Solution for short-term hydrothermal scheduling with a logarithmic size MILP formulation

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Abstract

Short-term hydrothermal scheduling (STHS) is a non-convex and non-differentiable optimization problem that is difficult to solve efficiently. One of the most popular strategy is to reformulate the complicated STHS by various linearization techniques that makes the problem easy to solve. However, in this process, a large number of extra continuous variables, binary variables and constraints will be introduced, which may lead to a heavy computational burden, especially for a large-scale problem. In this paper, a logarithmic size mixed-integer linear programming (MILP) formulation is proposed for the STHS, i.e., only a logarithmic number of binary variables and constraints are required to piecewise linearize the nonlinear functions of STHS. Based on such an MILP formulation, a global optimal solution is therefore can be solved efficiently. To eliminate the linearization errors and cope with the transmission loss, a differentiable non-linear programming (NLP) formulation, which is equivalent to the original non-differentiable STHS is derived. By solving this NLP formulation via the powerful interior point method (IPM), where the previous global optimal solution of MILP formulation is used as the initial point, a high-quality feasible optimal solution to the STHS can thus be determined. Simulation results show that the proposed logarithmic size MILP formulation is more efficient than the generalized one and when it is incorporated into the solution procedure, our solution methodology is competitive with currently state-of-the-art approaches.

Keywords: Short-term hydrothermal scheduling, piecewise linearize, logarithmic size, mixed-integer linear programming, non-linear programming

1. Introduction

Short-term hydrothermal scheduling (STHS) is considered as one of the important issues related to optimal economic operation in power systems. It refers to the attempt to manage the reservoir storage effectively by utilizing the available hydro resources as much as possible over a scheduled time horizon, while satisfying various system operation constraints, such that the total generation cost of thermal units is minimum [1]. Normally, STHS is boiled down to solving a mathematical programming problem, during which optimization methods are designed for the solution. However, due to the model is notorious for its non-convex and even more, non-differentiable, many difficulties and challenges will be encountered in the optimization process. For example, the non-convex hydropower generation function and the power transmission loss make the solution easily trapped in a poor local optima; when the valve-point effects (VPE) of thermal unit which makes the generation cost function non-differentiable is considered, the classical mathematical programming-based methods, also known as derivative-based optimization methods, are no longer suitable.

In the past decades, a wide range of optimization methods have been proposed for handling the STHS problem, such as network flow (NF) [2, 3], dynamic programming (DP) [4], Lagrangian relaxation (LR) [5, 6], mixed-integer linear programming (MILP)[7–14], semi-definite programming (SDP) [15, 16] and heuristic based algorithms [17–36]. Because of intractability of the problem, most of the classical approaches for the STHS problem consider only a subset of the real constraints and simplify the functions involved. For example, in [6, 15], the hydraulic subsystem is described as a simple linear model; in [10, 14, 16], the transmission system is not included; and in [4, 5, 16] the generation cost of thermal unit is considered as a quadratic function of the output power, where the VPE occured in actual operation is ignored. However, heuristic based algorithms, which are known for their flexibility and versatility, can perform excellently for various STHS formulations [37], even though they are non-convex and non-differentiable. Due to the stochastic nature of the optimization process, some intrinsic drawbacks of heuristic methods exist nevertheless. It is well known that they are quite sensitive to various parameter settings [35]. Besides,

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unlike deterministic mathematical programming-based optimization techniques, heuristics give unstable results, that is, the solution generated in each trial may be different [38].

By contrast, classical methods, also known as deterministic mathematical programming-based optimization techniques, can solve to robust solutions due to the solid mathematical foundation and the availability of powerful software tools. Therefore, the MILP-based approaches have recently gained increasing popularity for the STHS [7–14] because of the availability of the state-of-the-art MILP solvers and efficient modeling tools. In order to apply MILP-based approaches, some reformulation techniques will be adopted to convert the original nonlinear expressions of the STHS, which contains one or more variables, into the piecewise linear formulations. In [7–9], piecewise linear approximation is formulated through a set of one-dimensional functions. Such a one-dimensional method is favorable for practical use because it is simple to model. To get a better piecewise linear approximation, a more complex triangulation method is utilized in [10–12], as it could provide a more refined model. However, the more precise the model is, the more computational burden it will be encountered. To alleviate the computational burden, rectangle method is employed in [13, 14], achieving a good trade-off between computational efficiency and accuracy.

However, it is widely recognized that, when MILP-based approaches are applied, a large number of extra continuous variables, binary variables and constraints will be introduced in the reformulation, which may lead to a heavy computational burden, especially for a large-scale problem. In this paper, a logarithmic size MILP formulation is proposed for the STHS, i.e., only a logarithmic number of binary variables and constraints are required to piecewise linearize the nonlinear functions of STHS. The non-convex and non-differentiable thermal unit generation cost is piecewise linearized by the convex combination approach first, yielding an MILP formulation with a logarithmic number of binary variables and constraints. And then, the hydro generation function of two variables is subdivided into a number of triangles and approximated with a set of piecewise linear functions, which is more accurate than the rectangle partitions [13, 14]. Compared with the generalized triangulation [10, 11], "Union Jack" triangulation scheme [39] is utilized in this work, getting a logarithmic size piecewise linear functions for hydro generation function. Consequently, when transmission loss is not included, a logarithmic size MILP formulation for STHS, which can be solved to a global optimal solution directly and efficiently, is formulated.

Since MILP-based approach is adopted for the STHS, some piecewise linearization errors will be occured inevitably. It means that, the scheduling obtained by solving the MILP formulation individually is hard to fully satisfy the system operation constraints. To eliminate the errors caused by the linearizations, the original STHS will be solved after the optimization in MILP formulation. Due to the non-differentiable nature of the STHS, classical derivative-based optimization methods are not suitable any more. Fortunately, with the help of model reformulation, a non-linear programming (NLP) formulation of the STHS, which can be immediately solved using the polynomial-time interior point method (IPM), is derived.

Therefore, in the solution procedure, a logarithmic size MILP formulation of STHS is solved by a state-of-the-art MILP solver first, yielding a global optimal solution efficiently. And then, by solving the NLP formulation of STHS via IPM, where the MILP solution is used as the initial point of IPM, a high-quality feasible optimal solution to the STHS can thus be determined. While transmission loss is considered, this approach also works well. The validity and effectiveness of the proposed formulations and solution methodology are successfully demonstrated for three test systems.

The remainder of this paper is organized as follows. Section 2 describes the mathematical formulation of the STHS. Section 3 and Section 4 derive a logarithmic size MILP formulation and an NLP formulation for the STHS, respectively. Section 5 introduces the solution methodology. Section 6 presents simulation results and discussions. Section 7 offers conclusions.

2. Mathematical formulation of the STHS

Generally, the generation cost of a hydro unit is assume to be much less than that of a thermal unit [15]; thus, it can be negligible in the STHS problem. Hence, the STHS problem is to minimize the total generation cost of thermal units by utilizing the available hydro resources as much as possible over a scheduled time horizon, while satisfying various system operation constraints.

2.1. Objective function

Conventionally, the generation cost of each thermal unit can be modeled as a convex quadratic polynomial:

$$c_i^{\text{quad}}(P_{i,t}^{\text{T}}) = \alpha_i + \beta_i P_{i,t}^{\text{T}} + \gamma_i (P_{i,t}^{\text{T}})^2, \tag{1}$$

where $P_{i,t}^{\text{T}}$ is the power output of thermal unit i in period t and α_i, β_i and γ_i are positive coefficients for thermal unit i. When VPE is considered, a recurring rectified sinusoidal function

$$c_i^{\text{vpe}}(P_{i,t}^{\text{T}}) = e_i |\sin(f_i(P_{i,t}^{\text{T}} - P_{i,\min}^{\text{T}}))| \tag{2}$$

is added to the conventional generation cost, which makes the generation cost function non-convex and non-differentiable in nature[40]. Above, $P_{i,\min}^{\text{T}}$ is the minimum power output of thermal unit i, and e_i and f_i are positive coefficients of the VPE cost for thermal unit i. Consequently, the thermal unit generation cost considering VPE can be expressed as

$$c_i(P_{i,t}^{\text{T}}) = c_i^{\text{quad}}(P_{i,t}^{\text{T}}) + c_i^{\text{vpe}}(P_{i,t}^{\text{T}}). \tag{3}$$

The objective function of the STHS problem is minimization of total generation cost of multiple thermal units, which can be written as

$$\min \sum_{t=1}^{T}\sum_{i=1}^{N_{\mathrm{T}}} c_i(P_{i,t}^{\mathrm{T}}), \tag{4}$$

where N_{T} and T are the total numbers of thermal units and periods, respectively.

2.2. Constraints

The minimized STHS problem should be subject to the following constraints.

