

Adaptive Two-stage Stochastic Programming with an Application to Capacity Expansion Planning

Beste Basciftci, Shabbir Ahmed, Nagi Gebraeel

H. Milton Stewart School of Industrial and Systems Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332,
beste.basciftci@gatech.edu, shabbir.ahmed@isye.gatech.edu, nagi.gebraeel@isye.gatech.edu

Multi-stage stochastic programming is a well-established framework for sequential decision making under uncertainty by seeking policies that are fully adapted to the uncertainty. Often, e.g. due to contractual constraints, such flexible and adaptive policies are not desirable, and the decision maker may need to commit to a set of actions for a certain number of planning periods. Static or two-stage stochastic programming frameworks might be better suited to such settings, where the decisions for all periods are made here-and-now and do not adapt to the uncertainty realized. In this paper, we propose a novel alternative approach, where the stages are not predetermined but part of the optimization problem. In particular, each component of the decision policy has an associated *revision point*, a period prior to which the decision is predetermined and after which it is revised to adjust to the uncertainty realized thus far. We motivate this setting using the multi-period newsvendor problem by deriving an optimal adaptive policy. We label the proposed approach as *adaptive two-stage stochastic programming* and provide a generic mixed-integer programming formulation for finite stochastic processes. We show that adaptive two-stage stochastic programming is NP-hard in general. Next, we derive bounds on the value of adaptive two-stage programming in comparison to the two-stage and multi-stage approaches for a specific problem structure inspired by the capacity expansion planning problem. Since directly solving the mixed-integer linear program associated with the adaptive two-stage approach might be very costly for large instances, we propose several heuristic solution algorithms based on the bound analysis. We provide approximation guarantees for these heuristics. Finally, we present an extensive computational study on an electricity generation capacity expansion planning problem and demonstrate the computational and practical impacts of the proposed approach from various perspectives.

Key words: Stochastic programming, sequential decision making, mixed-integer programming, capacity expansion planning, approximation algorithm, energy.

1. Introduction

Optimization in sequential decision making processes under uncertainty is known to be a challenging task. Two-stage and multi-stage stochastic programming are fundamental techniques for modeling these processes, where stage refers to the decision times in planning. In two-stage programs, a set of decisions need to be determined at the beginning of the planning horizon resulting in *static* policies, whereas multi-stage programs allow total flexibility by deriving *fully adaptive* policies depending on the observed uncertainty. Although both approaches have its own pros and cons, the resulting policies may not be sufficient to address a wide range of business settings due to

the flexibility level of the corresponding processes. Specifically, one may need to have a fixed order of decisions before and after a specified time period as detailed below. To address these issues, we propose a *partially adaptive* stochastic programming approach that determines the best time to revise the decisions for the problems with limited flexibility.

There have been many problems in the literature that require partially adaptive policies for determining the best set of actions over a multi-period planning horizon. We will now motivate the applicability of partially adaptive approaches for three specific example settings:

- Capacity expansion management is a strategic level planning problem to determine the expansion times and amount of different resources in areas such as electricity expansion, production planning and network design. This problem involves uncertainties in system demand and investment costs. Setting the expansion decisions at the beginning of the planning results in static and restrictive policies, corresponding to two-stage models. Nevertheless, these actions might not be updated in each period as in multi-stage models since the expansion of resources may require commitments and lead time for establishing the necessary infrastructure. Therefore, fully adaptive policies obtained from multi-stage models may not be feasible either.

- Another example setting for the partially adaptive approaches involves portfolio optimization problems. To construct a portfolio, one may need to determine a fixed sequence of investment decisions for a period of time with a possible option to revise in future. Two-stage stochastic programs may not be sufficient as they entail rigid schedules by not allowing any revision option at all. On the other hand, it may not be appropriate to update decisions in each period due to additional transaction costs associated with rebalancing actions.

- A similar problem occurs in major overhaul decisions of components in maintenance scheduling. The overhaul schedules need to be determined ahead over a multi-year plan with a possibility of revision depending on the changes in components' conditions over time. Two-stage approaches result in restrictive schedules by not considering any change at all. On the other hand, it might be difficult or costly to observe the system's situation and update the schedules at each time period. Therefore, multi-stage approaches may not be applicable for this setting either.

As it can be observed from these three applications, static approaches may not be sufficient to address these problems by not allowing any revision in the schedules. Similarly, fully adaptive approaches may not be suitable either due to the problem characteristics and difficulty of obtaining system's state at each period. Therefore, in this paper, we will focus on the analysis of an adaptive two-stage approach by optimizing the revision points of each decision.

We first review the relevant studies in the literature that adopt partially adaptive policies, specifically developed for inventory and lot sizing problems. These problems focus on determining the inventory and production decisions under nonstationary demand and cost structures over a

multi-period planning horizon. Partially adaptive policies are motivated in these problem settings for establishing the coordination and synchronization of the supply chain systems over dynamic policies (Silver et al. 1988). Bookbinder and Tan (1988) introduces the *static-dynamic* uncertainty strategy for solving a probabilistic lot sizing problem by selecting the replenishment times at the beginning of the planning horizon and determining the corresponding order quantities at these time points. Tarim and Kingsman (2004) extends this concept by developing a mixed-integer linear programming formulation. Variants of this strategy have been studied in Ozen et al. (2012), Zhang et al. (2014), Tunc et al. (2018), Koca et al. (2018) to address the stochastic lot sizing problem under different demand and cost functions, and service level constraints.

In the capacity expansion planning literature, uncertainties over a multi-period planning horizon are addressed with different methods (see Van Mieghem (2003) for an extensive survey). Rajagopalan et al. (1998) studies capacity acquisition decisions and their timing to meet customer demand according to the technological breakthroughs by adopting a dynamic programming based solution methodology. Similarly, stochastic dynamic programming has been applied to this problem (Wang and Nguyen 2017, Lin et al. 2014), despite its disadvantages in incorporating the practical constraints. Stochastic programming is another fundamental methodology to address these problems by representing the underlying uncertainty through scenarios. For instance, Swaminathan (2000) and Riis and Andersen (2004) model this problem as a two-stage stochastic program by first determining the capacity expansion decisions and then adapting the capacity allocations with respect to the scenarios. On the other hand, Ahmed and Sahinidis (2003), Ahmed et al. (2003), and Singh et al. (2009) consider these problems as multi-stage stochastic mixed-integer programs and represent uncertainties through scenario trees. Despite this extensive literature, these studies neglect the need for partial flexibility in capacity expansion problems.

In the stochastic programming literature, intermediate approaches between two-stage and multi-stage models have been studied under different problem contexts to mainly address the computational complexity associated with the multi-stage models. As an example, *shrinking-horizon strategy* involves solving two-stage stochastic programs between predetermined time windows. Specifically, Dempster et al. (2000) and Balasubramanian and Grossmann (2004) consider this strategy to obtain an approximation to multi-period planning problems in oil industry, and multi-product batch plant under demand uncertainty for chemical processes, respectively. Shrinking-horizon strategy is also applied to airline revenue management problem in Chen and de Mello (2010) by proposing heuristics to determine the resolve points under specific assumptions regarding the stochastic process. Another two-stage approximation to multi-stage models is presented in Bodur and Luedtke (2018) using Linear Decision Rules by limiting the decisions to be affine functions of the uncertain parameters. As an alternative intermediate approach, Zou et al. (2018) proposes solving a generation

capacity expansion planning problem by first considering a multi-stage stochastic program until a predefined stage, and then representing it as a two-stage program. They also develop a *rolling horizon heuristic* as discussed in Ahmed (2016) to approximate the multi-stage model. Another line of research (Dupačová 2006) addresses how many stages to have in a multi-stage stochastic program by contamination technique which focuses on limiting the deviations from the underlying uncertainty distribution. These results are then extended to problems with polyhedral risk objectives for financial optimization problems in Dupačová et al. (2009). Several other studies (Dempster and Thompson 2002, Bertocchi et al. 2006) focus on numerical analyses for the choice of planing horizon and stages specifically for portfolio management. However, there has been little to no emphasis on optimizing the stage decisions by determining best time to observe uncertainty for a generic problem setting.

In this study, we propose a partially adaptive stochastic programming approach, in which the revision points are decision variables. Specifically, we consider a fixed sequence of decisions before and after a specific revision point, which is optimized for each decision. This procedure provides significant advantages for the settings where partially adaptive approaches become necessary, and two- and multi-stage models are not appropriate. Our contributions can be summarized as follows:

1. We propose an adaptive two-stage stochastic programming approach, in which we optimize the revision decisions over a static policy. We analyze this approach using a multi-period newsvendor problem, and provide a policy under the adaptive setting. We then develop a mixed-integer linear programming formulation for representing the proposed approach, and prove the NP-Hardness of the resulting stochastic program.

2. We provide analyses on the value of the proposed approach compared to two-stage and multi-stage stochastic programming methods with respect to the choice of the revision decisions. We focus our analyses on a specific structure that encompasses capacity expansion planning problem.

3. We propose solution algorithms for the adaptive two-stage program under the studied problem structure. We provide an approximation guarantee and demonstrate its asymptotic convergence.

4. We demonstrate the benefits of the adaptive two-stage approach on a generation capacity expansion planning problem. Our extensive computational study illustrates the relative gain of the proposed approach with up to 21% reduction in cost, compared to two-stage stochastic programming over different scenario tree structures. Our results also highlight the significant run time improvements in approximating the desired problem with the help of proposed solution algorithms. We also analyze a sample generation expansion plan to examine the practical implications of optimizing revision decisions.

The remainder of the paper is organized as follows: In Section 2, we motivate the adaptive two-stage approach using the newsvendor problem and provide an analytical analysis for its optimal

policy. In Section 3, we formally introduce the adaptive two-stage stochastic programming model. In Section 4, we study the proposed approach on a class of problems encompassing the capacity planning problem and present analytical results on its performance in comparison to the existing methodologies. In Section 5, we develop solution methodologies and derive their approximation guarantees by benefiting from our analytical results. In Section 6, we present an extensive computational study on a generation expansion planning problem in power systems. Section 7 concludes the paper with final remarks and future research directions.

