Vehicle Routing Problem with Steep Roads

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Abstract

Most routing decisions assume that the world is flat meaning that routing costs are modelled as a weighted sum of total distance and time spend by delivery vehicles. However, this assumption does not apply for urban logistics operations in cities with significant altitude differences. We fill a space in the VRP literature and model routing decisions including the impact of road grades in a fuel consumption cost model. A cost-efficient routing model over these geographies requires to distinguish different vehicle cargo levels and road grades in arc costs. To do so, we formulate the Vehicle Routing Problem in regions with steep roads (VRP-SR) as an integer linear program and implement a heuristic solution for this problem. In simulated experiments based on the geography of a real city, we estimate the potential cost benefits obtained by our solution compared to a benchmark assuming a flat geography. Our results suggest that we may reduce costs up to 13% for the city of Valparaiso, Chile. In addition, we obtain valuable managerial insights from our routing plans. We built routes that initially cluster customers with small altitudes differences to avoid abrupt altitude changes between consecutive customer visits with a loaded vehicle; our solutions may produce higher altitude changes between consecutive customers after the vehicle drops of a considerable amount of weight. We also notice situations in which it is cost-efficient to split a route into multiple routes; this may occur even if a single route can visit all customers to drop weight in between routes and climb hills with a lighter vehicle.

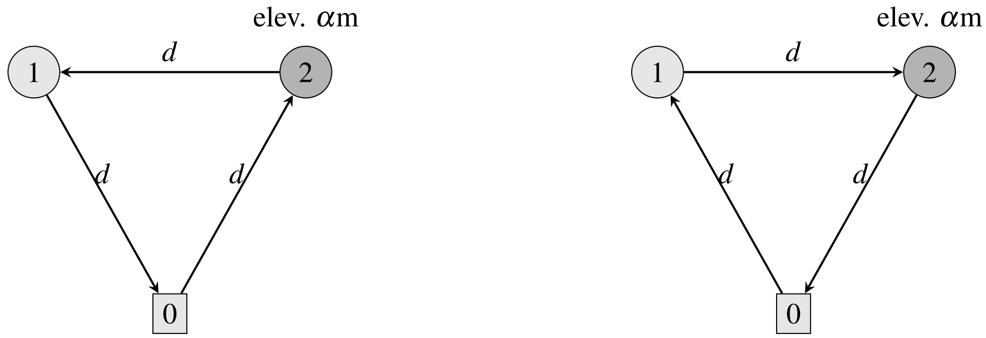
1 Introduction

Last-mile logistics deals with the transport of goods via a fleet of vehicles dispatched from urban distributions centers to geographically located customers within a city representing small-scale merchants and residential locations demanding relatively small volumes of product per delivery. According to [18], it is the most expensive part of the supply chain network and can represent up to 28% of a products total transportation cost measured from its manufacturing place. In this context, planning effective and cost-efficient urban distribution operations is essential.

To reduce costs, logistics service providers search to consolidate deliveries into routes executing multiple customer visits. The Vehicle Routing Problem (VRP) is a canonical representation of such a setting and models how to design a set of customer visiting routes starting and ending at a depot with the objective of minimizing the total operating costs. Vehicle routes are required to cover all customers demanding service and to satisfy a set of operational constraints such as a maximum vehicle capacity and a limited route duration.

The operating costs of the VRP are typically modeled as a weighted sum of total distance and time spend by all routes representing fuel consumption costs and driver wages, respectively. The scientific literature on VRPs is extensive [27], but VRP related research assumes that routes are executed over a flat and horizontal terrain. Also, only a small fraction of the VRP literature considers the effect of weight transported in fuel consumption, [3, 13, 22]. A more realistic fuel consumption model is found in [1] and depends on vehicle characteristics, speed, road grades and weight transported. The authors comment that it is cheaper to traverse roads downhill than uphill and account that the impact of weight transported over costs depends on road grades, but no routing decisions are discusses in this work. To our knowledge, no VRP model within the literature combines the effect of road grades and vehicle weight in the cost function model and, therefore, a classic VRP solution might be inefficient when routes are not planned considering the impact of steep roads.

Consider the example in Figure 1 presenting a single vehicle instance with a depot (0) and two customers (1 y 2). Both customers and the depot are located at the vertices of an equilateral triangle with side d km. Assume, for simplicity, that arc travel times are proportional to distance. The quantity of product demanded by customers 1 and 2 are q_1 ton and q_2 ton , respectively. Also, assume that $q_2 > q_1 > 0$ and that the vehicle has an unlimited freight transportation capacity. The depot and customer 1 are both at seal level (SL), while customer 2 is at α meters above SL. Also, consider that all roads represented by arcs are straight lines having constant road grades.



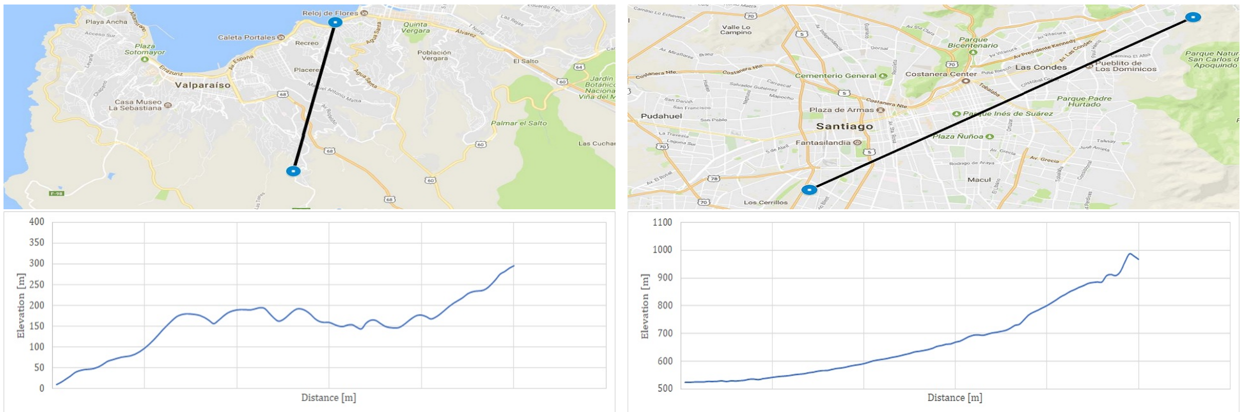
(a) Route $(0,2,1,0)$ (b) Route $(0,1,2,0)$

Figure 1: Feasible routes for a VRP instance with two customers

Both routes, *i.e.*, $(0,2,1,0)$ in Figure 1a and $(0,1,2,0)$ in Figure 1b, are optimal when distance (or time) is minimized. If we consider the effect of vehicle weight in fuel consumption, then we should first meet the heaviest demand and travel lighter most of the road reducing the ton-Km. In our example, we should deliver first to customer 2, who requested the heaviest load, and later to customer 1. In this case, the total ton-Km transported is $2d \cdot q_1 + d \cdot q_2 < 2d \cdot q_2 + d \cdot q_1$.

