Mileage Bands in Freight Transportation

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Abstract

Contract carriers in the trucking industry are known to offer shippers a per mile rate that decreases stepwise as the shipper's route lengthens, with a mileage band designated for each rate. While the use of quantity-based pricing discounts in supply chains has been well studied, there has been no research on how shippers should route under such a pricing scheme or how carriers should set such bands. In this paper, we provide methods for both. Route construction is complicated by the fact that the per mile rate cannot be determined until after the route has been created. With this consideration, we develop a model of this problem and then an algorithm for solving it that assists the shipper in constructing the lowest cost route. It is beneficial for the shipper to extend the length of certain routes to incur a lower per mile cost, and we find that most of these routes can be constructed to equal any mileage required to receive the lower rate. As an extended route generates unnecessary expense and energy use for the carrier, theoretical and analytical insights provide guidelines for a carrier to use in developing better mileage bands. These guidelines assist a carrier in constructing bands that maximize profit and minimize the cost associated with a shipper extending a route.

Keywords: Transportation; pricing; Traveling Salesman Problem; mileage bands

1. Introduction

Within the trucking industry, contract carriers use a variety of pricing schemes, typically utilizing rates proportional to distance and weight or volume. Another pricing practice involves the use of mileage bands. Mileage bands segregate route distances into windows, with the rate per mile (generally) decreasing as the mileage increases. For example, the rate for a route within the band of 500 to 599 miles may be \$1.65 per mile, decreasing to \$1.60 per mile for a route in the band of 600 to 699 miles. These rates are not marginal, rather they are applied to the mileage of the entire route. This rate scheme incentivizes a shipper to use the trucking company for more of their transportation needs, while allowing the trucking company to distribute the fixed costs associated with a route (e.g. cost of the truck, insurance, license, etc.) over a longer distance. Discussions with shippers have indicated that mileage bands are found in

Lower bound	Upper bound	$/mile	Percentage decrease
1900*	1999	1.514	-
2000	2099	1.456	3.83%
2100	2199	1.404	3.57%
2200	2299	1.375	2.07%
2300	2399	1.353	1.60%
2400	2499	1.332	1.55%
2500	2599	1.313	1.43%
2600	2699	1.296	1.29%
2700	2799	1.28	1.23%
2800	UP	1.265	1.17%

Table 1: Mileage band rates utilized by a trucking firm. *Any mileage below 1900 pays an additional penalty per mile for the difference between actual mileage and 1900.

settings where a shipper contracts a trucking company for dedicated use. With more trucking companies focusing on providing dedicated service (Smith, 2017), an increase in the use of mileage bands may be expected. An executive with a transportation solutions provider states, "we have a few shippers today that have that kind of rating structure. I would venture to say that the carrier/shipper negotiations are starting to get more creative in this regard." After completing service over a fixed time period, often one week, the trucking company calculates the total distance traveled for a specific shipper to determine which mileage band this distance falls within. This research was initially motivated by a company whose transportation provider utilizes the rates shown in Table 1. While some contract carriers in the trucking industry use mileage bands to price routes, research has widely ignored this practice.

This pricing scheme is of interest to both trucking companies and their customers. Forkenbrock (1999) discuss the transportation tapering principle, showing that the per mile operating costs incurred by a trucking company decrease with an increase in the length of the haul. Based on this principle, the per mile profit of a carrier using mileage bands does not necessarily decrease proportionately to the lower per mile rate they offer their shippers at longer distances. This draws more business from the shipper and creates a customer service. However, a shipper whose route distance falls just short of a bound is incentivized to adjust the route such that the distance is extended and they qualify for a lower per mile rate. They may enforce these adjustments by dictating a route be visited in an order that leads to the desired distance. While beneficial to the shipper, such longer, suboptimal routes have a detrimental impact on both the carrier and the environment. Energy is expended unnecessarily and, depending on the associated costs, the carrier may generate less profit.

This paper is the first, to our knowledge, to focus on the mileage band pricing scheme, and it looks at the scheme from both a shipper and carrier's perspective. From the perspective of the shipper, we define a new problem, the Traveling Salesman Problem with Mileage Bands. We propose an algorithm to solve this problem, by which a shipper may adjust the order of stops on a route to achieve the lowest cost based on mileage band pricing. Through this algorithm, we find that most routes with even a single digit number of stops can be adjusted to match any reasonable bound on distance, such that a shipper can easily create a route that incurs the lowest possible cost for them. From the perspective of the carrier, a set of theoretical guidelines outline how they may establish mileage bands that maximize profit or limit the likelihood a route is extended by a shipper. We present an algorithm that generates a set of mileage bands that maximize profit and minimize the cost associated with route extension. This algorithm provides several managerial insights that can assist a carrier in developing mileage bands that are most appropriate for their needs.

This paper is organized as follows. Section 2 places this work into the context of non-linear cost functions and quantity discounts within the supply chain realm. Section 3 presents the problem from the perspective of the shipper, with an algorithm that finds the route that minimizes his or her cost. Section 4 focuses on the carrier perspective, presenting theoretical and numerical insights into how a carrier may best select mileage bands to maximize profit. Finally, Section 5 provides conclusions and opportunities for continuing research in this area.

2. Literature Review

In most routing problems, the cost of a route is determined by converting the distance traveled on the route into a monetary or time cost, with this cost generally increasing linearly with distance (Toth & Vigo, 2001). Literature has also focused on transportation problems with both convex and nonconvex piecewise linear costs. However, all of this research utilizes variable costs over route segments or based on the volume or weight transported. There exists a gap in transportation-focused research, with little consideration for costs dependent on total route length, as required when using mileage bands.

The modeling component of this work is most closely related to the network flow problem where the cost of traveling on each arc is piecewise linear relative to the flow on the arc or the distance of the arc. This problem is formulated as a multi-objective, multi-modal, multi-commodity flow problem with time windows and concave costs by Chang (2008), where the cost on each route segment is dependent on the flow over that segment. Croxton et al. (2003) and Croxton et al. (2007) provide structural observations on several formulations of the problem with flow dependent costs. The transportation tapering principle of Forkenbrock (1999) is incorporated into a supply chain design model by Vieira et al. (2015), who use a continuous correctional factor to decrease the cost of a route segment as the distance increases. However, the piecewise linearity of these costs is a function of each individual segment, such that the cost is only a function of the flow on that segment, not the total flow through the network. With mileage bands, the cost associated with traveling over any segment can only be determined when the entire route is constructed.

Research also considers a broader perspective on economies of scale using the volume or weight of shipments moving through a network. Baumgartner et al. (2012) account for a wide range of supply chain related costs, including product cost, transport cost, tank rental cost, tank throughput cost, and inventory cost, and how these costs vary dependent on the volume of product being stored or transported. Moccia et al. (2011) present a multimodal network with consolidation that uses piecewise linear costs to model the fixed costs that may be shared through the combination of loads. Klincewicz (1990) solve a similar problem in using piecewise linear transportation costs based on volumes to decide whether to ship a product directly or via a consolidation terminal. The facility location problem is well known for utilizing techniques associated with economies of scale, where the costs associated with opening a facility are piecewise linear with the size of the facility or the volume of product moving through the facility. Holmberg (1994), Holmberg & Ling (1996) and Wollenweber (2008) used staircase shaped costs to represent the varying production levels at facilities that are under consideration in the problem.

Considering both facility location and routing in network design creates a richer problem, with past research applying piecewise linear costs to one or both components. Lapierre et al. (2004) introduce a hybrid metaheuristic to determine the number and the location of transshipment centers as well as the best transportation alternative - LTL, FTL, Parcel, or own fleet - on each segment, with piecewise linear costs based on which alternative is used and the shipment weight or volume. Similarly, Klincewicz (2002) use piecewise linear costs dependent on volume on each arc to determine the location of transportation hubs and how to direct flow through the resulting network. Melechovskỳ et al. (2005) solve the location-routing problem with non-linear piecewise costs associated with opening a depot and the corresponding flow passing through. We again highlight that the costs associated with these problems are known during construction of the solution, while with a mileage band pricing scheme the cost is unknown until a route or set of routes are completed.

Most closely related to a mileage band pricing scheme is the concept of volume quantity discounts (Dolan, 1987, Kohli & Park, 1989, Weng, 1995, Güder & Zydiak, 1997). This topic has been extensively covered from the perspective of marketing, manufacturing and economics. However, while ordering greater quantities results in increased holding costs, this is generally viewed as a favorable policy for both parties given that the buyer has a tangible increase in product on hand, while the seller should see increased profits. In the case of mileage discounts, extending the route does not necessarily have these same tangible benefits. Ultimately, additional resources are being used to execute the same route. While the shipper sees a reduction in cost, the carrier generally does not have an incentive to extend the route. The results presented in this paper may inform research on quantity discounts, but the perspective on the two problems is quite opposite.

3. Shipper Focused Route Optimization

In the traditional traveling salesman problem (TSP), a vehicle departs from a depot to visit a set N of stops over a period of time, with an objective of minimizing a cost function, most commonly associated with distance traveled. When costs are determined using mileage bands, this objective may vary depending on perspective. Assuming that the rate per mile decreases as the mileage increases between bands, the objective of the carrier is to minimize route distance. A route at this distance is within the band with the greatest possible rate per mile that may be collected from the shipper, while it also minimizes the costs that the carrier may incur while executing the route. However, from the perspective of the shipper, a solution where the route distance is minimized is not necessarily optimal. A route with a length that is close to the upper bound of a mileage band is more expensive than a longer route in the next mileage band, and the shipper has a financial incentive to extend the route. For this section, we solve the problem from the perspective of the shipper, such that cost, rather than distance, is minimized. We present a formulation for a modified TSP, a algorithm to quickly solve this model, and numerical results from testing the algorithm.

3.1. Traveling Salesman Problem with Mileage Bands

We consider the problem of finding the lowest cost route given that the rate per mile decreases between mileage bands as the route length increases. This is modeled over a network where every node N in the network must be visited, and we assume that the graph is complete. Let S represent the set of mileage bands, with B_s representing the lower bound for band $s \in S$ and R_s representing the rate per mile for band s (note that $B_0 = 0$ and $B_{|S|+1} = M$, where M is a very large number, in all cases). We assume the bounds on the mileage bands are such that the total route cost at the bounds increases as the bands increase, or $R_s B_s < R_{s+1} B_{s+1}$ for all $s \in S$. In this way, the route cannot be continuously extended to decrease cost. The variable x_{ij} is 1 if the vehicle travels from node i to j and is 0 otherwise, while d_{ij} represents the distance traveled between the two nodes. Then, the variable x_s is the total distance traveled, if that distance falls within mileage band s, where $x_s > 0$ only if $B_s \leq x_s < B_{s+1}$. The binary variable y_s takes a value of 1 if x_s falls within the bounds of band s and is 0 otherwise. We may then minimize the cost for the shipper using the following formulation:

$$\text{minimize} \sum_{s \in S} R_s x_s$$

subject to

$$\sum_{i \in N, i \neq j} x_{ij} = 1 \quad \forall j \in N \tag{1}$$

$$\sum_{j \in N, i \neq j} x_{ij} = 1 \quad \forall i \in N \tag{2}$$

$$\sum_{i,j \in N} d_{ij} x_{ij} = \sum_{s \in S} x_s \tag{3}$$

$$B_s y_s \leq x_s < B_{s+1} y_s \quad \forall s \in S \tag{4}$$

$$\sum_{s \in S} y_s \leq 1 \tag{5}$$

$$\sum_{i,j \in Q} x_{ij} \leq |Q| - 1 \quad \forall Q \subseteq N \tag{6}$$

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in N, \tag{7}$$

$$x_s \geq 0 \quad \forall s \in S \tag{8}$$

$$y_s \in \{0, 1\} \quad \forall s \in S \tag{9}$$

The objective function minimizes routing cost for the total route distance x_s at the per mile rate of R_s for band s. Constraints (1) and (2) require that the route visits each node in N once. Constraint (3) sums the distances of all segments traveled to determine the total route distance. Constraints (4) bound the total route distance within the appropriate band. Constraint (5) limits the solution to only one band. Constraints (6) are the subtour elimination constraints.

