

# Dynamic optimization for airline maintenance operations

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## Abstract

The occurrence of unexpected aircraft maintenance tasks can produce expensive changes in an airline's operation. When it comes to critical tasks, it might even cancel programmed flights. Despite of it, the challenge of scheduling aircraft maintenance operations under uncertainty has received limited attention in the scientific literature. We study a dynamic airline maintenance scheduling problem, which daily decides the set of aircraft to maintain and the set of pending tasks to execute in each aircraft. The objective is to minimize the expected costs of expired maintenance tasks over the operating horizon. To increase flexibility and reduce costs, we integrate maintenance scheduling with tail assignment decisions. We formulate our problem as a Markov Decision Process and design dynamic policies based on Approximate Dynamic Programming. In a case study based on a Latin American airline, we show the value of dynamic optimization by testing our best policies against a simple airline decision rule and a deterministic relaxation with perfect information. We suggest to schedule tasks requiring less resources first to increase utilization of residual maintenance capacity. Finally, we observe strong economies of scale when sharing maintenance resources between multiple airlines.

**Keywords:** Airline maintenance, Approximate Dynamic Programming, Task Scheduling, Tail Assignment

## 1 Introduction

Over the past decades, the air transport industry sector has pushed the global economy upwards growing approximately 7% annually (IATA 2018c) and allowing a fast movement of passengers and freight worldwide. Such a growth has driven strict safety standards in aircraft operations, which have positioned commercial air transport as the world's safest transportation mode (IATA 2018b). Within an airline, the maintenance area is responsible for planing and executing all preventive actions required to meet such safety standards including maintenance tasks for each aircraft, demanding skilled jobs, *e.g.*, aircraft mechanics, avionic systems experts, electricians, cabin experts.

From the economic perspective, airlines operate with very low margins, *e.g.*, 1.8% annual average in the last decade (IATA 2018a), and search for cost-efficient maintenance. According to the Bureau of Transportation Statistics (2017), maintenance operations roughly represent a 9% of an airline’s operating costs and more than \$12.5 billion dollars annually only taking into account the 10 largest US airlines. Within maintenance costs, wages of skilled workforce represent approximately 50% of the expenses (IATA’s Maintenance Cost Task Force 2013), and this percentage is expected to grow. As it is depicted in Figure 1, the expected supply of aviation mechanics in the US will not meet the industry requirements by 2027. An efficient maintenance scheduling operation is increasingly important as labour availability becomes scarce.

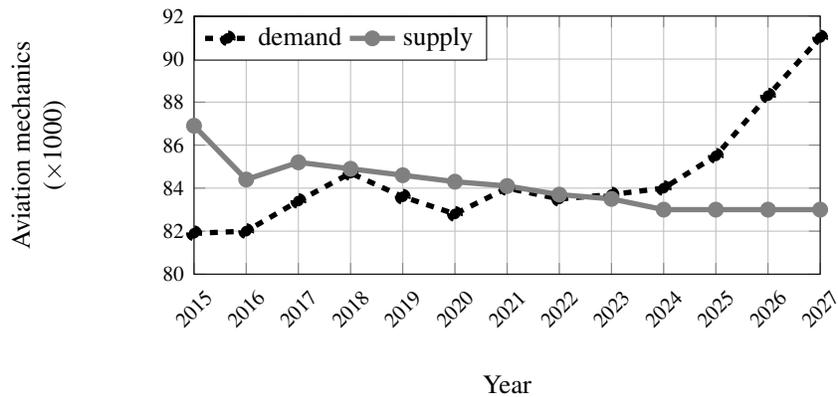


Figure 1: Expected yearly demand and supply of commercial aviation mechanics in the US air transport industry (Prentice et al. 2017)

Besides costs, maintenance activities can negatively impact customer service level delaying itineraries or even cancelling flights. Given each aircraft capital cost, airlines continuously search to increase aircraft utilization (IATA 2017) reducing ground time windows and increasing the probability that a slight maintenance delay could severely propagate into the future. A study by Cook et al. (2004) estimates that, for a large airline, the annual costs of flight delays directly caused by maintenance reached over \$20 million dollars. Thus, it becomes even more important to plan cost-efficient maintenance operations, while minimizing expected operational delays.

### 1.1 Airline Maintenance Scheduling Problem

We introduce an Airline Maintenance Scheduling Problem (AMSP), which daily decides the set of aircraft to maintain and set of tasks to execute in each aircraft. The objective is to minimize the expected costs of expired maintenance tasks over the operating horizon. We weight tasks differently in the objective to

balance the importance of each task. While some task delays are slightly undesirable, expired critical tasks might cancel programmed flights and leave an aircraft inoperative.

The AMSP belongs to the family of task scheduling problems (Pinedo), but differs from generic model formulations due to multiple specifics. First, the AMSP is a dynamic and stochastic problem, meaning that only a fraction of all tasks involved in the problem are known to the decision maker before the execution starts; remaining tasks are disclosed dynamically over time. This problem has incomplete information to make optimal decisions and, typically, the decision maker can only rely in probabilistic information regarding future events and potential task requirements. Second, the problem is not separable by aircraft. Each day, maintenance can only be executed at airports equipped for such operations, each having a limited number of hangars to perform maintenance. Third, the execution of each task consumes resources, *e.g.*, person-hours of multiple types of skilled and scarce human resources. Finally, each aircraft is assigned to a particular flight itinerary having different ground time windows available for maintenance operations.

## **1.2 State of the practice in a Latin American airline**

Nowadays, Latin American airlines solve the AMSP manually only helped by custom-made spreadsheets and consuming valuable hours of trained staff in their Department of Operations Planning. As discussed with the maintenance planning manager (H. Salazar, personal communication, April 21, 2017) from the Latin American airline LATAM, we observed that aircraft maintenance operation is planned a day ahead scheduling tasks according to a modified earliest due date (EDD) rule (Pinedo) until they consume all available hangar capacity and human resources; this process does not necessarily builds a cost-efficient operation. We also observed that the task scheduling rule used by LATAM aggregates daily resources available over time, does not consider each aircraft ground time window and could lead to infeasible task assignments. When this situation occurs for an aircraft's critical task, its itinerary must be cancelled and should stay on ground to receive emergency maintenance. To recover operations, the airline has to use backup aircraft reducing fleet utilization. Therefore, a cancelled itinerary costs a new itinerary and the additional loss of aircraft utilization.

## **1.3 An example of a small instance**

To illustrate the problem, let us consider the instance depicted in Figure 2 having three aircraft with five, three and one pending maintenance tasks each, and assume each task consumes an hour of a single technician

available from midnight to 9:00am. Any scheduling rule would, indeed, schedule all nine tasks in that night, but could produce an infeasible assignment when each aircraft has a specific ground time window. For instance, assume the itinerary of aircraft *A* has a ground time window between 3:00 and 9:00, *B*'s between 2:00 and 6:00, and *C*'s from midnight to 7:00 as shown in the upper part of Figure 3. Unavoidably, one task from *A* or *B* should be left unattended. A manual assignment would have scheduled an infeasible plan and would not have taken into account that one hour of a future night shift is required to complete maintenance.

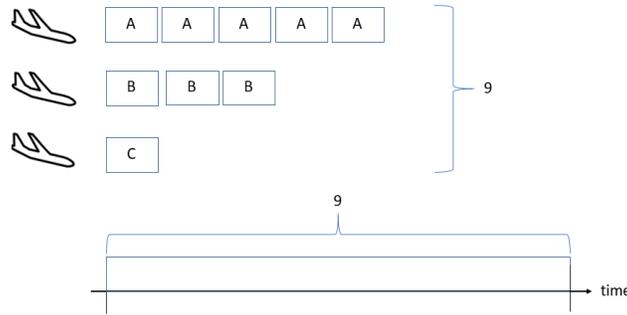


Figure 2: Example of a task assignment problem

The previous example is deterministic and works with one human resource type and one task type; it becomes even harder when tasks consume multiple resources, different task types are present, and several days are considered having incomplete information of future task realizations.

#### 1.4 Integrated Aircraft Maintenance Scheduling and Tail Assignment Problem

To improve efficiency, an additional option is to dynamically reassign the itinerary of each aircraft within the AMSP. Assigning itineraries to aircraft is a problem known as Tail Assignment (TA) and has been extensively studied (Grönkvist 2005, Maher et al. 2018). To illustrate the effect of integrating tail assignment decisions into the AMSP, let us re-consider the example of Figure 2. A solution scheduling all nine tasks in one night can be obtained by reassigning aircraft itineraries and, thus, all ground time windows. As shown in Figure 3, this is possible if aircraft *A* is reassigned to aircraft's *C* itinerary, aircraft *B* to *A*'s and aircraft *C* to *B*'s. We call the extended version of the AMSP as the The Aircraft Maintenance Scheduling Problem with Tail Assignment (AMSP-TA) and becomes even harder to solve manually.

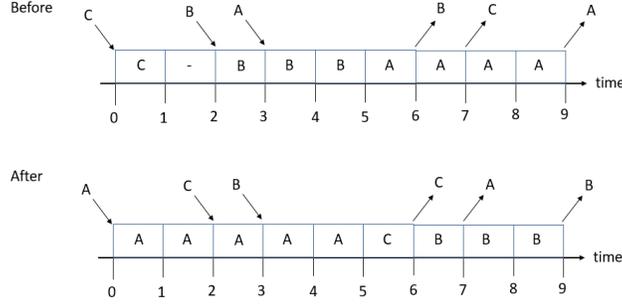


Figure 3: Small instance of the AMSP with original and reassigning aircraft's itineraries

## 1.5 Contribution

In this article, we design a dynamic decision support tool to plan a cost-efficient airline maintenance scheduling operation. In particular, we consider the following to be our main contributions:

- We begin studying a novel dynamic airline maintenance scheduling problem under uncertainty, which is formulated as a Markov Decision Process model.
- We design cost-efficient and computationally tractable dynamic policies based on Approximate Dynamic Programming techniques Powell (2011); such policies dynamically react to disclosed information and proactively anticipate future decision stages.
- We test our policies in a family of computationally simulated instances based on the information provided by a major Latin American Airline. Our best policy is compared against a lower bound optimizing a deterministic problem for each possible scenario realization (Brown et al. 2010) and to an airline's decision rule. Compared to the latter, we manage to avoid flight cancellations and significantly reduce expired tasks.
- Finally, we provide managerial insights derived from our solution's behaviour. As an example, we schedule first pending tasks consuming less resources to increase utilization of residual maintenance capacity.

## 1.6 Outline

The remainder of this article is organized as follows. In Section 2, we present the related literature review. Later, the AMSP-TA is formally presented in Section 3. Section 4 presents a deterministic version of our problem, while Section 5 presents our proposed dynamic policies and benchmarks. We describe our case

study and present computational results in Section 6. Finally, we conclude our work and suggest future research in Section 7.

