

An Integrated Car-and-ride Sharing System for Mobilizing Heterogeneous Travelers with Application in Underserved Communities

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The fast-growing carsharing and ride-hailing businesses are generating economic benefits and societal impacts in the modern society. However, both have limitation to cover demand from diverse populations, e.g., travelers in low-income, underserved communities. In this paper, we consider two types of travelers: Type 1 who rent shared cars and Type 2 who need shared rides. We propose an integrated car-and-ride sharing (CRS) system to enable self-sustaining, community-based shared transportation. We design a two-phase approach, where in Phase I, we match Type 1 drivers with Type 2 users to maximize demand coverage, and in Phase II we solve a stochastic mixed-integer program to optimize routes and schedules for matched Type 1 and Type 2 users under random travel time. The goal is to minimize the total travel cost plus expected penalty cost of users' waiting and system overtime, and we develop an efficient decomposition algorithm for improving the computational efficiency. We demonstrate the performance of a CRS system in Washtenaw County, Michigan by testing instances generated based on census data and diverse user demand distribution patterns. Our results show high demand fulfillment rates and effective matching and scheduling with low risk of waiting and overtime. The integrated system also achieves better performance of cost and service quality if we allow vehicle relocation.

Key words: Carsharing, ride-hailing, spacial-temporal network, stochastic vehicle routing, two-stage stochastic integer programming

History:

1. Introduction

Transportation is a scarce resource in metropolitan Detroit, Michigan: 40% of residents do not own cars, and another 40% do not have access to vehicles (Firth 2016). According to the Bureau of Labor Statistics by the U.S. Department of Labor, Detroit has the highest unemployment rate as 23.1% among the 50 largest cities in the country. Despite that surrounding suburban areas house more than five times of employment opportunities as the city of Detroit, only 9% of the jobs are taken by residents in Detroit. According to Data Driven Detroit (2015), in communities comprised of minority, low-income, elderly, and disabled, referred to as “underserved communities” in this paper, the unemployment rate gets worse. In Detroit areas, more than 10,000 residents face tremendous pressures in commuting to their jobs in suburban communities that do not offer public transit. Even worse, some suburban municipalities decide to reduce and eliminate much of its public bus service due to financial and safety concerns (McLaughlin 2015). The lack of transportation means becomes one major factor of social inequality and health-related disparities (see, e.g., Neutens 2015). It creates barriers to health-enhancing resources such as healthy food (Walker et al. 2010) and health care (Syed et al. 2013). Yang et al. (2006) characterize the importance of transportation for urban children receiving proper health care and Giang et al. (2008) show that poor transportation access to grocery stores can lead to worse health and worse economic well-being in underserved communities.

In recent years, shared-mobility has shown its flexibility and strength in providing convenience to personal travel and reducing traffic congestion on public roads by reducing car ownership (Shaheen et al. 2015, Martin 2016). With increasing concerns about climate change, congestion, and fossil fuel dependency, shared-mobility attracts more attention and is undergoing a fast rise in popularity and industrial growth (Chan and Shaheen 2012).

Two main mobility-sharing forms are carsharing and ride-hailing. The former provides users with access to car rental service on an hourly basis (Millard-Ball 2005). The latter, a service initially aiming to group travelers with common itineraries (Chan and Shaheen 2012), has evolved rapidly with the development of smartphone technologies. Although both carsharing and ride-hailing provide useful alternatives to owning personal cars, they focus on different user groups. Carsharing users usually rent cars for running errands with multiple short stops. To use carsharing service, they are required to meet driving eligibility requirements. Ride-hailing service targets on users with short, and often one-time ride needs. It especially benefits people who are not able to drive, unfamiliar with the city transit system, and/or commuting with poor transit access.

Residents in underserved communities are experiencing financial, technical, skill-based, informational and social barriers to the use of shared-mobility service (Ge et al. 2016). For both service providers and users, the adoption of the existing shared-mobility forms in underserved communities posts significant challenges. From service providers' perspective, income and crime rate have influence on the willingness of drivers to serve particular areas, and therefore residents there find fewer drivers and higher service price (Thebault-Spieker et al. 2015). Meanwhile, due to the lack of personal-owned vehicles, residents in low-income communities are disadvantaged in finding work of becoming a shared-mobility driver (Duggan and Smith 2013, Gross et al. 2012). They also post concerns for the shared-mobility concept, including safety and trust that need to be addressed differently from the current implementation of shared-mobility platforms. For example, residents in low-income communities are hesitant to pay bills using smartphones or share information about their home locations, and they prefer the service provided by people "who you know" (Dillahunt and Malone 2015).

1.1. Problem Description

Given a variety of concerns we listed above, in this paper we propose a car-and-ride sharing (CRS) system, in which carsharing and ride-hailing services are integrated and co-provided to boost the mobility of heterogeneous travelers based on their characteristics and special needs. We aim to design a financially and operationally self-sustained system, such that users with ride-hailing demands are served by drivers with carsharing demands. We aim to build an affordable, reliable, and incentive system that foster the connections within and in between communities.

For validating the design of CRS, we consider a service region that mainly consists of underserved populations, such as jobless, elderly, and disabled, to whom transportation is a scarce resource. We partition the service region into zones and shared cars are located *a priori* in some designated parking spaces in each zone. We classify two types of users: Type 1 who want shared cars for private use but also have spare time outside their travel time windows to serve as drivers, and Type 2 who have ride-hailing demand but cannot drive themselves. We aim to build a self-sustained CRS system to encourage Type 1 drivers to “serve” Type 2 users at their capability to receive profit for compensating their payment for car rental.

1.2. System Implementation and Justification

To implement CRS, we can build a reservation system that collects Type 1 drivers’ rental information along with their available time windows for serving others, as well as Type 2 users’ ride-hailing requests with origin-destination of their trips and time windows for pick-up, before the optimization and computation steps. Note that both carsharing and ride-hailing have two primary forms: Reservation-based and on-demand. From users’ perspective, on-demand service is more appealing since it provides an instant solution for their

travel needs. However, in this paper, we assume that all the requests are collected prior to the optimization phase due to the following reasons. First, on-demand ride-hailing service usually requires large number of drivers and passengers to achieve matching efficiency. We consider travel needs from underserved communities, and thus the demand is sparsely distributed. Second, Type 1 drivers who serve others to gain more vehicle access often know their available time sufficiently early before committing to the service. Third, the purposes of users' trips can be generally categorized as job commute, hospital visit, job interview, and grocery shopping in underserved communities, which all have fixed schedule and can be known in advance. Based on this, we propose a static reservation-based CRS system and assume that all supplies and demands are known in advance.

1.3. Solution Approach Overview

We consider a two-phase approach for the design and operations of the CRS system: In Phase I, we maximize the fulfillment of demand by matching Type 1 drivers with Type 2 users over a spatial-temporal network that describes all users' demand and availability, while maintaining the financial self-sustainability of the system; in Phase II, we optimize pick-up routes and schedules for matched Type 1 and Type 2 users under stochastic vehicle travel time and service (pick-up) time, which may result in users' waiting and overtime. We propose a two-stage stochastic mixed-integer programming formulation for Phase II, which balances vehicles' total travel cost and the expected penalty cost of users' waiting and vehicle-use overtime. To solve the large-scale formulation with many samples of uncertain travel time and service time, we propose a driver-based decomposition algorithm, which first determines a sequence of Type 2 users assigned to each Type 1 driver, and then for each driver the algorithm decides an optimal schedule to arrive at each Type 2 user's location, which can be quickly solved via linear programming. We conduct computational studies by

testing instances based on serving underserved populations in Washtenaw County, Michigan and consider diverse demand patterns of Type 1 and Type 2 users. We show that our decomposition algorithm well outperforms the standard scenario-based decomposition approach, and demonstrate the cost and service-quality performance of the CRS system.

1.4. Contributions and Main Results

This paper focuses on developing a new CRS system for serving heterogeneous travelers whose demand cannot be solely met by either carsharing or ride-hailing business models. The contributions of this paper are three-fold.

- We conduct a “proof-of-concept” study of a new design of shared-mobility that integrates both carsharing and ride-hailing services. We focus on underserved populations, who are currently being left behind by the shared-mobility industry. Taking into account their special transportation needs and time flexibility, the system will provide prominent solutions for practical use and step towards improving social equity.

- The classification of two types of travelers leads to self-sustained operations of the system via supply-demand matching. However, combining two types of services results in substantial operational challenges. We decompose the problem into two phases, and successfully reduce operational complexity while obtaining good-quality results shown by our computational studies. We also demonstrate the importance of vehicle relocation, as we can achieve higher demand fulfillment rates even we only allow round trips of shared cars.