3.2. Bounded TSP Algorithm

There already exist numerous highly effective TSP solvers that may quickly solve problem instances of the size that a freight carrier may experience (generally fewer than 100 stops in a week). We propose an exact algorithm that may utilize one of these solvers to find the minimum cost for a shipper. A constraint is added to the TSP that places a lower bound on distance, when necessary, and the solver determines the shortest route that satisfies the bound. We first introduce the *mileage break*, a metric similar to the weight break used with a pricing scheme based on mass. This is the distance within a band at which the cost of the route is equivalent to the cost of a route at the lowest bound of the next mileage band. For example, suppose the per mile rate for a route of 900-999 miles is \$1.50 per mile and the rate for a route of 1000-1099 miles is \$1.45 per mile. The mileage break for the 900-999 band is $\frac{1{,}000 * \$1.45}{\$1.50} = 966.7$ miles. The cost of any routes within this band greater than 966.7 miles may potentially be decreased by extending the route to the next band. We define the mileage break for band s as $Y_s = \frac{B_{s+1} R_{s+1}}{R_s}$.

Algorithm 1 provides the pseudocode used to solve this problem. The TSP is first solved and we determine the distance of the minimum length route, D^*, with no consideration for mileage bands. The mileage band, s, for distance D^* is determined. If this distance is less than the mileage break for that band, Y_s, the route is optimal for the shipper as the cost cannot be decreased. However, if the distance is greater than Y_s, a constraint is added that bounds the TSP such that the length of the new solution, D', must be greater than or equal to the bound on the next mileage band, B_{s+1}. The bounded TSP is solved and the distance of the new solution is D'. If the new distance is less than or equal to the mileage break of this band, $D' \leq Y_{s+1}$, and if the new route length results in a decrease in cost, such that $R_s D^* > R_{s+1} D'$, this route is optimal for the shipper. If the cost is greater, then D^* is optimal. However, if $D' > Y_{s+1}$, then there may be a lower cost solution in the next band. This cycle may continue as long as the cost of the original route is greater than the cost at the mileage break of successive bands, such that $R_s D^* > R_{s+i} Y_{s+i}$ for any band $i \geq 0$. A narrow band and large change in R_s between bands are required for a scenario where a route will be extended beyond the next band, but we include this here for completeness.

Algorithm 1 Bounded TSP

Solve TSP for unbounded problem to find a solution, Z, with the shortest route length, D^*.
Determine band s for length D^* and corresponding mileage break Y_s.
if $D^* \leq Y_s$ **then**
  Return Z.
else if $D^* > Y_s$ **then**
  Set $J = R_s D^*$ and iterator $i = 0$.
  while $R_s D^* > R_{s+i} Y_{s+i}$ **do**
    Add constraint $D' \geq B_{s+1+i}$ to TSP.
    Solve bounded TSP to find optimal solution, Z', with route length D'. If no solution exists, return Z.
    Set $J' = R_{s+1+i} D'$.
    If $J' \leq J$, then set $Z = Z'$.
    if $D' \leq Y_{s+1+i}$ **then**
      Return Z.
    else if $D' > Y_{s+1+i}$ **then**
      Increment i.
    end if
  end while
  Return Z.
end if

If we assume that there is only the bound $B_0 = 0$ on the IP formulation, this is the traditional TSP, equivalent to the first component of Algorithm 1. If a non-zero bound is introduced, the solution to the IP formulation is either the shortest unbounded route or the lowest cost route greater than the bound, dependent on which has the lowest cost. This is equivalent to Algorithm 1 for all bounds, such that we may now use the algorithm to determine the optimal route to achieve the lowest cost for a shipper.

3.3. Numerical Analysis

The algorithm is evaluated using the symmetric set of the well known TSPLIB cases. Trucking companies often use a one week time frame to determine the total distance that a vehicle traveled on a route for a shipper. Over a five day week, it is unlikely that a truck will be able to make more than 15 stops per day. Based on the transportation context of this problem, only those instances with fewer than 76 stops are tested. Of the TSPLIB cases, this leaves 19 unique instances ranging from 14 to 76 stops. For those instances where the optimal distance is greater than 6000, all distances are divided by ten. It is extremely rare for a driver to travel over 6000 miles, even in a week, and it is unlikely that a trucking firm would have mileage bands that extend to this distance. Each instance is solved using CPLEX 12 on a cluster of machines with 8 Intel Xeon CPUs running at 2.66 GHz with 32 GB RAM with a processing time limit of three hours.

The bands may be arbitrarily defined for these instances. Moving the bounds on a band will bring some route lengths closer to the upper bound and move some into the lower end of a band. We would like to determine the magnitude of savings relative to the change in cost per mile and where the route length falls within the band. With a sufficiently narrow band and a diverse set of instances, any initial bound should provide a description of how the bands impact savings. As 100 miles is used in industry as a band width, we set the bounds every 100 miles beginning with the first bound at 0.

For the results presented here, the per mile rates from Table 1 are used as these provide a range of realistic values. The percentage decrease in rate from band s to $s+1$ is $P_s = \frac{R_s - R_{s+1}}{R_s}$. Table 2 presents the percentage decrease in cost for those instances where savings could be found through route extension. These values are calculated as $\frac{R_s * D^* - R_{s+1} D'}{R_s * D^*} > 0$, using each R_s and R_{s+1} from Table 1. Results with an N/A indicate those that violate the inequality $R_s B_s < R_{s+1} B_{s+1}$. Those instances with a minimum route length just below a bound show the greatest opportunity for savings, greater than 1% in many cases. In the trucking industry, where many firms operate with profit margins below 5%, a 1% difference in total cost is significant for both the shipper and carrier. The cost savings increase with longer routes, as does the window within which routes should be extended, indicating that a shipper should request that a carrier bill them by collecting the routing distance over a longer period, perhaps a week rather than a day.

While revealing the magnitude of savings, quantifying the decrease in cost does not reflect the frequency with which a route may be extended. As the bands may be arbitrarily set, the position of a route length within the band and the resulting likelihood that the route should be increased to decrease cost are also arbitrary. For a shipper to consider extending a route, it must be feasible for the stops on the route to be arranged such that the length is greater than the bound on the next band, but sufficiently short to reduce total cost. If route construction is flexible enough, this is a plausible option. Alternatively, without that flexibility, reducing costs by moving to cheaper mileage bands is not realistic.

Note that in Table 2, the algorithm was unable to find a route that matched the bound on the band in only three of the 19 instances, and in those three instances the solution added at most three miles above the bound. The instance in which the route was three miles above the bound

				Percentage decrease in per mile rate between bands, P_s								
				3.83%	3.57%	2.07%	1.60%	1.55%	1.43%	1.29%	1.23%	1.17%
Instance	Minimized distance	Bound of next band	Bounded distance	Percentage decrease in total routing cost								
att48	1062.8	1100	1100	0.46%	0.20%	-	-	-	-	-	-	-
bayg29	1610	1700	1700	-	-	-	-	-	-	-	-	-
bays29	2020	2100	2100	0.02%	-	-	-	-	-	-	-	-
berlin52	754.2	800	800	-	-	-	-	-	-	-	-	-
brazil58	2539.5	2600	2600.2	1.53%	1.27%	-	-	-	-	-	-	-
burma14	3323	3400	3403	N/A	N/A	-	-	-	-	-	-	-
dantzig42	699	700	700	3.69%	3.43%	1.93%	1.46%	1.41%	1.29%	1.15%	1.09%	1.03%
eil51	426	500	500	-	-	-	-	-	-	-	-	-
eil76	538	600	600	-	-	-	-	-	-	-	-	-
fri26	937	1000	1000	-	-	-	-	-	-	-	-	-
gr17	2085	2100	2100	3.14%	2.88%	1.36%	0.89%	0.84%	0.72%	0.58%	0.52%	0.46%
gr21	2707	2800	2801	N/A	0.22%	-	-	-	-	-	-	-
gr24	1272	1300	1300	1.71%	1.45%	-	-	-	-	-	-	-
gr48	5046	5100	5100	N/A	N/A	N/A	0.55%	0.50%	0.37%	0.24%	0.18%	0.11%
hk48	1146.1	1200	1200	-	-	-	-	-	-	-	-	-
st7	675	700	700	0.27%	-	-	-	-	-	-	-	-
swiss42	1273	1300	1300	1.79%	1.53%	-	-	-	-	-	-	-
ulysses16	685.9	700	700	1.85%	1.59%	0.05%	-	-	-	-	-	-
ulysses22	701.3	800	800	-	-	-	-	-	-	-	-	-

Table 2: Percentage decrease in cost through the extension of a route to the next band using the change in per mile rates from Table 1

is also the smallest instance with 14 nodes. As the number of solutions grows exponentially with the number of nodes, given a sufficient number of nodes, the algorithm is able to find a solution that matches any bound within a reasonable distance. This indicates that if a shipper wishes to create a route at a specific bound, it is almost certain that such a route could be created.

As the number of nodes decreases, the number of potential solutions also decreases and it may become difficult to fit the route to a specific distance. To evaluate the potential to adjust those routes that have fewer than 25 stops, the number of nodes within each instance is reduced by 75%. The TSPLIB instances with an original number of 76 to 107 nodes are included in this analysis (an additional five instances), such that the largest instance has 26 stops. The instance burma14 is not included as removing 75% of nodes leaves it with three stops, which can only be traveled using one minimum route length. Three instances are created from each original instance, generating a total of 69 instances. If the number of nodes in the original instance is X, the nodes removed are the first $0.75X$ nodes, the last $0.75X$ nodes, and the middle $0.75X$ nodes. Four sets of bounds on the mileage bands were used, with a bound every 100, 150, 200, and 300 miles. Therefore, each original instance produces three instances tested with four bounds, generating 12 opportunities for the algorithm to find a route close to the bound.

Table 3 presents the aggregated results for all instances, with the average percentage by which the route length increased when a bound was added and the number of instances in which the route length was greater than the bound. Note that for the instances with four nodes, only two alternative route lengths may be calculated and in several instances a solution could not be found greater than a larger bound. Of the 252 instances, 98 do not have a solution equivalent to the corresponding bound. It does become more difficult to find a solution that exactly matches the bound as the number of nodes is reduced; however, this is still possible for the majority of

	Nodes	Average percentage increase in route length above bound	Number of instances with a route length greater than the bound (out of 12)
gr17	4	11.96%	*
ulysses16	4	18.99%	*
gr21	5	8.67%	12
ulysses22	5	24.21%	12
fri26	6	1.57%	7
gr24	6	1.12%	7
bayg29	7	0.82%	7
bays29	7	0.41%	10
dantzig42	10	0.00%	0
swiss42	10	0.88%	1
att48	12	0.05%	3
eil51	12	0.00%	0
gr48	12	0.64%	10
hk48	12	0.42%	7
berlin52	13	0.17%	6
brazil58	14	0.01%	7
st70	17	0.00%	0
eil76	19	0.00%	0
rat99	24	0.00%	0
eil101	25	0.00%	0
rd100	25	0.02%	1
lin105	26	0.01%	1
pr107	26	0.01%	7

Table 3: Instances with an increase in route length

instances with each bound, particularly for routes with ten or more nodes.

