

# Assessing the Cost of the Hazard-Decision Simplification in Multistage Stochastic Hydrothermal Scheduling

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## Abstract

Hydropower is one of the world's primary renewable energy sources whose usage has profound economic, environmental, and social impacts. We focus on the dispatch of generating units and the storage policy of hydro resources. In this context, an accurate assessment of the water opportunity-cost is crucial for driving the sustainable use of this scarce resource. Nevertheless, traditional computational tools use stochastic dual dynamic programming under the Hazard-Decision (HD) modeling simplification, where dispatch decisions are determined assuming perfect information about the current inflows. In practice, however, some dispatch decisions are made before water inflows are observed, i.e., under a Decision-Hazard (DH) scheme. This inconsistency generates an optimistic assessment of the opportunity cost of the water, inducing a sub-optimal use of energy resources. Thus, our objectives are: (i) to raise awareness of the HD issue bridging research and implementation with a clear recommendation to system operators and regulators; (ii) to incorporate the DH scheme into the long-term hydrothermal scheduling problem; (iii) to assess the economic impacts and other market distortions due to the actual energy usage policy based on the HD simplification. For a representative case-study with realistic data from the Brazilian power system, results show that the HD simplification introduces a regret cost of 12%. Furthermore, we show that incorporating the DH scheme into the hydrothermal planning model, the energy supply cost can be reduced up to 5.3%. We also show that spot-price distortions and expensive thermal generation could be mitigated in the DH scheme.

*Keywords:* Multistage hydrothermal scheduling, Stochastic dual dynamic

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## 1. Introduction

A key concept of economic theory is the opportunity cost of limited resources. In energy economics, the calculation of such costs plays a major role and has a profound influence on most activities and segments of energy sectors worldwide. From market design (Munoz et al. (2018)) and infrastructure planning (Sauma and Oren (2006) and Velloso et al. (2020)) to short-term operations (Hafiz et al. (2019) and Mancilla-David et al. (2020)) and bidding strategies in various markets (Liu et al. (2019) and Fanzeres et al. (2019)), the opportunity-cost of the relevant resources is key<sup>1</sup>. Therefore, an appropriate opportunity-cost assessment strategy has direct impacts on the economic, environmental, and social dimensions. In this context, a primary concern in energy systems with high storage capacity is the intertemporal management of limited energy resources (see Street et al. (2017), Papavasiliou et al. (2018), and Hafiz et al. (2019)). Notably, as per EIA (2019), hydropower constitute one of the world’s primary renewable energy sources. Its usage has profound economic, environmental, and social impacts. In this context, an accurate assessment of the water opportunity-cost is crucial for driving the sustainable use of this scarce resource, albeit relevant characteristic of this assessment has been neglect over the last two decades.

In this work, we focus on long-term hydrothermal scheduling (LTHS) problem, which determines the dispatch and storage policy for generating units and hydro reservoirs, targeting the total expected operating cost minimization. Among systems with substantial shares of hydro generation, most system operators and market players worldwide use the SDDP algorithm (Pereira and Pinto, 1991) to solve the LTHS problem. Motivated by computational benefits, the SDDP implementations for the LTHS use the HD simplifying assumption, i.e., they assume that the dispatch decisions of a given stage are made after observing the one-step-ahead inflow realization. This issue is a constant in the timeline of the LTHS literature, from the first publications (Pereira et al. (1984) and Pereira and Pinto (1991)), in which the method and application were presented, to the most recent developments

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<sup>1</sup>See also Memorandum to the California ISO Hobbs (2018).

(Zou et al. (2019) and Hjelmeland et al. (2019)), where stochastic dual dynamic integer programming (SDDiP) is presented and applied.

The success of the SDDP to solve practical LTHS problems is mainly due to the stagewise independence assumption. This assumption enables a unique cost-to-go function per stage (time periods), which can be efficiently assessed through sampling-based methods (Shapiro (2011)), with a convergence proof (Philpott and Guan (2008)). However, the extension of the SDDP framework to consider autocorrelation in the inflow scenarios is made by augmenting the state space to define the cost-to-go as a function of both reservoir levels and past inflows (see Shapiro (2011) and Löhdorf and Shapiro (2019)). Analogously, Diniz et al. (2011) and Helseth et al. (2016) also use the augmented state idea to respectively address liquefied natural gas constraints in the LTHS problem and reserves capacity sales within a competitive environment. On the methodological side, different implementation strategies, such as cut selection, parallelization, and other algorithmic parameter tuning were studied in de Matos et al. (2015). Finally, in Brigatto et al. (2017), the time inconsistency cost due to network simplifications is studied, while in Street et al. (2017) the co-optimization of energy and reserves is efficiently accounted for in the LTHS problem to address  $n - K$  security criteria. Notwithstanding, all previously reported works rely on the HD simplifying assumption, which produces an optimistic assessment of the opportunity cost of the water, reflecting a one-step-ahead anticipative decision process.

In the actual decision process, however, the HD premise is not appropriate for all dispatch decisions. The main objective of our work is to propose a framework where part of the dispatch decisions are made before observing the inflows realization, hereinafter referred to as the *decision hazard* (DH) scheme. The consideration of an explicit DH process into the SDDP procedure introduces additional computational complexities, but it better characterizes the actual decision process associated with the LTHS. As a result, the DH-based SDDP provides a more accurate assessment of the opportunity cost of the water, leading to more reliable system operation and consequential pricing signals. In this context, our work touches on a relevant and subtle information issue that comes from the genesis of the SDDP-based LTHS (Pereira and Pinto, 1991), and it is still present in many industrial and academic applications. We emphasize the importance of the proposed DH scheme by showing that the HD simplifying assumption biases the opportunity cost assessment with an optimistic view of future information avail-

ability. For systems with high storage capacity, this bias has the potential to significantly affect costs, operational decisions, and prices, thereby with profound impacts on the economic, environmental, and social dimensions.