- We extend the basic model to a stochastic integer program to capture the randomness of vehicle travel time and service time. We develop efficient decomposition algorithms for optimizing the large-scale stochastic model with finite samples. When applied to synthetic data, our proposed model achieves high operational efficiency (measured by waiting and overtime of the system), compared with the deterministic model using the expected values.

The proposed community-based CRS system can be applied beyond the scope of serving underserved populations as it provides a solution for communities where there is a mixture of travelers with different driving abilities and time flexibility.

1.5. Organization of the Paper

The remainder of this paper is organized as follows. In Section 2, we review the literature related to carsharing, ride-hailing, and their optimization models and algorithms. In Section 3, we describe the problem and formulate a two-phase approach to optimize the integrated CRS design and operations. We further present a stochastic programming variant of the Phase II model by considering random vehicle travel time and service time. In Section 4, we develop an efficient way to decompose the proposed models and derive valid cuts based on integer L -shaped method. In Section 5, we demonstrate the effectiveness of the CRS system and present computational results in various instances. We conclude the paper and present future research directions in Section 6.

2. Literature Review

For carsharing, Laporte et al. (2015) classify the literature of carsharing under five main topics: Station location, fleet dimensioning, station inventory, rebalancing incentives, and vehicle repositioning. Nourinejad and Roorda (2014) propose a dynamic optimization-simulation model to study the relationship between fleet size and reservation time (i.e., the time between reservation and picking up vehicles). Febbraro et al. (2012) propose an optimal relocation policy to address vehicle relocation problem. Nair and Miller-Hooks (2014) use a bilevel mixed-integer linear program to optimize station location and capacity, as well as vehicle inventories. To handle vehicle imbalance in the one-way carsharing system, Kek et al. (2009) introduce a spatial-temporal network to model the movement of vehicles and determine the workforce needed for relocation. Similar approaches have been used by

de Almeida Correia and Antunes (2012) to optimize parking locations, and by Fan (2014) to optimize the allocation and relocation in carsharing systems that allow one-way car rental.

For ride-hailing, Agatz et al. (2012) conduct a comprehensive survey of optimization approaches for fleet management and other related operational problems in dynamic ride-hailing. Pavone et al. (2012) focus on fluid approximation method and Zhang and Pavone (2016) propose queuing-based methods to model and analyze ride-hailing systems. Alonso-Mora et al. (2017) propose a general model for a dynamic real-time high-capacity ride-pooling system that solves an assignment problem on a graph of feasible trips and compatible vehicles. To match ride-hailing requests to available cars, as well as to route shared vehicles, most papers consider variants of the vehicle routing problem (VRP) for seeking static ride-hailing solutions. Toth and Vigo (2014) summarize the formulations and solution approaches of VRP variants, including multi-depot VRP, VRP with time windows, and VRP with simultaneous pickup and delivery. Exact algorithms for VRP are mainly based on branch-and-cut (B&C) (Lysgaard et al. 2004, Achuthan et al. 2003, Ralphs et al. 2003, Cordeau 2006) and branch-cut-and-price (BCP) (Fukasawa et al. 2006, Pecin et al. 2014). We also refer the interested readers to a survey paper by Bräysy and Gendreau (2005) for metaheuristic approaches for VRPs, which covers the tabu search algorithm, genetic and evolution strategies, simulated annealing, ant algorithms, and very large-scale neighborhood search. Furthermore, Dinh et al. (2017), Ghosal and Wiesemann (2018) respectively propose exact BCP and B&C algorithms for optimizing capacitated VRP with stochastic demand, both using chance-constrained programming formulations. Taş et al. (2013) develop heuristic approaches for VRP with soft time windows and stochastic travel time. In addition to routing cost, they also consider the cost of late and early arrivals, which are

related to the waiting time and idle time considered in our stochastic formulation discussed later.

For designing carsharing systems, He et al. (2016) optimize service zone selection for sharing electric vehicles under uncertain carsharing demand and fuel price. Lu et al. (2018) study a carsharing fleet allocation problem with stochastic one-way and round-trip demands. They propose a two-stage stochastic program to minimize the total cost of parking lots/permits and car allocation, as well as penalty cost from unfulfilled demand. Zhang et al. (2018) extend the models and solution approaches in Lu et al. (2018) for optimizing fleet allocation and service operations of electric vehicle sharing with vehicle-to-grid selling under random travel demand and electricity price. To the best of our knowledge, this paper is the first to combine carsharing system design with ride-hailing service optimization, which has special application domains as we have described in the above.

3. Problem Formulations

We consider a fleet of K vehicles for serving a set I of zones. Let $L = \{1, 2, \dots, |L|\}$ be a set of reservations received from Type 1 drivers. Each $l \in L$ is associated with a tuple, $(o_l, d_l, [s_l, t_l], [g_l, h_l])$, which includes the pick-up zone $o_l \in I$, return zone $d_l \in I$ of a rental car, time window $[s_l, t_l]$ during which a car is needed, and the time window $[g_l, h_l]$ during which Type 1 driver l is able to provide ride-hailing service to some Type 2 users. Let J be a set of ride-hailing demand from Type 2 (non-driver) users. Each $j \in J$ is associated with a tuple, $(o'_j, d'_j, e_j, g'_j, h'_j)$, where o'_j and d'_j represent the trip's origin and destination, respectively, e_j is the total service time needed (including driving time from o_j to d_j , time of loading passengers or goods, etc.), and $[g'_j, h'_j]$ is the available time window for picking up the corresponding Type 2 user at origin o'_j . We define a binary parameter vector $w = (w_{jl}, j \in J, l \in L)^T$, where $w_{jl} = 1$ if ride-hailing $j \in J$ can be served by carsharing trip

$l \in L$ and 0 otherwise. We set $w_{jl} = 1$ if $|[g_j^J, h_j^J] \cap [g_l^L, h_l^L]| \geq e_j$, meaning that Type 2 user $j \in J$ can be served within the time window specified by Type 1 driver $l \in L$.

For each Type 1 demand $l \in L$, we charge r_l^{car} per period of car use (which depend on the pick-up location and drop-off location). We also pay r_l^{drive} for each time period a driver serves Type 2 users. For each Type 2 demand $j \in J$, we charge r_j^{ride} per period dependent on the origin, destination, and time window. A car in use will incur a service cost c^{ser} (including maintenance, insurance, and other types of cost) per period to the system operation and will incur an idle cost c_i^{idle} (including c^{ser} and parking cost) per period if it sits idle at zone $i \in I$.

3.1. Phase I: Carsharing Planning and Operations

Given service requests from Type 1 drivers and Type 2 users, we first implement Phase I to decide which requests to fulfill and pass the solutions to Phase II. Specifically, we determine where to allocate K cars and decide which Type 1 and Type 2 users' requests to be passed to Phase II for further routing and scheduling. We construct a spatial-temporal network, $G = (N, A)$, to capture carsharing reservations from Type 1 drivers over T periods, where each node $n_{it} \in N$ represents a zone $i \in I$ at period $t \in \{0, 1, \dots, T\}$. We partition A into three types of arcs, $A = A^{\text{travel}} \cup A^{\text{idle}} \cup A^{\text{rel}}$, as follows.

- Travel arcs $(n_{it}, n_{i't'}) \in A^{\text{travel}}$ are created for each Type 1 demand $l \in L$ where $i = o_l$, $i' = d_l$, $t = s_l$ and $t' = t_l$. The amount of flows on arc $a_l \in A^{\text{travel}}$ indicates whether or not a vehicle being rented by Type 1 driver $l \in L$. We set $f_{a_l} = (t_l - s_l)(r_l^{\text{car}} - c^{\text{ser}}) - r_l^{\text{drive}}(h_l - g_l)$ as the unit flow revenue and $u_{a_l} = 1$ as the capacity of arc $a_l \in A^{\text{travel}}$, for all $l \in L$.

- Idle arcs $(n_{it}, n_{i,t+1}) \in A^{\text{idle}}$ are created for $i \in I$ and $t \in \{0, 1, \dots, T-1\}$. The amount of flows on arc $(n_{it}, n_{i,t+1}) \in A^{\text{idle}}$ indicates the number of vehicles being idle in zone i from t to $t+1$. We set unit flow revenue and capacity of arc $a \in A^{\text{idle}}$ as $f_a = -c_i^{\text{idle}}$ and $u_a = K$, respectively.

- Relocation arcs $(n_{it}, n_{i',t'}) \in A^{\text{rel}}$ are created for $i \neq i' \in I$ and $t' > t \in \{0, 1, \dots, T-1\}$.

