Coordination of a two-level supply chain with contracts

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Abstract

We consider the coordination of planning decisions of a single product in a supply chain composed of one supplier and one retailer, by using contracts. We assume that the retailer has the market power: he can impose his optimal replenishment plan to the supplier. Our aim is to minimize the supplier's cost without increasing the retailer's cost. To this end, the supplier (or a trusted third party) proposes to the retailer a contract, which is made of a replenishment plan and a side payment. This side payment compensates the increase of cost of the retailer due to the fact that the proposed replenishment plan may have a cost larger than the retailer's optimal replenishment plan. We evaluate how much the supplier can gain by using contracts under several scenarios which depend on the side payment coverage. From a theoretical point of view, in all the scenarios, contracts may decrease the cost of the supplier by an arbitrarily large factor. We do experiments which measures the gain that can be obtained in practice on various instances types. If side payment are allowed, experiments show that the use of contracts decreases significantly the cost of the supplier, and that side payment on the holding costs are sufficient. We show that if there is no side payment, or if there is no constraint on the side payment, then the problem can be solved in polynomial time. On the contrary, if the side payment is limited to the holding costs of the retailer, then the problem is NP-hard. We extend this study to the case where the information is asymmetric (the supplier - or the trusted third entity - does not know all the costs of the retailer): in this case, the situation is modeled by a screening game.

Keywords: Supply chain management, dynamic lot-sizing, contracts, complexity analysis, screening game.

1 Introduction

We address the problem of coordinating planning decisions in a supply chain for a single item. We consider a supply chain with two actors: one supplier and one retailer. In general, the supplier and the retailer are individual entities with conflicting objectives. Each of them wants to optimize their decisions in their best interest. However, in such supply chain, giant retailers, like Wal-Mart, Carrefour, Toys-R-us, or food stores [3, 5, 25], may have the market power. In that case, the retailers take decisions in their best interest, and then the supplier takes his decisions based on the retailer's optimal decisions. From the supplier's viewpoint, this solution can induce a poor cost for him, and so for the whole supply chain. Then, to obtain a better performance for the supplier, coordination mechanisms need to be design to better manage the planning decisions in such supply chains.

The planning decisions in this supply chain are as follows. The retailer has to satisfy a deterministic demand over a finite planning horizon of T periods. He has to determine a replenishment plan over the horizon by setting the ordering periods and quantities to meet the demands. The supplier has to determine a production plan in order to satisfy the retailer's replenishment plan. For each actor, ordering units (in the replenishment or in the production plan) induces a fixed cost and a unit ordering cost. Carrying units in the inventory induces a unit holding cost for each actor. The overall cost of each actor is given by the sum of his total ordering and holding costs over the planning horizon.

In this paper, we consider that the retailer has the market power. If the supplier and the retailer act individually, the retailer will impose his (or one of his) optimal replenishment plan(s). In this case, the cost of the supplier may be very high. However, if the actors decide to collaborate, the supplier may decrease his cost. To this end, a replenishment plan can be proposed by the supplier, who has to know the ordering and holding costs of the retailer, or by a trusted third party, at which both actors would give their ordering and holding costs. If the proposed replenishment plan is not optimal for the retailer and if there is no side payment, then the retailer will reject it, because of his leadership position. The aim is thus to propose a contract made of a replenishment plan and a payment such that the cost of the retailer, which is the cost of the proposed replenishment plan minus the payment he receives, is at most the cost he would have by choosing his optimal replenishment cost. In this paper, our aim is to compute a contract which minimizes the cost of the supplier. We analyze the following scenarios:

- No side payment is allowed between the actors of the supply chain (the replenishment plan has thus to be an optimal replenishment plan for the retailer).
- The supplier can assume part of the retailer's cost through side payment. We consider here two cases. In the first case, the side payment may concern all the costs of the retailer (the supplier is allowed to pay the retailer's holding and ordering costs, or part of it, over the horizon). In the second case, the side payment may only concern the holding costs (the supplier is allowed to pay the retailer's holding costs, or part of it, over the horizon). This second case may be particularly adapted when the retailer and the supplier share a same warehouse, since in this case it is easy for the supplier to pay the holding costs of the retailer.

As seen above, in the previous scenarios, we assume that the costs of the retailer are known from the supplier (or from a trusted third party). However, in some cases, the retailer may not reveal his ordering and holding costs, even to a trusted third party. In this case, we use screening games [21] to model this situation. The idea of screening game is the following one: we assume that the supplier knows the different "types" of retailers (and theirs ordering and holding costs), and the probability that a retailer is of a given type. The supplier does not know of which type his retailer is, and he will propose him several contracts. The retailer will choose one of these contracts (we will insure that there is a contract for which the cost is at most the cost of his optimal replenishment plan). Like in the previous case with complete information, our aim is to design contracts in order to minimize the cost of the supplier.

In this paper, we assume that the cost of the retailer will be his optimal cost, and we focus on minimizing the supplier's gain under this constraint. This will give us an upper bound on the gain which can be obtained by collaborating when the retailer has the market power. Other mechanisms, such as price discounts, holding cost subsidies, and revenue sharing have been studied for this problem [4, 12]. Note also that we consider that the gain of collaborating benefits only the supplier (since the aim is to minimize the supplier's cost while the retailer's cost is his optimal cost). All the results can be directly adapted if we decide to share the profit of the supplier (i.e. his cost without collaboration minus his cost with collaboration) between the supplier and the retailer. For example, the retailer could obtain a percentage of the profit that the supplier obtain by collaborating – this would gives him an incentive to accept to collaborate with the supplier or the trusted third party.

Contributions and map of the paper.

The purpose of our work is to evaluate how much the supplier can gain by designing contracts under complete or asymmetric information assumptions on the retailer's cost structure. We start the paper (Section 2) by introducing useful notations and recalling some results for the classical two-level lot-sizing problem, whose aim is to minimize the supply chain cost (which is the sum of the retailer's cost and the supplier's cost).

In Section 3, we study the problem where the supplier has a complete information about the retailer's costs parameters. We show that designing contracts which minimize the supplier's cost can be done in polynomial time if no side payment is allowed or if there is no constraint on the side payment. On the contrary, if the side payment is restricted to the retailer's holding costs, we prove that the problem is NP-hard. From a theoretical point of view, we show that cooperation can decrease the cost of the supplier by an arbitrarily large factor even without side payment; that using side payment on the holding costs can decrease the cost of the supplier by an arbitrarily large factor compared to the case without side payment; and that using side payment without restriction on the costs can decrease the cost of the supplier by an arbitrarily large factor compared to the case with side payment on the holding costs only. From a practical point of view, we do experiments that validate our approach, showing that contracts decrease significantly the supplier's costs (up to about 30%). Surprisingly, these experiments show that, in practice, allowing side payment only on the retailer's holding costs decrease the supplier's cost as much as if the side payment was on both on ordering and holding costs.

In Section 4, we address the problem where the supplier has not a complete information about the retailer's costs parameters. We model this situation by a screening game. We show that in this case the problem of designing contracts which minimize the supplier's cost is still polynomial when no side payment is allowed. We also show that the loss of information for the supplier has a cost: the supplier's cost can be arbitrarily high compared to the case where complete information is assumed. An experimental analysis is performed to evaluate the supplier's gain when the actors collaborate, and to compare the supplier's gain under complete and asymmetric information assumption. This analysis shows that the supplier's gain can be significant, and thus that cooperation between both actors is useful.

Finally, in Section 5, we extend our experimental study by analyzing the case where there is more than one retailer.

2 Preliminaries: minimizing the cost of the supply chain

Determining the optimal cost of the supply chain is equivalent to solve a two-level Uncapacitated Lot-Sizing (2ULS) problem, which is polynomially solvable [26]. Section 2.1 introduces notations used through this paper. It also gives the mathematical formulation for the 2ULS problem. Section 2.2 recalls the polynomial solving approach proposed by Zangwill [26]. This algorithm will be adapted for our problem in the following sections.

2.1 Notations and mathematical formulation

We denote by d_t the demand that the retailer has to satisfy at period t. The cumulative demand from period i to period j (with $1 \leq i \leq j \leq T$) is denoted by $d_{ij} = \sum_{k=i}^{j} d_k$. $\mathcal{T}$ denotes the set of all the periods $\{1, \ldots, T\}$. Ordering one unit at period t induces a fixed ordering cost f_t^R (resp. f_t^S) and a unit ordering cost p_t^R (resp. p_t^S) for the retailer (resp. supplier). Carrying one unit from period t to period $t+1$ induces a unit holding cost h_t^R (resp. h_t^S) for the retailer (resp. supplier).

We denote by x_t^R (resp. x_t^S) the retailer's (resp. supplier's) ordering quantity at period t, and by s_t^R (resp. s_t^S) the retailer's (resp. supplier's) inventory quantity at the end of period t. Let y_t^R (resp. y_t^S) be the setup binary variable at period t at the retailer (resp. supplier) level: this variable is equal to 1 if an order occurs at period t at the retailer (resp. supplier) level.

The cost of the retailer is $C^R(y^R, x^R, s^R) = \sum_{t \in \mathcal{T}} (f_t^R y_t^R + p_t^R x_t^R + h_t^R s_t^R)$. Likewise, the cost of the supplier is $C^S(y^S, x^S, s^S) = \sum_{t \in \mathcal{T}} (f_t^S y_t^S + p_t^S x_t^S + h_t^S s_t^S)$.

The 2ULS problem, which consists at minimizing $C^R(y^R, x^R, s^R) + C^S(y^S, x^S, s^S)$, can be viewed as a fixed-charge network flow problem (see Figure 1). The nodes 1 to T at both levels correspond to the periods. A dummy node is added to represent the total flow d_{1T} of the network. The demands are represented at the retailer level. The edges from the dummy node to the nodes $1, \ldots, T$ correspond to the ordering edges of the supplier. The flow in edge (d_{1T}, t) is x_t^S, the amount ordered at a period t. The cost of this flow is $f_t^S + x_t^S p_t^S$. At the supplier's level, the edge $(t, t+1)$ represents s_t^S, the stock at the end of period t. Its cost is $s_t^S h_t^S$. Similarly, the edge from node t at the supplier's level and node t at the retailer's level corresponds to x_t^R, the ordering of the retailer at period t. The cost of this flow is $f_t^R + x_t^R p_t^R$. At the retailer's level, the edge $(t, t+1)$ represents s_t^R, the stock at the end of period t. Its cost is $s_t^R h_t^R$.

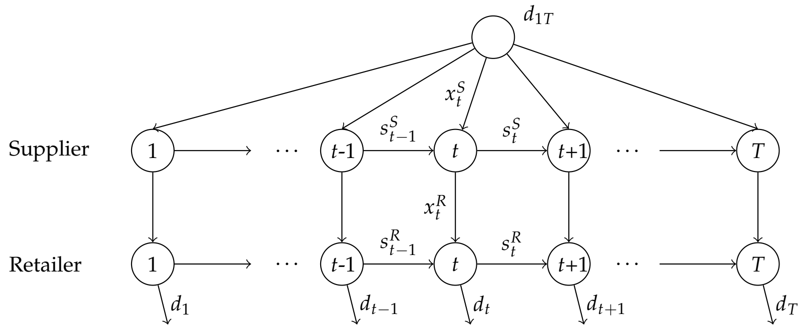

Figure 1: Fixed-charge network of the 2ULS problem.

The mathematical formulation of the 2ULS problem is given by:

$$\begin{aligned}
\min \quad & C^R(y^R, x^R, s^R) + C^S(y^S, x^S, s^S) \\
\text{s.t.} \quad & s^R_{t-1} + x^R_t = d_t + s^R_t \qquad \forall t \in \mathcal{T}, && (1)\\
& s^S_{t-1} + x^S_t = x^R_t + s^S_t \qquad \forall t \in \mathcal{T}, && (2)\\
& x^R_t \le d_{tT} y^R_t \qquad \forall t \in \mathcal{T}, && (3)\\
& x^S_t \le d_{tT} y^S_t, \qquad \forall t \in \mathcal{T}, && (4)\\
& x^S_t, s^S_t, x^R_t, s^R_t \ge 0 \qquad \forall t \in \mathcal{T} && (5)\\
& y^S_t, y^R_t \in \{0,1\} \qquad \forall t \in \mathcal{T} && (6)
\end{aligned}$$

Constraints (1) are the balance constraints at the retailer level for satisfying the demand. Constraints (2) are the balance constraints at the supplier level where the demand is the amount ordered at the retailer level. Constraints (3) and (4) force the setup variables to be equal to 1 if an order occurs at period t, *i.e.* $x^R_t > 0$ and $x^S_t > 0$ respectively.

