# Fleet Sizing and Empty Freight Car Allocation

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**Abstract**

Empty freight car allocation problems as well as fleet sizing problems depict highly important topics in the field of railway cargo optimization. Fleet sizing is mainly used in order to find the minimal number of freight cars (fixed costs) needed to operate the transportation network successfully (e.g. satisfy customer demands). After a consignment is transported to its destination, the unloaded freight cars must be reallocated. This reallocation process is linked to costs depending on the traveled distance.

In this paper we propose an integer linear program (ILP) for determining the optimal trade-off between the number of freight cars and the costs linked to empty vehicle allocation. The ILP is tested on a distributation network that is related to a specific part of Rail Cargo Austria's railway network. Following the results of this test, different deviations in the network (changes in cost structure, drive time and demand) are examined and compared to the basic scenario.

Fleet sizing, empty freight car allocation, distribution network, integer linear programming.

## 1 Introduction

Railway freight transportation is characterized by various types of costs. One of the major cost drivers is the company's rolling stock, i.e. the locomotives and freight cars. In terms of the freight cars, the costs can be described in a simplified way as the expenses connected to the fleet size (holding costs per car per day), plus the costs linked to the actual movement of the fleet in the network. Therefore, fleet sizing problems are usually concerned with the determination of the minimum required number of vehicles needed to satisfy the demand of a transportation network. In contrast, once a consignment is shipped to its' destination, the empty freight cars have to be reallocated. This reallocation process is linked to costs depending on traveled distance and can be minimized with the help of empty freight car distribution optimization.

In fact, the described types of cost drivers directly affect each other. The bigger the fleet, the more freight cars are located all over the network and therefore, the less empty vehicle kilometers are typically needed since some of the freight cars are close

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to the next customer. In turn, the smaller the fleet size, the longer the distance to the next customer and therefore, the distribution costs for empty cars increase. The aim of our proposed optimization model is to find the best trade-off between fleet size and empty vehicle kilometers in order to reduce the overall costs of a specific part of the distribution network of Rail Cargo Austria.

## 2 Literature Review

Several studies regarding empty vehicle allocation in the railway industry have already been conducted. White and Bomberault [1], Jordan and Turnquist [2], Kikuchi [3] , Joborn et al. [4] and Mendiratta and Turnquist [5] introduced different network flow models. A taxonomy of empty flows as well as a literature review regarding empty freight car flow models was carried out by Dejax and Crainic [6].

Fleet sizing problems have been studied in different fields of transport. Zuckerman and Tapiero [7] tried to determine the optimal fleet size for passenger transports, Jordan and Turnquist [8] developed a deterministic model for container fleet sizing, Beaujon and Turnquist [9] tried to find the optimal number and location of freight car pools and Sherali and Tuncbilek [10] compared a static and a dynamic model for fleet sizing for the automobile and railroad industries.

There also already exist a few optimization models that jointly consider fleet sizing and empty freight car allocation. Bojovic [11] formulated a general system theory approach for minimizing overall costs. Sayarshad and Ghoseiri [12] developed an integer linear program (ILP) for fleet sizing and empty freight car allocation with deterministic demands for solving small-size instances to optimality. Additionally, a simulated annealing heuristic was applied for solving real-world sized problems. Sayarshad and Tavakkoli-Moghaddam [13] proposed a model for optimizing fleet size and empty freight car allocation under uncertain demands. Both in [12] and [13] travel times were assumed to be deterministic and backordering of demand was included.

Since loading and unloading times may have a major impact on the overall time a freight car is idle, and therefore not available for further disposition, the service times will be taken into account in the model presented in this paper. Thus, considerations of service times represent one of the main differences to the models proposed in [12] and [13]. Furthermore, contrary to [13] we assume the demand of loaded freight cars to be deterministic. Additionally, demand has to be fulfilled 100% in every period, i.e. the possibility of backlogging demand is not considered in our model. Moreover, in terms of the number of distributed empty freight cars, a minimum and maximum limitation per shipment is applied, due to economic reasons as well as conditions of certain route sections. In view of the solution approach, we suggest an ILP formulation that is able to obtain optimal solutions for our large-scale real-world instances, related to a specific part of Rail Cargo Austria's distribution network.

