A fully mixed-integer linear programming formulation for economic dispatch with valve-point effects, transmission loss and prohibited operating zones

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Abstract

Economic dispatch (ED) problem considering valve-point effects (VPE), transmission loss and prohibited operating zones (POZ) is a very challenging issue due to its intrinsic non-convex, non-smooth and non-continuous natures. To achieve a near globally solution, a fully mixed-integer linear programming (FMILP) formulation is proposed for such an ED problem. Since the original loss function is highly coupled on n-dimensional spaces, it is usually hard to piecewise linearize entirely. To handle this difficulty, a reformulation trick is utilized, transforming it into a group of tractable quadratic constraints. By taking full advantage of the variables coupling relationships among univariate and bivariate functions, an FMILP formulation that requires as few binary variables and constraints as possible is consequently constructed for the ED with VPE and transmission loss. When the POZ restrictions are also considered, a distance-based technique is adopted to rebuild these constraints, making them compatible with the previous FMILP reformulation. With the help of a logarithmic size formulation technique, a further reduction can be made for the introduced binary variables and constraints. By solving such an FMILP formulation, a near globally solution is therefore gained efficiently. In order to search for a more excellent feasible solution, a non-linear programming (NLP) model for the ED will be given and solved based on the FMILP solution. The case study results show that the presented FMILP formulation is very effective in solving the ED problem that involves non-convex, non-smooth and non-continuous natures.

Keywords: Economic dispatch, non-convex, non-smooth, non-continuous, fully mixed-integer linear programming, non-linear programming

1. Introduction

Economic dispatch (ED) problem is a typical issue of secure and economic operation in power systems, which aims at reasonably arranging the power outputs of the units over the planning time periods, such that the total generation cost is minimized, while simultaneously fulfilling various system operation constraints. Normally, the classical ED problem is boiled down to solving a convex quadratic programming problem [1], so nowadays it can be solved efficiently in polynomial time via the state-of-the-art solvers like CPLEX [2], MOSEK [3], etc. However, when the more practical features such as valve-point effects (VPE), transmission loss and prohibited operating zones (POZ) are taken into account, the model becomes non-convex, non-smooth and non-continuous, which greatly increases the complexity of the problem. For instance, as a result of these features, multiple local minimums therefore emerge in the objective function, making it extremely hard to seek the global minimum. In addition, the intrinsic non-smooth nature will lead to the failure in applying the derivative-based mathematical programming method directly.

Recently, the heuristic algorithms which are known for their flexibility and versatility have gained increasing concern in solving the more practical ED problem, such as particle swarm optimization (PSO) [4–6], evolutionary programming (EP) [7], teaching learning based optimization (TLBO) [8], hybrid grey wolf optimizer (HGWO) [9], shuffled differential evolution (SDE) [10], ant lion optimizer (ALO) [11], one rank cuckoo search algorithm (ORCSA) [12], group search optimizer (GSO) [13], oppositional real coded chemical reaction optimization (ORCCRO) [14], modified symbiotic organisms search algorithm (MSOS) [15]. However, due to their stochastic natures, some limitations exist in many of the heuristic-based techniques. For example, their performances depend very much on the setting of the parameters and they need multiple independent trials to capture a satisfied result.

By contrast, the deterministic optimization methods are robust and the solutions obtained in each run are the same. So unlike stochastic search techniques, they only need to run once. Therefore, deterministic methods have attracted a great attention in recent years. In [16], Maclaurin sine

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series expansion is applied for the sinusoid function and then Lagrangian method is used to solve the ED problem. In [17], the cost function induced by valve-point effects is piecewise linearized and solved by mixed integer quadratic programming (MIQP). Based on an MIQP model, multi-step method, warm start technique and range restriction scheme are incorporated in [18] for the solution of dynamic economic dispatch (DED). In [19], the whole generation cost function is replaced by its linear approximations and then a hybrid approach that integrates mixed-integer linear programming with interior point method is put forward for DED. In [20], the ED problem is reformulated into a quadratically constrained quadratic programming (QCQP) form, thereby resulting an semidefinite programming (SDP) formulation. By using the convex iteration and branch-and-bound methods, it can be solved iteratively. For the case that mainly focuses on the ED with POZ, a compact formulation for the ED is presented in [21]; a semidefinite programming (SDP) scheme is put forward in [22]; an MIQP strategy that bases on a novel big-M method is proposed in [23] and an MIQP model based on unambiguous distance is presented in [24], etc.

When the VPE, transmission loss and POZ are all involved in the ED problem, seeking a good solution efficiently is a very challenging task due to its intrinsic non-convex, non-smooth and non-continuous features. A natural idea for solving such a complicated problem is to approximate each non-convex and non-smooth function by a piecewise linear one, and rebuild the model for the non-continuous regions at the same time. And then the tricky ED problem will be reformulated as a tractable mixed-integer linear programming (MILP) formulation, thereby taking full advantage of the extremely effective and mature MILP solvers. Although some analogous attempts have been made and achieved good results in the previous literatures, there are still room for further improvement. In [17, 19], the linear approximations are mainly aimed at the generation cost function; while in [18] only a small part of the quadratic terms in the loss functions are linearized and then a complicated adjustment will be needed in the solution procedure; in [23, 24], the reformulations primarily focus on the POZ components.

In this work, a fully mixed-integer linear programming (FMILP) formulation is proposed for the ED problem considering VPE, transmission loss and POZ. Since the original loss function is highly coupled on n-dimensional spaces, it is usually hard to piecewise linearize entirely. To handle this difficulty, a reformulation trick is utilized, transforming it into a group of tractable quadratic constraints. Different from most of the linearization techniques used for solving the ED, the univariate and bivariate functions are linearized simultaneously, and then the variables coupling relationships among these functions can be taken full advantage. And then an FMILP formulation that requires as few binary variables and constraints as possible is consequently constructed for the ED with VPE and transmission loss. When the POZ restrictions are also con-

sidered, a distance-based technique is adopted to rebuild these constraints, making them compatible with the previous FMILP reformulation. With the help of a logarithmic size formulation technique, a further reduction can be made for the introduced binary variables and constraints. By solving such an FMILP formulation, a near globally solution is therefore gained efficiently. In order to search for a more excellent feasible solution, a non-linear programming (NLP) model for the ED will be given and solved based on the FMILP solution. The case study results show that the presented FMILP formulation is very effective in solving the ED problem that involve non-convex, non-smooth and non-continuous natures.