## 2 Literature Review

Designing an airline schedule involves short, medium and long term plans. The last two involve strategic and tactical decisions and have been extensively addressed in the Operations Research literature. Building a complete airline schedule is difficult and, typically, the process is divided into four sequential steps: flight scheduling (Etschmaier and Mathaisel 1985, Gopalan and Talluri 1998, Eltoukhy et al. 2017), fleet assignment (Sherali et al. 2006, Eltoukhy et al. 2017), aircraft maintenance routing problem (AMRP) (Liang et al. 2015, Haouari et al. 2013) and crew scheduling and pairing problems (Deveci and Demirel 2018). Please refer to Barnhart et al. (2003) and Barnhart and Smith (2012) for a comprehensive literature review of this process.

Our work relates to the AMRP, which designs all sequences of flight legs, *i.e.*, routes, to operate in each aircraft. Also, each route should proactively plan time on ground for periodic maintenance checks on each aircraft. To simplify the problem, the AMRP literature has aggregated maintenance decisions limiting the number of hours an aircraft can operate between maintenance checks (*e.g.*, A-Check).

The AMRP is a tactical problem and is solved months before the operation starts and, therefore, it can only account for two maintenance checks (A-Check and B-Check); it does not consider each individual task required to be carried out in each aircraft and available human resources at the base. As a result, planned maintenance checks are frequently modified by operation planners dealing with daily detailed requirements (Sarac et al. 2006). In the scientific literature, the short-term maintenance plan involving each individual task is referred as the Aircraft Line Maintenance Problem (Van den Bergh et al. 2013) and directly interacts with aircraft routing at the operational level. Figure 4 presents an overview of the airline planning process and the interaction at the operational level between aircraft routing and line maintenance.

When line maintenance planning is not planned at the operational level, the occurrence of unexpected maintenance tasks may trigger expensive changes in an airline's operation (Biró et al. 1992, Sriram and Haghani 2003). When it comes to critical tasks, it might even cancel programmed flights. To build cost-efficient schedules, it becomes mandatory to plan daily interactions between maintenance planning and aircraft routing at the individual task level.

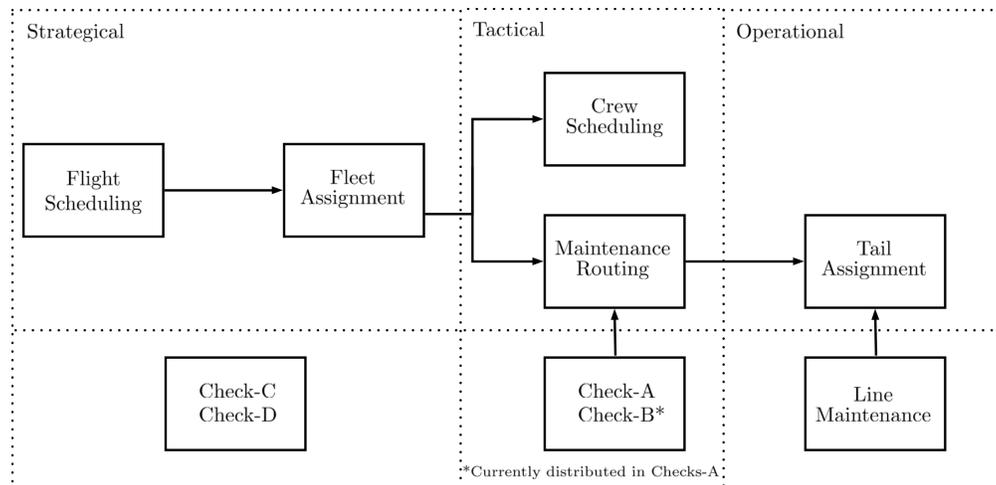


Figure 4: An example of an airline planning process. Adapted from Lapp and Cohn (2012) and Liang et al. (2015)

## 2.1 *Operational aircraft maintenance routing problem (OAMRP) and Tail assignment problem (TAP)*

Sriram and Haghani (2003) begin studying an operational AMRP and model re-assignments of aircraft to flight segments. The authors introduce the concept of line of flights (LOF), which consist in sequences of daily flights operated by each aircraft. It is assumed that an aircraft goes through maintenance when it stays overnight at a maintenance base.

Sarac et al. (2006) introduce the Operational Aircraft Maintenance Routing Problem (OAMRP) that assigns daily routes to each aircraft in a given fleet taking into account that a subset of aircraft must finish daily operations at a maintenance base. The objective of the problem is to maximize flight hours subject to hangar capacity and limited resources at the maintenance base; resources are consider in an aggregate level.

Haouari et al. (2013) and Al-Thani et al. (2016) study an OAMRP where the planned schedule is periodically repeated with a daily frequency. The model has limited aircraft capacity at the hangar and considers a fixed time to execute each maintenance (A-check). Al-Thani et al. (2016) expand the OAMRP and consider a weekly plan.

Murat Afsar et al. (2007) and Murat Afsar et al. (2009) extend the OAMRP and consider a dynamic operation under a rolling horizon framework; they restrict an aircraft's maintenance once a week and consider limited hangar capacity. Basdere and Bilge (2014) present a similar model, but also model fixed execution times required by maintenance.

The Tail Assignment (TAP) problem is similar to the OAMRP, but considers each aircraft at the individual level (Grönkvist 2005); it is a well known  $\mathcal{NP}$ -hard problem (Grönkvist 2006, Otten et al. 2006, Gabteni and Grönkvist 2009). Maher et al. (2018) study a one day robust TAP in which a subset of aircraft is selected for the day’s maintenance operation. Each future day, the schedule proactively secures enough LOFs ending in maintenance bases as aircraft requiring this service.

Instead of traditional maintenance classification (A, B, C, and D-checks), modern fleet maintenance programs adopt a task-driven approach (Ruther et al. 2017). Papakostas et al. (2010) design a decision support software that heuristically schedules a list of detailed tasks required in each aircraft within an airline’s fleet. Schedules are sequentially constructed for each aircraft, leaving the remaining maintenance plan unaltered; it does not necessarily reach cost-efficient solutions, when maintenance resources are shared among the fleet. Recently Safaei and Jardine (2018) propose to simultaneously solve an aircraft routing, tail assignment and maintenance scheduling problem. Given the amount of decisions involved, the problem aggregates maintenance requirements into a generalized set of constraints ensuring the assigned aircraft routes provide enough maintenance opportunities. Compared to our work, we assume LOFs to be fixed, model a dynamic schedule, and work with a detailed plan for each individual maintenance task.

Table 1 presents a broad summary of previous OAMRP and TAP research classified according to: (1) maintenance approach, which can be by periodic maintenance checks (MC) or by individual tasks (MT); (2) constraints modeled, such as limited aircraft capacity (AC) and daily (RD) or hourly (RH) maintenance resource; and uncertainty within the task disclosure process (UT).

Also, Table 1 illustrates the research gap filled with this article. First, we model each task and resource consumption at the individual level to increase realism. Instead of planing aircraft maintenance operations having a limited amount of daily resources and maintenance capacity, we plan maintenance considering individual tasks consuming detailed amounts of multiple types of resources limited per unit-time. To increase scheduling flexibility and resource utilization, we allow tasks to be carried out in different levels of resource consumption per time. For example, a task demanding six person-hours can be performed by two technicians in three hours or in six hours by one. All previous articles use fixed task processing times.

Finally, we consider a stochastic and dynamic problem where tasks get disclosed dynamically over time in which the decision maker has to proactively anticipate future task arrivals. We believe that this setting is better suited for real operations than deterministic problems assuming a known set of maintenance requirements.

Table 1: Classification of previous work in OAMRP/TAP

Paper	Maintenance		Constraints			UT
	MC	MT	AC	RD	RH	
El Moudani and Mora-Camino (2000)	✓					
Sriram and Haghani (2003)	✓		✓			
Grönkvist (2005)	✓		✓			
Sarac et al. (2006)	✓		✓	✓		
Otten et al. (2006)	✓					
Murat Afsar et al. (2007)	✓		✓			
Murat Afsar et al. (2009)	✓		✓			
Gabteni and Grönkvist (2009)	✓					
Papakostas et al. (2010)		✓		✓		
Keysan et al. (2010)	✓		✓			
Orhan et al. (2011)	✓					
Lapp and Wikenhauser (2012)						
Haouari et al. (2013)	✓		✓			
Basdere and Bilge (2014)	✓		✓			
Gavranis and Kozanidis (2015)	✓		✓	✓		
Al-Thani et al. (2016)	✓					
Ruther et al. (2017)		✓				
Khaled et al. (2018)	✓		✓			
Maher et al. (2018)	✓		✓			
Safaei and Jardine (2018)		✓		✓		
<b>Our work</b>		✓	✓		✓	✓

### 3 Problem Statement

Now, we formally state the Aircraft Maintenance Scheduling Problem with Tail Assignment (AMSP-TA), which dynamically plans maintenance tasks and assigns itineraries to aircrafts. The objective of the AMSP-TA is to minimize the total costs incurred by expected tasks expired over the operating horizon.

#### 3.1 Basic setting

Consider an airline operating a fleet  $A$  of homogeneous aircraft executing daily LOFs from a central airport (hub) during an operating horizon  $T := \{1, 2, \dots, H\}$  of  $H$  consecutive days. This setting models the domestic operation of a typical Latin American airline. The horizon is further divided into a finer set  $J$  of time periods, typically hours, *e.g.*,  $J := \{1, 2, \dots, 24 \cdot H\}$ .

Each day  $t \in T$ , the airline assigns a LOF  $i \in I_t$  to each aircraft in  $a \in A$  from a given set  $I_t$  (assume  $|A| \geq |I_t|$ ). Each day  $t$  at the hub, a LOF  $i \in I_t$  starts in the morning at  $f_i \in J$  and ends at night at  $e_i \in J$ .

Aircraft maintenance operations occur during the night shift before day  $t$  at the hub's maintenance base. In day  $t$ , we consider a maintenance period  $J_t = \{\theta_t, \dots, \rho_t\} \in J$  starting when the earliest LOF of day  $t - 1$  ends ( $\theta_t := \min_{i \in I_{t-1}} \{e_i\}$ ) and ending when the latest LOF of day  $t \in T$  begins ( $\rho_t := \max_{i \in I_t} \{f_i\}$ ). Also, define the auxiliary set of LOFs starting and ending at time period  $j \in J$  as  $I_{t,j}^+ := \{i \in I_t : f_i = j\}$  and  $I_{t,j}^- := \{i \in I_t : e_i = j\}$ , respectively.

