap RES, \forall t \in T | f(i,l) > 0 \wedge l = Elec.$$

$$\mathbf{F_t}(i,t) \leq \mathbf{y_{lb}}(i,t) \qquad\qquad \forall i \in TECH \backslash STO, \forall t \in T$$

$$\mathbf{z_{lb}}, \mathbf{p_f}, \mathbf{y_{lb}} \in \mathbb{R}^+$$

In (12) Γ_{lb} is the protection parameter, $\mathbf{z_{lb}}$ and $\mathbf{p_f}$ are the dual variables, δ_f is the maximum worst-case deviation from the nominal value of f.

Some types of end-use demand, instead, can be compatible. As an example, an additional mobility demand can be supplied by either public or private transportation technologies, or an additional low temperature heat demand can be satisfied by centralized or decentralized technologies. Thus, the method by Babonneau et al. (2012) can be applied to these layers with (13). The formulation, by an appropriate set definition, also avoids possible multiplications among variables which would introduce nonlinearities.

$$\sum_{l \in L | l \in EUT\ OF\ EUC(euc)} \mathbf{LB} - \mathbf{z_{lb}}(l,t)\Gamma_{lb} - \sum_{i \in TECH \backslash STO} \mathbf{p_f}(i,euc,t) - \sum_{eui \in EUI\ OF\ EUC(euc)} \sum_{s \in S} \mathbf{p_{dem}}(eui,s,t) \geq 0 \qquad \forall euc \in EUC, \forall t \in T \quad (13)$$

$$\mathbf{z_{lb}}(euc, t) + \mathbf{p_{dem}}(eui, s, t) \geq \delta_{dem}(eui, s) \frac{\%_{lighting}(t)}{t_{op}(t)}$$

$$\forall euc \in EUC, \forall eui \in EUI\ OF\ EUC(euc), \forall s \in S, \forall t \in T | eui = Lighting$$

$$\mathbf{z_{lb}}(euc, t) + \mathbf{p_{dem}}(eui, s, t) \geq \delta_{dem}(eui, s) \frac{\%_{sh}(t)}{t_{op}(t)}$$

$$\forall euc \in EUC, \forall eui \in EUI\ OF\ EUC(euc), \forall s \in S, \forall t \in T | eui = Heat\ LowT\ SH$$

$$\mathbf{z_{lb}}(euc, t) + \mathbf{p_{dem}}(eui, s, t) \geq \frac{\delta_{dem}(eui, s)}{\sum_{t \in T} t_{op}(t)}$$

$$\forall euc \in EUC, \forall eui \in EUI\ OF\ EUC(euc), \forall s \in S, \forall t \in T | eui \notin \{Lighting, Heat\ LowT\ SH\}$$

$$\mathbf{z_{lb}}, \mathbf{p_{dem}} \in \mathbb{R}^+$$

The sets of equations (12)-(13) define the robust counterpart of the layer balance constraint (10), in which the protection parameter Γ_{lb} controls the uncertainty levels of f and $endUses_{year}$. By increasing Γ_{lb} of a unitary step, for each period an additional uncertain parameter takes its worst case realization, allowing to account for demand and efficiency uncertainties in the planning of the energy system. It has to be noted that, as the uncertain parameters are not indexed over T, the parameter which changes is not necessarily the same for all t.

The application of the method by Babonneau et al. (2012) to (10) shows as well the advantages of concise model formulations for uncertainty studies. In fact, if the deterministic model is already rather complex, which is often the case in the energy field, obtaining the robust counterparts can be difficult - and lead to extremely intricate formulations - or, in some cases, practically impossible, as highlighted by Hajimiragha et al. (2011). Constraint (10) serves as representative example of this. In this case, although the constraint offers various challenges, such as the presence of different uncertain parameters, some of them linked to other constraints, its robust counterpart is obtained with the addition of only two sets of additional constraints. This is made possible by the fact of using a compact MILP formulation, together with an appropriate definition of sets.

Overall, the application to the case study reveals the interest of the method by Babonneau et al. (2012) for the RO of strategic energy planning models, offering tractable uncertainty sets in the place of an otherwise intractable combinatorial problem. However, it also shows that the use of the method is not automatic, but it requires thorough case-by-case evaluation to make sure of its consistency.

