Day-Ahead Contingency-Constrained Unit Commitment with Co-Optimized Post-Contingency Transmission Switching

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Abstract

This paper addresses the incorporation of transmission switching in the contingency-constrained unit commitment problem within the context of co-optimized electricity markets for energy and reserves. The proposed generation scheduling model differs from existing formulations due to the joint consideration of four major complicating factors. First, transmission switching actions are considered both in the pre- and post-contingency states, thereby requiring binary post-contingency variables. Secondly, generation scheduling and transmission switching actions are co-optimized. In addition, the time-coupled operation of generating units is precisely characterized. Finally, practical features of modern power systems, such as uncertain nodal net injections and the operation of energy storage, are also considered. The proposed model is cast as a challenging mixed-integer program for which the off-the-shelf software customarily used for simpler models may lead to intractability even for moderately-sized instances. In order to circumvent this computational issue, this paper presents an enhanced and novel application of an exact nested column-and-constraint generation algorithm featuring the inclusion of valid constraints to improve the overall computational performance. Numerical simulations based on the IEEE 118- and 300-bus systems demonstrate the effective performance of the proposed approach as well as its economic and operational advantages over existing models disregarding post-contingency transmission switching.

Nomenclature

The symbols used in this paper are defined in this section. Superscript "m" is used to represent new variables in the inner master problem. Superscripts "(k)" and "(m)" are used to denote the value of a variable at outer-loop iteration k and inner-loop iteration m, respectively.

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Dual Variables

β_{bt} Dual variable associated with the power balance equation at bus b in period t in the lower level of the oracle problem.

γ_{it}, χ_{it} Dual variables associated with the constraints imposing lower and upper bounds for $\tilde{p}_{it}$.

ω_{lt}, ζ_{lt} Dual variables associated with the constraints relating line flows and phase angles for line l in period t in the lower level of the oracle problem.

π_{lt}, σ_{lt} Dual variables associated with the constraints imposing lower and upper bounds for $\tilde{f}_{lt}$.

Functions

$\boldsymbol{f}(\cdot)$ Vector of linear functions defining the set of contingency states.

$C^P_{it}(\cdot)$ Production cost function offered by generator i in period t.

Parameters

$\Delta^{dn}_{bt}, \Delta^{up}_{bt}$ Maximum reduction and increase in the net injection at bus b in period t.

ϵ^i, ϵ^o Inner- and outer-loop convergence parameters.

η^p_s, η^s_s Discharging and charging efficiency rates of storage unit s.

$\overline{F}_l$ Rated capacity of transmission line l.

$\overline{R}^{dn}_{it}, \overline{R}^{up}_{it}$ Maximum down- and up-spinning reserve contributions of generator i in period t.

$\underline{P}_{it}, \overline{P}_{it}$ Lower and upper production limits of generator i in period t.

$\underline{SE}_s, \overline{SE}_s$ Lower and upper energy storage limits of storage unit s.

$\underline{SP}_s, \overline{SP}_s$ Lower and upper discharging limits of storage unit s.

$\underline{SS}_s, \overline{SS}_s$ Lower and upper charging limits of storage unit s.

A^c_{it} Parameter that is equal to 1 if generator i is available in period t under contingency state c, being 0 otherwise.

A^c_{lt} Parameter that is equal to 1 if transmission line l is available in period t under contingency state c, being 0 otherwise.

C^I Cost coefficient of power imbalance.

C^{dn}_{it}, C^{up}_{it} Down- and up-spinning reserve costs offered by generator i in period t.

C^p_{st}, C^s_{st} Cost rates offered by storage unit s in period t for discharging and charging power levels.

d_{bt} Power demand at bus b in period t.

K Number of unavailable system components.

LB, UB Lower and upper bounds for the total cost.

M_l Big-M parameter related to transmission line l.

RD_i, RU_i Ramp-down and ramp-up limits of generator i.

SD_i, SU_i Shutdown and start-up ramp limits of generator i.

W Conservativeness parameter.

x_l Reactance of line l.

Sets

$\mathcal{B}$	Set of bus indices b.
$\mathcal{B}^u$	Set of bus indices b with uncertain net power injection.
$\mathcal{C}$	Set of contingency state indices c. The pre-contingency state is represented by $c = 0$.
$\mathcal{C}_k$	Set of contingency state indices c considered at outer-loop iteration k.
$\mathcal{F}_i$	Feasibility set for the decision variables associated with generator i.
$\mathcal{I}$	Set of generator indices i.
$\mathcal{I}_b$	Set of indices i of generators located at bus b.
$\mathcal{L}$	Set of transmission line indices l.
$\mathcal{L}^{TS}$	Set of indices l of switchable transmission lines.
$\mathcal{S}$	Set of indices s of energy storage units.
$\mathcal{S}_b$	Set of indices s of storage units located at bus b.
$\mathcal{T}$	Set of time period indices t.
$b(i)$	Bus where generator i is located.
$fr(l)$	Origin bus index of line l.
$to(l)$	Destination bus index of line l.

Decision Variables

λ_t, ξ_{it}	Auxiliary variables used in the valid constraints.
Φ, Φ^{ap}	Levels of system power imbalance resulting from the oracle problem and the inner master problem.
$\Phi_{bt}^{-c}, \Phi_{bt}^{+c}$	Variables used in the linearization of the absolute value of the power imbalance at bus b in period t under contingency state c.
Φ^w	Worst-case system power imbalance.
$\theta_{bt}^c, \tilde{\theta}_{bt}$	Phase angles at bus b in period t under contingency state c and in the lower level of the oracle problem.
$\tilde{\Phi}_{bt}^-, \tilde{\Phi}_{bt}^+$	Variables used in the linearization of the absolute value of the power imbalance at bus b in period t in the lower level of the oracle problem.
a_{bt}^{dn}	Binary variable that is equal to 1 if the net injection at bus b is at its minimum value in period t, being 0 otherwise.
a_{bt}^{up}	Binary variable that is equal to 1 if the net injection at bus b is at its maximum value in period t, being 0 otherwise.
a_{it}	Binary variable that is equal to 1 if generator i is available in period t, being 0 otherwise.
a_{lt}	Binary variable that is equal to 1 if transmission line l is available in period t, being 0 otherwise.
c_{it}^{sd}, c_{it}^{su}	Shut-down and start-up costs of generator i in period t.
$f_{lt}^c, \tilde{f}_{lt}$	Power flows of line l in period t under contingency state c and in the lower level of the oracle problem.
$p_{it}^c, \tilde{p}_{it}$	Power outputs of generator i in period t under contingency state c and in the lower level of the oracle problem.

p^w Worst-case system production.

r_{it}^{dn}, r_{it}^{up} Down- and up-spinning reserve contributions of generator i in period t.

r_{st}^{dn}, r_{st}^{up} Down- and up-spinning reserve contributions of storage unit s in period t.

se_{st}^c Stored energy in storage unit s in period t under contingency state c.

sp_{st}^c, ss_{st}^c Levels of discharging and charging power of storage unit s in period t under contingency state c.

v_{it} Binary variable that is equal to 1 if generator i is scheduled in period t, being 0 otherwise.

$z_{lt}^c, \tilde{z}_{lt}$ Binary variables that are equal to 1 if transmission line l is connected in period t, being 0 otherwise, under contingency state c and in the lower level of the oracle problem.

1 Introduction

Unit commitment is one of the main tools used by power system operators to manage energy resources one day ahead. The goal of this optimization problem is to schedule and dispatch generators to meet future demand with minimum operating costs [1–3]. In addition to supplying the demand, system operators have to ensure security, i.e., the capability to withstand system component outages and net injection fluctuations. To that end, security-constrained unit commitment (SCUC) models [4] are run to co-optimize energy and ancillary services, such as reserves and ramping [5].