The paper is organized as follows. The model of the ED problem is briefly described in Section 2. Section 3 presents a fully MILP formulation for the solution of the ED problem. Section 4 provides some results from case study. Finally, conclusions are given in Section 5.

2. The model of the ED problem

The ED problem considering VPE, transmission loss and POZ is discussed in this paper, which can be boiled down to the following non-convex, non-smooth and non-continuous mathematical programming problem:

$$\min \; \sum_{i=1}^{N} c_i(P_i), \tag{1}$$

where

$$c_i(P_i) = \alpha_i + \beta_i P_i + \gamma_i P_i^2 + e_i|\sin(f_i(P_i - \underline{P}_i))|, \tag{2}$$

subjected to,

- power balance equation

$$\sum_{i=1}^{N} P_i = P_d + P_l, \tag{3}$$

- transmission loss constraint

$$P_l = \sum_{i=1}^{N} \sum_{j=1}^{N} (P_i B_{i,j} P_j), \tag{4}$$

- power generation restrictions

$$\underline{P}_i \leq P_i \leq \overline{P}_i, \quad \forall \, i, \tag{5}$$

- prohibited operating zones restrictions

$$\begin{cases} \underline{P}_i = \underline{P}_{i,1} \leq P_i \leq \overline{P}_{i,1}, \; or \\ \underline{P}_{i,j} \leq P_i \leq \overline{P}_{i,j}, \; \forall j \in \{2, ..., n_i - 1\}, \; or \\ \underline{P}_{i,n_i} \leq P_i \leq \overline{P}_{i,n_i} = \overline{P}_i, \; \forall \, i, \end{cases} \tag{6}$$

• ramp rate restrictions

$$DR_i \leq P_i - P_i^0 \leq UR_i, \quad \forall\, i, \tag{7}$$

where P_i is the power generation of unit i; $\underline{P}_i$ and $\overline{P}_i$ are the corresponding lower and upper bounds of the power generation P_i; α_i, β_i, γ_i, e_i and f_i are the cost coefficients of unit i; N is the number of units; P_d is the load demand; P_l is the transmission loss; $B_{i,j}$ is the (i,j)-th element of the loss coefficient matrix B; $\underline{P}_{i,j}$ and $\overline{P}_{i,j}$ are the lower and upper bounds of the jth allowed operating region in unit i; n_i is the number of the allowed operating regions for unit i; P_i^0 is the previous power generation of unit i; DR_i and UR_i are the ramp rate limits of unit i.

3. A fully MILP formulation for the ED problem

As can be seen from section 2, the non-convex and non-smooth natures in the original ED problem are resulted from the objective function (1) and the transmission loss constraint (4), and the non-continuous natures are resulted from the POZ restrictions (6). To tackle the non-convex and non-smooth objective function, a class of popular strategy is to approximate each non-convex and non-smooth terms appeared in the objective function by a family of linear functions, thereby achieving a tractable MILP or MIQP form for them [17–19]. However, for the non-convex transmission loss constraint, it is very difficult to piecewise linearize it entirely since the right side of this equality constraint is a highly coupling non-linear function on n-dimensional spaces. Therefore, a reformulation for the loss constraint is provided next. When the non-continuous POZ restrictions are included, a distance-based technique will be adopted to rebuild these POZ constraints, making them compatible with the previous MILP reformulation by using as few variables as possible.

3.1. A reformulation for the loss constraint

Let $E_{i,j} = P_i P_j$, then the complicated non-linear constraints (4) in the original ED problem can be equivalent to the following simple formulation:

$$P_l = \sum_{i=1}^{N} \sum_{j=1}^{N} (B_{i,j} E_{i,j}), \tag{8}$$

$$E_{i,j} = P_i P_j, \; \forall i,j. \tag{9}$$

To get such a simple formulation, each quadratic term in the loss constraint (4) is replaced by a new variable. At first glance, N^2 new variables, along with the same number of simple quadratic constraints that have serious impacts on the computational efficiency are brought in. However, if we make a careful observation, it can find that

$E_{i,j} = E_{j,i}$. So let

$$E = \begin{bmatrix} E_{1,1} & E_{1,2} & \dots & E_{1,N_b} \\ E_{1,2} & E_{2,2} & \dots & E_{2,N_b} \\ \vdots & \vdots & \ddots & \vdots \\ E_{1,N_b} & E_{2,N_b} & \dots & E_{N_b,N_b} \end{bmatrix}, \tag{10}$$

the constraints (8)-(9) can be rewritten as:

$$P_l = \mathrm{tr}(BE), \tag{11}$$

$$E_{i,j(i \leq j)} = P_i P_j, \; \forall i,j, \tag{12}$$

where "$\mathrm{tr}(\bullet)$" denotes the trace of "$\bullet$". Due to the symmetry of the variable matrix E, only $(N^2 + N)/2$ simple quadratic constraints (see (12)) will be appended into the new formulation. And now, the complicated loss constraint (4) is reformulated into a linear constraint (11) and a group of simple quadratic constraints (12). Though the quadratic constraints (12) are still non-convex, compared with the original loss constraint (4), piecewise linearization can be directly applied for them.