If we consider the cost-function model proposed in [1] as our objective to minimize, one can show that $(0,1,2,0)$ minimizes fuel consumption if 2 is placed at a high-enough elevation α above SL. For instance, the cost savings compared to route (0,2,1,0) are 9.2%, when $d = 2$ km, $q_1 = 5$ ton, $q_2 = 8$ ton, speed is 30 km/hr and $\alpha = 100$ meters. Intuitively, the vehicle incurs in less fuel consumption by gradually climbing the hill even if it incurs in additional ton-Km.

The previous exercise illustrates the value of considering the effect of slopes on fuel consumption. A cost-efficient routing model over geographies with varying altitudes requires to distinguish different vehicle cargo levels and road grades in arc costs. This problem is specially relevant to urban logistics operations cities with important differences in altitude, such as coastal cities located between mountains and the sea, *e.g.*, Valparaiso, Chile (Figure 2a), where some customers are located at SL and others might be at 300 meters above SL; also, it is important to cities located in valleys surrounded by mountains, such as such as Santiago, Chile (2b) with elevations between 500 and 1000 meters above SL.



(a) Valparaso, 5.6Km segment

(b) Santiago, 22.7Km segment

Figure 2: Map and altitude profiles of two segments in two Chilean cities

2 Literature Review

Now, we review the fuel consumption models present in the literature and classify all related routing problems.

2.1 Fuel consumption model

In [1, 2], the authors propose a fuel consumption model derived from the physics of vehicle dynamics. As stated in Equation 1, the author model establishes that a vehicle's fuel consumption rate $\frac{df}{dt}$ is approximately represented by

$$\frac{df}{dt} \approx \lambda \left(kNV + \frac{P}{\eta_1 \eta_2} \right) \tag{1}$$

, where η_1, η_2 are constants related to drivetrain and engine efficiency, respectively, N is the engine speed, V is the engine displacement, k is the engine friction factor, and λ is the fuel/air equivalence ratio. The value P represents the total tractive power demand requirement placed on the vehicle (in kW) and is defined as

$$P := \frac{\left(M \left(g \cdot \sin(\theta) + g \cdot C_r \cdot \cos(\theta) + a \right) + 0.5 \cdot C_d \cdot A \cdot \rho \cdot v^2 \right) v}{1000} \tag{2}$$

, where M is the vehicle weight in Kg, v is the vehicle speed in meters per second, a is the vehicle's acceleration (m/s^2), g is the gravity acceleration constant ($9.81\, m/s^2$), θ is the road grade in degrees, C_r is the rolling resistance coefficient, A is the frontal area of the vehicle (m^2), ρ is the air density (kg/m^3) and C_d is a drag coefficient.

If all parameters defining P remain constant over a segment of length d meters, the fuel consumption rate is constant over the segment and defined as $f = \frac{df}{dt} \cdot \frac{d}{v}$; this model assumes that speed v is constant ($a = 0$) and disregards additional fuel consumption driven by air conditioning. From such a model, we derive a fuel consumption function analogous to the formula used in [3]

$$f(d,v,M,\theta) = \max\{\beta_1 \cdot \frac{d}{v} + \beta_2 \cdot \mu(\theta) \cdot M \cdot d + \beta_3 \cdot d \cdot v^2, 0\}, \tag{3}$$

where $\beta_1 = \lambda kNV$, $\beta_2 = \frac{\lambda \cdot g}{1000\eta_1\eta_2}$, $\beta_3 = \frac{0.5\lambda C_d A\rho}{1000\eta_1\eta_2}$ and $\mu(\theta) = sin(\theta) + C_r cos(\theta)$, which is a function of road grade θ. The maximum operator avoids recovering energy in cases where the vehicles travels downhill and, therefore, does not consider technologies such as regenerative breaks and electric vehicles.

In Table 1, we describe and provide used values for all physics parameters required to model fuel consumption in this section.

Table 1: Set of Parameters.

Parameter	Description	Value
λ	Fuel/air equivalence ratio $(1/Lkj)$	3.08×10^{-5}
k	Engine friction factor $(kj/rev/L)$	0.2
N	Engine speed (rev/s)	36.67
V	Engine displacement (L)	6.9
g	Gravity acceleration (m/s^2)	9.8
C_r	Rolling resistance coefficient	0.01
C_d	Drag coefficient	0.7
A	Frontal area (m^2)	8
ρ	Air density (kg/m^3)	1.2041
η_1	Drivetrain efficiency	0.45
η_2	Engine efficiency	0.45

2.2 Fuel emission and vehicle routing

The VRP is a classic problem studied within the operations research field, please refer to [27] for multiple mathematical programming formulations, extensions and solution algorithms. In this research, we classify VRP related research that deal with more detailed objective cost functions.

Now, we refer to VRPs planning routes for conventional vehicles to minimize the emission of pollutants, since it is directly related with fuel consumption. In the literature, we find objective functions with arc costs depending on variables such as vehicle speed, load and distance. Some authors also optimize vehicle speed in each arc used by a planned route to minimize emissions. The work in [22] presents a model to estimate fuel consumption based on vehicle speed, load and distance for a set of routes within a given plan. A Tabu Search metaheuristic is used to improve a given routing plan. An experimental evaluation of the proposed method is performed. In [13], the authors present the *Emission Vehicle Routing Problem* (EVRP), which is a VRP variant with the objective of minimizing emissions and fuel consumption as the primary and secondary objective. The authors present a mathematical programming formulation for the EVRP and practical solution approaches based on improvement heuristics. This work uses the pollutant emission function presented by [17]. The authors compare routes planned under the EVRP and the VRP under multiple levels of congestion, and conclude that it may be possible to reduce emissions with a minimal or null increase in routing costs. The work in [19] propose a similar VRP model to minimize time in route and emission of pollutants.