One of the variants for SCUC which most accurately represents the real-world problem is contingency-constrained unit commitment (CCUC) [6–14]. Unlike other formulations potentially leading to suboptimal or infeasible solutions once contingencies occur [6], CCUC ensures reserve deliverability. This practical feature results from considering system operation under every contingency state. Furthermore, as described in [7, 9], and [10], CCUC provides a suitable modeling framework for considering deterministic security criteria based on those currently implemented in industry practice [1, 15]. Recent works also take into account renewable generation uncertainty, fast-acting units, and other practical aspects [8, 11–14].

An operational feature that has also been studied is the modification of the transmission network's topology through the so-called transmission switching (TS) or topology control, whereby lines can be switched on and off by the operator in each time period [16–26]. Although the idea of switching off a functioning transmission line might seem odd, it has been shown to potentially provide significant operating cost reductions and system reliability enhancements. This result stems from Kirchhoff's voltage law, which has to be met for every loop in the system. Thus, by removing certain loops, system operation is less constrained and cheap reserves may be unlocked.

This paper addresses the incorporation of TS into CCUC under a practical deterministic security criterion [1, 15] with emphasis on the practical issues related to security and complexity. Relevant works in the related literature [18, 19, 23] also adopt deterministic security criteria but have focused on pre-contingency TS. Thus, topology changes are only allowed in the normal state or base case, i.e., under no contingency. In [18], pre-contingency TS was first brought to the CCUC framework in day-ahead electricity markets. However, the computational effort required to co-optimize TS and generation scheduling was deemed impractical and heuristics were proposed to address the problem. In [19], a sequential and hence inexact approach was proposed to incorporate pre-contingency TS into the multi-period SCUC. In [23], two-stage robust optimization was proposed to co-optimize pre-contingency TS with generation scheduling over a single-period setting while considering corrective redispatch actions.

Unfortunately, the models described in [18, 19], and [23] disregard post-contingency TS actions, which represent real-time reactive topology changes in response to the occurrence of contingencies. The significance of post-contingency TS, also known as smart corrective switching, has been discussed in [20,22], and [24–26], among others. In [20,22], and [25], the benefits of post-contingency TS were examined within a different operational context, namely contingency analysis for dynamic security assessment. Note that, in such works, generation scheduling, reserve procurement, and pre-contingency TS were deemed as input parameters whereas a single snapshot of the system was examined. An optimal power flow with post-contingency TS was proposed in [24]. In [26], corrective switching was incorporated in a stochastic unit commitment model without considering contingencies or co-optimizing reserve offers. However, despite the practical advantages of corrective switching, little attention has been paid to CCUC with post-contingency

Table 1: Proposed Approach versus the Related Literature

Approach	Pre-Contingency TS	Post-Contingency TS	Co-Optimization of Generation Scheduling and TS	Multi-Period Setting with Inter-Temporal Constraints	Uncertain Nodal Net Injections	Energy Storage
[18]	✓	–	✓	–[a]	–	–
[19]	✓	–	–	✓	–	–
[21]	✓	✓[b]	✓	–	–	–
[23]	✓	–	✓	–	–	–
Proposed approach	✓	✓	✓	✓	✓	✓

[a] The approach proposed in [18] co-optimizes generation scheduling and TS under a multi-period setting with inter-temporal constraints, but the required computational effort was deemed impractical and a simplified time-decoupled version was implemented.

[b] The solution technique applied in [21] leads to intractability for the problem under consideration even for moderately-sized instances.

TS, [21] being the only relevant exception. In [21], using a deterministic security criterion, post-contingency TS was co-optimized with energy and reserve offers for the first time in the literature. For that purpose, a single-period contingency-dependent model was devised wherein system operation was explicitly characterized for all possible contingencies. Unfortunately, the straightforward application of off-the-shelf state-of-the-art software as proposed in [21] leads to intractability for instances considering a practical multi-period setting even for moderately-sized systems. Therefore, new techniques are yet needed to address the co-optimization of energy, reserves, and post-contingency TS actions in multi-period CCUC problems.

In this paper, we extend the state-of-the-art works on CCUC with TS [18, 19, 21, 23] from both a modeling and a methodological perspective, as summarized in Table 1. In this table, symbols "✓" and "–" indicate whether a particular aspect is considered or not.

On the modeling side, both pre- and post-contingency TS actions are jointly considered, unlike [18,19], and [23]. It should also be noted that, in contrast to the sequential approach of [19], TS and generation scheduling are co-optimized. Thus, the pre-contingency stage explicitly takes into account the best network reaction with post-contingency TS. As a consequence, system security, i.e., the capability to withstand the loss of system components, is enhanced. Moreover, the proposed model departs from the only available CCUC formulation considering post-contingency TS [21] by precisely accounting for the effect of TS on a multi-period setting with inter-temporal operational constraints, such as ramping limits, start-up and shutdown costs, and minimum up and down times, which were also neglected in [23]. As a result, the proposed model provides the optimal day-ahead energy and reserve scheduling policy considering both preventive and corrective transmission flexibility. Note that, as per Table 1, there is no work in the related literature that guarantees the exact co-optimization of generation scheduling and TS under a multi-period setting with inter-temporal constraints. Finally, extensions that include uncertain nodal net injections and the operation of energy storage are provided.

From a methodological perspective, the proposed approach addresses the complexity associated with the binary variables required to model the on/off statuses of transmission lines, thereby significantly differing from the related literature [18, 19, 21, 23]. Note that this practical issue is further stressed under the modeling framework adopted in this paper, wherein a multi-period setting is considered and TS is allowed under contingency. Due to its dimension, the resulting mixed-integer program is unsuitable for the direct application of off-the-shelf commercial solvers relying on state-of-the-art branch-and-cut algorithms, as done in [18] and [21]. Furthermore, the use of binary variables for post-contingency TS actions precludes the application of either the method adopted in [19], which is based on Benders decomposition [27], or the standard column-and-constraint generation algorithm (CCGA) [28] utilized in [23]. Note that the nonconvexity of the recourse problem representing the operation under contingency would render both decomposition-based methods essentially heuristics as the acknowledgement of optimality would be prevented.

As an alternative to the methods employed in [18, 19, 21], and [23], we use an exact master-subproblem decomposition technique consisting in an enhanced version of the nested CCGA. The standard nested CCGA [29] has been successfully applied to address other unit commitment models wherein binary recourse actions were related to the operation of energy storage [30] and fast-acting units [31]. The proposed solution approach exploits the countability of the set of possible post-contingency TS actions to devise an additional inner loop. Thus, finite convergence to optimality is guaranteed (see [29], Propositions 2 and 4, for a proof). Moreover, a measure of the distance to the optimum is provided along the iterative process. In order to improve the performance of the decomposition procedure, the nested CCGA is enhanced by adding a set of valid constraints to the outer-loop master problem.

The main contributions of this paper are threefold:

1. From a modeling perspective, a novel formulation is proposed to co-optimize generation scheduling and pre- and post-contingency TS under a multi-period setting, thereby extending the single-period model presented in [21] and the models disregarding post-contingency TS addressed in [18, 19], and [23]. The significance of this modeling contribution is backed by (i) the practical interest of day-ahead unit commitment models and smart corrective TS in current electricity markets, (ii) the drastic increase in the complexity of the resulting optimization problem, which renders the methodologies reported in the closely related literature for simpler models [18, 19, 21, 23] unsuitable for the resulting unit commitment formulation, and (iii) the possibility of modeling extensions to consider practical features present in modern power systems such as uncertain nodal net injections and energy storage.