2.2.1. Power balance equations

At each time period the total active power generation should meet the load demand and the transmission loss of the power system, expressed as

$$\sum_{i=1}^{N_{\mathrm{T}}} P_{i,t}^{\mathrm{T}} + \sum_{j=1}^{N_{\mathrm{H}}} P_{j,t}^{\mathrm{H}} = D_t + P_t^{\mathrm{L}},\ \forall\ t, \tag{5}$$

where $P_{j,t}^{\mathrm{H}}$ is the power output of hydro unit j in period t, N_{H} is the total number of hydro units, D_t is the load demand in period t and P_t^{L} is the transmission loss in period t, which can be calculated based on the B-coefficient method and expressed as a quadratic function of the power outputs [41]:

$$P_t^{\mathrm{L}} = \sum_{i=1}^{N_{\mathrm{T}}+N_{\mathrm{H}}}\sum_{j=1}^{N_{\mathrm{T}}+N_{\mathrm{H}}} P_{i,t}^{*} B_{i,j} P_{j,t}^{*},\ \forall\ t, \tag{6}$$

where $B_{i,j}$ is the (i,j)-th element of the matrix of the transmission loss coefficients, B; $P_{i,t}^{*}$ represents the power output of the ith thermal or hydro unit in period t.

The power output of hydro unit can be calculated using a nonseparable quadric function of reservoir storage volume and water discharge, which is given by

$$\begin{aligned} P_{j,t}^{\mathrm{H}} =& \xi_{j,1}V_{j,t}^2 + \xi_{j,2}Q_{j,t}^2 + \xi_{j,3}V_{j,t}Q_{j,t} \\ &+ \xi_{j,4}V_{j,t} + \xi_{j,5}Q_{j,t} + \xi_{j,6},\ \forall\ j,t, \end{aligned} \tag{7}$$

where $\xi_{j,1}, \xi_{j,2}, \xi_{j,3}, \xi_{j,4}, \xi_{j,5}$ and $\xi_{j,6}$ are the coefficients of hydro unit j; $V_{j,t}$ is the storage volume of reservoir j in period t; $Q_{j,t}$ is the discharge of hydro unit j in period t.

2.2.2. Output capacity limitations

The output of hydro and thermal units should lie in the determined intervals, which can be stated as

$$P_{i,\min}^{\mathrm{T}} \le P_{i,t}^{\mathrm{T}} \le P_{i,\max}^{\mathrm{T}},\ \forall\ i,t, \tag{8}$$
$$P_{j,\min}^{\mathrm{H}} \le P_{j,t}^{\mathrm{H}} \le P_{j,\max}^{\mathrm{H}},\ \forall\ j,t, \tag{9}$$

where $P_{i,\max}^{\mathrm{T}}$ is the maximum power output of thermal unit i; $P_{j,\min}^{\mathrm{H}}$ and $P_{j,\max}^{\mathrm{H}}$ are the minimum and maximum power outputs of hydro unit j, respectively.

2.2.3. Hydraulic network constraints

The reservoir storage volumes and water discharges should meet the physical limitations below

$$V_{j,\min} \le V_{j,t} \le V_{j,\max},\ \forall\ j,t, \tag{10}$$
$$Q_{j,\min} \le Q_{j,t} \le Q_{j,\max},\ \forall\ j,t, \tag{11}$$

where $V_{j,\min}$ and $V_{j,\max}$ are the minimum and maximum storage volumes of reservoir j, respectively; $Q_{j,\min}$ and $Q_{j,\max}$ are the minimum and maximum discharges of hydro unit j in period t, respectively.

For each reservoir, it must meet the water balance equation as follow

$$\begin{aligned} V_{j,t} =& V_{j,t-1} + I_{j,t} - Q_{j,t} - S_{j,t} \\ &+ \sum_{r\in R_j}(Q_{r,t-\tau_r} + S_{r,t-\tau_r}), \end{aligned} \tag{12}$$

where $V_{j,t}$ is the storage volume of reservoir j in period t; $I_{j,t}$ is the natural inflow of reservoir j in period t; $S_{j,t}$ is the spillage of reservoir j in period t; R_j is the index set of upstream reservoir(s) of reservoir j; τ_r is the transfer time of water from reservoir r to immediate downstream reservoir.

Lastly, the following initial and final reservoir storage volumes limits should be satisfied,

$$V_{j,0} = V_{j,\mathrm{init}},\ \forall\ j, \tag{13}$$
$$V_{j,24} = V_{j,\mathrm{end}},\ \forall\ j, \tag{14}$$

where $V_{j,0}$ and $V_{j,24}$ are the storage volumes of reservoir j in period 0 and 24, respectively; $V_{j,\mathrm{init}}$ and $V_{j,\mathrm{end}}$ are the initial and final storage volumes of reservoir j, respectively.

3. A logarithmic size MILP formulation for the STHS

3.1. A generalized MILP formulation

In this paper, the convex combination formulation [14] is adopted to piecewise linearise the nonlinear functions of STHS. Firstly, $L_i + 1$ break points are chosen over a generation interval $[P_{i,\min}^{\mathrm{T}}, P_{i,\max}^{\mathrm{T}}]$, such that $P_{i,\min}^{\mathrm{T}} = p_{i(0)}^{\mathrm{T}} \le p_{i(1)}^{\mathrm{T}} \le \cdots \le p_{i(L_i)}^{\mathrm{T}} = P_{i,\max}^{\mathrm{T}}$, where L_i is calculated by

$$L_i = \lceil \hat{L}\cdot f_i \cdot (P_{i,\max}^{\mathrm{T}} - P_{i,\min}^{\mathrm{T}})/\pi \rceil.$$

Here $\hat{L}$ is the number of equal segments on each $\sin(x)$ where x belongs to $[0,\pi]$. So for any given $P_{i,t}^{\mathrm{T}} \in [P_{i,\min}^{\mathrm{T}}, P_{i,\max}^{\mathrm{T}}]$, it can be expressed uniquely as a convex combination of at most two consecutive break points by introducing some continuous variables $\lambda_{i,t(l)} \in [0,1], (l = 0,1,...,L_i)$; the corresponding function value $c_i(P_{i,t}^{\mathrm{T}})$ can thus be approximated by the convex combination of the function values in these break points. Consequently, the generation cost $c_i(P_{i,t}^{\mathrm{T}})$ can be approximated as

$$c_i(P_{i,t}^{\mathrm{T}}) = \sum_{l=0}^{L_i} \lambda_{i,t(l)} c_i(p_{i(l)}^{\mathrm{T}}), \tag{15}$$

with some additional constraints

$$P_{i,t}^{\mathrm{T}} = \sum_{l=0}^{L_i} \lambda_{i,t(l)} p_{i(l)}^{\mathrm{T}}, \tag{16}$$

$$\sum_{l=0}^{L_i} \lambda_{i,t(l)} = 1, \lambda_{i,t(l)} \geq 0, \tag{17}$$

$$\sum_{l=1}^{L_i} x_{i,t(l)} = 1, x_{i,t(l)} \in \{0,1\}, \tag{18}$$

$$\lambda_{i,t(l-1)} \leq x_{i,t(l-1)} + x_{i,t(l)}, (l = 1, ..., L_i + 1), \tag{19}$$

where $x_{i,t(0)} = x_{i,t(L_i+1)} = 0$ in (19). Constraint (18) imposes that only one $x_{i,t(l)}$ takes the value 1, and then constraints (17) and (19) impose that at most two consecutive $\lambda_{i,t(l)}$ values are not 0.

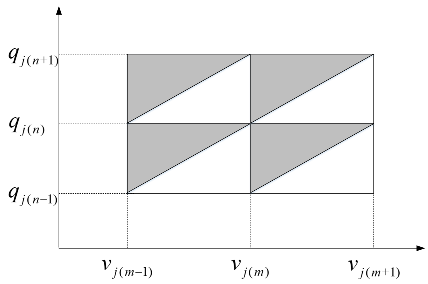

Figure 1: Geometric representation of the generalized triangulation

As seen from (7), $P_{j,t}^{\mathrm{H}}$ is a function with respect to $V_{j,t}$ and $Q_{j,t}$, which is defined on the rectangle $[V_{j,\min}, V_{j,\max}] \times [Q_{j,\min}, Q_{j,\max}]$. This two-variables function, which could be non-convex, can be approximated linearly using the following triangle method [10, 42, 43]. Firstly, $M_j + 1$ and $N_j + 1$ break points are chosen over intervals $[V_{j,\min}, V_{j,\max}]$ and $[Q_{j,\min}, Q_{j,\max}]$, such that $V_{j,\min} = v_{j(0)} \leq v_{j(1)} \leq \cdots \leq v_{j(M_j)} = V_{j,\max}$ and $Q_{j,\min} = q_{j(0)} \leq q_{j(1)} \leq \cdots \leq q_{j(N_j)} = Q_{j,\max}$. Using these break points as vertices, $M_i \times N_i$ rectangles can thus be obtained. For each rectangle, two triangles can be produced after connecting the left lower and right upper vertices(see Fig. 1). For any given $(V_{j,t}, Q_{j,t})$, it can be expressed as a convex combination of the vertices of the triangle containing $(V_{j,t}, Q_{j,t})$ by introducing some continuous variables $\lambda_{i,t(m,n)} \in [0,1], (m = 0, 1, ..., M_i, n = 0, 1, ..., N_i)$; the corresponding function $P_{j,t}^{\mathrm{H}}$ can thus be approximated by the convex combination of the function values at these vertices. Consequently, the hydro generation function $P_{j,t}^{\mathrm{H}}$ can be approximated as