In a DH scheme for LTHS, we assume that the dispatch of base thermo-electric units to save water for the next stage is performed before observing the inflows. In contrast, hydroelectric and fast units dispatch are corrective actions made after observing inflows realization. To solve a DH-LTHS problem, we compare two solution methodologies. The first is an adapted version of the SDDP algorithm considering a full two-stage stochastic program as the sub-problem for each stage. This approach is hereinafter referred to as fullDH-SDDP. The second is based on an *augmented state dynamic model* (ASDM) to encompass decisions made before observing inflows realization. This approach is henceforth referred to as *augmented-state decision-hazard SDDP* (ASDH-SDDP). We present numerical results for a representative case study of the Brazilian system where we show significant cost savings, lower expensive thermal generation, and reduced spot-price volatility.

That said, the contributions of this work are summarized as follows:

1. **To raise the awareness of the HD simplification issue in the long-term hydrothermal scheduling problem, bridging research and implementation with a clear recommendation to system operators, planners, and regulators.** Based on industry practice, we acknowledge that base (slow) thermal generators are dispatched in advance while hydroelectric and fast units are adaptively dispatched as corrective actions.
2. **To incorporate the DH scheme into the multistage problem and compare solution approaches.** We consider two different solution methodologies: (i) the fullDH-SDDP, where we adapt the SDDP algorithm to handle a two-stage stochastic program as the sub-problem for each stage, and (ii) ASDH-SDDP, where we augment the state of the system to encompass decisions made before observing inflows realization.
3. **To assess via numerical simulations the practical consequences of the HD simplification for a representative case-study of the Brazilian power system.** As numerical findings, we obtain: (i) the actual cost of the inconsistent policy is 12% higher than expected under the HD assumption (regret cost); (ii) the energy supply cost can be reduced up to 5.3% by incorporating the DH scheme into the hy-

drothermal planning model; (iii) the DH policy increases the cheaper preventive thermal dispatch of slow units to reduce unnecessary expensive dispatch of fast ones; and, as a result (iv) spot-price distortions (price spikes and high volatility) are reduced.

This paper is organized as follows: In Section 2, we briefly explain the classical formulation of the LTHS problem based on the HD simplification. In Section 3, we propose a DH-based LTHS model and show how this decision scheme affects the SDDP implementation. In Section 4, the performance metrics are introduced to compare different policies based on the HD and DH schemes. In Section 5, a case study based on the Brazilian power system is presented, and the conclusions are addressed in section 6.

## 2. Problem formulation under the Hazard-Decision simplification

For didactic purposes, we assume a compact model for the system dispatch constraints. In such a model, the power flow (direct current model – DC) and reservoirs storage (system state) transition equations are explicitly represented. The remaining operative constraints, such as the maximum and minimum bounds, Kirchhoff’s Voltage Law, etc., are represented through a polyhedral set  $\mathcal{X}_t$  (we refer to Brigatto et al. (2017) for further details).

The SDDP procedure assumes that the state of the system is known at the beginning of the period  $t$  and aims to evaluate the cost-to-go function through the minimal expected cost of operating the system from  $t$  until the end of the horizon. In our application, the state of the system at the beginning of  $t$  is composed of the stored volume in each reservoir at the end of the previous period,  $t - 1$ , represented by the vector  $v_{t-1}$ . Finally, according to Pereira and Pinto (1991), the evaluation of the cost-to-go function for each stage,  $t$ , and state,  $v_{t-1}$ , of the system is based on the expectation of the immediate and future operational costs.

Under the HD simplification, the immediate cost is obtained by the inner product between the vector of unitary costs,  $c_t$ , and the dispatched thermoelectric generation vector,  $g_{t,\omega}$ , i.e.,  $c_t^\top g_{t,\omega}$ , for each inflow scenario,  $w_{t,\omega}$ . As usual, we consider a high-cost virtual generator to represent unserved energy. Following Shapiro (2011) and Löhndorf and Shapiro (2019), we also consider a periodic autoregressive model of order one for the inflows,  $w_{t,\omega} = \phi_t + \Phi_t w_{t-1} + \varepsilon_{t,\omega}$ , where  $(\phi_t, \Phi_t)$  represent the periodic coefficients of the model. In this framework, we assume a discrete sample space  $\Omega_t$  for

each stage  $t \in T$ , where  $\omega \in \Omega_t$  represents a scenario associated with the autoregressive noise  $\varepsilon_{t,\omega}$ .

The future cost is represented by the cost-to-go function for the next stage,  $\mathcal{Q}_{t+1}(v_{t,\omega}, w_{t,\omega})$  evaluated at the resulting state of the system at the end of period  $t$ , i.e., the hydro reservoir volumes  $v_{t,\omega}$  and the current inflow realization  $w_{t,\omega}$ . The aforementioned description leads to the dynamic programming formulation for the HD multistage stochastic model used in LTHS planning:

$$\begin{aligned} \mathcal{Q}_t(v_{t-1}, w_{t-1}) = & \\ \min_{\{X_{t,\omega}\}_{\omega \in \Omega_t}} & \sum_{\omega \in \Omega_t} p_\omega [c_t^\top g_{t,\omega} + \mathcal{Q}_{t+1}(v_{t,\omega}, w_{t,\omega})] & (1) \\ \text{s.t.} & Af_{t,\omega} + Bg_{t,\omega} + Pu_{t,\omega} = d_t & \forall \omega \in \Omega_t & (2) \\ & v_{t,\omega} = v_{t-1} - H(u_{t,\omega} + s_{t,\omega}) + w_{t,\omega} & \forall \omega \in \Omega_t & (3) \\ & w_{t,\omega} = \phi_t + \Phi_t w_{t-1} + \varepsilon_{t,\omega} & \forall \omega \in \Omega_t & (4) \\ & (g_{t,\omega}, u_{t,\omega}, s_{t,\omega}, f_{t,\omega}, v_{t,\omega}) \in \mathcal{X}_t & \forall \omega \in \Omega_t. & (5) \end{aligned}$$