As depicted in Figure 5, an assignment of aircraft  $a \in A$  to LOFs  $i_1 \in I_{t-1}$  and  $i_2 \in I_t$  defines its ground time window (GTW)  $\{e_{i_1}, \dots, f_{i_2}\}$ , where maintenance in day  $t$  may occur. Each day  $t$ , the base has capacity to work on  $n < |A|$  aircraft and has  $c_{p,j} \geq 0$  workers (person-hours) available for each resource  $p \in P$  per period  $j \in J_t$ .

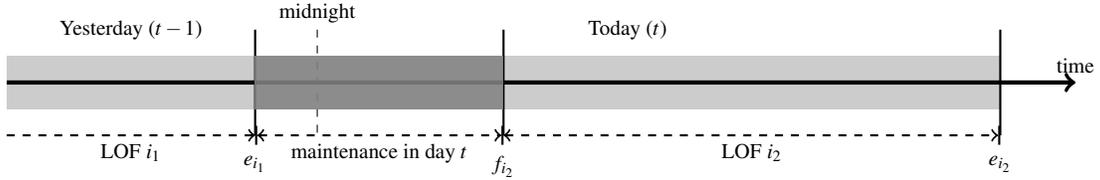


Figure 5: Evolution of day  $t$  for each aircraft

### 3.2 Task arrival process

Aircraft maintenance tasks get disclosed as follows. At each day  $t \in T$ , a random set  $N_{a,t}$  of new tasks required for aircraft  $a \in A$  becomes known. Each task  $k \in N_{a,t}$  must be executed from day  $t$ , is due by day  $b_k$ , has a minimum processing time of  $\ell_k$  hours, and requires  $\beta_{k,p} \geq 0$  person-hours from each resource  $p \in P$ . Future task arrivals are unknown, but we assume possible to simulate realizations of the stochastic process.

Each task  $k$  is critical ( $\gamma_k = 1$ ) or normal ( $\gamma_k = 0$ ). When a critical task  $k$  is left unattended after day's  $t = b_k$  maintenance, the related aircraft  $a$  is forced to be on ground (AOG) in day  $b_k$  at a cancellation cost  $\$ \alpha$ . A cancellation clears all remaining pending critical tasks in aircraft  $a$ , which are assumed to be executed during the day  $b_k$  on ground without spending resources from the night shift. Conversely, when a normal task is past due, the aircraft continues its operation meaning that the task gets outsourced at a penalty cost  $\$ \delta_k$ ; in our experiments we assume  $\delta_k$  equal to the total hours of resources required times a cost per outsourced hour  $\delta$ , *i.e.*,  $\delta_k := \delta \cdot \sum_{p \in P} \beta_{k,p}$ .

### 3.3 State & action space

The decision dynamics of the AMSP-TA go as follows. At each day  $t \in T$ , the system is at a state  $S_t = (m_t, O_t) \in \mathcal{S}_t$ , where  $\mathcal{S}_t$  represents the feasible state space. Vector  $m_t := \{m_{1,t}, \dots, m_{a,t}, \dots, m_{|A|,t}\}$  provides information regarding each aircraft GTW start period defining the earliest time period for maintenance operations in day  $t$ . The vector of sets  $O_t := \{O_{1,t}, \dots, O_{a,t}, \dots, O_{|A|,t}\}$  model *open* (disclosed and pending) tasks for each aircraft  $a$  before maintenance in day  $t$  starts; note that  $O_{a,t} \subseteq \cup_{t'=1}^t N_{a,t'}$ .

In state  $S_t \in \mathcal{S}_t$  and day  $t \in T$ , the decision maker has to execute three decisions:

- First, it has to assign a LOFs to each aircraft, *i.e.*, tail assignment. Define the binary variable  $x_{a,i,t} \in \{0, 1\}$  for each day  $t$ , aircraft  $a \in A$  and LOF  $i \in I_t$ , which is equal to 1 when  $a$  is assigned to  $i$  in day  $t$  and 0, otherwise. An AOG occurs when  $a$  is left unassigned; this decision is captured by variable  $u_{a,t} := 1 - \sum_{i \in I_t} x_{a,i,t}$ .
- Second, it has to choose which aircraft to maintain defined by the binary variable  $z_{a,t}$ , which is equal to 1 when  $a$  goes to maintenance in day  $t$ .
- Finally, the decision maker has to feasibly select a subset of open maintenance tasks to execute in each aircraft  $a$  assigned to maintenance. Let  $y_{k,t} \in \{0, 1\}$  be a binary variable, which is equal to 1 if task  $k \in O_{a,t}$  is executed and 0, otherwise; this decision defines if task  $k$  expired in day  $t$ , encoded in variable  $q_k \in \{0, 1\}$ .

The action space  $\mathbb{X}_t(S_t)$  in state  $S_t$  is formally defined in (1) as a set of solutions  $X_t = (\mathbf{x}_t, \mathbf{y}_t, \mathbf{z}_t, \mathbf{q}, \mathbf{u}_t)$  to the integer polytope

$$\mathbb{X}_t(S_t) := \left\{ (\mathbf{x}_t, \mathbf{y}_t, \mathbf{z}_t, \mathbf{q}, \mathbf{u}_t) : \exists \mathbf{r}, \mathbf{w} \text{ such that:} \right.$$

$$\begin{aligned}
\sum_{a \in A} x_{a,i,t} &\leq 1, & \forall i \in I_t, & \quad (1a) \\
u_{a,t} + \sum_{i \in I_t} x_{a,i,t} &= 1, & \forall a \in A, & \quad (1b) \\
q_k + y_{k,t} &\geq 1, & \forall k \in \cup_{a \in A} O_{a,t} : b_k = t, & \quad (1c) \\
u_{a,t} &\geq q_k, & \forall k \in O_{a,t} : \gamma_k = 1, \forall a \in A, & \quad (1d) \\
y_{k,t} &\leq z_{a,t}, & \forall k \in O_{a,t}, \forall a \in A, & \quad (1e) \\
\sum_{a \in A} z_{a,t} &\leq n, & & \quad (1f) \\
\ell_k \cdot y_{k,t} &\leq \sum_{j \in J_t} w_{a,j}, & \forall k \in O_{a,t}, \forall a \in A, & \quad (1g) \\
\sum_{k \in O_{a,t}} \beta_{k,p} \cdot y_{k,t} &= \sum_{j \in J_t} r_{a,p,j}, & \forall a \in A, \forall p \in P, & \quad (1h) \\
\sum_{a \in A} r_{a,p,j} &\leq c_{p,j}, & \forall p \in P, \forall j \in J_t, & \quad (1i) \\
r_{a,p,j} &\leq w_{a,j} \cdot c_{p,j}, & \forall p \in P, \forall j \in J_t, \forall a \in A, & \quad (1j) \\
1 &= w_{a,m_{a,t}+1} + u_{a,t}, & \forall a \in A, & \quad (1k) \\
w_{a,j} &= \mathbb{I}(j < \rho_t) w_{a,j+1} + \sum_{i \in I_{t,j}^+} x_{a,i,t}, & \forall a \in A, \forall j \in J_t : m_{a,t} < j \leq \rho_t, & \quad (1l) \\
x_{a,i,t} &\in \{0, 1\}, & \forall a \in A, \forall i \in I_t, & \quad (1m) \\
y_{k,t}, q_k &\in \{0, 1\}, & \forall k \in \cup_{a \in A} O_{a,t}, & \quad (1n) \\
u_{a,t}, z_{a,t} &\in \{0, 1\}, & \forall a \in A, & \quad (1o) \\
r_{a,p,j} &\geq 0, & \forall a \in A, \forall p \in P, \forall j \in J_t, & \quad (1p) \\
w_{a,j} &\in \{0, 1\}, & \forall a \in A, \forall j \in J_t \} & \quad (1q)
\end{aligned}$$

, where the auxiliary binary variable  $w_{a,j}$  indicates if aircraft  $a$  is assigned to maintenance operations in period  $j$  and auxiliary variable  $r_{a,p,j}$  indicates the consumption of resource  $p$  in period  $j$  due to maintenance to  $a$ . Constraints (1a) ensure that each daily itinerary  $i$  is assigned to at most one aircraft; (1b) assign each aircraft  $a$  to an itinerary  $i$  or set it AOG in day  $t$ ; (1c) impose that each task with deadline  $b_k = t$  should be executed or marked as expired; (1d) force an AOG status in aircraft  $a$  when any of its critical tasks expire; and constraints (1e) force that tasks can only be executed on aircraft assigned to the maintenance base. The single constraint (1f) ensures the maximum aircraft capacity of the maintenance base. The set of constraints (1g) ensures that we only execute tasks with minimum duration  $\ell_k$  in larger GTWs, while the set (1h) ensures that the workload of all scheduled tasks in each aircraft meets the total resource consumption for that aircraft and day. Note that the set of constraints (1g) and (1h) allows to carry out tasks in different

levels of resource consumption per time. Constraints (1i) guarantee to satisfy overall availability of each resource per hour; (1j) ensures that an aircraft resource consumption may be positive at a given time period only if it is assigned to maintenance. Flow constraints (1k)-(1l) define the maintenance block assigned to each aircraft. Remaining constraints define the nature of each variable in the program.

### 3.4 Daily operation cost

The cost incurred by the system in day  $t$  is defined by

$$C(S_t, X_t) = \alpha \sum_{a \in A} u_{a,t} + \sum_{k \in O_{a,t} : \gamma_k = 0} \delta_k \cdot q_k. \quad (2)$$

The first term in the right-hand-side of Equation (2) models the cost related AOGs (cancelled flights), while the second term models the cost incurred by expired normal tasks.

### 3.5 State transitions

Once an action  $X_t \in \mathbb{X}_t(S_t)$  is taken in state  $S_t = (m_t, O_t)$ , the next decision epoch occurs in day  $t + 1$  and state  $S_{t+1} = (m_{t+1}, O_{t+1})$ . For each aircraft  $a$ , its set of open tasks  $O_{a,t+1} := \{k \in O_{a,t} : y_{k,t} = 0\} \cup N_{a,t+1}$  is updated clearing all previously served tasks and including newly disclosed ones. For each aircraft  $a$ , the new earliest maintenance period  $m_{a,t+1} := \sum_{i \in I_t} x_{a,i,t} \cdot e_i$  is defined as the return period of the aircraft's previously executed LOF.

### 3.6 MDP statement and Bellman equations

Now, we define the AMSP-TA as a Markov Decision Process (MDP) (Puterman 2014). The objective is to find a policy  $\pi^* = \{X_1^*, X_2^*, \dots, X_H^*\} \in \Pi$  over the set of all feasible Markovian and deterministic policies  $\Pi$ , such that the total expected cost over the operating horizon is minimized. Formally, our problem is modelled as

$$C^*(S_1) = \min_{\pi \in \Pi} \mathbb{E} \left[ \sum_{t=1}^H C(S_t, X_t^\pi(S_t)) | S_1 \right]. \quad (3)$$

, where  $S_1$  is the system's initial state.