The *Pollution Routing Problem* (PRP) presented in [3] is another extension of the VRP with the objective to minimize total time spent by the routing plan and fuel consumption. It considers vehicle load and speed effect over the cost of each arc and, moreover, it optimizes vehicle speeds over each selected arc. No study related to road grades in conducted. Heuristic and exact algorithms for the PRP are presented in [5, 6, 7]. Also, the research effort in [21, 20] extends the PRP to an heterogeneous fleet of vehicles.

Variable speed and congestion has also been studied within the VRP and PRP literature. The work in [19] propose a VRP model to minimize time in route and emission of pollutants considering two operational periods having different constant vehicle speeds: free flow and congestion. Also, [14] formulate a *time-dependent* PRP (TDPRP), in which consider varying speed limits for the PRP. A similar model is studied in [24] to minimize CO_2 emissions. Recently, [11, 29] integrate time dependence on speed limits with heterogeneous fleet in routing decisions.

2.3 Road grade and routing

No research effort within the vehicle routing literature directly integrates road grades into route costs, but some indirectly approach related topics. The work of [28] propose a methodology to select different vehicle types to different geographies depending on vehicle performance data. Later, these geographically located vehicles are assigned to previously planned vehicle routes; in this work, the effect of road grades is considered at an aggregate level to define geographies. Finally, [9, 10] consider a snow plowing arc routing problem with constraints to avoid plowing roads uphill and favor ones downhill.

In Table 2, we summarize all previous research found in the VRP literature with non-traditional routing costs. Each article is classified according to; whether it explicitly considers vehicle routing decisions or not (Routing); whether it optimizes vehicle speed or takes it as an input (Speed); whether it considers vehicle weight effects over costs or not; whether it considers road grade effects over costs or not (Grade); inclusion of time dependent speed limits due to congestion (TD); and heterogeneous fleet considerations (HF). Based on this literature reviewed, we are not aware of research effort considering the effect of road grades in fuel consumption simultaneaously with vehcile routing decisions.

Table 2: Brief classification of literature review

Reference	Routing	Speed	Weight	Grade	TD	HF
Figliozzi 13	✓				✓	
Bektaş and Laporte 3	✓	✓	✓			
Kuo and Wang 22		✓	✓			
Erdoğan and Miller-Hooks 12	✓					
Demir et al. 6	✓	✓	✓			
Jabali et al. 19	✓	✓			✓	
Franceschetti et al. 14	✓	✓	✓		✓	
Demir et al. 7	✓	✓	✓			
Dabia et al. 5	✓	✓	✓			
Kopfer et al. 21	✓		✓			✓
Koç et al. 20	✓	✓	✓			✓
Dussault et al. 10	✓			✓		
Velázquez-Martínez et al. 28		✓	✓	✓		✓
Ehmke et al. 11	✓		✓		✓	✓
Xiao and Konak 29	✓		✓		✓	✓
Qian and Eglese 24	✓	✓			✓	
VRP-SR	✓		✓	✓		

2.4 Contribution and scope

In this article, we study a Vehicle Routing Problem in regions with steep roads (VRP-SR) and fill a space in the VRP literature integrating a fuel consumption model, that includes varying profiles of road grades per road segment and varying vehicle loads into a vehicle routing problem model. Our objective is to quantify the potential savings in operational costs obtained by planning vehicle routes considering these effects in the cost function compared to one assuming a flat geography. Specifically, our main contributions are:

1. We formally state the VRP-SR as an integer linear programming model (MIP) and begin the study of

routing problems in networks with steep roads.

2. In computationally simulated experiments using the geography of Valparaso, Chile, we empirically verify that the inclusion of road grades in route planning can result in average cost savings of 3.6%; these savings may increase up to 13.2% for some instances.

3. In addition, we obtain valuable managerial insights from our routing plans. (1) First, we build routes that initially cluster customers with small altitudes differences to avoid abrupt changes between consecutive customer visits with a loaded vehicle; our solutions only produce higher altitude changes between consecutive customers after the vehicle drops of a considerable amount of weight. (2) We also notice situations in which it is cost-efficient to split a route into two or more routes, even if a single route can visit all customers, to drop some weight in between routes and climb hills having a lighter vehicle.

In our work, we suppose that it is infeasible to decide on vehicle speeds, which are considered given as a parameter. We believe that this assumption is realistic in an urban environment, in which congestion levels are significant. Also, we assume for simplicity that a vehicle's speed is constant and static over each arc and that we operate a fleet of homogeneous vehicles; the study of heterogeneous fleet and dynamic vehicle speeds are left to future research efforts.

2.5 Outline

The remainder of this article is organized as follows. We formulate the VRP-SR in section 3. A solution algorithm is described in Section 4. We present a case study and draw empirical insights for the VRP-SR over two important Chilean cities in Section 5. Finally, Section 6 provides conclusions and discuss potential future research.

3 Problem statement

Now, we formally define the VRP-SR. First, we requiere to compute the cost incurred by a vehicle when it travels in between a pair of customers; such a cost should not only depend on travel time spent and distance traversed, but also on road gradient and vehicle load. Later, we formulate an Integer Linear Program (ILP) model, which takes previous cost information as input and designs cost-efficient vehicle routes for the VRP-SR.

3.1 Network and data representation

We consider an urban area represented by a directed graph $G = (N,A)$ obtained from a Geographic Information System (GIS), *e.g.*, Google Maps or Openstreet Maps, among others. The set of nodes N represents intersections within the city, and the set of arcs $A \subseteq N \times N$ represents all feasible vehicle road segments (arc) connecting intersections. We suppose that G is given with enough detail to assume that each arc $a \in A$ has a constant road grade θ_a. Also, we assume that vehicle speed v_a and distance d_a are given for each $a \in A$.