Throughout this work,  $X_{t,\omega} = \{g_{t,\omega}, u_{t,\omega}, s_{t,\omega}, f_{t,\omega}, v_{t,\omega}, w_{t,\omega}\}$  represent the set of decision vectors at the end of period  $t$  and scenario  $\omega$  containing, respectively, the generation of each thermoelectric unit, the water discharged into the turbines of each hydroelectric unit, the water spilled by each hydroelectric unit, the power flow transferred through each transmission line, the volume of water stored in each reservoir of the system, as well as the auxiliary variable  $w_{t,\omega}$  used to consider the inflow autoregressive equation.

The expression (1) comprises the minimization of the immediate and future expected costs, as previously described in this section. The expression (2) considers the nodal energy balance equation, where  $A$  represents the network incidence matrix,  $B$  accounts for the total thermal generation in each bus of the system,  $P$  considers the productivity of each hydroelectric unit to account for the total hydro generation in each bus, and  $d_t$  is the vector of the nodal net demand of period  $t$  in each bus of the system. The expression (3) accounts for the state transition and mass conservation functions, where  $H$  (matrix with only  $+1, 0, -1$  elements) translates the cascading topology by assigning the total water discharged and spilled by each hydroelectric unit to the corresponding downstream reservoirs. Finally, the expression (5) accounts for the operational constraints, such as the Kirchhoff's Voltage Law, and bounds Brigatto et al. (2017).

According to the HD scheme, dispatch decisions are made under the perfect information of the water inflow scenario at period  $t$ . Under this hypothesis, the cost-to-go function defined in (1)-(5) can be decomposed into scenario-based functions as

$$Q_t(v_{t-1}, w_{t-1}) = \sum_{\omega \in \Omega_t} p_\omega Q_t(v_{t-1}, w_{t-1}, \varepsilon_{t,\omega}), \quad (6)$$

where the function

$$Q_t(v_{t-1}, w_{t-1}, \varepsilon_{t,\omega}) = \min_{X_{t,\omega}} c_t^\top g_{t,\omega} + Q_{t+1}(v_{t,\omega}, w_{t,\omega}) \quad (7)$$

$$\text{s.t. } Af_{t,\omega} + Bg_{t,\omega} + Pu_{t,\omega} = d_t \quad (8)$$

$$v_{t,\omega} = v_{t-1} - H(u_{t,\omega} + s_{t,\omega}) + w_{t,\omega} \quad (9)$$

$$w_{t,\omega} = \phi_t + \Phi_t w_{t-1} + \varepsilon_{t,\omega} \quad (10)$$

$$(g_{t,\omega}, u_{t,\omega}, s_{t,\omega}, f_{t,\omega}, v_{t,\omega}) \in \mathcal{X}_t \quad (11)$$

represents the optimal value for each objective term of (1)-(5), which associated with each inflow scenario  $w_{t,\omega}$ .

Because  $Q_t(v_{t-1}, w_{t-1}, \varepsilon_{t,\omega})$  is a convex and piecewise linear function of  $(v_{t-1}, w_{t-1})$  for all values of  $\varepsilon_{t,\omega}$ , a lower approximation, based on the maximum within a set of supporting planes, can be built. Thereby, the standard SDDP algorithm, which relies on successive improvements of such a lower approximation by means of the inclusion of new supporting planes, can be directly applied.

### 3. Long Term Hydrothermal Scheduling under a Decision-Hazard scheme

In this section, we propose a DH scheme by allowing the dispatch of a subset of thermoelectric units within set  $J$  to be determined before inflows realization, which can be accounted for into the optimization model through the consideration of non-anticipativity constraints. We propose using an additional decision vector, namely,  $g_t^o$ , whose components,  $\{g_t^o[j]\}_{j \in J}$ , contain the dispatch decisions made at the beginning of stage  $t$ , under the uncertainty of the inflows. In the classical literature of two-stage stochastic programming (see Birge and Louveaux (2011)), the decisions made under uncertainty are also known as here-and-now decisions. In the same literature, the remaining decisions, those that can be made after the observation (with

perfect information) of the uncertainty parameters, are called wait-and-see decision. Oftentimes, here-and-now decisions are referred to as preventive actions, whereas wait-and-see decisions are also referred to as corrective actions.

Hence, a new set of non-anticipative constraints is incorporated to model the here-and-now decisions in the dynamic model, leading to the following DH formulation

$$\mathcal{Q}_t(v_{t-1}, w_{t-1}) = \min_{g_t^o, \{X_{t,\omega}\}_{\omega \in \Omega_t}} \sum_{\omega \in \Omega_t} p_\omega [c_t^\top g_{t,\omega} + \mathcal{Q}_{t+1}(v_{t,\omega}, w_{t,\omega})] \quad (12)$$

$$\text{s.t.} \quad Af_{t,\omega} + Bg_{t,\omega} + Pu_{t,\omega} = d_t \quad \forall \omega \in \Omega_t \quad (13)$$

$$v_{t,\omega} = v_{t-1} - H(u_{t,\omega} + s_{t,\omega}) + w_{t,\omega} \quad \forall \omega \in \Omega_t \quad (14)$$

$$w_{t,\omega} = \phi_t + \Phi_t w_{t-1} + \varepsilon_{t,\omega} \quad \forall \omega \in \Omega_t \quad (15)$$

$$g_{t,\omega}[j] = g_t^o[j] \quad \forall j \in J, \omega \in \Omega_t \quad (16)$$

$$(g_{t,\omega}, u_{t,\omega}, s_{t,\omega}, f_{t,\omega}, v_{t,\omega}) \in \mathcal{X}_t \quad \forall \omega \in \Omega_t. \quad (17)$$

Model (12)-(17) is a two-stage stochastic programming model that extends the HD formulation (1)-(5) to consider a DH stochastic dynamic problem. Expression (16) accounts for the non-anticipativity constraints implying that here-and-now dispatch decisions for the units in  $J$  do not anticipated the current inflows realization of the current stage  $t$ .