$$P_{j,t}^{\mathrm{H}} = \sum_{m=0}^{M_j} \sum_{n=0}^{N_j} \lambda_{j,t(m,n)} p_{j(m,n)}^{\mathrm{H}} \tag{20}$$

with some additional constraints

$$V_{j,t} = \sum_{m=0}^{M_j} \sum_{n=0}^{N_j} \lambda_{j,t(m,n)} v_{j(m)}, \tag{21}$$

$$Q_{j,t} = \sum_{m=0}^{M_j} \sum_{n=0}^{N_j} \lambda_{j,t(m,n)} q_{j(n)}, \tag{22}$$

$$\sum_{m=0}^{M_j} \sum_{n=0}^{N_j} \lambda_{j,t(m,n)} = 1, \lambda_{j,t(m,n)} \geq 0, \tag{23}$$

$$\begin{aligned} &\sum_{m=1}^{M_j} \sum_{n=1}^{N_j} (y_{j,t(m,n)} + z_{j,t(m,n)}) = 1, \\ &y_{j,t(m,n)} \in \{0,1\},\ z_{j,t(m,n)} \in \{0,1\}, \end{aligned} \tag{24}$$

$$\begin{aligned} \lambda_{j,t(m-1,n-1)} \leq\ & y_{j,t(m-1,n-1)} + y_{j,t(m,n-1)} \\ & + y_{j,t(m,n)} + z_{j,t(m-1,n-1)} \\ & + z_{j,t(m-1,n)}, + z_{j,t(m,n)}, \\ (m = 1, ..., & M_j + 1, n = 1, ..., N_j + 1) \end{aligned} \tag{25}$$

where $p_{j(m,n)}^{\mathrm{H}}$ denotes the hydro generation at $(v_{j(m)}, q_{j(n)})$ and

$$\begin{aligned} y_{j,t(0,*)} = y_{j,t(*,0)} = y_{j,t(M_j+1,*)} = y_{j,t(*,N_j+1)} = 0, \\ z_{j,t(0,*)} = z_{j,t(*,0)} = z_{j,t(M_j+1,*)} = z_{j,t(*,N_j+1)} = 0. \end{aligned}$$

Above-mentioned $y_{j,t(m,n)}$ and $z_{j,t(m,n)}$ are binary variables that are associated to the upper and lower triangle in the (m,n)-th rectangle. Constraint (24) imposes that, among all triangles, only one is adopted for the convex combination. Then, constraints (23) and (25) impose that only the $\lambda_{j,t(m,n)}$ associated with the three vertices in one of the triangle are not 0.

As a result, a generalized MILP formulation, denoted as G-MILP, for the STHS without transmission loss is formulated,

$$\begin{aligned} \min\ & \sum_{t=1}^{T} \sum_{i=1}^{N_{\mathrm{T}}} \sum_{l=0}^{L_i} \lambda_{i,t(l)} c_i(p_{i(l)}^{\mathrm{T}}) \\ s.t.\ & (5), (8) - (14), (16) - (25), \end{aligned} \tag{26}$$

where $P_t^{\mathrm{L}} = 0$ in (5). In this formulation, a number of extra continuous variables, binary variables and constraints are introduced, which will cause a heavy computational burden, especially for a large-scale problem. In order to save computational effort, a new piecewise linear modeling technique presented in [44], which requires only a logarithmic number of binary variables and constraints to approximate the one and two variables nonlinear functions, is employed for the STHS. This is because the computational efficiency of an MILP formulation depends much more on the number of binary variables and constraints than the number of continuous variables [45], reducing constraints and binary variables can make a more significant impact than reducing continuous variables on the computational efficiency. Next, we will follow the basic idea in [44] to construct a logarithmic size MILP formulation for the STHS.

3.2. *A logarithmic size MILP formulation*

In this subsection, the subscripts of thermal units and periods are dropped to simplify the expressions. Let us introduce the following additional notations.

$\mathcal{J} = \{0, 1, ..., L\}$, index set of break points.

$\mathcal{I} = \{1, ..., L\}$, index set of intervals.

$\bar{\mathcal{I}} = \{1, ..., L-1\}$.

$\mathcal{S}_i = \{i-1, i\}, \forall i \in \mathcal{I}$.

$\mathcal{I}(j) = \{i \in \mathcal{I} : j \in \mathcal{S}_i\}$.

$\mathcal{L}(|\mathcal{I}|) = \{1, ..., \log_2 |\mathcal{I}|\}$.

$\triangle^{\mathcal{J}} = \{\lambda \in \mathbb{R}^{|\mathcal{J}|} : \sum_{j \in \mathcal{J}} \lambda_j = 1, \lambda_j \geq 0\}$.

Let $B : \mathcal{I} \to \{0,1\}^{\log_2 |\mathcal{I}|}$ be a bijective function such that for all $i \in \bar{\mathcal{I}}$ the vectors $B(i)$ and $B(i+1)$ differ in at most one component. Then

$$\begin{cases} \sum\limits_{j \in \mathcal{J}^+(k,B)} \lambda_j \leq x_k, \\ \sum\limits_{j \in \mathcal{J}^0(k,B)} \lambda_j \leq (1 - x_k), \\ \lambda \in \triangle^{\mathcal{J}}, x_k \in \{0,1\}, \forall k \in \mathcal{L}(|\mathcal{I}|), \end{cases} \tag{27}$$

where $\mathcal{J}^+(k,B) = \{j : \forall i \in \mathcal{I}(j), k \in \sigma(B(i)), j \in \mathcal{J}\}$ and $\mathcal{J}^0(k,B) = \{j : \forall i \in \mathcal{I}(j), k \notin \sigma(B(i)), j \in \mathcal{J}\}$, imposes that, at most two consecutive λ variables can take a non-zero value.

Above-mentioned $\sigma(B(i))$ is the support set of vector $B(i)$, which is defined by $\sigma(B(i)) \equiv \{k : B(i)_k \neq 0\}$. Based on the definition of $\mathcal{I}(j)$, we have

$$\mathcal{I}(0) = \{1\}, \mathcal{I}(j) = \{j, j+1\} (j \in \bar{\mathcal{I}}), \mathcal{I}(L) = \{L\}.$$

Then,

$$\begin{aligned} \mathcal{J}^+(k,B) =& \{j : k \in \sigma(B(j)) \cap \sigma(B(j+1)), j \in \bar{\mathcal{I}}\} \bigcup \\ & \{j : k \in \sigma(B(\mathcal{I}(j))), j \in \{0, L\}\}, \end{aligned}$$

$$\begin{aligned} \mathcal{J}^0(k,B) =& \{j : k \notin \sigma(B(j)) \cup \sigma(B(j+1)), j \in \bar{\mathcal{I}}\} \bigcup \\ & \{j : k \notin \sigma(B(\mathcal{I}(j))), j \in \{0, L\}\}. \end{aligned}$$

i.e.,

$$\begin{aligned} \mathcal{J}^+(k,B) =& \{j : B(j)_k = B(j+1)_k = 1, j \in \bar{\mathcal{I}}\} \bigcup \\ & \{j : B(1)_k = 1, j \in \{0\}\} \bigcup \\ & \{j : B(L)_k = 1, j \in \{L\}\}, \end{aligned}$$

$$\begin{aligned} \mathcal{J}^0(k,B) =& \{j : B(j)_k = B(j+1)_k = 0, j \in \bar{\mathcal{I}}\} \bigcup \\ & \{j : B(1)_k = 0, j \in \{0\}\} \bigcup \\ & \{j : B(L)_k = 0, j \in \{L\}\}. \end{aligned}$$

Next, we will give an example to illustrate how to construct the formulation (27) and how it works.