2. Methodologically, the novel application of an enhanced version of an exact nested column-and-constraint generation algorithm is presented. Thus, for the first time in the literature, multi-period CCUC with co-optimized pre- and post-contingency TS actions is solved with guaranteed finite convergence to optimality. Moreover, in order to speed up the performance of the decomposition procedure, a set of valid constraints is added. The beneficial effect of the proposed enhancement is backed by the reduction in both the computational effort and the number of iterations required to converge.

3. A study of the benefits of pre- and post-contingency TS (henceforth dubbed PPTS) is presented for several benchmarks including the IEEE 118- and 300-bus systems in a day-ahead setting. This study illustrates the potential reductions in costs and worst-case levels of power imbalance provided by PPTS when compared to the cases considering no TS and only pre-contingency TS. This contribution is relevant for two reasons. First, successful numerical experience is reported with case studies that are far larger than those customarily examined in the state-of-the-art references [18, 19, 21, 23]. Secondly, existing approaches do not allow conducting such a study with current computing capabilities.

The remainder of this paper is organized as follows. In Section 2, the CCUC problem with PPTS is formulated. In Section 3, the solution methodology is presented. Section 4 discusses possible extensions of the proposed formulation to allow for features such as uncertain nodal net injections and energy storage. Section 5 provides numerical results illustrating the benefits of PPTS and the computational gains of the proposed decomposition technique. Finally, conclusions are drawn in Section 6.

2 Problem Formulation

The proposed model is cast as a mixed-integer program driven by the minimization of the sum of the offer costs and the worst-case power imbalance cost:

$$\underset{\substack{\theta^c_{bt},\Phi^w,\Phi^{-c}_{bt},\Phi^{+c}_{bt},\\ c^{sd}_{it},c^{su}_{it},f^c_{lt},p^c_{it},\\ r^{dn}_{it},r^{up}_{it},v_{it},z^c_{lt}}}{\text{Minimize}} \quad \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\left[C^P_{it}(p^0_{it},v_{it}) + C^{up}_{it}r^{up}_{it} + C^{dn}_{it}r^{dn}_{it} + c^{su}_{it} + c^{sd}_{it}\right] + C^I\Phi^w \tag{1}$$

subject to:

$$\Phi^w \geq \sum_{b\in\mathcal{B}}\sum_{t\in\mathcal{T}}(\Phi^{-c}_{bt} + \Phi^{+c}_{bt});\ \forall c\in\mathcal{C} \tag{2}$$

$$\sum_{i\in\mathcal{I}_b} p^c_{it} + \sum_{l\in\mathcal{L}|to(l)=b} f^c_{lt} - \sum_{l\in\mathcal{L}|fr(l)=b} f^c_{lt} = d_{bt} + \Phi^{-c}_{bt} - \Phi^{+c}_{bt};\ \forall b\in\mathcal{B},\forall t\in\mathcal{T},\forall c\in\mathcal{C} \tag{3}$$

$$-M_l(1-A^c_{lt}z^c_{lt}) \leq f^c_{lt} - \frac{1}{x_l}(\theta^c_{fr(l)t} - \theta^c_{to(l)t});\ \forall l\in\mathcal{L},\forall t\in\mathcal{T},\forall c\in\mathcal{C} \tag{4}$$

$$f^c_{lt} - \frac{1}{x_l}(\theta^c_{fr(l)t} - \theta^c_{to(l)t}) \leq M_l(1-A^c_{lt}z^c_{lt});\ \forall l\in\mathcal{L},\forall t\in\mathcal{T},\forall c\in\mathcal{C} \tag{5}$$

$$-A^c_{lt}z^c_{lt}\overline{F}_l \leq f^c_{lt} \leq A^c_{lt}z^c_{lt}\overline{F}_l;\ \forall l\in\mathcal{L},\forall t\in\mathcal{T},\forall c\in\mathcal{C} \tag{6}$$

$$z^c_{lt} = 1;\ \forall l\in\mathcal{L}\backslash\mathcal{L}^{TS},\forall t\in\mathcal{T},\forall c\in\mathcal{C} \tag{7}$$

$$v_{it}A^c_{it}\underline{P}_{it} \leq p^c_{it} \leq v_{it}A^c_{it}\overline{P}_{it};\ \forall i\in\mathcal{I},\forall t\in\mathcal{T},\forall c\in\mathcal{C} \tag{8}$$

$$A^c_{it}(p^0_{it} - r^{dn}_{it}) \leq p^c_{it} \leq A^c_{it}(p^0_{it} + r^{up}_{it});\ \forall i\in\mathcal{I},\forall t\in\mathcal{T},\forall c\in\mathcal{C} \tag{9}$$

$$0 \leq r^{up}_{it} \leq \overline{R}^{up}_{it};\ \forall i\in\mathcal{I},\forall t\in\mathcal{T} \tag{10}$$

$$0 \leq r^{dn}_{it} \leq \overline{R}^{dn}_{it};\ \forall i\in\mathcal{I},\forall t\in\mathcal{T} \tag{11}$$

$$p^0_{it} - p^0_{it-1} \leq RU_i v_{it-1} + SU_i(v_{it} - v_{it-1}) + \overline{P}_{it}(1-v_{it});\ \forall i\in\mathcal{I},\forall t\in\mathcal{T} \tag{12}$$

$$p^0_{it-1} - p^0_{it} \leq RD_i v_{it} + SD_i(v_{it-1} - v_{it}) + \overline{P}_{it}(1-v_{it-1});\ \forall i\in\mathcal{I},\forall t\in\mathcal{T} \tag{13}$$

$$\{c^{sd}_{it}\}_{t\in\mathcal{T}},\{c^{su}_{it}\}_{t\in\mathcal{T}},\{v_{it}\}_{t\in\mathcal{T}} \in \mathcal{F}_i;\ \forall i\in\mathcal{I} \tag{14}$$

$$\Phi^{-c}_{bt} \geq 0, \Phi^{+c}_{bt} \geq 0;\ \forall b\in\mathcal{B},\forall t\in\mathcal{T},\forall c\in\mathcal{C} \tag{15}$$

$$z^c_{lt} \in \{0,1\};\ \forall l\in\mathcal{L}^{TS},\forall t\in\mathcal{T},\forall c\in\mathcal{C} \tag{16}$$

$$v_{it} \in \{0,1\};\ \forall i\in\mathcal{I},\forall t\in\mathcal{T}. \tag{17}$$

The objective function minimized in (1) comprises the offered costs of pre-contingency power generation, up- and down-spinning reserve allocation, start-ups, and shutdowns, as well as the cost of the worst-case power imbalance. Constraints (2) ensure that Φ^w represents the worst-case power imbalance. In (3)–(6), the effect of the transmission network is characterized using a dc load flow model, as is customary in currently implemented day-ahead electricity markets [32] and in the closely related literature [18, 19, 21, 23]. Power balances are modeled in (3). Based on [33], expressions (4) and (5) represent line flows while taking into account PPTS and line availability, whereas line flow capacity limits are set in (6). As per (7), non-switchable lines are always switched on. In (8), power outputs are bounded. In (9), the relationship between power outputs and reserve contributions is characterized. It should be noted that generator availability is considered in (8) and (9). In (10) and (11), bounds on up- and down-spinning reserve contributions are respectively imposed. Expressions (12) and (13) model the ramping limitations. Note that, by accounting for the minimum and maximum possible power outputs in the post-contingency stage, coherent post-contingency ramping is ensured [34]. Start-up and shutdown offer costs as well as minimum up and down times are formulated in (14) in a compact way; more details can be found in [35]. Constraints (15) ensure the non-negativity of the variables used in the linearization of the absolute value of the power imbalance. Finally, TS and generation scheduling are modeled by binary variables in (16) and (17), respectively. In this formulation, the contingency state $c = 0$ represents the pre-contingency state, wherein all generators and transmission lines are available.