The node $d \in N$ represents the depot location, where a dispatcher is assumed to have m identical vehicles with weight Q_0 and an integer load capacity Q; in our experiments we use $Q_0 = 5,5$ and $Q = 13$ tons. The fleet is represented in set $K = \{1,\dots,m\}$. Also, a subset of nodes $C \subseteq N$ represents all customer locations requiring service. Each visit to $i \in C$, demands a delivery of q_i product units and a service time of h_i units. Define $C_0 = C \cup \{0\}$.

3.2 Preprocessing costs

To reduce the problem size and avoid working with vast GIS graphs, we preprocess network information and build a subgraph defined by a given set of customer locations $C \subset N$. Also, define $C_0 := C \cup \{d\}$. Later, we compute load dependent costs $c_{ij}(q)$ for each node pair $i, j \in C_0$ and vehicle load level $q \in \{0,\dots,Q\}$ via a Shortest Path Problem over the arcs in G. Each $c_{ij}(q)$ represents the cheapest vehicle travel cost from node i to j for a vehicle transporting a load of weight q. In the computation of $c_{i,j}(q)$ we consider two sources of operational costs:

- Fuel costs, equal to the total fuel consumed over all road segments used times a fuel cost per unit consumed $\$p_f = 500$ (per liter).
- Driver wages, assumed equal to total driving and service time multiplied by a wage per time $\$p_d = 2,520$ (per hour);

both constants are representative values of Chilean fuel and labor costs. Then, the cost $c_{ij}(q)$ for each origin-destination pair (i, j) and vehicle load $q \in [0,Q]$ is formally defined as

$$c_{i,j(q)} = p_d \cdot \mathbb{I}_{j\neq 0} \cdot h_j + \text{ShortestPath}(\mu^q, i, j), \tag{4}$$

where ShortestPath(μ^q, i, j) is the cost of the shortest path from i to j over the vector of arc costs μ^d defined as

$$\mu_a^q := p_f \cdot f(d_a, v_a, Q_0 + q, \theta_a) + p_d \cdot \frac{d_a}{v_a} \tag{5}$$

for each arc $a \in A$ and depending on the fuel consumption model (3) and travel times. The indicator function $\mathbb{I}_{j \neq 0}$ is equal to 1 when the destination node differs from the depot and accounts for service time costs. By assuming integer load levels we can may compute $c_{ij}(q)$ for each $q \in \{0, \ldots, Q\}$ and $i, j \in C$ efficiently as $\mathscr{O}\{Q|C|^2\}$ calls of a regular Shortest Path algorithm, *e.g.*, Dijkstra's procedure [8].

3.3 Problem formulation

We now formulate the VRP-SR as an ILP with load-dependent arc costs and variables. Let the binary variable $x_{i,j,q}$ equal to 1 when any vehicle travels from node $i \in C_0$ to $j \in C_0$ loaded with $q \in \{q_i, \ldots, Q\}$ units of product and 0 otherwise. The ILP model defined in Equations (6) formally defines the VRP-SR

$$\min \sum_{i,j \in C_0 : i \neq j} \sum_{q=0}^{Q} c_{i,j}(q) \cdot x_{i,j,q}, \tag{6a}$$

$$\text{s.t.} \sum_{i \in C} x_{i,0,0} \leq m, \tag{6b}$$

$$\sum_{q=q_j}^{Q} \sum_{i \in C_0} x_{i,j,q} = 1, \qquad \forall j \in C, \forall q \in \{q_i, \ldots, Q\}, \tag{6c}$$

$$\sum_{i \in C_0} x_{i,j,q} = \sum_{i \in C_0} x_{j,i,q-q_j}, \qquad \forall j \in C, \forall q \in \{q_i, \ldots, Q\}, \tag{6d}$$

$$x_{i,j,q} \in \{0,1\}, \qquad \forall i, j \in C_0 : i \neq j, \forall q \in \{q_j, \ldots, Q\}, \tag{6e}$$

where the objective of the problem defined in Equation (6a) is to minimize total operational costs. Constraint (6b) limits available fleet size establishing that no more that m vehicles can depart from the depot. The family of constraints defined in (6c) imposes to visit each customer exactly once and constraints (6d) establish flow conservation of vehicle routes and track vehicle load consumption at each customer node; these last constraints also eliminate subtours similarly to a capacity-indexed VRP formulation [23]. Finally, constraints (6e) guarantee the binary nature of arc variables.

3.4 Benchmark model over a flat terrain

We also define $c_{i,j}^{FLAT}(q)$ as the cost between any origin-destination pair (i, j) over an identical, but flat terrain. We obtain such costs by computing (4) analogously to $c_{i,j}(q)$ but artificially imposing that $\theta_a = 0$ for each $a \in A$ in (5); all remaining parameters are kept unaltered.

The resulting VRP-SR using costs $c_{i,j}^{FLAT}(q)$ in the objective function will be referred as "The flat VRP" model, which emulates planning routes without considering road grades in fuel consumption model. Such a model will be our benchmark. Compared to a regular VRP, The flat VRP is a richer routing model considering vehicle load effects in fuel consumption. In our experiments, we will compare the full version of the VRP-SR against this flat VRP to isolate benefits obtained by considering road grades in route planning.