3.2. Evaluation of the robust solutions

As previously discussed, uncertainty in the constraints is linked to feasibility: by increasing protection against worst case, constraint violations are reduced at the price of a higher objective value. For example, a robust energy system will normally be more expensive, but more likely to satisfy the demand. This trade-off is measured by the PoR, which is defined as the difference between the objective (cost) of a given robust solution and the objective of the deterministic problem. Measuring the PoR "*is useful if the objective is certain, since in that case PoR is the amount that has to be paid for being robust against*

Table 4 Ranges of uncertainty (relative to the nominal values) for the uncertain parameters in the constraints: conversion efficiency of technologies (η) and yearly energy demand ($endUses_{year}$) in the different sectors (Moret et al. 2017).

Parameter	Range of variation	
	min	max
$endUses_{year}$(HOUSEHOLDS)	-6.9%	4.3%
$endUses_{year}$(SERVICES)	-7.4%	4.1%
$endUses_{year}$(INDUSTRY)	-10.5%	5.9%
$endUses_{year}$(TRANSPORT)	-3.4%	3.4%
$\eta_{mature,standard}$	-5.7%	5.7%
$\eta_{mature,customized}$	-20.6%	20.6%
$\eta_{new,standard}$	-20.8%	20.8%
$\eta_{new,customized}$	-28.7%	28.7%

Table 5 Simulation results for uncertainty in the constraints: PoR against statistics of constraint violations ξ ($\overline{x}$: mean, σ: standard deviation). First test: all decision variables fixed. Second test: only investment decisions are fixed (free resources). Uncertain parameters are the efficiencies (f) and the end-use demand ($endUses_{year}$), sampled from entire range. $\mathbf{C_{tot}}$ includes fixed costs for existing hydroelectric dams, existing electricity grid and efficiency measures. Renewables include photovoltaics, wind, new hydro dam and run-of-river plants, deep geothermal, solar thermal. [†][Mpkm/h] for passenger, [Mtkm/h] for freight mobility end-uses

Γ_{lb}	$\mathbf{C_{tot}}$ [MCHF/y]	PoR [MCHF/y]	Renewables [TWh/y]	Test 1			Test 2		
				$\%_{\mathbf{viol}}$ [%]	$\overline{x}_\xi$ [GW]†	σ_ξ [GW]†	$\%_{\mathbf{viol}}$ [%]	$\overline{x}_\xi$ [GW]†	σ_ξ [GW]†
0	13834	0	2.96	99.9%	0.254	0.140	53.3%	0.093	0.078
1	14491	657	7.06	24.9%	0.064	0.061	1.05%	0.041	0.034
2	14633	799	1.42	8.50%	0.067	0.062	0.05%	0.011	0.009
3	14712	878	1.89	2.73%	0.063	0.056	0%	-	-
4	14769	935	10.97	0.81%	0.048	0.045	0%	-	-
5	14812	978	13.59	0.31%	0.042	0.033	0%	-	-
6	14849	1015	12.90	0.04%	0.019	0.009	0%	-	-
7	14873	1039	16.24	0%	-	-	0%	-	-
8	14888	1054	17.49	0.01%	0.009	0.000	0%	-	-
20	14904	1070	18.00	0%	-	-	0%	-	-

constraint violations" (Gorissen et al. 2015). Thus, the effect of robustness in the constraints is evaluated via simulation while keeping the deterministic objective function, i.e. $\Gamma_{obj} = 0$. As an example, the focus of the analysis is on the layer balance constraint (Γ_{lb}), using the uncertainty ranges reported in Table 4 (from (Moret et al. 2017)).

In this case, 10 different energy system configurations are obtained by gradually increasing the value of Γ_{lb} from the deterministic case ($\Gamma_{lb} = 0$) to the fully robust solution ($\Gamma_{lb} = |J_{lb}|$). Then, two sets of simulations are carried out. In the first one, all the decision variables of the MILP problem are fixed to the values obtained in output of the RO runs. This means that both the investment and the operation strategy of the system are fixed in all the simulation runs. The MILP model is run $n_{sample} = 10000$ times sampling the values of the uncertain parameters (f, $endUses_{year}$) from uniform distributions over the entire uncertainty range. In the second set of simulations, to better represent the reality of energy systems, only investment-related decision variables (such as the installed capacity of technologies, $\mathbf{F}$) are fixed, while operating decision variables (such as $\mathbf{F_t}$) are left free, i.e. determined by the optimization.