In (1)–(17), contingency states associated with the prescribed security criterion are characterized through parameters A^c_{it} and A^c_{lt}. Thus, a compact formulation for the security criterion is as follows:

$$\boldsymbol{f}\Big(\{A^c_{it}\}_{i\in\mathcal{I}}, \{A^c_{lt}\}_{l\in\mathcal{L}}\Big) \geq \mathbf{0};\ \forall t \in \mathcal{T}, \forall c \in \mathcal{C} \tag{18}$$

where $\boldsymbol{f}(\cdot)$ is a set of linear constrained functions. As an example, for an $n - K$ criterion, expressions (18) become:

$$\sum_{i\in\mathcal{I}} A^c_{it} + \sum_{l\in\mathcal{L}} A^c_{lt} \geq |\mathcal{I}|+|\mathcal{L}|-K;\ \forall t \in \mathcal{T}, \forall c \in \mathcal{C}. \tag{19}$$

Admittedly, the proposed model leads to results that may be optimistic and, hence, a complete study of the scheduling problem should also consider the effect of reactive power and post-switching stability. This generalization would, however, render the problem essentially intractable through optimization, thereby requiring the use of repeated simulations. These modeling limitations notwithstanding, the solution of the generation scheduling problem based on the dc load flow model while disregarding stability issues is relevant to the system operator as it provides a first estimate of a cost-effective and secure solution [18, 19, 21, 23, 32].

3 Solution Methodology

Directly solving problem (1)–(17) may be computationally intractable due to the need to explicitly model system operation under all contingency states associated with the pre-specified security criterion. In the recent CCUC literature, the concept of umbrella contingencies [36, 37] has been widely utilized to enable the use of master-subproblem decomposition techniques [27, 28] for problems structurally similar to (1)–(17). Such decomposition methods have become the standard solution procedures for CCUC [8–14]. In this context, the subproblem, or oracle problem, is a bilevel program responsible for finding the worst-case contingency state for a given schedule provided by the preceding master problem. Such a contingency state is then inserted into the master problem, which outputs a new schedule. The algorithm converges through the update of the upper and lower bounds obtained at each iteration.

Unfortunately, the presence of binary variables associated with post-contingency TS in the lower level of the oracle problem makes problem (1)–(17) unsuitable for the standard single-loop master-oracle structures [27, 28] used for CCUC [8–14]. As a salient methodological feature, we propose a novel and enhanced application of the nested CCGA [29], which is an exact decomposition technique that consists of two CCGA loops.

3.1 Outer Loop

The outer loop represents the master-oracle structure that is iterated until convergence to determine the solution of the original problem (1)–(17). The outer loop converges once the bounds provided by the master problem and the oracle problem are within a pre-specified tolerance ϵ^o.

3.1.1 Master Problem

The master problem is a relaxation of the original problem (1)–(17) where, at each outer-loop iteration k, $\mathcal{C}$ is replaced with the subset of contingency states $\mathcal{C}_k$ comprising those identified by the oracle problem so far. Solving the master problem yields decisions $p_{it}^{0(k)}$, $v_{it}^{(k)}$, $r_{it}^{up(k)}$, and $r_{it}^{dn(k)}$, which represent the optimal schedule for the set of states $\mathcal{C}_k$. Since the master problem constitutes a relaxation of the original problem, its solution allows computing a lower bound for the optimal value of the objective function (1):

$$LB^{(k)} = \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \left[C_{it}^{P}(p_{it}^{0(k)}, v_{it}^{(k)}) + C_{it}^{up} r_{it}^{up(k)} + C_{it}^{dn} r_{it}^{dn(k)} + c_{it}^{su(k)} + c_{it}^{sd(k)} \right] + C^{I} \Phi^{w(k)}. \quad (20)$$

3.1.2 Oracle Problem

The goal of the oracle problem is to identify the worst-case contingency state for a given schedule obtained by the preceding master problem. To that end, availability parameters A_{it}^{c} and A_{lt}^{c} are replaced with binary variables a_{it} and a_{lt} and the worst-case setting is implemented by a bilevel programming framework [9–12, 14]. As a result, system operation under contingency is implicitly modeled and indices c are dropped. In the bilevel oracle problem, the upper level is responsible for finding the contingency state maximizing the power imbalance, while the lower level obtains the optimal system reaction. The oracle problem for (1)–(17) is presented below. For the sake of clarity, a tilde is used to denote the decision variables modeling system operation under contingency, whereas dual variables are shown in parentheses.

$$\Phi^{(k)} = \underset{a_{it}, a_{lt}}{\text{Max}} \quad \underset{\substack{\tilde{\theta}_{bt}, \tilde{\Phi}_{bt}^{-}, \tilde{\Phi}_{bt}^{+}, \\ \tilde{f}_{lt}, \tilde{p}_{it}, \tilde{z}_{lt}}}{\text{Min}} \quad \sum_{b \in \mathcal{B}} \sum_{t \in \mathcal{T}} \left(\tilde{\Phi}_{bt}^{-} + \tilde{\Phi}_{bt}^{+} \right) \quad (21)$$

subject to:

$$a_{it} \in \{0, 1\}; \; \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \quad (22)$$

$$a_{lt} \in \{0, 1\}; \; \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \quad (23)$$

$$\boldsymbol{f}(\{a_{it}\}_{i \in \mathcal{I}}, \{a_{lt}\}_{l \in \mathcal{L}}) \geq \boldsymbol{0}; \; \forall t \in \mathcal{T} \quad (24)$$

$$\sum_{i \in \mathcal{I}_b} \tilde{p}_{it} + \sum_{l \in \mathcal{L} | to(l) = b} \tilde{f}_{lt} - \sum_{l \in \mathcal{L} | fr(l) = b} \tilde{f}_{lt} = d_{bt} + \tilde{\Phi}_{bt}^{-} - \tilde{\Phi}_{bt}^{+} : (\beta_{bt}); \; \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \quad (25)$$

$$- M_l (1 - a_{lt} \tilde{z}_{lt}) \leq \tilde{f}_{lt} - \frac{1}{x_l} (\tilde{\theta}_{fr(l)t} - \tilde{\theta}_{to(l)t}) : (\omega_{lt}); \; \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \quad (26)$$

$$\tilde{f}_{lt} - \frac{1}{x_l} (\tilde{\theta}_{fr(l)t} - \tilde{\theta}_{to(l)t}) \leq M_l (1 - a_{lt} \tilde{z}_{lt}) : (\zeta_{lt}); \; \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \quad (27)$$

$$-a_{lt}\tilde{z}_{lt}\overline{F}_l \leq \tilde{f}_{lt} \leq a_{lt}\tilde{z}_{lt}\overline{F}_l : (\pi_{lt}, \sigma_{lt}); \forall l \in \mathcal{L}, \forall t \in \mathcal{T} \tag{28}$$

$$\tilde{z}_{lt} = 1;\ \forall l \in \mathcal{L} \backslash \mathcal{L}^{TS}, \forall t \in \mathcal{T} \tag{29}$$

$$a_{it}(p_{it}^{0(k)} - r_{it}^{dn(k)}) \leq \tilde{p}_{it} \leq a_{it}(p_{it}^{0(k)} + r_{it}^{up(k)}) : (\gamma_{it}, \chi_{it});\ \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \tag{30}$$

$$\tilde{\Phi}_{bt}^{-} \geq 0, \tilde{\Phi}_{bt}^{+} \geq 0;\ \forall b \in \mathcal{B}, \forall t \in \mathcal{T} \tag{31}$$

$$\tilde{z}_{lt} \in \{0, 1\};\ \forall l \in \mathcal{L}^{TS}, \forall t \in \mathcal{T}. \tag{32}$$

In (21), the max-min structure allows identifying the contingency state resulting in the maximum power imbalance while taking into consideration the best reaction. Expressions (22) and (23) characterize the variables representing the availability of generators and transmission lines, respectively. In (24), the prescribed security criterion is enforced using the vector $\boldsymbol{f}(\cdot)$ explained in Section 2. Constraints (25)–(27) correspond to the dc power flow model. Bounds for post-contingency line flows are set in (28). In (29), non-switchable lines are forced to be switched on. Constraints (30) impose bounds on post-contingency power outputs based on allocated reserves. Finally, (31) and (32) define the variables modeling power imbalance and post-contingency TS, respectively.