3.5 Example of solution over a test instance

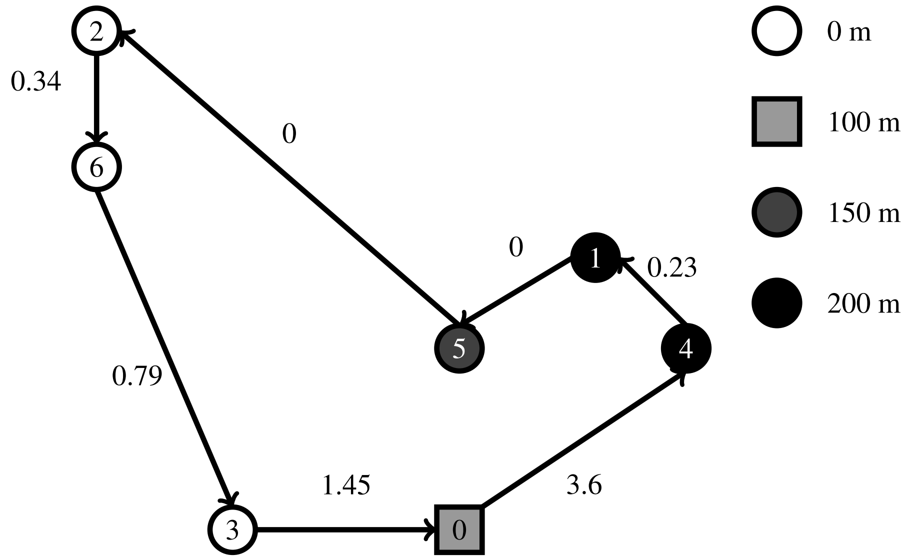
As follows, we present an optimal solution over a small instance illustrated in Figure (3) to present potential savings. Six customer nodes are connected to each other via single arc roads with constant grades and euclidean distances; customer node have four different altitudes given by 0,100,150 and 200 meters above sea level. Each altitude is represented in by a different node color in the grayscale; the derker the color, the higher is the node. The demand for each customer is 1 unit, arc speed is 40 Km/hour, and vehicle capacity is 6 units. Also, $p_f = \$500$ per liter o fuel and $p_c = \$2,520$ per driver hour.

Figures (3a) and (3b) present an optimal solution of a flat VRP and of the full VRP-SR model, respectively. Over each arc, we provide its fuel consumption, while the number in parenthesis is fuel consumption if the vehicle traverses each arc in the opposite direction.

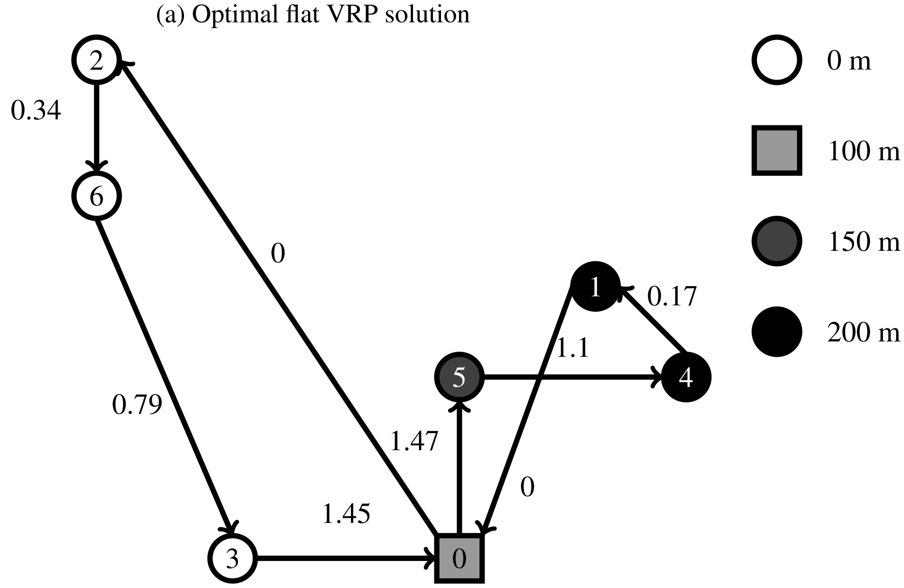
We observe that the flat VRP solution consolidates all customers in a single route, while the solution considering road grades uses two routes, even when it is feasible to cover all customers in just one. Intuitively it breaks the route into two to avoid carrying additional weight while traversing arcs uphill and used the depot as a intermediate storage point. Compared to the flat VRP, the VRP-SR also clusters customers with similar altitudes in different routes and re-sequences the customer visiting order to avoid steep roads and visit a node at an intermediate altitude before climbing up to the highest customers at 200m above SL.

Table 3 indicates the real operational cost, fuel consumption and distance traveled for both solutions considering the effect of road grades.

In this example, an optimal route to the flat VRP model increases operational costs by 8%. We also



(a) Optimal flat VRP solution



(b) Optimal VRP-SR solution

Figure 3: Test Instance

Table 3: Total operating cost, fuel consumption and distance between flat VRP and VRP-SR solutions over test instance.

	flat VRP	VRP-SR	Percentage difference (%)
fuel consumption (liters)	6.4	5.3	-17
distance (km)	11	14	28
total operating cost ($)	4,493	4,121	-8

observe that there exists a trade-off between distance traveled and fuel consumed. By splitting the operation into two routes and re-sequencing customer visits the VRP-SR model increases distance traveled in 28%, however, it also reduces fuel consumption in 17%. The overall costs savings will depend on the relative cost of labor and fuel. For instance, in a scenario where labor is free (autonomous vehicles), the problem will be reduced to optimize fuel consumption and solving the VRP-SR will be crucial. Alternatively, in a scenario with expensive labor, the main objective will be to minimize time (or distance) traveled. In this case, the flat VRP model may be enough.

Unfortunately, the VRP-SR model stated in (6) is difficult and its solution complexity increases exponentially as a function of the vehicle capacity Q and number of customer $|C|$. Therefore, we require a heuristic procedure to deal with real-sized instances, which is designed in Section 4.

4 Solution algorithm

We build a solution algorithm that obtains a good feasible solution to any VRP-SR instance. Our algorithm is based on heuristic frameworks found in the literature exploiting the VRP structure of a feasible solution. It combines solution construction heuristics, intra-route and inter-route local search moves and route partitioning strategies adapted to the VRP-SR. To do this last, all potential candidate solutions to local search moves are computed considering load dependent arc costs. Compared to the VRP, we account for two additional difficulties when implementing heuristics for the VRP-SR First, a local search move is sequence dependent and cannot be evaluated only based on the arcs involved in the move; the cost of all routes involved in a move is recomputed from a point onwards. Second, the maximum vehicle load Q may be too large and could result in an intractably large number of arc cost $c_{ij}(q)$ evaluations for each $q \in \{0,\dots,Q\}$ and $i, j \in C_0 : i \neq j$. To avoid depending on the size of Q, we reduced evaluations and only computed costs for a predefined load grid $\{q_1 = 0, q_2 \dots, q_k, \dots, q_K = Q\}$ of size K. Later, we used $c_{ij}(q'_k)$ to approximate the exact value $c_{ij}(q)$ with $k' = \max\{k \in \{1,\dots,K\} : q_k \leq q\}$; also $c_{ij}(q) \approx c_{ij}(Q)$ when $q > Q$.