2. A motivating example: The newsvendor problem

2.1. Formulation and optimal policy

In this section, we illustrate the adaptive two-stage approach on a newsvendor problem with T periods. The decision maker determines the order amount in each period t , namely x_t , while minimizing the total expected cost over the planning horizon. Demand in period t , denoted by d_t for $t = 1, \dots, T$, is assumed to be random and independently distributed across periods. We consider a unit holding cost at the end of each period t as h_t , and assume that stockouts are backordered with a cost b_t . We incur an ordering cost c_t per unit in each period t and assume that initial inventory at hand is zero. We also assume that the cost structure satisfies the relationship $c_t - b_t \leq c_{t+1} \leq c_t + h_t$ for $t = 1, \dots, T - 1$, because otherwise it might become more profitable to backorder demand or hold inventory. Additionally, we set $b_T \geq c_T$ to avoid backordering at the end of the planning horizon.

Our goal is to formulate an adaptive program in which we determine the order schedule until a specified time period t^* , then observe the underlying uncertainty and determine the remainder of the planning horizon accordingly. We first note that by defining inventory amount at the end of period t as I_t and considering the inventory relationship $I_t = I_{t-1} + x_t - d_t$, we can rule out the inventory variable using the relationship $I_t := \sum_{t'=1}^t (x_{t'} - d_{t'})$. Consequently, dynamic programming formulation of the adaptive multi-period newsvendor problem with a revision point at t^* can be formulated as follows:

$$\min_{x_1, \dots, x_{t^*-1}} \left\{ \sum_{t=1}^{t^*-1} \left(c_t x_t + E[h_t \max\{\sum_{t'=1}^t (x_{t'} - d_{t'}), 0\}] - b_t \min\{\sum_{t'=1}^t (x_{t'} - d_{t'}), 0\}] \right) + E[Q_{t^*}(\sum_{t=1}^{t^*-1} (x_t - d_t))] \right\}, \quad (1)$$

where

$$Q_{t^*}(s) = \min_{x_{t^*}, \dots, x_T} \left\{ \sum_{t=t^*}^T \left(c_t x_t + E[h_t \max\{s + \sum_{t'=t^*}^t (x_{t'} - d_{t'}), 0\}] - b_t \min\{s + \sum_{t'=t^*}^t (x_{t'} - d_{t'}), 0\}] \right) \right\}. \quad (2)$$

THEOREM 1. Order quantity for the adaptive two-stage approach (1) can be represented in the following form:

$$\tilde{F}_{1,t}(X_{1,t}) = \frac{-c_t + c_{t+1} + b_t}{h_t + b_t}, \quad t = 1, \dots, t^* - 1, \quad (3)$$

$$\tilde{F}_{t^*,t}(s_{t^*} + X_{t^*,t}) = \frac{-c_t + c_{t+1} + b_t}{h_t + b_t}, \quad t = t^*, \dots, T, \quad (4)$$

where $X_{i,j} = \sum_{t=i}^j x_t$, $D_{i,j} = \sum_{t=i}^j d_t$, $s_{t^*} = X_{1,t^*-1} - D_{1,t^*-1}$, $c_{T+1} = 0$, and $\tilde{F}_{i,j}$ is the cumulative distribution function of $D_{i,j}$.

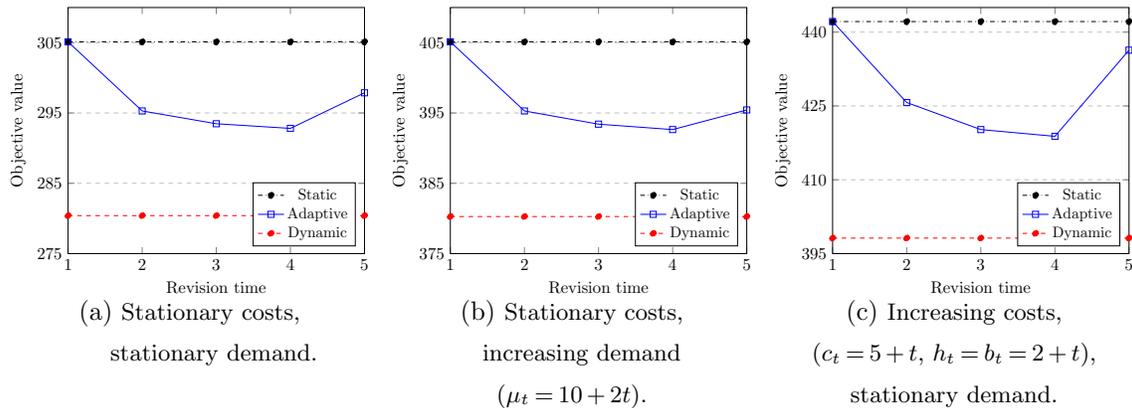
Proof: See Appendix A.1. \square

Using Theorem 1, we can show that the optimal adaptive two-stage solution follows an order up-to policy. Let $\{X_{1,t}^*\}_{t=1}^T$ be the cumulative order quantities obtained by Theorem 1. We have i) $X_{1,t}^* = \tilde{F}_{1,t}^{-1}(\frac{-c_t + c_{t+1} + b_t}{h_t + b_t})$ for $t = 1, \dots, t^* - 1$, and ii) $X_{t^*,t}^* = \tilde{F}_{t^*,t}^{-1}(\frac{-c_t + c_{t+1} + b_t}{h_t + b_t}) - s_{t^*}$ for $t = t^*, \dots, T$. Next, we derive the order amount of each period as follows: $x_1^* = X_{1,1}^*$, and $x_t^* = \max\{X_{1,t}^* - X_{1,t-1}^*, 0\}$ for $t = 2, \dots, t^* - 1$. At time t^* , we observe the cumulative net inventory of that period, s_{t^*} . Then, we derive the remaining ordering policy as $x_{t^*}^* = \max\{X_{t^*,t^*}^*, 0\}$, and $x_t^* = \max\{X_{t^*,t}^* - \sum_{t'=t^*}^{t-1} x_{t'}^*, 0\}$ for $t = t^* + 1, \dots, T$. We note that when t^* is set to 1, and s represents the initial inventory, then the adaptive approach converts into a fully static setting where the decision maker determines the order schedules until the end of the planning horizon ahead of the planning.

2.2. Illustrative example

To demonstrate the importance of the time to revise our decisions, we illustrate the performance of the adaptive approach under different revision times. We consider $T = 5$, and assume demand in each period is normally and independently distributed with $d_t \sim N(\mu_t = 10, \sigma_t^2 = 4)$, and cost values are set to $c_t = 5$, $h_t = 2$ for $t = 1, \dots, 5$ in stationary setting. We let $b_t = h_t$ for the first 4 periods, and set $b_5 = c_5 + 1$ to ensure backordering is costly in the last period.

We demonstrate how policies are affected from different revision times by evaluating them under 1000 different demand scenarios in Figure 1, and compare them with static and dynamic order up-to policies (see Zipkin (2000)). Specifically, order schedule is determined ahead of the planning in static setting, and ordering decisions can be revised in each period by observing the underlying demand in dynamic policies. We illustrate three cases depending on the cost and demand parameters. As expected, we observe that the fully static and dynamic cases result in the highest and lowest costs, respectively. For the adaptive approach, revising at 4^{th} period gives the least cost in all settings, and objective function value is significantly affected by the choice of the revision time.

Figure 1 Objective values under different revision times.


3. Adaptive Two-Stage Stochastic Programming Formulation, Complexity and Value

In this section, we first propose a generic formulation of the adaptive two-stage approach. Then, we compare the adaptive two-stage approach with the existing stochastic programming methodologies along with their respective decision structures. Finally, we show that solving the adaptive two-stage stochastic programming is NP-hard.

3.1. Generic formulation of adaptive two-stage approach

We first describe a generic formulation for the adaptive two-stage approach, and then extend it to a stochastic setting. We consider a sequential decision making problem with T periods, in which we take into account two sets of decisions. The *state* variables $\{x_t(\xi_{[t]})\}_{t=1}^T$ represent the primary decisions given the data vector $\xi_{[t]}$, namely the data available until period t . These variables are used for linking decisions of different time periods to each other. The *stage* variables $\{y_t(\xi_{[t]})\}_{t=1}^T$ correspond to the secondary decisions that are local to period t . We assume that each state and stage variable have dimensions of I and J , respectively. We formalize the adaptive two-stage approach by allowing one revision decision for each state variable throughout the planning horizon. More specifically, the decision maker determines her decisions for the state variable $x_{it}(\xi_{[t]})$ until its revision time t_i^* for every $i \in \{1, \dots, I\}$. Then, she observes the underlying data until that period, namely $\xi_{[t_i^*]}$, and revises the decisions of the corresponding state variable for the remainder of the planning horizon. Combining the above, we formulate the adaptive two-stage program as follows:

$$\min_{x, \hat{x}, y, r} \sum_{t=1}^T f_t(x_t(\xi_{[t]}), y_t(\xi_{[t]})) \quad (5a)$$

$$\text{s.t. } (x_1(\xi_{[1]}), x_2(\xi_{[2]}), \dots, x_t(\xi_{[t]}), y_t(\xi_{[t]})) \in \mathcal{Z}_t \quad t = 1, \dots, T, \quad (5b)$$

$$r_{it_i^*} = 1 \Leftrightarrow \begin{cases} x_{it}(\xi_{[t]}) = \hat{x}_{it} & t = 1, \dots, t_i^* - 1, \\ x_{it}(\xi_{[t]}) = \hat{x}_{it}(\xi_{[t_i^*]}) & t = t_i^*, \dots, T, \end{cases} \quad i = 1, \dots, I, \quad (5c)$$

$$\sum_{t=1}^T r_{it} = 1 \quad i = 1, \dots, I, \quad (5d)$$

$$r_{it} \in \{0, 1\} \quad t = 1, \dots, T, i = 1, \dots, I, \quad (5e)$$

where the variable r_{it} is 1 if the state variable i has been revised at period t , and the function f_t corresponds to the objective function at period t . The set \mathcal{Z}_t in (5b) represents the set of constraints corresponding to stage t . We consider a linear relationship for the constraints as described in the form (6), where the state decisions until period t is linked with the local decisions of that period. We note that these constraints can be also written in Markovian form by eliminating the state variables until period $t - 1$:

$$\sum_{l=0}^{t-1} C_{t,t-l} x_{t-l}(\xi_{[t-l]}) + D_t y_t(\xi_{[t]}) \geq d_t. \quad (6)$$

We define the auxiliary decision variable \hat{x} for constructing the adaptive relationship depending on the revision time of each state variable, as illustrated in constraint (5c). Thus, if $x_{it}(\xi_{[t]})$ is revised at stage t_i^* , then underlying data $\xi_{[t_i^*]}$ is observed at that period, and the decisions for the remainder of the planning horizon depend on those observations. Constraints (5d) and (5e) ensure that each state variable is revised once during the planning horizon.