Example 1

Let $\mathcal{J} = \{0, ..., 4\}, \mathcal{I} = \{1, ..., 4\}$ and $(\lambda_j)^4_{(j=0)} \in \triangle^{\mathcal{J}}$. Firstly, a bijective function $B : \{1,2,3,4\} \to \{0,1\}^2$ can be constructed as shown in Fig. 2. According to Fig. 2, the bijective function B can be obtained (see Table 1). Based on the definition of $\mathcal{J}^+(k,B)$ and $\mathcal{J}^0(k,B)$, we have

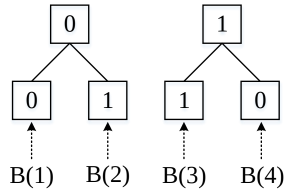

Figure 2: The construction of bijective function B

Table 1: Bijective function B

B(1)	B(2)	B(3)	B(4)
0	0	1	1
0	1	1	0

$$\begin{aligned} &\mathcal{J}^+(1,B) = \{3,4\}, \ \mathcal{J}^0(1,B) = \{0,1\}, \\ &\mathcal{J}^+(2,B) = \{2\}, \quad \ \mathcal{J}^0(2,B) = \{0,4\}. \end{aligned}$$

Then, a logarithmic number of binary variables $x_k (k \in \{1,2\})$ and constraints

$$\begin{aligned} \lambda \in \triangle^{\mathcal{J}}, \ \lambda_3 + \lambda_4 \leq x_1, \ \lambda_0 + \lambda_1 \leq (1 - x_1), \\ \lambda_2 \leq x_2, \ \lambda_0 + \lambda_4 \leq (1 - x_2), \end{aligned}$$

impose that at most two consecutive λ variables can take a non-zero value. The Fig. 3 illustrates how these constraints work. ■

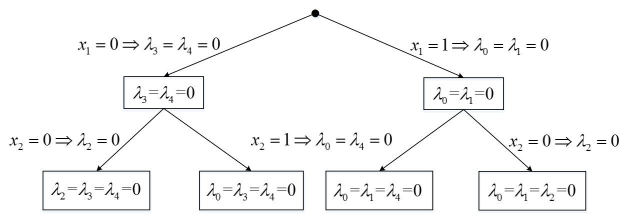

Figure 3: The procedure for enforcing at most two consecutive λ are not 0

Note that, in the above discussion, we assume that, $|\mathcal{I}|$ is a power of two. If $|\mathcal{I}|$ is not a power of two, we only need to complete $\mathcal{I}$ to an index set of size $2^{\lceil \log_2 |\mathcal{I}| \rceil}$ and set the corresponding λ to 0 in the formulation (27).

This technique can be extended to non-separable two variables function (7) easily. Different from the generalized linearization in subsection 3.1(see Fig. 1), the "Union Jack" triangulation scheme [39] which is depicted in Fig. 4 is utilized to partition $[V_{j,\min}, V_{j,\max}] \times [Q_{j,\min}, Q_{j,\max}]$. In this procedure, the rectangle induced by the triangulation is chosen at first, and then one of the two triangles inside this rectangle is selected.

Now let us introduce the following additional notations.

$\mathcal{J}_1 = \{0, 1, ..., M\}$, index set of break points.

$\mathcal{J}_2 = \{0, 1, ..., N\}$, index set of break points.
$\mathcal{I}_1 = \{1, ..., M\}$, index set of intervals.
$\mathcal{I}_2 = \{1, ..., N\}$, index set of intervals.
$\mathcal{L}(|\mathcal{I}_1|) = \{1, ..., \log_2|\mathcal{I}_1|\}$.
$\mathcal{L}(|\mathcal{I}_2|) = \{1, ..., \log_2|\mathcal{I}_2|\}$.
$\triangle^{\mathcal{J}_1\times\mathcal{J}_2} = \{\lambda \in \mathbb{R}^{|\mathcal{J}_1|\times|\mathcal{J}_2|} : \sum_{m\in\mathcal{J}_1}\sum_{n\in\mathcal{J}_2}\lambda_{m,n} = 1, \lambda_{m,n} \geq 0\}$.

In this part, $|\mathcal{I}_1|$ and $|\mathcal{I}_2|$ are also assumed to be the power of two. By applying the above-mentioned technique to each component, the first phase results to the following set of constraints

$$\begin{cases} \sum_{n=0}^{N}\sum_{m\in\mathcal{J}_1^+(k,B_1)} \lambda_{m,n} \leq x_k^1, \\ \sum_{n=0}^{N}\sum_{m\in\mathcal{J}_1^0(k,B_1)} \lambda_{m,n} \leq (1 - x_k^1), \\ \sum_{m=0}^{M}\sum_{n\in\mathcal{J}_2^+(k,B_2)} \lambda_{m,n} \leq x_k^2, \\ \sum_{m=0}^{M}\sum_{n\in\mathcal{J}_2^0(k,B_2)} \lambda_{m,n} \leq (1 - x_k^2), \\ \lambda \in \triangle^{\mathcal{J}_1\times\mathcal{J}_2}, x_k^1 \in \{0,1\}, \forall k \in \mathcal{L}(|\mathcal{I}_1|), \\ x_k^2 \in \{0,1\}, \forall k \in \mathcal{L}(|\mathcal{I}_2|), \end{cases} \tag{28}$$

where $\mathcal{J}_1^+(k, B_1), \mathcal{J}_2^+(k, B_2)$ and $\mathcal{J}_1^0(k, B_1), \mathcal{J}_2^0(k, B_2)$ are the specializations of $\mathcal{J}^+(k, B)$ and $\mathcal{J}^0(k, B)$.

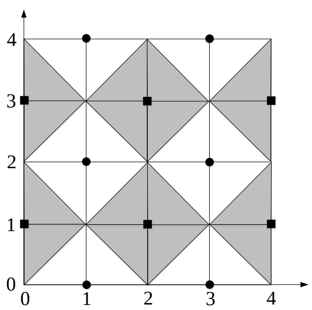

Figure 4: Geometric representation of the logarithmic triangulation

In the second phase, the dichotomy scheme depicted in Fig. 4 is used to select the triangles colored white or the ones colored gray, which induces the following set of constraints

$$\begin{cases} \sum_{(m,n)\in\mathcal{E}} \lambda_{m,n} \leq x^3, \\ \sum_{(m,n)\in\mathcal{O}} \lambda_{m,n} \leq (1 - x^3), x^3 \in \{0,1\}, \end{cases} \tag{29}$$

where $\mathcal{E} = \{(m, n) \in \mathcal{J}_1 \times \mathcal{J}_2 :\ m$ is even and n is odd$\}$ and $\mathcal{O} = \{(m, n) \in \mathcal{J}_1 \times \mathcal{J}_2 :\ m$ is odd and n is even$\}$.

The following example 2 illustrates how these constraints work.

Example 2

Let $\mathcal{J}_1 = \{0, ..., 4\}$, $\mathcal{I}_1 = \{1, ..., 4\}$ and $\mathcal{J}_2 = \{0, 1, 2\}$, $\mathcal{I}_2 = \{1, 2\}$. Based on the bijective function B constructed in example 1 and the definitions of $\mathcal{E}$ and $\mathcal{O}$, we have

$$\mathcal{J}_1^+(1, B_1) = \{3, 4\}, \mathcal{J}_1^0(1, B_1) = \{0, 1\},$$
$$\mathcal{J}_1^+(2, B_1) = \{2\}, \mathcal{J}_1^0(2, B_1) = \{0, 4\},$$
$$\mathcal{J}_2^+(1, B_2) = \{2\}, \mathcal{J}_2^0(1, B_2) = \{0\},$$
$$\mathcal{E} = \{(0, 1), (2, 1), (4, 1)\},$$
$$\mathcal{O} = \{(1, 0), (1, 2), (3, 0), (3, 2)\}.$$

Therefore, a logarithmic number of binary variables and constraints can be constructed as follows

$$\lambda \in \triangle^{\mathcal{J}_1\times\mathcal{J}_2},$$
$$\sum_{j=0}^{2}\lambda_{3,j} + \sum_{j=0}^{2}\lambda_{4,j} \leq x_1^1, \ \sum_{j=0}^{2}\lambda_{0,j} + \sum_{j=0}^{2}\lambda_{1,j} \leq (1 - x_1^1),$$
$$\sum_{j=0}^{2}\lambda_{2,j} \leq x_2^1, \ \sum_{j=0}^{2}\lambda_{0,j} + \sum_{j=0}^{2}\lambda_{4,j} \leq (1 - x_2^1),$$
$$\sum_{i=0}^{4}\lambda_{i,2} \leq x_1^2, \ \sum_{i=0}^{4}\lambda_{i,0} \leq (1 - x_1^2),$$
$$\lambda_{0,1} + \lambda_{2,1} + \lambda_{4,1} \leq x^3,$$
$$\lambda_{1,0} + \lambda_{1,2} + \lambda_{3,0} + \lambda_{3,2} \leq 1 - x^3.$$

The Fig. 5 illustrates how these constraints work. ■

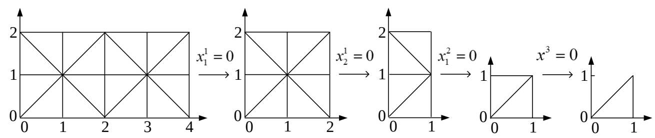

Figure 5: The procedure for the logarithmic triangulation

So the formulas (18)-(19) which have $|\mathcal{I}|$ binary variables and $|\mathcal{I}| + 2$ constraints, can be replaced by the formula (27), which is modeled with only $\log_2|\mathcal{I}|$ binary variables and $2\log_2|\mathcal{I}|$ constraints, and the formulas (24)-(25) which have $\mathcal{T}$ binary variables and $\frac{\mathcal{T}}{2} + |\mathcal{I}_1| + |\mathcal{I}_2| + 2$ constraints, can be replaced by the formulas (28) and (29), which are modeled with only $\log_2\mathcal{T}$ binary variables and $2\log_2\mathcal{T}$ constraints. Here $\mathcal{T}$ is the number of triangles in the partition, where $\mathcal{T} = 2|\mathcal{I}_1||\mathcal{I}_2|$. When $|\mathcal{I}_1| = |\mathcal{I}_2|$, $\frac{\mathcal{T}}{2} + |\mathcal{I}_1| + |\mathcal{I}_2| + 2 = \frac{\mathcal{T}}{2} + \sqrt{2\mathcal{T}} + 2$, and then their functional relationship can be seen in Fig. 6. Obviously, comparing to the linear size formulation, the logarithmic size formulation can reduce a large number of binary variables and constraints, especially for a more refined model.