The amount of flows on $(n_{it}, n_{i',t'}) \in A^{\text{rel}}$ indicates the number of vehicles being relocated from zone i at t to zone i' at t' . We set unit flow revenue and capacity of arc $a \in A^{\text{rel}}$ as $f_a = -c^{\text{rel}}(t' - t)$ and $u_a = K$, respectively.

We define integer decision vector $x = (x_i, i \in I)^T$ where x_i is the number of cars initially located in zone $i \in I$, integer decision vector $y = (y_a, a \in A)^T$ where y_a indicates the amount of flows on arc $a \in A$, and binary decision vector $z = (z_{jl}, j \in J, l \in L)^T$ where $z_{jl} = 1$ indicates that Type 2 user $j \in J$ is served by Type 1 driver $l \in L$, and $z_{jl} = 0$ otherwise. We let $\delta^+(n_{it})$, $\delta^-(n_{it})$ be the sets of arcs to which node n_{it} is their tail and head nodes, respectively, and formulate an integer program **P1** as follows.

$$[\mathbf{P1}] \quad \text{maximize}_{x,y,z} \quad \sum_{a \in A} f_a y_a + \sum_{j \in J} r_j^{\text{ride}} \sum_{l \in L} e_j z_{jl} \quad (1a)$$

$$\text{subject to} \quad \sum_{i \in I} x_i \leq K \quad (1b)$$

$$\sum_{a \in \delta^+(n_{it})} y_a - \sum_{a \in \delta^-(n_{it})} y_a = \begin{cases} x_i & \text{if } t=0 \\ 0 & \text{if } t \in \{1, \dots, T-1\}, \forall i \in I \\ -x_i & \text{if } t=T \end{cases} \quad (1c)$$

$$y_a \leq u_a \quad \forall a \in A \quad (1d)$$

$$\sum_{j \in J} z_{jl} e_j \leq y_{a_l} (h_l - g_l) \quad \forall l \in L \quad (1e)$$

$$z_{jl} \leq w_{jl} \quad \forall j \in J, \forall l \in L \quad (1f)$$

$$\sum_{l \in L} z_{jl} \leq 1 \quad \forall j \in J \quad (1g)$$

$$x \in \mathbb{Z}_{\geq 0}^{|I|}, y \in \mathbb{Z}_{\geq 0}^{|A|}, z \in \{0, 1\}^{|J| \times |L|}. \quad (1h)$$

The objective function (1a) maximizes the potential total profit from both carsharing and ride-hailing operations, which is also equivalent to maximize the total number of Type 1 and Type 2 requests served in the system. Constraint (1b) allows the total number of vehicles used in the CRS system to be no more than K . Constraints (1c) are flow balance constraints formulated for the spatial-temporal network. Constraints (1d) are arc capacity

constraints. Constraints (1e) ensure that the accepted carsharing requests from all Type 1 drivers can potentially provide sufficient time to serve accepted ride-hailing requests from all Type 2 users. Constraints (1f) ensure that $z_{jl} = 1$ only if a Type 1 driver $l \in L$ can serve a Type 2 user $j \in J$. Constraints (1g) ensure that each ride-hailing request will be served at most once.

3.2. Phase II: Ride-hailing Routing and Scheduling

After solving P1, we obtain solutions as (i) the set of Type 1 drivers to accept and (ii) their matched Type 2 users. We then construct a Phase II model to find the routing and scheduling decisions for both Type 1 and Type 2 requests that can be accepted. Note that our final service acceptance decisions will be made after Phase II and then both types of users will be notified whether we can provide carsharing or ride-hailing service to them.

We define a network based on the CRS service region as $G' = (V, E)$ where V is the node set and $E = \{(u, v) : u, v \in V, u \neq v\}$ is the edge set. Let $V = V_0 \cup V_1 \cup V_2$ where $V_0 = \{o_l : l \in L\}$ and $V_1 = \{d_l : l \in L\}$ are the sets of pickup and return locations for approved Type 1 carsharing requests from solving P1, respectively. Set $V_2 = \{v_j : j \in J\}$ contains each Type 2 ride-hailing request. We assume that all the arcs can be traveled along both directions but can have different travel time, and therefore G' is directed. We use c_{uv} to denote the estimated travel time between two nodes such that $(u, v) \in E$. For each $v_j \in V_2$, $j \in J$, we calculate the estimated travel time in the following way: We set $c_{u,v_j} = c_{u,o_j}$ and $c_{v_j,u} = c_{d_j,u}$ for all $u \in V$, where o_j , d_j represent origin and destination of Type 2 user $j \in J$.

We define binary decision vector $\alpha = (\alpha_{uv}^l, (u, v) \in E, l \in L)^T$ such that $\alpha_{uv}^l = 1$ indicates that Type 1 driver $l \in L$ travels along arc (u, v) , and 0 otherwise. We define binary vector $\beta = (\beta_j, j \in J)^T$ such that $\beta_j = 1$ indicates that Type 2 user $j \in J$ is served, and 0 otherwise.

We define continuous decision vector $\gamma = (\gamma_v, v \in V)^T$ such that $\gamma_v \geq 0$ is the planned time of a vehicle arriving at location $v \in V$. We formulate the Phase II problem as a variant of the VRP with time windows and multiple depots as follows.

$$[\mathbf{P2}] \quad \text{maximize}_{\alpha, \beta, \gamma} \quad \sum_{j \in J} r_j^{\text{ride}} e_j \beta_j \quad (2a)$$

$$\text{subject to} \quad \sum_{l \in L} \sum_{u \in V} \alpha_{uv}^l = \beta_j \quad \forall v_j \in V_2, \quad (2b)$$

$$\sum_{v: (o_l, v) \in E} \alpha_{ov}^l - \sum_{v: (v, o_l) \in E} \alpha_{vo_l}^l = 1 \quad \forall o_l \in V_0, l \in L \quad (2c)$$

$$\sum_{u: (u, d_l) \in E} \alpha_{ud_l}^l - \sum_{u: (d_l, u) \in E} \alpha_{d_l u}^l = 1 \quad \forall d_l \in V_1, l \in L \quad (2d)$$

$$\sum_{u: (u, v) \in E} \alpha_{uv}^l - \sum_{u: (v, u) \in E} \alpha_{vu}^l = 0 \quad \forall v \in V_2, l \in L \quad (2e)$$

$$\alpha_{uv}^l = 0 \quad \forall u \in V_0, v \in V, u \neq o_l, l \in L \quad (2f)$$

$$\gamma_{o_l} + c_{o_l v} - T(1 - \alpha_{o_l v}^l) \leq \gamma_v \quad \forall o_l \in V_0, (o_l, v) \in E, \quad (2g)$$

$$\gamma_{v_j} + e_j + c_{v_j u} - T \left(1 - \sum_{l \in L} \alpha_{v_j u}^l \right) \leq \gamma_u \quad \forall v_j \in V_2, (v_j, u) \in E, \quad (2h)$$

$$g_l \leq \gamma_{o_l} \leq \gamma_{d_l} \leq h_l \quad \forall l \in L, \quad (2i)$$

$$g'_j \leq \gamma_{v_j} \leq h'_j \quad \forall v_j \in V_2, \quad (2j)$$

$$\alpha \in \{0, 1\}^{|E| \times |L|}, \beta \in \{0, 1\}^{|J|}, \quad (2k)$$

where the objective function (2a) maximizes the total profit from providing ride-hailing services. Constraints (2b) ensure that $\beta_j = 1$ if the origin o'_j for Type 2 user $j \in J$ has been visited. Constraints (2c)–(2e) balance vehicle flows for each approved Type 1 driver $l \in L$. Constraints (2f) forbid flows for approved Type 1 driver $l \in L$ to visit nodes representing pickup/return location for Type 1 driver $l' \in L$, for all $l' \neq l$. Constraints (2g)–(2h) formulate the arrival time at each origin v of Type 2 user to be greater than the arrival time

at node u (which is either pickup location of a Type 1 driver or destination location of another Type 2 user) plus travel and service time, if a trip travels from u to v directly. Constraints (2i)–(2j) ensure that departure and return time for Type 1 driver, as well as departure time for each Type 2 user fall into the corresponding time windows.

3.3. Phase II Model Variant under Stochastic Travel Time and Service Time

In practice, uncertainties exist and affect the operations of the proposed CRS system. For example, travel time and service time can be random due to varying road conditions, weather, and traffic. Therefore, the primary task in this section is to propose a two-stage stochastic programming formulation that incorporates random travel and service time for Phase II.