As a next step, we focus on the case where data is random, in which we minimize the expected objective function value over the planning horizon. We denote the random data vector between stages t and t' as $\boldsymbol{\xi}_{[t,t']}$, and its realized value as $\xi_{[t,t']}$. For simplifying the notation, we denote the variable $x_t(\xi_{[t]})$ as x_t . In terms of the decision dynamics, we represent the adaptive two-stage stochastic case in a form where we determine the revision time of each state variable in the first level, and the second level turns into a multi-stage stochastic program once the revision times are fixed. Specifically, when the state variable x_{it} has been assigned to a revision time t_i^* , we determine $\{x_{it}\}_{t=1}^{t_i^*-1}$ at the beginning of the planning horizon. Next, we observe the underlying uncertainty, namely $\xi_{[t_i^]}$, and determine the decisions x_{it} for the remainder of the planning horizon, accordingly. The resulting adaptive two-stage stochastic program can be represented in the following form:

$$\begin{aligned} \min_r \min_{(x_1, y_1) \in F_1} \{ & f_1(x_1, y_1) + \mathbb{E}_{\boldsymbol{\xi}_{[2,T]}} \left[\mathbb{E}_{\xi_{[1]}} \left[\min_{(x_2, y_2) \in F_2(x_1, \xi_2, r)} \{ f_2(x_2, y_2) + \dots \right. \right. \right. \\ & \left. \left. \left. + \mathbb{E}_{\boldsymbol{\xi}_{[T,T]}} \left[\mathbb{E}_{\xi_{[T-1]}} \left[\min_{(x_T, y_T) \in F_T(x_{T-1}, \xi_T, r)} \{ f_T(x_T, y_T) \} \right] \right] \right] \right] \}, \end{aligned}$$

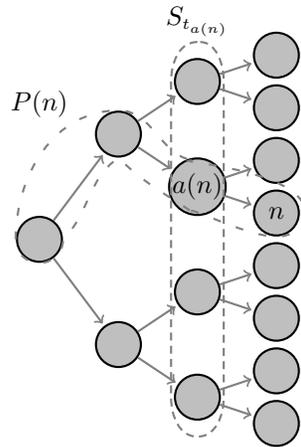
where $F_t(x_{t-1}, \xi_t, r)$ represents the feasible region of stage t , and $\mathbb{E}_{\boldsymbol{\xi}_{[t,T]}} \left[\mathbb{E}_{\xi_{[t-1]}} \right]$ is the expectation operator at stage t by realizing the observations until that period. As the revision decisions are fixed for the inner problem, we can represent the partially adaptive relationship in (5c) within the constraint set of each stage.

3.2. Scenario Tree Formulation

In order to represent the adaptive two-stage approach described in Section 3.1, we approximate the underlying stochastic process by generating finitely many samples. Specifically, scenario tree is a fundamental method to represent uncertainty in sequential decision making processes (Ruszczynski and Shapiro 2003), where each node corresponds to a specific realization of the underlying uncertainty. To construct a scenario tree, sampling approaches similar to the ones proposed for Sample Average Approximation can be adopted to approximate the underlying uncertainty. Theoretical bounds and confidence intervals regarding the construction of these trees and optimality of the solutions are studied extensively in literature (see e.g. Pflug (2001), Shapiro (2003), Kuhn (2005)).

In the remainder of this paper, we consider a scenario tree \mathcal{T} with T stages to model the uncertainty structure in a multi-period problem with T periods. We illustrate a sample scenario tree in Figure 2, and represent each node of the tree as $n \in \mathcal{T}$. We define the set of nodes in each period $1 \leq t \leq T$ as S_t , and the period of a node n as t_n . Each node n , except the root node, has an ancestor, which is denoted as $a(n)$. The unique path from the root node to a specific node n is represented by $P(n)$. We note that each path from the root node to a leaf node corresponds to a scenario, in other words each $P(n)$ gives a scenario when $n \in S_T$. We denote the subtree rooted at node n until period t as $\mathcal{T}(n, t)$ for $t_n \leq t \leq T$. To shorten the notation, when the last period of the subtree is T , we let $\mathcal{T}(n) := \mathcal{T}(n, T)$ for all $n \in \mathcal{T}$. The probability of each node n is given by p_n , where $\sum_{n \in S_t} p_n = 1$ for all $1 \leq t \leq T$.

Figure 2 Scenario tree structure.



Utilizing the scenario tree structure introduced above, a general form multi-stage stochastic program can be formulated as

$$\min_{x,y} \sum_{n \in \mathcal{T}} p_n (a_n^\top x_n + b_n^\top y_n) \tag{7a}$$

$$\text{s.t.} \quad \sum_{m \in P(n)} C_{nm} x_m + D_n y_n \geq d_n \quad \forall n \in \mathcal{T}, \quad (7b)$$

$$x_n \in \mathcal{X}_n, y_n \in \mathcal{Y}_n, \quad \forall n \in \mathcal{T}, \quad (7c)$$

where the decision variables corresponding to node $n \in \mathcal{T}$ are given as x_n, y_n , and the parameters of this node is represented as $(a_n, b_n, C_n, D_n, d_n)$. We denote the *state variables* as $\{x_n\}_{n \in \mathcal{T}}$, and the *stage variables* as $\{y_n\}_{n \in \mathcal{T}}$, where *stage variables* y_n are local variables to their associated stage t_n . Constraints referring to the variables x_n and y_n for each node $n \in \mathcal{T}$ are compactly represented by the sets \mathcal{X}_n and \mathcal{Y}_n . We let the dimensions of the variables $\{x_n\}_{n \in \mathcal{T}}$ and $\{y_n\}_{n \in \mathcal{T}}$ be I and J , respectively. We note that constraint (7b) is analogous to constraint (6).

Two-stage stochastic programs are less adaptive compared to multi-stage approaches since they determine a single static solution for the stage variables per each period, irrespective of the specific realizations of that period. Consequently, two-stage stochastic programming version of the formulation (7) can be represented as

$$\min_{x,y} \quad \sum_{n \in \mathcal{T}} p_n (a_n^\top x_n + b_n^\top y_n), \quad (8a)$$

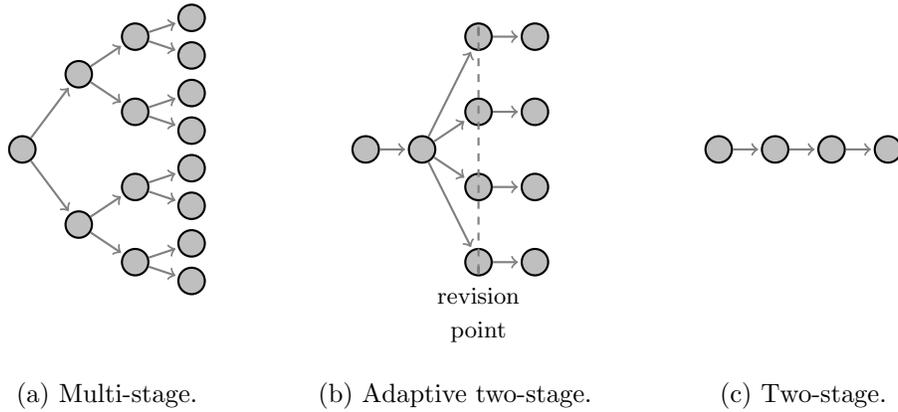
$$\text{s.t.} \quad (7b), (7c)$$

$$x_m = x_n \quad \forall m, n \in S_t, t = 1, \dots, T, \quad (8b)$$

where constraint (8b) ensures that the state variables $\{x_n\}_{n \in \mathcal{T}}$ are determined at the beginning of the planning horizon, and same across different scenarios. We allow $\{y_n\}_{n \in \mathcal{T}}$ decisions to have a multi-stage decision structure by allowing revisions with respect to the underlying uncertainty.

In this study, we focus on problem settings that require an intermediate approach between the multi-stage and two-stage stochastic programming by demanding a fixed sequence of decisions with a possible option to revise in future. Specifically, we propose the adaptive two-stage approach by optimizing the time to revise the decisions over a static policy. We illustrate this concept over scenario trees in Figure 3 by visualizing the decision structures of stochastic programming approaches for the state variable $\{x_n\}_{n \in \mathcal{T}}$. In multi-stage stochastic programming, decisions are specific to each node, whereas two-stage approaches provide a simpler method by resulting in one decision per each time period. We illustrate the adaptive two-stage decision structure by considering one revision point throughout the planning horizon. More specifically, we have a static decision structure before the revision stage. Once we revise our decisions, the remainder trees rooted from each node of the revision stage are compressed to have a single decision for the remaining planning horizon corresponding to that node. We note that the critical point of the adaptive two-stage approach is the choice of the revision point with respect to the underlying uncertainty.

Figure 3 Decision structures for $\{x_n\}_{n \in \mathcal{T}}$ in different stochastic programming approaches.