Hence, a logarithmic size MILP formulation, denoted as L-MILP, for the STHS can be formulated,

$$\begin{aligned} \min \ & \sum_{t=1}^{T}\sum_{i=1}^{N_{\mathrm{T}}}\sum_{l=0}^{L_i} \lambda_{i,t(l)} c_i(p_{i(l)}^{\mathrm{T}}) \\ s.t. \ & (5), (8) - (14), (16), (17), \\ & (20) - (23), (27) - (29). \end{aligned} \tag{30}$$

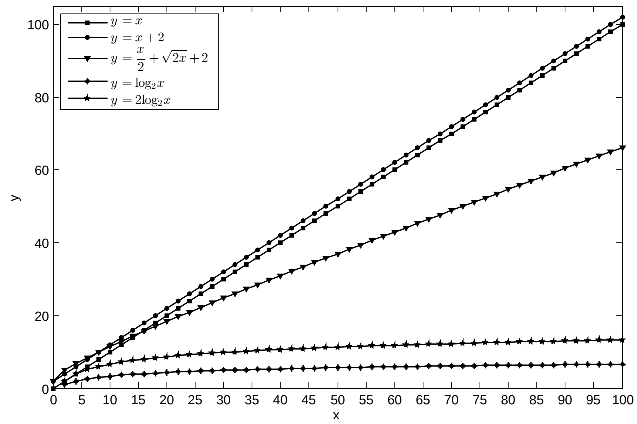

Figure 6: The functional relationship of the binary variables and constraints in two kinds of formulations

4. An NLP formulation for the STHS

Since the non-convex and non-differentiable thermal generation cost function and the nonlinear hydro generation function of the STHS are both replaced with their linear approximations, some errors will be occured inevitably in the MILP formulation. So the scheduling obtained by solving the MILP formulation individually will be hard to fully satisfy the system operation constraints. To eliminate these linearization errors, the original STHS will be solved after the optimization of the MILP formulation, and then the solution feasibility can thus be guaranteed.

However, as seen from section 2, STHS is a non-convex and non-differentiable optimization problem which is intractable. Due to its non-differentiable nature, classical mathematical programming-based methods, also known as derivative-based optimization methods, become invalid. To conquer this difficulty, an auxiliary variable $s_{i,t}$ is utilized to replace the $|\sin(f_i(P^{\mathrm{T}}_{i,t} - P^{\mathrm{T}}_{i,\min}))|$ and then, the objective function given in (4) can be rewritten to

$$\min \sum_{t=1}^{T}\sum_{i=1}^{N_{\mathrm{T}}}(\alpha_i + \beta_i P^{\mathrm{T}}_{i,t} + \gamma_i (P^{\mathrm{T}}_{i,t})^2 + e_i s_{i,t}) \tag{31}$$

$$s.t. \quad s_{i,t} \geq \sin(f_i(P^{\mathrm{T}}_{i,t} - P^{\mathrm{T}}_{i,\min})), \tag{32}$$

$$s_{i,t} \geq -\sin(f_i(P^{\mathrm{T}}_{i,t} - P^{\mathrm{T}}_{i,\min})), \ \forall\, i, t. \tag{33}$$

By introducing some slack variables $u_{i,t}$ and $w_{i,t}$, the inequality constraints given in (32) and (33) can be converted into the following equality constraints [46],

$$s_{i,t} - u_{i,t} - w_{i,t} = 0, \tag{34}$$

$$\sin(f_i(P^{\mathrm{T}}_{i,t} - P^{\mathrm{T}}_{i,\min})) + u_{i,t} - w_{i,t} = 0, \tag{35}$$

$$u_{i,t} \geq 0, \quad w_{i,t} \geq 0, \ \forall\, i, t. \tag{36}$$

As a result, the original non-convex and non-differentiable STHS is equivalent to the following differentiable NLP formulation that can be optimized using the powerful IPM immediately,

$$\min \sum_{t=1}^{T}\sum_{i=1}^{N_{\mathrm{T}}}(\alpha_i + \beta_i P^{\mathrm{T}}_{i,t} + \gamma_i (P^{\mathrm{T}}_{i,t})^2 + e_i s_{i,t}) \quad s.t.\ (5)-(14), (34)-(36). \tag{37}$$

Remark: *Although the NLP formulation for the STHS can be directly solved using the IPM, but it is well known that IPM is a local optimization method. If the STHS problem is solved using the IPM in a single step, the optimization can easily become trapped in a poor local optimum due to its non-convex nature and multiple local minima.*

5. Solution methodology for the STHS

As it can be seen in previous sections, when transmission loss is not considered, the STHS problem can reformulate as a logarithmic size MILP formulation that can be optimized using a state-of-the-art MILP solver directly and efficiently. And then, a global optimal solution within a preset tolerance can be achieved via an enumeration algorithm. Though the obtaining schedule result satisfies all the constraints in the MILP formulation, it may lead to power imbalance in the actual operation due to the linearization errors. To eliminate the errors caused by linearization, after the optimization in MILP, the original STHS, i.e. the NLP formulation given in (37), will be solved again to guarantee the feasibility of the solution.

Consequently, a deterministic solution methodology that combining MILP-based technique and NLP-based approach, denoted as MILP-NLP, is proposed to solve the STHS problem, which is summarized as follows.

Step 1: Solve the L-MILP formulation (30) by using the MILP-based approach to obtain a global optimal solution within a preset tolerance for the STHS problem.

Step 2: Solve the NLP formulation (37) by using the IPM, where the initial point is set equal to the solution obtained in step 1, to obtain a high-quality local feasible optimal solution for the STHS problem.

When transmission loss is considered, some transmission loss constraints will be involved in the NLP formulation (37), which implies that more outputs are required to balance the power balance equations. And because in each period the transmission loss is small compared with the load demand, the global optimal solution yielding in step 1 can be regarded as an approximated solution for the STHS with transmission loss [46]. Therefore, when the NLP formulation is solved in step 2, based on such an approximated solution obtained in step 1, the outputs of the units will be fine-tuned via IPM, to satisfy the new constraints, and then, a high-quality feasible optimal solution can be expected.

6. Simulation results and discussions

In this section, several test systems that are widely studied for STHS over a scheduled time horizon of 24 h are adopted to assess the validity and effectiveness of the proposed formulations and solution methodology. Firstly, a system with three thermal units and four cascaded hydro reservoirs is carried out, suggesting that solving the proposed L-MILP formulation is more efficient than the G-MILP formulation and our solution methodology MILP-NLP is an effective approach for non-convex and non-differentiable STHS problem. Here, two cases, with and without transmission loss constraints, are considered. And then, two larger systems, one consisting of ten thermal units and four cascaded hydro reservoirs, another consisting of forty thermal units and ten cascaded hydro reservoirs, are implemented to demonstrate the potential of MILP-NLP approach for solving large-scale problems. All test cases are executed on an Intel Core 2.5 GHz notebook with 8 GB of RAM. The models are coded in MATLAB R2014a with YALMIP [47] and are optimized using C-PLEX 12.6.1 [48] to solve the L-MILP formulation given in (30) and IPOPT 3.12.6 [49] to solve the NLP formulation given in (37).

6.1. Test system 1

This test system contains three thermal units and four cascaded hydro reservoirs, each of which has one hydro unit. The system data can be found in [22].