Note that, at each outer-loop iteration k, the following upper bound for the optimal cost can be derived:

$$UB^{(k)} = \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \left[C_{it}^{P}(p_{it}^{0(k)}, v_{it}^{(k)}) + C_{it}^{up} r_{it}^{up(k)} + C_{it}^{dn} r_{it}^{dn(k)} + c_{it}^{su(k)} + c_{it}^{sd(k)} \right] + C^{I} \Phi^{(k)}. \tag{33}$$

3.2 Inner Loop

At each outer-loop iteration k, an inner master problem and an inner subproblem are iterated until convergence to determine the solution of the bilevel oracle problem (21)–(32). The inner loop converges once the bounds provided by the inner master problem and the inner subproblem are within a prescribed tolerance ϵ^{i}.

3.2.1 Inner Subproblem

The inner subproblem comprises the lower-level optimization of the oracle problem for a given contingency state, i.e., the minimization in (21) subject to constraints (25)–(32) where a_{it} and a_{lt} are replaced with the optimal values provided by the previous inner master problem. At each inner-loop iteration m, the solution to the inner subproblem provides a lower bound for the optimal value of the objective function optimized in the oracle problem. The optimal line switching decisions resulting from the inner subproblem at inner-loop iteration m, $\tilde{z}_{lt}^{(m)}$, are fed to the following inner master problem.

3.2.2 Inner Master Problem

The inner master problem represents a single-level relaxation of (21)–(32). Following the methodology presented in [29], the inner master problem at outer-loop iteration k and inner-loop iteration j is formulated as follows:

$$\underset{\substack{\beta_{bt}^{m}, \gamma_{it}^{m}, \zeta_{lt}^{m}, \pi_{lt}^{m}, \sigma_{lt}^{m}, \\ \Phi^{ap}, \chi_{it}^{m}, \omega_{lt}^{m}, a_{it}, a_{lt}}}{\text{Maximize}} \quad \Phi^{ap} \tag{34}$$

subject to:

$$\Phi^{ap} \leq \sum_{t \in \mathcal{T}} \left\{ \sum_{b \in \mathcal{B}} \beta_{bt}^{m} d_{bt} + \sum_{l \in \mathcal{L}} \left[\omega_{lt}^{m} M_l \left(a_{lt} \tilde{z}_{lt}^{(m)} - 1 \right) \right. \right.$$

$$+ \zeta_{lt}^m M_l \Big(a_{lt} \tilde{z}_{lt}^{(m)} - 1 \Big) - \pi_{lt}^m a_{lt} \tilde{z}_{lt}^{(m)} \overline{F}_l - \sigma_{lt}^m a_{lt} \tilde{z}_{lt}^{(m)} \overline{F}_l \Bigg]$$

$$+ \sum_{i \in \mathcal{I}} \Bigg[\gamma_{it}^m a_{it} \Big(p_{it}^{0(k)} - r_{it}^{dn(k)} \Big) - \chi_{it}^m a_{it} \Big(p_{it}^{0(k)} + r_{it}^{up(k)} \Big) \Bigg] \Bigg\}; \ m = 1, \ldots, j \qquad (35)$$

Constraints (22)–(24) (36)

$$\omega_{lt}^m \geq 0, \zeta_{lt}^m \geq 0, \pi_{lt}^m \geq 0, \sigma_{lt}^m \geq 0; \ \forall l \in \mathcal{L}, \forall t \in \mathcal{T}, m = 1, \ldots, j \qquad (37)$$

$$\gamma_{it}^m \geq 0, \chi_{it}^m \geq 0; \ \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, m = 1, \ldots, j \qquad (38)$$

$$\beta_{b(i)t}^m + \gamma_{it}^m - \chi_{it}^m \leq 0; \ \forall i \in \mathcal{I}, \forall t \in \mathcal{T}, m = 1, \ldots, j \qquad (39)$$

$$\beta_{to(l)t}^m - \beta_{fr(l)t}^m + \omega_{lt}^m - \zeta_{lt}^m + \pi_{lt}^m - \sigma_{lt}^m = 0; \ \forall l \in \mathcal{L}, \forall t \in \mathcal{T}, m = 1, \ldots, j \qquad (40)$$

$$-1 \leq \beta_{bt}^m \leq 1; \ \forall b \in \mathcal{B}, \forall t \in \mathcal{T}, m = 1, \ldots, j \qquad (41)$$

$$\sum_{l \in \mathcal{L} | fr(l) = b} \frac{\omega_{lt}^m - \zeta_{lt}^m}{x_l} + \sum_{l \in \mathcal{L} | to(l) = b} \frac{\zeta_{lt}^m - \omega_{lt}^m}{x_l} = 0; \ \forall b \in \mathcal{B}, \forall t \in \mathcal{T}, m = 1, \ldots, j. \qquad (42)$$

Expression (35) contains nonlinearities in the form of products of decision variables. Using the algebraic results presented in [38], these bilinear terms can be linearized and, thus, the inner master problem can be cast as a single-level mixed-integer linear program. The optimal values of variables a_{it} and a_{lt} obtained from the resolution of problem (34)–(42) are used as parameters in the subsequent inner subproblem. Since the inner master problem is a relaxation of (21)–(32), its solution allows computing an upper bound for the optimal value of the objective function optimized in the oracle problem.

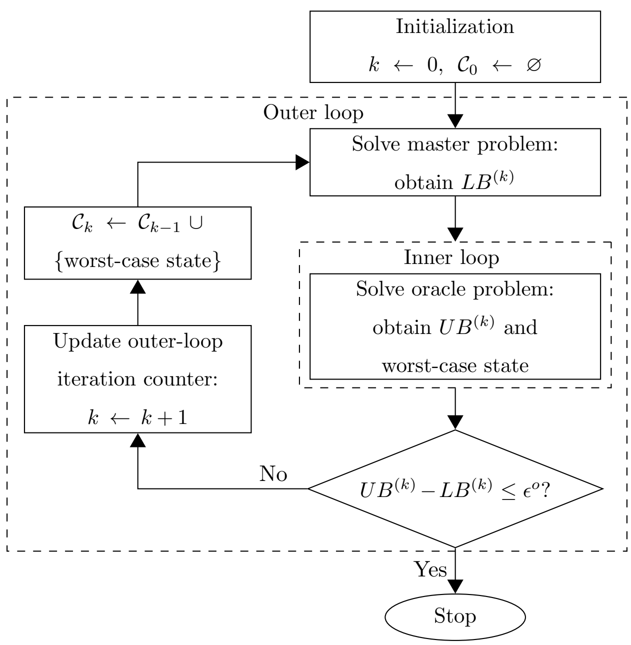

Figure 1: Flowchart of the proposed solution methodology.