Let \tilde{c}_{uv} be the random travel time along arc $(u, v) \in E$ and \tilde{e}_j be the random service time for ride $j \in J$. Under uncertainty, one or both of the following scenarios could happen: (i) Type 1 driver may arrive late to pick-up the scheduled Type 2 user and (ii) Type 1 driver may return the shared vehicle later than the scheduled returning time. Therefore, our goal is to optimize the start time of each ride to maximize the revenue from ride-hailing operation minus the expected penalty cost due to Type 2 users' waiting and system overtime. (We define the overtime as the sum of the late time of returning vehicles by all Type 1 drivers.) We denote p^w and p^o as the penalty cost of per unit waiting time and overtime, respectively.

Let \bar{e}_j be an estimated service time for Type 2 user $j \in J$, which can be taken as the mean value of \tilde{e}_j . We revise **P2** and present its stochastic variant as:

$$[\mathbf{SP2}] \quad \underset{\alpha, \beta, \gamma}{\text{maximize}} \quad \sum_{j \in J} r_j^{\text{ride}} \bar{e}_j \beta_j - \mathbb{E}(Q(\alpha, \gamma, \tilde{c}, \tilde{e})) \quad (3)$$

subject to: (2b)–(2k)

where $Q(\alpha, \gamma, \tilde{c}, \tilde{e})$ is the total penalty cost of random waiting time and overtime given solution (α, γ) and uncertainty (\tilde{c}, \tilde{e}) , and $\mathbb{E}(\cdot)$ denotes the expectation of random variable \cdot . To approximate $\mathbb{E}(Q(\alpha, \gamma, \tilde{c}, \tilde{e}))$, we apply the Sample Average Approximation (SAA) method (Kleywegt et al. 2002). The idea is to generate a finite set of samples following the Monte Carlo sampling approach and approximate the expectation of its sample average function.

Let Ω denote the set of all sampled scenarios, and the probability of realizing each scenario is $1/|\Omega|$ when applying the SAA approach, i.e., $\mathbb{E}(Q(\alpha, \gamma, \tilde{c}, \tilde{e})) = \sum_{\omega \in \Omega} \frac{1}{|\Omega|} Q(\alpha, \gamma, \tilde{c}(\omega), \tilde{e}(\omega))$, where $\tilde{c}(\omega)$, $\tilde{e}(\omega)$ are realizations of \tilde{c} , \tilde{e} in scenario ω , respectively. We define auxiliary decision variables $W_j(\omega)$ as waiting time for Type 2 ride request $j \in J$, and $O_l(\omega)$ as overtime for Type 1 driver service $l \in L$ for each scenario $\omega \in \Omega$. Given a (α, γ) -solution and realized value $(\tilde{c}(\omega), \tilde{e}(\omega))$ of (\tilde{c}, \tilde{e}) in scenario $\omega \in \Omega$, we specify the sample-based linear program for computing the value of $Q(\alpha, \gamma, \tilde{c}(\omega), \tilde{e}(\omega))$ as

$$\text{minimize } \sum_{j \in J} p^w W_j(\omega) + \sum_{l \in L} p^o O_l(\omega) \quad (4a)$$

$$\text{subject to } \gamma_u + \tilde{c}(\omega)_{uv} - T(1 - \alpha_{uv}^l) \leq \gamma_v + W_j(\omega) \quad \forall (u, v) = (o_l, o'_j) \in E, \quad (4b)$$

$$\gamma_{o'_j} + W_j(\omega) + \tilde{e}_j(\omega) + \tilde{c}_{d'_j, o'_j}(\omega) - T \left(1 - \sum_{l \in L} \alpha_{d'_j, o'_j}^l \right) \leq \gamma_{o'_j} + W_{j'}(\omega) \quad \forall (d'_j, o'_j) \in E, \quad (4c)$$

$$\gamma_{o'_j} + W_j(\omega) + \tilde{e}_j(\omega) + \tilde{c}_{d'_j, d_l}(\omega) - T(1 - \alpha_{d'_j, d_l}^l) \leq \gamma_{d_l} + O_l(\omega) \quad \forall (d'_j, d_l) \in E, \quad (4d)$$

$$W_j(\omega) \geq 0 \quad \forall j \in J, \quad (4e)$$

$$O_l(\omega) \geq 0 \quad \forall l \in L. \quad (4f)$$

Here the objective function (4a) minimizes the total penalty cost of all Type 2 users' waiting time and all Type 1 drivers' overtime of returning vehicles. Let $S_j(\omega)$ be the actual service starting time for Type 2 user $j \in J$ and $S'_l(\omega)$ be the actual vehicle return time for Type 1 driver $l \in L$ for each sampled scenario $\omega \in \Omega$, respectively. We have

- $S_j(\omega) = \gamma_v + W_j(\omega)$ for $v \in V_2$ representing Type 2 user $j \in J$, and sampled scenario $\omega \in \Omega$;

- $S'_l(\omega) = \gamma_v + O_l(\omega)$ for $v \in V_1$ representing Type 1 driver $l \in L$, and sampled scenario $\omega \in \Omega$.

Note that $S_j(\omega)$ and $S'_l(\omega)$ are related to the right-hand sides of constraints (4b)–(4c) and (4d), respectively. Therefore, for each Type 1 driver $l \in L$, the corresponding constraint (4b) calculates the actual service start time of its firstly served Type 2 user j and ensures that it is no earlier than the planned time that driver l departs origin o_l plus the travel time from o_l to o'_j . Similarly, each constraint in (4c) propagates this time relationship for all the subsequent Type 2 users who will be served by Type 1 driver l , and ensures that their actual service start time will be no earlier than the actual service start time of their predecessors plus the realized service time and travel time before driver l arrives. Lastly, constraint (4d) calculates the actual time of returning vehicle by Type 1 driver $l \in L$ and ensures that it is no earlier than the time of completing service in the last Type 2 user's location plus the travel time to the return location.

4. Solution Approaches

To solve **SP2** efficiently, we propose an algorithm based on the integer L -shaped method (Laporte and Louveaux 1993). We decompose the problem into a relaxed master problem, which contains variables of all the decisions made before realizing the values of (\tilde{c}, \tilde{e}) , and a series of subproblems with recourse decisions. Unlike the standard decomposition approach that creates a subproblem for each scenario, we will reformulate our problem by first deciding routes for individual drivers and then creating driver-based subproblems. Specifically, the master problem matches and assigns sequences of Type 2 users to each Type 1 driver. Subproblems are then formulated for each Type 1 driver, to find an optimal schedule to pick up assigned Type 2 users, and each subproblem aims to minimize the expected penalty cost of the assigned Type 2 users' total waiting time and the corresponding Type 1 driver's

overtime use of a vehicle. We initially set the lower bound of the expected penalty cost for each Type 1 driver as zero. Then we iteratively generate cuts from each subproblem and add them to the master problem to improve the lower bound.

We define decision variables $\theta = (\theta_l, l \in L)^T$, such that θ_l is the optimal objective value (i.e., the optimal expected penalty cost of waiting time and overtime) of the subproblem formulated for Type 1 driver l , for each $l \in L$. We formulate a relaxed master problem in the current iteration as:

$$[\mathbf{MP}] \quad \underset{\alpha, \beta, \pi, \theta}{\text{maximize}} \quad \sum_{j \in J} r_j^{\text{ride}} \bar{e}_j \beta_j - \sum_{l \in L} \theta_l \quad (5a)$$

subject to: (2b)–(2k)

$$L(\alpha, \theta_l) \geq 0 \quad \forall l \in L \quad (5b)$$

where in (5b), $L(\alpha, \theta_l) \geq 0$ denotes the set of cuts for improving the lower bound of θ_l , generated from solving subproblems (described later) in previous iterations. In the above model, constraints (2b)–(2f) enforce sufficient vehicles to cover matched Type 2 users as explained in the previous section.

After solving **MP**, we obtain a tentative optimal solution $\bar{\alpha}$ that can be used to recover the routing sequence for each driver $l \in L$ as follows. We use the non-zero values of $\bar{\alpha}_{uv}^l$ for $v \in V_2$ to obtain the set of Type 2 users assigned to driver $l \in L$ and then perform a depth-first to recover the path to visit assigned Type 2 users for each Type 1 driver. For each $l \in L$, let $(\sigma^l(1), \sigma^l(2), \dots, \sigma^l(n_l))$ be such a sequence with n_l denoting the total number of Type 2 users assigned to driver l . Let $\sigma^l(0)$ and $\sigma^l(n_l + 1)$ be the depots where Type 1 driver $l \in L$ pick-up and return the vehicle. The only decisions left are the planned arrival time at each Type 2 user's pick up location and the planned time for returning each vehicle. Alternatively, for each Type 1 driver $l \in L$, we can decide the time to allocate in between $\sigma^l(n)$ and $\sigma^l(n + 1)$ for $n = 0, \dots, n_l$.