It is important to note that not every shipper will extend a route when it is feasible and financially beneficial for them to do so. Some shippers may find the risk associated with adjusting a schedule to extend a route to be much greater than any potential cost savings. These changes may result in the violation of time windows, increased in-transit inventory costs, or delivery errors. The $75 cost savings from extending a route five miles may be too risky for one shipper, while in a more extreme scenario certain shippers may find the $5 cost savings from extending a route 40 miles to be worth considering. This model does not make a decision for the shipper, rather it provides information that the shipper may use to make the decision themselves.

4. Carrier Focused Mileage Band Construction

Based on these results, a shipper may have both the incentive and the flexibility to extend route distances to decrease costs. However, from an environmental perspective, and from the viewpoint of the carrier, extending a route is inefficient and costly. To limit these costs, the carrier should set the mileage bands to minimize the frequency with which shippers extend their routes. In this section, we provide a detailed analysis of how a carrier may best create the bounds on the mileage bands such that their total profit is maximized while the lost profit associated with shippers extending routes is minimized.

4.1. Model Formulation

Considering the carrier's perspective, we assume that they are creating mileage bands to maximize profit, which is a function of revenue generated from the shipper less the operating cost (e.g. fuel, driver wages, etc.) of executing the route to serve the shipper. We also assume that every shipper that may reduce the rate charged by the carrier by extending a route will do so and, based on the results in Section 3.3, that the shipper may extend the route to a length that matches the bound on the next greater band. It is likely that not every shipper with a financial incentive will extend a route, particularly if that incentive is limited. However, the model we propose indicates how a carrier may create mileage bands limiting the likelihood that a shipper will have any incentive to extend a route, while also maximizing the profits of the carrier regardless of what the shipper does.

We presume the carrier is free to determine all parameters associated with the band scheme, most importantly the bound on each band, B_s, the number of bands, $|S|$ and the rate per mile for each band, R_s. For this analysis, we assume that all R_s values are selected before the bands are set, and that R_s is monotonically decreasing as s increases, such that the highest rate band is $s = 1$. We define the per mile operating costs that the carrier incurs as C_s. These costs may remain constant across all bands, although they most likely decrease as bands increase (Forkenbrock, 1999). The bounds, B_s, are the fundamental variables in this model.

We assume that the bands are determined based on a set of historical routes, L, that the carrier has executed in the past for one or multiple shippers. Each route $i \in L$ has a known length D_i. As many contract carriers operate on a regular schedule, the routes that these carriers follow are often repetitive and the carrier may develop a distribution based on the length of routes previously traveled for a particular shipper or a set of shippers. Let H be the range of the route lengths that the carrier has executed. We also assume that there is a minimum width, w, and maximum width, W, on each band, and that there is a minimum number of bands, V.

The profit that the carrier gains from executing a route without extension is $k_i = D_i(R_s - C_s)$, where i is in band s. If a shipper extends a route when it is beneficial for them to do so, the resulting decrease in profit for the carrier is $l_i = D_i(R_s - C_s) - B_{s+1}(R_{s+1} - C_{s+1})$, where $D_i(R_s - C_s)$ is the profit of the route without extension and $B_{s+1}(R_{s+1} - C_{s+1})$ is the profit of the route if it is extended. Note that $l_i \geq 0$ for all i, such that additional profit is not generated for route distances that are below the mileage break, where $D_i(R_s - C_s) < B_{s+1}(R_{s+1} - C_{s+1})$. We assume that the decrease in operating cost is less than the decrease in rate across all bands, such that $R_s - C_s > R_{s+1} - C_{s+1}$ for all s, and that $R_s - C_s$ is constant across the entire band s. As C_s may change at a different rate from R_s, there may be instances where the shipper extends the route without a loss in profit for the carrier, when $D_iR_s > B_{s+1}R_{s+1}$ but the drop in C_s is large enough that $D_i(R_s - C_s) < B_{s+1}(R_{s+1} - C_{s+1})$. We focus on the carrier perspective here and are only interested in those instances when a route extension results in lost profit for the carrier. The net profit for a carrier for any route is then $k_i - l_i$. Similar to the formulation in

Section 3.1, y_{is} is 1 if route i falls in band s and 0 otherwise. We present the following model that sets the mileage band bounds to maximize the carrier's total net profit:

$$\text{maximize} \sum_{i \in L} (k_i - l_i)$$

subject to

$$k_i \leq D_i(R_s - C_s) + M(1 - y_{is}) \quad \forall i \in L, s \in S, \tag{10}$$

$$l_i \geq D_i(R_s - C_s) - B_{s+1}(R_{s+1} - C_{s+1}) - M(1 - y_{is}) \quad \forall i \in L, s \in S, \tag{11}$$

$$D_i y_{is} < B_{s+1} \quad \forall i \in L, s \in S, \tag{12}$$

$$B_s \leq D_i(M - M y_{is} + 1) \quad \forall i \in L, s \in S, \tag{13}$$

$$\sum_{s \in S} y_{is} = 1, \quad \forall i \in L, \tag{14}$$

$$w \leq B_{s+1} - B_s \leq W \quad \forall s \in S : s < |S|, \tag{15}$$

$$y_{is} \in \{0, 1\} \quad \forall i \in L, s \in S, \tag{16}$$

$$B_s \geq 0 \quad \forall s \in S \tag{17}$$

$$k_i, l_i \geq 0 \quad \forall i \in L \tag{18}$$

The objective maximizes the profit gained from route i, k_i and minimizes the profit lost from a shipper lengthening route i, l_i. Constraints (10) define the profit gained by bounding it from above by the profit generated from executing each route, where we assume that M is a very large number. Constraints (11) define the profit lost by bounding it from below with the difference between the profit generated from executing the route and the potentially lower profit from executing a route the length of the lower bound on the next greater band. Constraints (12) and (13) bound each route length within the appropriate band, such that routes equivalent to a bound are in the greater band. Constraints (14) ensure that each route is within one band. Constraints (15) regulate each band width to make sure they are not larger or smaller than the preestablished limitations.

To better understand the band structure, we consider how the two components of the objective, profit gained and lost profit from route extension, independently impact the band widths, finding that they lead to widely different mileage bands. First, we consider maximizing the k_i profit component, which is done by maximizing each bound.

Theorem 4.1. *Maximizing $\sum_{i \in L} k_i$ is equivalent to maximizing $\sum_{s \in S} B_s$.*

Proof: As $k_i \leq D_i(R_s - C_s) + M(1 - y_{is})$ and D_i is a constant for each i, k_i can only increase if the corresponding value $R_s - C_s$ changes, such that $D_i(R_s - C_s) < D_i(R'_s - C'_s)$ or $D_i(R_s - C_s) < D_i(R_{s-1} - C_{s-1})$. As $R_s - C_s$ is fixed for each band s, the only way to increase this value for any route i is to move the route to the more expensive band $s - 1$. To move a route of fixed length

D_i to the band $s-1$, the bound B_s must be increased to a value greater than D_i. Therefore, $D_i(R_s - C_s)$ is maximized by maximizing B_s. As this holds for every i, $\sum_{i \in L} k_i$ is maximized when $\sum_{s \in S} B_s$ is maximized. □

Therefore, when solely maximizing profit gained, the widest bands should be those with the highest profit per mile, lowest in the range of bands. The binding constraint on band width is then the maximum width, $B_{s+1} - B_s \leq W$, and bands are only less than this width when constrained by the range, H, and the minimum number of bands, V.

The route extension component of the objective function involving lost profit is more complex as it is dependent on whether a route should be extended. As $l_i \geq 0$ for all i, l_i only has a non-zero value when $D_i(R_s - C_s) > B_{s+1}(R_{s+1} - C_{s+1})$. To analyze this component, we introduce an alternative mileage break that indicates the distance at which route extension leads to a decrease in profit, $Y'_s = \frac{B_{s+1}(R_{s+1} - C_{s+1})}{R_s - C_s}$, as well as the percentage decrease in per mile profit, $P'_s = \frac{(R_s - C_s) - (R_{s+1} - C_{s+1})}{R_s - C_s}$. The route is then extended when the distance D_i is between Y'_s and B_{s+1}. Therefore, to minimize the expected value of $\sum_{i \in L}(D_i(R_s - C_s) - B_{s+1}(R_{s+1} - C_{s+1}))$, the number of routes with distance D_i between Y'_s and B_{s+1} must be minimized. This is dependent on the distribution of the route lengths and the space between Y'_s and B_{s+1}. Note that as $Y'_s(R_s - C_s) = B_{s+1}(R_{s+1} - C_{s+1})$ and $R_{s+1} - C_{s+1} = (R_s - C_s)(1 - P'_s)$, $Y'_s = B_{s+1}(1 - P'_s)$.

For our analysis, we consider two common distributions on route length, uniform and triangular. We first assume a carrier has executed a series of routes with lengths that are uniformly distributed over the support $[a, b]$ (note that this support defines the route length range H). Let X be the distance of a potential route and $F(X)$ be the probability that X falls between Y'_s and B_{s+1}, such that $F(X) = \frac{B_{s+1} - Y'_s}{b-a}$. By minimizing $F(X)$, the carrier minimizes the probability that a shipper extends the route, thereby minimizing the expected lost profit from route extension. We now show that minimizing $F(X)$ is done by minimizing each bound.