We formalize the adaptive two-stage approach in (9) by allowing one revision decision for each state variable throughout the planning horizon. The revision times are denoted by the variable t^* .

$$\begin{aligned} \min_{x,y,t^*} \quad & \sum_{n \in \mathcal{T}} p_n \left(\sum_{i=1}^I a_{in} x_{in} + \sum_{j=1}^J b_{jn} y_{jn} \right) & (9a) \\ \text{s.t.} \quad & (7b), (7c) \end{aligned}$$

$$x_{im} = x_{in} \quad \forall m, n \in S_t, \quad t < t_i^*, i = 1, \dots, I, \quad (9b)$$

$$x_{im} = x_{in} \quad \forall m, n \in S_t \cap \mathcal{T}(j), \quad j \in S_{t_i^*}, \quad t \geq t_i^*, i = 1, \dots, I, \quad (9c)$$

$$t_i^* \in \{1, \dots, T\} \quad \forall i \in I. \quad (9d)$$

Here, constraints (9b) and (9c) refer to the adaptive two-stage relationship under the revision decisions t_i^* , as illustrated in Figure 3 and similar to constraint (5c).

As constraints (9b) and (9c) depend on the decision variable t_i^* , we obtain a nonlinear stochastic programming formulation in (9). To linearize this relationship, we introduce an auxiliary binary variable r_{it} for each $i \in I$, which is 1 if the decisions $\{x_{in}\}$ are revised at nodes $n \in S_t$, and 0 otherwise. Combining the above, we can reformulate the adaptive two-stage stochastic program as follows:

$$\min_{x,y,r} \quad \sum_{n \in \mathcal{T}} p_n \left(\sum_{i=1}^I a_{in} x_{in} + \sum_{j=1}^J b_{jn} y_{jn} \right) \quad (10a)$$

$$\text{s.t.} \quad (7b), (7c)$$

$$\sum_{t=1}^T r_{it} = 1 \quad i = 1, \dots, I, \quad (10b)$$

$$x_{im} \geq x_{in} - \bar{x} \left(1 - \sum_{t'=t+1}^T r_{it'} \right) \quad \forall m, n \in S_t, t = 1, \dots, T-1, i = 1, \dots, I, \quad (10c)$$

$$x_{im} \leq x_{in} + \bar{x}(1 - \sum_{t'=t+1}^T r_{it'}) \quad \forall m, n \in S_t, t = 1, \dots, T-1, i = 1, \dots, I, \quad (10d)$$

$$x_{im} \geq x_{in} - \bar{x}(1 - r_{it}) \quad \forall m, n \in S_{t'} \cap \mathcal{T}(l), l \in S_t, t' \geq t, t = 1, \dots, T, i = 1, \dots, I, \quad (10e)$$

$$x_{im} \leq x_{in} + \bar{x}(1 - r_{it}) \quad \forall m, n \in S_{t'} \cap \mathcal{T}(l), l \in S_t, t' \geq t, t = 1, \dots, T, i = 1, \dots, I, \quad (10f)$$

$$r_{it} \in \{0, 1\} \quad \forall i = 1, \dots, I, t = 1, \dots, T. \quad (10g)$$

Here, constraint (10b) ensures that each state variable is revised once throughout the planning horizon. Constraints (10c) and (10d) guarantee that the decision of each state variable until its revision point is determined at the beginning of the planning horizon. Constraints (10e) and (10f) correspond to the decisions after the revision point by observing the underlying uncertainty at that time. We note that the parameter \bar{x} denotes an upper bound value on the decision variables $\{x_{in}\}_{n \in \mathcal{T}}$ for each $i = 1, \dots, I$, whose specific value depends on the problem structure.

REMARK 1. Although optimization model in (10) provides a stochastic mixed-integer linear programming formulation for the adaptive two-stage approach, it requires the addition of exponentially many linear constraints in terms of the number of periods for representing the desired relationship. Additionally, constraints (10c)–(10f) involve big- M coefficients which may weaken the corresponding linear programming relaxation. Consequently, it is computationally challenging to directly solve the formulation (10) when the size of the tree becomes larger (see, for instance, Tables 2 and 3 in Section 6). This computational complexity associated with the adaptive two-stage formulation has motivated us to understand the theoretical and empirical performances of the proposed methodology on specific problem structures.

3.3. Complexity

To understand the difficulty of this problem, we identify its computational complexity.

THEOREM 2. Solving the adaptive two-stage stochastic programming model in (9) is NP-Hard.

Proof: See Appendix A.2. \square

The proof of Theorem 2 demonstrates that the hardness of the adaptive two-stage problem comes from the choice of the revision times, i.e. t_i^* for all $i \in I$. In particular, the adaptive two-stage problem considered in the proof reduces to a linear program with polynomial size when revision time decisions are fixed, becoming polynomially solvable.

3.4. Value of adaptive two-stage stochastic solutions

In this section, we present a way to assess the performances of the stochastic programming approaches according to their adaptiveness level to uncertainty under a given revision decision $t^* \in \mathbb{Z}_+^I$. Specifically, we have the following relationship for the vector $t^* \in \{1, \dots, T\}^I$

$$V^{MS} \leq V^{ATS}(t^*) \leq V^{TS},$$

where V^{MS} , $V^{ATS}(t^*)$, and V^{TS} correspond to the objective values of the multi-stage, adaptive two-stage under given revision decision t^* , and two-stage models, respectively. We note that this relation holds as the two-stage program provides a feasible solution for the adaptive two-stage program under any t^* vector, and the solution of the adaptive two-stage program under any t^* vector is feasible for the multi-stage program.

In order to evaluate the performance of the adaptive two-stage approach, we aim deriving bounds for the $V^{MS} - V^{ATS}(t^*)$, and $V^{TS} - V^{ATS}(t^*)$ for a given t^* vector. We refer to the bound $V^{TS} - V^{ATS}(t^*)$ as *value of adaptive two-stage* (VATS) in the remainder of this paper. We note that in Section 2.2, we illustrate the value of the adaptive two-stage approach in comparison to static and fully dynamic policies over a multi-period newsvendor problem. Here, static and fully dynamic policies align with two-stage and multi-stage stochastic programming approaches, respectively. Our analysis demonstrates that the optimal objective value is significantly affected by the choice of the revision time, making it critical to determine the best time to realize the underlying uncertainty.

4. Analysis of Capacity Expansion Planning Problem

In this section, we study a special problem structure where analytical bounds can be derived to compare the solution performances of two-stage, multi-stage and adaptive two-stage approaches. The structure studied is relevant for a class of problems including capacity expansion and investment planning. In the remainder of this section, we first present the problem formulation for the adaptive two-stage approach under given revision points. Then, we provide a theoretical analysis of the performance of the proposed methodology with respect to the choice of the revision decisions.

4.1. Problem formulation

The capacity expansion problem determines the capacity acquisition decisions of the set of resources \mathcal{I} by allocating the corresponding capacities to the tasks \mathcal{J} , while satisfying the demand of items \mathcal{K} and capacity constraints. A stochastic capacity expansion problem with T periods can be written as follows:

$$\min_{x,y} \sum_{n \in \mathcal{T}} p_n (a_n^\top x_n + b_n^\top y_n) \tag{11a}$$

$$\text{s.t. } A_n y_n \leq \sum_{m \in P(n)} x_m \quad \forall n \in \mathcal{T}, \tag{11b}$$

$$B_n y_n \geq d_n \quad \forall n \in \mathcal{T}, \tag{11c}$$

$$x_n \in \mathbb{Z}_+^{|\mathcal{I}|}, y_n \in \mathbb{R}_+^{|\mathcal{J}|} \quad \forall n \in \mathcal{T}, \tag{11d}$$

where the decision variables x_n , y_n , and the parameter p_n represent the capacity acquisition decisions, capacity allocation decisions, and probability corresponding to the node n , respectively. By

adopting the scenario tree structure, we consider the problem parameters as $(a_n, b_n, A_n, B_n, d_n)$ corresponding to the node n . Objective (11a) minimizes the total cost by considering capacity acquisition and allocation decisions. Constraint (11b) guarantees that the assigned capacity in each period is less than the available capacity, and constraint (11c) ensures that the demand is satisfied.

We note that the proposed multi-stage stochastic problem (11) can be reformulated as in Huang and Ahmed (2009) by considering its specific substructure. The resulting formulation can be stated as follows:

$$\min_y \sum_{n \in \mathcal{T}} p_n b_n^\top y_n + \sum_{i \in \mathcal{I}} Q_i(y) \quad (12a)$$

$$\text{s.t.} \quad (11c)$$

$$y_n \in \mathbb{R}_+^{|\mathcal{J}|} \quad \forall n \in \mathcal{T}, \quad (12b)$$

where, for each $i \in \mathcal{I}$,

$$Q_i(y) = \min_{x_i} \sum_{n \in \mathcal{T}} p_n a_{in} x_{in}, \quad (13a)$$

$$\text{s.t.} \quad \sum_{m \in P(n)} x_{im} \geq [A_n y_n]_i \quad \forall n \in \mathcal{T}, \quad (13b)$$

$$x_{in} \in \mathbb{Z}_+ \quad \forall n \in \mathcal{T}. \quad (13c)$$

In the proposed reformulation, the first problem (12) determines the capacity allocation decisions, whereas the second problem (13) considers the capacity acquisition decisions given the allocation decisions, which is further decomposed with respect to each item $i \in \mathcal{I}$. We let $I = |\mathcal{I}|$ in the remainder of this section.

We note that under a given sequence of allocation decisions $\{y_n\}_{n \in \mathcal{T}}$, we can obtain the optimal capacity acquisition decisions corresponding to each item i by solving (13). This problem can be restated by letting $\delta_{in} = [A_n y_n]_i$ as follows:

$$\min_{x_i} \sum_{n \in \mathcal{T}} p_n a_{in} x_{in} \quad (14a)$$

$$\text{s.t.} \quad \sum_{m \in P(n)} x_{im} \geq \delta_{in} \quad \forall n \in \mathcal{T} \quad (14b)$$

$$x_{in} \in \mathbb{Z}_+ \quad \forall n \in \mathcal{T}. \quad (14c)$$

We first observe that the feasible region of the formulation (14) gives an integral polyhedron under integer $\{\delta_{in}\}_{n \in \mathcal{T}}$ values (Huang and Ahmed 2009). Furthermore, it is equivalent to a simpler version of the stochastic lot sizing problem without a fixed order cost.