The results of both tests are reported in Table 5. To measure the magnitude of the constraint violations, a positive variable ξ is added to the robust constraints and also to the objective function with a high

penalty cost. As expected, by increasing the protection level Γ_{lb}, constraint violations are sharply reduced, both in terms of frequency, and in terms of mean ($\overline{x}$) and standard deviation (σ). The first test shows that constraint violations start to become negligible at low values of the protection parameter ($\Gamma_{lb} \geq 4$). Thus, to obtain good protection levels it is not needed to be fully robust, which further confirms the interest of the approach by Bertsimas and Sim (2004). The second test, in which the operation strategy of the system is left free to adapt to the realizations of the uncertainty, reveals that in a real situation the deterministic solution has still a high risk of infeasibility, and that this risk disappears as soon as uncertainty is taken into account. Also, similarly to what observed in the case of uncertainty in the objective, the increase of Γ_{lb} is associated to a higher penetration of renewables in the system.

Overall, the tests reveal that uncertainty in the constraints, which is seldom considered in the literature, can have a significant impact on the definition of the energy strategy. Also, the proposed robust formulation can identify appropriate trade-offs between the suboptimality of robust solutions and the risk of constraint violations. In fact, the results suggest that in real energy system applications, low levels of protection against uncertainty - and, thus, low additional costs - are sufficient to highly reduce the risk of infeasibilities (e.g. inability to satisfy the demand).

4. Decision support method

The formulations and results presented in the previous sections offer a complete framework to apply RO to strategic energy planning models, as well as an insight on how the energy strategy changes when uncertainty is separately accounted for in the objective function and in the other constraints. However, these mathematical methods need to be linked to the energy planning practice. As an example, Grossmann et al. (2015), reviewing the use of the method by Bertsimas and Sim (2004) in the field of process system design, comment that "*it is not trivial*" for a user to adequately specify the uncertainty budget (Γ). In fact, if probabilistic results - as the ones presented in Bertsimas and Sim (2004) - can help identifying appropriate bounds for the constraints, in the case of the objective the translation of RO results into energy planning decisions is not obvious.

Thus, the purpose of this section is to evaluate how the presented methods can support the decision-making practice in the energy field. To do this, a decision support method is proposed to address uncertainty both in the objective function and in the constraints.

4.1. First feasibility, then optimality

Simulation is the most appropriate way of comparing different investment strategies to support decision-making (Gorissen et al. 2015). In fact, if robust solutions are obtained considering worst-case realizations of uncertainty, simulation studies can challenge these solutions by using different uncertainty sets, which can better represent real-life conditions.

In the previous sections, uncertainty in the objective function and in the constraints has been separately considered. In fact, although equivalent in terms of mathematical formulation, the two are fundamentally
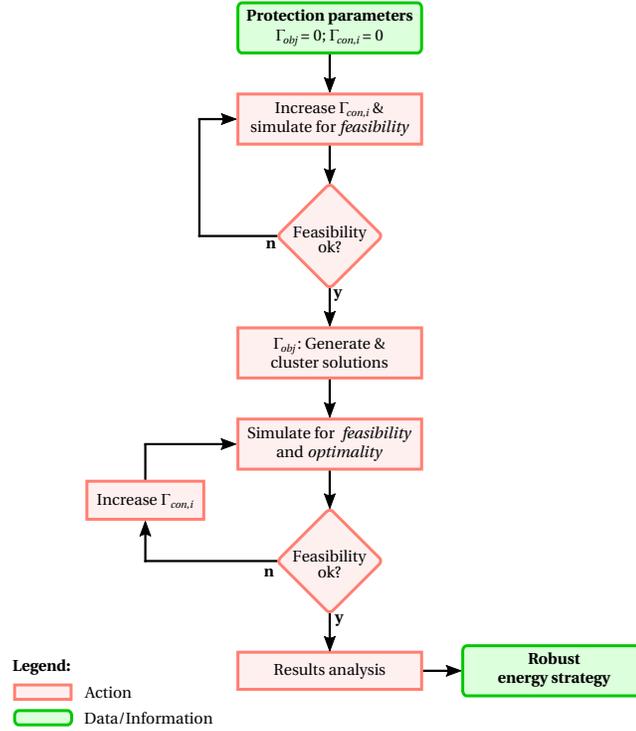
Figure 4   Flowchart of the proposed decision support method: uncertainty is first considered in the constraints to ensure feasibility, and then in the objective to evaluate optimality.