Case 1: STHS without transmission loss

In this test case, transmission loss is not taken into account for the STHS. First, we directly solve the L-MILP formulation (30) using CPLEX to 0.01% optimality. In this formulation, the segment parameters $\hat{L}_i$, M_i, and N_i are set to 6, 13 and 13, respectively. After optimization, a global optimal solution for this formulation is obtained in 1.30s with a total generation cost of \$39754.08. Taking this solution as the initial point in step 2, we solve the NLP formulation (37) using IPOPT with the default options, obtaining a high-quality feasible optimal solution for the STHS with an optimal value of \$40004.90 in 1.61s. Whereas, if the G-MILP formulation (26) is utilized in step 1, more than 500s will be required to find a feasible solution, which clearly indicates that using the L-MILP formulation in step 1 can make a significant computational saving.

Table 2 compares the results for the total generation cost obtained using the MILP-NLP approach with other methods for this case. We can see that the proposed MILP-NLP can obtain a solution with a smaller total generation cost. Since a computationally efficient L-MILP formulation is incorporated into the solution procedure, compared with other reported methods, our MILP-NLP can solve to a better solution with less CPU time (see in Table 2).

The optimal dispatch results for case 1 of test system 1 are given in Table 3 to check the feasibility. Not surprisingly, our solution can always fully satisfy all the constraints of the original STHS problem. And the circumstances such as power balance violations reported in [50] does not occur. This is primarily the result of the rigorous theoretical foundations of the IPM.

Table 2: Summary results for Case 1 of test system 1

Method	Total generation cost (\$)			CPU Time (s)
	Minimum	Average	Maximum	
EP [17]	45063.00	-	-	-
PSO [24]	44740.00	-	-	232.73
MDE [19]	42611.14	-	-	125.00
CSA [28]	42440.57	-	-	109.12
TLBO [29]	42385.88	42407.23	42441.36	-
IQPSO [25]	42359.00	-	-	-
QTLBO [30]	42187.49	42193.46	42202.75	21.60
MHDE [20]	41856.50	-	-	31.00
DNLPSO [27]	41231.00	41783.00	42367.00	125.00
MCDE [1]	40945.75	41380.54	41977.04	50.8
RCGA-AFSA [32]	40913.83	41235.73	41362.58	21.00
TLPSOS [14]	40298.28	-	-	102
MDNLPSO [27]	40179.00	40637.00	41182.00	123.00
PSO [36]	40145.00	-	-	-
MILP-NLP	40004.90	-	-	2.91

Case 2: STHS with transmission loss

In this test case, transmission loss is also considered for the STHS. In step 1, the same L-MILP formulation is solved as in case 1. And then, we solve the NLP formulation, where transmission loss constraints are also included in step 2, obtaining a high-quality feasible optimal solution with an optimal value of \$41199.14 in 1.70s.

The results for the total generation cost obtained using MILP-NLP approach and other methods are shown in Table 4. It is obvious that, when transmission loss is considered, the MILP-NLP can also solve to a solution with a smaller total generation cost in shorter time.

The optimal dispatch results for case 2 of test system 1 are given in Table 5 for the feasibility verification.

Table 4: Summary results for Case 2 of test system 1

Method	Total generation cost (\$)			Time (s)
	Minimum	Average	Maximum	
SPSO [26]	44980.32	46112.85	49166.68	139.7
DE [22]	44862.28	-	-	24
QEA [18]	44686.31	-	-	19
RCGA [32]	43465.24	43643.37	43717.27	32
MDE [19]	43435.41	-	-	-
ADE [22]	43222.41	-	-	24
CABC [31]	43104.56	-	-	32
SPPSO [26]	42740.23	43622.14	44346.97	32.7
RQEA [18]	42715.69	-	-	21
MHDE [20]	42679.87	-	-	40
CDE [22]	42452.99	-	-	29
AABC [31]	42381.24	-	-	16
GSO [34]	42316.39	42339.35	42379.18	617.36
QOGSO [34]	42120.02	42130.15	42145.37	625.07
RCGA-AFSA [32]	41707.96	41832.36	41894.63	25
ACDE [22]	41593.48	-	-	29
MCDE [1]	41586.18	42022.67	42365.84	100.05
DRQEA [18]	41435.76	-	-	18
PSO [36]	41339.00	-	-	-
ACABC [31]	41281.75	-	-	18
MILP-NLP	41199.14			3.00

6.2. Test system 2

This test system contains ten thermal units and four cascaded hydro reservoirs. The system data is taken from [21]. Here transmission loss is not taken into account.

Similar to above two cases, we solve the L-MILP formulation first, with the parameters set the same as the cases

Table 3: Dispatch results for Case 1 of test system 1

t	Water discharge ($\times 10^4$ m^3)				Hydro generation (MW)				Thermal generation (MW)			Total (MW)
	$Q_{1,t}$	$Q_{2,t}$	$Q_{3,t}$	$Q_{4,t}$	$P^{\mathrm{H}}_{1,t}$	$P^{\mathrm{H}}_{2,t}$	$P^{\mathrm{H}}_{3,t}$	$P^{\mathrm{H}}_{4,t}$	$P^{\mathrm{T}}_{1,t}$	$P^{\mathrm{T}}_{2,t}$	$P^{\mathrm{T}}_{3,t}$	
1	9.2353	6.0000	21.9196	6.0000	82.4919	50.1640	31.2163	129.0269	102.6735	124.9079	229.5196	750.0000
2	6.4305	6.0000	24.4706	6.0000	65.0257	51.2960	8.5071	125.7437	175.0000	124.9079	229.5196	780.0000
3	6.6402	6.0000	22.3062	6.0000	66.9283	52.9340	18.8447	121.6253	175.0000	124.9079	139.7598	700.0000
4	8.2707	6.0000	18.8180	6.0000	77.7945	54.5000	34.5422	115.8221	102.6735	124.9079	139.7598	650.0000
5	8.5466	6.0000	17.7364	6.0000	78.7015	55.5040	37.7075	130.7458	102.6735	124.9079	139.7598	670.0000
6	8.5120	6.0995	16.5540	7.5861	78.0231	56.6681	41.9634	166.2444	102.6735	124.9079	229.5196	800.0000
7	9.1058	7.2418	15.9724	11.2162	80.9580	63.9063	43.9920	219.1348	102.6735	209.8158	229.5196	950.0000
8	7.7809	6.0000	15.6689	9.6777	73.5699	55.8277	44.9644	208.7213	102.6735	294.7237	229.5196	1010.0000
9	9.3780	8.2307	17.0822	15.1439	83.0104	70.4755	41.0611	268.5362	102.6735	294.7237	229.5196	1090.0000
10	8.6436	7.3635	17.5698	14.9809	79.7466	65.9178	38.9415	268.4773	102.6735	294.7237	229.5196	1080.0000
11	8.9821	7.9285	16.9958	16.3632	82.5601	70.0284	40.4857	280.0090	102.6735	294.7237	229.5196	1100.0000
12	7.3677	6.0000	19.4395	15.2782	72.9836	57.9357	31.2685	271.1356	102.6735	294.7237	319.2794	1150.0000
13	9.2154	8.8359	18.4823	17.0822	85.0492	75.9697	35.9768	286.0875	102.6735	294.7237	229.5196	1110.0000
14	9.0613	8.8162	18.1225	17.5698	84.8781	75.9509	37.3795	289.7826	102.6735	209.8158	229.5196	1030.0000
15	8.0205	7.8684	19.1978	16.9958	78.9517	70.8640	32.7585	285.4170	102.6735	209.8158	229.5196	1010.0000
16	9.6055	10.3009	16.3786	19.4395	88.6589	83.0726	43.6755	302.5842	102.6735	209.8158	229.5196	1060.0000
17	9.0654	10.0354	15.8631	18.4823	85.5973	80.1509	45.9420	296.3008	102.6735	209.8158	229.5196	1050.0000
18	7.8675	9.3639	16.1597	18.1225	77.9853	74.8809	46.4245	293.7925	102.6735	294.7237	229.5196	1120.0000
19	9.6344	11.9061	14.7104	20.0000	88.2409	83.1250	50.8885	305.1798	103.2304	209.8158	229.5196	1070.0000
20	7.7841	11.0805	14.8022	20.0000	76.6333	77.9434	51.9853	301.4292	102.6735	209.8158	229.5196	1050.0000
21	5.0000	7.8239	11.2589	17.1117	54.6031	62.2333	55.1875	280.8751	102.6735	124.9079	229.5196	910.0000
22	5.8190	10.9151	12.1448	20.0000	62.1067	76.7757	57.7506	296.0257	102.6735	124.9079	139.7598	860.0000
23	5.1939	11.5376	12.5353	20.0000	56.8194	76.8192	58.7757	290.2445	102.6735	124.9079	139.7598	850.0000
24	9.8395	14.6521	12.9728	20.0000	90.6747	80.3745	58.9976	284.4000	20.8854	124.9079	139.7598	800.0000