2.2 Solving the 2ULS problem

The Zero Inventory Ordering (ZIO) [24] property is a well-known property in lot-sizing. It is fulfilled by a given problem if, for each instance of this problem, there is an optimal solution where at each period t there is at most an ordering or a non zero incoming stock (i.e. for $A \in \{R, S\}$, if $s^A_{t-1} \neq 0$ then $x^A_t = 0$, and if $x^A_t \neq 0$ then $s^A_{t-1} = 0$). Zangwill [26] has shown that the ZIO property holds at each level of the 2ULS problem: there exists an optimal solution such that at each period t an order occurs only if the incoming stock is null. Based on this ZIO property, Zangwill proposes a dynamic programming algorithm which solves the 2ULS problem in $\mathcal{O}(T^3)$ [26]. More recently, Melo and Wolsey [16] provide a dynamic programming algorithm that improves the time complexity to $\mathcal{O}(T^2 \log T)$. Even if the best time complexity is provided by the algorithm proposed by Melo and Wolsey, we present the dynamic programming scheme proposed by Zangwill since it is the one that we use to derive a polynomial time algorithm for one of the problems addressed in this paper.

For the sake of clarity, we assume that the retailer (resp. supplier) level corresponds to the first (resp. second) level. The subscripts and superscripts used for the retailer and the supplier will be replaced by 1 and 2 respectively in what follows.
Let $1 \le i \le 2$. We define $G_i(t, \alpha, \beta)$ as the minimum cost at level i for supplying $d_{\alpha\beta}$ units available at period t, when $1 \le t \le \alpha \le \beta \le T$. $G_i(t, \alpha, \beta)$ is equal to $+\infty$ if the inequality $1 \le t \le \alpha \le \beta \le T$ does not hold.

Three cases have to be considered for computing $G_i(t, \alpha, \beta)$ at a given period t: either the quantity $d_{\alpha\beta}$ is stored until period $t+1$ at level i, or it is dispatched at period t at level $i-1$, or part of the quantity $d_{\alpha\beta}$ is stored until period $t+1$ at level i and the complementary part is dispatched at period t at level $i-1$. The recursion formula below considers these cases:
- if $d_\alpha > 0$,

$$\begin{aligned}
G_i(t, \alpha, \beta) = \min\{ & h^i_t d_{\alpha\beta} + G_i(t+1, \alpha, \beta), f^{i-1}_t + p^{i-1}_t d_{\alpha\beta} + G_{i-1}(t, \alpha, \beta), \\
& \min_{\alpha+1 \le \gamma \le \beta} (f^{i-1}_t + p^{i-1}_t d_{\alpha\gamma-1} + G_{i-1}(t, \alpha, \gamma-1) + h^i_t d_{\gamma\beta} + G_i(t+1, \gamma, \beta))\}
\end{aligned}$$

- if $d_\alpha = 0$, $\quad G_i(t, \alpha, \beta) = \begin{cases} G_i(t, \alpha+1, \beta), & \text{if } \alpha < \beta \\ 0, & \text{if } \alpha = \beta \end{cases}$

The initial conditions for the recursion are defined as:

$$G_1(t, \alpha, \beta) = \begin{cases} 0, & \text{if } t = \alpha = \beta \\ h^1_t d_{\max(t+1,\alpha),\beta} + G_1(t+1, \max(t+1, \alpha), \beta), & \text{otherwise} \end{cases}$$

A dummy third level is considered in order to setup the total quantity d_{1T} that can be ordered at the second level. The inventory cost parameter h^3_t is null over the planning horizon. The optimal cost of the 2ULS problem is given by $G_3(1, 1, T)$.

In the following section, we assume that the supplier has a complete information about the retailer's cost parameters f^R, p^R and h^R.

3 Coordination with complete information

In this section, the aim of the supplier is to design a contract (x^R, z) where x^R is the replenishment plan proposed to the retailer and z is a side payment. The aim of the supplier is to minimize his cost (i.e. the cost of the replenishment plan plus the side payment). In order to ensure that the retailer will accept the contract, the retailer's cost must not exceed his optimal cost, that we will denote by C^R_{opt} in the sequel. This constraint is known as the *Individual Rationality* (IR) constraint. The retailer's optimal replenishment plan (and thus the cost C^R_{opt}) is computed by solving an Uncapacitated Lot-Sizing (ULS) problem [24]. The supplier's total cost is equal to $C^S(y^S, x^S, s^S) + z$ while the retailer's total cost is equal to $C^R(y^R, x^R, s^R) - z$.

3.1 Literature review

Most of the works related to the coordination of a supply chain under complete information assumption involve the Economic Ordering Quantity (EOQ) problem which is a continuous time model with an infinite planning horizon and a stationary demand. The interested reader is referred to the comprehensive survey by Li and Wang [15]. Many works show the efficiency of coordination mechanism by providing contracts to achieve coordination. Cachon [4] proposes a survey of the works on designing contracts that coordinate multi-level supply chains. However, few papers deal with the problem with dynamic demand. Recently, Geunes *et al.* [12] consider a two-level supply chain composed of a supplier and a retailer. They use a Stackelberg game where the supplier announces his ordering and/or holding costs to the retailer in order to persuade the retailer to change his replenishment plan. They show that price discount mechanisms can decrease the supplier's cost.

3.2 Designing contracts without side payment

When the retailer has several optimal replenishment plans, the supplier chooses among the retailer's replenishment plans of cost C^R_{opt} one that minimizes his cost. We denote by P^{CI}_{WP} the problem of designing a contract without side payment. Its mathematical formulation is given below:

$$
\begin{aligned}
\min \quad & C^S(y^S, x^S, s^S) \\
\text{s.t.} \quad & (1), (2), (3), (4), (5), (6) \\
& y^R_t \le x^R_t \qquad \forall t \in \mathcal{T}, && (7) \\
& C^R(y^R, x^R, s^R) \le C^R_{opt} && \text{(IR)}
\end{aligned}
$$

Since the retailer's costs are not considered in the objective function, Constraints (7) are introduced to force the retailer's setup variables to be equal to 0 at period t if no order occurs at t. This problem is equivalent to minimizing the supply chain cost where the retailer's total cost cannot exceed C^R_{opt}.

The collaboration between the actors of the supply chain can lead to a significant decrease of the supplier's cost. The following example shows that the factor can be arbitrarily large: $T = 2$, $d = (0, 1)$, $f^R = p^R = h^R = f^S = (0, 0)$, $p^S = (0, M)$ and $h^S = (2M, 0)$. There are two optimal replenishment plans $\hat{x}^R = (0, 1)$ and $\bar{x}^R = (1, 0)$ of cost 0. The optimal production plan for the supplier to satisfy $\hat{x}^R$ is $\hat{x}^S = (0, 1)$. The optimal plan for the supplier to satisfy $\bar{x}^R$ is $\bar{x}^S = (1, 0)$: in the former case, the supplier's cost is M while it is 0 in the latter one. Therefore, from the supplier point of view, it is better to satisfy the replenishment plan $\bar{x}^R$. If the actors collaborate, the supplier can propose the contract $(\bar{x}^R, 0)$ to the retailer. The induced cost is 0 instead of M if no collaboration is assumed.

Intuitively, we can solve this problem by determining all the optimal replenishment plans of the retailer that satisfy the ZIO property. However, the complexity of such solution is $\mathcal{O}(T^T)$. In the following, we propose a polynomial dynamic programming algorithm that solves P^{CI}_{WP}. The algorithm is based on the following decomposition of dominant solutions. There exists an optimal solution such that the ZIO property holds at both levels. Moreover, at the supplier level, if $d_{\alpha\beta}$ units are available at period t, where $t \le \alpha \le \beta$, either these units are stored at the supplier level until period $t + 1$, or they are completely sent to the retailer at period t, or part of these units is stored at the supplier level until period $t + 1$ and the other part is sent to the retailer at period t. At the retailer level, since the ZIO property holds, if $d_{\alpha\beta}$ units are ordered at period t then $s^R_{t-1} = s^R_\beta = 0$ and no order takes place at a period k such that $t < k \le \beta$. Thus, knowing the retailer's cost for satisfying the demands $d_{\alpha\beta}$ from an order at period t, it is possible to compute the retailer's cost over the entire horizon by computing independently the retailer's minimum cost for satisfying the demands $d_{1,t-1}$ and $d_{\beta T}$.

We first compute the retailer's minimum cost $I(t,k)$ for satisfying the demands from period t to period k, with $1 \leq t \leq k \leq T$. We set the cost $I(t,k)$ to $+\infty$ if $k > t$. For a fixed value t, the cost $I(t,k)$ can be computed by solving an ULS problem in $\mathcal{O}(T\log T)$ [1, 10, 23]. Therefore, the cost $I(t,k)$ can be computed in $\mathcal{O}(T^2\log T)$ for all t,k.

Let $H_i(t,\alpha,\beta)$, with $1 \leq t \leq \alpha \leq \beta \leq T$, be the minimum cost of supplying a quantity $d_{\alpha\beta}$ of units available at level i at period t such that the total cost of the retailer does not exceed his optimal cost C^R_{opt}, and $+\infty$ otherwise. We recall that the retailer (resp. supplier) level corresponds to the first (resp. second) level. Similarly to Zangwill's algorithm, a dummy third level is added such that its inventory cost h^3_t is null for all periods t. The computation of $H_i(t,\alpha,\beta)$ follows the one presented in Section 2.2 by setting an infinite cost to the solutions that violate the bound constraint on the retailer's total cost. The cost $H_i(t,\alpha,\beta)$ is computed by the same recursion formula provided in Section 2.2, except for the case where $d_\alpha > 0$ at the second level.

Figures 2, 3 and 4 represent the three cases described above for satisfying the demands from period α to period β where $d_{\alpha\beta}$ units are available at a period t at the supplier level. The nodes correspond to the periods. The vertical inflows of each node correspond to the ordering quantities and the horizontal inflows correspond to the inventory quantities. The components of each term in the computation of $H_2(t,\alpha,\beta)$ when $d_\alpha > 0$ (described thereafter) are depicted in the figures.

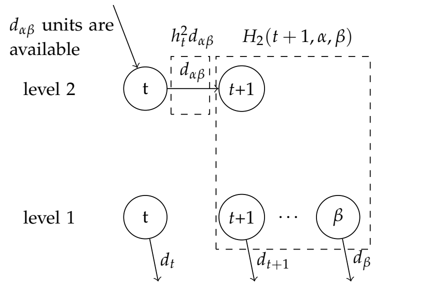

Figure 2: Illustration of $H_2(t,\alpha,\beta)$ computation of when $d_{\alpha\beta}$ units are stored until period $t+1$ at the supplier level.

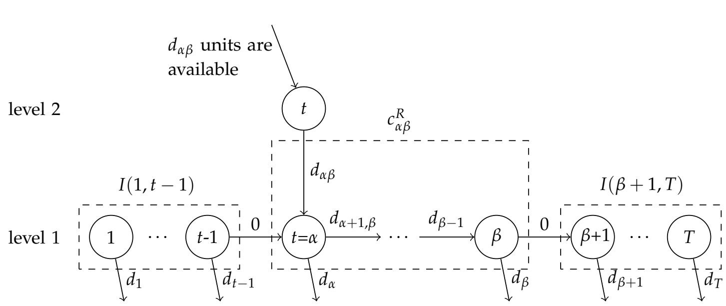

Figure 3: Illustration of $H_2(t,\alpha,\beta)$ computation when $t = \alpha$ and $d_{\alpha\beta}$ units are sent to the retailer at period t.

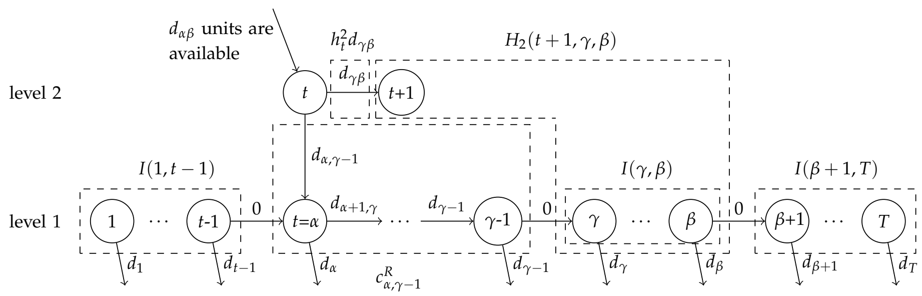

Figure 4: Illustration of $H_2(t,\alpha,\beta)$ computation when $t = \alpha$, $d_{\alpha,\gamma-1}$ units are sent to the retailer at period t and $d_{\gamma\beta}$ units are stored until period $t+1$ at the supplier level.