Recall that $S_i(\omega)$ is the actual service start time at node i if $i \in V_2$ or actual vehicle return time at node i if $i \in V_1$. The subproblem for each Type 1 driver $l \in L$ is equivalent to the following linear program:

$$[\mathbf{SUBP}_l] \quad \underset{W, O, S, \gamma}{\text{minimize}} \quad \frac{1}{|\Omega|} \left(\sum_{i=1}^{n_l} p^w W_{\sigma^l(i)}(\omega) + p^o O_l(\omega) \right) \quad (6a)$$

subject to: (2i), (2j)

$$S_{\sigma^l(1)}(\omega) \geq \gamma_{\sigma^l(0)} + c_{\sigma^l(0), \sigma^l(1)}(\omega) \quad \omega \in \Omega, \quad (6b)$$

$$S_{\sigma^l(i+1)}(\omega) \geq S_{\sigma^l(i)}(\omega) + e_{\sigma^l(i)}(\omega) + c_{\sigma^l(i), \sigma^l(i+1)}(\omega) \quad \omega \in \Omega, \quad i = 1, \dots, n_l, \quad (6c)$$

$$S_{\sigma^l(i)}(\omega) = \gamma_{\sigma^l(i)} + W_{\sigma^l(i)}(\omega) \quad i = 1, \dots, n_l, \quad \omega \in \Omega, \quad (6d)$$

$$S_{\sigma^l(n_l+1)}(\omega) = \gamma_{\sigma^l(n_l+1)} + O_l(\omega) \quad \omega \in \Omega, \quad (6e)$$

$$W_{\sigma^l(i)}(\omega) \geq 0 \quad \forall i = 1, \dots, n_l, \quad \omega \in \Omega \quad (6f)$$

$$O_l(\omega) \geq 0 \quad \forall l \in L, \quad \omega \in \Omega, \quad (6g)$$

where constraint (6b) ensure that the actual service start time at the first assigned Type 2 user is not earlier than the actual time Type 1 driver leaves depot, plus the travel time from depot to the Type 2 user's pick-up location. Constraints (6c) ensure that the actual service start time at the $(i+1)^{\text{th}}$ node (or actual vehicle return time if such a node represents a depot) of Type 1 driver l is no earlier than the actual service start time at the i^{th} Type 2 user plus the time for completing the service at the i^{th} Type 2 user and the travel time from the i^{th} user to the $(i+1)^{\text{th}}$ node. For all scenarios $\omega \in \Omega$, constraints (6d) let the actual time of serving each assigned Type 2 user be the planned arrival time plus the waiting time of the user. Constraints (6e) let the actual time for returning the vehicle be the planned time plus the overtime. Both waiting time and overtime variables are nonnegative according to (6f) and (6g). These constraints are analogous to constraints (4b)–(4d). Each subproblem \mathbf{SUBP}_l will output an optimal schedule for Type 1 driver $l \in L$ to serve the assigned Type 2 users (given by \mathbf{MP}), and also the minimum expected penalty cost given the current

visiting sequence. Moreover \mathbf{SUBP}_l are linear programs without big-M coefficients, which can be solved very efficiently to return cuts to \mathbf{MP} . We describe the overall algorithm as follows, which is detailed in Algorithm 1.

Algorithm 1 An integer L -shaped method for solving $\mathbf{SP2}$.

- 1: Initialize \mathbf{MP} as (5a)–(5b) and set $L(\alpha, \theta_l) \geq 0 = \emptyset$ for all $l \in L$.
 - 2: Solve \mathbf{MP} by branch-and-bound to obtain a tentative solution $(\bar{\alpha}, \bar{\theta})$.
 - 3: Recover route sequence $(\sigma^l(1), \sigma^l(2), \dots, \sigma^l(n_l))$ for each $l \in L$ from $\bar{\alpha}$.
 - 4: Solve \mathbf{SUBP}_l for each route sequence of driver $l \in L$, and let $\hat{\theta}_l$ be the optimal objective for \mathbf{SUBP}_l .
 - 5: **if** $\sum_{l \in L} \bar{\theta}_l < \sum_{l \in L} \hat{\theta}_l$ **then**
 - 6: **for** $l \in L$ **do**
 - 7: Add Cut (7) to the cut set $L(\alpha, \theta_l) \geq 0$ in \mathbf{MP} .
 - 8: **goto** Step 2.
 - 9: **else**
 - 10: Store the solution $\bar{\gamma}$ from each \mathbf{SUBP}_l as planned service start time for each assigned Type 2 user to driver l .
 - 11: Return solution $(\bar{\alpha}, \bar{\gamma}, \bar{\theta})$ as an optimal solution to the overall $\mathbf{SP2}$ problem.
-

After solving \mathbf{MP} , we obtain an integer solution $\bar{\alpha}$ as well as solutions $\bar{\theta}_l$ for all $l \in L$. Let $\hat{\theta}_l$ be the optimal objective obtained by solving \mathbf{SUBP}_l for each $l \in L$. If $\bar{\theta}_l < \hat{\theta}_l$, following the integer L -shaped method (see, e.g., Laporte et al. 2002), we propose to add the following cut to the \mathbf{MP} and re-solve it:

$$\theta_l \geq \hat{\theta}_l \left(\sum_{u,v: \bar{\alpha}_{uv}^l=1} \alpha_{uv}^l - \sum_{u,v} \alpha_{uv}^l + 1 \right). \quad (7)$$

THEOREM 1. *Cut (7) is a valid inequality for **MP** and enforces that no same values of $\bar{\alpha}, \bar{\theta}_l$ can be obtained in future iterations.*

Proof. Inequality (7) only takes effect when $\sum_{u,v:\bar{\alpha}_{uv}=1} \alpha_{uv}^l - \sum_{u,v} \alpha_{uv}^l \geq 0$, which is equivalent to $\alpha_{uv}^l = 1$ for all u, v with $\bar{\alpha}_{uv}^l = 1$. Each subproblem **SUBP** $_l$ outputs the optimal scheduling for Type 1 driver l and calculates the minimum expected penalty cost, θ_l . The variable representing recourse penalty cost for the current visiting sequence, should be bounded below by $\hat{\theta}_l$. Therefore, (7) is valid for any optimal (α, θ_l) , and the same value of $(\bar{\alpha}, \bar{\theta}_l)$ will not repeat if it is not optimal. \square

THEOREM 2. *Algorithm 1 converges in finite steps.*

Proof. In each iteration, we have $\bar{\theta}_l < \hat{\theta}_l$ only if current visiting sequence for driver $l \in L$ has not been explored, otherwise cut (7) enforces $\bar{\theta}_l \geq \hat{\theta}_l$. Since we have a finite number of visiting sequences for a given Type 2 to Type 1 assignment, the algorithm converges in finite steps.

REMARK 1. To solve **SP2**, alternatively, we can decompose the problem by scenario (instead of decomposing the model by driver l proposed in Algorithm 1) and construct a relaxed master problem that matches Type 1 drivers to Type 2 users and determine pick-up routes and schedules, and then formulate subproblems to compute the penalty cost of Type 2 users' waiting time and Type 1 drivers' overtime in each scenario. However, this traditional way of decomposing the problem cannot produce sufficiently good routes and schedules in the master problem under uncertain travel time and service time, which will only be realized in the subproblems. Furthermore, comparing to constraints in **SP2** that determine feasible routes and schedules, the driver-based decomposition algorithm as Algorithm 1 avoids “big-M” constraints in both **MP** and **SUBP** $_l$, for all $l \in L$. Due to

these two reasons, our decomposition approach significantly improves the computation of **SP2**, which we will demonstrate later in numerical studies.

We may also further decompose **SUBP_l** by scenario. Since **SUBP_l** only contains continuous decision variables and is a linear program that can be quickly solved to obtain cut (7), we directly solve it without further decomposition in our numerical studies.

5. Numerical Results

We conduct numerical studies and demonstrate the performance of proposed models using randomly generated instances based on statistical and census data of underserved populations in Washtenaw County, Michigan.