The two-stage version of the single-resource problem (14) can be represented as follows:

$$\min_{x_i} \sum_{n \in \mathcal{T}} p_n a_{in} x_{in} \quad (15a)$$

$$\text{s.t.} \quad (14b), (14c)$$

$$x_{im} = x_{in} \quad \forall m, n \in S_t. \quad (15b)$$

As an alternative to the formulations (14) and (15), we consider the adaptive two-stage stochastic programming approach where the capacity allocation decision of each item $i \in \mathcal{I}$ is determined at the beginning of planning until stage $t_i^* - 1$. Then, the underlying scenario tree at time t_i^* is observed, and the decisions until the end of the planning horizon are determined at that time. The resulting problem can be formulated as follows:

$$\min_{x_i} \sum_{n \in \mathcal{T}} p_n a_{in} x_{in} \quad (16a)$$

$$\text{s.t.} \quad (14b), (14c)$$

$$x_{im} = x_{in} \quad \forall m, n \in S_t, \quad t < t_i^* \quad (16b)$$

$$x_{im} = x_{in} \quad \forall m, n \in S_t \cap \mathcal{T}(j), \quad j \in S_{t_i^*}, \quad t \geq t_i^*. \quad (16c)$$

We note that the above formulation considers the case under a given t_i^* value. We can equivalently represent (16) by defining a condensed scenario tree based on the t_i^* value. Let the condensed version of tree \mathcal{T} for item i be $\mathcal{T}_i(t_i^*)$. We denote the set of nodes that are condensed to node $n \in \mathcal{T}_i(t_i^*)$ by $\hat{C}_n \subset \mathcal{T}$, where \hat{C}_n is obtained by combining the nodes with the same decision structure as in constraints (16b) and (16c). Then, the resulting formulation is

$$\min_{x_i} \sum_{n \in \mathcal{T}_i(t_i^*)} \hat{p}_n \hat{a}_{in} x_{in} \quad (17a)$$

$$\text{s.t.} \quad \sum_{m \in P(n)} x_{im} \geq \hat{\delta}_{in} \quad \forall n \in \mathcal{T}_i(t_i^*) \quad (17b)$$

$$x_{in} \in \mathbb{Z}_+ \quad \forall n \in \mathcal{T}_i(t_i^*), \quad (17c)$$

where $\hat{p}_n = \sum_{m \in \hat{C}_n} p_m$, $\hat{a}_{in} = \sum_{m \in \hat{C}_n} p_m a_{im}$, and $\hat{\delta}_{in} = \max_{m \in \hat{C}_n} \{\delta_{im}\}$.

PROPOSITION 1. The coefficient matrix of the formulation (17) is totally unimodular.

Proof: See Appendix A.3. \square

4.2. Deriving VATS for Capacity Expansion Planning

To evaluate the performance of the adaptive two-stage approach, we aim deriving bounds for the $V^{MS} - V^{ATS}(t^*)$, and $V^{TS} - V^{ATS}(t^*)$ for the capacity expansion problem (11) under a given

revision vector t^* . We first study the single item version of the subproblem (13) under different stochastic programming formulations in Section 4.2.1, and present a sensitivity analysis on these bounds in Section 4.2.2. Then, we extend these results to the complete capacity expansion problem in Section 4.2.3.

4.2.1. VATS for the Single-Resource Problem In this section, we derive the VATS for the single-resource problem (14) using its linear programming relaxations under two-stage, adaptive two-stage and multi-stage models. We let v^M , v^T , $v^R(t^*)$ be the optimal value of the linear programming relaxations of the formulations (14), (15), and (16) respectively. As we focus on the single-resource problem, we omit the resource index i in this section for brevity.

To construct the VATS, we examine the problem parameters with respect to the underlying scenario tree. Specifically, we define the minimum and maximum costs over the scenario tree as $a_* = \min_{n \in \mathcal{T}} \{a_n\}$ and $a^* = \max_{n \in \mathcal{T}} \{a_n\}$, respectively. We denote the maximum demand over the scenario tree as $\delta^* = \max_{n \in \mathcal{T}} \{\delta_n\}$, and the expected maximum demand over scenarios as $\bar{\delta} = \sum_{n \in S_T} p_n \max_{m \in P(n)} \{\delta_m\}$. Additionally, we examine the problem parameters based on the choice of the revision time. In particular, we define the minimum and maximum cost parameters before and after the revision time t^* as follows:

$$\begin{aligned} \underline{a}^-(t^*) &= \min_{n \in \mathcal{T}: t_n < t^*} \{a_n\}, & \underline{a}^+(t^*) &= \min_{n \in \mathcal{T}: t_n \geq t^*} \{a_n\}, \\ \bar{a}^-(t^*) &= \max_{n \in \mathcal{T}: t_n < t^*} \{a_n\}, & \bar{a}^+(t^*) &= \max_{n \in \mathcal{T}: t_n \geq t^*} \{a_n\}. \end{aligned}$$

For the demand parameters, we let

$$\delta^-(t^*) = \max_{m \in \mathcal{T}(1, t^*-1)} \{\delta_m\}, \quad \delta^+(t^*) = \sum_{n \in S_{t^*}} p_n \max_{m \in \mathcal{T}(1, t^*-1) \cup \mathcal{T}(n)} \{\delta_m\}.$$

Here, the parameter $\delta^-(t^*)$ represents the maximum demand value before the revision time t^* . The parameter $\delta^+(t^*)$ corresponds to the expected maximum demand over the tree until the revision time t^* and the subtree rooted at each node of the revision stage. By comprehending the underlying uncertainty through these definitions, we obtain our main result for analyzing the single-resource problem.

THEOREM 3. We derive the following bounds for $v^T - v^R(t^*)$ and $v^R(t^*) - v^M$:

$$\begin{aligned} a_* \delta^* - (\bar{a}^-(t^*) - \bar{a}^+(t^*)) \delta^-(t^*) - \bar{a}^+(t^*) \delta^+(t^*) &\leq v^T - v^R(t^*) \\ &\leq a^* \delta^* - (\underline{a}^-(t^*) - \underline{a}^+(t^*)) \delta^-(t^*) - \underline{a}^+(t^*) \delta^+(t^*), \end{aligned} \tag{18}$$

and

$$\begin{aligned} (\underline{a}^-(t^*) - \underline{a}^+(t^*)) \delta^-(t^*) + \underline{a}^+(t^*) \delta^+(t^*) - a^* \bar{\delta} &\leq v^R(t^*) - v^M \\ &\leq (\bar{a}^-(t^*) - \bar{a}^+(t^*)) \delta^-(t^*) + \bar{a}^+(t^*) \delta^+(t^*) - a_* \bar{\delta}. \end{aligned} \tag{19}$$

To obtain the value of the adaptive two-stage approach in Theorem 3, we first identify the bounds on the optimum values v^M , v^T , $v^R(t^*)$.

PROPOSITION 2. We can derive the following bound for $v^R(t^*)$:

$$(\underline{a}^-(t^*) - \underline{a}^+(t^*))\delta^-(t^*) + \underline{a}^+(t^*)\delta^+(t^*) \leq v^R(t^*) \leq (\bar{a}^-(t^*) - \bar{a}^+(t^*))\delta^-(t^*) + \bar{a}^+(t^*)\delta^+(t^*).$$

Proof: See Appendix A.4. \square

We note that we can obtain bounds for the linear programming relaxation of the two-stage equivalent of the formulation (14) using Proposition 2 by selecting the revision period t^* as 1.

COROLLARY 1. For v^T , we have $a_*\delta^* \leq v^T \leq a^*\delta^*$.

Combining Proposition 2 and Corollary 1, we can obtain the VATS for the single-resource problem in (18) as in the first part of Theorem 3.

PROPOSITION 3. (Huang and Ahmed 2009) For v^M , we have $a_*\bar{\delta} \leq v^M \leq a^*\bar{\delta}$.

Combining Propositions 2 and 3, we obtain the value of the multi-stage model against adaptive two-stage model for the single-resource problem in (19) as in the second part of Theorem 3.

4.2.2. Sensitivity Analysis on the Single-Resource Problem In order to gain some insight regarding the effect of the revision point decisions on the performance of the analytical bounds, we consider two cases with respect to the values of cost and demand parameters.

Demand Sensitivity We first consider the case where the cost parameters $\{a_n\}_{n \in \mathcal{T}}$ are almost equal to each other across the scenario tree, i.e. $\{a_n\}_{n \in \mathcal{T}} \approx a$. In that case, the bounds (18) and (19) reduce to the following expressions:

$$v^T - v^R(t^*) \approx a(\delta^* - \delta^+(t^*)), \quad v^R(t^*) - v^M \approx a(\delta^+(t^*) - \bar{\delta}). \quad (20)$$

Consequently, the value of the adaptive formulation depends on how much $\delta^+(t^*)$ differs from the maximum demand value, δ^* . Similarly, the value of the multi-stage stochastic program against the adaptive two-stage approach depends on the difference between $\delta^+(t^*)$ and the maximum average demand, $\bar{\delta}$. These results highlight the importance of the variability of the demand on the values of the stochastic programs. If there is not much variability in the demand across scenarios, then the three approaches result in similar solutions as the bounds in (20) go to zero. As variability of the scenarios increases, the corresponding bounds might change accordingly.

Furthermore, the bounds (20) demonstrate that the values of the adaptive approach might be highly dependent to the selection of t^* value. In particular, best performance gains for the adaptive approach, compared to two-stage and multi-stage cases, can be obtained when $\delta^+(t^*)$ is minimized.

By this way, we obtain the solution of the adaptive program which gives the least loss compared to the multi-stage stochastic programs, and the most gain compared to the two-stage stochastic programs, with respect to the proposed bounds. Let us denote the best revision point in terms of the demand bounds as $t^{DB} := \arg \min_{2 \leq t \leq T} \delta^+(t)$.