Although the SDDP approach could be adapted to consider a cost-to-go function defined by a two-stage stochastic program (Valladão et al., 2019), here denoted as fullDH-SDDP, the backward problem would not be suitable for scenario decomposition as per expression (6). In addition, the forward iteration would require the solution of a full two-stage stochastic problem to define the final state at each stage,  $\{v_t^{(k)}\}_{k=1, \dots, i-1}$ . The previously described implementation issues associated with the consideration of non-anticipativity constraints in (12)-(17) constitute a major incompatibility with the primary feature that allows the state-of-the-art SDDP procedure to solve large-scale problems in a reasonable computational time.

To overcome the aforementioned implementation issues associated with the scenario-separability property not observed in the DH dynamic model (12)-(17), we adopt an alternative approach where the dispatch decisions  $\{g_t^o[j]\}_{j \in J}$  of each stage sub-problem be fixed and defined as state variables

of the current stage. This approach yields an ASDM formulation for the dynamic equations. Additionally, the decision of those variables is passed to the previous stage as scenario-dependent dispatch variables. Thereby, in any given stage  $t$ , the future dispatch decisions  $\{g_{t+1,\omega}^o\}_{\omega \in \Omega_{t+1}}$ , are also decision variables of the current subproblem. Thus, the new ASDM for the DH scheme is as follows:

$$\begin{aligned} \mathcal{Q}_t(v_{t-1}, w_{t-1}, g_t^o) = & \\ \min_{\{g_{t+1,\omega}^o, X_{t,\omega}\}_{\omega \in \Omega_t}} & \sum_{\omega \in \Omega_t} p_\omega [c_t^\top g_{t,\omega} + \mathcal{Q}_{t+1}(v_{t,\omega}, w_{t,\omega}, g_{t+1,\omega}^o)] \quad (18) \\ \text{s.t.} \quad & Af_{t,\omega} + Bg_{t,\omega} + Pu_{t,\omega} = d_t \quad \forall \omega \in \Omega_t \quad (19) \\ & v_{t,\omega} = v_{t-1} - H(u_{t,\omega} + s_{t,\omega}) + w_{t,\omega} \quad \forall \omega \in \Omega_t \quad (20) \\ & w_{t,\omega} = \phi_t + \Phi_t w_{t-1} + \varepsilon_{t,\omega} \quad \forall \omega \in \Omega_t \quad (21) \\ & g_{t,\omega}[j] = g_t^o[j] \quad \forall j \in J, \omega \in \Omega_t \quad (22) \\ & (g_{t,\omega}, g_{t+1,\omega}^o, u_{t,\omega}, s_{t,\omega}, f_{t,\omega}, v_{t,\omega}) \in \mathcal{X}'_t \quad \forall \omega \in \Omega_t. \quad (23) \end{aligned}$$

It is noteworthy that model (18)–(23) differs from (12)–(17) in the following aspects: 1) unlike (30),  $g_t^o$  is a right-hand-side vector of coefficients defined by the state of the system (it is not a decision vector of period  $t$ ); 2) new decision vectors,  $\{g_{t+1,\omega}^o\}_{\omega \in \Omega_{t+1}}$ , accounting for the newly added state variables are considered in the model and the associated bounds accounted for in the extended version of the polyhedral set  $\mathcal{X}'_t$  in (23).

The ASDM (18)–(23) is a scenario-decomposable dynamic model, thereby suitable for standard SDDP implementations. In this sense, following the same steps found in the previous section, problem (18)–(23) can be decomposed per scenario. As a result, we can reformulate

$$\mathcal{Q}_t(v_{t-1}, w_{t-1}, g_t^o) = \sum_{\omega \in \Omega_t} p_\omega \mathcal{Q}_t(v_{t-1}, w_{t-1}, g_t^o, \varepsilon_{t,\omega}), \quad (24)$$

where

$$\mathcal{Q}_t(v_{t-1}, w_{t-1}, g_t^o, \varepsilon_{t,\omega}) = \quad (25)$$

$$\min_{g_{t+1,\omega}^o, \{X_{t,\omega}\}_{\omega \in \Omega_t}} c_t^\top g_{t,\omega} + \mathcal{Q}_{t+1}(v_{t,\omega}, w_{t,\omega}, g_{t+1,\omega}^o) \quad (26)$$

$$\text{s.t.} \quad Af_{t,\omega} + Bg_{t,\omega} + Pu_{t,\omega} = d_t \quad (27)$$

$$v_{t,\omega} = v_{t-1} - H(u_{t,\omega} + s_{t,\omega}) + w_{t,\omega} \quad (28)$$

$$w_{t,\omega} = \phi_t + \Phi_t w_{t-1} + \varepsilon_{t,\omega} \quad (29)$$

$$g_{t,\omega}[j] = g_t^o[j] \quad \forall j \in J \quad (30)$$

$$(g_{t,\omega}, g_{t+1,\omega}^o, u_{t,\omega}, s_{t,\omega}, f_{t,\omega}, v_{t,\omega}) \in \mathcal{X}'_t. \quad (31)$$

Model (26)-(31) is an augmented-state version of the dynamic equation (12)-(17) and can be solved using the standard SDDP algorithm. Note that, in our model, the set of here-and-now decisions,  $J$ , is a modeling choice, thereby generalizing the existing HD approach.