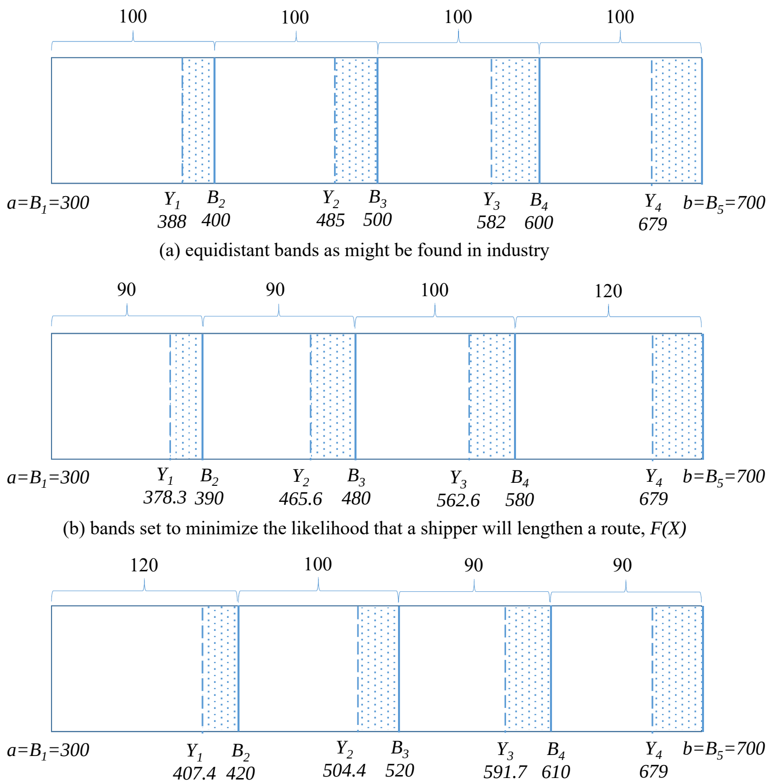
Theorem 4.2. *Given a support $[a, b]$ for a uniformly distributed set of route lengths, minimizing $F(X)$ is equivalent to minimizing B_s, for any $s \in S$.*

Proof: As $F(X) = \frac{B_{s+1} - Y'_s}{b-a}$ and $Y'_s = B_{s+1}(1 - P'_s)$, $F(X)$ may be written as:

$$\begin{aligned} F(X) &= \frac{(B_{s+1} - Y'_s)}{b-a} \\ &= \frac{B_{s+1} - B_{s+1}(1 - P'_s)}{b-a} \\ &= \frac{P'_s * B_{s+1}}{b-a} \end{aligned}$$

As a, b and P'_s are fixed parameters, minimizing B_{s+1}, or B_s, will minimize $F(X)$. □

Therefore, when solely minimizing the lost profit associated with route extension, we must minimize the probability that a route will be extended, which is done by minimizing each bound. Thus, the narrowest bounds are those lowest in the range of bands, which is the complete opposite of how the bands are constructed to maximize profit.



(a) equidistant bands as might be found in industry

(b) bands set to minimize the likelihood that a shipper will lengthen a route, *F(X)*

(c) bands set to maximize profit

Figure 1: Comparison of three mileage band settings for a uniform distribution

Consider the example in Figure 1, with $H = [300, 700]$, $w = 90$, $W = 120$, $V = 4$ and $P'_s = 3\%$ for all bands. The shaded regions are where a shipper would want to extend the route to the next band. Figure 1(a) is a set that is commonly used in industry, equally spaced at 100 miles per band. To minimize the likelihood that a shipper will lengthen a route, $F(X)$, the second figure shows that the first two bands are minimized at 90 miles and the remaining bounds increase in order to fill the remainder of the range. As the value for each bound has decreased, so has the corresponding probability $F(X)$. The final figure presents the bands with the profit maximized, with the first band as the largest set at 120 miles, and the greater bands only decreasing in width due to the limitations of the range. Considering a simple set of route lengths, where there is a route every 10 miles over this range (i.e., 310, 320, 330...) and the profit per mile of the first band is \$0.15, the profit gained and profit lost due to route extension associated with each set of bands is found in Table 4. The bands that minimize the probability of route extension have the lowest profit gained, but also the lowest profit lost associated with route extension. The bands that maximize profit have the greatest profit, but also the highest lost profit from route extension. The equally spaced bands have neither the highest profit gained nor the lowest profit lost.

The expected route lengths that a carrier executes may be found to more closely resemble

	(a)	(b)	(c)
Profit gained	\$2772	\$2761	\$2777
Profit lost	\$3.80	\$3.60	\$5.40

Table 4: Profit gained and lost profit due to route extension for example in Figure 1

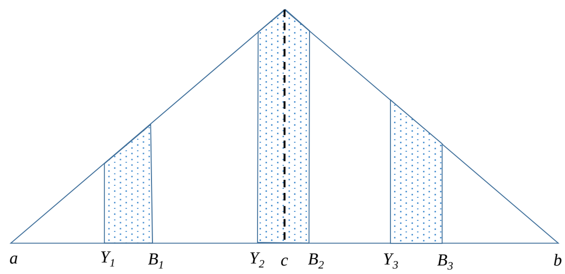


Figure 2: The three mileage band scenarios for a triangular distribution

a triangular distribution. This distribution is defined by a lower limit a, upper limit b, and a mode c. When considering the profit maximizing component of the objective function, Theorem 4.1 still holds as it is not dependent on the distribution. However, the construction of bands changes when minimizing the profit lost from route extension. In considering the probability $F(X)$ that route X has a length between Y_s' and B_{s+1}, each mileage band may fall under one of the following scenarios: 1) the values of Y_s' and B_{s+1} are both less than or equal to the mode c; 2) both are greater than c; or 3) Y_s' is less than or equal to c and B_{s+1} is greater than c. Figure 2 presents mileage bands that each depict one of these scenarios. We now show that minimizing $F(X)$ is dependent on where the band falls within the distribution.

Theorem 4.3. *Given a triangularly distributed set of route lengths with lower limit a, upper limit b, and mode c, $F(X)$ is minimized when:*

1. $Y_s' \leq c$ *and* $B_{s+1} \leq c$*: minimizing* B_{s+1}*, for any* $s \in S$.
2. $Y_s' > c$ *and* $B_{s+1} > c$*:*
   (a) *maximizing* B_{s+1}*, for any* $s \in S$*, when* $B_{s+1} > \frac{b}{2-P_s'}$
   (b) *minimizing* B_{s+1}*, for any* $s \in S$*, when* $B_{s+1} < \frac{b}{2-P_s'}$
3. $Y_s' \leq c$ *and* $B_{s+1} > c$*:*
   (a) *maximizing* B_{s+1}*, for any* $s \in S$*, when* $B_{s+1} > \frac{(ab-ac)P_s'+(a-b)c}{(c-b)P_s'^2+(b-c)2P_s'+a-b}$;
   (b) *minimizing* B_{s+1}*, for any* $s \in S$*, when* $B_{s+1} < \frac{(ab-ac)P_s'+(a-b)c}{(c-b)P_s'^2+(b-c)2P_s'+a-b}$.

The proof of this theorem may be found in the Appendix. The result for bands less than c is very similar to that for the uniform distribution, with narrow bands lower in the distribution. This is primarily because if the bound is closer to the mode, the probability $F(X)$ increases, even if the distance between Y_s' and B_{s+1} is the same as bounds lower in the distribution. Thus, each band that is below c is likely to have a width at or close to w.

For bands greater than c, the bounds on the bands should generally be maximized with the most narrow bands grouped higher in the distribution. A symmetric triangular distribution will always have a mode $c \geq b/2$, such that $B_{s+1} > \frac{b}{2-P'_s}$ for all $P'_s > 0$ and B_s should be maximized to minimize $F(X)$. Also, as $Y'_s = B_{s+1} * (1 - P'_s) = \frac{b*(1-P'_s)}{2-P'_s} \geq \frac{b}{2}$, $Y'_s \geq c$ whenever $c \geq b/2$. As with the other half of the distribution, as the bound is closer to the mode, the probability $F(X)$ increases, even if the distance between Y'_s and B_{s+1} is the same as bounds higher in the distribution. This leaves the widest bands in the middle of the distribution, just above c in a symmetric distribution or $\frac{b}{2-P'_s}$ in an asymmetric one. When the mode of the distribution is less than $b/2$, the bounds less than $\frac{b}{2-P'_s}$ should follow a pattern similar to that for the first scenario.

The third portion of this theorem indicates that a bound should generally not be placed close to the mode. The value $\frac{(ab-ac)P'_s+(a-b)c}{(c-b)P'^{2}_{s}+(b-c)2P'_s}$ is always greater than the mode c and it is less than $\frac{b}{2-P'_s}$ under most realistic conditions. As with the second scenario, bounds above this value should be increased to decrease $F(X)$, while bands below should be decreased. When the bound is decreased to c, the conditions for the first part of the theorem hold and the bound should be further minimized. As the bound is increased to $\frac{c}{1-P'_s}$, the conditions for the second part of the theorem hold and the bound should be maximized.

Consider the examples in Figure 3. As in Figure 1, $H = [300, 700]$, $w = 90$, $W = 120$, $V = 4$ and $P'_s = 3\%$ for all bands. Note the value $\frac{b}{2-P'_s} = 355$, below the mode. The shaded regions are where a shipper would want to extend the route to the next band. The examples on the left are a symmetric triangular distribution, while an asymmetric distribution is on the right. Figure 1(a) depicts bands set as commonly used in industry, equally spaced at 100 miles per band. To minimize $F(X)$ for the symmetric distribution the two bounds below the mode are minimized to create bands of 90 miles, while the bound just above the mode is maximized to create a band of 120 miles, as shown in Figure 1(b). For the asymmetric distribution, the two extreme bounds on either end of the distribution are minimized, while the 120 mile band is shifted right to again straddle the mode. If the 100 and 120 mile bands were swapped, this would lead to a larger $F(X)$ value for the 100 mile band as the shaded region would be moved closer to the mode, while the probability for the 120 mile band would be the same as currently pictured for the 100 mile band. The bands when maximizing profit, as depicted in Figure 3(c), are the same as they were for the uniform distribution. While there are more routes in the 100 mile band than in the 120 mile band, the profit is still maximized by having the widest band at the highest cost per mile as reducing the bound on the first band would also shift all of the other bands to a lower cost per mile. Testing with a set of triangularly distributed route lengths produces results that are similar to those found in Table 1.

4.2. Numerical Analysis

These theoretical results indicate that the two components of the objective function lead to sets of mileage bands that are counter to each other. We now consider how the bands are constructed in balancing these contradictory components. We apply the model to a set of problem instances that test a variety of parameters.

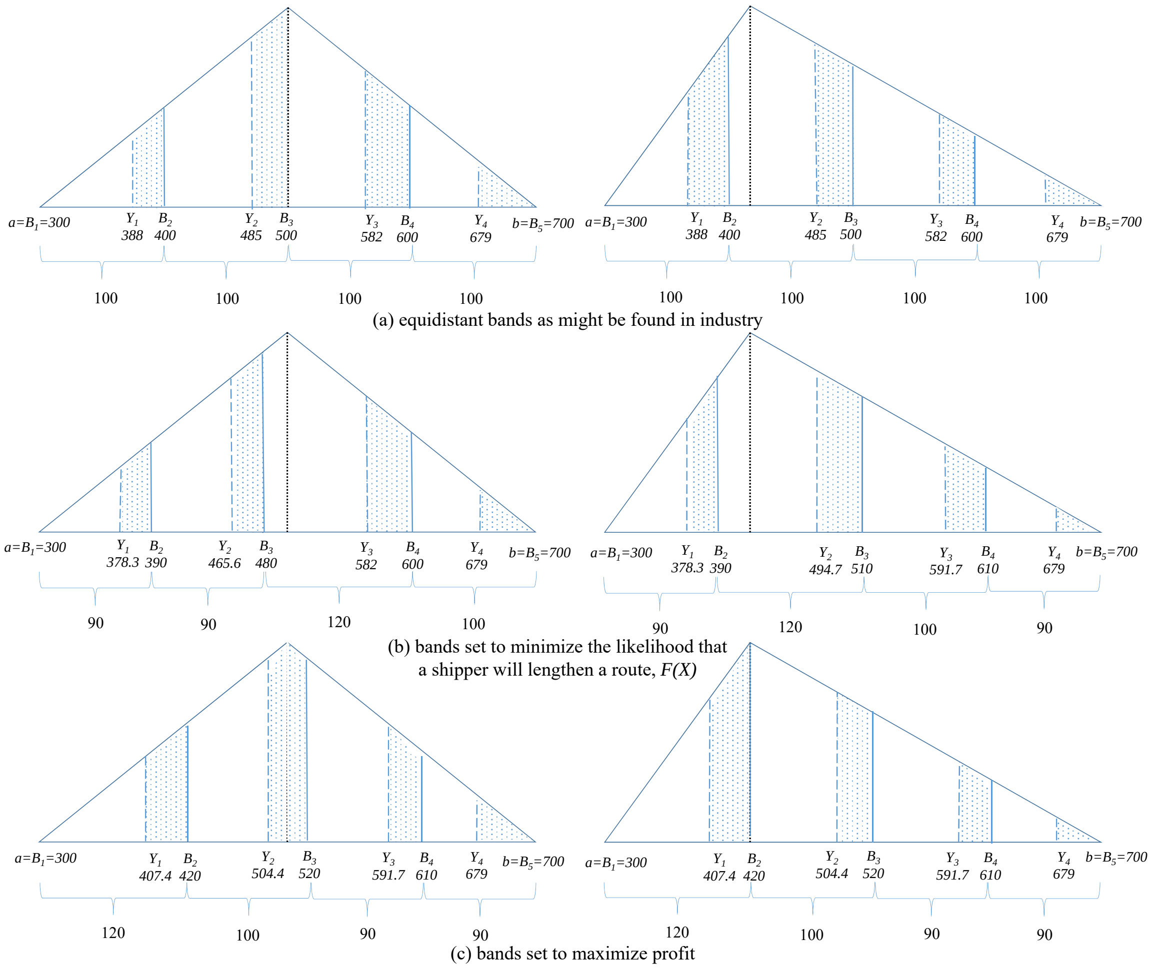


Figure 3: Comparison of three mileage band settings for a triangular distribution, symmetric on the left, asymmetric on the right