3.2. A preliminary fully MILP formulation for the ED problem

After the reformulation, each function (2) appeared in the objective function (1) and each simple quadratic function (12) appeared in the loss constraint (4) that cause the problem non-convex and non-smooth can be approximated by a family of linear functions. To simplify the notations and facilitate the descriptions, the piecewise linear approximations of these functions are boiled down to linearize the following generalized system

$$f_c(x) = c(x), \; f_c(y) = c(y), \; f_c(z) = c(z), \tag{13}$$

$$f_l(x) = x^2, \; f_l(y) = y^2, \; f_l(z) = z^2, \tag{14}$$

$$f_l(x,y) = xy, \; f_l(x,z) = xz, \; f_l(y,z) = yz, \tag{15}$$

where

$$\underline{x} \leq x \leq \overline{x}, \; \underline{y} \leq y \leq \overline{y}, \; \underline{z} \leq z \leq \overline{z},$$

and the f_c and f_l correspond to the cost function and the loss function given in (2) and (12), respectively.

For the univariate functions or bivariate functions such as (13)-(15), various piecewise linearization techniques that emerge in [25–27] can be utilized for their linear approximations. In most of the previous studies, the applications of these techniques primarily focus on the approximation for some independent univariate functions (such as [17–19]) or bivariate functions (such as [28]). For a more specific case, such as the function system (13)-(15) appeared in the ED problem, the impacts of the coupling variables among different functions have not been extensively explored yet. In this paper, we take full advantage of these coupling relationships and propose a fully MILP formulation for the ED problem.

To get such a fully MILP form for the ED problem, convex combination model [29] is applied to piecewise linearize both univariate and bivariate functions. For univariate functions $f_c(x)$ and $f_l(x)$, $l_x + 1$ break points are

selected first, such that $\underline{x} = x_0 \leq x_1 \leq, ..., \leq x_{l_x} = \overline{x}$. By introducing the same number of continuous variables $\lambda_{x,i} \in [0,1]$ ($i \in \mathcal{S}_x \triangleq \{0, 1, ..., l_x\}$), the functions $f_c(x)$ and $f_l(x)$ in $[\underline{x}, \overline{x}]$ can be approximated by a convex combination form:

$$f_c(x) \approx \sum_{i=0}^{l_x}[\lambda_{x,i} f_c(x_i)], \qquad (16)$$

$$f_l(x) \approx \sum_{i=0}^{l_x}[\lambda_{x,i} f_l(x_i)], \qquad (17)$$

$$x = \sum_{i=0}^{l_x}(\lambda_{x,i} x_i), \qquad (18)$$

$$\sum_{i=0}^{l_x} \lambda_{x,i} = 1, \lambda_{x,i} \geq 0, \ \forall i \in \mathcal{S}_x, \qquad (19)$$

$$\sum_{i=1}^{l_x} u_{x,i} = 1, u_{x,i} \in \{0,1\}, \ \forall i \in \mathcal{S}'_x \triangleq \mathcal{S}_x \setminus \{0\}, \qquad (20)$$

$$\lambda_{x,i} \leq u_{x,i} + u_{x,i+1}, \ \forall i \in \mathcal{S}_x, \qquad (21)$$

where $u_{x,0} = u_{x,l_x+1} = 0$ in (21). Constraint (20) forces that just an $u_{x,i}$ can be equal to 1, and together with constraints (19) and (21) force that at most two adjacent $\lambda_{x,i}$ are non-zero. Similarly, the functions $f_c(y)$, $f_l(y)$, $f_c(z)$ and $f_l(z)$ can also be approximated by the above convex combination form if we replace the "x" given in (16)-(21) with "y" or "z". Due to lack of space, they do not list here. For the completeness of the subsequent description, we use "$(*)†$" to denote a constraint set of "x, y, z" corresponding to the constraint "$(*)$".

For the bivariate functions $f_l(x, y) = xy$, $f_l(x, z) = xz$ and $f_l(y, z) = yz$, which are defined on the rectangle $[\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}]$, $[\underline{x}, \overline{x}] \times [\underline{z}, \overline{z}]$ and $[\underline{y}, \overline{y}] \times [\underline{z}, \overline{z}]$, respectively, the "Union Jack" triangulation [30] (shown in Fig.1) is applied to their piecewise linearization. This approximation procedure will be accomplished in two stages. Firstly, one rectangle consisting of two triangles is selected. And then, only one of this two triangles will be picked to complete the approximation.
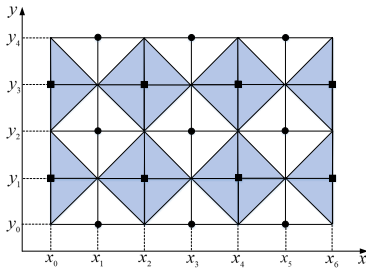

Figure 1: Geometric representation for "Union Jack" triangulation

Note that, the break points in $[\underline{x}, \overline{x}]$, $[\underline{y}, \overline{y}]$ and $[\underline{z}, \overline{z}]$ have been selected in previous approximations for the univariate functions. They should also be used for the partitions of the bivariate functions that have the same variables. So

$l_x \times l_y$, $l_x \times l_z$ and $l_y \times l_z$ rectangles can be produced in $[\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}]$, $[\underline{x}, \overline{x}] \times [\underline{z}, \overline{z}]$ and $[\underline{y}, \overline{y}] \times [\underline{z}, \overline{z}]$, respectively, and then the "Union Jack" triangulation can be realized easily.