It is worth mentioning that the proposed ASDM increases the dimension of the state in  $|J|$ , which in our case, is a subset of the thermoelectric units. However, for a more practical and efficient implementation of the ASDM, a moderate increase in the state can be achieved by adding to the state only the aggregated (total) thermoelectric generation for each electrical subsystem in  $s \in S$ . In such case, vector  $g_t^{(o)}$ , whose components  $\{g_t^o[s]\}_{s \in S}$  account for the total generation within each subsystem, can be defined with a reduced dimension (because, in general,  $|S| \ll |J|$ ). To adapt model (26)-(31) to such a case, one is only required to replace (22) by

$$\sum_{j \in J_s} g_{t,\omega}[j] = g_t^o[s], \quad \forall s \in S, \quad (32)$$

where  $J_s$  represents the subset of units of  $J$  that belongs to subsystem  $s$ . Hence, by replacing (22) with (32), the dispatch within each subsystem will naturally follow the minimum cost order, where cheaper generators are prioritized.

#### 4. Performance measures and policy definitions

As argued in the introduction, the implementability of a policy relies on decision models that deliver non-anticipative decisions. To measure the isolated impact of a policy that simplifies the DH scheme, we propose two quantitative metrics: 1) the *time-inconsistency GAP* (**TIGAP**) owing to the HD simplification, which estimates the additional expected cost incurred by the inconsistency between planned and implemented decisions (see Rudloff et al. (2014) and Brigatto et al. (2017) for more details), and 2) the *value of the DH policy* (**VDHP**), which resembles, within our context, the well-known concept of *value of the stochastic solution* (VSS) (see Birge and Louveaux (2011) for a quick reference). Hence, three policies must be simulated for a large set

of scenarios, and their expected costs estimated. They are introduced and explained in the following paragraphs.

1) ***HD planning policy (HDplan)*** – this policy is simulated based on model (1)-(5) assuming the HD hypothesis, i.e., the assessment of its expected cost assumes that the system operator has the perfect information of the uncertainty realization within stage  $t$  when deciding vector  $g_t^{(o)}$ . The expected operational cost evaluated under such policy, namely,  $C^{(\text{HDplan})}$ , constitutes an optimistic estimate for the expected operational cost throughout the study horizon. Therefore, this cost constitutes a lower-bound reference used to assess the value of the one-step-ahead perfect information. Although not implementable in practice, this policy is generally used as a reference for planning studies. Additionally, the opportunity costs obtained from it (embedded in its cost-to-go function) are the inputs for the actual inconsistent implemented policy as discussed in the next item.

2) ***HD implementation policy (HDimp)*** – the HDplan policy is one-step-ahead anticipative, thereby not implementable in practice. To generate implementable decisions based on the HD cost-to-go function, we emulate the current industry practice, where non-anticipative decisions are made based on a one-step-ahead deterministic model using the expected value of inflows for the current period. Hence, at the beginning of each simulated inflow scenario, a vector of non-anticipative decisions is obtained using the model (7)-(11) for the expected inflow scenario. Subsequently, the non-anticipative here-and-now decisions of a given stage are made fixed by means of a constraint in this model. Thus, the remaining corrective actions are obtained by solving the constrained version of the model for each scenario of the current stage<sup>2</sup>. The state is updated accordingly, and the process is repeated until the end of the horizon. In this process, future planned decisions implicitly represented by the cost-to-go function of the HD-based model may differ from those obtained with the deterministic implementation approach. As per Rudloff et al. (2014) and Brigatto et al. (2017), this yields a time-inconsistent policy since the future planned decisions (under HD simplification) will differ from the actual implemented ones (under the one-step-ahead deterministic approach). As the HD implementation policy is suboptimal, the expected cost estimates for the

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<sup>2</sup>In this constrained model, only wait-and-see decisions are optimized and defined for each scenario as corrective actions, whereas the vector of here-and-now decisions is kept fixed and equal to the value obtained with the deterministic model.

HD implementation policy,  $C^{(\text{HDimp})}$ , should be greater than or equal to the expected cost given by the HD planning policy, i.e.,  $C^{(\text{HDimp})} \geq C^{(\text{HDplan})}$ .<sup>3</sup>

3) **DH policy** – this policy is simulated based on model (12)-(17), which can be efficiently implemented based on (18)–(23) as per Section 3. It constitutes an improvement with regard to the HD implementation policy as it incorporates into the cost-to-go function the actual model that will be used in the future. Therefore, assuming that the implementation model is (12)-(17), the expected cost estimate for this policy, namely,  $C^{(\text{DH})}$ , lies in between the cost estimate for the two previously introduced policies, i.e.,  $C^{(\text{HDplan})} \leq C^{(\text{DH})} \leq C^{(\text{HDimp})}$ .

The evaluation of each one of the three aforementioned policies depends on the different dynamic models used. However, in any of the three cases, each policy-expected cost estimate can be evaluated based on the sample average dispatch cost obtained as the result of a final simulation step within each policy configuration and cost-to-go function. Therefore, if  $g_{t,\omega}^{(p)}$  is the dispatch decision obtained by policy  $p \in \{\text{HDplan}, \text{HDimp}, \text{DH}\}$  at period  $t$  and scenario  $\omega$ , each of the aforementioned expected cost estimates can be calculated by the following formula:

$$C^{(p)} = \frac{1}{N} \sum_{t \in T} \sum_{\omega \in \hat{\Omega}_t^N} c_t^\top g_{t,\omega}^{(p)}. \quad (33)$$

In (33),  $\hat{\Omega}_t^N$  represents a large set of  $N$  independently generated scenarios, randomly sampled from  $\Omega_t$ , and  $g_{t,\omega}^{(p)}$  represents the optimal dispatch decisions, for each policy  $p \in \{\text{HDplan}, \text{HDimp}, \text{DH}\}$ , period  $t$ , and sampled scenario  $\omega \in \hat{\Omega}_t^N$ .