The primary input to the model is the set of route lengths that the carrier has operated (or expects to operate). As no previous research has been conducted on this problem, we generate sets of route lengths that mimic what a carrier experiences, while providing an opportunity to test how band widths fluctuate under a variety of conditions. The route lengths are randomly generated using both uniform and triangular distributions over several ranges, H: [500,1500], [1500,2500] and [2500,3500] miles. As the probability $F(X)$ is proportional to B_{s+1} and P'_s, increasing the range within which the route lengths are found will increase $F(X)$, if the change in profit per mile is held constant. If the band width constraints, w and W, were to increase proportionally as the range increases, the same results would be produced for each range. Therefore, to test the impact of route length range on mileage band construction, we maintain the same minimum, $w = 75$, and maximum, $W = 200$, band width for each range. The minimum number of bands, V, is 10. These values are selected as they are representative of industry parameters. They also allow for a comparison to be made with a set of mileage bands that is used in industry in which each 1000 mile range is divided into ten equal bands of 100 miles each.

The number of route lengths is also varied, with fewer route lengths available representing a greater spread of routes within a range or limited information on the expected route lengths. This variation was tested for $H = [1500, 2500]$, with the problem instances composed of 25, 50, or 100 route lengths within this range. All problem instances using the other two ranges have 50 route lengths per instance.

To determine the profit per mile for the bands, we consider both the rate, R_s, and the operating cost, C_s. Based on a survey of industry, the American Transportation Research Institute indicates an average marginal operating cost of \$1.50 per mile for TL carriers in 2015 (Torrey & Murray, 2016). Carriers are less willing to provide rate information. The trucking load board DAT indicates that the average national van rate in November 2017 is \$2.06 per mile (DAT Solutions, 2017), while other trucking boards provide widely varying rates. The carrier rates provided to the shipper that was the basis of this study, shown in Table 1, indicate an initial rate of \$1.514. When considering the range of both rates and costs, per mile profits can vary significantly. Because of these variations, per mile profit in the range of [\$0.05-\$1.50] was initially tested for the first, lowest band. Results indicate that there is virtually no difference in bands as this initial value changes. Therefore, we use the median of the various profit values found in our research, \$0.15, for all analysis reported here.

Numerous values for the rate at which profits decrease from band to band, P'_s, are also considered. The change in P'_s may be linear if the change in C_s is constant while R_s decreases linearly, if a decreasing change in R_s is counteracted by an increasing change in C_s, or vice versa. We test the problem instances with a constant value of $P'_s = 1\%, 2\%$ and 3% between bands. Values above 3% maintain a similar pattern and are not reported for brevity. As R_s and C_s may change at different rates, we also test instances with both increasing and decreasing P'_s. These include a rate that increases by 0.5% from one band to the next with $P'_1 = 0.5\%, P'_2 = 1\%$... (referred to as IR 0.5%), a rate that decreases by 0.5% with $P'_1 = 4.5\%$ (DR 0.5%), a rate that increases by 1% with $P'_1 = 1\%$ (IR 1%), and a rate that decreases by 1% with $P'_1 = 9\%$ (DR 1%). Finally, the values for the percentage decrease in R_s shown in Table 1 are tested on each problem instance, with the assumption that C_s is constant such that $P'_s = P_s$.

A carrier may be interested in limiting route extension as they perceive other intangible costs, such as the environmental impact or operational inefficiency. To evaluate the emphasis that a carrier wishes to place on limiting route extension, we add a weighting parameter, α, on the l_i component of the objective function, such that it becomes:

$$\text{maximize} \sum_{i \in L} (k_i - \alpha l_i)$$

This parameter takes the values $\alpha = \{1, 2, 3, 4, 5\}$. Adjusting this value should indicate how the mileage bands and profits change as more emphasis is placed on minimizing route extension. We also test conditions where only profit gained is maximized with no consideration for route extension, with $\alpha = 0$, and conditions under which only the lost profit from route extension is minimized, where the objective is simply to minimize $\sum_{i \in L} l_i$.

To establish a baseline for the band widths, problem instances with a set of routes evenly distributed across the range are tested for both uniform and triangular distributions. The uniform instance is constructed such that there are 50 route lengths, one every 20 miles over the range (the first route length is equivalent to the range parameter a, while the last is 20 miles below the parameter b). The triangular instance is constructed using the random variate generating functions, $D_U = a + \sqrt{U(b-a)(c-a)}$ for $0 < U < F(c)$ and $X = b - \sqrt{(1-U)(b-a)(b-c)}$ for $F(c) \leq U < 1$, where $F(c) = \frac{c-a}{b-a}$ and U ranges from 0.02 to 1 in increments of 0.02.

The model is solved using CPLEX 12 on a cluster of machines with 8 Intel Xeon CPUs running at 2.66 GHz with 32 GB RAM. Processing time is not a factor. We next examine the computational results in order to address the following:

1. Determine the value for α that most limits route extension with the least impact on profit,
2. Examine how the sparseness of routes over the range impacts the profit associated with mileage bands,
3. Examine how the average route length impacts the profit associated with mileage bands, and,
4. Examine how the change in profit per mile impacts the profit associated with mileage bands.

In the course of this analysis, we show that the mileage bands created by the model outperform bands of equidistant width by both generating more profit and minimizing the frequency with which routes may be extended by the shipper.

4.2.1. Analysis of α

The α weighting parameter that increases the emphasis on limiting route extension has the most significant impact on the model. For each instance, we compare the change in profit gained relative to the change in lost profit due to route extension as the α value increases. Throughout this analysis, we will refer to lost profit due to route extension as the cost of route extension, as it is a cost incurred by the carrier. We determine the largest profit generated across all α values for an instance and calculate the decrease for each of the other values. For example, if the largest profit for one instance is found to be \$10,000 with $\alpha = 1$, while the profit is \$9,000 when the weight is $\alpha = 5$, the decrease in profit with $\alpha = 5$ is \$1,000. The cost of route extension reported is the actual value l_i, not the value multiplied by the α weight.

Figure 4 presents a comparison of the average change in profit and average cost due to route extension for all uniformly and triangularly distributed instances. The two distributions have very similar results. With $\alpha = 0$, profit is maximized while the cost of route extension is also largest. As α increases, both profit and the cost of route extension decrease. For the smaller values of α the decrease in cost outweighs the decrease in profit, indicating that an emphasis should be placed on limiting route extension, particularly if there are additional costs beyond the lost profit. The impact tapers off as α increases above a value of three, amplified when there is no weight on the profit component. Thus, a carrier should consider a limited emphasis on the cost of route extension, with an α value between 1 and 3 appropriate for these instances.

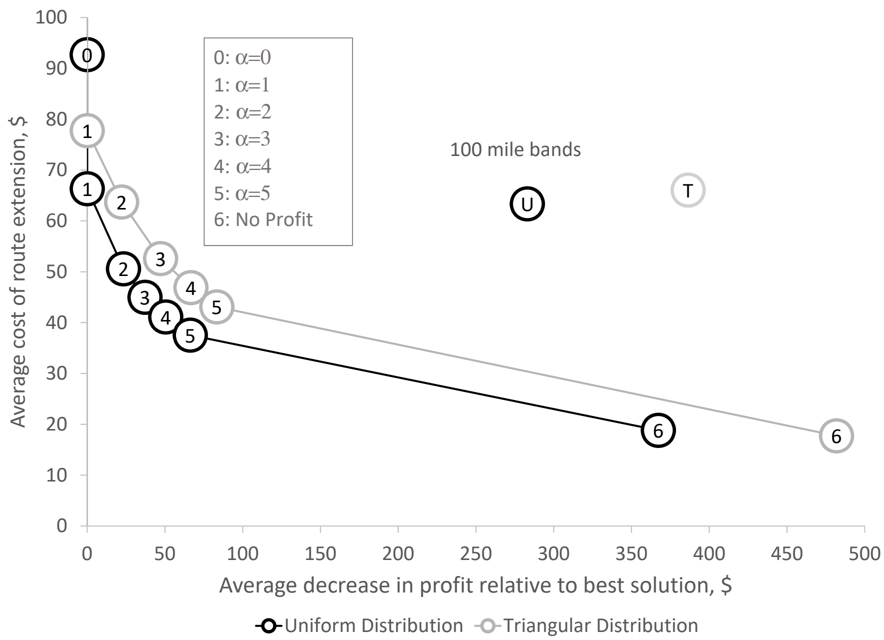


Figure 4: Comparison of average decrease in profit from best solution and cost of route extension for all uniformly and triangularly distributed instances

For an additional comparison, the profit values were calculated for a set of bands used in industry. Each band width is fixed at 100 miles, such that there are ten bands within each range. In Figure 4, the metrics for these two sets of instances are indicated by a U for the uniformly distributed instances and T for the triangularly distributed instances. Using equally spaced bands results in an overall decrease in profit and a possible increase in cost due to route extension. These equidistant bands do result in a lower cost due to route extension relative to those instances that only maximize profit, but at a considerable decrease in profit. Carriers using these equidistant band widths would likely be able to generate additional profit by adjusting the widths as described here.

Figure 5 presents the average width of each of the bands, distinguished by α weight, and the theoretical bands based on maximizing profit and minimizing the cost of extended routes. The number of bands was equivalent to the minimum $V = 10$ for all instances, as increasing the bands decreased profit and increased the opportunities for a route to be extended. Those instances with small α values have the widest bands low in the range, allowing for the most routes to be charged the highest per mile rates. They closely mimic the theoretical bands that maximize profit. The bands low in the range narrow with an increase in α; however, even with a myopic focus on minimizing the cost of route extension, the band widths high in the range do not approach the maximum of 200 miles, as seen in the theoretical bands. This is due to the variation in the route lengths. Profit maximization is not dependent on how route lengths are distributed within the range, simply increasing profits by maximizing the lowest bounds. Minimizing the cost of extending routes takes into consideration where the opportunities for this extension exist. So, if several routes are grouped together low in the range, this band is widened to prevent any of these routes from being extended, rather than creating a wider band higher in the range as theoretically predicted.