Based on these partitions, in the first stage, some continuous variables $\lambda_{xy,i,j}, \lambda_{xz,i,k}, \lambda_{yz,j,k} \in [0,1]$ ($i \in \mathcal{S}_x, j \in \mathcal{S}_y, k \in \mathcal{S}_z$) are introduced, yielding the following convex combination forms for the functions $f_l(x, y)$, $f_l(x, z)$ and $f_l(y, z)$, respectively:

$$f_l(x, y) \approx \sum_{i=0}^{l_x} \sum_{j=0}^{l_y}[\lambda_{xy,i,j} f_l(x_i, y_j)], \qquad (22)$$

$$x = \sum_{i=0}^{l_x} \sum_{j=0}^{l_y}(\lambda_{xy,i,j} x_i), \qquad (23)$$

$$y = \sum_{i=0}^{l_x} \sum_{j=0}^{l_y}(\lambda_{xy,i,j} y_j), \qquad (24)$$

$$\sum_{i=0}^{l_x} \sum_{j=0}^{l_y} \lambda_{xy,i,j} = 1, \lambda_{xy,i,j} \geq 0, \forall i \in \mathcal{S}_x, j \in \mathcal{S}_y; \qquad (25)$$

$$f_l(x, z) \approx \sum_{i=0}^{l_x} \sum_{k=0}^{l_z}[\lambda_{xz,i,k} f_l(x_i, z_k)], \qquad (26)$$

$$x = \sum_{i=0}^{l_x} \sum_{k=0}^{l_z}(\lambda_{xz,i,k} x_i), \qquad (27)$$

$$z = \sum_{i=0}^{l_x} \sum_{k=0}^{l_z}(\lambda_{xz,i,k} z_k), \qquad (28)$$

$$\sum_{i=0}^{l_x} \sum_{k=0}^{l_z} \lambda_{xz,i,k} = 1, \lambda_{xz,i,k} \geq 0, \forall i \in \mathcal{S}_x, k \in \mathcal{S}_z; \qquad (29)$$

$$f_l(y, z) \approx \sum_{j=0}^{l_y} \sum_{k=0}^{l_z}[\lambda_{yz,j,k} f_l(y_j, z_k)], \qquad (30)$$

$$y = \sum_{j=0}^{l_y} \sum_{k=0}^{l_z}(\lambda_{yz,j,k} y_j), \qquad (31)$$

$$z = \sum_{j=0}^{l_y} \sum_{k=0}^{l_z}(\lambda_{yz,j,k} z_k), \qquad (32)$$

$$\sum_{j=0}^{l_y} \sum_{k=0}^{l_z} \lambda_{yz,j,k} = 1, \lambda_{yz,j,k} \geq 0, \forall j \in \mathcal{S}_y, k \in \mathcal{S}_z. \qquad (33)$$

Since that, in a system solution, one variable can only correspond to an unique value. So based on (18), (23) and

(27), it can be obtained that:

$$x = \sum_{i=0}^{l_x} (\lambda_{x,i} x_i)$$

$$= \sum_{i=0}^{l_x} \sum_{j=0}^{l_y} (\lambda_{xy,i,j} x_i) = \sum_{i=0}^{l_x} [(\sum_{j=0}^{l_y} \lambda_{xy,i,j}) x_i] \qquad (34)$$

$$= \sum_{i=0}^{l_x} \sum_{k=0}^{l_z} (\lambda_{xz,i,k} x_i) = \sum_{i=0}^{l_x} [(\sum_{k=0}^{l_z} \lambda_{xz,i,k}) x_i].$$

And in the mechanism of convex combination model, any given variable value will be expressed as a convex combination of at most two adjacent break points. So, combining with (34), it can be obtained that:

$$\begin{cases} \lambda_{x,i} = \sum_{j=0}^{l_y} \lambda_{xy,i,j}, \\ \lambda_{x,i} = \sum_{k=0}^{l_z} \lambda_{xz,i,k}, \forall i \in \mathcal{S}_x. \end{cases} \qquad (35)$$

Similarly, the following relationships hold.

$$\begin{cases} \lambda_{y,j} = \sum_{i=0}^{l_x} \lambda_{xy,i,j}, \\ \lambda_{y,j} = \sum_{k=0}^{l_z} \lambda_{yz,j,k}, \forall j \in \mathcal{S}_y. \end{cases} \qquad (36)$$

$$\begin{cases} \lambda_{z,k} = \sum_{i=0}^{l_x} \lambda_{xz,i,k}, \\ \lambda_{z,k} = \sum_{j=0}^{l_y} \lambda_{yz,j,k}, \forall k \in \mathcal{S}_z. \end{cases} \qquad (37)$$

Normally, for the independent bivariate functions, apart from some continuous variables, meanwhile a number of extra binary variables and constraints will be introduced in this stage. However, based on the relationships given in (35)-(37) and the below nonnegative restrictions:

$$\lambda_{xy,i,j}, \lambda_{xz,i,k}, \lambda_{yz,j,k} \geq 0, \forall i \in \mathcal{S}_x, j \in \mathcal{S}_y, k \in \mathcal{S}_z, (38)$$

for the bivariate functions (15), no further binary variables and constraints are required for this stage and the selection of the rectangle will be realized via the constraints (19)†-(21)†.

In stage two, the dichotomy strategy shown in Fig. 1 is adopted to select the white or blue triangles, which leads to a series constraints as follows:

$$\begin{cases} \sum_{(i,j)\in\mathcal{E}_{xy}} \lambda_{xy,i,j} \leq u_{xy}, \\ \sum_{(i,j)\in\mathcal{O}_{xy}} \lambda_{xy,i,j} \leq (1-u_{xy}), u_{xy} \in \{0,1\}, \end{cases} \qquad (39)$$

$$\begin{cases} \sum_{(i,k)\in\mathcal{E}_{xz}} \lambda_{xz,i,k} \leq u_{xz}, \\ \sum_{(i,k)\in\mathcal{O}_{xz}} \lambda_{xz,i,k} \leq (1-u_{xz}), u_{xz} \in \{0,1\}, \end{cases} \qquad (40)$$

$$\begin{cases} \sum_{(j,k)\in\mathcal{E}_{yz}} \lambda_{yz,j,k} \leq u_{yz}, \\ \sum_{(j,k)\in\mathcal{O}_{yz}} \lambda_{yz,j,k} \leq (1-u_{yz}), u_{yz} \in \{0,1\}, \end{cases} \qquad (41)$$

where

$$\begin{cases} \mathcal{E}_{xy} \triangleq \{(i,j) \in \mathcal{S}_x \times \mathcal{S}_y : i \text{ is even and } j \text{ is odd}\}, \\ \mathcal{O}_{xy} \triangleq \{(i,j) \in \mathcal{S}_x \times \mathcal{S}_y : i \text{ is odd and } j \text{ is even}\}; \end{cases}$$

$$\begin{cases} \mathcal{E}_{xz} \triangleq \{(i,k) \in \mathcal{S}_x \times \mathcal{S}_z : i \text{ is even and } k \text{ is odd}\}, \\ \mathcal{O}_{xz} \triangleq \{(i,k) \in \mathcal{S}_x \times \mathcal{S}_z : i \text{ is odd and } k \text{ is even}\}; \end{cases}$$

$$\begin{cases} \mathcal{E}_{yz} \triangleq \{(j,k) \in \mathcal{S}_y \times \mathcal{S}_z : j \text{ is even and } k \text{ is odd}\}, \\ \mathcal{O}_{yz} \triangleq \{(j,k) \in \mathcal{S}_y \times \mathcal{S}_z : j \text{ is odd and } k \text{ is even}\}. \end{cases}$$