Based on the cost of the three aforementioned policies, the evaluation metrics can be assessed as follows:

$$TIGAP = C^{(\text{HDimp})} - C^{(\text{HDplan})}, \quad (34)$$

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<sup>3</sup>For practical implementation purposes, instead of using the HDplan cost-to-go functions obtained from the SDDP to simulate the HD implementation policy, we recommend a more accurate assessment based on the fast algorithm proposed in Brigatto et al. (2017). In this case, the fast algorithm is initialized with the HDplan cost-to-go functions and improved with new cuts on points of the state space likely to be visited when using the implementation model.

$$VDHP = C^{(HDimp)} - C^{(DH)}. \quad (35)$$

The TIGAP is a measure of the expected regret, or frustration, in terms of the additional expected cost incurred when adopting modeling simplifications in the planning stage. Hence, this metric can be used to test the planning model bias and can be regarded as a measure of how optimistic the planning stage is with regard to the implementation model within a controlled experiment (where no other effect is considered). Finally, VDHP is a measure of the expected benefit (in terms of cost savings) by considering the non-anticipativity constraints in the planning stage. The value of this metric can be seen as the benefit of an improved assessment of the opportunity cost of the water due to a more realistic decision model.

## 5. Case study

The objectives of this case study are threefold: 1) to raise the awareness of the HD issue using the TIGAP metric, 2) quantify the potential benefits of a DH scheme through the VHDP improvement index, and 3) demonstrate that the proposed ASDM is an effective model for solving the DH multi-stage LTHS problem. Hence, we used realistic data from the Brazilian power system, which currently adopts a similar expected-value-based approach as described for the HD implementation policy. The methods were developed with Julia Language version 0.5.1 using the Xpress solver. All computational experiments were executed on an Intel(R) Core(TM) i7-3960X CPU @ 3.3 GHz with 64 GB of RAM memory.

To achieve objectives 1 and 2 of this case study, we devised a controlled experiment where constraints (13)-(17) are always feasible for the real system. Hence, all decisions simulated in the implementation step, for both the HD implementable policy and DH policy, are feasible policies. This assessment framework allows us to isolated the negative effects of the HD hypothesis without introducing bias from other modeling simplifications. In this context, the present case study can be regarded as an extension of the issues raised in Brigatto et al. (2017). Finally, to achieve the third objective of this case study, we compare the solution time of two SDDP-based algorithms for solving the LTHS problem under the proposed DH scheme. The first algorithm, namely, fullDH-SDDP, relies on the implementation of an adapted SDDP algorithm (Valladão et al., 2019) whose subproblem is the full two-stage DH model (12)-(17). In contrast, the ASDH-SDDP approach is based on the augmented-state DH version of the SDDP model (18)–(23).

The energetic model adopted in this work uses official data from the Brazilian system. It comprises four distinct subsystems as used in the official dispatch model, namely, Southeast (SE), South (S), Northeast (NE), and North (N). The hydroelectric plants are aggregated into four different equivalent reservoirs, each of which located in one of the subsystems. The system comprises 111 thermal plants distributed through the subsystems, from which the 55 cheaper and slower generators are considered to be dispatched as non-anticipative resources (2 nuclear, 11 coal, and 42 gas-fired). Thus, the dispatch decision for the hydroelectric and faster (and more expensive) thermoelectric units are considered as corrective recourse actions. Table 1 summarize the thermoelectric data, where the total maximum and minimum generation levels ( $G_{max}$  and  $G_{min}$ ) are expressed in terms of average MW per month. The complete dataset can be found in Street et al. (2017).

Table 1: Thermoelectric data per subsystem.

	Preventive (here-and-now) units			Corrective (wait-and-see) units		
	Number of Units	Total $G_{max}$	Total $G_{min}$	Number of Units	Total $G_{max}$	Total $G_{min}$
SE	24	8,788	1,600	13	2,057	430
S	10	2,498	738	3	277	34
NE	8	2,860	0	30	2,953	0
N	13	2,451	0	10	1,000	233
Total	55	16,597	2,338	56	6,287	697

The planning horizon considered in this study encompasses 5 years, in a monthly basis. Although omitted in the model, a present value factor related to a discount rate of 0.5% per month (i.e.,  $1.005^{-1}$ ) is applied. Following previous works (see Street et al. (2017); Brigatto et al. (2017) and references therein), the inflow scenarios utilized in this study are based on monthly observations for the four subsystems from 1991 to 2015, amounting to 25 observations per month. A periodic autoregressive model of order one was also implemented according to Maceira et al. (2002) and Shapiro (2011) to incorporate the inter-temporal effects of water inflow. The simulation is based on a bootstrap procedure of the residuals following the finds of Castro et al. (2015).

We adopted the same parameters used in Brigatto et al. (2017) and Street et al. (2017), namely, 1000 initial forward-and-backward iterations<sup>4</sup> respectively, followed by a convergence test based on intermediate evaluation steps at each 100 iterations. Each convergence check is based on a t-test for the difference of means applied to the sampled average cost estimated in two consecutive evaluation steps and based on the respective lower bound differences. The sampled average cost used in the t-test is estimated based on a simulation with 3000 newly generated scenarios. The convergence of the lower bound is tested for a 1% gap. We refer the reader to the appendix of Brigatto et al. (2017) for the description of the stopping criterion used in this paper.

Regarding the first two objectives of this case study, Table 2 presents the cost for the three policies used to devise the proposed evaluation metrics: (34) and (35).