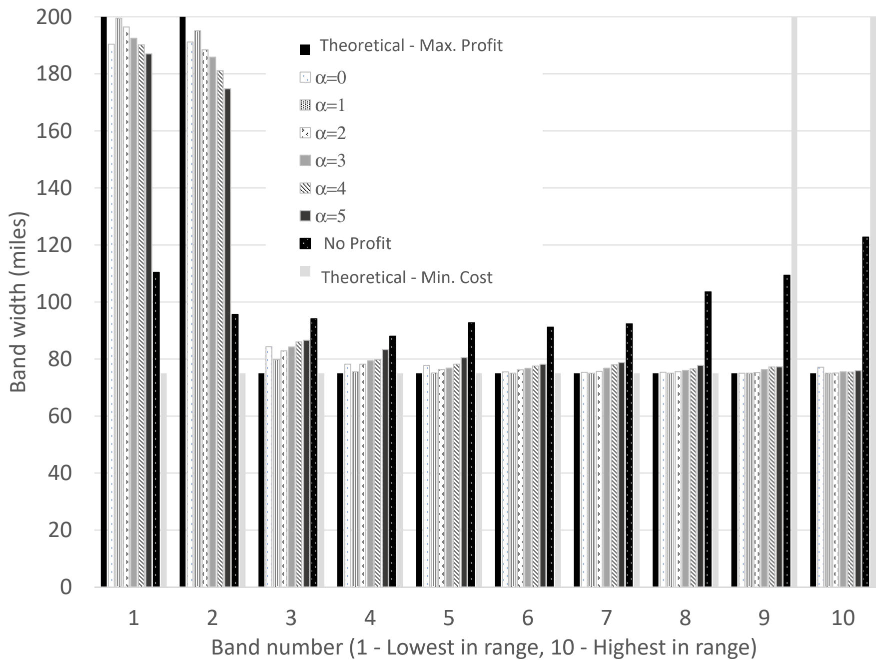


Figure 5: Average width of the bands for all uniformly distributed instances and theoretical bands for maximizing profit and minimizing cost of route extension

To emphasize this point, the model is tested on the evenly distributed uniform instance, with a route length every 20 miles over the range. Figure 6 presents the band widths for this instance. The results for those instances with a small α are very similar to those found in Figure 5, with the lower bands approaching or reaching the maximum limit of 200 miles. The instance with $\alpha = 0$ has a slightly smaller first band as there are multiple optimal solutions under these conditions and the model selected one in which the first band width was not set at the maximum of 200 miles. However, even with $\alpha = 5$, there is not enough emphasis on minimizing the cost of route extension to mimic the theoretical bands. We tested three additional values of $\alpha = 10, 100, 500$, with the bands for $\alpha = 500$ equal to the bands when profit is not considered. For these two instances, band 9 is shorter than the theoretical band, primarily due to the slightly uneven spacing at the end of the distribution (further highlighting the sensitivity of the band widths to the route lengths).

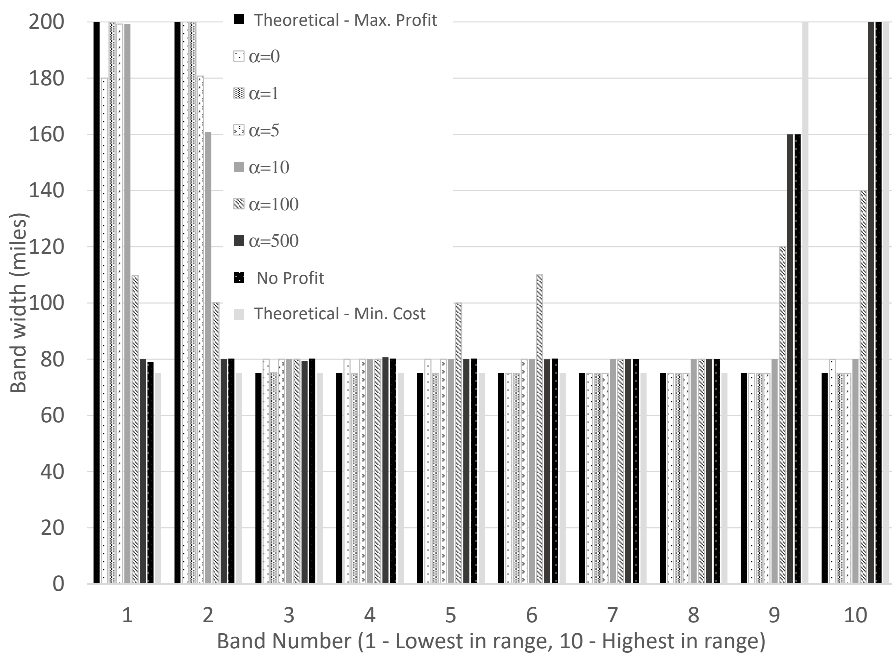


Figure 6: Average width of the bands for instances that are evenly uniformly distributed

Figure 7 presents the average width of each of the ten bands, distinguished by α weight, for the symmetric triangularly distributed instances. The results are similar to those found in Figure 5. Again, the instances with small α values have the widest bands low in the range, with those bands becoming more narrow as α increases, and even with a myopic focus on minimizing the cost of extending a route, the middle bands are not as wide as the theoretical bands. This is again due to the variation associated with the randomly generated route lengths. Note that with the theoretical bands, band 6 straddles the mode c. The results with evenly triangularly distributed route lengths, in Figure 8, are similar to those presented in Figure 6, in that they closely replicate the theoretical bands, but only when α is greatly increased.

We now consider the impact that several other parameters have on the profit and cost values. Unless otherwise indicated, the band widths are similar to those already presented for the remaining instances, so they are not presented here for brevity. The following results are reported only for the uniformly distributed instances, as the triangularly distributed instances reflect similar patterns.

4.2.2. Analysis of route length density

A carrier may serve routes that are sparsely distributed over the range, H, or they may simply have a shorter history of routes from which to draw a distribution. This impacts the structure of the bands and the resulting profits. As the number of routes used for an instance varies from 25 to 100, the profits also vary considerably. To make a valid comparison, both metrics are normalized by dividing the profit or cost for each instance by the largest profit or

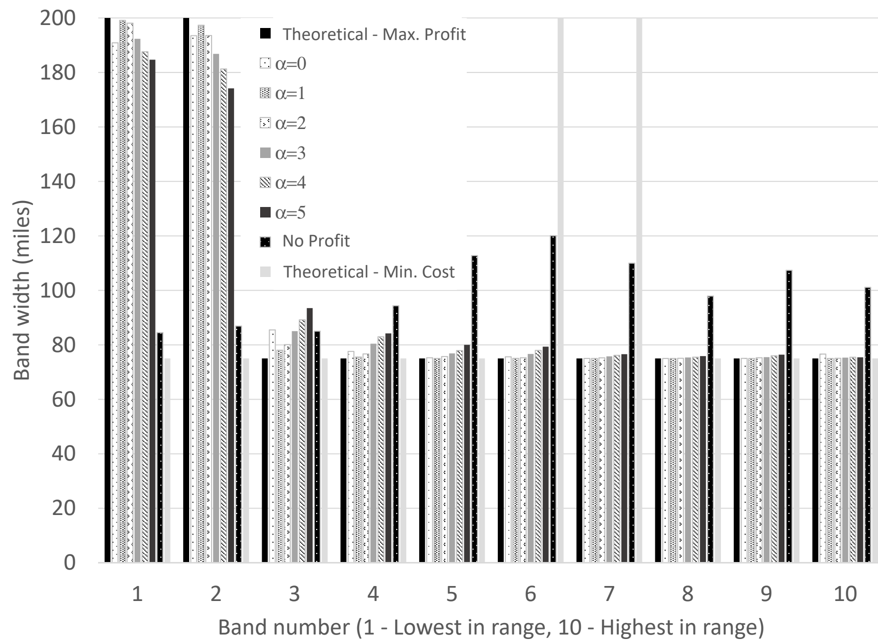


Figure 7: Average width of the bands for all symmetric triangularly distributed instances and theoretical bands for maximizing profit and minimizing cost of route extension

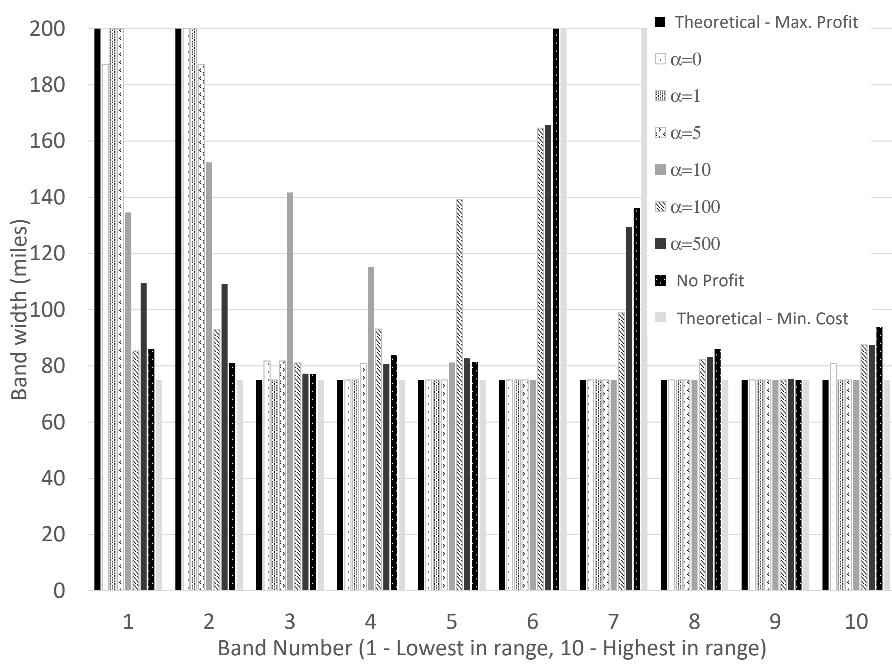


Figure 8: Average width of the bands for instances that are evenly triangularly distributed

cost, respectively, among all α values for that instance, such that 100% indicates the largest decrease in profit or increase in cost and 0% the smallest.

Figure 9 presents a comparison of the instances with a varying number of routes spread across the range $H = [1500, 2500]$. As the number of route lengths within the distribution decreases, a larger α leads to a greater drop in cost of route extension, but also a greater decrease in profit. For those instances where the route lengths are sparsely distributed, it is easier for the bounds to be spaced to limit opportunities for route extension. However, as there are fewer routes that amount to less profit, adjusting the bands has a bigger impact on that profit. These results indicate that a carrier that executes routes with lengths that are more dispersed across the range is likely to limit costs due to route extension, but with a slightly greater risk of lost profit.