Such an optimal policy should solve the MDP's Bellman equations for each  $t \in T$  and  $S_t \in \mathcal{S}$

$$V_t(S_t) = \min_{X_t \in \mathbb{X}_t(S_t)} \{C(S_t, X_t) + \mathbb{E}[V_{t+1}(S_{t+1}) | S_t, X_t]\} \quad (4)$$

, where  $V_t(S_t) = \mathbb{E} [\sum_{t'=t}^H C(S_{t'}, X_{t'}^*(S_{t'})) | S_t]$  is the system's optimal expected cost-to-go from day  $t$  onwards. The optimal expected cost satisfies  $C^*(S_1) = V_1(S_1)$ .

Solving (4) exactly is difficult because of the curse of dimensionality; the cardinality of the state space grows exponentially with the number of tasks and fleet size, the action space is an integer polytope having exponentially many feasible points, and the expectation expressions have an exponential number of terms depending on the number of potential tasks per day. Indeed, simply evaluating a given policy is hard. Given these difficulties, we focus on designing policies based on Approximate Dynamic Programming (ADP) for the AMSP-TA.

## 4 A deterministic AMSP-TA model

Although the AMSP-TA is dynamic and stochastic, it is useful to solve its deterministic version, *i.e.*, a perfect information relaxation (PIR) (Brown et al. 2010). In the deterministic case, each set  $N_{a,t}$  of realized maintenance tasks for day  $t \in T$  and aircraft  $a \in A$  is completely known to the decision maker before the operation starts. Let  $C^{PIR}(N, S_1)$  be the minimum deterministic problem cost for a given realization  $N := \{N_{a,t} : \forall a \in A, \forall t \in T\}$  of aircraft maintenance tasks and initial state  $S_1$ ; this value is a (potentially unreachable) lower bound to the minimum cost of the dynamic-stochastic system executed in the realization  $N$ . Therefore, we have that

$$C^*(S_1) \geq \mathbb{E}_N[C^{PIR}(N, S_1)], \forall S_1 \in \mathcal{S}. \quad (5)$$

In practice, we compute  $C^{PIR}(N, S_1)$  solving the Integer Programming model (IP) defined in (6). A lower bound to  $C^*(S_1)$  can be approximated via Monte Carlo simulation over a sample  $\Omega$  of task arrival realizations, *i.e.*,  $\mathbb{E}_N[C^{PIR}(N, S_1)] \approx \frac{1}{|\Omega|} \sum_{\omega \in \Omega} C^{PIR}(N(\omega), S_1)$ .

To state the IP, we define the auxiliary set of executable tasks in day  $t \in T$  for each aircraft  $a \in A$  as  $K_{a,t} := \{k \in \cup_{v=1}^t N_{a,v} : t \leq b_k\}$ ; the set of all tasks related to  $a$  as  $K_a = \cup_{v=1}^H N_{a,v}$ ; the set of all tasks  $K = \cup_{a \in A} K_a$ ; and the feasible set of days to work on task  $k \in N_{a,t}$  as  $T_k = \{t, \dots, b_k\}$ . The set  $J_t$  is further divided into three sets: the set of hours when LOFs start  $J_t^+$ ; the set of hours when LOFs end  $J_t^-$ ; and the set of hours when LOFs do not start nor end  $J_t^0$ .

$$\min_{\mathbf{q}, \mathbf{r}, \mathbf{u}, \mathbf{w}, \mathbf{x}, \mathbf{y}, \mathbf{z}} \sum_{a \in A} \left( \sum_{t \in T} \sum_{j \in J_{t-1}^-} \alpha \cdot u_{a,t,j} + \sum_{k \in K_a: \gamma_k=0} \delta_k \cdot q_k \right) \quad (6a)$$

$$\text{s.t. } \sum_{a \in A} x_{a,i,t} \leq 1, \quad \forall i \in I_t, \forall t \in T, \quad (6b)$$

$$u_{a,1,m_{a,1}} + \sum_{i \in I_1} x_{a,i,1} = 1, \quad \forall a \in A, \quad (6c)$$

$$\sum_{j \in J_{t-1}^-} u_{a,t,j} + \sum_{i \in I_t} x_{a,i,t} = 1, \quad \forall a \in A, \forall t \in T \setminus \{1\}, \quad (6d)$$

$$q_k + \sum_{t \in T_k} y_{k,t} \geq 1, \quad \forall k \in K, \quad (6e)$$

$$\sum_{t \in T_k} \sum_{j \in J_{t-1}^-} u_{a,t,j} \geq q_k, \quad \forall a \in A, k \in K_a: \gamma_k = 1, \quad (6f)$$

$$y_{k,t} \leq z_{a,t}, \quad \forall a \in A, \forall k \in K_a, \forall t \in T_k, \quad (6g)$$

$$\sum_{a \in A} z_{a,t} \leq n, \quad \forall t \in T, \quad (6h)$$

$$\ell_k \cdot y_{k,t} \leq \sum_{j \in J_t} w_{a,j}, \quad \forall a \in A, \forall k \in K_a, \forall t \in T_k, \quad (6i)$$

$$\sum_{k \in K_{a,t}} \beta_{k,p} \cdot y_{k,t} = \sum_{j \in J_t} r_{a,p,j}, \quad \forall a \in A, \forall p \in P, \forall t \in T, \quad (6j)$$

$$\sum_{a \in A} r_{a,p,j} \leq c_{p,j}, \quad \forall p \in P, \forall t \in T, \forall j \in J_t, \quad (6k)$$

$$r_{a,p,j} \leq w_{a,j} \cdot c_{p,j}, \quad \forall a \in A, \forall p \in P, \forall t \in T, \forall j \in J_t, \quad (6l)$$

$$1 = w_{a,m_{a,1}+1} + u_{a,1,m_{a,1}}, \quad \forall a \in A, \quad (6m)$$

$$\sum_{i \in I_{1,\theta_2}^-} x_{a,i,1} + u_{a,1,d_a} = w_{a,\theta_2+1} + u_{a,2,\theta_2}, \quad \forall a \in A, \quad (6n)$$

$$\sum_{i \in I_{t-1,\theta_t}^-} x_{a,i,t-1} + \sum_{j \in J_{t-2}^-} u_{a,t-1,j} = w_{a,\theta_t+1} + u_{a,t,\theta_t}, \quad \forall a \in A, \forall t \in T \setminus \{1,2\}, \quad (6o)$$

$$\sum_{i \in I_{t-1,j}^-} x_{a,i,t-1} + w_{a,j} = w_{a,j+1} + u_{a,t,j}, \quad \forall a \in A, \forall t \in T \setminus \{1\}, \forall j \in J_{t-1}^- \setminus \{\theta_t\}, \quad (6p)$$

$$w_{a,j} = \mathbb{I}(j < \rho_t) w_{a,j+1} + \sum_{i \in I_{t,j}^+} x_{a,i,t}, \quad \forall a \in A, \forall t \in T, \forall j \in J_t^+, \quad (6q)$$

$$w_{a,j} = w_{a,j+1}, \quad \forall a \in A, \forall t \in T, \forall j \in J_t^0, \quad (6r)$$

, where the binary variable  $u_{a,t,j}$  is equal to 1 when  $a$  is left unassigned (AOG) in day  $t$  at period  $j$  and 0, otherwise; for the deterministic case we require to consider the period when the AOG occurs. All other parameters and variables are identical as in 3.

The objective function considers the same two components as in the dynamic problem, but added for the whole planning horizon. Constraints (6b) ensure that each daily LOF  $i$  is assigned to at most one aircraft; (6c) and (6d) assign each aircraft  $a$  to a LOF  $i$  or set it AOG that day; (6e) impose that each task is executed before due date or marked expired; (6f) force an AOG status when any critical task expires for aircraft  $a$ ; and constraints (6g) force tasks to be carried out only when aircraft  $a$  is assigned to the maintenance base. Constraints (6h) limit aircraft capacity at the maintenance base each day. The family of constraints (6i) ensures that we only execute tasks with minimum duration  $\ell_k$  in larger GTWs, while the set (6j) ensures that the workload of all scheduled tasks in each aircraft meets the total resource consumption for that aircraft and day. Constraints (6k) guarantee to satisfy overall availability of each resource per hour; (6l) ensures that an aircraft has positive resource consumption at a given time period only when it is assigned to maintenance. Flow constraints (6m)-(6r) define the maintenance block assigned to each aircraft. Remaining constraints define the nature of each variable in the program.

## 5 Dynamic Policies

In this section, we design heuristic dynamic policies for the dynamic-stochastic AMSP-TA presented in Section 3; these policies approximately solve the Bellman equations stated in (4).

### 5.1 Myopic Policy (MP)

Each day  $t$  in state  $S_t$ , we obtain a myopic policy (MP) by executing decisions taking the optimal action  $X_t$  to the Bellman equations assuming that the expected future cost, *i.e.*,  $Q_t(S_t, X_t) := \mathbb{E}[V_{t+1}(S_{t+1})|S_t, X_t]$ , is zero. A major advantage of MP is that only requires solving a small deterministic IP for each day and, therefore, runs relatively fast compared to time consuming online solutions; we use the MP as a benchmark.

### 5.2 Value Function Approximation (VFA)

Instead of simply disregarding the expected future cost of today's decisions in (4), an alternative approach is to estimate its contribution via a parametric linear function depending on features from state  $S_t$  and chosen

action  $X_t$ . Formally, to approximate  $Q_t(S_t, X_t)$  we use

$$Q_t^{VFA}(S_t, X_t) = \sum_{f \in F} \lambda_f \cdot \phi_f(S_t, X_t) \quad (7)$$

, where each  $\phi_f(S_t, X_t)$  models a selected feature  $f \in F$  from the state and decision pair  $(S_t, X_t)$ . Each parameter  $\lambda_f$  is calibrated via optimal learning techniques running Approximate Value Iteration (AVI) and linear regression techniques in simulated system runs; please refer to Powell (2011) for a further explanation on VFA techniques. For VFA, parameter calibration is performed offline, *i.e.*, before the real operation starts, and online solution times are comparable to MP. In preliminary experiments, we tried multiple sets of features and obtained the best results with the total workload of normal pending tasks  $\{k \in \cup_{a \in A} O_{a,t} : \gamma_k = 0, y_{k,t} = 0\}$  left for day  $t + 1$  after a decision  $X_t$  is taken in state  $S_t$ .