different. On the one hand, uncertainty in the constraints is linked to *feasibility*, i.e. the goal is obtaining "*a solution that will be feasible for any realization taken by the unknown coefficients*"; thus, its evaluation (using PoR) is meaningful when the objective is certain. On the other hand, uncertainty in the objective is linked to *optimality*, i.e. the goal is obtaining "*a solution that performs well for any realization taken by the unknown coefficients*" (Gabrel et al. 2014). However, energy planners are interested in solutions that are both feasible *and* cost-effective, which requires to simultaneously consider uncertainty in the objective function *and* in the constraints. Thus, in this section, a method is proposed to consider both types of uncertainty, with the final aim of aiding decision-making.

The fundamental idea behind the proposed method can be summarized as: *first feasibility, then optimality*. In other words, feasibility (e.g. ability to satisfy the demand) is a *condicio sine qua non*, i.e. it makes sense to evaluate optimality only for solutions which satisfy the desired feasibility criteria. This has also the advantage of limiting the number of solutions to simulate: in fact, if in the case of the objective it is relevant to evaluate solutions obtained at all protection levels (corresponding to different cost scenarios), in the case of the constraints there is no interest in considering solutions at higher protection levels than the one ensuring the desired degree of feasibility.

The steps of the method are illustrated by the flowchart in Figure 4 considering a problem with uncertainty in the objective function (with protection parameter Γ_{obj}) and in N constraints (with protection parameters $\Gamma_{con,i}, \forall i = 1, \ldots, N$). Starting with the deterministic solution ($\Gamma_{obj} = 0, \Gamma_{con,i} = 0$), the protection level in the constraints is gradually increased until the desired level of feasibility is reached (keeping

$\Gamma_{obj} = 0$). Feasibility is evaluated through simulations in which *all* the uncertain parameters in the constraints are made to vary. Once feasibility is ensured, the $\Gamma_{con,i}$ are kept fixed and different solutions are generated for uncertainty in the objective by varying Γ_{obj}. Out of the generated solutions, a representative set - selected, for example, using *k-medoids* clustering - is simulated accounting for uncertainty of *all* model parameters. Of course, as feasibility is initially assessed only in the case of $\Gamma_{obj} = 0$, the new solutions generated by considering uncertainty in the objective might introduce feasibility issues. In this case, $\Gamma_{con,i}$ can be further increased. Finally, statistical analysis on the simulation results leads to the selection of the final robust energy strategy.

The method is applied to the MILP energy system case study. Based on the developments presented in the previous sections, the model has one protection parameter for the objective (Γ_{obj}, accounting for the uncertainty of the cost-related parameters i_{rate}, c_{inv}, c_{maint}, c_{op}, n) and two protection parameters for the other constraints: $\Gamma_{c_{p,t}}$ (accounting for the uncertainty of $c_{p,t}$) and Γ_{lb} (accounting for the uncertainty of f and $endUses_{year}$). First, uncertainty in the constraints is evaluated. On the one hand, numerical tests reveal that $\Gamma_{c_{p,t}}$ does not significantly impact feasibility. On the other hand, the results in Table 5 reveal that Γ_{lb} has a strong influence: for a real energy system situation (test 2 in Table 5), feasibility is sharply improved already for $\Gamma_{lb} = 1$.