Table 2: Evaluation Metrics (in M\$)

$C^{(\text{HDplan})}$	$C^{(\text{HDimp})}$	$C^{(\text{DH})}$	TIGAP	VDHP
12,467.7	13,960.4	13,222.3	1,492.7	738.1

The results depict a non-negligible impact of disregarding non-anticipativity constraints on the system operation. Based on the TIGAP metric, the expected regret (additional cost) for the system operator quantifying the next five years' expected cost based on an HD-based planning tool is M\$ 1,492.7, i.e., approximately 12%. In contrast to that, the adoption of a consistent DH policy allows an expected reduction of M\$ 738.14 (5.3%) with regard to the cost of the HD implementation policy (the policy that represents the status quo in our experiment). This amount represents approximately 50% of the inconsistency gap. It is worth emphasizing that the VDHP represents the expected over-cost, or sub-optimality cost, due to the adoption of the HD simplification in the planning step. Moreover, both the TIGAP and VDHP evaluation metrics are based on the sample average statistics, suitable for a

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<sup>4</sup>We consider one sample per forward iteration and all the residuals as scenarios in each backward iteration. Within this strategy, each forward step uses the information of all local approximations built on previous steps, which is an effective strategy when parallel computing is not used.

t-test to verify the statistical significance. For a sample size of 3,000 simulated inflows, the t-test rejected the null hypothesis, where the true mean is assumed equal to zero, with p-values lower than 0.1%. Therefore, the results in Table 2 are statistically significant with respect to a significance level of 0.1%.

The relevant operational differences among the three policies, {HDplan, HDimp, DH}, are depicted in Figures 1 and 2, where the average preventive (here-and-now) and corrective (wait-and-see) thermoelectric dispatches are presented, respectively. Additionally, in Figure 3, the average spot prices for the primary subsystem (SE) are presented for the three policies. In these figures, the red dots represent the HD planning policy, the dotted black lines refer to the HD implementation policy, and the continuous black lines are associated with the results of the DH policy.

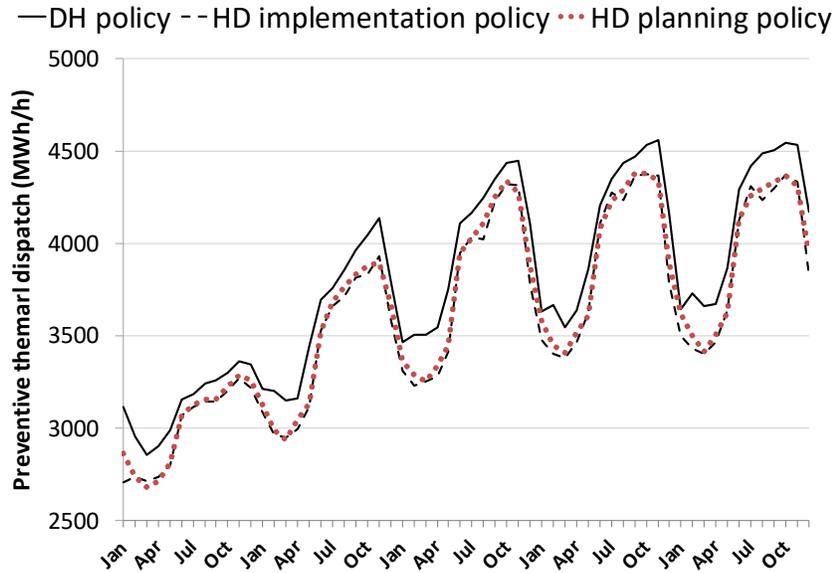


Figure 1: Preventive (here-and-now) dispatch of slow thermometric units - SE system

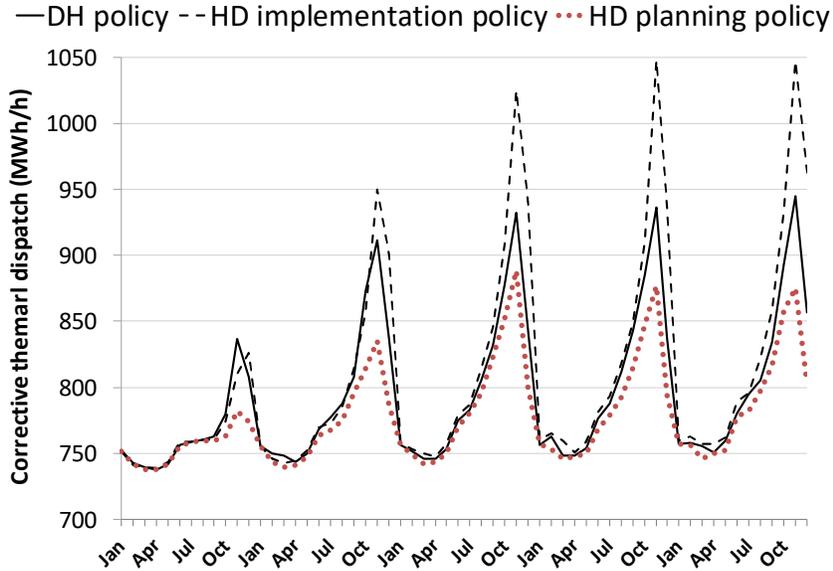


Figure 2: Corrective (wait-and-see) dispatch of fast thermometric units - SE system