## 3 Mathematical Formulation

The network is divided into origin nodes $i \in I$ and destination nodes $j \in J$ of empty freight cars. Note that the origin nodes $i \in I$ are at the same time destinations of loaded freight cars. The total number of time periods in which demand could occur is denoted by $\mathcal{T} = \{1, ..., T\}$. In order to balance out the predefined demand for loaded freight cars $D_{j,i,t}$, shipped from $j$ to $i$ in $t$, a certain amount of empty freight cars $X_{i,j,t}$ has to be distributed from $i$ to $j$ in $t$. Since freight cars could be distributed at the end of time horizon $\mathcal{T}$ and therefore arrive or be in service in $t > T$, we define another time horizon $\mathcal{T}_{DS} = \{1, ..., T_{DS}\}$ with $T_{DS} \geq \max\{T + \alpha_{j,i} + \delta_i\,, T + \beta_{i,j} + \tau_j\}$. In contrast to the longer time horizon $\mathcal{T}_{DS}$ for distribution and service of freight cars, the time horizon for inventory is denoted as $\mathcal{T}_I = \{0, ..., T_I\}$ with $T_I \geq T$. In turn, the fleet size $F$ can be defined as sum of all freight cars in inventory in $t = 0$. The total sets of input parameters as well as integer decision variables are summarized in Tables 1 and 2 respectively.

Now we are able to formulate our fleet sizing and empty freight car allocation problem as the following integer linear program (ILP):

$$\min \sum_{t\in\mathcal{T}} \sum_{j\in J} \sum_{i\in I} d_{i,j} \cdot X_{i,j,t} \cdot cv + F \cdot cf \cdot T \tag{1}$$

$$\text{subject to: } F = \sum_{j}^{J} VS_{j,0} + \sum_{j\in J} V_{j,0} + \sum_{i\in I} YS_{i,0} + \sum_{i\in I} Y_{i,0}, \tag{2}$$

$$S_{i,j,(t+\beta_{i,j})} = X_{i,j,t}, \qquad \forall\, i \in I, j \in J, t \in \mathcal{T}, \tag{3}$$

$$B_{(t+\tau_j),j} = \sum_{i\in I} S_{i,j,t}, \qquad \forall\, j \in J, t \in \mathcal{T}, \tag{4}$$

$$VS_{t,j} = VS_{t-1,j} + B_{(t+\tau_j),j} - B_{t,j}, \qquad \forall\, j \in J, t \in \mathcal{T}, \tag{5}$$

$$V_{t,j} = V_{t-1} + B_{t,j} - \sum_{i\in I} D_{t,j,i}, \qquad \forall\, j \in J, t \in \mathcal{T}, \tag{6}$$

$$V_{t,j} + VS_{t,j} \leq mV_j, \qquad \forall\, j \in J, t \in \mathcal{T}_I, \tag{7}$$

$$P_{(t+\alpha_{j,i}),j,i} = D_{t,j,i}, \qquad \forall\, j \in J, i \in I, t \in \mathcal{T}, \tag{8}$$

$$L_{i,(t+\delta_i)} = \sum_{j\in J} P_{t,j,i}, \qquad \forall\, i \in I, t \in \mathcal{T}, \tag{9}$$

$$YS_{i,t} = YS_{i,t-1} + L_{i,(t+\delta_i)} - L_{i,t}, \qquad \forall\, i \in I, t \in \mathcal{T}, \tag{10}$$

$$Y_{i,t} = Y_{i,t-1} + L_{i,t} - \sum_{j\in J} X_{i,j,t}, \qquad \forall\, i \in I, t \in \mathcal{T}, \tag{11}$$

$$Y_{i,t} + YS_{i,t} \leq mY_i, \qquad \forall\, i \in I, t \in \mathcal{T}_I, \tag{12}$$