We illustrate the effect of the revision times on the bound values (20) for an instance of the single-resource problem described in Appendix B. Specifically, we have unit cost for $\{a_n\}_{n \in \mathcal{T}}$ values, and $\{\delta_n\}_{n \in \mathcal{T}}$ values are sampled from a probability distribution. We present the bound values in Table 1, where $\delta^* = 41$ and $\bar{\delta} = 34.06$, considering the scenario tree in Figure 8. We observe that $\delta^+(t^*)$ is minimized at period 3, i.e. $t^{DB} = 3$, maximizing the objective gap of the adaptive two-stage approach with two-stage model and minimizing the corresponding gap with multi-stage model. These results demonstrate that we can analytically determine the revision points for the single-resource problem when costs remain same during the planning horizon.

Table 1 Bound comparison with respect to the revision time.

	$t^* = 1$	$t^* = 2$	$t^* = 3$	$t^* = 4$	$t^* = 5$
$v^T - v^R(t^*)$	0.00	1.50	5.25	2.63	0.00
$v^R(t^*) - v^M$	6.94	5.44	1.69	4.31	6.94
$\delta^+(t^*)$	41.00	39.50	35.75	38.38	41.00

Next, we examine how $\delta^+(t^*)$ changes under different δ_n structures. Let \hat{t}^D be the first stage that we observe the maximum demand value over the scenario tree, i.e. $\hat{t}^D = \min\{t \in \{1, \dots, T\} : \delta_n = \delta^*, n \in S_t\}$.

PROPOSITION 4. Under a general demand structure for δ_n with cost values $\{a_n\}_{n \in \mathcal{T}} \approx a$, if $\hat{t}^D = 1$, then $t^{DB} \in \{2, \dots, T\}$. Otherwise, $t^{DB} \in \{2, \dots, \hat{t}^D\}$.

Proof: See Appendix A.5. \square

For specific forms of demand patterns, we can further refine Proposition 4.

PROPOSITION 5. Let stage $t_n \in \{2, \dots, T\}$ has N_t many independent realizations for the demand values $\{\delta_n\}_{n \in \mathcal{T}}$ where cost values $\{a_n\}_{n \in \mathcal{T}} \approx a$. Then, $t^{DB} = \hat{t}^D$.

Proof: See Appendix A.6. \square

Cost Sensitivity Next, we consider the case where the demand parameters $\{\delta_n\}_{n \in \mathcal{T}}$ are almost equal to each other throughout the scenario tree, i.e. $\{\delta_n\}_{n \in \mathcal{T}} \approx \delta$. In that case, the bounds (18) and (19) reduce to the following

$$\max\{(a_* - \bar{a}^-(t^*))\delta, 0\} \lesssim v^T - v^R(t^*) \lesssim (a^* - \underline{a}^-(t^*))\delta, \quad (21)$$

$$\max\{(\bar{a}^-(t^*) - a^*)\delta, 0\} \lesssim v^R(t^*) - v^M \lesssim (\bar{a}^-(t^*) - a_*)\delta. \quad (22)$$

We observe that the value of the adaptive formulation depends on the choice of the revision point as the bounds in (21) and (22) are functions of t^* . In order to select the best revision point for the adaptive two-stage approach, we aim maximizing the lower bound on its objective difference between two-stage in (21) and minimize the upper bound on the corresponding difference between multi-stage in (22). Thus, this results in finding the revision point $2 \leq t^* \leq T$ that minimizes $\bar{a}^-(t^*) - a_*$. Since a_* is not dependent to the choice of the revision point, it is omitted in the remainder of our analysis. We denote the best revision point in terms of the cost bounds as $t^{CB} := \arg \min_{2 \leq t \leq T} \bar{a}^-(t)$.

We consider several specific cases to illustrate the revision point decision t^{CB} based on the cost values. If cost values are increasing in each stage of the scenario tree, then $\bar{a}^-(t)$ is monotonically increasing in t . Thus, the revision point needs to be as early as possible. On the other hand, if cost values are decreasing in each stage, then the value of the adaptive approach has the same value in all time periods as $\bar{a}^-(t)$ is same for every $2 \leq t \leq T$. Thus, revision time does not affect the analytical bounds for this setting. We can generalize this result as follows by letting the maximum cost value over the scenario tree as $\hat{t}^C = \min\{t \in \{1, \dots, T\} : a_n = a^*, n \in S_t\}$.

PROPOSITION 6. Under a general cost structure for a_n with demand values $\{\delta_n\}_{n \in \mathcal{T}} \approx \delta$, $\bar{a}^-(t)$ is monotonically nondecreasing until the period \hat{t}^C , and it remains constant afterwards.

The proof of Proposition 6 is immediate by following the definition of $\bar{a}^-(t)$. This proposition shows that if the maximum cost value is in the root node of the scenario tree, then the value of the adaptive approach is not affected by the revision decision. Otherwise, the revision point needs to be selected as early as possible before observing the highest cost value of the scenario tree.

4.2.3. VATS for the Capacity Expansion Planning Problem In this section, we extend our results on the single-resource subproblem (16) to the capacity expansion planning problem (11) under a given revision decision for each resource. We derive analytical bounds on the objective of the adaptive two-stage approach in comparison to two-stage and multi-stage stochastic models. To derive the desired bounds, we utilize the linear programming relaxations of the capacity expansion planning problem.

PROPOSITION 7. Let $\{y_n^{TLP}\}_{n \in \mathcal{T}}$ be the optimal capacity allocation decisions to the linear programming relaxation of the two-stage version of the stochastic capacity expansion planning problem (11). Then,

$$V^{TS} - V^{ATS}(t^*) \geq \sum_{i=1}^I (v_i^T - v_i^R(t_i^*) - \max_{n \in \hat{\mathcal{T}}_i(t_i^*)} \{[\delta_{in}] - \delta_{in}\} a_{i1}), \quad (23)$$

where v_i^T and $v_i^R(t_i^*)$ are the optimal objective values of the linear programming relaxations of the models (15) and (16) under $\delta_{in} = [A_n y_n^{TLP}]_i$.

Proof: See Appendix A.7. \square

PROPOSITION 8. Let $\{y_n^{MLP}\}_{n \in \mathcal{T}}$ be the optimal capacity allocation decisions to the linear programming relaxation of the multi-stage stochastic capacity expansion planning problem (11). Then,

$$V^{ATS}(t^*) - V^{MS} \leq \sum_{i=1}^I (v_i^R(t_i^*) - v_i^M + \max_{n \in \mathcal{T}_i(t_i^*)} \{[\delta_{in}] - \delta_{in}\} a_{i1}), \quad (24)$$

where v_i^M and $v_i^R(t_i^*)$ are the optimal objective values of the linear programming relaxations of the models (14) and (16) under $\delta_{in} = [A_n y_n^{MLP}]_i$.

Proof: See Appendix A.8. \square

Using the bound in Proposition 8 and the relationship $V^{MS} \leq V^{ATS}(t^*)$ for any revision vector t^* , we can evaluate the performance of the adaptive two-stage approach in comparison to its value under a given t^* vector.

COROLLARY 2. Let V^{ATS} denote the optimal objective of the adaptive two-stage capacity expansion problem. Then,

$$V^{ATS}(t^*) - V^{MS} \leq \sum_{i=1}^I (v_i^R(t_i^*) - v_i^M + \max_{n \in \mathcal{T}_i(t_i^*)} \{[\delta_{in}] - \delta_{in}\} a_{i1}), \quad (25)$$

where $\delta_{in} = [A_n y_n^{MLP}]_i$.

We note that these theoretical results will be used in the solution methodology introduced in the next section for developing heuristic approaches with approximation guarantees.

5. Solution Methodology

The adaptive two-stage formulation imposes computational challenges when the revision times of resources are decision variables. In this section, we propose three approximation algorithms to solve the adaptive two-stage equivalent of the capacity expansion planning problem (11). Our first two algorithms are based on the bounds derived in Propositions 7 and 8 by selecting the revision points that provide most gain against two-stage and least loss against multi-stage models. In Algorithm 1, we identify the revision point of each resource by maximizing the lower bound of the gain in objective in comparison to two-stage stochastic model. Similarly, in Algorithm 2, we determine the revision points by minimizing the upper bound of the loss in objective in comparison to multi-stage stochastic model.

Algorithm 1 Algorithm Two-stage Relax (**TS-Relax**)

- 1: Solve the linear programming relaxation of the two-stage version of the stochastic capacity expansion planning problem (11) and obtain $\{y_n^{TLP}\}_{n \in \mathcal{T}}$.
- 2: Compute $\delta_{in} = [A_n y_n^{TLP}]_i$ for all $i = 1, \dots, I, n \in \mathcal{T}$.
- 3: **for all** $i = 1, \dots, I$ **do**
- 4: Find t_i^* that maximizes the lower bound on $v_i^T - v_i^R(t_i^*) - \max_{n \in \hat{\mathcal{T}}_i(t_i^*)} \{[\delta_{in}] - \delta_{in}\} a_{i1}$ using the bounds (18).
- 5: Solve the adaptive two-stage version of the stochastic capacity expansion planning problem (11) for $\{x_n, y_n\}_{n \in \mathcal{T}}$ given the $\{t_i^*\}_{i=1}^I$ values.

Algorithm 2 Algorithm Multi-stage Relax (**MS-Relax**)

- 1: Solve the linear programming relaxation of the multi-stage stochastic capacity expansion planning problem (11) and obtain $\{y_n^{MLP}\}_{n \in \mathcal{T}}$.
- 2: Compute $\delta_{in} = [A_n y_n^{MLP}]_i$ for all $i = 1, \dots, I, n \in \mathcal{T}$.
- 3: **for all** $i = 1, \dots, I$ **do**
- 4: Find t_i^* that minimizes the upper bound on $v_i^R(t_i^*) - v_i^M + \max_{n \in \hat{\mathcal{T}}_i(t_i^*)} \{[\delta_{in}] - \delta_{in}\} a_{i1}$ using the bounds (19).
- 5: Solve the adaptive two-stage version of the stochastic capacity expansion planning problem (11) for $\{x_n, y_n\}_{n \in \mathcal{T}}$ given the $\{t_i^*\}_{i=1}^I$ values.