Thus, Γ_{lb} is fixed to 1, and uncertainty in the objective is evaluated. For consistency with the results presented in Figure 3 (in which $\Gamma_{lb} = 0$), the same values of Γ_{obj} are used to select the 15 representative solutions. The representative energy system configurations, listed in the upper part of Table 6, are simulated $n_{sample} = 10000$ times, sampling the values of the uncertain parameters (all uncertain parameters) from uniform distributions over the entire uncertainty range. To represent the reality of energy systems, only investment-related decision variables (such as the installed capacity of technologies, **F**) are fixed, while operating decision variables are reoptimized.

The results are shown in Figure 5, in which the different robust solutions are compared to the deterministic case ($\Gamma_{obj} = 0$, $\Gamma_{lb} = 0$). In terms of optimality, simulation results are in line with what observed in Figure 3: robust solutions are on average more expensive, but feature a lower variability and offer a better protection against worst-case. The main difference is in terms of feasibility: considering uncertainty in the constraints by fixing $\Gamma_{lb} = 1$ causes a dramatic reduction in constraint violations compared to the deterministic solution.

Interestingly, introducing uncertainty in the objective slightly increases the frequency of constraint violations of some solutions, reaching 2.21% for $\Gamma_{obj} = 37$. If this is above the feasibility threshold desired by the decision-maker, the protection level in the constraints can be further increased. As an example, by fixing $\Gamma_{lb} = 2$ (lower part of Table 6 and dashed lines in Figure 5), infeasibility is reduced to negligible values, while maintaining similar performance in terms of cost.

Thus, by considering uncertainty in both the objective and constraints, the proposed method can offer solutions which are both feasible and cost-effective. Also, by first ensuring feasibility in the constraints

Table 6   Characterization of the representative energy system configurations for $\Gamma_{lb} = 1$ and $\Gamma_{lb} = 2$. $\%_{\mathbf{Dhn}}$ is the share of heat demand supplied by district heating. Renewables include photovoltaics, wind, new hydro dam and run-of-river plants, deep geothermal, solar thermal. †At least one power plant is integrated gasification combined cycle (IGCC).