5.1. Experiment Design

We generate test instances based on the most updated United States Census data for Washtenaw County in Michigan, collected in April 2010 (see United States Census Bureau 2010). In the dataset, Washtenaw County is divided into 100 different census tracts, and each contains the information of population and location (i.e., longitude and latitude of the geographical center). We assume that each census tract is represented by its geographical center and construct a corresponding network with 100 nodes. The travel time between any of two nodes can be obtained by accessing Google Map API¹. In the original dataset, we can get the population for each census tract grouped by different ages. Under safety concerns, we enforce the age requirement for Type 1 drivers as 21 to 65 years old, which matches the minimum age requirement of current carsharing service providers (e.g., Zipcar requires drivers to be at least 21 years old²). To simulate Type 2 users in underserved populations, we consider the populations with age over 50 and with disability.

¹ <https://developers.google.com/maps/documentation/distance-matrix/>

² <https://support.zipcar.com/hc/en-us/articles/220333808-Eligibility-Requirements>

• **Problem Size** To pick the carsharing service zones in set I , we sort the 100 nodes based on the population of Type 1 drivers in each node and select the first $|I|$ nodes. We set $|I| = 5$, and assume that the operational hours of our system is from 7 am to 7 pm, and thus $T = 12$ hours.

• **Type 1 Driver Data** We simulate the set of Type 1 driver reservations, L , as follows. We consider $|L| = 40, 60$. For each Type 1 driver reservation $l \in L$, we simulate the pick-up and return locations, o_l and d_l , from the node set I based on the density of populations that are jobless or whose annual incomes are below a certain threshold. For car pick-up time s_l of each Type 1 driver $l \in L$, we sample s_l uniformly from 7 am to 4 pm with 1-hour time interval. The time of private use of a car is then uniformly sampled from 0–2 hours with 1-hour time interval. We simulate Type 1 drivers’ different flexibility levels of serving Type 2 users and consider two types of service-hour distributions: For Case (i), we generate the service hours for Type 1 drivers from $\{1, 2, 3, 4\}$ with equal probability of 0.25. For Case (ii), we sample the service hours for Type 1 drivers from $\{1, 2, 3, 4\}$ with the probability of 0.1, 0.2, 0.3, and 0.4, respectively. For the rest of the paper, we refer to Case (i) as the case with “regular drivers” and to Case (ii) as the case with “flexible drivers”. Except for the tests in Section 5.2.2 and Section 5.2.3, we report the results for the case with “flexible drivers”.

• **Type 2 User Data** We consider instances with $|J| = 40, 60, 80$ for Type 2 ride-hailing demand. For each Type 2 user $j \in J$, we sample the origin node, o'_j , based on the population density of target Type 2 users (age over 50 years old and disabled) and sample the destination node, d'_j , uniformly over the 100 census tracts. At the same time, we avoid $o'_j = d'_j$ in all our sampled instances by re-sampling the destination node if it happens. We uniformly generate g'_j for each Type 2 user over the entire operational hours (i.e., 7 am to 7 pm) and assign a 30-minute time window for each pick-up, i.e., $h'_j = g'_j + 30, \forall j \in J$.

• **Stochastic Travel Time** In all our test instances, for each Type 2 reservation $j \in J$, we set $e_j = c_{o'_j, d'_j}$ plus a constant loading/unloading time that is the same for all $j \in J$ and thus can be omitted in the model. Following the standard VRP literature such as Polus (1979), Laporte et al. (1992), we consider Gamma distribution for sampling the random travel time between pairs of locations. We assume that $\tilde{c}_{ij} = c_{ij}(0.8 + \xi)$ where ξ follows a Gamma distribution with shape parameter α and scale parameter λ , and c_{ij} is the deterministic travel time obtained from Google Map API for arc $(i, j) \in E$. Then following the definition of Gamma distribution, we have

$$E(\tilde{c}_{ij}) = (0.8 + \alpha\lambda)c_{ij},$$

$$Var(\tilde{c}_{ij}) = \alpha\lambda^2c_{ij}$$

We choose parameters $\alpha = 0.2, \lambda = 1$ to generate all the scenarios.

• **Other Parameters** For the rest of parameters, we set $r^{\text{car}} = \$8$ per hour and $r^{\text{ride}} = \$20$ per hour to match the price of current carsharing and ride-hailing service, e.g., Zipcar and Uber, in Washtenaw County. Note that the pricing strategy for ride-hailing company is composed by two parts, mileage (\$0.95/mile) and service time (\$0.15/minute) in Washtenaw County³. Here we combine them together to obtain reasonable price settings. We set $r^{\text{drive}} = \$16$ per hour to reasonably incentivize Type 1 drivers to provide ride-hailing service. We let $c^{\text{ser}} = \$1$ per hour and $c^{\text{idle}} = \$1$ per hour. The penalty cost parameters are set as $p^{\text{w}} = \$0.1$ per minute per user and $p^{\text{o}} = \$0.1$ per minute per car for model **SP2**.

Table 1 summarizes the important parameters used in the numerical studies. For each test instance described above, we generate five replications and report the average statistics unless otherwise noted.

³ <https://www.uber.com/fare-estimate/>

Table 1 Summary of important parameters

Basic Settings			
$ I $	5	$ L $	{40,60}
T	12	$ J $	{40,60,80}
$ \Omega $	500		
Stochastic Travel Time			
c_{ij}	average travel time obtained from Google Map API		
ξ	Follow Gamma Distribution with parameters $\alpha = 0.2$, $\lambda = 1$		
\tilde{c}_{ij}	$c_{ij}(0.8 + \xi)$		
Other Parameters			
r^{car}	\$8/hour	c^{idle}	\$1/hour
r^{ride}	\$20/hour	p^{w}	\$0.1/hour
r^{drive}	\$16/hour	p^{o}	\$0.1/hour
c^{ser}	\$1/hour		

All instances are programmed using Java 10. We call the solver Gurobi 8.0 to optimize all mixed-integer linear programming models. All programs are run on a desktop computer with Microsoft Windows 10 64-bit operating system, an Intel Core i7-6700K Central Processing Unit (CPU) with 4.0 GHz, and 32.0 GB RAM.

5.2. Computational Results

For our numerical experiments, we analyze our proposed model from the aspects of computational time, quality of service, revenue and cost, and out-of-sample tests.

5.2.1. Computational Time We present the computational results of the proposed model for various test instances, with five replications for each parameter combination (with the same number of users but different input data, e.g., locations, pick-up time windows). For each instance, we report the maximum (max), minimum (min), and average (avg) CPU time. The average CPU time is calculated based on solved replications. We choose parameters as described in Section 5.1 and 500 scenarios for the stochastic programming model **SP2**. We set the CPU time limit as 30 minutes for computing each replication.

In Table 2, we report the CPU time (in seconds) of models **P1**, **P2**, and **SP2** for solving each test instance with different number of available vehicles (K), number of Type 1

Table 2 CPU time (in seconds) or optimality gap for models **P1**, **P2**, and **SP2**

K	$ L $	$ J $	P1			P2			SP2 (Direct)			SP2 (L -shaped)		
			max	min	avg	max	min	avg	max	min	avg	max	min	avg
20	40	40	0.02	0.01	0.01	2.18	1.56	1.91	38.37%*	0.57%*	10.30%*	0.20%*	113.03	317.51
20	40	60	0.02	0.01	0.01	59.02	5.78	23.05	N/A	N/A	N/A	1.43%*	0.59%*	0.98%*
20	40	80	0.02	0.02	0.02	176.65	18.05	76.65	N/A	N/A	N/A	5.03%*	2.65%*	3.44%*
30	60	40	0.02	0.01	0.02	18.58	5.36	9.65	N/A	1.33%*	37.87%*	0.14%*	120.42	247.97
30	60	60	0.02	0.02	0.02	31.34	12.23	20.51	N/A	N/A	N/A	0.68%*	0.22%*	0.45%*
30	60	80	0.03	0.02	0.03	483.93	33.91	256.95	N/A	N/A	N/A	6.81%*	1.53%*	4.00%*

*: optimality gap is reported; N/A: no feasible solutions found within the 30-minute time limit

reservations ($|L|$), and number of Type 2 reservations ($|J|$). We report optimality gap for **SP2** when reaching the time limit. For **SP2**, we report results for both commercial optimization solver (see Columns “SP2 (Direct)”) and our proposed solution approach (see Columns “SP2 (L -shaped)”).

According to Table 2, model **P1** is easy to solve whereas **P2** and **SP2** are relatively difficult to optimize for both deterministic and stochastic cases. Besides, **SP2** is more challenging to solve compared to **P2**, as 4 out of 6 test instance sizes cannot be solved to optimality within time limit when the travel time and service time are both random.