In addition to these, we propose another approximation algorithm in Algorithm 3 by first solving a relaxation of the adaptive two-stage stochastic program to identify the revision points. Then, we obtain the capacity expansion and allocation decisions under the resulting revision decisions.

Algorithm 3 Algorithm Adaptive Two-stage Relax (**ATS-Relax**)

- 1: Solve a relaxation of the adaptive two-stage version of the stochastic capacity expansion planning problem (11) where $\{x_n\}_{n \in \mathcal{T}}$ decisions are continuous, and obtain $\{r_{i,t}^{ALP}\}$ for $i = 1, \dots, I, t = 1, \dots, T$.
- 2: Let $t_i^* = \sum_{t=1}^T tr_{i,t}^{ALP}$ for all $i = 1, \dots, I$.
- 3: Solve the adaptive two-stage version of the stochastic capacity expansion planning problem (11) for $\{x_n, y_n\}_{n \in \mathcal{T}}$ given the $\{t_i^*\}_{i=1}^I$ values.

We note that Corollary 2 provides an upper bound to compare the objectives of the true adaptive two-stage program with the adaptive program under a given revision point. Using this result, we can demonstrate the approximation guarantees of Algorithms 1 - 3 according to their choices of revision decisions. We can further improve this bound for Algorithm 3 as follows.

PROPOSITION 9. Let $V^{ATS-Relax}$ and $t^{ATS-Relax}$ denote the objective and the revision vector of the adaptive two-stage program under the solutions found in Algorithm 3. Then,

$$V^{ATS-Relax} - V^{ATS} \leq \sum_{i=1}^I \left(\max_{n \in \hat{\mathcal{T}}_i(t_i^{ATS-Relax})} \{(\lceil \delta_{in} \rceil - \delta_{in})\} a_{i1} \right), \quad (26)$$

where $\delta_{in} = [A_n y_n^{LP}]_i$ and y^{LP} is the capacity allocation decisions found in Step 1 of Algorithm 3.

Proof: See Appendix A.9. \square

We can obtain a simpler upper bound on the optimality gap of the ATS-Relax Algorithm derived in (26). Specifically, we have $V^{ATS-Relax} - V^{ATS} \leq \sum_{i=1}^I a_{i1}$ as $\lceil \delta_{in} \rceil - \delta_{in} \leq 1$ for all resources i and nodes n . We note that this optimality gap is irrespective of the number of stages, scenario tree structure and problem data. The gap only depends on the capacity acquisition cost of each resource at the first stage. Combining the above, we derive the asymptotic convergence of the ATS-Relax Algorithm as follows.

COROLLARY 3. The Algorithm 3 asymptotically converges to the true adaptive program, i.e.

$$\lim_{T \rightarrow \infty} \frac{V^{ATS-Relax} - V^{ATS}}{T} = 0.$$

6. Computational Analysis

We illustrate our results on an application to generation capacity expansion planning, which contains the problem structure studied in Section 4. In Section 6.1, we present the experimental setup and the details of the problem formulation. In Section 6.2, we provide an extensive computational study by demonstrating the value of the adaptive two-stage approach and comparing the performances of the different solution algorithms on various scenario tree structures. We also present a detailed optimal solution for a specific generation expansion problem and provide insights regarding the proposed approach.

6.1. Experimental Setup and Optimization Model

Generation capacity expansion planning is a well-studied problem in literature to determine the acquisition and capacity allocation decisions of different types of generation resources over a long-term planning horizon. Our aim is to optimize the yearly investment decisions of different generation resources while producing energy within the available capacity of each generation resource and satisfying the overall system demand in each subperiod. We consider four types of subperiods within a year, namely peak, shoulder, off-peak and base, depending on their demand level. We assume a restriction on the number of generation resources to purchase until the end of the planning horizon. We consider six different types of generation resources for investment decisions, namely nuclear,

coal, natural gas-combined cycle (NG-CC), natural gas-gas turbine (NG-GT), wind and solar. We utilize the data set presented in Min et al. (2018), which is based on a technical report of U.S. Department of Energy (Black 2012). This data set is used for computing capacity amounts and costs associated with each type of generation resource.

Using the predictions in Liu et al. (2018), we consider a 10% reduction in the capacity acquisition costs of the renewable generation resources in each year. Additionally, we assume 15% and 10% yearly increases in total for fuel prices and operating costs of natural gas and coal type generation resources, respectively. For other types of generators, we take into account 3% increase in fuel prices (Min et al. 2018). To promote the renewable resources, we allow at most 20% capacity expansion for the traditional generation types in terms of their initial capacities. We assume that the generators are available for production in the period that the acquisition decision is made, similar to Jin et al. (2011), Zou et al. (2018).

The source of uncertainty of our model is the demand level of each subperiod in set \mathcal{K} throughout the planning horizon. We adopt the scenario generation procedure presented in Singh et al. (2009) for constructing the scenario tree. At the beginning of the planning horizon, we start with an initial demand level for each subperiod, using the values in Min et al. (2018). Then, we randomly generate a demand increase multiplier for each node, except the root node, to estimate that node's demand value based on its ancestor node's demand. The details of the scenario tree generation algorithm is presented in Algorithm 4 in Appendix C.

Following the studies Singh et al. (2009), Zou et al. (2018), we represent the generation capacity expansion planning problem as a stochastic program. We reformulate this problem as an adaptive two-stage program, where the capacity acquisition decisions can be revised once within the planning horizon. The sets, decision variables, and parameters are defined as follows:

Sets:

- \mathcal{I} Set of generation types for expansion
- \mathcal{K} Set of subperiod types within each period

Parameters:

- T Number of periods
- n_i^0 Number of generation resource type i at the beginning of planning
- n_i^{max} Maximum number of generation resource type i at the end of planning
- m_i^{max} Maximum capacity of a type i generation resource in MWh
- $m_i'^{max}$ Maximum effective capacity of a type i generation resource in MWh
- l_i Peak contribution ratio of a type i generation resource

c_{in}	Capacity acquisition cost per MWh of a generation resource type i at node n
f_{in}	Fix operation and maintenance cost per MWh of a generation resource type i at node n
g_{in}	Fuel price per MWh of a generation resource type i at node n
k_{in}	Generation cost per MWh of a generation resource type i at node n
h_{kt}	Number of hours of subperiod k in period t
d_{kn}	Hourly demand at subperiod k at node n
w	Penalty per MWh for demand curtailment
r	Yearly interest rate

Decision variables:

x_{in}	Number of generation resource type i acquisition at node n
u_{ikn}	Hourly generation amount of generator type i at subperiod k at node n
v_{kn}	Demand curtailment amount at subperiod k at node n
t_i^*	Revision point for acquisition decisions of generation resource type i

We then formulate the generation expansion planning problem as follows:

$$\min_{x,u,v,t^*} \sum_{n \in \mathcal{T}} \sum_{i \in \mathcal{I}} p_n \frac{1}{(1+r)^{t_n-1}} \left(\left(c_{in} + \sum_{t=t_n}^T \frac{f_{in}}{(1+r)^{t-t_n}} \right) m_i^{max} x_{in} + \sum_{k \in \mathcal{K}} (g_{in} + k_{in}) h_{kt_n} u_{ikn} \right) + \sum_{n \in \mathcal{T}} \sum_{k \in \mathcal{K}} p_n \frac{1}{(1+r)^{t_n-1}} w h_{kt_n} v_{kn} \quad (27a)$$

$$\text{s.t. } u_{ikn} \frac{1}{m_i^{max}} \leq n_i^0 + \sum_{m \in P(n)} x_{im} \quad \forall n \in \mathcal{T} \quad (27b)$$

$$\sum_{i \in \mathcal{I}} l_i u_{ikn} + v_{kn} \geq d_{ns} \quad \forall n \in \mathcal{T}, \quad \forall k \in \mathcal{K} \quad (27c)$$

$$\sum_{m \in P(n)} x_{im} \leq n_i^{max} \quad \forall n \in S_T, \quad \forall i \in \mathcal{I} \quad (27d)$$

$$x_{im} = x_{in} \quad \forall m, n \in S_t, \quad t < t_i^*, \quad \forall i \in \mathcal{I} \quad (27e)$$

$$x_{im} = x_{in} \quad \forall m, n \in S_t \cap \mathcal{T}(j), \quad j \in S_{t_i^*}, \quad t \geq t_i^*, \quad \forall i \in \mathcal{I} \quad (27f)$$

$$x_{in} \in \mathbb{Z}_+, t_i^* \in \{1, \dots, T\}, u_{ikn}, v_{kn} \in \mathbb{R}_+ \quad \forall n \in \mathcal{T}. \quad (27g)$$

The objective function (27a) aims to minimize the expected costs of capacity acquisition, allocation and demand curtailment. In order to compute the capacity acquisition costs, we consider the building cost of the generation unit in addition to its maintenance and operating costs for the upcoming periods. For computing the cost of capacity allocation decisions, we use fuel prices and production cost associated with each type of generation resource. All of the costs are then

discounted to the beginning of the planning horizon. Constraint (27b) ensures that the production amount in each subperiod is restricted by the total available capacity for each type of generation resource. Constraint (27c) guarantees that system demand is satisfied, and constraint (27d) restricts the acquisition decision throughout the planning horizon. The provided formulation (27) is a special form of the capacity expansion problem studied (12). In particular, constraints (27b) and (27d) correspond to constraint (11b), and constraint (27c) refers to constraint (11c). We note that constraint (27d) becomes redundant in the linear programming relaxation of the subproblem (13) due to Proposition 2 in Zou et al. (2018).

Constraints (27e) and (27f) represent the adaptive two-stage relationship for the capacity acquisition decisions such that the acquisition decision of each resource type i can be revised at t_i^* . We note that the presented formulation is nonlinear, however it can be reformulated as a mixed-integer linear program by defining additional variables for the revision decisions, as shown in (10). In our computational experiments, we solve the resulting mixed-integer linear program.