$$W_{i,j} \leq X_{i,j,t} \leq \widetilde{W}_{i,j}, \qquad \forall\, i \in I, j \in J, t \in \mathcal{T}, \tag{13}$$

$$X_{i,j,t}, F \in \mathbb{Z}_{\geq 0}, \qquad \forall\, i \in I, j \in J, t \in \mathcal{T}, \tag{14}$$

$$S_{i,j,t}, P_{t,j,i}, L_{i,t}, l_{i,t}, b_{t,j}, B_{t,j} \in \mathbb{Z}_{\geq 0}, \qquad \forall\, i \in I, j \in J, t \in \mathcal{T}_{DS}, \tag{15}$$

$$Y_{i,t}, YS_{i,t}, V_{t,j}, VS_{t,j} \in \mathbb{Z}_{\geq 0}, \qquad \forall\, i \in I, t \in \mathcal{T}_I, \tag{16}$$

| Input-Parameters | Description |
|---|---|
| $d_{i,j}$ | distance in kilometers from origin $i$ to destination $j$ |
| $cv$ | costs per empty vehicle kilometer |
| $cf$ | renting costs per freight car per day |
| $mV_j$ | maximum capacity of freight cars at destination node $j$ |
| $mY_i$ | maximum capacity of freight cars at origin node $i$ |
| $\widetilde{W}_{i,j}$ | max. number of empty freight cars sent from $i$ to $j$ per distribution |
| $W_{i,j}$ | min. number of empty freight cars sent from $i$ to $j$ per distribution |
| $D_{j,i,t}$ | demand shipped from $j$ to $i$ in $t$ |
| $\alpha_{j,i}$ | drive time from $j$ to $i$ |
| $\beta_{i,j}$ | drive time from $i$ to $j$ |
| $\delta_i$ | service time for unloading in $i$ |
| $\tau_j$ | service time for loading in $j$ |

Table 1: Input parameters

| Decision-Variables | Description |
|---|---|
| $X_{i,j,t}$ | number of empty freight cars sent from $i$ to $j$ in $t$ |
| $S_{i,j,t}$ | number of empty freight cars sent from $i$ arriving at $j$ in $t$ |
| $B_{j,t}$ | sum of new loaded freight cars at $j$ in $t$ |
| $P_{j,i,t}$ | number of loaded freight cars sent from $j$ arriving at $i$ in $t$ |
| $L_{i,t}$ | sum of new empty freight cars at $i$ in $t$ |
| $Y_{i,t}$ | inventory of empty freight cars at $i$ in $t$ |
| $YS_{i,t}$ | inventory of loaded freight cars in service (unloading) at $i$ in $t$ |
| $V_{j,t}$ | inventory of loaded freight cars at $j$ in $t$ |
| $VS_{j,t}$ | inventory of empty freight cars in service (loading) at $j$ in $t$ |
| $F$ | fleet size |

Table 2: Decision variables

Our objective function (1) ensures the minimization of overall costs. Accordingly, the optimal trade off between costs for empty vehicle kilometers per freight car sent from $i$ to $j$ and costs for freight car fleet per day is determined. Equation (2) defines the fleet size as sum of starting inventories in period $t = 0$. Constraint (3) states that empty freight cars dispatched from $i$ to $j$ in $t$ will arrive at destination $j$ after drive time $\beta_{i,j}$. Equation (4) ensures that the sum of all empty freight cars sent from $i$ to $j$ arriving in $t$ are ready for redistribution in $j$ after service time $\tau_j$ for loading. The inventory for empty freight cars currently in service (loading) $VS_{t,j}$, as well as the inventory for the loaded freight cars $V_{t,j}$ at $j$ in $t$ are defined by (5) and (6) respectively. Additionally, the inequalities in (7) restrict the overall inventory by the maximum capacity in node $j$. Similarly to Equation (3) for empty freight cars, Constraint (8) states that loaded freight cars dispatched from $j$ to $i$ in $t$ (demand), will arrive at $i$ after the drive time $\alpha_{j,i}$. Equation (9) ensures that the sum of loaded freight cars sent from $j$ to $i$ arriving in $t$ are ready for redistribution in $i$ after the service time $\delta_i$ for unloading. Constraints (10) – (12) model the inventory change and the capacity restrictions at the origin node $i$ in $t$ and the inequalities in (13) define the limitations on the number of distributed empty freight cars from $i$ to $j$ in $t$. Constraints (14) – (16) ensure that all decision variables are non-negative integers.