In Table 2, the CPU time for **P2** increases as the number of Type 1 and Type 2 reservations increases. For example, given $|L| = 40$, the average CPU time increases from 1.91 seconds to 76.65 seconds as $|J|$ grows; given $|J| = 60$, the average CPU time increases from 9.65 seconds to 256.95 seconds as $|L|$ grows from 40 to 80. We also observe the effectiveness of the proposed solution approach, as both CPU time and optimality gap have been significantly improved from those given by general-purpose solvers.

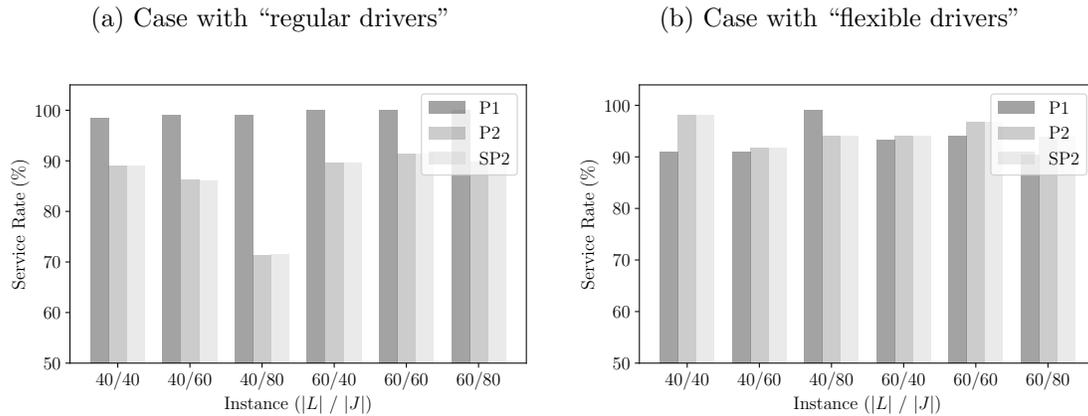
Although some instances cannot be solved to optimality when testing **SP2**, the optimality gap can be small when reaching the CPU time limit, i.e., when we set the optimization gap tolerance to 5%, most of the cases can be solved within the time limit. In later sections, we will see that the number of served trips output by **SP2** is similar to that of **P2** while achieving a higher quality of scheduling. The difficulty of closing the optimality

gap for **SP2** is due to the schedule adjustment that minimizes the expected penalty cost. Therefore, despite that **SP2** cannot be solved to optimality, we can still use the output solutions of matched Type 1 drivers and Type 2 users, and the sequence of serving Type 2 users to schedule the operations in Phase II. Based on the reasonable instance size of the designed system, our tests show the feasibility and practicability of the proposed models.

5.2.2. Quality of Service Here we show the quality of service (QoS) of the proposed models using the results of the percentage of approved demands. As mentioned in Section 5.1, we consider two cases of Type 1 drivers: “regular drivers” with service hours generated from $\{1, 2, 3, 4\}$ hours with equal probability and “flexible drivers” with service hours picked from $\{1, 2, 3, 4\}$ hours with probability $\{0.1, 0.2, 0.3, 0.4\}$, i.e., the latter case of drivers are more time flexible and prefer providing service with extended hours.

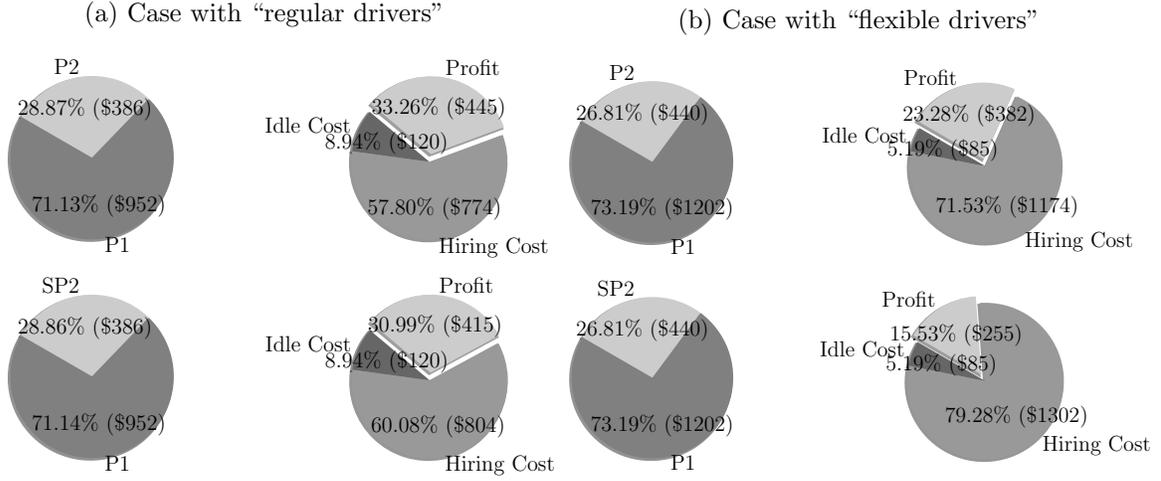
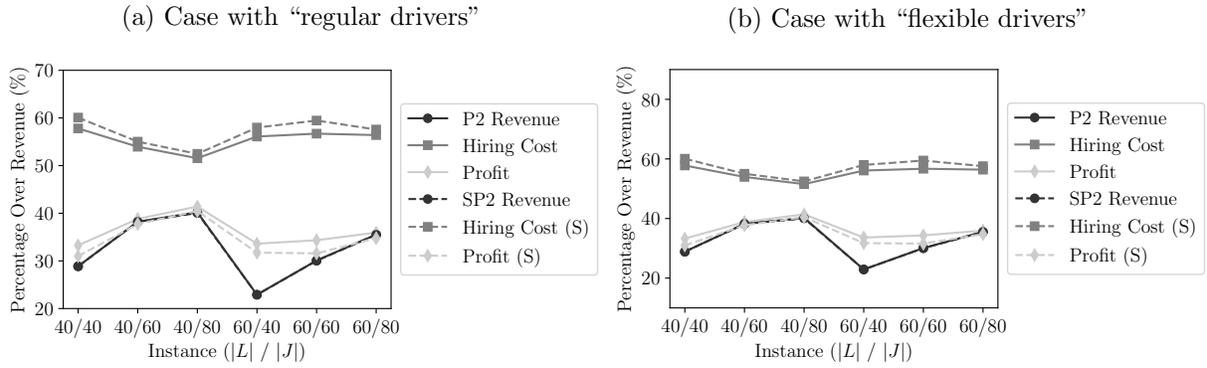
Figure 1a shows the average demand service rate across different models with “regular drivers” and Figure 1b shows the average demand service rate across different models with “flexible drivers”. For both cases, the demand service rate for Type 1 drivers is kept at high levels (over 90%). At the same time, both **P2** and **SP2** yield similar demand service rate for Type 2 users. However, for the case of “regular drivers”, the demand service rate for Type 2 users varies from 70% to 90% due to the lack of available drivers/service hours: When the ratio between Type 2 users and Type 1 driver rises, the demand service rate for Type 2 users drops significantly (see the instance with $|L| = 40, |J| = 80$). On the other hand, the demand service rates for both Type 1 drivers and Type 2 users are very high (over 90%) across all instances for the case with “flexible drivers”. Based on the results, the proposed model would be particularly helpful for the underserved communities whose residents are more flexible in terms of service hours.

Figure 1 Average demand service rate by proposed models



5.2.3. Revenue and Cost We first show the revenue and cost composition for a given instance and then extend the discussion across all instances. Figure 2 depicts the revenue and cost composition generated by proposed models for the instance with $|L| = 40, |J| = 40$ for both "regular drivers" (Figure 2a) and "flexible drivers" (Figure 2b). In each figure, the top row depicts the case where we only consider deterministic traveling time, and the bottom row demonstrates the results when stochastic traveling time is taken into account. The amounts of revenue generated from serving Type 2 users are similar under **P2** and **SP2**. However, as we consider the variation of traveling and service time, **SP2** utilizes slightly more drivers to provide reliable service (and correspondingly, the hiring cost proportion increases by 1% and 6%, for the two cases, respectively). Comparing Figure 2a and Figure 2b, the total revenue increases in the latter case from \$1338 to \$1642. At the same time, the hiring cost to serve Type 2 users also increases considerably whereas the profit proportion drops.