The selected features are defined in Equation (8); each computes the total pending workload of normal tasks left with  $f = \{1, 2, 3, \text{ or } \geq 4\}$  days left before due date.

$$\phi_f := \sum_{p \in P} \sum_{a \in A} \sum_{k \in O_{a,i} : \gamma_k = 0} \mathbb{I}(b_k - t = f) \cdot (1 - y_{k,t}) \cdot \beta_{k,p} \quad \forall f \in \{1, 2, 3, 4\} \quad (8)$$

We choose four periods after observing in preliminary experiments that deterministic solutions from the perfect information relaxation stated in Section 4 used less than 5 days of task anticipation in more than 95% of the cases.

An alternative to optimal learning is to set intuitive values for each  $\lambda_f$  heuristically; we refer to this policy as HVFA. In preliminary experiments over deterministic models, we observed that the fewer days left before a task's deadline, the more expensive it is to leave it open. Based on these preliminary observations, we set each  $\lambda_f$  equal to a fixed percentage of a task's outsourcing cost per hour. Specifically, we set  $\lambda = (0.5 \cdot \delta; 0.3 \cdot \delta; 0.2 \cdot \delta; 0.1 \cdot \delta)$ .

### 5.3 Rolling horizon (RH)

An alternative decision policy to VFA is a Rolling Horizon (RH) policy (Powell 2011). In day  $t$  and state  $S_t$ , RH estimates forecasts  $\bar{N}_{a,t'}$  to each stochastic set  $N_{a,t'}$  for aircraft  $a \in A$  and future day  $t' \in \{t + 1, \dots, t + R\}$  over a truncated horizon of  $R$  days starting from day  $t + 1$ . Later, it solves a deterministic rolling horizon problem (RHP) from day  $t$  to  $t + R$  with  $N_{a,t}, \bar{N}_{a,t+1}, \dots, \bar{N}_{a,t+R}$  and the current system state  $S_t$  as inputs. The action  $X_t$  planned for day  $t$  is obtained from RHP's optimal solution; plans for future "forecasted" days are

discarded. Finally, the system advances to day  $t + 1$  and repeats the policy. Figure 6 illustrates an example of a seven-day rolling horizon ( $R = 6$ ).

A RH policy approximates the value of  $Q_t(S_t, X_t)$  with the deterministic costs generated by the RH deterministic solution from day  $t + 1$  to  $t + R$ ; it proactively plans considering an “average” development of future events and, compared to VFA, is simpler and adapts to the current state of the system without aggregation, but is myopic to potential future actions of the decision maker and increases online computational costs.

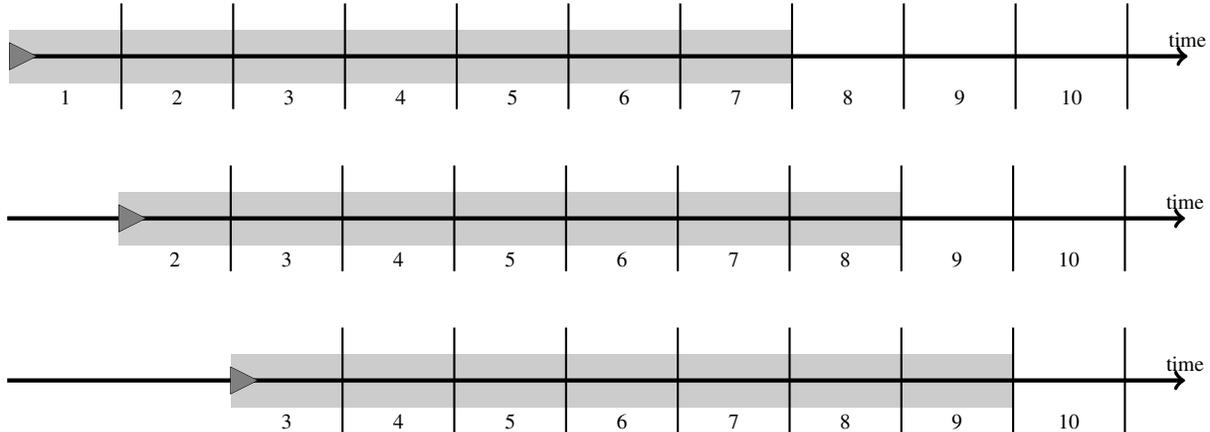


Figure 6: Example of a RH policy with  $R = 6$

In our setting, we use  $R = 6$  to forecast a week of the airline operation. This fits a weekly plan of LOFs well, which is a common practice (Basdere and Bilge 2014, Ruther et al. 2017). Also, we found that, in preliminary results over deterministic problems, a one-week RH policy is close to optimality.

#### 5.4 Hybrid VFA-RH policy

We follow the advice of Powell (2011) and Ulmer et al. (2018) and produce a hybrid policy combining an offline VFA with an online RH policy. A drawback of VFA is that artificially imposes a linear form on the future expected costs depending on heuristically selected features. Conversely, RH truncates the estimate of future expected costs to  $R$  days. We deal with both drawbacks by adding to the RHP an approximate cost-to-go function  $Q_{t+R}^{VFA}$  to estimate expected future costs from day  $t + R + 1$ . In this way, we estimate detailed future costs between days  $t + 1$  and  $t + R$  and approximate long term costs linearly via VFA. We expect that the marginal improvement of a VFA approximation after a seven-day rolling horizon procedure is smaller than implementing it directly over a myopic policy. Also, the calibration process could suffer from higher variability.

## 5.5 Airline decision rule

Finally, we build a policy emulating the manual decision process in a Latin American Airline. In this case, tail assignment and maintenance decisions are taken sequentially.

First, aircraft are sorted starting from higher loads of pending critical tasks. Assume that this ordering is represented by list  $\mathcal{L} := \{a_1, a_2, \dots\}$ . To execute tail assignment in day  $t$ , the first (typically older) aircraft in  $\mathcal{L}$  get assigned to LOFs with longer GTWs starting later in the day, while the latest (typically newer and healthier) aircraft in  $\mathcal{L}$  get assigned to LOFs with shorter GTWs. After LOFs get assigned, the decision rule begins assigning pending critical tasks. For each ordered  $a \in \mathcal{L}$ , the algorithm assigns all pending critical tasks of  $a$  in day  $t$ ; if this is infeasible then the aircraft  $a$  is left AOG. Available resources are updated accordingly. Later, the algorithm iterates over each  $a \in \mathcal{L}$  and, if possible, assigns all pending normal tasks expiring in day  $t$ . If resources are still available on day  $t$  at the maintenance base, then the decision rule repeats the above procedure considering pending critical and normal tasks expiring within the next 3 days, and so on.

## 6 Case Study

In this section, we present a case study to address the following key questions. First, how much is the cost of task arrival uncertainty and what are the benefits of planning maintenance operations with sophisticated dynamic policies. Intuitively, when more variable and dynamic task arrivals are present, maintenance operations must be executed with less information and more beneficial dynamic decisions can be.

Second, how does the system's task congestion level affects task outsourcing decisions, how the proposed policy adapts to different congestion levels, and what is the marginal benefit of an additional resource unit at the maintenance base.

Third, how much are the savings obtained by integrating maintenance planning with tail assignment decisions (AMSP-TA).

Finally, how does a centralized plan sharing all maintenance resources compares against decentralized assignment, where resources are divided in fleet groups; this division of resources is observed in Latin American airline holdings separating maintenance for each different country.

We start presenting the different benchmarks tested in section 6.1. In section 6.2 we discuss the analyzed experiments and data sets. Finally, in Section 6.3 we analyze results.

## 6.1 Tested policies

Table 2 summarizes all heuristic policies tested in this study. For each particular instance, we simulated 50 realizations of the task arrival process and use these common sample paths to estimate perfect information bounds (4) and each policies’ expected cost via Monte Carlo sampling.

Table 2: Policies tested in the case study

Policy	Acronym	Reference
Myopic Policy	MP	Section 5.1
Airline decision rule	ADR	Section 5.5
VFA with AVI parameter calibration	VFA	Section 5.2
Heuristic VFA	HVFA	Section 5.2
Rolling Horizon	RH	Section 5.3
Hybrid VFA-RH policy	VFA-RH	Section 5.4
Hybrid HVFA-RH policy	HVFA-RH	Section 5.4

## 6.2 Experimental design

We study the performance of our policies in five experiments. An experiment defines specific configuration of costs, daily LOFs to operate, fleet size, maintenance base capacity, resources and task arrival rates. We refer to a particular instance that mimics the domestic maintenance operation faced by a mayor Chilean airline as the “base experiment”.

### Base experiment design.

We take a 30 day operation and choose Santiago city airport as the maintenance base, since approximately 94% of all domestic flights depart or arrive from it (Junta Aeronáutica Civil 2018). For the base experiment, the fleet size is set to 30, which is similar to the domestic fleet of the airline studied.

For each aircraft, task arrivals per day are simulated via a Poisson distribution with rate  $\mu := \mathbb{E}[N_{a,t}] = 3$  representing the airline average.

Each task  $k$  within the stochastic set  $N_{a,t}$  has an expiration date set as  $b_k = t + \lfloor v_k \rfloor$ , where  $v_k$  is generated from a triangular distribution with parameters  $\eta_1 = 1$ ,  $\eta_2 = 10$  and  $\eta_3 = 30$  days. The probability of creating a critical task is set to 40% ( $P(\gamma_k = 1) = 0.4$ ) as we observe in the airline’s data.

We also set aircraft capacity at the maintenance base equal to 20, *i.e.*,  $\frac{2}{3}$  of the fleet and consider two resources, mechanics and avionics, both in person-hours. For the base experiment, we set available resources

equal to the average daily requirements *i.e.*, congestion 100%. As defined in Equation (9), the total number of person-hours available per resource is set equal to the sum over all task arrival rates for each day and aircraft times an average resource consumption per task. These resources are evenly distributed over each night shift, assuming a constant number of technicians over time.

$$\sum_{j \in J_t} c_{p,j} = \mu \cdot |A| \cdot \mathbb{E}_k [\beta_{k,p}], \quad \forall p \in P \quad (9)$$

For each task, the person-hours (PH) required per resource are simulated following a discrete empirical distribution obtained from the airline's data; see Figure 7. We set the minimum task maintenance time as

$$l_k = \left\lceil \frac{\beta_{k,p}}{\mathbb{E}_k [\beta_{k,p}]} \right\rceil.$$

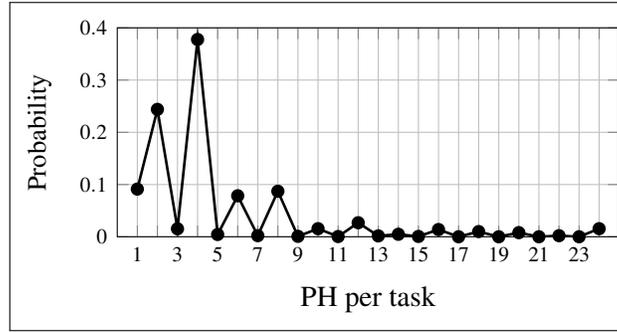


Figure 7: Empirical frequency of person-hour requirements per task in avionics and mechanics

To set all LOFs, we use the information of all domestic arrivals and departures from Santiago city airport in year 2017. Figure 8 illustrates the year's empirical frequency of arrivals and departures per hour. Arrivals after midnight are presented as hours 25, 26 and 27.