		Electricity production							Other						
Γ_{obj}	Γ_{lb}	CCGT [GW$_e$]	Coal [GW$_e$]	PV [GW$_e$]	Wind [GW$_e$]	New dams [GW$_e$]	New river [GW$_e$]	Imports [TWh$_e$/y]	Boilers [GW$_{th}$]	CHP [GW$_{th}$]	Elec HPs [GW$_{th}$]	Solar Th. [GW$_{th}$]	Deep Geo [GW$_{th}$]	$\%_{Dhn}$ [-]	Renewables [TWh/y]
0	1	2	0	0	0	0.440	0	0	21.91	1.50	2.19	0	0.83	0.314	7.1
6	1	3	0	0	0	0.402	0	0	22.09	0.28	0.86	0	0.32	0.120	3.2
7	1	2	0	0	5.30	0.439	0	0	13.82	0.84	8.66	0	0.23	0.120	13.3
8	1	2	0	0	5.30	0.424	0	0.77	12.39	0.92	10.05	0	0.18	0.120	12.9
9	1	1.5	0.5	0	5.30	0.329	0	8.05	11.09	0.20	11.93	0	0.32	0.120	13.7
14	1	0.5	1	0	5.30	0.172	0	11.81	11.08	0.20	11.94	0	0.32	0.120	13.4
19	1	0	1.5†	0	5.30	0.129	0	11.25	12.01	0.26	11.46	0	0.67	0.181	15.5
27	1	0	1	3.01	5.26	0.440	0.850	9.72	13.21	0.20	10.60	0	1.38	0.252	26.4
30	1	0	1†	2.96	5.30	0.440	0.850	9.28	13.04	0.20	10.23	1.16	1.31	0.241	27.2
37	1	1	0.5†	1.89	5.30	0.440	0.850	5.46	11.23	0.58	9.89	3.72	0.83	0.151	25.7
44	1	1.5	0	1.92	5.30	0.440	0.754	4.03	12.92	0.50	9.47	3.52	0.83	0.230	25.8
56	1	1	0	0.83	5.30	0.413	0.327	1.74	13.17	1.62	9.93	1.54	0.74	0.293	20.7
68	1	1	0	0.47	5.30	0.440	0.202	1.00	12.75	1.88	10.29	0.89	0.64	0.277	18.6
91	1	1	0	0.02	5.30	0.425	0.007	0.04	10.39	2.22	10.91	0.03	0.23	0.121	13.3
200	1	1	0	0	5.30	0.424	0	0	10.33	2.26	11.06	0	0.25	0.132	13.4
0	2	1	0	0	0	0.440	0	0.01	21.73	2.40	2.27	0	0.07	0.322	1.42
6	2	1.5	0	0	0	0.380	0	0	21.06	1.82	0.89	0	0	0.129	0.78
7	2	2	0	0	5.30	0.440	0	0.12	11.88	1.18	10.65	0	0.25	0.158	13.4
8	2	2	0	0	5.30	0.440	0	0	11.79	1.26	10.93	0	0.28	0.178	13.6
9	2	1	0	0	5.30	0.000	0	15.90	12.63	0.20	11.53	0	0.64	0.231	15.4
14	2	0.5	1†	0	5.30	0.053	0	12.54	14.07	0.20	11.12	0	0.85	0.318	17.1
19	2	0	1.5†	0	5.30	0.275	0	12.21	14.08	0.18	10.78	0.00	1.20	0.318	19.3
27	2	0	1†	3.27	5.30	0.440	0.850	9.48	13.02	0.22	10.40	0.77	1.38	0.254	27.5
30	2	0	1†	3.20	5.30	0.440	0.850	9.44	13.00	0.22	10.38	0.77	1.40	0.253	27.5
37	2	1	0.5†	2.08	5.30	0.440	0.850	5.47	11.53	0.62	9.78	3.60	1.04	0.187	27.0
44	2	1	0	1.90	5.30	0.429	0.748	4.01	12.76	0.98	9.22	3.54	0.83	0.243	25.9
56	2	1	0	0.84	5.30	0.337	0.366	1.77	13.28	1.72	9.72	1.54	0.76	0.301	20.9
68	2	1	0	0.44	5.30	0.349	0.194	0.93	13.17	2.10	10.19	0.83	0.60	0.322	17.9
91	2	1	0	0.02	5.30	0.269	0.008	0.04	10.60	2.52	10.79	0.03	0.32	0.167	13.7
200	2	1	0	0	5.30	0.278	0	0	10.57	2.52	10.80	0	0.32	0.165	13.6

and then evaluating cost-optimality, it limits the computational requirements and respects the conceptual difference between the two cases. Interestingly, the results show that relatively low protection levels in the constraints are sufficient to ensure the reliability of the system, at the price of a marginal loss in terms of average optimality. This additionally confirms the interest of a probabilistic approach to RO, as interesting energy system configurations both in terms of feasibility and cost-effectiveness - and with a high penetration of renewables and efficient technologies - are obtained at medium protection levels.