3.3 Algorithm Overview

Fig. 1 presents a flowchart of the proposed solution methodology. The key idea is to add to $\mathcal{C}_k$ the worst-case state identified by the oracle problem at each outer-loop iteration k. Thus, for a small value of k, $|\mathcal{C}_k| \ll |\mathcal{C}|$. The main advantage of this methodology is that attaining the optimal solution to the original problem (1)–(17) generally requires considering a small subset of contingency states $\mathcal{C}_k$. Therefore, although the master problem could potentially end up being identical to the original problem, as $\mathcal{C}_k = \mathcal{C}$ should all possible contingency states be examined, in practice only a small subset of $\mathcal{C}$ is utilized. The nested iterative process is stopped when the difference between the upper and lower cost bounds is less than or equal to a pre-specified outer-loop tolerance ϵ^o. For further details on the standard nested CCGA, we refer the interested reader to [29].

3.4 Performance Enhancement

As shown in Fig. 1, the number of variables and constraints in the outer-loop master problem iteratively grows. Therefore, the computational effort required to attain convergence is strongly related to the total number of outer-loop iterations. In addition, the relaxation of the original problem provided by the master problem at the first outer-loop iterations tends to be loose due to the empty or small contingency set $\mathcal{C}_k$.

As a relevant performance enhancement technique, valid constraints associated with generator outages in a simplified single-bus model are incorporated in the master problem. This addition substantially increases the quality of the relaxation along the first iterations. As a result, the total number of outer-loop iterations is reduced, yielding significant speed-ups.

The simplified problem considering a single-bus model and generator outages can be formulated as [7]:

$$\underset{\substack{\Phi^w, p_{it}^0, c_{it}^{sd},\\ c_{it}^{su}, r_{it}^{up}, v_{it}}}{\text{Minimize}} \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\left[C_{it}^P(p_{it}^0, v_{it}) + C_{it}^{up} r_{it}^{up} + c_{it}^{su} + c_{it}^{sd}\right] + C^I \Phi^w \tag{43}$$

subject to:

$$\sum_{i\in\mathcal{I}} p_{it}^0 = \sum_{b\in\mathcal{B}} d_{bt};\ \forall t \in \mathcal{T} \tag{44}$$

$$p_{it}^0 + r_{it}^{up} \le \overline{P}_{it} v_{it};\ \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \tag{45}$$

$$p_{it}^0 \ge \underline{P}_{it} v_{it};\ \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \tag{46}$$

$$\text{Constraints (10), (12)–(14), and (17)} \tag{47}$$

$$\Phi^w \ge \sum_{b\in\mathcal{B}}\sum_{t\in\mathcal{T}} d_{bt} - p^w \tag{48}$$

$$p^w = \underset{a_{it}}{\text{Minimize}} \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}} a_{it}(p_{it}^0 + r_{it}^{up}) \tag{49}$$

subject to:

$$\sum_{i\in\mathcal{I}} a_{it} \ge |\mathcal{I}| - K : (\lambda_t);\ \forall t \in \mathcal{T} \tag{50}$$

$$0 \le a_{it} \le 1 : (\xi_{it});\ \forall i \in \mathcal{I}, \forall t \in \mathcal{T}. \tag{51}$$

Using the duality theory of linear programming, expressions (48)–(51) can be equivalently cast as:

$$\Phi^w \ge \sum_{t\in\mathcal{T}}\left[\sum_{b\in\mathcal{B}} d_{bt} - (|\mathcal{I}| - K)\lambda_t + \sum_{i\in\mathcal{I}} \xi_{it}\right] \tag{52}$$

$$\lambda_t - \xi_{it} \le p_{it}^0 + r_{it}^{up};\ \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \tag{53}$$

$$\xi_{it} \geq 0;\ \forall i \in \mathcal{I}, \forall t \in \mathcal{T} \tag{54}$$

$$\lambda_t \geq 0;\ \forall t \in \mathcal{T}. \tag{55}$$

For every generator outage, expressions (52)–(55) (or (48)–(51), likewise) guarantee that the sum of up-spinning reserve contributions of all available generators is at least equal to the pre-contingency generation of the out-of-service generators. As this condition also holds for the original CCUC model, expressions (52)–(55) form a set of valid constraints that can be added to the outer-loop master problem without cutting off the optimal solution.

4 Extensions

In order to place the focus on the incorporation of TS into CCUC, the approach described in Sections 2 and 3 disregards both the uncertainty associated with nodal net injections and the operation of energy storage systems. The extension of the proposed approach to account for such additional features is described next.

4.1 Uncertain Nodal Net Injections

Uncertain renewable generation and demand can be incorporated using the two-stage robust optimization framework described in [14], which essentially involves the extension of the set of contingency states $\mathcal{C}$. To that end, according to [14] and [39], let $\mathcal{B}^u$ be the set of buses with uncertainty such that their net injections are within intervals $[d_{bt} - \Delta_{bt}^{dn}, d_{bt} + \Delta_{bt}^{up}]$. Based on [14] and [40], the worst-case uncertainty realization will always be a combination of nominal values and upper and lower bounds from these intervals. Therefore, by considering all the possible combinations, $\mathcal{C}$ can be extended to account for both contingencies and uncertain nodal net injections. Hence, this extension would only require modifying problem (1)–(17) by replacing (3) with:

$$\sum_{i \in \mathcal{I}_b} p_{it}^c + \sum_{l \in \mathcal{L}|to(l)=b} f_{lt}^c - \sum_{l \in \mathcal{L}|fr(l)=b} f_{lt}^c = d_{bt} + \Phi_{bt}^{-c} - \Phi_{bt}^{+c};\ \forall b \in \mathcal{B} \backslash \mathcal{B}^u, \forall t \in \mathcal{T}, \forall c \in \mathcal{C} \tag{56}$$

$$\sum_{i \in \mathcal{I}_b} p_{it}^c + \sum_{l \in \mathcal{L}|to(l)=b} f_{lt}^c - \sum_{l \in \mathcal{L}|fr(l)=b} f_{lt}^c = d_{bt} + \Phi_{bt}^{-c} - \Phi_{bt}^{+c} + A_{bt}^{up,c} \Delta_{bt}^{up} - A_{bt}^{dn,c} \Delta_{bt}^{dn};\ \forall b \in \mathcal{B}^u, \forall t \in \mathcal{T}, \forall c \in \mathcal{C} \tag{57}$$

where (56) and (57) model the power balance for the buses without and with uncertain nodal net injections, respectively.

In the solution methodology, such an extension would require modifying 1) the master problem with expressions similar to (56) and (57), and 2) the oracle problem (21)–(32) by adding binary variables a_{bt}^{up} and a_{bt}^{dn} as well as the following constraints:

$$a_{bt}^{up}, a_{bt}^{dn} \in \{0,1\};\ \forall b \in \mathcal{B}^u, \forall t \in \mathcal{T} \tag{58}$$

$$a_{bt}^{up} + a_{bt}^{dn} \leq 1;\ \forall b \in \mathcal{B}^u, \forall t \in \mathcal{T} \tag{59}$$

$$\sum_{b \in \mathcal{B}^u} \left(a_{bt}^{up} + a_{bt}^{dn} \right) = W;\ \forall t \in \mathcal{T} \tag{60}$$

$$\sum_{i \in \mathcal{I}_b} \tilde{p}_{it} + \sum_{l \in \mathcal{L}|to(l)=b} \tilde{f}_{lt} - \sum_{l \in \mathcal{L}|fr(l)=b} \tilde{f}_{lt} = d_{bt} + \tilde{\Phi}_{bt}^{-} - \tilde{\Phi}_{bt}^{+} : (\beta_{bt});\ \forall b \in \mathcal{B} \backslash \mathcal{B}^u, \forall t \in \mathcal{T} \tag{61}$$

$$\sum_{i \in \mathcal{I}_b} \tilde{p}_{it} + \sum_{l \in \mathcal{L}|to(l)=b} \tilde{f}_{lt} - \sum_{l \in \mathcal{L}|fr(l)=b} \tilde{f}_{lt} = d_{bt} + \tilde{\Phi}_{bt}^{-} - \tilde{\Phi}_{bt}^{+} + a_{bt}^{up} \Delta_{bt}^{up} - a_{bt}^{dn} \Delta_{bt}^{dn} : (\beta_{bt});\ \forall b \in \mathcal{B}^u, \forall t \in \mathcal{T}. \tag{62}$$

The binary nature of a_{bt}^{up} and a_{bt}^{dn} is modeled in (58). Constraints (59) ensure that the net injection at each bus with uncertainty cannot simultaneously be at its upper and lower bound. Expressions (60) control the conservativeness of the model by only allowing W buses to experience net injection fluctuations. Finally, (61) and (62) model nodal power balances.