Therefore, a fully MILP form, for the function system (13)-(15) (i.e., for the ED problem with VPE and transmission loss) can be formulated:

$$(16)\dagger - (21)\dagger, (22), (26), (30), (35) - (41). \qquad (42)$$

Since we take full advantage of the coupling relationships among the univariate functions and bivariate functions, the extra binary variables and constraints introduced for this linear approximation process can be reduced greatly.

When the non-continuous POZ restrictions are also included, to make the model compatible with the previous MILP reformulation by using as few variables as possible, a distance-based technique presented in [24] is adopted to transform the POZ constraints (6) into the following MILP form:

$$\underline{P}_{i,1} + \sum_{j=1}^{n_i-1} \tilde{u}_{i,j} \underline{d}_{i,j} \leq P_i \leq \overline{P}_{i,1} + \sum_{j=1}^{n_i-1} \tilde{u}_{i,j} \overline{d}_{i,j}, \qquad (43)$$

$$\sum_{j=1}^{n_i-1} \tilde{u}_{i,j} \leq 1, \ \tilde{u}_{i,j} \in \{0,1\}, \qquad (44)$$

where $\underline{d}_{i,j}$ and $\overline{d}_{i,j}$ are the distances computed by

$$\underline{d}_{i,j} = \underline{P}_{i,j+1} - \underline{P}_{i,1}, \ \ \overline{d}_{i,j} = \overline{P}_{i,j+1} - \overline{P}_{i,1}.$$

Consequently, a preliminary fully MILP formulation for the ED problem with VPE, transmission loss and POZ can be achieved by incorporating the forms (43) and (44) into formulation (42).

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3.3. An enhance fully MILP formulation for the ED problem

To further reduce the number of the additional binary variables and constraints, a modeling technique presented in [31] that requires just a logarithmic number of binary variables and constraints can be employed for the foregoing piece-wise linearization process.

Let $B : \mathcal{S}'_x \to \{0,1\}^{\log_2|\mathcal{S}'_x|}$ be a bijective function such that for all $i \in \mathcal{S}'_x \setminus \{l_x\}$ the vectors $B(i)$ and $B(i+1)$ is difference at most one component. Then the formulas

$$
\begin{cases}
\displaystyle\sum_{i\in\mathcal{S}_x^+(s,B)} \lambda_{x,i} \leq u_{x,s}, \\[2mm]
\displaystyle\sum_{i\in\mathcal{S}_x^0(s,B)} \lambda_{x,i} \leq (1 - u_{x,s}), \\[2mm]
u_{x,s} \in \{0,1\}, \forall s \in \{1,...,\log_2|\mathcal{S}'_x|\},
\end{cases}
\tag{45}
$$

$$
\sum_{i=0}^{l_x} \lambda_{x,i} = 1, \lambda_{x,i} \geq 0, \ \forall i \in \mathcal{S}_x,
\tag{46}
$$

force that, at most two adjacent $\lambda_{x,i}$ are non-zero. Here,

$$
\begin{aligned}
\mathcal{S}_x^+(s,B) \triangleq &\{i : B(i)_s = B(i+1)_s = 1, l \in \mathcal{S}'_x \setminus \{l_x\}\} \bigcup \\
&\{i : B(i+1)_s = 1, i \in \{0\}\} \bigcup \\
&\{i : B(i)_s = 1, i \in \{l_x\}\},
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{S}_x^0(s,B) \triangleq &\{i : B(i)_s = B(i+1)_s = 0, i \in \mathcal{S}'_x \setminus \{l_x\}\} \bigcup \\
&\{i : B(i+1)_s = 0, i \in \{0\}\} \bigcup \\
&\{i : B(i)_s = 0, i \in \{l_x\}\},
\end{aligned}
$$

Note that, above $|\mathcal{S}'_x|$, i.e., l_x, is supposed to be a power of two. Otherwise, we can complete the set $\mathcal{S}'_x$ to a size $2^{\lceil \log_2|\mathcal{S}'_x|\rceil}$ and then the formulas (45)-(46) can be achieved after setting the extra $\lambda_{x,i}$ to zero. Therefore, the formulas (20)-(21) that involve l_x binary variables and $l_x + 2$ constraints can be enhanced to a more compact form (45) that involve just $\log_2 l_x$ binary variables and $2\log_2 l_x$ constraints. When the "x" given in (45)-(46) is replaced with "y" or "z", the similar results can also be obtained.