Table 5: Dispatch results for Case 2 of test system 1

t	Water discharge ($\times 10^4$ m^3)				Hydro generation (MW)				Thermal generation (MW)			Total (MW)	Loss (MW)
	$Q_{1,t}$	$Q_{2,t}$	$Q_{3,t}$	$Q_{4,t}$	$P^{\mathrm{H}}_{1,t}$	$P^{\mathrm{H}}_{2,t}$	$P^{\mathrm{H}}_{3,t}$	$P^{\mathrm{H}}_{4,t}$	$P^{\mathrm{T}}_{1,t}$	$P^{\mathrm{T}}_{2,t}$	$P^{\mathrm{T}}_{3,t}$		
1	9.4183	6.0175	21.0278	6.0000	83.3797	50.2788	36.1580	129.0269	102.6735	124.9079	229.5196	755.9443	5.9443
2	9.6006	6.6809	19.8024	6.0000	84.0892	55.6332	37.7520	125.7437	128.6221	124.9079	229.5196	786.2676	6.2676
3	6.9200	6.0000	21.9509	6.0000	68.1272	52.5591	23.5725	121.6253	175.0000	124.9079	139.7598	705.5517	5.5517
4	8.7319	6.0681	18.9982	6.0000	79.2733	54.5983	37.4206	115.8221	102.6735	124.9079	139.7598	654.4555	4.4555
5	8.5297	6.3078	18.4370	6.3569	77.2744	57.1758	38.8791	134.3918	102.6735	124.9079	139.7598	675.0622	5.0622
6	8.6072	6.8029	17.8121	8.4475	77.1515	60.7090	41.4080	171.2475	102.6735	124.9079	229.5196	807.6169	7.6169
7	7.7284	6.0000	17.5245	7.8968	71.9732	55.0676	42.3597	176.1912	175.0000	209.8158	229.5196	959.9271	9.9271
8	8.1467	6.3306	16.0100	10.5479	74.8542	57.7741	46.7192	215.4912	102.6735	294.7237	229.5196	1021.7554	11.7554
9	7.9369	6.2355	18.4754	11.7460	74.2279	57.9663	38.3366	234.4268	175.0000	294.7237	229.5196	1104.2009	14.2009
10	9.1558	8.4206	18.1264	15.8331	81.8924	72.4830	38.4571	275.9498	102.6735	294.7237	229.5196	1095.6990	15.6990
11	9.4314	8.7621	16.9505	17.5245	84.1736	74.5719	41.5973	289.4452	102.6735	294.7237	229.5196	1116.7047	16.7047
12	7.5402	6.3334	17.4891	16.0100	73.5314	59.8485	39.9227	277.4437	102.6735	294.7237	319.2794	1167.4229	17.4229
13	9.7388	9.7543	18.5710	18.4754	86.8855	79.8443	37.4040	296.2541	102.6735	294.7237	229.5196	1127.3047	17.3047
14	9.7818	10.1283	17.6726	18.1632	87.7427	81.0836	41.1920	294.0446	102.6735	209.8158	229.5196	1046.0717	16.0717
15	8.4989	8.7095	18.0309	17.4156	81.1566	73.7926	40.3824	288.1524	102.6735	209.8158	229.5196	1025.4929	15.4929
16	6.9593	7.3385	18.5129	16.9871	71.1105	65.7774	39.7890	285.3499	175.0000	209.8158	229.5196	1076.3622	16.3622
17	9.9234	11.2351	15.3680	19.2524	89.6721	84.1591	50.1068	300.7183	102.6735	209.8158	229.5196	1066.6652	16.6652
18	8.8301	10.5232	15.7122	19.1623	83.4531	78.3831	50.0858	298.6435	102.6735	294.7237	229.5196	1137.4823	17.4823
19	6.1114	8.8523	15.7677	17.6011	64.2692	68.9760	50.7413	288.3419	175.0000	209.8158	229.5196	1086.6637	16.6637
20	9.3710	12.2598	14.0783	20.0000	85.9415	81.1667	54.9729	302.6758	102.6735	209.8158	229.5196	1066.7658	16.7658
21	5.0000	8.4339	12.8578	18.8622	54.4194	64.3818	56.8230	292.0769	102.6735	124.9079	229.5196	924.8021	14.8021
22	7.2797	13.0391	13.2715	20.0000	73.2468	81.0965	58.2711	294.4058	102.6735	124.9079	139.7598	874.3614	14.3614
23	6.7587	13.5011	13.8152	20.0000	69.6742	78.1603	59.0164	289.7555	102.6735	124.9079	139.7598	863.9476	13.9476
24	5.0000	8.2655	16.4732	18.8357	55.0200	56.9056	55.6589	277.7155	102.6735	124.9079	139.7598	812.6413	12.6413

in test system 1. After optimization, a global optimal solution is obtained in 1.59s with a total generation cost of \$157896.54. Taking this solution as the initial point in step 2, we solve the NLP formulation using IPOPT with the default options, obtaining a high-quality feasible optimal solution with an optimal value of \$158127.84 in 5.14s. However, if the G-MILP formulation is solved in step 1, more than 1000s will be consumed for searching a feasible solution.

The total generation cost obtained using the MILP-NLP approach are compared with those of other methods in Table 6. We can also see that MILP-NLP can achieve a solution with a much smaller total generation cost in a short time. The optimal dispatch results for this test system are given in Tables 7 and 8 for the feasibility checking.

Table 6: Summary results of test system 2

Method	Total generation cost (\$)			CPU Time (s)
	Minimum	Average	Maximum	
SPSO [26]	189350.63	190560.31	191844.28	108.10
MDE [26]	177338.60	179676.35	182172.01	86.50
DE [21]	170964.15	-	-	96.40
IDE [23]	170576.50	170589.60	170608.30	727.06
GSO [34]	170511.26	170547.56	170586.91	652.35
QOGSO [34]	170293.21	170321.57	170349.34	661.79
SPPSO [26]	167710.56	168688.92	170879.30	24.80
MCDE [1]	165331.70	166116.4	167060.60	178.5
RCCRO [33]	164138.65	164140.40	164182.35	22.02
ORCCRO[33]	163066.03	163068.77	163134.54	15.74
IPCSO [35]	162714.00	162813.00	162953.00	22.65
PSO [36]	161292.00	-	-	-
MILP-NLP	158127.84	-	-	6.73

Table 7: Hydrothermal generation (MW) for test system 2

t	Hydro generation				Thermal generation										Total
	$P^{\mathrm{H}}_{1,t}$	$P^{\mathrm{H}}_{2,t}$	$P^{\mathrm{H}}_{3,t}$	$P^{\mathrm{H}}_{4,t}$	$P^{\mathrm{T}}_{1,t}$	$P^{\mathrm{T}}_{2,t}$	$P^{\mathrm{T}}_{3,t}$	$P^{\mathrm{T}}_{4,t}$	$P^{\mathrm{T}}_{5,t}$	$P^{\mathrm{T}}_{6,t}$	$P^{\mathrm{T}}_{7,t}$	$P^{\mathrm{T}}_{8,t}$	$P^{\mathrm{T}}_{9,t}$	$P^{\mathrm{T}}_{10,t}$	
1	75.7365	50.1641	0.0005	201.8992	229.5196	199.5996	94.7998	119.7331	174.5997	139.7331	104.2753	84.8665	98.0603	177.0125	1750.0000
2	89.4156	64.4318	15.1807	188.7722	229.5196	199.5996	94.7998	119.7331	174.5997	139.7331	104.2753	84.8665	98.0603	177.0125	1780.0000
3	82.7581	57.8707	11.8917	175.1464	229.5196	199.5996	94.7998	119.7331	124.7331	139.7331	104.2753	84.8665	98.0603	177.0125	1700.0000
4	85.5214	64.3014	21.2213	157.2936	229.5196	199.5996	94.7998	119.7331	124.7331	139.7331	104.2753	84.8665	98.0603	126.3417	1650.0000
5	64.5149	52.8821	7.3585	172.9114	229.5196	199.5996	94.7998	119.7331	124.7331	139.7331	104.2753	84.8665	98.0603	177.0125	1670.0000
6	84.4386	72.2115	35.4962	185.6541	229.5196	199.5996	94.7998	119.7331	174.5997	139.7331	104.2753	84.8665	98.0603	177.0125	1800.0000
7	76.2897	60.0420	29.4497	197.4594	319.2794	274.3995	94.7998	119.7331	174.5997	139.7331	104.2753	84.8665	98.0603	177.0125	1950.0000
8	75.3090	59.5410	31.7536	206.7706	319.2794	274.3995	94.7998	119.7331	174.5997	139.7331	104.2753	134.7331	98.0603	177.0125	2010.0000
9	80.6242	67.9145	37.1837	217.7853	319.2794	274.3995	94.7998	119.7331	224.4662	139.7331	104.2753	134.7331	98.0603	177.0125	2090.0000
10	75.2941	60.7576	34.3158	223.1400	319.2794	274.3995	94.7998	119.7331	224.4662	139.7331	104.2753	134.7331	98.0603	177.0125	2080.0000
11	79.6874	66.5305	37.7361	229.5538	319.2794	274.3995	94.7998	119.7331	224.4662	139.7331	104.2753	134.7331	98.0603	177.0125	2100.0000
12	77.1155	63.9041	37.4469	235.1746	319.2794	274.3995	94.7998	119.7331	224.4662	189.5997	104.2753	134.7331	98.0603	177.0125	2150.0000
13	77.4971	64.9193	39.8851	241.2061	319.2794	274.3995	94.7998	119.7331	224.4662	139.7331	104.2753	134.7331	98.0603	177.0125	2110.0000
14	79.6493	67.6859	41.7888	254.1167	319.2794	274.3995	94.7998	119.7331	174.5997	139.7331	104.2753	84.8665	98.0603	177.0125	2030.0000
15	71.3585	60.9329	40.6619	250.2875	319.2794	274.3995	94.7998	119.7331	174.5997	139.7331	104.2753	84.8665	98.0603	177.0125	2010.0000
16	81.1664	71.6981	45.4635	274.9128	319.2794	274.3995	94.7998	119.7331	174.5997	139.7331	104.2753	84.8665	98.0603	177.0125	2060.0000
17	75.6856	67.5241	45.2393	274.7917	319.2794	274.3995	94.7998	119.7331	174.5997	139.7331	104.2753	84.8665	98.0603	177.0125	2050.0000
18	78.2060	70.6592	47.4940	287.0149	319.2794	274.3995	94.7998	119.7331	174.5997	139.7331	104.2753	134.7331	98.0603	177.0125	2120.0000
19	74.7376	69.7546	48.9206	289.8280	319.2794	274.3995	94.7998	119.7331	174.5997	139.7331	104.2753	84.8665	98.0603	177.0125	2070.0000
20	65.1629	65.5876	50.1174	282.3729	319.2794	274.3995	94.7998	119.7331	174.5997	139.7331	104.2753	84.8665	98.0603	177.0125	2050.0000
21	70.2404	71.3936	52.4883	293.6781	229.5196	199.5996	94.7998	119.7331	174.5997	139.7331	104.2753	84.8665	98.0603	177.0125	1910.0000
22	70.0901	66.3652	54.5167	296.6950	229.5196	199.5996	94.7998	119.7331	124.7331	139.7331	104.2753	84.8665	98.0603	177.0125	1860.0000
23	57.9166	67.3167	56.1426	296.2910	229.5196	199.5996	94.7998	119.7331	124.7331	139.7331	104.2753	84.8665	98.0603	177.0125	1850.0000
24	62.5969	70.4382	57.8260	296.0812	229.5196	199.5996	94.7998	119.7331	124.7331	139.7331	45.0000	84.8665	98.0603	177.0125	1800.0000