An outline of our procedure is described in Algorithm 1, which is referred as the VRP-SR heuristic. It starts building an initial sequence r_0 visiting all customers with the depot at both endpoints, *i.e.*, a macro route; in our experiments we used the *farthest insertion* heuristic [15]. The macro route visiting sequence is later improved via local search operations running algorithm IntraLS, which heuristically solves a single vehicle VRP-SR problem (see section 4.1). A macro route r_0 may be infeasible and violate vehicle capacity constraints. In algorithm Partition, we optimally partition r_0 into a capacity feasible set of routes following the node visiting sequence of r_0 (see section 4.2); such a procedure may be seen as a route-first and cluster-second heuristic [4]. To avoid local optima, our solution is later improved via additional inter-route and intra'route local search moves. Finally, our procedure outputs a feasible solution $\{r_1, \ldots, r_q\}$.

Algorithm 1: VRP-SR heuristic

input: VRP-SR instance;
build an initial macro-route r_0 ;
run local search: $r_0 \leftarrow \text{IntraLS}(r_0)$;
partition macro-route: $(r_1, \ldots, r_q) \leftarrow \text{Partition}(r_0)$;
run inter-route local search: $\{r_1, \ldots, r_q\} \leftarrow \text{InterLS}(r_1, \ldots, r_q)$;
for *i=1,...,q* **do**
    run route local search: $r_i \leftarrow \text{IntraLS}(r_i)$;
end
return $\{r_1, \ldots, r_q\}$;

4.1 Intra-route local search algorithm

Now, we briefly discuss the heuristic referred as IntraLS(r), which searches improvements for a given route $r = \{0, i_1, i_2, \ldots, i_p, 0\}$ with $p \leq n$ customers. The procedure tests route candidates within three sets of polynomially sized route neighborhoods discussed in [25], which are adapted for the VRP-SR; these neighborhoods have at most quadratically many solutions as a function of p and are defined by three local search moves: (1) all possible two-arc exchanges within route r, *i.e.*, 2-opt; (2) all possible relocations of a subsequence of k nodes in r, *i.e.*, Or-opt; and (3) all possible swaps between two nodes in the route, *i.e.*, a node switch. The algorithm runs all possible best improving local search moves over r and updates the best route available. When no improvement is found, we also run random route partial destruction and reconstruction to avoid local optimality.

4.2 Route partitioning algorithm

We optimally partition a macro route $r_0 = \{0, i_1, i_2, \ldots, i_n, 0\}$ covering n customers into multiple feasible routes by solving an auxiliary shortest path problem over an acyclic network that determines the optimal partition points of a given macro route. First, we build an auxiliary graph $G(r_0)$ with nodes $\{0, 1, \ldots, n\}$ for a given macro route r_0. The arc set of $G(r_0)$ is defined as follows:

- We add an arc $(0, j)$ to $G(r_0)$ for each $j = 1, \ldots, n$ representing a capacity feasible subroute in r_0 from

0 to i_j plus a trip back to the depot from j. The cost of arc $(0, j)$ is set equal to the corresponding cost of the subroute , *i.e.*, route $\{0, i_1, \ldots, i_j, 0\}$.

- Also, we also add an arc (k, j) to $G(r_0)$ for each $k = 1, .., n$ and, $j = k+1, \ldots, n+1$ representing a capacity feasible subroute in r_0 from i_{k+1} to i_j plus a trip from the depot to $_{k+1}$ and one back to the depot from i_j. The cost of arc (k, j) is set equal to the corresponding cost of the subroute $\{0, i_{k+1}, \ldots, i_j, 0\}$.

Then a shortest path from 0 to n encodes a minimum cost partition of r_0 into capacity feasible routes. To improve the heuristic, we also test optimal partitions for the macro routes defined by $\{i_j, i_{j+1}, \ldots, i_n, i_1, i_2, \ldots, i_{j-1}\}$ for each $j = 2, \ldots, n$ and select the best solution available.

4.3 Inter-route local search algorithm

Finally, we also run local search moves between each pair of routes for a given feasible route set $\{r_1, \ldots, r_q\}$. InterLS performs nodes swaps over pairs of routes that reduce cost and keep the solution capacity feasible.

5 Computational experiments

In this section, we estimate the potential benefits of including road grades in the cost function of a VRP. We present a comparison of the results obtained from the application of our model, which explicitly considers the vehicle weight and road grades in fuel consumption, versus a benchmark model, in which road grades are not considered, also referred as the flat-VRP. To do so, we conducted computationally simulated experiments using geographical data draw from Valparaiso, which is a particularly mountainous coastal city in Chile. To illustrate the reader, a similar geography may be found in the US city of San Francisco, CA.

We start by explaining the instance-generating procedure. Later, we discuss experimental results from these simulations.

5.1 Scenarios and instances studied

We obtained the network topology $G = (N, A)$ of Valparaiso, Chile from a strategic network representation of the city provided by [26], in which the set of intersections (nodes) is depicted in Figure 4. Later, we used the Google Maps Elevation API [16] to obtain the altitude of each node above the sea level.