Consequently, an enhance fully MILP formulation, denoted as FMILP, for the ED problem with VPE, transmission loss and POZ can be constructed when the objective function, transmission loss and POZ constraints of the original ED model given in Section 2 are replaced with the following MILP formulation:

$$
\begin{cases}
(16)\dagger - (19)\dagger, (22), (26), (30), \\
(35) - (41), (43) - (44), (45)\dagger.
\end{cases}
\tag{47}
$$

3.4. Solution for the ED problem

When the non-convex and non-smooth functions appeared in the ED problem all are replaced with their linear approximations, and the non-continuous POZ constraints are rebuilt to a compatible MILP form, an FMILP formulation for the ED problem is obtained based on (47). Optimizing such an FMILP formulation, a global optimum within a preset tolerance can be achieved immediately. Since the linearization technique is applied, after the optimization, the solution gained by solving the FMILP may not be able to fully satisfy the power balance. So, a verification step is implemented. Using the FMILP solution to recalculate the transmission loss, the unbalance can thus be evaluated by

$$
\triangle P = |\sum_{i=1}^{N} P_i - P_d - P_l|.
\tag{48}
$$

Generally, the unbalance $\triangle P$ will inevitably produce due to the fact that all the solutions used in this recalculation are rounded off. Therefore, it is hard to get a perfect balance in practice. So if $\triangle P$ is less than a given unbalance tolerance, the solution gained by the FMILP is considered to be a near globally solution of the original ED problem. Otherwise, a further search starting from this solution will be carried out by solving an equivalent ED model, getting a more excellent feasible solution.

Since the original ED problem is non-smooth and non-continuous in nature and the classical derivative-based methods become unavailable. With the help of the reformulation technique presented in [19] for the non-smooth objective function and the compact formulation presented in [21] with regard to the non-continuous POZ constraints, the original non-smooth and non-continuous ED problem can be converted into the following NLP model:

$$
\begin{aligned}
\min \ & \sum_{i=1}^{N} (\alpha_i + \beta_i P_i + \gamma_i P_i^2 + e_i s_i) \\
s.t. \ & s_i - v_i - w_i = 0, \\
& \sin(f_i(P_i - \underline{P}_i)) + v_i - w_i = 0, \\
& v_i \geq 0, \quad w_i \geq 0, \ \forall \ i, \\
& \prod_{j=1}^{n_i} (P_i - \underline{P}_{i,j})(P_i - \overline{P}_{i,j}) \leq 0, \ \forall \ i, \\
& (3) - (5), (7).
\end{aligned}
\tag{49}
$$

As is well known, IPOPT [32] is one of the most powerfull NLP solvers that is designed to find (local) solutions. It can start the search in a special region on account of its warm-starting capability. So, starting the search from the FMILP solution by solving the NLP model (49) with IPOPT, a more excellent feasible solution for the original ED one can hope for.

4. Case study

The performance of the FMILP formulation in solving the ED problem considering VPE, transmission loss and POZ is evaluated on five widely tested systems in different scales:

Case I: a 6-unit system taken from [33].
Case II: a 13-unit system taken from [34].
Case III: a 20-unit system taken from [35].
Case IV: a 40-unit system taken from [9].
Case V: a 140-unit system taken from [36].

In this case study, firstly, the FMILP formulation will be solved to an appropriate tolerance by CPLEX 12.6.1 [2] and then a global solution for this formulation can be achieved. Based on such a solution, a verification step will be executed. If the unbalance is less than a given tolerance 0.002, this solution is considered to be a near globally solution of the original ED problem. Starting from such a solution, the equivalent NLP model will be optimized by IPOPT 3.12.6, to achieve a more excellent feasible solution. If the unbalance is more than a given tolerance 0.002, based on the FMILP solution, the NLP model will be optimized to correct the linearization errors. For the convenience of the following discussion, this solution methodology briefly describes as FMILP-based approach. To illustrate the superiority of the FMILP formulation in exploring optimal solution, the solving results will be compared with those obtained by using some of the up-to-date methods. All tests are carried out on a notebook containing an Intel Core i7-6500U CPU with 2.5 GHz and 8 GB RAM. And the codes are accomplished based on the modeling tool YALMIP [37] in MATLAB R2014a.

4.1. Case I: a 6-unit system

In this case, the system involves 6 units. In the optimization process for the FMILP formulation, the piecewise parameter, i.e., the segments on each sin function cycle, is set to 6 and the relative MIP gap tolerance for CPLEX is set to 0.01%. After solving the FMILP formulation, an optimal solution for this formulation is attained and this solution is also considered as a solution of the original ED problem on account of its trivial unbalance. Starting the search from such a solution by solving the NLP formulation with IPOPT at its default settings, a more excellent feasible solution that consumes lower generation cost is gained. Table 1 presents a detailed summary of total generation costs obtained by FMILP-Based approach and some other methods for the 6-unit system.

Table 1: Summary results for the 6-unit system

Method	Generation cost ($)			CPU time (s)
	Minimum	Average	Maximum	
GA [33]	996.0369	-	1117.1285	0.5780
H-GA [33]	984.9365	-	992.4815	0.0150
PSO [38]	925.7581	926.3880	928.4270	0.3529
MSG-HS [38]	925.6406	926.8510	928.5990	0.6215
BSA [35]	925.4139	925.5542	926.2994	0.655
FPSOGSA [34]	925.4137	925.4175	925.4213	1.4108
FMILP-based	925.4137	-	-	0.2500

As we can see from Table 1, the proposed FMILP-based method can get the lowest generation cost in reasonable time. Although in this case, FPSOGSA [34] can seek to a solution with the same generation cost in comparison

with FMILP-based, but the FMILP-based is more stable because the result in every run is changeless and thus the optimization procedure only needs to execute once.

In Table 2, the power generations for the 6-unit system with $P_d = 283.4$ MW are provided. Using these power generations that have been rounded off, the transmission loss can be recalculated by (4), and then the system loss $P_l = 11.106443$ MW. Based on this results, one can verify that, the solution gained by using FMILP-Based method is strictly feasible, i.e., the total generation P_{Total} can fulfil the load demand P_d and transmission loss P_l entirely.

Table 2: Scheduling solution (MW) for the 6-unit system

P_1	P_2	P_3	P_4	P_5	P_6	P_{Total}
199.599650	20.000000	23.981024	18.872991	18.207326	13.845452	**294.506443**
P_l: **11.106443**	P_d: **283.4**	$\triangle P$: **0**				

4.2. Case II: a 13-unit system

In this case study, a larger system that involves 13 units is considered. The piecewise parameter in this case is set to 4. Table 3 shows a detailed summary of the generation costs obtained by using FMILP-Based approach and some of the other methods for the 13-unit system in load demand level $P_d = 2520$ MW.