5. Conclusions and future work

This paper proposes a complete framework to incorporate *all* uncertainties in strategic energy planning optimization models using the robust formulation by Bertsimas and Sim (2004). First, uncertainty in the objective function and in the other constraints is separately considered. On the one hand, for uncertainty in the objective, a novel robust formulation is proposed and demonstrated, extending Bertsimas and Sim (2004) to consider the the case of multiplied uncertain parameters. On the other hand, the method proposed by Babonneau et al. (2012) is adopted to systematically consider uncertainty in the other constraints; this additionally shows the advantages of using concise formulations when aiming at incorporating uncertainty in optimization models. Then, the robust formulations for the constraints (related to feasibility) and for the objective (related to the optimality) are put together to propose a decision support method for energy systems, with the goal of identifying solutions which are both reliable and cost-effective, while limiting the computational requirements. The presented formulations and methods are validated on the real case study of a national energy system model. The results reveal that the deterministic energy strategy, heavily relying on fossil fuels, changes dramatically when uncertainty
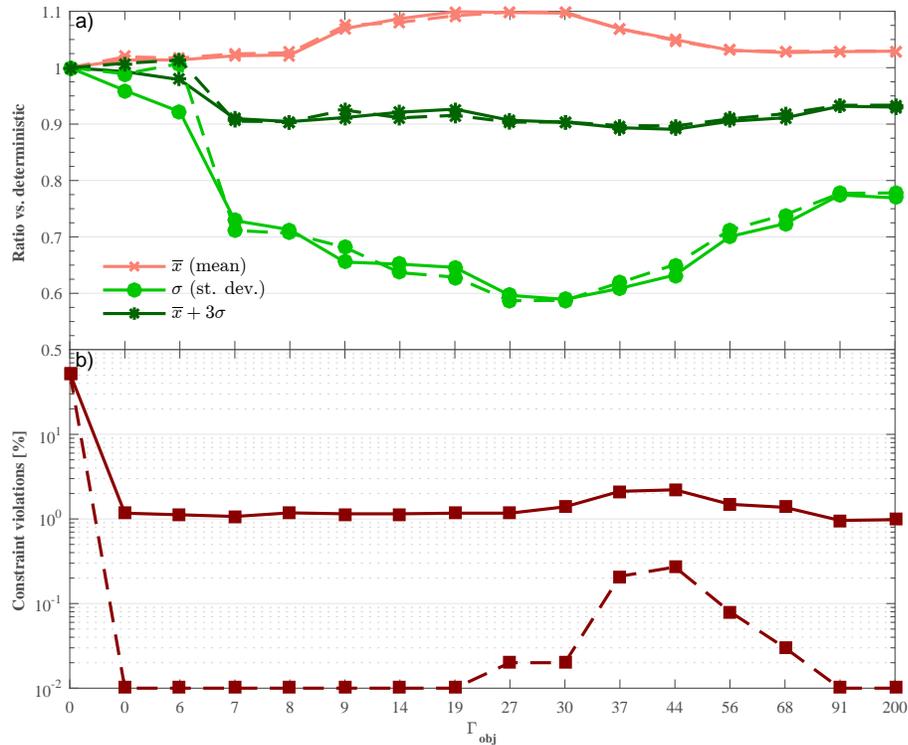
Figure 5 Simulation results for the robust solutions with uncertainty in the objective and in the constraints: a) mean ($\overline{x}$), standard deviation (σ), $\overline{x} + 3\sigma$ and for the objective value and b) frequency of constraint violations. Reference (on the y axis) is the deterministic solution ($\Gamma_{obj} = 0$, $\Gamma_{lb} = 0$). Full lines: $\Gamma_{lb} = 1$. Dashed lines: $\Gamma_{lb} = 2$. Solutions are simulated on all uncertain parameters (sampled from entire uncertainty range) and only investment decisions are fixed (free resources).

is accounted for. In general, robust solutions are characterized by a significantly higher penetration of renewables and efficient technologies. Simulation studies reveal that robust investment strategies are on average marginally more expensive than the deterministic solution, but offer more reliability (e.g. low risk of not meeting demand) and stability over time, due to a reduced dependency on highly volatile fossil fuel prices. The simulations additionally highlight the interest of generating robust solutions at intermediate protection levels, which confirms the validity of probabilistic approaches to RO, such as the one by Bertsimas and Sim (2004).

Future works are envisioned for investigating the application of the latest RO developments to the MILP formulation used in this work. Although the inclusion of recourse in RO is to date an open challenge, recent developments - such as linear decision rules (LDR) (Kuhn et al. 2011) and adjustable robust optimization (ARO) (Ben-Tal et al. 2009) - make it possible to include recourse in robust formulations. Second the framework could be also extended to distributionally robust optimization (Wiesemann et al. 2014), a paradigm where the uncertain problem data are governed by a probability distribution that is itself subject to uncertainty.

Appendix. Proof of Proposition 1.