The recursion formula for computing $H_2(t,\alpha,\beta)$ are given by:

- if $d_\alpha = 0$, $\quad H_i(t,\alpha,\beta) = \begin{cases} H_i(t,\alpha+1,\beta), & \text{if } \alpha < \beta. \\ 0, & \text{if } \alpha = \beta. \end{cases}$

- if $d_\alpha > 0$,

$$H_3(t,\alpha,\beta) = \min\{ \; H_3(t+1,\alpha,\beta), f^2_t + p^2_t d_{\alpha\beta} + H_2(t,\alpha,\beta), \min_{\alpha+1 \leq \gamma \leq \beta} (f^2_t + p^2_t d_{\alpha\gamma-1} + H_2(t,\alpha,\gamma-1) + H_3(t+1,\gamma,\beta)\}.$$

$$
\begin{aligned}
H_2(t,\alpha,\beta) = \min\{\, & h_t^2 d_{\alpha\beta} + H_2(t+1,\alpha,\beta),\\
& \underbrace{f_t^1 + p_t^1 d_{\alpha\beta} + H_1(t,\alpha,\beta)}_{c^R_{\alpha\beta}} + \mathbb{1}(I(1,t-1) + c^R_{\alpha\beta} + I(\beta+1,T)),\\
& \min_{\alpha+1\leq\gamma\leq\beta} (\underbrace{f_t^1 + p_t^1 d_{\alpha\gamma-1} + H_1(t,\alpha,\gamma-1)}_{c^R_{\alpha,\gamma-1}} + h_t^2 d_{\gamma\beta} + H_2(t+1,\gamma,\beta)\\
& \qquad + \mathbb{1}(I(1,t-1) + c^R_{\alpha,\gamma-1} + I(\gamma,\beta) + I(\beta+1,T)))\}. \qquad (8)
\end{aligned}
$$

where $\mathbb{1}(c)$ is equal to 0 if $c \leq C^R_{opt}$ and $+\infty$ otherwise. In Equation (8), the cost $\mathbb{1}(I(1,t-1) + c^R_{\alpha\beta} + I(\beta+1,T))$ of the second term represents the retailer's minimum cost when $d_{\alpha\beta}$ is dispatched at period t to the retailer. The cost $\mathbb{1}(I(1,t-1) + c^R_{\alpha,\gamma-1} + I(\gamma,\beta) + I(\beta+1,T))$ of the third term represents the retailer's minimum cost when $d_{\alpha,\gamma-1}$ is dispatched to the retailer at period t. Moreover, in the third term, the minimum cost $H_2(t+1,\gamma,\beta)$ is either different from $+\infty$ and in this case, the retailer's cost for satisfying $d_{\gamma\beta}$ computed by $H_2(t+1,\gamma,\beta)$ is equal to $I(\gamma,\beta)$ plus the supplier's holding cost, or it is equal to $+\infty$ which implies that the solution where $s^R_{\gamma-1} = s^R_\beta = 0$ was already pruned because it is not optimal for the retailer.

The initialization of the recursion is given by the cost $H_1(t,\alpha,\beta) = G_1(t,\alpha,\beta)$. The optimal cost of P^{CI}_{WP} is equal to $H_3(1,1,T)$. Since the precomputation of $I(t,k)$ is done in $\mathcal{O}(T^2 \log T)$, the time complexity of the dynamic programming algorithm is $\mathcal{O}(T^3)$.

3.3 Designing contracts with side payment on the retailer's holding cost

The problem studied in this section where side payment is allowing on the retailer's holding cost is denoted by P^{CI}_{HC}. The mathematical formulation of P^{CI}_{HC} is given by:

$$
\begin{aligned}
\min\ & z + C^S(y^S, x^S, s^S)\\
\text{s.t. } & (1),(2),(3),(4),(5),(6)\\
& C^R(y^R, x^R, s^R) - z \leq C^R_{opt} && \text{(IR)}\\
& z \leq \sum_{t\in\mathcal{T}} h_t^R s_t^R && (9)\\
& z \geq 0
\end{aligned}
$$

Constraint (9) ensures that the side payment z covers at most the retailer's holding cost.

By summing up Constraints (IR) and (9), we obtain $\sum_{t\in\mathcal{T}} (f_t^R y_t^R + p_t^R x_t^R) \leq C^R_{opt}$. Since the supplier wants to minimize his total cost, the side payment z is exactly equal to the lower bound $C^R(y^R,x^R,s^R) - C^R_{opt}$ provided by (IR). Thus, the retailer's cost will be equal to C^R_{opt}. The formulation of P^{CI}_{HC} reduces to:

$$
\begin{aligned}
\min\ & C^S(y^S, x^S, s^S) + C^R(y^R, x^R, s^R) - C^R_{opt}\\
\text{s.t. } & (1),(2),(3),(4),(5),(6)\\
& \sum_{t\in\mathcal{T}} (f_t^R y_t^R + p_t^R x_t^R) \leq C^R_{opt} && (10)
\end{aligned}
$$

P^{CI}_{HC} comes down to the 2ULS problem with an additional constraint that imposes that the retailer's ordering cost cannot exceed C^R_{opt}.

Note that the supplier is be able to decrease his cost by a factor arbitrarily large compared to the case where no side payment is allowed. For example, consider the following instance: $T = 2$, $d = (0,1)$, $f^R = p^R = f^S = (0,0)$, $p^S = (0,M)$, $h^R = (1,0)$ and $h^S = (2M,0)$. The optimal replenishment plan is given by $\bar{x}^R = (0,1)$: the retailer's optimal cost is 0. The optimal production plan for the supplier to satisfy $\bar{x}^R$ is given by $\bar{x}^S = (0,1)$ of cost M. Assuming collaboration between the actors, the supplier can consider the replenishment plan $x^R = (1,0)$ of cost 1. The optimal production plan for the supplier to satisfy x^R is $x^S = (1,0)$ of cost 0. The supplier may propose the contract $(x^R, 1)$ to the retailer, inducing a decrease of his cost by M-1.

The following result settles the complexity of P^{CI}_{HC}.

Theorem 1. *P^{CI}_{HC} is NP-hard.*

Proof. We show that P^{CI}_{HC} is NP-hard through a reduction from the knapsack problem which is NP-hard [11]. An instance of the knapsack problem is given by: a set $\mathcal{N} = \{1, \dots, N\}$ of N items, a utility v_i for all $i \in \mathcal{N}$, a weight w_i for all $i \in \mathcal{N}$, a capacity $W \in \mathbb{N}$ such that $W < \sum_{i=1}^{N} w_i$ and a total value goal $V \in \mathbb{N}$. The recognition version of the knapsack problem requires a *yes/no* answer to the following question: is there a subset $\mathcal{S} \subset \mathcal{N}$ such that $\sum_{i \in S} w_i \leq W$ and $\sum_{i \in S} v_i \geq V$?

The recognition version of P^{CI}_{HC} is defined by the following question: is there a production plan x^S and a replenishment plan x^R that satisfy the demands such that $C^R_O \leq C^R_{opt}$, where C^R_O is the retailer's ordering cost, and $C^R(y^R, x^R, s^R) + C^S(y^S, x^S, s^S) \leq K$ where K is a bound on the total cost?

We define an instance $\mathcal{I}$ of P^{CI}_{HC} as follows (see Figure 5). We set $T = 3N$ and denote by $\mathcal{T}_1$, $\mathcal{T}_2$, $\mathcal{T}_3$ the set of periods $\{1, 4, \dots, 3i-2, \dots, 3N-2\}$, $\{2, 5, \dots, 3i-1, \dots, 3N-1\}$, $\{3, 6, \dots, 3i, \dots, 3N\}$, respectively. The retailer setup costs are given by $f^R_t = 0$ for all $t \in \mathcal{T}_1$, $f^R_t = w_{(t+1)/3}$ for all $t \in \mathcal{T}_2$ and $f^R_t = NM$ for all $t \in \mathcal{T}_3$, where $M = \max_i\{W/N + \alpha_i, v_i + w_i + \alpha_i\}$ and $\alpha_i = \max\{0, W/N - w_i\}$. The retailer holding costs are set to $h^R_t = W/N$ for all $t \in \mathcal{T}_1$, $h^R_t = \alpha_{(t+1)/3}$ for all $t \in \mathcal{T}_2$ and $h^R_t = NM$ for all $t \in \mathcal{T}_3$. Finally, the retailer ordering cost p^R_t is set to zero for all periods t. The supplier costs are set to $f^S_t = M - W/N - \alpha_{(t+1)/3}$ for all $t \in \mathcal{T}_1$, $f^S_t = M - v_{(t+1)/3} - w_{(t+1)/3} - \alpha_{(t+1)/3}$ for all $t \in \mathcal{T}_2$ and $f^S_t = NM$ for all $t \in \mathcal{T}_3$. The supplier holding cost is $h^S_t = NM$ for all periods t. Finally, the supplier ordering cost p^S_t is set to zero for all periods t. The demand d_t is null for all $t \in \mathcal{T}_1 \cup \mathcal{T}_2$, and equal to 1 for all $t \in \mathcal{T}_3$. Finally, we set the bound K to $NM - V$.

An illustration of the instance is given in Figure 5. The nodes represent the periods at each level. We associate the fixed ordering cost to each vertical edge and the holding cost to each horizontal edge. In addition, the external demands at the retailer level are representing by outflow edges at each period.

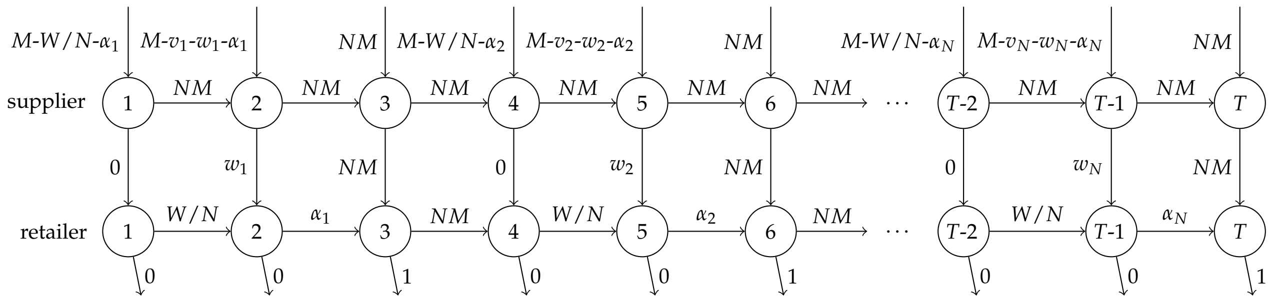

Figure 5: Instance $\mathcal{I}$ of P^{CI}_{HC} where the fixed ordering cost is associated to a vertical edge and the holding cost to a horizontal edge.

We prove that the recognition version of the knapsack problems has a *yes* answer if and only if the above described instance of the recognition version of P^{CI}_{HC} is a *yes* answer as well.

We first start by showing that the retailer's optimal cost C^R_{opt} is equal to W for instance $\mathcal{I}$. We show in what follows that the retailer would order only at a period $t \in \mathcal{T}_1 \cup \mathcal{T}_2$ and at most one unit per order. The cost for satisfying a demand d_t where $t \in \mathcal{T}_3$ from an order at period t is equal to $f^R_t = NM$. The one for satisfying d_t from an order at period $t-1 \in \mathcal{T}_2$ is equal to $f^R_{t-1} + h^R_{t-1} = w_{(t+1)/3} + \alpha_{(t+1)/3} < NM$. Finally, the retailer's cost for ordering the demand d_t at period $t-2 \in \mathcal{T}_1$ is equal to $h^R_{t-2} + h^R_{t-1} = W/N + \alpha_{(t+2)/3} < NM$. Then, in order to minimize his cost, the retailer will place an order either at period $t-1 \in \mathcal{T}_1$ or $t-2 \in \mathcal{T}_2$. Moreover, since $h^R_t = NM$ for $t \in \mathcal{T}_3$, the retailer will only store between a period $t \in \mathcal{T}_1$ and a period $k \in \mathcal{T}_3$, and he stores at most one unit.
If $w_i < W/N$, the retailer orders one unit at period $3i-1 \in \mathcal{T}_2$ which induces a cost of $w_i + \alpha_i = W/N$ (if the retailer orders at period $3i-2 \in \mathcal{T}_1$, his total total is equal to $f^R_{3i-2} + h^R_{3i-2} + h^R_{3i-1} = W/N + \alpha_i > W/N$ since $\alpha_i = W/N - w_i > 0$). In the case where $w_i \geq W/N$, the retailer orders one unit at period $3i-2 \in \mathcal{T}_1$ which induces a cost of W/N since $\alpha_i = 0$. Since there are N demands to satisfy, there are N ordering periods of cost W/N. Then, the retailer's optimal cost C^R_{opt} is equal to W.