As we can see from Table 3, apart from MSOS [15], FMILP-Based approach can solve to a lower generation cost than other reported methods. Although the optimum is the same in MSOS [15] and FMILP-Based approach, by comparison, FMILP-Based approach performances better than MSOS on account of the fact that it consumes less CPU time.

Table 3: Summary results for the 13-unit system

Method	Generation cost ($)			CPU time (s)
	Minimum	Average	Maximum	
ST-HDE [39]	24560.0800	24706.6300	24872.4400	2.9783
ICA-PSO [40]	24540.0600	24561.4600	24589.4500	0.0520
FPSOGSA [34]	24515.3554	24516.6823	-	-
MSOS [15]	24515.2258	24515.2258	24515.2258	12.8000
FMILP-based	24515.2258	-	-	4.2450

The power generations for the 13-unit system are given in Table 4 for verification. Using these data shown in Table 4, the system loss can be recomputed, getting that $P_l = 40.811358$ MW. Based on this results, a slight unbalance 0.000002 that is less than the given tolerance is taken place, showing that, the solution gained by FMILP-Based approach can meet the power balance equation commendably.

Table 4: Scheduling solution (MW) for the 13-unit system

P_1	P_2	P_3	P_4	P_5	P_6	P_7
628.318530	299.199300	299.199300	159.733100	159.733100	159.733100	159.733100
P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{Total}
159.733100	159.733100	77.399912	113.495890	92.399912	92.399912	**2560.811356**
P_l: **40.811358**	P_d: **2520**	$\triangle P$: **0.000002**				

7

4.3. Case III: a 20-unit system

In this case study, a more larger system that involves 20 units is tested. The piecewise parameter in this case is set to 6. Table 5 lists a concise summary of the generation costs gained by using FMILP-Based approach and other methods for the 20-unit system with load demand $P_d = 2500$ MW.

Table 5: Summary results for the 20-unit system

| Method | Generation cost ($) | | | CPU time (s) |
	Minimum	Average	Maximum	
EHNN [41]	62610.0000	-	-	0.1100
NR [41]	62489.5000	-	-	0.4000
BBO [42]	62456.7926	62456.7928	62456.7935	0.0293
BSA [35]	62456.6925	62457.1517	62458.1272	14.4770
LI [43]	62456.6391	-	-	0.0338
GSO [13]	62456.6364	-	-	30.45
CQGSO [13]	62456.6349	-	-	11.13
HM [43]	62456.6341	-	-	0.0064
FMILP-based	62456.6330	-	-	3.1540

From Table 5, it can observe that the proposed FMILP-based approach can solve to a solution with the lowest generation cost in a reasonable time consumption. The power generations for this system are also provided in Table 6 for the feasibility checking. Based on this solution, one can verify that the solution achieved by the FMILP-based approach for this case fulfil the constraints commendably.

Table 6: Scheduling solution (MW) for the 20-unit system

P_1	P_2	P_3	P_4	P_5	P_6	P_7
512.781759	169.101513	126.890725	102.867240	113.682901	73.572038	115.290031
P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}
116.399860	100.404903	106.027368	150.238527	292.765834	119.114220	30.831524
P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{Total}
115.805940	36.254365	66.859196	87.971229	100.802660	54.304831	**2591.966664**
P_l: **91.966666**	P_d: **2500**	$\triangle P$: **0.000002**				

4.4. Case IV: a 40-unit system

In this case study, a more larger system that involves 40 units is adopted. In this system, VPE exists in all of the units and 40×40 quadratic terms are embed in the complicated non-convex and non-linear transmission loss constraint, making the problem hard to tackle. After the reformulation, only 41×20 separated quadratic terms are needed to handle. The piecewise parameter in this case is set to 4 and the summary results of the generation costs achieved by using FMILP-based approach and other methods are given in Table 7.

Table 7: Summary results for the 40-unit system

| Method | Generation cost ($) | | | CPU time (s) |
	Minimum	Average	Maximum	
GA-API [44]	139864.9600	-	-	-
SDE [10]	138157.4600	-	-	-
BBO [14]	137026.8200	137116.5800	137587.8200	0.2000
DE/BBO [14]	136950.7700	136966.7700	137150.7700	0.16
ORCCRO [14]	136855.1900	136855.19	136855.1900	0.07
HGWO [9]	136681.0000	-	-	1.4610
KHA [45]	136670.3701	136671.2293	136671.8648	-
MSOS [15]	136442.4827	136445.0400	136449.3511	102.2400
FMILP-based	136440.6847	-	-	75.97

Obviously, FMILP-based approach can explore a lower total generation cost than all the other methods reported in Table 7. And the detailed power generations for the 40-unit system are offered in Table 8. It can be seen from Table 8, when the provided data is used in the verification, a minute unbalance is emerged. But overall, this unbalance is less than the given tolerance, which means that the optimal results obtained by FMILP-based approach can also fulfil the restriction conditions.

Table 8: Scheduling solution (MW) for the 40-unit system

P_1	P_2	P_3	P_4	P_5	P_6	P_7
114.000000	114.000000	120.000000	179.733100	87.799905	140.000000	300.000000
P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}
300.000000	289.963920	279.599650	243.599650	94.000000	484.039160	484.039160
P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}
484.039160	484.039160	489.279370	489.279370	511.279370	511.279370	523.279370
P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{27}	P_{28}
523.279370	523.279370	523.279370	523.279370	523.279370	10.000000	10.000000
P_{29}	P_{30}	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}
10.000000	87.799905	190.000000	190.000000	190.000000	200.000000	200.000000
P_{36}	P_{37}	P_{38}	P_{39}	P_{40}	-	P_{Total}
164.799825	110.000000	110.000000	110.000000	550.000000	-	**11472.246295**
P_l: **972.244629**	P_d: **10500**	$\triangle P$: **0.001666**				

4.5. Case V: a 140-unit system

In this case study, a much larger system that involves 140 units is tested. This is a lossless system and the VPE exists in 12 of the units and POZ exists in 4 of the units. In this case, the piecewise parameter is set to 10. The summary results of the generation costs achieved by using FMILP-based approach and other methods are given in Table 9.