To simulate initial conditions, we run a one-month warm-up period. During this month, tasks are scheduled in its expiration date. Thus, at the end of this month (beginning of the planning horizon), the system will have pending tasks with a short expiration date, *i.e.*, the expected number of pending tasks at the end of the warm-up period is  $\mathbb{E}[O_{t=0}] = \left(\frac{\eta_1 + \eta_2 + \eta_3}{3} + \frac{1}{2}\right) \cdot |A| \cdot \mu$ .

The daily cost of an aircraft on ground (AOG) is set equal to  $\alpha = \$100,000$ , while the unit cost of an outsourced PH is  $\delta = \$75$ . These values produce that, on average, producing an AOG is 17 times more expensive than expired normal tasks.

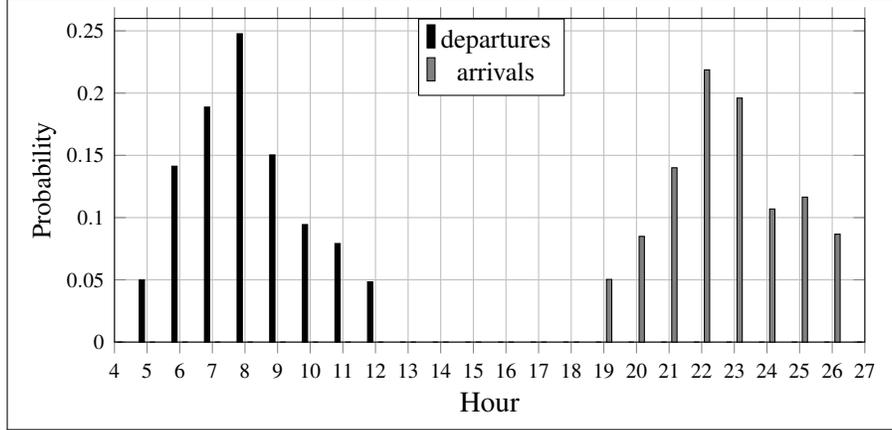


Figure 8: Hourly distribution of LOFs departures and arrivals.

### Extended experiments.

From the base experiment, we build four extended experiments:

- Experiment 1: Reduced task execution flexibility. For each task, we reduce the average time between disclosure and expiration by modifying the triangular distribution of  $v_k$  to  $\eta = (0.5; 5; 15)$ .
- Experiment 2: Modified congestion level. We increase the supply of resources available at the maintenance base to congestion levels of 97.5%, 95%, 92.5% and 90%.
- Experiment 3: AMSP without TAP integration. In this experiment, we study the impact of assigning LOFs to each aircraft before the operation starts, while tasks are still dynamically scheduled each day. To assign LOFs, we initially solve a deterministic AMSP-TA problem with average task information available in day 1. These LOFs are assigned *a priori* to each aircraft and, later, a dynamic AMSP-TA policy assuming fixed ground time windows is executed.
- Experiment 4: Decentralized maintenance plans.

In this experiment, we divide the fleet and available maintenance resources into two groups having half of the airline’s fleet and maintenance resources each and run four different experiments. The first experiment solves an independent AMSP-TA problem for each group, completely separating resources and fleet. A second experiment solves a single AMSP-TA, but restricts LOFs assignments to be performed within each group while maintenance resources are shared over the unified fleet. We run a third experiment, where a restricted AMSP-TA is solved limiting the chance to share resources

but having a flexible TA over the fleet. The final experiment is the base experiment, where both LOF and maintenance resources are assigned having full flexibility.

### 6.3 Results

Now, we present results and initially discuss the base experiment to select our best policy. Later, we discuss results obtained in each extended experiment.

We implemented our solution in the Python programming language v3.6.3. and used Gurobi v7.5.1. each time we required to solve an MIP; we set 3 minute limit per solution. We ran all experiments in an Intel Core i7-7500U 2.9Ghz processor having 16GB of RAM.

#### **Base experiment.**

We computed four statistics for each policy: (1) number of AOG, (2) expired tasks, as the percentage of tasks past due over the total arrived with deadline within the operational period, (3) cost percentage increase over the perfect information relaxation (gap) and (4) the average daily decision time. We set a 3 hour limit to solve each PIR problem and take the best lower bound available, when no optimal solution is found.

Table 3 presents base experiment results for each policy averaged over 50 realizations of the task arrival process. As expected, MP is outperformed by all our policies and ADR is the second worst in terms of gap, but significantly reduces AOGs. Compared to MP, VFA reduces gap approximately 24 times, prevents AOGs and decreases the percentage of expired tasks in more than 3.7 times. VFA also outperforms the HVFA policy in terms of AOG and gap; this shows the benefits of using simulation-based calibrated cost function parameters.

For RH, the overall percentage of expired tasks decreases to roughly 2.5%, which is comparable to the percentage observed in the PIR bound. Our best policy is the hybrid HVFA-RH policy, which has a gap below 3%. Unexpectedly, HVFA-RH outperforms the calibrated VFA-RH, but there is no statistical difference when gap confidence intervals are compared for both policies meaning there is no value in refining parameter calibration seven days ahead.

All feasible policies in Table 3 take on average less than 2 minutes to plan a daily schedule; this is compatible with operational decision-making. As expected, decision times increase as the policy's complexity increases. If an airline requires an instantaneous decision, then VFA can be an excellent candidate, otherwise, we suggest to select HVFA-RH.

Table 3: Average results for the base experiment over all policies

Policy	AOG	Expired tasks (%)	gap (%)	Daily decision time (s)
MP	12.16	20.97	3,147.00	0.12
ADR	0.02	15.95	531.52	0.038
HVFA	1.74	4.16	389.39	0.61
VFA	0	5.65	126.32	0.52
RH	0	2.57	6.66	50.47
HVFA-RH	0	2.44	2.64	121.66
VFA-RH	0	2.43	3.71	116.14
PIR	0	2.33	0	-

Figure 9 presents the average task anticipation frequencies observed for our best policies. We refer to “task anticipation” as the number of days elapsed between a task execution and its expiration date. All policies equipped with RH procedures manage to proactively plan tasks similar to the PIR bound. These results suggest that there is marginal value in anticipating future costs of events occurring one week ahead, and explains the good results of a RH policy truncated to a 7-day horizon.

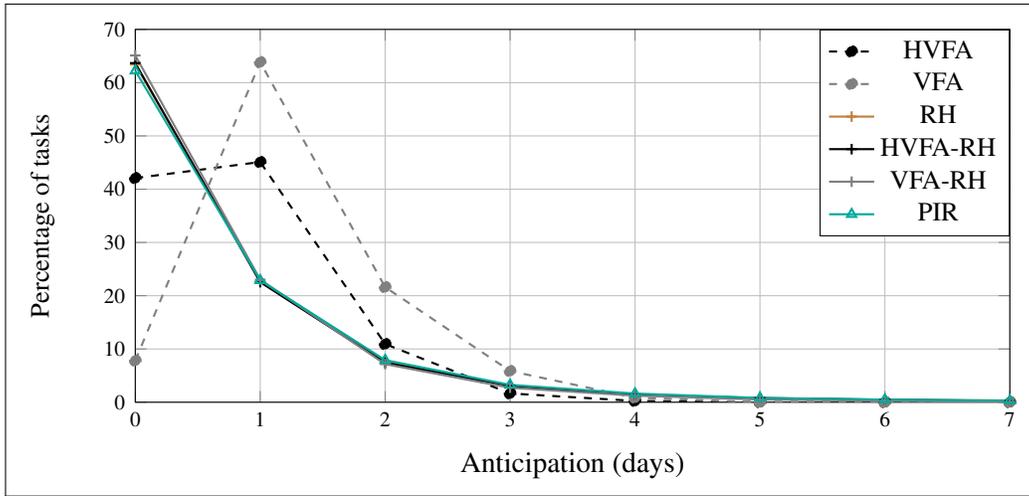


Figure 9: Average task anticipation frequency for the PIR and the best dynamic policies.

Given our initial results, we now focus the analysis on our best policy: HVFA-RH.

**Experiment 1: Reduced task execution flexibility.**

Table 4 presents average results for experiment 1 for policy HVFA-RH. Compared to the base experiment, we observe a 4% increase in the percentage of expired tasks, which is explained by the lost degree of

flexibility in task scheduling over time. The average gap increases showing that the dynamic problem becomes harder compared to the PIR, but our best policy keeps that gap under 8%.

Table 4: Average results for the HVFA-RH policy over the base experiment versus the extended experiment with reduced task execution flexibility.

Experiment	AOG	Expired Tasks (%)	gap (%)
Base experiment	0	2.44	2.64
Extended experiment 1	0	2.53	7.54

### Experiment 2: Modified congestion levels.

Figure 10 (a) illustrates the average number of outsourced resources over different system congestion levels for the HVFA-RH policy and the PIR bound. As expected, fewer maintenance resources need to be outsourced as congestion decreases. We note that the marginal value of an additional resource PH increases with congestion, *i.e.*, increasing curve slope in Figure 10 (a). A decision maker might compare this slope to the marginal cost of resources to optimally decide the capacity of the maintenance base. In Figure 10 (b), we observe that HVFA-RH increases its absolute gap compared to the PIR bound as congestion levels increase, but relative gap reduces as the absolute outsourcing costs increase.