Suppose that we have a solution $\mathcal{S}$ to the knapsack problem. Let us show that there is a solution to the corresponding instance of P_{HC}^{CI}. Figure 6 represents the solution of P_{HC}^{CI} corresponding to an instance of the knapsack problem where $N = 3$, $w_1 + w_3 \leq W$ and $v_1 + v_3 \geq V$. Only the active edges are represented in the figure, *i.e.* the edges corresponding to an ordering period or a strictly positive inventory. For each item $i \in \mathcal{S}$, the supplier and the retailer order one unit at period $t = 3i - 1 \in \mathcal{T}_2$ ($y_t^S = y_t^R = 1$, $x_t^S = x_t^R$=1). The retailer stores the unit until period $3i$ to satisfy the demand at period $3i$ ($s_t^R = 1$). For each item $i \notin \mathcal{S}$, the supplier and the retailer order one unit at period $t = 3i - 2 \in \mathcal{T}_1$ in order to satisfy the demand at period $3i$ ($y_t^S = y_t^R = 1$ and $x_t^S = x_t^R = 1$). The retailer stores the unit until period $3i$ to satisfy the demand at period $3i$ ($s_t^R = s_{t+1}^R = 1$). Since $p_t^R = 0$ for $t \in \mathcal{T}$ and $f_t^R = 0$ for $t \in \mathcal{T}_1$, the retailer's ordering cost is equal to $C_O^R = \sum_{i\in\mathcal{S}} f_{3i-1}^R y_{3i-1}^R = \sum_{i\in\mathcal{S}} w_i$ which is less than or equal to $W = C_{opt}^R$. The retailer's total cost is equal to $C^R = C_O^R + \sum_{i\in\mathcal{S}}(w_i + \alpha_i) + \sum_{i\in\mathcal{S}} h_{3i-1}^R s_{3i-1}^R + \sum_{i\notin\mathcal{S}}(h_{3i-2}^R s_{3i-2}^R + h_{3i-1}^R s_{3i-1}^R) = \sum_{i\in\mathcal{S}}(w_i + \alpha_i) + \sum_{i\notin\mathcal{S}}(W/N + \alpha_i)$. Since $p_t^S = 0$ for $t \in \mathcal{T}$, the supplier's total cost is equal to $C^S = \sum_{i\in\mathcal{S}} f_{3i-1}^S y_{3i-1}^S + \sum_{i\notin\mathcal{S}} f_{3i-2}^S y_{3i-2}^S = \sum_{i\in\mathcal{S}}(M - v_i - w_i - \alpha_i) + \sum_{i\notin\mathcal{S}}(M - W/N - \alpha_i)$. Thus, the total cost of the supply chain is equal to $C^R + C^S = \sum_{i\in\mathcal{S}}(w_i + \alpha_i) + \sum_{i\notin\mathcal{S}}(W/N + \alpha_i) + \sum_{i\in\mathcal{S}}(M - v_i - w_i - \alpha_i) + \sum_{i\notin\mathcal{S}}(M - W/N - \alpha_i) = \sum_{i\in\mathcal{S}}(M - v_i) + \sum_{i\notin\mathcal{S}} M = \sum_{i\in\mathcal{N}} M - \sum_{i\in\mathcal{S}} v_i \geq NM - V$. Therefore, there exists a solution of P_{HC}^{CI} for instance $\mathcal{I}$.

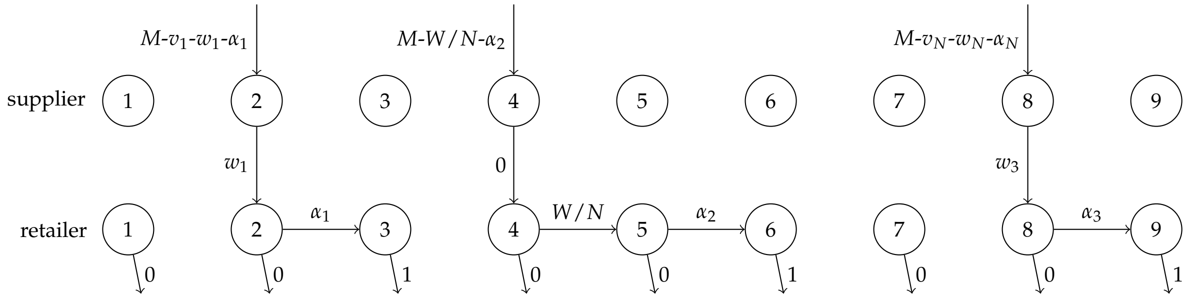

Figure 6: Solution of P_{HC}^{CI} where $N = 3$, $w_1 + w_3 \leq W$ and $v_1 + v_3 \geq V$

Conversely, suppose that there exists a solution of P_{HC}^{CI} for instance $\mathcal{I}$. Since $h^S = NM$, if the supplier sets an order, then the retailer will order at the same period ($y_t^S = y_t^R$ and $x_t^S = x_t^R$ for all t), otherwise, there will be a period t where $s_t^S > 0$ and the cost will be greater than $NM - V$. Moreover, since the retailer can order at most one unit at a period $t \in \mathcal{T}_1 \cup \mathcal{T}_2$ (and thus store at most one unit), then in order to satisfy a demand at period $t \in \mathcal{T}_3$, the supplier and the retailer order at most one unit from period $t - 2 \in \mathcal{T}_1$ or $t - 1 \in \mathcal{T}_2$. Let us show that the subset of ordering periods $\mathcal{S} \subset \mathcal{T}_2$ gives a positive answer to the knapsack problem. In this solution, item i is selected if and only if there is an order at period $3i - 1 \in \mathcal{S}$. Since $p_t^R = 0$ for $t \in \mathcal{T}$ and $f_t^R = 0$ for all $t \in \mathcal{T}_1$, we have $C_O^R = \sum_{t\notin\mathcal{S}} f_t^R y_t^R + \sum_{t\in\mathcal{S}} f_t^R y_t^R = \sum_{t\in\mathcal{S}} w_{(t+1)/3}$. By definition of P_{HC}^{CI}, $C_O^R \leq C_{opt}^R$. Therefore, $C_O^R = \sum_{t\in\mathcal{S}} w_{(t+1)/3} \leq W$. Moreover, since $p_t^R = p_t^S = 0$ for $t \in \mathcal{T}$ and $f_t^R = 0$ for all $t \in \mathcal{T}_1$, $C^R(y^R, x^R, s^R) + C^S(y^S, x^S, s^S) = \sum_{t\notin\mathcal{S}}(f_t^S y_t^S + f_t^R y_t^R + h_t^R s_t^R + h_{t+1}^R s_{t+1}^R) + \sum_{t\in\mathcal{S}}(f_t^S y_t^S + f_t^R y_t^R + h_t^R s_t^R) = \sum_{t\notin\mathcal{S}} M + \sum_{t\in\mathcal{S}}(M - v_{(t+1)/3}) = NM - \sum_{t\in\mathcal{S}} v_{(t+1)/3}$. Similarly, by definition of P_{HC}^{CI}, we have $C^R(y^R, x^R, s^R) + C^S(y^S, x^S, s^S) \leq NM - V$. This implies that $\sum_{t\in\mathcal{S}} v_{(t+1)/3} \geq V$. Thus, the subset $\mathcal{S}$ gives a positive answer to the knapsack problem. □

3.4 Designing contracts with side payment on the retailer's total cost

The problem of designing a contract with side payment on the retailer's total cost is denoted by P_{TC}^{CI} where CI stands for complete information assumption. The supplier has to ensure that the contract is individually rational. The mathematical formulation of P_{TC}^{CI} is given by:

$$\begin{aligned} \min \quad & C^S(y^S, x^S, s^S) + z \\ \text{s.t.} \quad & (1), (2), (3), (4), (5), (6) \\ & C^R(y^R, x^R, s^R) - z \leq C_{opt}^R \qquad \text{(IR)} \\ & z \geq 0 \end{aligned}$$

Constraint (IR) represents the individual rationality constraint which ensures that the retailer's cost associated to the contract (x^R, z) will not exceed his optimal cost C_{opt}^R.

A key observation is that the supplier can decrease his cost by a factor arbitrarily large compared to the case where the side payment is on the retailer's holding cost as illustrated in the following example. The parameters of the instance, denoted by $\mathcal{I}_c$, are given by $T = 3$, $d = (0, 1, 1)$, $f^R = f^S = (0, 0, 0)$, $p^R = (1, 0, 1)$, $h^R = (1, 1, 1)$, $p^S = (0, M, M)$ and $h^S = (2M, 2M, 2M)$.

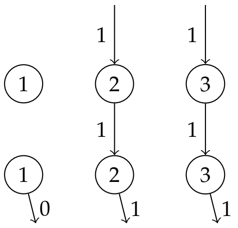

Figure 7: Optimal solution for the instance $\mathcal{I}_c$, $C^R = 1$, $C^S = 2M$.

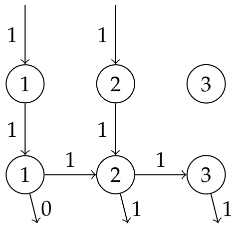

Figure 8: Solution with side payment on the retailer's holding cost, $C^R = 3$, $C^S = M$.

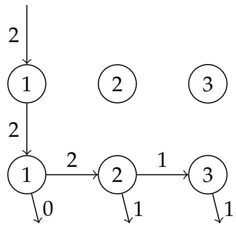

Figure 9: Solution with side payment on the retailer's total cost, $C^R = 5$, $C^S = 0$.

The optimal replenishment plan is $\bar{x}^R = (0, 1, 1)$ of cost 1. The optimal production plan for the supplier to satisfy $\bar{x}^R$ is $\bar{x}^S = (0, 1, 1)$ of cost $2M$ (see Figure 7). If the actors collaborate with a side payment on the retailer's holding cost (see Figure 8), the supplier can choose the replenishment plan $\tilde{x}^R = (1, 1, 0)$ of cost 3. The optimal production plan for the supplier to satisfy $\tilde{x}^R$ is $\tilde{x}^S = (1, 1, 0)$ of cost M. By proposing the contract $(\tilde{x}^R, 2)$ to the retailer, the supplier's cost is equal to $M + 2$. If the actors collaborate with a side payment on the retailer's total cost (see Figure 9), the supplier can consider the replenishment plan $\bar{x}^R = (2, 0, 0)$ of cost 5. The production plan $\tilde{x}^S = (2, 0, 0)$ of cost 0 to satisfy $\bar{x}^R$ is optimal for the supplier. In that case, the supplier can propose the contract $(\bar{x}^R, 4)$ to the retailer, inducing a cost of 4 instead of $M + 2$ when the side payment is on the retailer's holding cost.

Since the supplier wants to minimize his total cost, the side payment z is exactly equal to the lower bound $C^R(y^R, x^R, s^R) - C^R_{opt}$ given by Constraint (IR). By replacing z by its lower bound, we obtain the mathematical formulation of the 2ULS problem. So, P^{CI}_{TC} is a 2ULS problem which is solvable in polynomial time (see Section 2.2). The supplier's cost is then equal to $C^S(y^S, x^S, s^S) + C^R(y^R, x^R, s^R) - C^R_{opt}$. The retailer's cost is exactly equal to his optimal cost C^R_{opt}. In this case, the contract $(x^R_{opt}, C^R(y^R_{opt}, x^R_{opt}, s^R_{opt}) - C^R_{opt})$ proposed by the supplier achieves the optimal cost of the supply chain, where x^R_{opt} is the optimal replenishment plan for P^{CI}_{TC}.