PROPOSITION 1. *The annualization factor in the first half of the objective function can be approximated as follows*

$$\mathbf{Obj_1} = \sum_{j \in TECH} \frac{i_{rate}(i_{rate}+1)^{n(j)}}{(i_{rate}+1)^{n(j)}-1} c_{inv}(j)\mathbf{F}(j) \approx \sum_{j \in TECH} \left(\alpha \frac{i_{rate}}{2} + \frac{1}{n(j)} \right) c_{inv}(j)\mathbf{F}(j), \tag{14}$$

Proof. Given a total investment C, the *constant annuity* $A_t = A, \quad \forall t = 1, \ldots, n$, discounting the investment over the n-year lifetime, is calculated by (15).

$$C = \sum_{t=1}^{n} \frac{A_t}{(1+i_{rate})^t} = A \sum_{t=1}^{n} \frac{1}{(1+i_{rate})^t} = A \frac{(i_{rate}+1)^n - 1}{i_{rate}(i_{rate}+1)^n} = \frac{A}{\tau} \tag{15}$$

The annuity A is the sum of two components ($A = M_t + I_t$): M_t, related to the amortization of the capital; I_t, related to the payment of the interests. In the case of a constant annuity, as in (15), M_t increases over the time horizon, while I_t decreases. Under the assumption of *constant amortization*, the capital cost is equally spread over the n-year time horizon, hence $M_t = C/n, \forall t$. In this case, the capital left at the beginning of each period is $C - (t-1)M_t$ (16). Assuming that the payments are made at the end of each period, the amount of due interests I_t at the end of each period t is calculated based on the left capital (I_t decreases over time). In this way, the total amount of interests paid over the whole time horizon I_{tot} is calculated using (17).

$$I_t = \left(C - (t-1)M_t \right)i_{rate} \qquad \forall t \tag{16}$$

$$I_{tot} = Ci_{rate} + (C - M_t)i_{rate} + (C - 2M_t)i_{rate} + \cdots + \left(C - (n-1)M_t \right)i_{rate} \tag{17}$$

$$= Ci_{rate} + \frac{n-1}{n}Ci_{rate} + \frac{n-2}{n}Ci_{rate} + \cdots + \frac{1}{n}Ci_{rate}$$

$$= \frac{1}{n}Ci_{rate}\sum_{t=1}^{n} t = \frac{1}{n}Ci_{rate}\frac{n(n+1)}{2} = \frac{n+1}{2}Ci_{rate}$$

Since A is constant, then I_{tot} is also equally distributed over the n years, as in (18),

$$A = M_t + I_t = \frac{C}{n} + \frac{I_{tot}}{n} = \frac{C}{n} + \frac{n+1}{2n}Ci_{rate} \tag{18}$$

which, for high values of n, gives:

$$\tau = \frac{A}{C} = \frac{M_t + I_t}{C} \approx \frac{i_{rate}}{2} + \frac{1}{n}. \tag{19}$$

Following this simplification, (19) offers an approximation of the annualization factor τ as the linear sum of its two components. To reduce the margin of error in the approximation, a correction factor α is proposed in (20). Given N different lifetimes n, $\Delta_\tau(n)$ $(n = 1, \ldots, N)$ is the difference between the annualization factor calculated with (20) and with (15). Factor α is determined by imposing $\sum_n \Delta_\tau(n) = 0$ such that,

$$\tau = \tau_r + \tau_n = \alpha \frac{i_{rate}}{2} + \frac{1}{n} \tag{20}$$

$$\sum_{n=1}^{N} \Delta_{\tau,n} = \sum_{n=1}^{N} \Big(\tau(n)[\text{Eq. (20)}] - \tau(n)[\text{Eq. (15)}] \Big) = \sum_{n=1}^{N} \left[\alpha \frac{i_{rate}}{2} + \frac{1}{n} - \frac{i_{rate}(i_{rate}+1)^n}{(i_{rate}+1)^n - 1} \right] = 0 \qquad (21)$$

$$\implies \alpha = \frac{1}{N} \sum_{n=1}^{N} \left[\frac{i_{rate}(i_{rate}+1)^n}{(i_{rate}+1)^n - 1} - \frac{1}{n} \right] \frac{2}{i_{rate}}$$

As an example, for the nominal values of $i_{rate} = 3.215\%$ and considering lifetimes $n \in [10, 80]$ (typical values for energy technologies), $\alpha = 1.25$. The approximation in (20) is a better approximation of the exact calculation of τ; thus, it is used to derive the robust formulation of the studied MILP problem. $\quad\square$

Endnotes

1. The code is publicly available at `https://github.com/stefanomoret/SES_MILP`

2. *Layers* are defined as all the elements in the system that need to be balanced in each period, such as resources and end-uses demand.

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