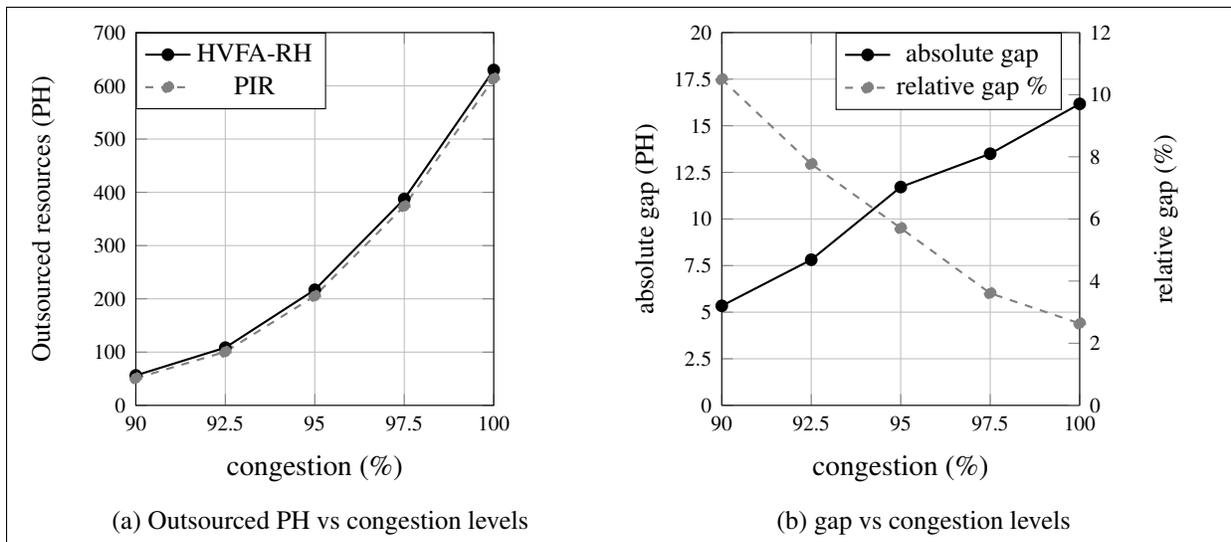


Figure 10: Average number of resources outsourced and gap versus different congestion levels for the HVFA-RH policy.

Figure 11 depicts how task anticipation changes over two different system congestion levels suggest-

ing that HVFA-RH anticipates more tasks in a less congested system (90%) compared to a congested one (100%). Intuitively, our policy adapts its decision rule in congested environments with relatively fewer resources available.

If we distinguish task anticipation for each task size, *i.e.*, task resource consumption level, then we observe that our policy executes earlier, smaller tasks ( $\leq 6$  PH) compared to medium ( $6 < \text{PH} \leq 25$ ) and large size tasks ( $\text{PH} \geq 25$ ). As suggested in previous results, our policy schedules first all urgent tasks. Later, as seen in Figure 11, it anticipates less-urgent tasks consuming fewer resources to increase utilization of residual maintenance capacity.

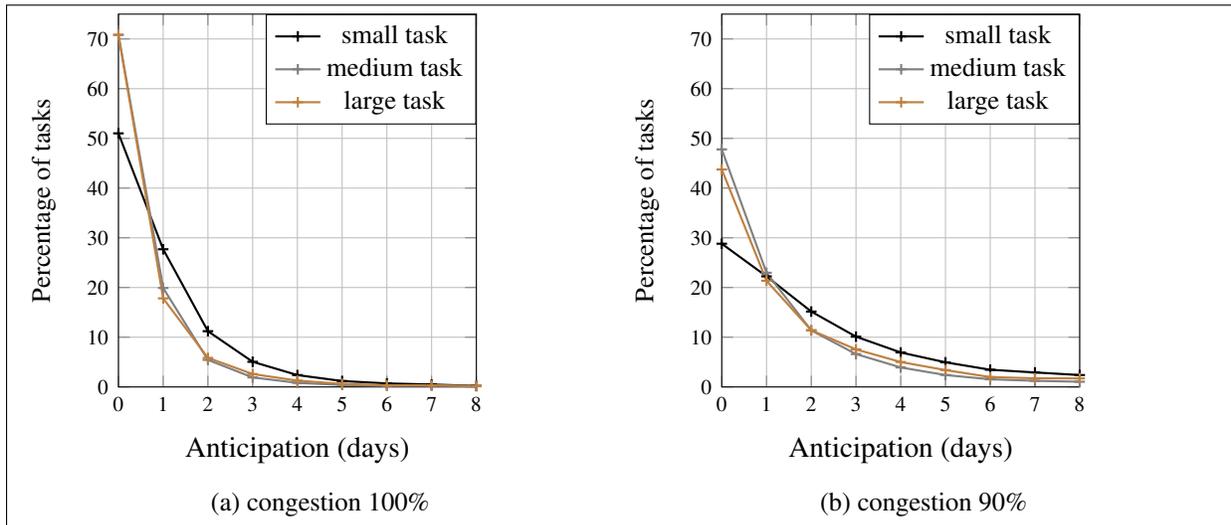


Figure 11: Average number of days a task is executed before its expiration date over different congestion levels and resource consumption requirements.

### Experiment 3: AMSP without TAP integration.

Table 5 measures the costs of executing tail assignment decisions *a priori* before the operation starts. First, the average number of expired tasks increases by 15.1%. Also, we obtain one realization with an AOG showing the potential costs of such a loss in decision flexibility.

Table 5: Average results with and without integrated task scheduling and tail assignment for the HVFA-RH policy.

Integrated decisions	AOG	Expired Tasks (% increase)
Yes	0	65.28
No	0.02	75.12 (15.1%)

#### Experiment 4: Decentralized maintenance plans

Table 6 presents average results under the four previously defined settings of decentralized decision-making.

As observed in the first two rows, no significant differences are observed when tail assignment decisions are split in two different groups. We believe that in smaller groups having 15 aircraft each there is enough flexibility in tail assignment to make efficient maintenance decisions.

Conversely, the system incurs in a major cost when maintenance resources are decentralized. The above implies a 77% increase in the average number of expired tasks suggesting that there exists significant economies of scale in maintenance operations. Accordingly, one might recommend smaller airlines to outsource or share maintenance operations with their competitors to reduce costs. If we divide both LOFs assignments and resources into two groups, then the number of expired tasks increases even higher than when both effects are added up separately; this suggests that each option recovers from a potential loss incurred when losing the other.

Table 6: Average results in decentralized experiments for HVFA-RH.

Decentralized		AOG	Expired Tasks (% increase)
TAP	Resources		
No	No	0	65.28
Yes	No	0	63.16 (-3.2%)
No	Yes	0	115.46 (76.9%)
Yes	Yes	0	180.60 (176.7%)

## 7 Conclusion

In this article, we begin studying a complex operational maintenance problem, which we refer as the Aircraft Maintenance Scheduling Problem with Tail Assignment (AMSP-TA). Instead of traditional deterministic approaches building aggregate maintenance plans, we model dynamic decision-making with stochastic disclosure of maintenance tasks over time at the individual level. We assume that each task consumes a fixed vector of maintenance resources, which are available in limited quantities per time period at the maintenance base. To increase flexibility, we integrate maintenance scheduling with tail assignment decisions and allow tasks to be carried out in different intensities of resource consumption per time. We present an MDP model for such a setting and design dynamic policies based on ADP, such as rolling horizon (RH) and value function approximation techniques (VFA).

We empirically test our policies in a set of computationally simulated experiments based on information from a major Latin American Airline. In our experiments, all our policies decision times are compatible with a daily operation. Our best policy is a hybrid VFA-RH combining both procedures; this policy outperforms the airline's decision rule and simpler policies based on VFA or RH.

The following takeaways are obtained from this study. First, our policy proactively plans tasks similar to the relaxed problem with deterministic information; we also observed that task anticipation increases when system congestion levels increase, and that our policy adapts accordingly to different congestion levels. Second, we estimate the marginal benefit of an additional maintenance resource unit and suggest that it can be measured as the potential reduction of outsourced resourced. Third, we empirically verify that an integrated tail assignment (TA) and maintenance task scheduling decision-making reduces the total expected costs and is essential to build cost-efficient maintenance plans. We also observe the existence of economies of scale when sharing maintenance resources across the system; this suggests smaller airlines to outsource or share maintenance operations with their competitors to reduce costs. Finally, we suggest from our experiments that most critical tasks should be executed on its expiration date and less-urgent tasks consuming fewer resources should be anticipated to increase utilization of remaining residual maintenance capacity.

The AMSP opens a large variety of new and unsolved problems. For example, one can extend the AMSP to multiple hubs and maintenance bases or study a potential integration with aircraft routing decisions to redesign LOFs withing the AMSP-TA. We could also distinguish between a task's disclosure date and earliest execution time to measure the cost of different anticipation levels in the task arrival process.

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## References

- Nayla Ahmad Al-Thani, Mohamed Ben Ahmed, and Mohamed Haouari. A model and optimization-based heuristic for the operational aircraft maintenance routing problem. *Transportation Research Part C: Emerging Technologies*, 72:29–44, 2016. ISSN 0968090X. doi: 10.1016/j.trc.2016.09.004. URL <http://dx.doi.org/10.1016/j.trc.2016.09.004>.
- Cynthia Barnhart and Barry Smith, editors. *Quantitative Problem Solving Methods in the Airline Industry*, volume 169. Springer US, 1 edition, 2012. ISBN 978-1-4614-1607-4. doi: 10.1007/978-1-4614-1608-1. URL <http://link.springer.com/10.1007/978-1-4614-1608-1>.
- Cynthia Barnhart, Peter Belobaba, and Amedeo R. Odoni. Applications of Operations Research in the Air Transport Industry. *Transportation Science*, 37(4):368–391, 2003. ISSN 0041-1655. doi: 10.1287/trsc.37.4.368.23276. URL <http://pubsonline.informs.org/doi/10.1287/trsc.37.4.368.23276>.
- Mehmet Basdere and Ümit Bilge. Operational aircraft maintenance routing problem with remaining time consideration. *European Journal of Operational Research*, 235(1):315–328, 2014. ISSN 03772217. doi: 10.1016/j.ejor.2013.10.066.
- M. Biró, I. Simon, and C. Tánzos. Aircraft and maintenance scheduling support, mathematical insights and a proposed interactive system. *Journal of Advanced Transportation*, 26(2):121–130, 1992. ISSN 20423195. doi: 10.1002/atr.5670260204.
- David B. Brown, James E. Smith, and Peng Sun. Information Relaxations and Duality in Stochastic Dynamic Programs. *Operations Research*, 58(4-part-1):785–801, aug 2010. ISSN 0030-364X. doi: 10.1287/opre.1090.0796. URL <http://pubsonline.informs.org/doi/abs/10.1287/opre.2014.1322> <http://pubsonline.informs.org/doi/abs/10.1287/opre.1090.0796>.
- Bureau of Transportation Statistics. Form 41: Air Carrier Financial Reports, 2017. URL <https://www.transtats.bts.gov/>.
- Andrew J. Cook, Graham Tanner, and Stephen Anderson. Evaluating the True Cost To Airlines of One Minute of Airborne or Ground Delay. Technical report, University of Westminster, 2004.
- Muhammet Deveci and Nihan Çetin Demirel. A survey of the literature on airline crew scheduling. *Engineering Applications of Artificial Intelligence*, 74(March):54–69, 2018. ISSN 09521976. doi: 10.1016/j.engappai.2018.05.008. URL <https://doi.org/10.1016/j.engappai.2018.05.008>.
- Walid El Moudani and Félix Mora-Camino. A dynamic approach for aircraft assignment and maintenance scheduling by airlines. *Journal of Air Transport Management*, 6(4):233–237, 2000. ISSN 09696997. doi: 10.1016/S0969-6997(00)00011-9.
- Abdelrahman E.E. Eltoukhy, Felix T.S. Chan, and S.H. Chung. Airline schedule planning: a review and future direc-