3.5 Experimental analysis

The experimental analysis aims to evaluate the decrease of the supplier's cost, we have carried out an experimental analysis. The tests were performed using concert technology of CPLEX 12.6 on a machine running under Debian x86_64 GNU/Linux with Intel Xeon(R) E5-1630 v3 3.70GHz processor and 8Gb of RAM memory.

The instance generation scheme follows the one proposed in [27]. Unless otherwise mentioned, the cost parameters used in our experiments are not stationary and generated using a uniform distribution. We recall that a cost parameter is stationary if it is constant over the planning horizon. For our experiments, we define four classes of instances A, B, C, D described in Table 1. The class A (resp. B) represents the case where h^S is small (resp. large) and h^R is small. The class C (resp. D) represents the case where h^S is small (resp. large) and h^R is large. In order to compare the results under both the complete and the asymmetric information assumptions (see Section 4.5), parameter p^R is generated in $[1, 20] \cup [70, 90]$. We consider T periods with $T \in \{5, 10, 20, 30\}$, the demands are randomly generated from the uniform distribution $[0, 300]$ and the cost parameters are randomly chosen in the discrete intervals described in Table 1 where $\delta = 500$. These settings comes from preliminary experiments to find relevant values[1]. The supplier's fixed ordering costs are given by $f^S_t = \delta p^S_t$ for all $t = 1, \ldots, T$. For each class of instances, we randomly generate 100 quintuplets (p^S, h^S, f^R, h^R, d) and for each quintuplet, we randomly generate two costs p^R such that the first one is in $[1, 20]$ and the second one in $[70, 90]$. Thus, in total, we have

[1]Different cost structures have been considered for p^S in a preliminary work. The results show that: for $p^S \in [1, 10]$ or $p^S \in [1, 50]$, the supplier's gain when p^S is stationary is significantly smaller than the one obtained when p^S is not stationary. For $p^S \in [40, 50]$, the supplier's gain does not depend on the stationarity assumption of p^S. Moreover, when $p^S \in [1, 10]$ or $p^S \in [40, 50]$, the supplier's gain is smaller than the one observed when $p^S \in [1, 50]$. Preliminary results also show that other values of δ do not change the qualitative results of the analysis.

200 instances for each class of instances. For example, an instance of the class A is generated such that: at each period t, d_t is randomly chosen in $[0, 300]$, h_t^S is a random integer in $[0, 6]$, h_t^R is a random integer in $[0, 6]$, $f_t^R = \delta c$ where c is randomly chosen in $[1, 100]$, p^S is randomly chosen in $[1, 50]$, p_t^R is a random integer in $[1, 20]$.

Table 1: Description of the classes of instances.

Class	h^S	p^S	h^R	p^R	f^R
A B	[0,6] [12,18]	[1,50]	[0,6]	[1,20] ∪ [70,90]	δ×[1,100]
C D	[0,6] [12,18]	[1,50]	[12,18]	[1,20] ∪ [70,90]	δ×[1,100]

To asses the supplier's gain in percentage, we compute $(1 - c_P^S/c^S) \times 100$ where c_P^S is the supplier's cost for each of the addressed problems (P_{WP}^{CI}, P_{HC}^{CI} or P_{TC}^{CI}), and c^S is the supplier's cost when the retailer imposes his optimal replenishment plan. If several optimal plans exist, we choose the one that is the least profitable for the supplier. The results are provided in Tables 2 and 3 where the column *Mean* (resp. *Max*) represents the average (resp. maximum) value of the supplier's gain over the 200 instances for each class of instances.

Table 2: Comparison of the average of the supplier's gains (%) between P_{WP}^{CI}, P_{HC}^{CI} and P_{TC}^{CI} where $T = 5$.

	P_{WP}^{CI}		P_{HC}^{CI}		P_{TC}^{CI}	
Class	Moy	Max	Moy	Max	Moy	Max
A	0.0	0.0	1.2	37.3	2.0	37.3
B	0.0	0.0	3.5	81.8	5.7	81.8
C	0.0	0.0	0.6	23.6	0.6	23.6
D	0.0	0.0	7.2	75.5	7.4	75.5

Table 2 presents the average of the supplier's gains for P_{WP}^{CI}, P_{HC}^{CI} and P_{TC}^{CI} where $T = 5$. We observe that the contract without side payment does not allow the supplier to decrease his cost: the maximum value of his gains is equal to 0% for all the classes. When side payment is allowed between the actors, the averages of his gains are higher when h^R is large (classes B and D): the average of his gains is at least equal to 3.5% for P_{HC}^{CI} and at least equal to 5.7% for P_{TC}^{CI}. Moreover, the results show that the difference of the average of the supplier's gains between P_{HC}^{CI} and P_{TC}^{CI} is at most 2.2 percentage points. Then, despite the fact that when side payment is allowed on the retailer's total cost, the supplier can decrease his cost by a factor arbitrarily large compared to the case where side payment is allowed on the retailer's holding cost, the experiments show that the supplier's gains obtained for P_{HC}^{CI} are almost as interesting as the one obtained for P_{TC}^{CI}.

Table 3: Average of the supplier's gains (%) for P_{WP}^{CI}, P_{HC}^{CI} and P_{TC}^{CI} when $T = 10$ and $T = 20$.

	$T = 10$						$T = 20$					
	P_{WP}^{CI}		P_{HC}^{CI}		P_{TC}^{CI}		P_{WP}^{CI}		P_{HC}^{CI}		P_{TC}^{CI}	
Class	Moy	Max	Moy	Max	Moy	Max	Moy	Max	Moy	Max	Moy	Max
A	0.0	0.0	5.9	54.6	6.9	54.6	0.0	0.0	12.2	63.3	12.2	63.3
B	0.0	0.0	15.7	74.7	17.3	74.7	0.0	0.0	27.3	75.4	27.9	75.4
C	0.0	0.0	2.2	31.9	2.2	31.9	0.0	2.3	3.0	27.1	3.0	27.1
D	0.0	0.0	11.5	56.9	11.5	56.9	0.0	0.0	14.0	40.5	14.0	40.5

Table 3 presents the average of the supplier's gains for each class of instances when side payment is allowed on the retailer's holding cost for larger values of T. Similarly to the case where $T = 5$, the supplier does not decrease his cost when side payment is not allowed even for a larger horizon, the supplier's gain is not null (2.3% when $T = 20$). The results show that even for large horizons, the average of the supplier's gains when side payment is allowed on the retailer's holding cost is as significant as the one where side payment is allowed on the retailer's total cost. The difference is at most equal to 2.4 percentage points when $T = 10$ and it is at most equal to 0.6 percentage points when $T = 20$. When h^S is small and h^R is large (class C), the supplier cannot decrease his cost significantly compared to the other classes. When $T = 20$, the average of the supplier's gains for the class C is equal to 3% whereas it is equal to 12.2% for the other classes. This observation can be explained by the fact that the supplier does not have an incentive to have more units in the retailer's inventory and to take over the retailer's holding costs since $h^R > h^S$. Conversely, the average of the supplier's gains is higher for the class B when $h^R < h^S$. When $T = 20$, the observed average is equal to 27.3% whereas it is at most equal to 14% for the other classes.

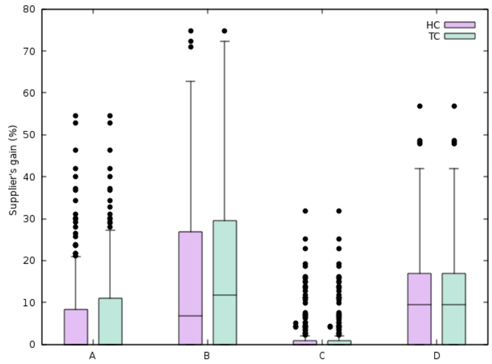

Figure 10: The supplier's gain of P^{CI}_{HC} and P^{CI}_{TC} for each class of instances when $T = 10$.

Figure 10 gives a representation of the results as boxplots for P^{CI}_{HC} and P^{CI}_{TC}. A boxplot can be analyzed in the following way: the middle line represents the median, the borders of the boxplot represent the 25th and the 75th quantile denoted by Q_1 and Q_3 respectively, the "whiskers" represent the maximum and the minimum value such that it is less than Q_3 plus 1.5 times the length of the boxplot and greater than Q_1 minus 1.5 times the length of the boxplot respectively, the outliers represented by points correspond to the values that are greater than Q_3 plus 1.5 times the length of the boxplot or smaller than Q_1 minus 1.5 times the length of the boxplot. For half of the instances of the class B, the supplier's gain is at least equal to 6.8% when side payment is allowed on the retailer's holding cost and to 11.8% when side payment is allowed on the retailer's total cost. Among these instances, for 25% of them, the gain obtained for P^{CI}_{HC} is above 27% and the one obtained for P^{CI}_{TC} is above 29.3%. For the class A of instances, the boxplots give a better information about the distribution of the supplier's gain. Indeed, the boxplots show that the median is equal to 0% for both problems whereas in Table 3 , the average is at least equal to 5.9%.

4 Coordination with asymmetric information

In this section, we consider that the retailer has private information about his cost structure. The supplier assumes that a retailer has N possible types. These types represent different retailer's profiles that the supplier may supply. We denote by $\mathcal{N} = \{1, \ldots, N\}$ the set of types. The real type of the retailer is necessarily in the set $\mathcal{N}$. Moreover, the supplier knows the probability P_i for a retailer to be of type $i \in \mathcal{N}$ ($\sum_{i \in \mathcal{N}} P_i = 1$). Contrary to the case where the supplier has a complete information about the retailer's cost parameters, a set of contracts will be proposed to the retailer, each one is associated to a type. Then, the retailer may choose one contract in this set or reject all the contracts. This process is called a screening game [21].

Under the asymmetric information assumption, it is possible that the retailer does not report his type truthfully. Indeed, a retailer of type i may have an incentive to choose a contract designed for a retailer of type j that minimizes his cost. We say that there is no partial verification when for each type, the set of types that the retailer of type i can report is the entire set $\mathcal{N}$. When there is no partial verification, the revelation principle [18] states that there always exists a set of optimal contracts for which the retailer will truthfully choose the contract that corresponds to his real type. Therefore, we can focus our study on the set of contracts which ensures that the retailer will report his true type. We say that these contracts are Incentive Compatible (IC).

4.1 Literature review

Many authors study the EOQ problem in a two-player supply chain context with asymmetric information. Corbett and De Groote [6] consider an EOQ problem where the supplier does not know the retailer's holding cost. In order to influence the retailer's behavior, the supplier proposes a set of optimal contracts with quantity discounts to the retailer. However, for the EOQ problem, if the supplier knows the retailer's ordering cost and his optimal ordering quantity, he can deduce the retailer's holding cost. Thus, the information asymmetry in this setting is not meaningful [22]. Sucky [22] study the EOQ problem where the supplier does not know the retailer's ordering and holding costs. He proposes to use a screening model: the retailer has two types associated to the unknown cost parameters in order to improve the supply chain cost. The supplier offers a set of contracts composed of a side payment and a replenishment plan in order to influence the retailer's decision. His model leads to a non-convex problem for which he characterizes sets of contracts that are candidate to be an optimal solution. Pishchulov and Richter [19] consider the same model and complete Sucky's analysis of the optimal sets of contracts. More recently, Kerkkamp *et al.* [13] consider a general number of retailer's types for the EOQ problem contrary to the works of [19, 22]. They consider that the retailer has only one private cost parameter that corresponds to his holding cost. The purpose of their work is to design a menu of contracts for each type of retailer by using a screening game. The authors propose structural properties of the optimal solutions by looking at a directed graph associated to the individual rational and the incentive compatible constraints. They give a sufficient condition to guarantee that the menu of contracts in an optimal solution is composed of a unique contract for each retailer's type. In the specific case of two retailer's types, they give a complete characterization of the unique optimal menu of contracts. Contrary to their work, we investigate the design of contracts using a screening game in a dynamic setting.