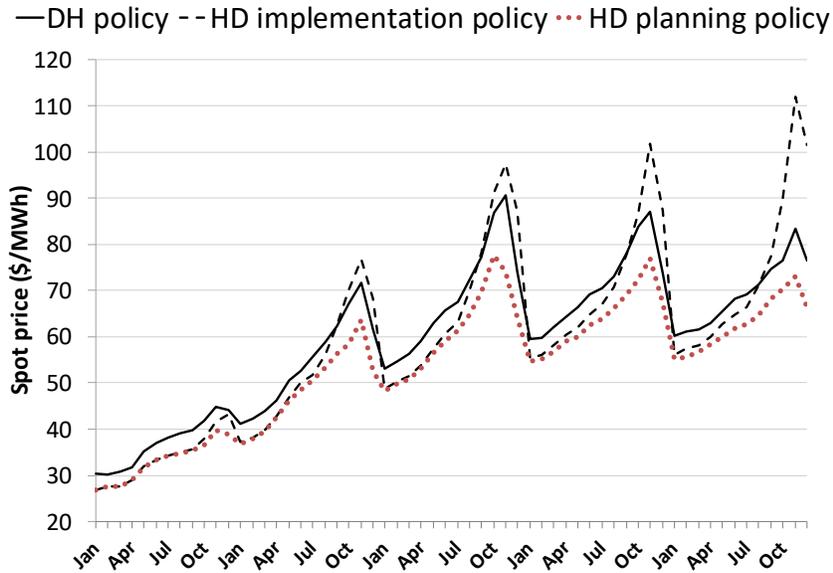


Figure 3: Spot Prices - SE Subsystem

Figure 1 reveals a relevant structural increase in the cheaper preventive dispatch of the slow thermoelectric units for the DH policy in comparison to

the HD-based ones. Notwithstanding, the most salient difference between HD planning and the HD implementation policies is observed in the expensive corrective (wait-and-see) dispatch levels, depicted in Figure 2. The unrealistic flexibility within the anticipative HD planning policy artificially reduces the corrective dispatch levels (most expensive units), which would not be achievable within an implementable policy accounting for non-anticipative decisions. As a consequence of this implementation issue, spot price discrepancies are observed and follow a similar pattern observed in the corrective dispatch levels, as shown in Figure 3. Finally, it is worth mentioning that the DH policy mitigates these two issues while yielding an expected overall operational cost saving of 5.3% with regard to the HD implementation scheme. It worth mentioning that the depicted results refer to expected values and these issues are even more prominent on higher quantiles.

Finally, regarding the third objective of this case study section, Table 3 compares the computational times for solving the DH multistage LTHS problem with the two DH-based algorithms: the fullDH-SDDP and ASDH-SDDP. This comparison is made based on the computing time required to solve the HD planning policy. As shown, the incorporation of non-anticipativity constraints increases the total computational burden with regard to the HD reference as per the results for both DH-based algorithms. However, the relevant increase in the computational time, of 21.2 times, exhibited by the fullDH-SDDP algorithm can be significantly mitigated to a factor of 4.9 (approximately 23.1% of the fullDH-SDDP time), by the proposed ASDH-SDDP. Additionally, it is relevant to mention that the ASDH-SDDP inherits all structural features of the standard SDDP, including the possibility of parallel implementations. We argue that the systemic benefits largely outweigh the additional computational burden, especially because this is a long-term problem that allows for longer computing times.

Table 3: Computational time (sec)

Algorithm	Time until convergence	Increasing factor
HD reference	1,758	1.0
fullDH-SDDP	37,181	21.2
ASDH-SDDP	8,680	4.9

## 6. Conclusions

In this work, we assessed Hazard-Decision (HD) simplification in SDDP implementations for long-term hydrothermal scheduling (LTHS) problems. Our thrust is to raise awareness of the HD simplification on LTHS, arguing that it leads to a sub-optimal operation with an optimistic assessment of the opportunity cost of the water. Moreover, we proposed an alternative decision process model, based on the decision-hazard (DH) scheme, that better characterizes the actual LTHS dispatch process. Based on industry practice, we consider that base (slow) thermal generators are dispatched in advance as preventive resources (here-and-now decisions), whereas hydroelectric and fast thermal units are adaptively dispatched as corrective actions (wait-and-see decisions). In our model, the set of here-and-now decisions is a modeling choice, thereby generalizing the existing HD approach. We incorporated the DH scheme into the multistage problem and compared two solution approaches: the fullDH-SDDP, which considers an adapted version of the SDDP to handle the full two-stage model as sub-problem, and the ASDH-SDDP, which uses an augmented state dynamic model suitable for the classical SDDP approach.

Computational experiments with a representative case study of the Brazilian system show that the implemented actions based on a HD scheme may significantly deviate from the planned decisions. Simulations reveal a 12% regret (overcost or disappointment) when implementing a policy assuming the HD simplifying assumption. Additionally, more volatile and increased levels of expensive thermoelectric units used as corrective actions are observed when implementing the HD-based policy following industry practices. The spot price distortions followed the same pattern. As per our results, the proposed DH scheme mitigates such effects with a cost reduction of 5.3 %. We compared two solution methodologies for the proposed DH scheme and concluded that, for our case study, the ASDH-SDDP was at least four times faster than the fullDH-SDDP methodology.

We highlight as a relevant future research topic the study and assessment of cost impacts of other simplifications in the LTHS problem such as unit commitment constraints, nonlinear generation curves, and operational constraints.

## 7. Recommendations and discussion

Within the limitations of our case study hypotheses and data, results clearly illustrate the potential adverse effects of the hazard-decision (HD) simplification on the long-term hydrothermal scheduling. Thus, we strongly recommend system operators, planners, regulators, and market players, relying on HD implementations, to conduct specific studies based on official/proprietary models and data to quantify the effect of this simplification accurately. Therefore, our findings have immediate applicability providing a concrete step towards sustainable development of actual power systems with high storage worldwide, e.g., Brazil, California (USA), Chile, Colombia, El Salvador, Mexico, New Zealand, NordPool participants, Ontario (Canada), Portugal, Spain, Vietnam, just to mention a few. For these systems, an appropriate opportunity-cost assessment strategy has profound impacts on the economic, environmental, and social dimensions. On the economic side, for instance, Latin American countries such as Brazil use simulated results based on the HD simplification to compute relevant reliability and economic indexes defining the competitiveness of generators in new capacity expansion auctions. On the environmental side, the HD simplification increases the dispatch of most expensive and fast units, which, in general, are those with the highest carbon emission rates. Finally, in the social dimension, unnecessarily increased energy prices intensify the so-called fuel poverty, which refers to the inability to meet basic energy needs.

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