In the FMILP-based approach, the linear approximate model of the original problem, i.e., the FMILP formulation, is solved, thereby a near globally solution can be achieved. As can be seen from Table 9, though the generation cost gained by the presented FMILP-based approach is a little more than that of the CCPSO, CTPSO and C-QGSO, FMILP-based method can make a significant reduction in computational effort.

Table 9: Summary results for the 140-unit system

| Method | Generation cost ($) | | | CPU time (s) |
	Minimum	Average	Maximum	
GSO [13]	1728151.1680	1745514.9975	1753229.5636	53.80
CCPSO [36]	1657962.7300	1657962.7300	1657962.7300	150.00
CTPSO [36]	1657962.7300	1657964.0600	1658002.7900	100.00
CQGSO [13]	1657962.7270	1657962.7410	1657962.7760	31.67
FMILP-based	1657964.7087	-	-	0.08

The power generations for this system are also provided in Table 10 for the feasibility checking. Based on this solution, one can see that the unbalance is only 0.000001, indicating that solution achieved by the FMILP-based approach for this case can fulfil the constraints perfectly.

5. Conclusions

ED problem considering VPE, transmission loss and POZ is a complicated issue due to its intrinsic non-convex, non-smooth and non-continuous natures. In order to

Table 10: Scheduling solution (MW) for the 140-unit system

P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}
119.000000	164.000000	190.000000	190.000000	168.539816	190.000000	490.000000	490.000000	496.000000	496.000000	496.000000	496.000000	506.000000	509.000000
P_{15}	P_{16}	P_{17}	P_{18}	P_{19}	P_{20}	P_{21}	P_{22}	P_{23}	P_{24}	P_{25}	P_{26}	P_{27}	P_{28}
506.000000	505.000000	506.000000	506.000000	505.000000	505.000000	505.000000	505.000000	505.000000	505.000000	537.000000	537.000000	549.000000	549.000000
P_{29}	P_{30}	P_{31}	P_{32}	P_{33}	P_{34}	P_{35}	P_{36}	P_{37}	P_{38}	P_{39}	P_{40}	P_{41}	P_{42}
501.000000	499.000000	506.000000	506.000000	506.000000	506.000000	500.000000	500.000000	241.000000	241.000000	774.000000	769.000000	3.000000	3.000000
P_{43}	P_{44}	P_{45}	P_{46}	P_{47}	P_{48}	P_{49}	P_{50}	P_{51}	P_{52}	P_{53}	P_{54}	P_{55}	P_{56}
250.000000	250.000000	250.000000	250.000000	250.000000	250.000000	250.000000	250.000000	165.000000	165.000000	165.000000	165.000000	180.000000	180.000000
P_{57}	P_{58}	P_{59}	P_{60}	P_{61}	P_{62}	P_{63}	P_{64}	P_{65}	P_{66}	P_{67}	P_{68}	P_{69}	P_{70}
103.000000	198.000000	312.000000	312.000000	163.000000	95.000000	511.000000	511.000000	490.000000	254.800000	490.000000	490.000000	130.000000	339.439510
P_{71}	P_{72}	P_{73}	P_{74}	P_{75}	P_{76}	P_{77}	P_{78}	P_{79}	P_{80}	P_{81}	P_{82}	P_{83}	P_{84}
137.000000	388.327412	195.000000	211.100000	207.093261	247.600000	394.000000	330.000000	531.000000	531.000000	542.000000	56.000000	115.000000	115.000000
P_{85}	P_{86}	P_{87}	P_{88}	P_{89}	P_{90}	P_{91}	P_{92}	P_{93}	P_{94}	P_{95}	P_{96}	P_{97}	P_{98}
115.000000	207.000000	207.000000	175.000000	175.000000	175.000000	175.000000	575.400000	547.500000	836.800000	837.500000	682.000000	720.000000	718.000000
P_{99}	P_{100}	P_{101}	P_{102}	P_{103}	P_{104}	P_{105}	P_{106}	P_{107}	P_{108}	P_{109}	P_{110}	P_{111}	P_{112}
720.000000	964.000000	958.000000	947.900000	934.000000	935.000000	876.500000	880.900000	873.700000	877.400000	871.700000	864.800000	882.000000	94.000000
P_{113}	P_{114}	P_{115}	P_{116}	P_{117}	P_{118}	P_{119}	P_{120}	P_{121}	P_{122}	P_{123}	P_{124}	P_{125}	P_{126}
94.000000	94.000000	244.000000	244.000000	244.000000	95.000000	95.000000	116.000000	175.000000	2.000000	4.000000	15.000000	9.000000	12.000000
P_{127}	P_{128}	P_{129}	P_{130}	P_{131}	P_{132}	P_{133}	P_{134}	P_{135}	P_{136}	P_{137}	P_{138}	P_{139}	P_{140}
10.000000	112.000000	4.000000	5.000000	5.000000	50.000000	5.000000	42.000000	42.000000	41.000000	17.000000	7.000000	7.000000	26.000000

P_{total}: **49341.999999** P_d: **49342** $\triangle P$: **0.000001**

search a near globally solution, a fully MILP formulation is presented for the ED problem. That is, the non-convex and non-smooth objective function and the non-convex transmission loss constraint both are approximated to the corresponding MILP forms, and the non-continuous POZ constraints are rebuilt to a compatible MILP form, thereby achieving a fully MILP formulation that can be solved to a near globally optimum effectively. To get a more excellent feasible solution, an equivalent model for the ED is given. And then, it will be solved from the FMILP solution, to capture a more better solution. The introduced modeling techniques and the solution idea can be easily extended to a variety of modifications for the non-convex, non-smooth and non-continuous optimal economic dispatch in power systems.

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