The following works address the problem with asymmetric information and dynamic demand. Mobini *et al.* [17] consider the two-level supply chain problem for minimizing the supplier's cost. They use a screening model by considering several types contrary to most of the papers in the literature which only consider two types. In the mathematical formulation of the problem, the number of incentive compatible constraints which ensure that the retailer will always reveal his real type is quadratic. The purpose of their paper is to reduce the number of incentive compatible constraints. In the case where the private costs are stationary, they show that only a linear number of incentive compatible constraints should be considered in the model. Dudek and Stadtler [8] propose to coordinate a two-level supply chain with one supplier, one retailer and several items using a negotiation based model. Each actor only knows his own cost parameters. They consider a centralized method where the planning decisions are synchronized by a coordinator in order to minimize the supplier cost. The synchronization is made from information exchange about ordering proposals and impacts on local costs. A side payment is allowed between the actors in order to compensate the increase of the retailer's cost. They extend their work by considering one supplier and several retailers [9]. Lee and Kumara [14] study the problem with one supplier and several retailers. They coordinate the actor's planning decisions by using auction markets. The actors do not have to reveal their ordering costs, their ordering capacity and the external demand during the auction process. At each step of the auction, the revealed information for each actor concern the inventory (for example the inventory costs).

4.2 Mathematical formulation

In order to differentiate the types, we add a subscript to the notations defined in Section 2. The ordering (resp. replenishment) quantity at period t for the type i is given by x^S_{it} (resp. x^R_{it}). The inventory quantity at the end of period t at the supplier (resp. retailer) level is denoted by s^S_{it} (resp. s^R_{it}) for the type i. The binary variables associated to the setup at period t for the supplier (resp. retailer) is given by y^S_{it} (resp. y^R_{it}) for the type i. The side payment for the retailer of type i is z_i. The supplier wants to minimize his expected cost $\sum_{i\in\mathcal{N}} P_i(C^S_i + z_i)$ where $C^S_i = \sum_{t\in\mathcal{T}}(f^S_t y^S_{it} + p^S_t x^S_{it} + h^S_t s^S_{it})$ is the supplier's total cost associated to the retailer of type i. We denote by $C^R_{ij} = \sum_{t\in\mathcal{T}}(f^R_{it} y^R_{jt} + p^R_{it} x^S_{jt} + h^R_{it} s^R_{jt})$ the retailer's total cost when the retailer's real type is i and the type selected by the retailer is j. The optimal cost of the retailer of type i is denoted by $C^{R_i}_{opt}$.

The mathematical formulation of the coordination problem under asymmetric information is given by:

$$
\begin{aligned}
\min \sum_{i\in\mathcal{N}} & P_i(z_i + C_i^S) \\
\text{s.t. } & s^R_{i,t-1} + x^R_{it} = d_t + s^R_{it} \qquad \forall i\in\mathcal{N}, \forall t\in\mathcal{T}, && (11)\\
& s^S_{i,t-1} + x^S_{it} = x^R_{it} + s^S_{it} \qquad \forall i\in\mathcal{N}, \forall t\in\mathcal{T}, && (12)\\
& x^R_{it} \le d_{tT} y^R_{it} \qquad \forall i\in\mathcal{N}, \forall t\in\mathcal{T}, && (13)\\
& x^S_{it} \le d_{tT} y^S_{it} \qquad \forall i\in\mathcal{N}, \forall t\in\mathcal{T}, && (14)\\
& y^R_{it} \le x^R_{it} \qquad \forall i\in\mathcal{N}, \forall t\in\mathcal{T}, && (15)\\
& C^R_{ii} - z_i \le C^{R_i}_{opt} \qquad \forall i\in\mathcal{N}, && \text{(IR)}\\
& C^R_{ii} - z_i \le C^R_{ij} - z_j \qquad \forall i\in\mathcal{N}, \forall j\in\mathcal{N}, && \text{(IC)}\\
& x^S_{it}, x^R_{it}, s^S_{it}, s^R_{it} \ge 0 \qquad \forall t\in\mathcal{T}, \forall i\in\mathcal{N}, \\
& y^S_{it}, y^R_{it} \in \{0,1\} \qquad \forall t\in\mathcal{T}, \forall i\in\mathcal{N}, \\
& z_i \ge 0 \qquad \forall i\in\mathcal{N}
\end{aligned}
$$

Constraints (11) and (12) are the inventory balance constraints for each type $i\in\mathcal{N}$ at each level of the supply chain, at each period $t\in\mathcal{T}$. Constraints (13) and (14) ensure that the setup variable is equal to 1 if there is an order for each type $i\in\mathcal{N}$, at each period $t\in\mathcal{T}$. Since we do not minimize the retailer's cost, Constraints (15) ensure that the setup variable is equal to 0 if there is no order at period $t\in\mathcal{T}$ for a type $i\in\mathcal{N}$. Constraints (IR) represent the individual rationality constraint for each type $i\in\mathcal{N}$. Constraints (IC) represent the incentive compatible constraints which ensure that a retailer of type i will not have an incentive to choose a contract designed for another retailer's type j.

Similarly to the case with complete information, if the side payment is only on the retailer's holding cost, we add the following constraint to the mathematical formulation:

$$
z_i \le \sum_{t\in\mathcal{T}} h^R_{it} s^R_{it} \qquad \forall i\in\mathcal{N}
$$

4.3 Designing contracts without side payment

The problem of minimizing the supplier's cost without side payment ($z_i = 0$ for all i) under the asymmetric information assumption is referred to P^{AI}_{WP} where AI stands for asymmetric information. Since side payment is not allowed, the contracts proposed by the supplier are necessarily incentive compatible. Indeed, the supplier wants to minimize his expected cost by determining a contract $(x_{R_i}, 0)$ for each type i that minimizes his cost such that the cost of the replenishment plan x_{R_i} is equal to $C^{R_i}_{opt}$. Thus, a retailer of type i has no incentive to choose a contract that is designed for another type. The problem is then decomposed into N independent subproblems. Each subproblem is a coordination problem with complete information P^{CI}_{WP} addressed in Section 3.2 which gives the optimal replenishment plans $x^{R_i}_{opt}$ of cost $C^{R_i}_{opt}$ that minimizes the supplier's cost. The supplier will then propose N contracts $(x^{R_i}_{opt}, 0)$ for each $i\in\mathcal{N}$ to the retailer. Then, P^{AI}_{WP} can be solved in $\mathcal{O}(NT^3)$.

4.4 Designing contracts with side payment

The problem of minimizing the supplier's cost with side payment on the retailer's holding (resp. total) cost under the asymmetric information assumption is referred to as P^{AI}_{HC} (resp. P^{AI}_{TC}) where AI stands for asymmetric information. As shown when complete information is assumed (see Sections 3.3 and 3.4), the supplier's cost can be arbitrarily large. This is true even if the side payment is only assumed on the retailer's holding cost.

Since P^{CI}_{HC} is NP-hard, P^{AI}_{HC} is at least NP-hard. The complexity of P^{AI}_{TC} is an open question. However, if the incentive compatible constraint is relaxed, this problem can be decomposed into N problems for which complete information can be assumed and then the problem is solvable in polynomial time (Sections 3.4). We show that when the incentive compatible constraint is assumed, the cost of the supplier can be arbitrarily high compared to the case with complete information. In the case of a minimization problem, we recall that a α-approximate mechanism ($\alpha > 1$) is a mechanism which computes for each instance $\mathcal{I}$, a solution $x_{\mathcal{I}}$ of cost $c(x_{\mathcal{I}})$ such that $c(x_{\mathcal{I}}) \leq \alpha c(x^*)$ where x^* is an optimal solution of the problem. This mechanism is truthful if the incentive compatible constraint is satisfied.

Property 1. *For any $c > 1$, there is no c-approximate truthful mechanism for P^{AI}_{HC} and P^{AI}_{TC}.*

Proof. Let $c > 1$. We consider the following instance where the retailer's holding cost is private: $T = 2$, $N = 2$, $d = (0, 1)$, $f^R_i = p^R_i = (0, 0)$ for $i \in \{1, 2\}$, $h^R_1 = (1, 0)$, $h^R_2 = (c, 0)$, $p^S = (0, 0), f^S = (0, c^2)$ and $h^S = (c^2, 0)$. The probability that the retailer is of type 1 is $P_1 = 1 - 1/c$, and thus the probability that the retailer is of type 2 is $P_2 = 1 - P_1 = 1/c$. The optimal replenishment plan for each retailer's type is $x^{R_1}_{opt} = x^{R_2}_{opt} = (0, 1)$ of cost $C^{R_1}_{opt} = C^{R_2}_{opt} = 0$. The supplier's cost for satisfying this replenishment plan is equal to c^2. When the supplier knows the retailer's cost, the supplier will propose contracts to the retailer such that the retailer's cost is his optimal cost and the supplier's cost is minimized. If the retailer's type is 1, the supplier will propose a contract (x^R_1, z_1) such that $x^R_1 = (1, 0)$ and $z_1 = 1$. The retailer's cost including the side payment is equal to 0, and the supplier's cost is equal to 1, i.e. the cost of his optimal plan $x^S = [1, 0]$ to satisfy x^R_1 which is equal to 0, plus the side payment z_1. Likewise, if the retailer's type is 2, the supplier will propose the following contract (x^R_2, z_2) where $x^R_2 = (1, 0)$ and $z_2 = c$. The retailer's cost is then equal to 0 and the supplier's cost is equal to c. Therefore, assuming complete information, the supplier's expected cost is equal to $P_1 + P_2 c = (1 - 1/c)1 + 1 < 2$.

When asymmetric information on the retailer's cost is assumed, the supplier should consider the following contracts. We only consider the contracts that are interesting for the supplier with respect to his cost:

- Contract 1: the supplier proposes the replenishment plan $x^R = (0, 1)$ which is the optimal replenishment plan for each type of retailer. Thus, no side payment is needed. The cost of the supplier is then equal to c^2.
- Contract 2: the supplier proposes the replenishment plan $x^R = (1, 0)$ with a side payment equal to c. Both types of retailer have an incentive to choose this contract; the retailer of type 1 will have a negative cost and the retailer of type 2 will have a cost equal to 0. The supplier's cost related to this contract is equal to c.
- Contract 3: the supplier proposes the replenishment plan $x^R = (1, 0)$ with the side payment equal to 1. In that case, the only retailer that will have an incentive to choose this contract is the retailer of type 1, his cost will be equal to 0. The supplier's cost is then equal to 1.

The best solution that satisfies Constraints (IC) for the supplier is to propose Contract 2 to the retailer. The supplier's expected cost is then equal to $P_1 c + P_2 c = c$. Indeed, the supplier could also propose another set of contracts such as Contract 1 (to retailers of type 2) and Contract 3 (to retailers of type 1). But, in this case, the supplier's expected cost will be equal to $P_1 1 + P_2 c^2 = (1 - 1/c) + c > c$. Therefore, assuming asymmetric information, the supplier's expected cost is equal to c whereas it is less than 2 under complete information assumption. The supplier increases his cost by a factor larger than $c/2$ under the asymmetric information assumption. Then, when c tends to the infinity, this ratio can be as large as wanted. □

Note that both the fixed ordering cost and the unit ordering cost are null in the instance considered in the proof. Then, the above proposition holds even if the side payment is allowed only on the retailer's holding cost.

4.5 Experimental analysis

An experimental analysis is provided in order to evaluate the supplier's expected gain with regard to the different scenarios. We consider that either f^R, p^R or h^R is private and use a mathematical formulation for the two-level lot-sizing problem proposed by Wolsey and Pochet [20] to solve the addressed problems. The tests are performed using concert technology of CPLEX 12.6.