# 4 Computational Experiments

In this section we test the proposed ILP using a set of input parameters motivated by a real-world monthly demand scenario of Rail Cargo Austria with 31 periods and a demand of 3994. The total network consists of 14 destination nodes (1-14) and 14 origin nodes (A-N). Vehicle renting costs $cf$ are set to 32 per day and the costs for an empty vehicle kilometer are $cv = 1$ . This setting is used for computing a baseline scenario. Furthermore, the impact of deviations from this baseline scenario concerning cost structure, drive time between certain nodes and a potential production stop are tested. Additionally, effects on computing time caused by changes with respect to time horizon and the number of nodes in the network are examined. The time limit was set to 1 hour. All experiments were performed using IBM ILOG CPLEX Optimization Studio Version 12.8.0 on a Windows 7 64-bit machine equipped with an Intel Core i5-6300U CPU (2x2400MHz) and 4 GB RAM.

Table 3 summarizes the results of the mentioned experiments regarding different deviations in the network. With the help of the first three scenarios, concerning changes of cost structure, it can be seen how the optimal fleet size and empty kilometers needed affect each other, as rising costs for empty vehicle kilometers cause an increase of the fleet size in order to reduce the number of empty kilometers needed. In terms of deviations of network structure and demand it can be seen that small changes of parts of the network can affect the solution on the whole transportation network. With the help of the results stated in Table 4, we examine the relation between the problem size and computing time needed to solve the ILP. While smaller instances can be solved in a few seconds, computing time for bigger instances increases strongly. However, with the proposed ILP it is still possible to solve the considered real-world problem with 28 nodes and 31 time periods in reasonable time.

| Fleet Size | Empty Kilometers | Objective Value | Deviation |
|---:|---:|---:|---:|
| 876 | 631,941 | 1,500,933 | $cf$: 32 , $cv$: 1 (baseline) |
| 912 | 603,278 | 1,281,806 | $cf$: 24 , $cv$: 1 |
| 974 | 566,460 | 2,099,128 | $cf$: 32 , $cv$: 2 |
| 1184 | 477,368 | 1,835,632 | $cf$: 24 , $cv$: 2 |
| 913 | 613,415 | 1,519,111 | Double drive time on A-2 and F-8 |
| 877 | 618,667 | 1,488,651 | Production stop at A in t=9..15 |

Table 3: Effects of network deviations

| Origin Nodes | Destination Nodes | Periods | Demand | CPU Time | Objective Value |
|---|---|---|---|---|---|
| 14 | 14 | 15 | 1904 | 00:04 | 958,543 |
| 14 | 14 | 31 | 3994 | 04:00 | 1,500,933 |
| 14 | 14 | 40 | 5127 | 16:48 | 1,798,127 |
| 14 | 14 | 80 | 5756 | TO | 3,094,560 (0.13%) |
| 20 | 20 | 31 | 4574 | 34:24 | 1,798,270 |
| 30 | 30 | 31 | 5926 | TO | 2,395,112 (0.14%) |

Table 4: Problem size and computing times. The time out (TO) is set to 1 hour. In case of TO the upper bound and gap (%) are stated.

# 5 Conclusion

In this paper we presented an integer linear programming formulation for fleet sizing and empty vehicle allocation in railway freight transportation developed for Rail Cargo Austria. With the help of the proposed model, the best trade-off between renting costs for freight cars and costs for empty vehicle kilometers can be determined. Potential influences of changes in cost structure, drive times, demand, fleet size as well as empty vehicle kilometers were verified. Furthermore, the effects of the network size and time horizon on the computing time were tested. It was shown that bigger instances derived from a real-world scenario can be solved to optimality in an adequate time.

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