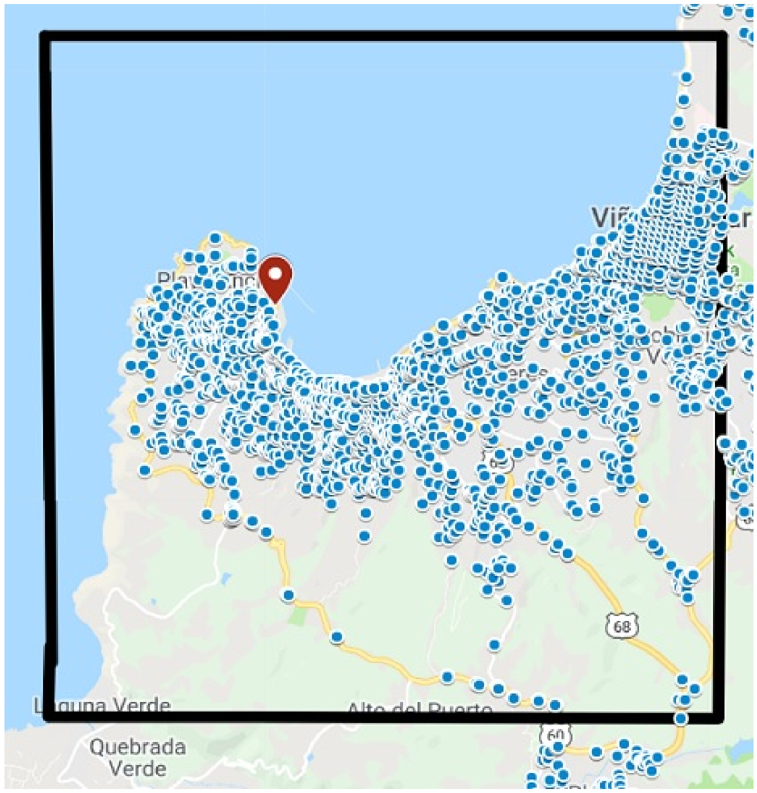


Figure 4: Zone of study.

Given this network and altitude information, we simulated 20 different instances for each number $n = |C|$ of customers within the set $\{10, 20, 50, 100\}$. The depot is always located at sea level by the city port and depicted in red in Figure 4. For each instance, the set C is randomly selected through a geographical sampling stratified by heights among all nodes N in the network, so that the altitudes observed in the set customer locations C is representative of the altitudes of the network's locations. Figure 5 illustrates the proportion of customers that are selected according to the range of heights for the city of Valparaiso.

For comparison purposes, we considered two scenarios:

1. the uncapacitated scenario (UCS), in which weight loads are assumed to be negligible, thus vehicles always travel with the same total weight and may visit an unlimited number of customers per route; and

2. the capacitated scenario (CS), which assumes that all vehicles has a maximum capacity of 13 tons, and, to simplify our setting, we assume that each customer demands a load of weight equal to 1 ton. Therefore, thirteen is the maximum number of customers that can be visited per route. Moreover, in

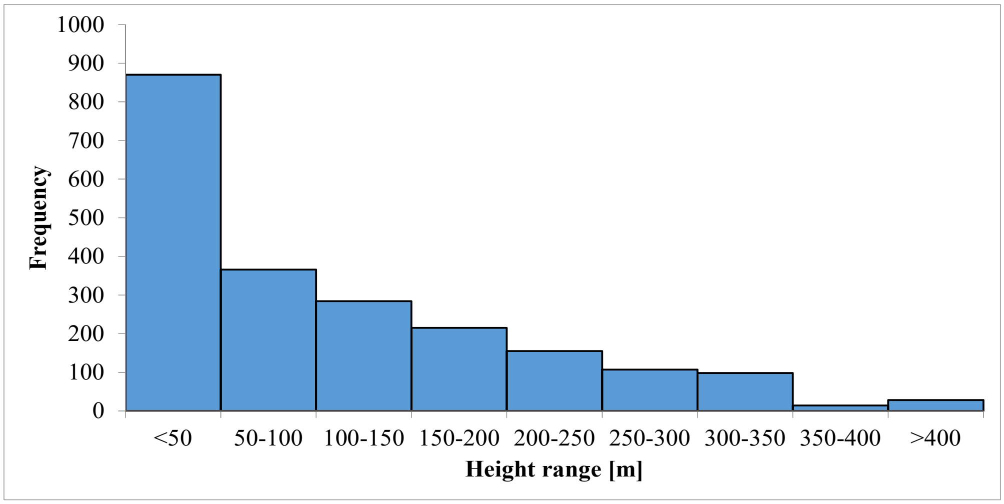


Figure 5: Histogram of customer locations by altitute in Valparaiso, Chile.

this case when computing cost between customers, we used a grid of size $K = 10$.

In addition, for all scenarios, we assume that each vehicle travels at a constant speed of 30 Km/hr.

5.2 Analysis of results

In Table 4, we summarize the experimental results for the two scenarios studied, among the 20 instances for each problem size. We present the maximum, minimum, average and standard deviation of the percentage cost savings obtained when routes are planned with our proposed method (VRP-SR) versus the benchmark model (Flat-VRP), in which slopes are neglected. For each instance, these saving are computed as the percentage cost difference of the solution obtained when solving the VRP-SR compared to the solution for the flat-VRP; both solutions are evaluated using the "real" VRP-SR costs with road grades and without grid approximation errors.

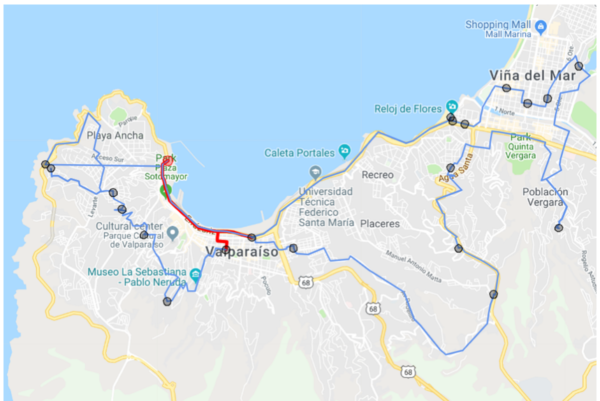
Our results suggest that in the capacitated scenarios (CS), the average savings are significant, being on average 3.6% and ranging between 2.7% up to 4.8% depending on the problem size. These savings tend to be larger, on average, in small-sized instances, and may reach up to 13.2% for particular cases. When vehicle loads are neglected (UCS), the saving are smaller and tend to be insignificant as the instance size decreases.