We generate the instances based on the classes described in Table 1. For every class of instances, we create 100 instances with $T \in \{10, 20, 30\}$, $N \in \{2, 10\}$. The probability P_i for $i \in \mathcal{N}$ is randomly generated from the uniform distribution $[0, 1]$, and then normalized in order to have $\sum_{i \in \mathcal{N}} P_i = 1$. The demands are randomly generated from the uniform distribution $[0, 300]$.
The supplier's fixed ordering cost is generated such that $f^S_t = \delta p^S_t$ where $\delta = 500$. When the retailer's costs f^R, h^R are not private, they are randomly generated (uniform distribution) in the discrete interval

described in Table 1. If the cost p^R is not private, it is randomly generated from the uniform distribution $[1, 100]$. We generate the private cost such that the retailer has several profiles. If f^R is the retailer's private cost, we generate the fixed ordering cost for the different types such that for type 1, $f_1^R = \delta c$ where c is randomly generated from the uniform distribution $[c_1^{R\min}, c_1^{R\max}] = [1, 20]$. Type 1 represents a retailer with a fixed ordering cost that is low. The subsequent types are generated such that the fixed ordering cost is increasing. For each type $i \in \{2, \ldots, N\}$, f_i^R is randomly generated from the uniform distribution $[c_{i-1}^{R\max} + 100/N, c_{i-1}^{R\max} + 20 + 100/N]$: 20 represents the length of the interval and $100/N$ enable us to generate different profiles of retailers. For example, we consider an instance where $N = 2$, f^R is the private cost and the instance is in the class A. The costs are generated in the following way: h^S, p^S, h^R are randomly chosen in the interval described in Table 1, p^R is randomly chosen in $[1, 100]$, f_1^R is generated such that $f_{1t}^R = \delta c$ where c is randomly chosen in $[1, 20]$ and f_2^R is generated such that $f_{2t}^R = \delta c$ where c is randomly chosen in $[70, 90]$. Similarly, if p^R (resp. h^R) is private, p_1^R (resp. h_1^R) is randomly generated from the uniform distribution $[p_1^{R\min}, p_1^{R\max}] = [1, 20]$ (resp. $[h_1^{R\min}, h_1^{R\max}] = [0, 6]$), and p_i^R is randomly generated from the uniform distribution $[p_{i-1}^{R\max} + 100/N, p_{i-1}^{R\max} + 20 + 100/N]$ (resp. $[h_{i-1}^{R\max} + 5, h_{i-1}^{R\max} + 7 + 5]$) for each type $i \in \{2, \ldots, N\}$. In order to compare the results when complete information is assumed and the ones obtained assuming asymmetric information, when p^R is private and $N = 2$, the instances are generated from the instances used for the experimental analysis with complete information in Section 3.5: from one pair of instances where $p^R \in [1, 20]$ or $p^R \in [70, 90]$ under complete information, we create an instance with asymmetric information where the retailer of type 1 has a cost $p^R \in [1, 20]$ and the retailer of type 2 has a cost $p^R \in [70, 90]$. Observe that in the case where h^R is private, we do not have to differentiate the case where h^R is high or not because of the retailer's profiles. Thus, we can only consider the classes of instances A and B.

The supplier's expected gain, in percentage, is defined by $(1 - \mathrm{E}[c_P^S]/\mathrm{E}[c^S]) \times 100$ where $\mathrm{E}[c_P^S]$ is the supplier's expected cost where P stands for P_{WP}^{AI}, P_{HC}^{AI} or P_{TC}^{AI}, and $\mathrm{E}[c^S]$ is the supplier's expected cost when the retailer imposes his optimal replenishment plan that is the least profitable for the supplier. The results are summarized in Tables 4, 5 and 6. The column *Mean* (resp. *Max*) represents the average (resp. maximum) value of the supplier's expected gain over the 100 instances for each class of instances.

Table 4: Average of the supplier's expected gains (%) when p^R is private.

		$T = 10$						$T = 20$					
		P_{WP}^{AI}		P_{HC}^{AI}		P_{TC}^{AI}		P_{WP}^{AI}		P_{HC}^{AI}		P_{TC}^{AI}	
N	Class	Mean	Max	Mean	Max	Mean	Max	Mean	Max	Mean	Max	Mean	Max
2	A	0.0	0.0	5.6	43.9	6.4	43.9	0.0	0.0	12.1	60.6	12.1	60.6
2	B	0.0	0.0	15.0	63.9	16.2	63.9	0.0	0.0	27.2	63.9	27.7	63.9
2	C	0.0	0.0	2.2	29.5	2.2	29.5	0.0	1	2.6	15.6	2.6	15.6
2	D	0.0	0.0	10.8	41.3	10.8	41.3	0.0	0.0	13.8	34	13.8	34
10	A	0.0	0.0	5.8	47.5	6.8	47.5	0.0	1.4	9.5	31.7	9.6	31.7
10	B	0.0	0.0	11.0	51.6	12.5	51.6	0.0	0.3	22	54.8	22.6	54.8
10	C	0.0	0.2	2.2	20.4	2.2	20.4	0.0	0.0	2.5	12.2	2.5	12.2
10	D	0.0	1.7	7.0	33.2	7.0	33.2	0.0	0.0	11.8	37	11.8	37

Table 4 and Figures 11, 12 report the results when p^R is private. Similarly to the case with complete information, the supplier's gain is near 0% when side payment is not allowed between the actors (the highest gain is equal to 1.7% and there are positive gains only for the classes C and D). We compare the results obtained for 2-type instances with the ones observed assuming complete information. We can notice that, the asymmetric information assumption does not deteriorate significantly the supplier's gains compared to the complete information case: the difference is at most 1 percentage point. Therefore, although Proposition 1 states that the supplier's cost with asymmetric information can increase by a factor arbitrarily large compared to the case with complete information, these experiments show that, in practice, the supplier's cost with asymmetric information does not increase significantly. We also observed that the average of the supplier's expected gains associated to P_{HC}^{AI} is near the one associated to P_{TC}^{AI}. When $T = 20$, there is a difference of at most 1.5 percentage points and when $T = 20$, the difference is at most equal to 0.6 percentage points. Moreover, for the class A of instances and $N = 2$, the boxplots show that for 25% of the instances, the supplier's expected gain associated to P_{HC}^{AI} is between 8% and 18.5% and it is greater than between 9.9% and 22.5% for P_{TC}^{AI}. As for the case with complete information, the results obtained with the class C of instances are the lowest (when $N = 2$, we observed

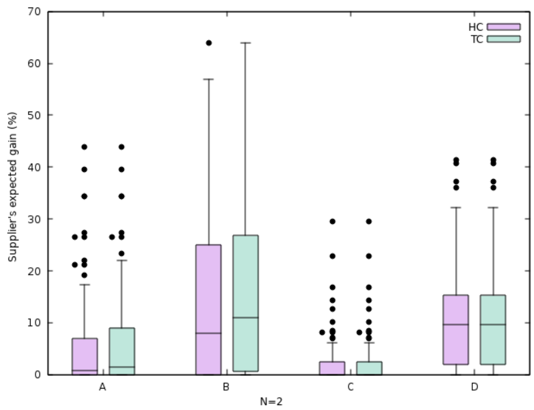

Figure 11: The supplier's expected gain of P_{HC}^{AI} and P_{TC}^{AI} for each class of instances where p^R is private, $N = 2$ and $T = 10$.

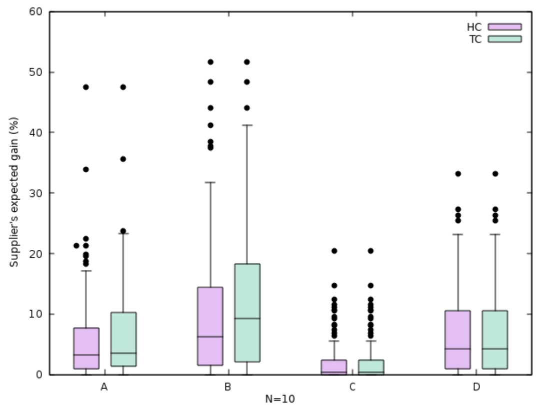

Figure 12: The supplier's expected gain of P_{HC}^{AI} and P_{TC}^{AI} for each class of instances where p^R is private, $N = 10$ and $T = 10$.

an expected gain of 2.6% for the class C and at least 12.1% for the other classes). The best results are obtained for the class B of instances (27.2% when $T = 20$ and $N = 2$). The number of types seems to have a little impact on the supplier's expected gain. For the class B and $T = 20$, there is at most a difference of 5.2 percentage points between the average of the supplier's expected gains when $N = 2$ (it is equal to 27.2%) and $N = 10$ (it is equal to 22%).

Table 5: Average of the supplier's expected gains (%) when f^R is private.

		$T = 10$						$T = 20$					
		P_{WP}^{AI}		P_{HC}^{AI}		P_{TC}^{AI}		P_{WP}^{AI}		P_{HC}^{AI}		P_{TC}^{AI}	
N	Class	Moy	Max	Moy	Max	Moy	Max	Moy	Max	Moy	Max	Moy	Max
2	A	0.0	0.0	2.5	42.3	3.0	42.3	0.0	0.0	9.5	63.2	9.6	63.2
2	B	0.0	0.0	7.0	74.6	9.0	74.6	0.0	0.0	16.9	59.3	17.4	59.3
2	C	0.0	0.0	2.9	31.1	3.0	31.1	0.0	0.0	3.8	29.2	3.8	29.2
2	D	0.0	0.0	8.4	56	8.4	56	0.0	0.9	11.7	52.3	11.7	52.3
10	A	0.0	0.0	1.1	10.2	1.2	10.2	0.0	0.0	4.8	37.4	4.8	37.4
10	B	0.0	0.0	2.3	22.5	3.1	22.5	0.0	0.0	11.3	69.1	12.3	69.1
10	C	0.0	0.0	2.0	18.8	2.0	18.8	0.0	0.1	3.1	17.5	3.1	17.5
10	D	0.0	0.0	5.3	33.7	5.3	33.7	0.0	0.0	10.8	34.7	10.8	34.7

Table 6: Average of the supplier's expected gains (%) when h^R is private.

		$T = 10$						$T = 20$					
		P_{WP}^{AI}		P_{HC}^{AI}		P_{TC}^{AI}		P_{WP}^{AI}		P_{HC}^{AI}		P_{TC}^{AI}	
N	Class	Moy	Max	Moy	Max	Moy	Max	Moy	Max	Moy	Max	Moy	Max
2	A	0.0	0.0	2.5	22.3	2.6	22.3	0.0	0.0	4.1	30.1	4.2	30.1
2	B	0.0	0.0	6.7	42.9	7.4	42.9	0.0	0.0	10.7	50.4	10.8	50.4
10	A	0.0	0.0	0.6	8.3	0.6	8.3	0.0	0.0	1.0	11.1	1.0	11.1
10	B	0.0	0.0	2.0	14.2	2.2	14.2	0.0	0.0	3.7	15.5	3.7	15.5

When the retailer's holding cost h^R is private, the averages of the supplier's expected gain are not as high as for the other cost parameters. For $T = 20$ and $N = 2$, the best average of the supplier's expected gains is equal to 10.8% whereas it is equal to 27.7% when p^R is private and 17.4% when f^R is private. Moreover, the number of types seems to have an important impact on the supplier's expected gain unlike the observation made when p^R is private. For $T = 20$, the higher average is equal to 3.7% when $N = 10$ and it is equal to 10.8% when $N)2$. Indeed, the retailer's holding cost increases with the number of types since a type represents a retailer's profile. However, when the retailer's holding cost is high, the supplier does not improve his cost significantly. Thus, if there are lot of types with high holding costs, the supplier's expected gain will not be significant.

5 Extension to the multi-retailer case

In this section, we extend the analysis by considering a two-level supply chain with one supplier and multiple retailers. Each retailer has to satisfy a demand for a single product. The problem of satisfying these demands in order to minimize the cost of the supply chain is the so-called the One-Warehouse Multi-Retailers (OWMR) problem that is known to be NP-hard [2]. The problems of designing contracts under complete and asymmetric information are refereed to as OWMR-Supp-CI and OWMR-Supp-AI respectively in order to minimize the supplier's cost.

5.1 Problem definition

As described for the single-retailer case, we consider the following contracts: the first one assumes no side payment between the supplier and the retailers, the second (resp. third) one allows a side payment on the retailer's holding (resp total) cost. We extend the screening game to multi-retailer case. We consider M retailers and denote by $\mathcal{M}$ the set of retailers $\{1, \ldots, M\}$. Each retailer m has N_m possible types. We denote by $\mathcal{N}_m$ the set of types of the retailer m. The supplier knows that the retailer m is of type $i \in \mathcal{N}_m$ with a probability P_i^m ($\sum_{i \in \mathcal{N}_m} P_i^m = 1$). Similarly to the single-retailer case, the supplier proposes to each retailer m a set of contracts (one contract for each type $i \in \mathcal{N}_m$). We have to consider every combination of types among the retailers. We denote by $\mathcal{C}$ the set of all the combinations. To illustrate these possible combinations, let us consider the following example for $M = 2$. The retailer 1 has $N_1 = 2$ types and the retailer 2 has $N_2 = 3$ types. Then, the set of combinations is $\mathcal{C} = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3)\}$ where (i,j) means that the retailer 1 is of type i and the retailer 2 is of type j. The probability to have a combination c is given by $P_c = \prod_{m \in \mathcal{M}} P_{c_m}^m$, where c_m is the type of the retailer m in the combination $c \in \mathcal{C}$. Moreover, the contracts still have to be individual rational and incentive compatible to ensure that each retailer m will not have an incentive to lie.