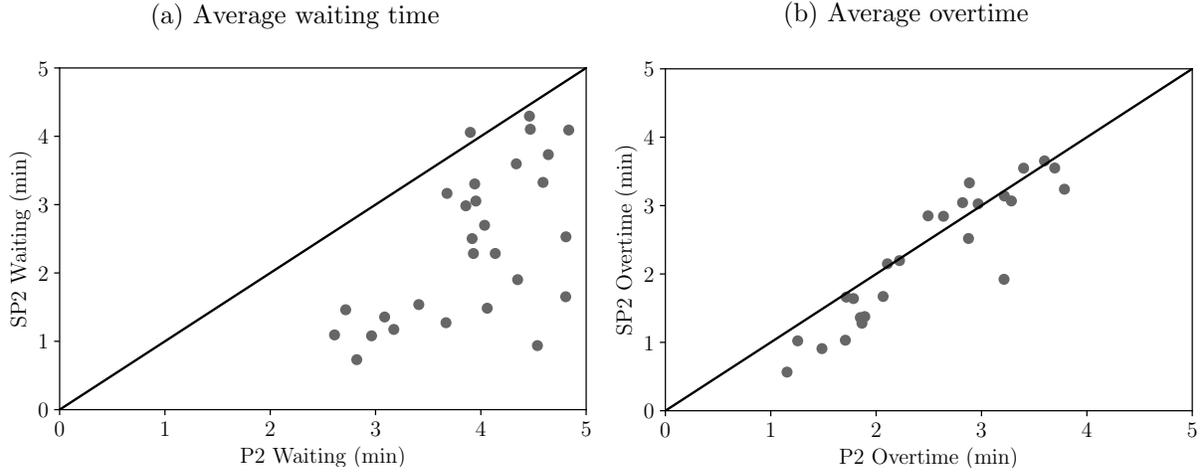
Figure 3 depicts the changes in revenue and cost compositions across all test instances. Both cases show similar effects: Given fixed $|L|$, the proportion for the revenue from serving Type 2 users and also the overall profit increase as $|J|$ increases. However, the proportion of hiring cost for Type 1 drivers decreases as more Type 2 requests appear. Therefore, we

Figure 2 Average revenue and cost compositions for instance with $|L| = 40, |J| = 40$ by proposed models.**Figure 3** Average revenue and cost percentage for all instances by proposed models

infer that our system produces efficient matching and scheduling to accommodate more Type 2 users. Relatively, the proportion of hiring cost is higher for **SP2** than **P2** and the proportion of profit is lower for **SP2** than **P2**. The finding suggests that we need to sacrifice some profit to compensate for a higher quality of scheduling service (which will be further shown in Section 5.2.4). Overall, the proposed system is in good financial health.

5.2.4. Out-of-Sample Tests We conduct out-of-sample tests to evaluate proposed models by measuring Type 2 users’ waiting and Type 1 drivers’ overtime. We generate the set of out-of-sample test scenarios Ω' following the same distribution of stochastic travel time discussed in Section 5.1 and use $|\Omega'| = 1000$.

Figure 4 Average waiting time and overtime per user in proposed system

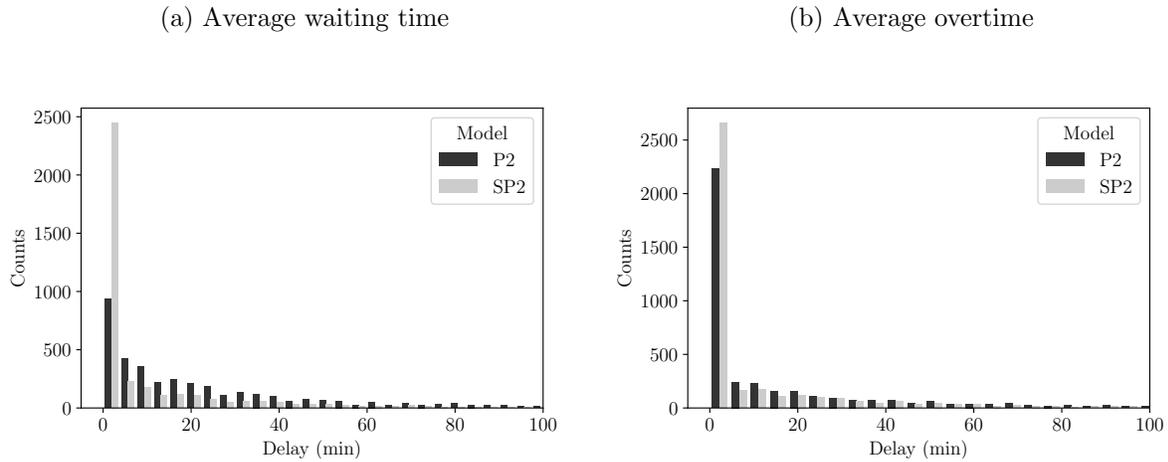


After performing out-of-sample testing, let $s(l)$ be the scheduled time to return the car by Type 1 driver $l \in L$ and $s'(j)$ be the scheduled starting service time for Type 2 user $j \in J$. Values $s(l)$, $l \in L$ and $s'(j)$, $j \in J$ can be obtained from the optimal solutions of α -variables in **P2** and **SP2**. Let $t(l, \omega)$ be the actual time of returning the car by Type 1 driver $l \in L$ in $\omega \in \Omega'$ and $t'(j, \omega)$ be the actual service starting time for Type 2 user $j \in J$ in $\omega \in \Omega'$, which will be computed based on the routing sequence and random travel and service time in scenario $\omega \in \Omega'$. Then for each test scenario ω , we calculate and report:

- $\text{wait}(j, \omega) = \max \{0, t'(j, \omega) - s'(j)\}$ for $j \in J$.
- $\text{overtime}(l, \omega) = \max \{0, t(l, \omega) - s(l)\}$ for $l \in L$.

Figures 4a and 4b demonstrate the out-of-sample test results on average waiting time per Type 2 user and average overtime per Type 1 driver by proposed models. We report the results for all test instances and replications. In Figure 4a, for **P2**, the average waiting time ranges from 2.5 to 5 minutes per customer, and reduces to 1 to 4 minutes for **SP2**. For most of the cases, the average waiting time decreases significantly as the dots appear far below the 45-degree line. However, the performance of models for average overtime per Type 1 driver is mixed. In Figure 4b, the average overtime per driver ranges from

Figure 5 90%-Tail distributions of average waiting time and overtime per user for instance with $|L| = 40$ and $|J| = 60$



1 to 4 minutes for both models and the dots are lying around the 45-degree line, which indicates the slightly better performance of the one of **SP2**. The reason that we do not see an obvious advantage of **SP2** in the overtime performance is that we set the same penalty coefficients for waiting time and overtime. It leads the model to treat waiting time and overtime indifferent and therefore focus on minimizing the overall penalties. If stakeholders weigh more on the overtime, one can adjust the unit penalty coefficients accordingly.

We also demonstrate the detailed out-of-sample performance for one specific instance. Figure 5 shows partial distributions of the average waiting and overtime per user in one test instance with 40 Type 1 drivers and 60 Type 2 users. Due to the significant amount of zero waiting and overtime per user (in more than 90% of all the out-of-sample scenarios), we report the tail distributions of the highest 10% waiting and overtime outcomes. Both Figure 5a and Figure 5b show long-tail effect that the waiting and overtime are concentrated on small values for most cases. Comparing the performance of the **P2** and **SP2**, **SP2** yields relatively short waiting and overtime than **P2**. Similar observations can be drawn in other instances as well.

6. Conclusions

In this paper, we designed a new shared-mobility system to serve transportation needs of underserved populations. We integrated both carsharing and ride-hailing, and developed a two-phase approach to design and operate such a system. We evaluated the models on various instances based on synthetic data in Washtenaw County, Michigan, focusing on serving jobless, elderly, and disabled populations. Numerical results indicated the computational tractability of our proposed solution approaches. Furthermore, the quality of service of the system was maintained at high levels.

We further extended the basic model to a two-stage stochastic programming model to capture the randomness of vehicle travel time and service time. Numerical comparisons with a deterministic counterpart using expected values of the random parameters showed the advantages of our models. Both in-sample and out-of-sample test results demonstrated the effectiveness of matching and scheduling using our approach, where the risk of waiting and overtime both decreased.

For future research, our models and results can be extended as follows. First, in this paper, we minimize the expected cost of overtime and waiting time. Instead, a robust optimization model that focuses on the worst-case analysis can be used for ensuring reliable operations. Second, we used census data to test our models. We are now working with nonprofit organizations and city governments and have participated in the design of a website (<http://communityride.us>) to be used for real-world deployment of the CRS system. We aim to make more transportation data of underserved communities available to the public through further investigation of this topic.

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