4.2 Energy Storage

The proposed approach can also accommodate energy storage units. For the sake of simplicity, this extension is described for ideal storage units [41], but note that more sophisticated models [30] can be used. With the addition of generic energy storage units, the optimization goal (1) is extended to include the costs of charging and discharging as well as up- and down-spinning reserves offered by these units:

$$\begin{aligned}\underset{\substack{\theta_{bt}^c, \Phi^w, \Phi_{bt}^{-c}, \Phi_{bt}^{+c},\\ c_{it}^{sd}, c_{it}^{su}, f_{lt}^c, p_{it}^c, r_{it}^{dn},\\ r_{st}^{dn}, r_{it}^{up}, r_{st}^{up}, se_{st}^c,\\ sp_{st}^c, ss_{st}^c, v_{it}, z_{lt}^c}}{\text{Minimize}} \quad & \sum_{i \in \mathcal{I}} \sum_{t \in \mathcal{T}} \left[C_{it}^P(p_{it}^0, v_{it}) + C_{it}^{up} r_{it}^{up} + C_{it}^{dn} r_{it}^{dn} + c_{it}^{su} + c_{it}^{sd} \right] \\ & + \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} \left[C_{st}^s ss_{st}^0 + C_{st}^p sp_{st}^0 + C_{st}^{up} r_{st}^{up} + C_{st}^{dn} r_{st}^{dn} \right] + C^I \Phi^w. \end{aligned} \tag{63}$$

Similarly, nodal power balance equations (3) are modified to:

$$\sum_{i \in \mathcal{I}_b} p_{it}^c + \sum_{l \in \mathcal{L} | to(l) = b} f_{lt}^c - \sum_{l \in \mathcal{L} | fr(l) = b} f_{lt}^c = d_{bt} - \sum_{s \in \mathcal{S}_b} sp_{st}^c + \sum_{s \in \mathcal{S}_b} ss_{st}^c + \Phi_{bt}^{-c} - \Phi_{bt}^{+c}; \ \forall b \in \mathcal{B}, \forall t \in \mathcal{T}, \forall c \in \mathcal{C}. \tag{64}$$

Finally, the following constraints related to storage operation are added:

$$\underline{SS}_s \leq ss_{st}^0 + r_{st}^{dn} \leq \overline{SS}_s; \ \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \tag{65}$$

$$\underline{SP}_s \leq sp_{st}^0 + r_{st}^{up} \leq \overline{SP}_s; \ \forall s \in \mathcal{S}, \forall t \in \mathcal{T} \tag{66}$$

$$0 \leq ss_{st}^c \leq r_{st}^{dn}; \ \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall c \in \mathcal{C} \backslash \{0\} \tag{67}$$

$$0 \leq sp_{st}^c \leq r_{st}^{up}; \ \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall c \in \mathcal{C} \backslash \{0\} \tag{68}$$

$$\underline{SE}_s \leq se_{st}^c \leq \overline{SE}_s; \ \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall c \in \mathcal{C} \tag{69}$$

$$se_{st}^c = se_{st-1}^c + \eta_s^s ss_{st}^c - \frac{1}{\eta_s^p} sp_{st}^c; \ \forall s \in \mathcal{S}, \forall t \in \mathcal{T}, \forall c \in \mathcal{C}. \tag{70}$$

Constraints (65) and (66) impose the rated limits for the pre-contingency charging and discharging power levels as well as the allocated reserves. In (67) and (68), the post-contingency charging and discharging power levels are bounded. Expressions (69) model the energy storage limits. Finally, the storage dynamics across periods is modeled in (70).

In the solution methodology, the above modifications affect the master problem and the lower level of the oracle problem. More specifically, following the modifications provided in this section, the master problem should include the modeling of energy storage in the pre-contingency stage, while the lower level of the oracle problem should account for the operation of energy storage in the post-contingency stage. For the sake of brevity, we omit the formulation of the modified problems solved along the iterative process.

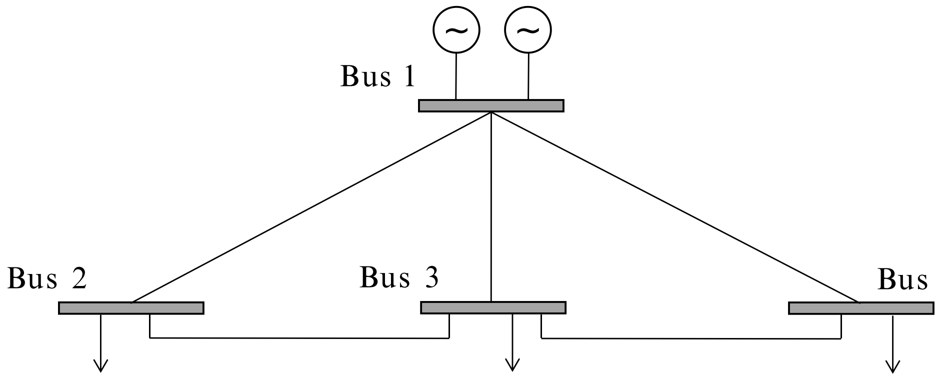

Figure 2: Illustrative 4-bus system.

Table 2: Pre-Contingency Statuses of Line 3-4 for the 4-Bus System

	Hour																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
No TS	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
PreTS	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	1	0	0	0	1	1	1	1
PPTS	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

5 Numerical Results

In order to demonstrate the effective performance of PPTS, three case studies were analyzed. The first benchmark is an illustrative example relying on the 4-bus system described in [21] with the addition of inter-temporal constraints. The topology of this system is shown in Fig. 2. The second case study is a modified version of the IEEE 118-bus system. The modifications include the increase in nodal consumption and the reduction of certain line capacities in order to stress the system. The third case study is a modified version of the IEEE 300-bus system that includes uncertain renewable generation and energy storage units. In all case studies, the standard $n-1$ security criterion was adopted over a 24-hour time span. For the 4-bus system, the security criterion accounted for the outage of every single generator and transmission line. For the 118-bus system, contingencies were considered for the 17 generators with rated power capacity above 200 MW and for the 12 tie lines connecting the three areas in which the system can be split [42]. For the 300-bus system, contingencies were considered for the 20 generators with rated power capacity above 500 MW and for the 7 tie lines connecting the main areas of the system. For all systems, it was assumed that producers offer linear cost functions and that the cost of power imbalance C^I is 10 times the variable cost coefficient of the most expensive generator. Note that the resulting schedules and dispatch levels remain unaltered for larger values of this coefficient. For the sake of reproducibility, system data are provided in [43].

In all case studies, the benefits of PPTS have been analyzed through the comparison with two formulations, namely 1) a model disregarding TS, denoted by No TS, and 2) a model solely considering pre-contingency TS, as done in [18], which is referred to as PreTS. The enhanced nested CCGA presented in Section 3, denoted by E-NCCGA, was employed to address PPTS, while No TS and PreTS were solved through a standard single-loop CCGA, i.e., the nested CCGA excluding the inner loop. The execution of the decomposition procedures was stopped when either a solution was found within a 1% optimality tolerance or a timeout limit of 24 hours was reached. All tests were conducted utilizing the Julia language and CPLEX 12.8 on an Intel Core i7-490K processor at 4.00 GHz and 32 GB of RAM.