5.2 Mathematical formulation

The mathematical formulations of the problems OWMR-Supp-CI and OWMR-Supp-AI are derived from the multi-commodity formulation proposed by Cunha and Melo [7] for the OWMR problem.

Ordering units at period t incurs a fixed ordering cost $f_t^{R,m}$ and a unit ordering cost $p_t^{R,m}$ for the retailer m. Carrying units from period t to period $t+1$ incurs a unit holding cost $h_t^{R,m}$ for the retailer m. The demand of a retailer m at period t is denoted by d_t^m.
Let $w_{ckt}^{S,m}$ be the ordering quantity of the supplier for the retailer m of type c_m to satisfy the demand of the retailer m at period t. Let $w_{ikt}^{R,m}$ be the ordering quantity of the retailer m of type i to satisfy his demand at period t. Let $\sigma_{ckt}^{S,m}$ (resp. $\sigma_{ikt}^{R,m}$) be the quantity of units in the inventory of the supplier (resp. retailer m of type i) at the end of period k to satisfy the demand of the retailer m of type c_m (resp. his demand) at period t. Let y_{ct}^S (resp. $y_{it}^{R,m}$) be the setup variable of the supplier associated to the combination c (resp. retailer m of type i). We denote by z_i^m the side payment that the supplier has to assume for the retailer m of type i. The optimal cost of the retailer m of type i is denoted by $C_{opt}^{R,m,i}$. The total cost of the retailer m of type i who chooses a contract designed for a type j is given by $C_{ij}^{R,m} = \sum_{t=1}^{T} (f_{it}^{R,m} y_{jt}^{R,m} + p_{it}^{R,m} x_{jt}^{R,m} + h_{it}^{R,m} s_{jt}^{R,m})$.

The mathematical formulation of the problem OWMR-Supp-AI when side payment is on the retailer's holding cost is given by:

$$\min \sum_{c \in \mathcal{C}} P_c \Big(\sum_{t=1}^{T} (f_t^S y_{ct}^S + p_t^S x_{ct}^S + h_t^S s_{ct}^S) + \sum_{m=1}^{M} z_{c_m}^m \Big)$$

$$\text{s.t. } \sigma_{i,k-1,t}^{R,m} + w_{ikt}^{R,m} = \delta_{kt} d_t^c + (1-\delta_{kt}) \sigma_{ikt}^{R,m} \quad \forall m \in \mathcal{M}, \forall i \in \mathcal{N}_m, \forall 1 \le k \le t \le T, \tag{16}$$

$$\sigma_{c,k-1,t}^{S,m} + w_{ckt}^{S,m} = w_{c_m kt}^{R,m} + \sigma_{ckt}^{S,m} \quad \forall c \in \mathcal{C}, \forall m \in \mathcal{M}, \forall 1 \le k \le t \le T, \tag{17}$$

$$w_{ikt}^{R,m} \le d_t^c y_{ik}^{R,m} \quad \forall m \in \mathcal{M}, \forall i \in \mathcal{N}_m, \forall 1 \le k \le t \le T, \tag{18}$$

$$w_{ckt}^{S,m} \le d_t^c y_{ck}^S \quad \forall c \in \mathcal{C}, \forall m \in \mathcal{M}, \forall 1 \le k \le t \le T, \tag{19}$$

$$y_{ik}^{R,m} \le w_{ikt}^{R,m} \quad \forall m \in \mathcal{M}, \forall i \in \mathcal{N}_m, , \forall 1 \le k \le t \le T, \tag{20}$$

$$\sum_{t=k}^{T} w_{ikt}^{R,m} = x_{ik}^{R,m} \quad \forall m \in \mathcal{M}, \forall i \in \mathcal{N}_m, , \forall k \in \mathcal{T}, \tag{21}$$

$$\sum_{m=1}^{M} \sum_{t=k}^{T} w_{ckt}^{S,m} = x_{ck}^S \quad \forall c \in \mathcal{C}, \forall k \in \mathcal{T}, \tag{22}$$

$$\sum_{t=k}^{T} \sigma_{ikt}^{R,m} = s_{ik}^{R,m} \quad \forall m \in \mathcal{M}, \forall i \in \mathcal{N}_m, , \forall k \in \mathcal{T}, \tag{23}$$

$$\sum_{m=1}^{M} \sum_{t=k}^{T} \sigma_{ckt}^{S,m} = s_{ck}^S \quad \forall c \in \mathcal{C}, \forall k \in \mathcal{T}, \tag{24}$$

$$C_{ii}^{R,m} - z_i^m \le C_{opt}^{R,m,i} \quad \forall m \in \mathcal{M}, \forall i \in \mathcal{N}_m, \tag{25}$$

$$C_{ii}^{R,m} - z_i^m \le C_{ij}^{R,m} - z_j^m \quad \forall m \in \mathcal{M}, \forall i, j \in \mathcal{N}_m, \tag{26}$$

$$z_i^m \le \sum_{t=1}^{T} h_{it}^{R,m} s_{it}^{R,m} \quad \forall m \in \mathcal{M}, \forall i \in \mathcal{N}_m, \tag{27}$$

$$w_{ckt}^{S,m}, \sigma_{ckt}^{S,m} \ge 0 \quad \forall c \in \mathcal{C}, \forall m \in \mathcal{M}, \forall 1 \le k \le t \le T,$$

$$y_{ct}^S \in \{0,1\} \quad \forall c \in \mathcal{C}, \forall t \in \mathcal{T},$$

$$w_{ikt}^{R,m}, \sigma_{ikt}^{R,m} \ge 0 \quad \forall m \in \mathcal{M}, \forall i \in \mathcal{N}_m, \forall 1 \le k \le t \le T,$$

$$y_{it}^{R,m} \in \{0,1\} \quad \forall m \in \mathcal{M}, \forall i \in \mathcal{N}_m, , \forall t \in \mathcal{T},$$

$$z_i^m \ge 0 \quad \forall m \in \mathcal{M}$$

where δ_{kt} is equal to 1 if $k = t$ and 0 otherwise. Constraints (16) are the balance constraints at the retailer level for satisfying the demand of each type of retailers. Constraints (17) are the balance constraints at the supplier level for satisfying the replenishment plans of the retailers for each combination. Constraints (18)-(19) set the setup variables to 1 if there is an ordering. Constraints (20) ensure that the setup variables of the retailers are equal to 0 if there is no ordering. Constraints (21)-(24) link the multi-commodity variables to the classical variables of lot-sizing problems. Constraints (25) are the individual rationality constraints which ensure that the contracts are optimal for each type of retailers. Constraints (26) are the incentive compatible constraints. Constraints (27) ensure that the side payment is only on the retailer's holding cost.

If the side payment is not allowed between the actors, the mathematical formulation of the problem is the same by setting all the variables z_i^m to 0 and removing Constraints (26). If side payment is on the retailer's total cost, the mathematical formulation of the problem is given by the previous formulation without Constraints (26).

Note that for the problem OWMR-Supp-CI, each retailer m has exactly one type. Then, there is only one combination and the incentive compatible constraint is not useful. The mathematical formulation of the problem OWMR-Supp-CI is equivalent to the one of the problem OWMR-Supp-AI without Constraint (26).

Under the complete information assumption, the mathematical formulation of the problem OWMR-Supp-CI is the same than the one for the OWMR problem with in addition the IR constraint for each retailer. Under the asymmetric information assumption, the mathematical formulation of the problem

OWMR-Supp-AI is an extension of the one proposed for the OWMR problem where the IR constraint for each retailer and the IC constraint for each combination of retailer's types have to be taken into account.

5.3 Experimental analysis

The first part of the experiments deals with the complete information assumption. Since the OWMR variants are NP-hard, we restrict the analysis to instances with $M = 3$ and $T = \{10, 20, 30\}$ in order to solve them optimally. The instances are generated using the same generation scheme than the one used for one retailer (see Section 3.5). The supplier's costs and each retailer's cost are generated according to the classes of instances described in Table 1. The results are given in Tables 7 and 8. We denote by WP, HC and TC the problem of coordination without side payment, with side payment on the retailer's holding cost and with side payment on the retailer's total cost.

Table 7: Average of the supplier's gains (%) with complete information where $M = 3$.

	$T = 10$						$T = 20$					
	WP		HC		TC		WP		HC		TC	
Class	Mean	Max	Mean	Max	Mean	Max	Mean	Max	Mean	Max	Mean	Max
A	0.0	0.0	9.3	61.3	11.4	61.3	0.0	0.0	13.9	58.1	14.2	58.1
B	0.0	0.0	17.7	65.2	21.4	65.2	0.0	0.0	30.2	69.9	31.1	69.9
C	0.0	0.0	4.4	24.7	4.4	24.7	0.0	0.0	4.1	24.5	4.1	24.5
D	0.0	0.0	13.9	48.8	13.9	48.8	0.0	0.0	12.9	35.6	12.9	35.6

Table 7 reports the results for the OWMR-Supp-CI problem when side payment is allowed on the retailer's holding cost. The observations provided for the single-retailer case still hold for the multi-retailer setting. The problem of coordination when side payment is not allow between the actors does not improve the supplier's gain (his gain is equal to 0% for all the instances). In the multi-retailer case, the difference of the supplier's gain between the problems HC and TC is more higher (the higher difference is equal to 3.7 percentage points for the class of instances B when $T = 10$).

Table 8: Average of the supplier's expected gains(%) with asymmetric information and side payment on the retailer's holding cost, where $M = 3$ and $N_i = 2$ for all i. We set a time limit of one hour.

	p^R private				f^R private				h^R private			
	$T = 10$		$T = 20$		$T = 10$		$T = 20$		$T = 10$		$T = 20$	
Class	Mean	Max	Mean	Max	Mean	Max	Mean	Max	Mean	Max	Mean	Max
A	8.6	52.9	13.4	49.2	3.4	30.4	9.9	53.7	3.2	20.6	4.8	22.4
B	16.3	61.4	28.9	65.2	9.2	50.0	20.6	55.1	6.9	26.5	11.8	31.9
C	3.8	20.3	3.7	18.6	3.9	26.8	4.3	16.7	-	-	-	-
D	12.7	35	12	31.4	10.3	36.8	12.9	30.0	-	-	-	-

In the following part, we assume that a cost parameter is private. We run the tests with $M = 3$, $N_i = 2$ for all $i = 1, 2, 3$ and $T = \{10, 20, 30\}$ to get the optimal expected gains. We generate the instances following the single-retailer generation scheme. The supplier's costs are generated according to the class of instances described in Table 1. The retailer's costs are generated following the procedure described for the single retailer case under asymmetric information assumption (Section 4.5). The results are given in Tables 8. The results are similar to the case with one retailer, then only the results for the problem with side payment on the retailer's holding cost is given. Similarly to the complete information case, the supplier cannot decrease his cost without side payment.

6 Conclusion and future research

In this paper, we study the coordination of planning decisions in a two-level supply chain by using contracts. Different variants of the problem have been discussed regarding complete or asymmetric information assumption. We design contracts where side payment can be allowed or not between the actors, a retailer that has the power market and a supplier in order to decrease the supplier's cost. We show that for the single-retailer problem with complete information, designing optimal contracts that minimize the supplier's cost can be done in polynomial time when side payment is not allowed or side payment is allowed on the retailer's total cost. When side payment is only allowed on the retailer's holding cost, we prove that the problem is NP-hard. Under the asymmetric information assumption, we show that designing optimal contracts without side payment is still polynomial. However, in the case where side payment is allowed on the retailer's total cost, the complexity issue is open. We prove that under the asymmetric information assumption, there is no constant factor approximation algorithm which is truthful for the problems with side payment on the retailer's cost. An experimental analysis shows that under complete or asymmetric information, contracts without side payment are not effective in decreasing the supplier's cost. On the contrary, when side payment is allowed, the benefit of the supplier can be substantial. Moreover, we have seen that contracts with side payment on the retailer's holding cost are sufficient to decrease significantly the supplier's cost. We extended the study for a multi-retailer setting. The analysis shows that the same trends are observed.

Certain open research questions remain. The complexity of the problem under the asymmetric information assumption when side payment is allowed is a challenging issues. In the multi-retailer case, it is also possible to consider the problem where side payment can be allowed between the retailers. In addition, our study concerns a single product. It can be interesting to study a supply chain with several products where side payments are shared between all the products or a subset of products. A strong assumption in our study is that the retailer has the market power. However, we can also consider the problem of designing contracts when the supplier is the leader.

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