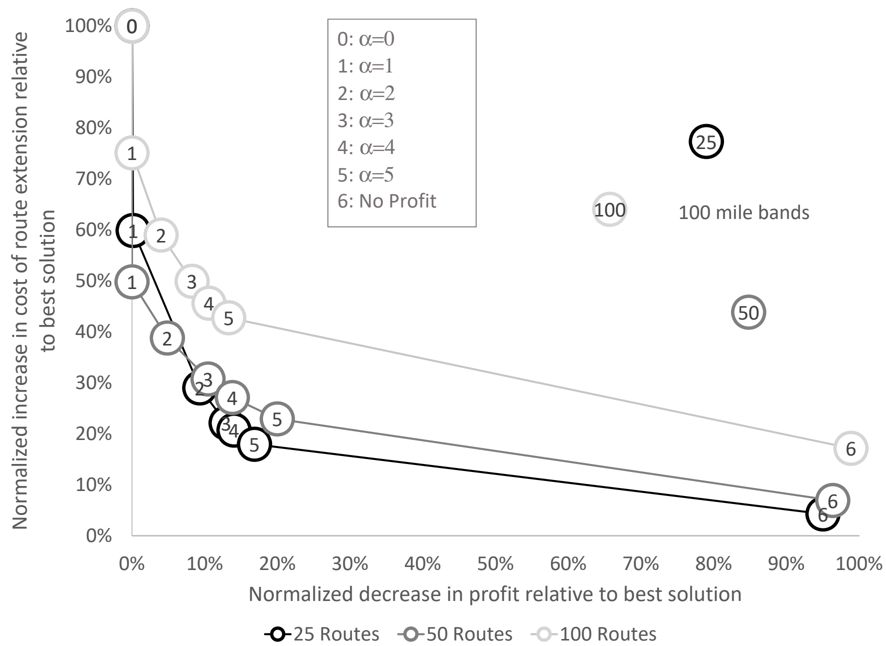


Figure 9: Comparison of normalized decrease in profit and increase in cost of route extension relative to respective best solutions for uniformly distributed instances with 25, 50 and 100 route lengths for $H = [1500, 2500]$

4.2.3. Analysis of average route length

Another factor that a carrier may consider in creating mileage bands is the average length of the routes executed, which was analyzed by testing three different ranges. As the profit of the route is determined by multiplying the length and the per mile rate, if the carrier executes shorter routes, the cost for increasing route lengths to cheaper bands is not as significant. When routes are longer, those costs have more of an impact.

Figure 10 presents a comparison of the normalized decrease in profit and increase in cost of route extension for the three different values of H. Similarly to the instances with sparsely distributed route lengths, the instances in the range [500,1500] exhibit a greater reduction in cost from route extension, but without a corresponding decrease in total profit. For shorter route lengths, the window between the mileage break, Y_s', and bound B_{s+1} is smaller relative to the other ranges and there are fewer opportunities for a shipper to extend a route to decrease cost. Reducing this window has a greater impact than reducing the number of routes over the range, as evidenced by the limited decrease in profit for these shorter routes. Alternatively, it is more difficult to limit the cost of route extension as route length increases. A carrier that regularly executes long routes is more prone to losing profit due to route extension and having that impact total profit. It is all the more important that such a carrier build appropriately spaced mileage bands. From the shipper perspective, just as longer routes provided more flexibility to adjust the route to the distance of a bound, longer routes also increase the probability that the route should be extended for cost savings.

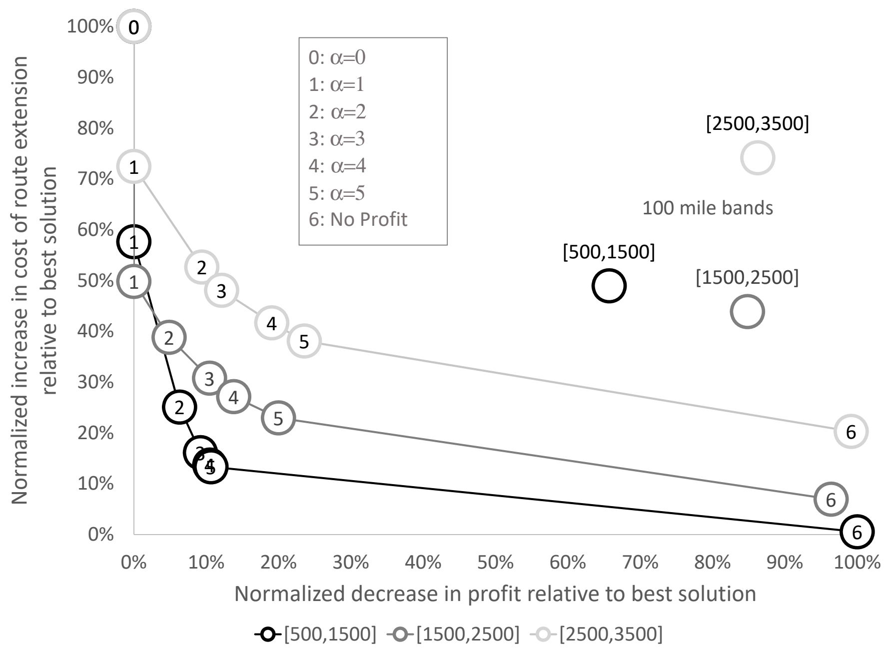


Figure 10: Comparison of normalized decrease in profit and increase in cost of route extension relative to respective best solutions for uniformly distributed instances with $H = [500, 1500], [1500, 2500]$ and $[2500, 3500]$

4.2.4. Analysis of change in per mile profit

Finally, we consider how the decrease in per mile profit between bands impacts the width of the bands. Figure 11 presents the impact on profits for the three linear decreases in P_s', 1%-3%, and the real world values ("Actual") in Table 1. As P_s' increases, the gap between Y_s' and B_{s+1} grows, increasing the probability that a shipper will extend a route to reduce cost. This is reflected in the results. With a rate of $P_s' = 1\%$, this gap is the smallest, just as it was for the shortest routes in the previous section, and the greatest cost reduction may be found with the smallest decrease in profit. Conversely, when $P_s' = 3\%$, it becomes difficult to limit the cost due to route extension as there are more routes that qualify for this extension. The results for

$P_s' = 2\%$ and the real world values are similar as the average of the latter values is close to 2%. These results indicate that a carrier should expect that shippers will be more likely to extend their routes if given the opportunity through larger rate reductions. While not surprising, this does confirm that a carrier should be judicious in the reduction they offer if they hope to prevent route extension.

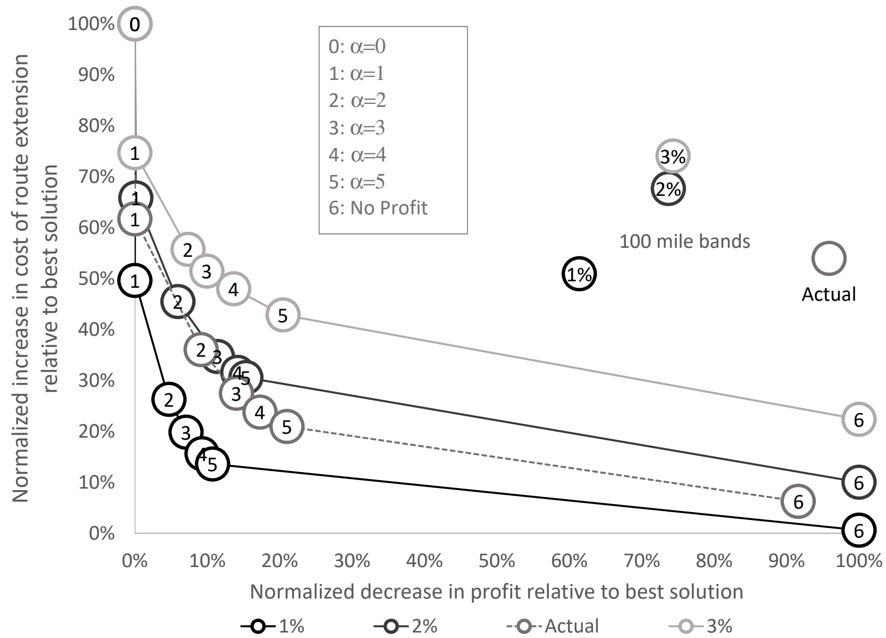


Figure 11: Comparison of normalized decrease in revenue and increase in cost of route extension relative to respective best solutions for uniformly distributed instances with linear and actual per mile profit decrease between bands

The results differ with increasing and decreasing values for P_s', shown in Figure 12, where IR 0.5% has P_s' values that range from 0.5% to 4.5% (DR 0.5% from 4.5% to 0.5%), while IR 1% ranges from 1% to 9% (DR 1% from 9% to 1%). The values with $P_s' = 3\%$ for each band are also included to provide context relative to the results in Figure 11. The results for IR 0.5% are similar to those found with $P_s' = 3\%$. The cumulative decrease between the initial profit of \$0.15 for the first band and the profit of the last band at \$0.1193 is 20%, compared to 24% with $P_s' = 3\%$, which is reflected in the slightly lower decrease in profit with IR 0.5%. However, the change in profit per mile between the latter bands with IR 0.5% is greater than 3%, which allows for the extension of more routes, increasing the cost.

The instances with a decreasing change in profit per mile exhibit a larger drop in the cost of route extension relative to the corresponding instances with an increasing rate. However, they also reflect a considerably greater decrease in profit. With the largest drop in per mile profit for the first band, rather than the last band, narrowing the wide bands that are low in the range has more of an impact on reducing the cost of route extension, but with a greater decrease in profits. Figure 13 presents the bands for the instances using DR 1%. The two lowest bands

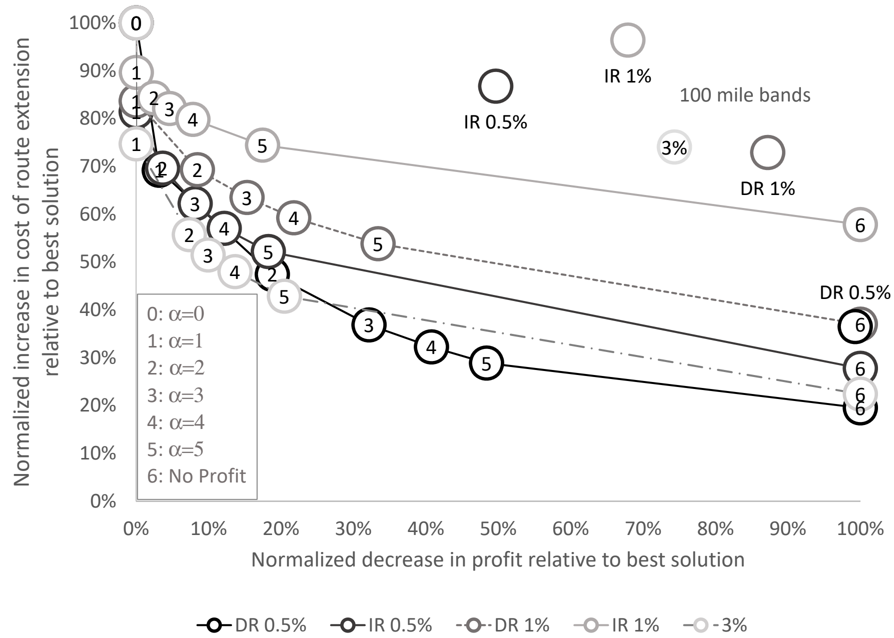


Figure 12: Comparison of normalized decrease in revenue and increase in cost of route extension relative to respective best solutions for uniformly distributed instances with increasing and decreasing per mile profit between bands

are visibly more narrow than in previous results, while the third and fourth bands are wider. With such a large drop in profit per mile, decreasing the width of the first two bands reduces the likelihood that the highest profit routes will be extended to bands with a lower profit. This reduces the cost of route extension with a considerable decrease in profit. The bands using DR 0.5% are similar, with a less significant change. These results indicate that if the profit per mile between bands changes at a varying rate, it will impact the band widths, particularly if the rate is decreasing. A carrier may consider decreasing the bands under these conditions, but this is likely to reduce profit.