5.1 4-Bus Example

The 4-bus system is useful to illustrate the benefits of PPTS, which are a consequence of the significant impact of corrective switching on system operation. This result is shown in Table 2, which lists the pre-contingency statuses of line 3-4 along the scheduling horizon provided by the three models. Unlike No TS and PreTS, the co-optimization of post-contingency TS leads to switching off this line throughout the time span in the pre-contingency stage. The impact of post-contingency TS on solution quality is evidenced by the results displayed in Table 3. Note that "System Cost" refers to the total cost driving the optimization process, i.e., the sum of the costs of pre-contingency power generation, up- and down-spinning reserves, start-ups, and shutdowns, which are also shown in this table, plus the worst-case power imbalance cost. Table 3 also provides the resulting worst-case percent levels of power imbalance in terms of system load. Regarding the potential to reduce power imbalance, PPTS decreased the worst-case levels attained by No TS and PreTS, namely 16.3% and 8.4%, respectively, down to 0%. Moreover, PPTS also reduced the system cost by 54.6% and 38.6% when compared to No TS and PreTS, respectively.

5.2 IEEE 118-Bus System

As shown in Table 3, for the 118-bus system, PPTS successfully ensured power balance for all possible contingency states within the pre-specified security criterion, while No TS and PreTS resulted in worst-case power imbalances of 1.4% and 1.0%, respectively. These results further corroborate the fact that PPTS allows obtaining schedules that better withstand contingencies. Moreover, the economic significance of the robustness granted by PPTS is revealed by the reductions in the system cost by 40.5% and 36.6% over those respectively featured by No TS and PreTS. As evidenced in Table 3, these reductions are a consequence of obtaining an operational schedule of similar production and reserve costs but with no risk of power imbalance under the considered credible contingencies.

Next, with the goal of illustrating the computational advantages of the proposed E-NCCGA, we have applied two alternative methods to solve PPTS for the 118-bus system. First, we utilize off-the-shelf branch-and-cut software directly applied to problem (1)–(17), as done in [18] and [21]. This procedure is hereinafter referred to as BC. The second method is based on the standard nested CCGA and is denoted by NCCGA.

Table 4 lists the computing times required by each approach under the above-described stopping criteria. For the two decomposition-based methods, namely NCCGA and the proposed E-NCCGA, the number of outer-loop iterations is also reported. The results evidence that the use of decomposition significantly outperforms the direct application of commercial software, as this moderately-sized multi-period instance is intractable for BC. In addition, the inclusion of valid constraints in the outer-loop master problem not only did substantially decrease the computing time by 29.6% but also reduced the required number of outer-loop iterations, which is a relevant result for practical application purposes.

Table 3: Impact of Post-Contingency TS on Solution Quality

Case		No TS	PreTS	PPTS
4-Bus System	System Cost ($)	754,285	557,519	342,227
	Production Cost ($)	262,944	262,944	262,944
	Up-Spinning Reserve Cost ($)	53,130	65,699	78,883
	Down-Spinning Reserve Cost ($)	8,585	8,742	0
	Start-up Cost ($)	400	400	400
	Shutdown Cost ($)	0	0	0
	Φ^w (%)	16.3	8.4	0.0
118-Bus System	System Cost ($)	3,390,408	3,182,480	2,018,330
	Production Cost ($)	1,891,174	1,850,098	1,848,431
	Up-Spinning Reserve Cost ($)	95,685	122,970	136,809
	Down-Spinning Reserve Cost ($)	0	0	0
	Start-up Cost ($)	15,640	18,380	20,540
	Shutdown Cost ($)	5,200	7,850	8,500
	Φ^w (%)	1.4	1.0	0.0
300-Bus System	System Cost ($)	9,281,979	9,279,278	8,901,470
	Production Cost ($)	7,753,399	7,747,414	7,739,167
	Up-Spinning Reserve Cost ($)	571,478	575,500	579,772
	Down-Spinning Reserve Cost ($)	21,946	22,269	22,266
	Start-up Cost ($)	2,453	1,897	1,901
	Shutdown Cost ($)	1,958	1,453	1,508
	Φ^w (%)	2.3	2.3	1.4

Table 4 also shows that the proposed E-NCCGA required 55.1 min to attain the high-quality near-optimal solution reported for PPTS. Bearing in mind that a regular computer was used, such a computational effort is acceptable as it is well within the prescribed time frame for day-ahead operation [44–46].

5.3 IEEE 300-Bus System with Wind Farms and Energy Storage

The proposed approach has been tested on a larger benchmark including the features of modern power systems discussed in Section IV. This test system is a modified version of the IEEE 300-bus system comprising 300 buses,

Table 4: Impact of Decomposition and Valid Constraints

Case	Approach	Computing Time (min)	Outer-Loop Iterations
118-Bus System	BC	Timeout	–
	NCCGA	78.3	12
	E-NCCGA	55.1	10
300-Bus System	BC	Timeout	–
	NCCGA	26.5	6
	E-NCCGA	17.9	5

418 lines, 69 generating units, two wind farms, and two energy storage units. The results from this case study are summarized in Tables 3 and 4.

Table 3 indicates that PPTS is capable of obtaining an operational schedule that has similar costs for production, reserves, start-ups, and shutdowns when compared to PreTS while significantly reducing the levels of power imbalance under the considered contingencies. Moreover, the results in Table 4 further corroborate the benefits of the proposed enhancement in the solution methodology, with E-NCCGA outperforming NCCGA by requiring a 32.5% lower computational effort and one fewer outer-loop iteration. Note also that solving such an instance in 17.9 min is well within industry standards and therefore deemed acceptable for a day-ahead setting [44–46].

6 Conclusion

This paper has addressed the contingency-constrained unit commitment problem for the co-optimization of energy, reserves, as well as pre- and post-contingency transmission switching. For the first time in the literature, all the aforementioned features have been considered in a multi-period setting. Straightforwardly solving the proposed formulation with off-the-shelf commercial software exceeds current computing capabilities even for moderately-sized instances. To address this issue, an exact decomposition method based on the nested column-and-constraint generation algorithm is applied. The solution methodology involves an outer loop wherein the original problem is decomposed into a master-oracle structure. The resulting bilevel oracle problem is responsible for obtaining the contingency state yielding the largest power imbalance for a given schedule. The presence of lower-level binary variables in the oracle problem is handled by an inner loop involving an inner master problem and an inner subproblem. Moreover, the computational performance of the standard nested column-and-constraint generation algorithm is improved by the incorporation of a set of valid constraints. Finally, the addition of practical features of modern power systems such as uncertain nodal net injections and energy storage is discussed.

The reported numerical experience allows drawing four main conclusions.

1. The incorporation of post-contingency transmission switching benefits system operation by reducing system costs and worst-case power imbalance levels.

2. From a computational perspective, the proposed solution technique significantly outperforms the direct application of off-the-shelf commercial software adopted in previous related works.

3. The computational effort required to attain high-quality near-optimal solutions is within industry standards for benchmarks such as the IEEE 118- and 300-bus systems over a 24-hour time span.

4. The computational advantage of the proposed enhancement is backed by the substantial reduction in the computing time due to the decrease in the number of outer-loop iterations that are required for convergence.

Future work will explore the computational savings that may be gained from the use of tailored solution algorithms as well as parallel and cloud computing. Other interesting avenues of research are the consideration of an ac load flow model and the characterization of the uncertainty associated with system component outages. In addition, addressing the practical issue of post-switching stability is an important future research topic.

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