Figure 6 compares the solution obtained by the Flat-VRP to the VRP-SR for one particular instances

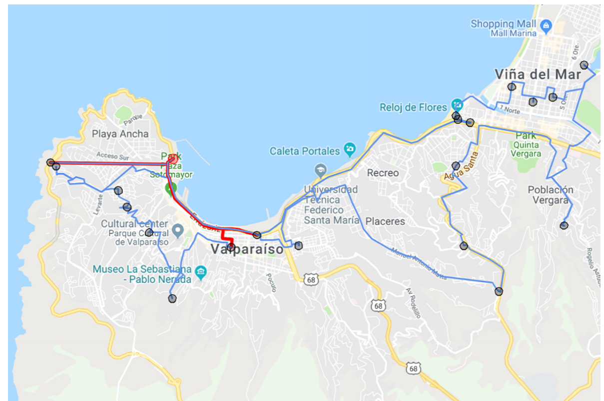
Table 4: Cost savings when slopes are considered: Uncapacitated and Capacitated Scenarios.

		Savings (%) between VRP-SR versus Flat-VRP	
		Uncapacitated Scenario (UCS)	Capacitated Scenario (CS)
$N = 100$	max	3.8	6.2
$(p - value_{UCS} = 0.233)$	min	0	0
$(p - value_{CS} = 0.029)$	average	1.2	2.7
	σ	0.9	1.6
$N = 50$	max	3.0	7.1
$(p - value_{UCS} = 0.227)$	min	0.3	1.1
$(p - value_{CS} = 0.006)$	average	1.4	3.5
	σ	0.9	1.9
$N = 20$	max	3.0	9.1
$(p - value_{UCS} = 0.293)$	min	0	0
$(p - value_{CS} = 0.067)$	average	1.3	3.4
	σ	0.8	2.5
$N = 10$	max	2.3	13.2
$(p - value_{UCS} = 0.404)$	min	0.1	0.5
$(p - value_{CS} = 0.065)$	average	0.9	4.8
	σ	0.6	3.7

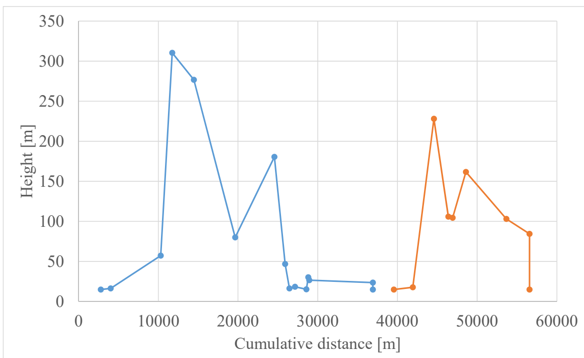
with 20 customers. The first customer visit in each route is depicted in red to illustrate each route's direction. Figures 6c and 6d present the customer altitude profile for both solutions following the customer visiting sequence as a function of the distance traveled along the route. The solution obtained from the VRP-SR model uses three routes, one more than the solution desgined by the flat-VRP model. In this instance, the set of customers visited in the first route are equivalent for both solutions, but the sequence is completely different. The solution of the VRP-SR, in Figure 6b, produces a route that initially clusters customers in low altitude terrain to avoid abrupt altitude changes between consecutive customer visits, particularly with a loaded vehicle. In general, a VRP-SR solution tends to evade climbing hills with a fully loaded vehicle, since this will result in higher fuel consumption. Moreover, as seen in this example, the VRP-SR tends to cluster customers in altitude levels to avoid changes between consecutive customers. In addition, the solution of the VRP-SR discussed in 6 makes more dispatches and thus increases the total distance traveled to reduce fuel consumption. This suggests that, sometimes, it can be advantageous to split a route into two, to leave cargo at the depot and visit high-altitude customers with a lighter vehicle. Table 5 summarizes the total cost and distance traveled by the Flat-VRP and VRP-SR solutions for this instance, in which a reduction of 8.4% is achieved with an increment of 5.6% in distance traveled; again, this highlights the trade-off between distance traveled and fuel consumption.



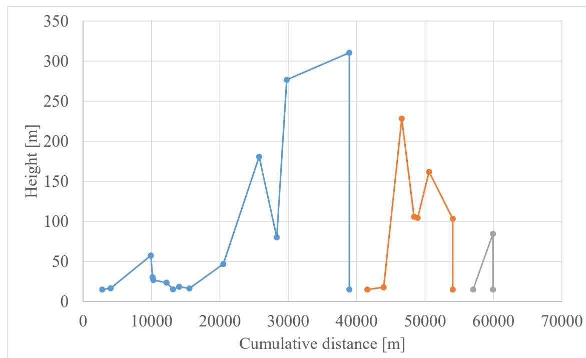
(a) Flat-VRP



(b) VRP-SR



(c) Flat-VRP height profile



(d) VRP-SR height profile

Figure 6: Comparison between Flat-VRP and VRP-SR including height profiles on an instance with 20 customers.

Table 5: Comparison of cost and distance between Flat-VRP and VRP-SR on an instance with 20 customers.

	Flat-VRP	VRP-SR	Δ (%)
Total cost (CLP $)	23,396	21,431	-8.4
Total travelled distance (km)	56.5	59.9	5.6

6 Conclusions

In this article, we introduce an extension to the classic VRP model that consideres the impact of road grades in the fuel consumption and, therefore, plans routes with more accurate cost information. We also present a mathematical formulation of this vehicle routing problem with steep roads (VRP-SR) and implement a heuristic solution method for it, which allowed us to evaluate the impact of including road grade effects on route plans. In our computational experiments based on altitude data and real road networks from the city of Valparaiso, Chile, we obtained savings up to 13.2%. This suggests that this refinement in routing cost may be significant depending on the city's geography.

In general, the routes designed when solving the VRP-SR tend to sacrifice distance traveled to reduce fuel consumption. Also, these solutions tend to cluster customers in zones with similar altitudes and avoid abrupt changes along the route, specially with a loaded vehicle. In particular, the routes generated avoid climbing steep roads with a heavily loaded vehicle. Moreover, our solutions obtained suggest that it is sometimes beneficial to split routes and climb hills with lighter vehicles, even when vehicle load capacity is not binding.

This article opens a rich family of related VRP problems. Future research may include studying the VRP-SR setting with variable arc speeds and heterogeneous fleet problems, among others.

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