5. Conclusions

This research presents a model and an algorithm through which shippers may find the lowest cost route when the carrier uses mileage bands for pricing. The Bounded TSP algorithm shows that even a route with few nodes may be adjusted to match the bound of a band, maximizing the opportunity for the shipper to reduce cost. This adjustment comes at the cost of a carrier traveling an additional distance on a route. In order to limit this additional travel, while also continuing to offer mileage band incentives, guidelines are presented for a carrier to improve the methodology by which mileage bands are created. Theoretical guidelines are provided for both uniform and triangular distributions on expected shipper route distances. An algorithm is also presented that creates bands based on a specific set of routes. The results from testing this algorithm provide managerial insight into how a carrier may best set band widths to maximize

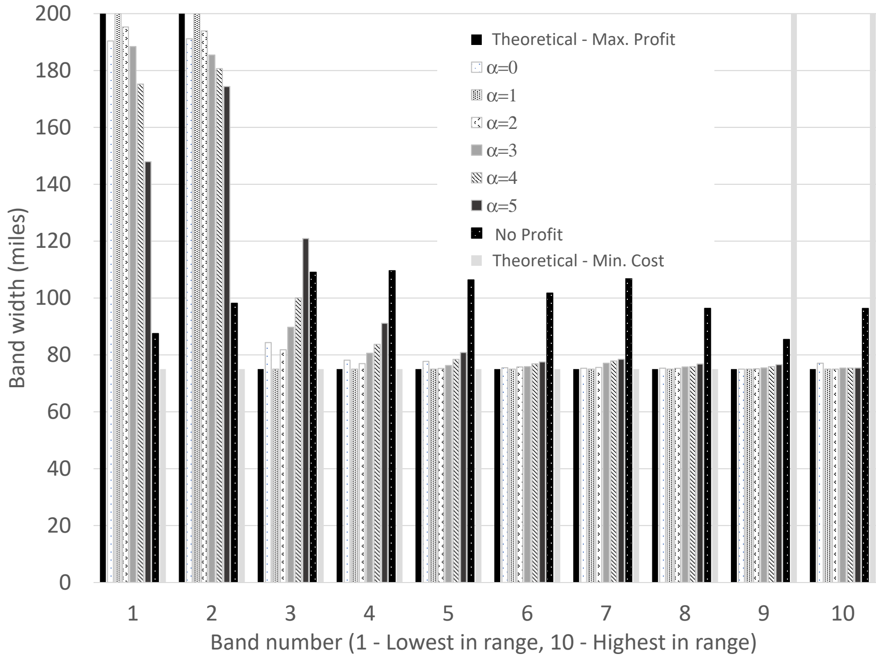


Figure 13: Average width of the bands for all uniformly distributed instances with P_s' set using DR 1% and theoretical bands for maximizing profit and minimizing cost of route extension

profit and minimize the cost of shippers extending routes.

Based on these results, the following are recommended for a carrier creating mileage bands:

1. avoid the equidistant bands that are most commonly used, as these lead to a decrease in profit and potentially allow for shippers to extend routes.
2. place some emphasis on limiting route extension. An α value between 1 and 3 produced the greatest reduction in cost from route extension without a comparable decrease in profit.
3. if executing shorter routes, or routes that vary considerably in length, a focus on limiting route extension can be particularly beneficial.
4. the greater the decrease in per mile profit from band to band, the more opportunity for a shipper to extend a route. A larger decrease should be balanced with a larger band. A decreasing rate in per mile profit as bands increase may be countered with smaller bands low in the range.

Considerable research on piecewise linear costs focuses on volume quantity discounts without considering how the price breaks should be distributed. The research here may be applied to that area. Alternatively, research in that area considers how demand changes with price, which should be considered for this problem. As the carrier modifies the mileage bands, the demand may vary from that previously forecast. Game theory may be useful in developing an equilibrium model that balances the carrier's needs with those of the shipper. There may also be opportunities for the carrier to offer different mileage bands to each shipper based on the sophistication of the shipper. If a shipper regularly generates routes that are suboptimal in terms of length, while also just above a bound on a mileage band, they may reduce the bound on the band to incentivize the shipper to create shorter routes.

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APPENDIX

Proof of Theorem 4.3:

1. As $F(X) = \frac{(B_{s+1}-a)^2-(Y_s'-a)^2}{(b-a)(c-a)}$, $Y_s'(R_s - C_s) = B_{s+1}(R_{s+1} - C_{s+1})$ and $R_{s+1} - C_{s+1} = (R_s - C_s)(1 - P_s')$, $F(X)$ may be defined as:

$$\begin{aligned} F(X) &= \frac{(B_{s+1}-a)^2-(Y_s'-a)^2}{(b-a)(c-a)} \\ &= \frac{B_{s+1}^2 - 2aB_{s+1} - Y_{s^2}' + 2aY_s'}{(b-a)(c-a)} \\ &= \frac{B_{s+1}^2(2P_s' - P_{s^2}') - B_{s+1}(2aP_s')}{(b-a)(c-a)} \end{aligned}$$

We then find the extreme point at:

$$\begin{aligned} \frac{dF(X)}{dB_{s+1}} &= \frac{2B_{s+1}(2P_s' - P_{s^2}') - (2aP_s')}{(b-a)(c-a)} = 0 \\ B_{s+1} &= \frac{a}{2-P_s'} \end{aligned}$$

The largest value that B_{s+1} may take under this condition is a, which is the lower limit on B_{s+1}. This is shown to be the minimum value of $F(X)$ by observing that $\frac{d^2F(X)}{dB_{s+1}^2} = 2 - P_s'$, which is positive for every value of P_s'. Therefore, decreasing B_{s+1} to a decreases $F(X)$.

2. As $F(X) = \frac{(b-Y_s')^2-(b-B_{s+1})^2}{(b-a)(b-c)}$, $Y_s'(R_s - C_s) = B_{s+1}(R_{s+1} - C_{s+1})$ and $R_{s+1} - C_{s+1} = (R_s - C_s)(1 - P_s')$, $F(X)$ may be defined as:

$$\begin{aligned} F(X) &= \frac{(b-Y_s')^2-(b-B_{s+1})^2}{(b-a)(b-c)} \\ &= \frac{Y_{s^2}' + 2bB_{s+1} - B_{s+1}^2 - 2bY_s'}{(b-a)(b-c)} \\ &= \frac{B_{s+1}(2bP_s') - B_{s+1}^2(2P_s' - P_{s^2}')}{(b-a)(b-c)} \end{aligned}$$

We then find the extreme point at:

$$\begin{aligned} \frac{dF(X)}{dB_{s+1}} &= \frac{2bP_s' - 2B_{s+1}(2P_s' - P_{s^2}')}{(b-a)(b-c)} = 0 \\ B_{s+1} &= \frac{b}{2-P_s'} \end{aligned}$$

This is shown to be the maximum value of $F(X)$ by observing that $\frac{d^2F(X)}{dB_{s+1}^2} = -(2 - P_s')$, which is negative for every value of P_s'. Therefore, $F(X)$ is maximized for a bound at $\frac{b}{2-P_s'}$, decreasing as the bands are pushed in either direction from this value.

3. The proof is similar to that for part 2.

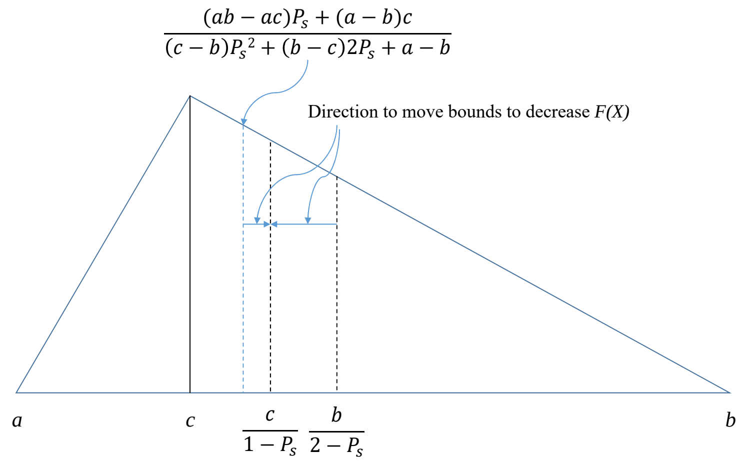

Figure 14: Circumstance depicting when $\frac{b}{2-P'_s} > \frac{c}{1-P'_s} > \frac{(ab-ac)P'_s+(a-b)c}{(c-b)P'_{s}{}_2+(b-c)2P'_s}$

Using the fact that $F(X) = 1 + \frac{(b-B_{s+1})^2*(a-c)+(Y'_s-a)^2(c-b)}{(b-a)(b-c)(c-a)}$, $Y'_s(R_s - C_s) = B_{s+1}(R_{s+1} - C_{s+1})$ and $R_{s+1} - C_{s+1} = (R_s - C_s)(1 - P'_s)$, we find that:

$$\begin{aligned}\frac{dF(X)}{dB_{s+1}} &= cP'_{s^2}B_{s+1} + aB_{s+1} + 2bP'_sB_{s+1} - bB_{s+1} \\ &= -bP'_{s^2}B_{s+1} - 2cP'_sB_{s+1} + acP'_s - baP'_s + bc - ac.\end{aligned}$$

We set this derivative to 0 to solve for $B_{s+1} = \frac{(ab-ac)P'_s+(a-b)c}{(c-b)P'_{s}{}_2+(b-c)2P'_s+a-b}$. As $b > c > a$, the second derivative $\frac{2(c-b)(1-P'_s)^2+2(a-c)}{(b-a)(b-c)(c-a)}$ must always be negative, such that the extreme point is a maximum. Therefore, $F(X)$ decreases as B_{s+1} moves in either direction from this value. □

The one exception to these results would be a situation where both $\frac{b}{2-P'_s}$ and $\frac{c}{1-P'_s}$ are greater than $\frac{(ab-ac)P'_s+(a-b)c}{(c-b)P'_{s}{}_2+(b-c)2P'_s}$, while $\frac{b}{2-P'_s} > \frac{c}{1-P'_s}$. This is represented in Figure 14. Part 2 of the Theorem indicates that the bound should decrease from $\frac{b}{2-P'_s}$ to decrease $F(X)$, while part 3 indicates that the bound should increase from $\frac{(ab-ac)P'_s+(a-b)c}{(c-b)P'_{s}{}_2+(b-c)2P'_s}$. The conditions for both parts meet at $\frac{c}{1-P'_s}$. However, it can be shown computationally that these conditions can never occur simultaneously, such that $\frac{(ab-ac)P'_s+(a-b)c}{(c-b)P'_{s}{}_2+(b-c)2P'_s}$ can never be less than both $\frac{b}{2-P'_s}$ and $\frac{c}{1-P'_s}$.