Table 8: Water discharge ($\times 10^4$ m^3) for test system 2

t	$Q_{1,t}$	$Q_{2,t}$	$Q_{3,t}$	$Q_{4,t}$
1	8.0055	6.0000	26.2599	13.2347
2	10.7803	8.2300	23.2996	13.1765
3	9.4171	6.9744	23.0370	13.2787
4	10.2806	7.8931	21.3050	13.2192
5	6.6617	6.0000	23.3397	13.0000
6	10.4427	9.3415	18.2583	13.0000
7	8.7648	7.3649	19.5241	13.0000
8	8.5524	7.3150	18.9438	13.0000
9	9.5521	8.8183	17.6656	13.0001
10	8.3449	7.4387	18.3056	13.0000
11	8.9335	8.3312	17.3874	13.0000
12	8.3840	7.8738	17.6590	13.0000
13	8.3092	8.0480	17.2836	13.1968
14	8.5009	8.4704	16.9952	14.1509
15	7.1169	7.1917	17.5643	13.3860
16	8.5299	9.1268	16.1470	15.9005
17	7.6483	8.5305	16.4122	15.7180
18	8.0212	9.5300	15.7149	17.2595
19	7.5274	9.7040	15.3794	17.6456
20	6.2637	9.0107	15.1265	16.7436
21	6.9132	10.4160	10.0000	18.5773
22	6.8680	9.4010	10.3195	19.7062
23	5.3444	9.8652	10.6491	20.6700
24	5.8374	11.1248	11.1506	22.5442

6.3. Test system 3

To show the potential of MILP-NLP for solving the STHS problem, a larger test system that contains forty thermal units and ten cascaded hydro reservoirs is simulated. The characteristics of the forty thermal units are taken from [51]. The hydraulic system is constructed based on [52]. The characteristics of the hydraulic network and load demands are provided in Appendix.

First, we directly solve the L-MILP formulation using CPLEX to 0.2% optimality. In this case, the segment parameters $\hat{L}_i$, M_i, and N_i are set to 6, 8 and 8, respectively. After optimization, a global optimal solution is obtained in 25.55s with a total generation cost of \$2200819.68. Taking this solution as the initial point in step 2, we solve the NLP formulation using IPOPT with the default options, a feasible optimal solution with an optimal value of \$2201437.44 can be calculated in 18.83s. But if the G-MILP formulation is solved in step 1, more than 1000s will be expended.

7. Conclusion

In this paper, a deterministic MILP-NLP approach is proposed for the complicated non-convex and non-differentiable STHS problem. To save computational effort, a logarithmic size L-MILP formulation which can be solved to a global optimal solution directly and efficiently, is constructed first. However, if the STHS is directly solved by the L-MILP formulation in a single step, some piecewise linearization errors will be occured inevitably. Therefore, a differentiable NLP formulation which is equivalent to the original STHS will be solved to search a high-quality feasible optimal solution for the STHS. The simulation results show that, the proposed L-MILP formulation can provide a significant computational advantage and it is very promising for solving numerous variants of the STHS problem. When it is incorporated into the solution procedure, MILP-NLP is competitive with currently state-of-the-art approaches.

Appendix

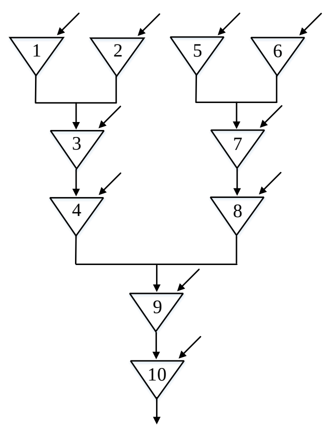

Figure 7: Hydraulic network of test system 3

Table 9: Transfer time to immediate downstream reservoir (h)

r	1, 5	2, 6	3, 7, 9	4, 8, 10
τ_r	2	3	4	0

Table 10: Hydro power generation coefficients

j	$\xi_{j,1}$	$\xi_{j,2}$	$\xi_{j,3}$	$\xi_{j,4}$	$\xi_{j,5}$	$\xi_{j,6}$
1, 5	-0.0042	-0.42	0.030	0.90	10.0	-50
2, 6	-0.0040	-0.30	0.015	1.14	9.5	-70
3, 7, 9	-0.0016	-0.30	0.014	0.55	5.5	-40
4, 8, 10	-0.0030	-0.31	0.027	1.44	14.0	-90

Table 11: Reservoir inflows ($\times 10^4$ m^3)

Reservoir	Hour											
	1	2	3	4	5	6	7	8	9	10	11	12
1, 5	10	9	8	7	6	7	8	9	10	11	12	10
2, 6	8	8	9	9	8	7	6	7	8	9	9	8
3, 7, 9	8.1	8.2	4	2	3	4	3	2	1	1	1	2
4, 8, 10	2.8	2.4	1.6	0	0	0	0	0	0	0	0	0
Reservoir	Hour											
	13	14	15	16	17	18	19	20	21	22	23	24
1, 5	11	12	11	10	9	8	7	6	7	8	9	10
2, 6	8	9	9	8	7	6	7	8	9	9	8	8
3, 7, 9	4	3	3	2	2	2	1	1	2	2	1	0
4, 8, 10	0	0	0	0	0	0	0	0	0	0	0	0

Table 12: Reservoir characteristics ($\times 10^4$ m^3)

j	$V_{j,\min}$	$V_{j,\max}$	$V_{j,\text{init}}$	$V_{j,\text{end}}$	$Q_{j,\min}$	$Q_{j,\max}$	$P_{j,\min}$	$P_{j,\max}$
1, 5	80	150	100	120	5	15	0	500
2, 6	60	120	80	70	6	15	0	500
3, 7	100	240	170	170	10	30	0	500
4, 8	70	160	120	140	13	25	0	500
9	100	400	170	190	10	60	0	500
10	70	320	120	160	13	50	0	500

Table 13: Load demands (MW)

t	1	2	3	4	5	6	7	8
load	7620	8150	8790	9250	9510	9530	9650	9070
t	9	10	11	12	13	14	15	16
load	8590	8100	8620	9230	9620	9670	9700	9530
t	17	18	19	20	21	22	23	24
load	9710	9600	9320	9100	8820	8480	8100	7730

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