- tions. *Industrial Management & Data Systems*, 117(6):1201–1243, 2017. ISSN 0263-5577. doi: 10.1108/IMDS-09-2016-0358. URL <http://www.emeraldinsight.com/doi/10.1108/IMDS-09-2016-0358>.
- Maximilian M. Etschmaier and Dennis F. X. Mathaisel. Airline Scheduling: An Overview. *Transportation Science*, 19(2):127–138, 1985. ISSN 0041-1655. doi: 10.1287/trsc.19.2.127. URL <http://pubsonline.informs.org/doi/abs/10.1287/trsc.19.2.127>.
- Sami Gabteni and Mattias Grönkvist. Combining column generation and constraint programming to solve the tail assignment problem. *Annals of Operations Research*, 171(1):61–76, 2009. ISSN 02545330. doi: 10.1007/s10479-008-0379-1.
- Andreas Gavranis and George Kozanidis. An exact solution algorithm for maximizing the fleet availability of a unit of aircraft subject to flight and maintenance requirements. *European Journal of Operational Research*, 242(2):631–643, 2015. ISSN 03772217. doi: 10.1016/j.ejor.2014.10.016. URL [https://ac.els-cdn.com/S0377221714008376/1-s2.0-S0377221714008376-main.pdf?\\_tid=16e981f6-b296-11e7-](https://ac.els-cdn.com/S0377221714008376/1-s2.0-S0377221714008376-main.pdf?_tid=16e981f6-b296-11e7-)
- Ram Gopalan and Kalyan T Talluri. Mathematical models in airline schedule planning : A survey. *Annals of Operations Research*, 76:155–185, 1998. ISSN 02545330. doi: 10.1023/A:1018988203220.
- Mattias Grönkvist. *The tail assignment problem, PhD Thesis*. Doctor of philosophy, Chalmers University of Technology and Göteborg University, 2005.
- Mattias Grönkvist. Accelerating column generation for aircraft scheduling using constraint propagation. *Computers and Operations Research*, 33(10):2918–2934, 2006. ISSN 03050548. doi: 10.1016/j.cor.2005.01.017.
- Mohamed Haouari, Shengzhi Shao, and Hanif D. Sherali. A Lifted Compact Formulation for the Daily Aircraft Maintenance Routing Problem. *Transportation Science*, 47(4):508–525, 2013. ISSN 0041-1655. doi: 10.1287/trsc.1120.0433. URL <http://pubsonline.informs.org/doi/abs/10.1287/trsc.1120.0433>.
- IATA. Economic performance of the airline industry. Technical report, 2017. URL <https://www.iata.org/whatwedo/Documents/economics/IATA-Economic-Performance-of-the-Industry-mid-year>
- IATA. Economic performance of the airline industry. Technical report, 2018a.
- IATA. Safety Report 2017. Technical Report April, 2018b. URL <https://aviation-safety.net/airlinesafety/industry/reports/IATA-safety-report-2017.pdf>.
- IATA. Air Passenger Market Analysis - June 2018. Technical report, 2018c.
- IATA's Maintenance Cost Task Force. Labour rate and productivity calculations for commercial aircraft maintenance. Technical report, 2013. URL <http://www.iata.org/whatwedo/workgroups/Documents/MCTF/2013-labor-rate.pdf>.
- Junta Aeronáutica Civil. Informe estadístico de Tráfico Aéreo 2017. Technical report, Santiago de Chile, 2018.
- Gizem Keysan, George L. Nemhauser, and Martin W. P. Savelsbergh. Tactical and Operational Planning of Scheduled Maintenance for Per-Seat, On-Demand Air Transportation. *Transporta-*

- tion Science*, 44(3):291–306, 2010. ISSN 0041-1655. doi: 10.1287/trsc.1090.0311. URL <http://pubsonline.informs.org/doi/abs/10.1287/trsc.1090.0311>.
- Oumaima Khaled, Michel Minoux, Vincent Mousseau, Stéphane Michel, and Xavier Ceugniet. A compact optimization model for the tail assignment problem. *European Journal of Operational Research*, 264(2):548–557, 2018. ISSN 03772217. doi: 10.1016/j.ejor.2017.06.045. URL <http://dx.doi.org/10.1016/j.ejor.2017.06.045>.
- Marcial Lapp and Amy Cohn. Modifying lines-of-flight in the planning process for improved maintenance robustness. *Computers and Operations Research*, 39(9):2051–2062, 2012. ISSN 03050548. doi: 10.1016/j.cor.2011.08.024. URL <http://dx.doi.org/10.1016/j.cor.2011.08.024>.
- Marcial Lapp and Florian Wikenhauser. Incorporating aircraft efficiency measures into the tail assignment problem. *Journal of Air Transport Management*, 19(1):25–30, 2012. ISSN 09696997. doi: 10.1016/j.jairtraman.2011.12.006. URL <http://dx.doi.org/10.1016/j.jairtraman.2011.12.006>.
- Zhe Liang, Yuan Feng, Xiaoning Zhang, Tao Wu, and Wanpracha Art Chaovalitwongse. Robust weekly aircraft maintenance routing problem and the extension to the tail assignment problem. *Transportation Research Part B: Methodological*, 78(2015):238–259, 2015. ISSN 01912615. doi: 10.1016/j.trb.2015.03.013. URL <http://dx.doi.org/10.1016/j.trb.2015.03.013>.
- Stephen J. Maher, Guy Desaulniers, and François Soumis. The daily tail assignment problem under operational uncertainty using look-ahead maintenance constraints. *European Journal of Operational Research*, 264(2):534–547, 2018. ISSN 03772217. doi: 10.1016/j.ejor.2017.06.041. URL <http://dx.doi.org/10.1016/j.ejor.2017.06.041>.
- H. Murat Afsar, Marie Laure Espinouse, and Bernard Penz. A two-step heuristic to build flight and maintenance planning in a rolling-horizon. *Proceedings - ICSSSM'06: 2006 International Conference on Service Systems and Service Management*, 2:1251–1256, 2007. doi: 10.1109/ICSSSM.2006.320688.
- H. Murat Afsar, Marie Laure Espinouse, and Bernard Penz. Building flight planning for an airline company under maintenance constraints. *Journal of Quality in Maintenance Engineering*, 15(4):430–443, 2009. ISSN 13552511. doi: 10.1108/13552510910997788.
- Ilkay Orhan, Muzaffer Kapanoglu, and t. Hikmet Karakoc. Concurrent Aircraft and Maintenance Scheduling. *Journal of Aeronautics and Space Technologies*, 5(1):73–79, 2011.
- Lars Otten, Mattias Grönkvist, and Devdatt Dubhashi. Randomization in Constraint Programming for Airline Planning. In Frédéric Benhamou, editor, *Principles and Practice of Constraint Programming - CP*, pages 406–420. Springer, Berlin, Heidelberg, 2006. ISBN 3540462678.
- N. Papakostas, P. Papachatzakis, V. Xanthakis, D. Mourtzis, and G. Chryssolouris. An approach to operational

- aircraft maintenance planning. *Decision Support Systems*, 48(4):604–612, 2010. ISSN 01679236. doi: 10.1016/j.dss.2009.11.010. URL <http://dx.doi.org/10.1016/j.dss.2009.11.010>.
- Michael L Pinedo. *Scheduling*. Springer. ISBN 978-3-319-26578-0. doi: 10.1007/978-3-319-26580-3.
- Warren B. Powell. *Approximate Dynamic Programming: Solving the Curses of Dimensionality: Second Edition*. Wiley Series in Probability and Statistics. John Wiley & Sons, Inc., Hoboken, NJ, USA, aug 2011. ISBN 9781118029176. doi: 10.1002/9781118029176. URL <http://doi.wiley.com/10.1002/9781118029176>.
- Brian Prentice, Derek Costanza, and John Smiley. Not enough mechanics. Technical report, Oliver Wyman, 2017.
- Martin L. Puterman. *Markov decision processes: discrete stochastic dynamic programming*. John Wiley & Sons, 2014. ISBN 1118625870. doi: 10.15713/ins.mmj.3. URL <https://books.google.com/books?hl=en&lr=&id=VvBjBAAAQBAJ&pgis=1>.
- Sebastian Ruther, Natashia Boland, Faramroze G. Engineer, and Ian Evans. Integrated aircraft routing, crew pairing, and tail assignment: branch-and-price with many pricing problems. *Transportation Science*, 51(1):177–195, 2017. ISSN 15265447. doi: 10.1287/trsc.2015.0664.
- Nima Safaei and Andrew K.S. Jardine. Aircraft routing with generalized maintenance constraints. *Omega (United Kingdom)*, 80:111–122, 2018. ISSN 03050483. doi: 10.1016/j.omega.2017.08.013. URL <https://doi.org/10.1016/j.omega.2017.08.013>.
- Abdulkadir Sarac, Rajan Batta, and Christopher M. Rump. A branch-and-price approach for operational aircraft maintenance routing. *European Journal of Operational Research*, 175(3):1850–1869, 2006. ISSN 03772217. doi: 10.1016/j.ejor.2004.10.033.
- Hanif D. Sherali, Ebru K. Bish, and Xiaomei Zhu. Airline fleet assignment concepts, models, and algorithms. *European Journal of Operational Research*, 172(1):1–30, 2006. ISSN 03772217. doi: 10.1016/j.ejor.2005.01.056.
- Chellappan Sriram and Ali Haghani. An optimization model for aircraft maintenance scheduling and re-assignment. *Transportation Research Part A: Policy and Practice*, 37(1):29–48, 2003. ISSN 09658564. doi: 10.1016/S0965-8564(02)00004-6.
- Marlin W. Ulmer, Justin C. Goodson, Dirk C. Mattfeld, and Marco Hennig. Offline-Online Approximate Dynamic Programming for Dynamic Vehicle Routing with Stochastic Requests. *Transportation Science*, 2018. URL <https://doi.org/10.1287/trsc.2017.0767>.
- Jorne Van den Bergh, Philippe De Bruecker, Jeroen Beliën, and Jonas Peeters. Aircraft maintenance operations: state of the art. *FEB@HUBRUSSEL RESEARCH PAPER*, 9:34, 2013. URL <https://lirias.kuleuven.be/bitstream/123456789/426843/1/>.