6.2. Computational results

In this section, we first demonstrate the value of the adaptive two-stage approach in comparison to two-stage stochastic programming under different scenario tree structures and demand characteristics. Secondly, we examine the performances of the different algorithms introduced in Section 5 for solving the adaptive two-stage problem. Finally, we analyze a generation capacity expansion plan under the adaptive two-stage approach to discuss its practical implications.

To construct our computational testbed, we generate scenario trees for demand values using Algorithm 4 when number of branches at each period, namely M , is equal to 2 or 3. We examine scenario trees with number of stages T ranging from 3 to 10. Consequently, the number of nodes in the scenario trees are in the range of 7 to 29524, corresponding to $(M, T) = (2, 3)$, and $(M, T) = (3, 10)$, respectively. We solve the optimization problem (27) under five different randomly generated scenario trees for each (M, T) pair. We report the average of these replications to represent the performances of the proposed methods under various instances. We conduct our experiments on an Intel i5 2.20 GHz machine with 8 GB RAM. We implement the algorithms in Python using Gurobi 7.5.2 with a relative optimality gap of 0.1% and time limit of 2 hours.

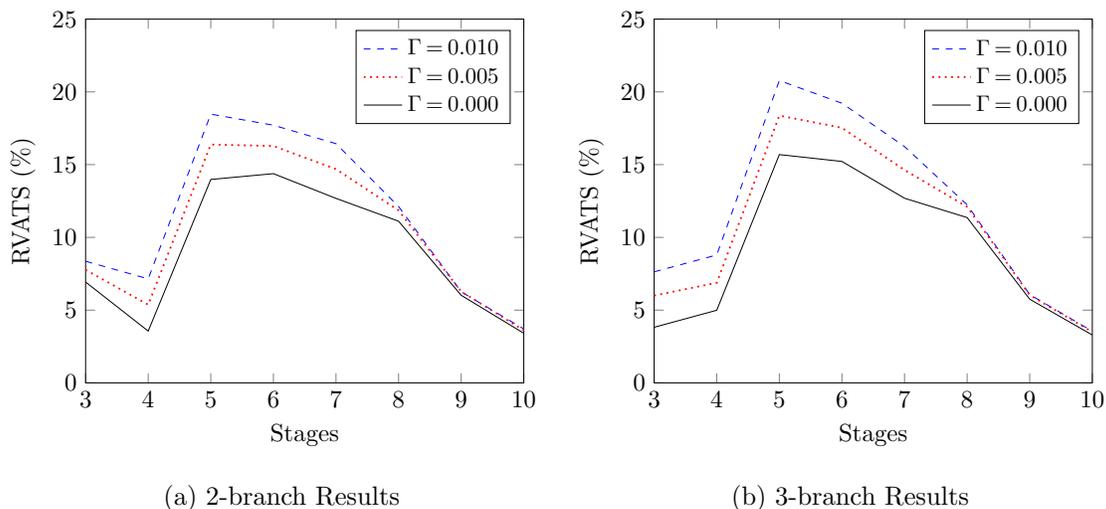
6.2.1. Value of Adaptive Two-Stage Approach To demonstrate the performance of the adaptive two-stage approach in comparison to two-stage stochastic programming, we define the relative value of adaptive two-stage approach (RVATS) by extending the definition of VATS. Specifically, we let

$$\text{RVATS (\%)} = 100 \times \frac{(V^{TS} - V^{ATS})}{V^{TS}}, \quad (28)$$

where V^{TS} and V^{ATS} are the objective values corresponding to the two-stage model and adaptive two-stage model, respectively.

We illustrate the RVATS under various scenario tree structures in Figure 4. Figures 4a and 4b show the behavior of the adaptive two-stage approach under scenario trees with 2 and 3 branches, each with three different demand patterns, respectively. For constructing the demand patterns, we examine the cases with an increasing variance under a constant mean for demand multipliers. Specifically, we consider the demand multipliers in Algorithm 4 as $\underline{\alpha}_t = 1.00 - \Gamma t$ and $\bar{\alpha}_t = 1.20 + \Gamma t$ for all stages $t = 2, \dots, T$ where $\Gamma = 0, 0.005, 0.01$. To compute RVATS, we utilize V^{ATS} for all stages in 2-branch trees and until stage 7 in 3-branch trees. For larger trees for 3-branch case, we report a lower bound on the RVATS due to the computational difficulty of solving ATS to optimality, as discussed in Section 4. In particular, we replace V^{ATS} in Equation 28 with $V^{TS-Relax}$, where $V^{TS-Relax}$ represents the objective value of the adaptive two-stage model under the solution of the Algorithm TS-Relax.

Figure 4 Value of Adaptive Two-Stage on Instances with Different Variability



We observe RVATS between 3-19% over 2-branch, and 3-21% over 3-branch scenario trees, demonstrating the significant gain of revising decisions over static policies. The largest gain is obtained when the number of stages is equal to 5 for both cases. As the number of stages increases, the adaptive two-stage approach becomes more and more similar to the two-stage stochastic programming. Consequently, the relative gain of the proposed approach decreases for the problems with longer planning horizons.

We note that the scenario tree has more variability in each stage when the parameter Γ increases. We observe higher gain of the adaptive two-stage approach for the scenario trees with larger

variability levels. Additionally, 3-branch scenario trees have higher RVATS values than 2-branch trees due to the larger variance of demand values in each stage. These results demonstrate an even better performance of the adaptive two-stage approach in comparison to two-stage when the scenario tree has larger variability.

6.2.2. Performance of Solution Algorithms In this section, we examine the performance of the solution algorithms for the capacity expansion planning problem from different perspectives. Tables 2 and 3 provide the computational results of the solution algorithms for 2-branch and 3-branch scenario trees, respectively. The computational experiments are under the default demand multiplier setting for the scenario trees when $\Gamma = 0.005$, i.e. $\underline{\alpha}_t = 1.00 - 0.005t$ and $\bar{\alpha}_t = 1.20 + 0.005t$ for all stages $t = 2, \dots, T$. The column “Time” represents the solution time in terms of seconds, and the column “Gain” corresponds to the percentage gain of the studied method in comparison to two-stage stochastic model. We note that we repeat the experiments with larger values of Γ and obtain consistent results.

The algorithms MS-Relax and ATS-Relax provide optimality gaps for the adaptive two-stage program. In particular, these algorithms obtain a lower bound to the adaptive two-stage program by solving relaxations of the multi-stage model and adaptive two-stage model in Step 1 of the Algorithm 1 and Algorithm 3. Since both algorithms construct a feasible solution, they provide an upper bound for the desired problem. Combining the lower and upper bounds, we construct optimality gap for the adaptive two-stage approach. The gap values are reported in the column “Gap” in Tables 2 and 3. These values provide performance metrics for the solution algorithms in approximating the adaptive two-stage model.

We first note that the ATS approach provides the most gain against the two-stage approach, as expected. Among the approximation algorithms, Algorithm ATS-Relax has the highest gain despite its computational disadvantage. The gain of Algorithm ATS-Relax is very close to the gain of ATS demonstrating the success of the proposed algorithm in approximating the adaptive two-stage approach. Our computational results also highlight the asymptotic convergence of Algorithm ATS-Relax as shown in Corollary 3. Specifically, as number of stages increases, the optimality gap provided by Algorithm ATS-Relax becomes closer to zero. However, we observe significant computational benefits of adopting Algorithms TS-Relax and MS-Relax as they provide notable speedups in comparison to ATS. We also observe that the computational complexity of the ATS and ATS-Relax approaches are more sensitive to the instance size in comparison to Algorithms TS-Relax and MS-Relax.

The size of the scenario tree and consequently the problem size increase as the number of stages and the number of branches get larger. We observe that the computational complexity of the ATS

Table 2 Performance of Solution Algorithms on 2-branch Results

Stage	ATS		TS-Relax		MS-Relax			ATS-Relax		
	Time(s)	%Gain	Time(s)	%Gain	Time(s)	%Gain	%Gap	Time(s)	%Gain	%Gap
3	0.16	7.77	0.16	6.01	0.17	4.77	5.47	0.24	7.56	1.25
4	0.58	5.37	0.22	0.79	0.27	4.95	2.80	0.56	5.31	1.18
5	1.42	16.38	0.31	10.70	0.30	14.80	3.76	1.19	16.36	0.06
6	15.34	16.28	0.61	11.41	0.57	13.74	7.84	5.17	16.26	0.04
7	34.05	14.69	1.19	13.00	1.17	11.91	9.56	16.49	14.69	0.02
8	103.25	11.85	3.15	11.22	2.21	9.65	8.48	35.19	11.84	0.03
9	234.10	6.28	5.14	5.79	4.42	4.86	5.46	94.01	6.27	0.02
10	930.34	3.61	12.37	3.10	9.76	2.91	3.09	370.83	3.59	0.03

Table 3 Performance of Solution Algorithms on 3-branch Results

Stage	ATS		TS-Relax		MS-Relax			ATS-Relax		
	Time(s)	%Gain	Time(s)	%Gain	Time(s)	%Gain	%Gap	Time(s)	%Gain	%Gap
3	0.48	6.00	0.30	5.32	0.32	4.48	3.85	0.71	5.37	1.57
4	1.52	6.88	0.55	1.20	0.50	6.31	2.67	1.70	6.72	1.23
5	28.30	18.37	1.07	13.73	0.99	16.77	3.90	8.91	18.34	0.10
6	162.22	17.53	3.01	11.04	3.02	11.22	12.06	40.94	17.51	0.02
7	721.34	14.63	13.53	14.57	10.21	11.63	11.64	331.39	14.61	0.02
8	7200.00	-	73.17	12.10	77.97	8.80	10.48	7200.00	-	-
9	7200.00	-	447.11	6.03	265.36	4.14	6.65	7200.00	-	-
10	7200.00	-	2409.83	3.50	1410.35	2.59	3.62	7200.00	-	-

and ATS-Relax approaches are more sensitive to the instance size in comparison to Algorithms TS-Relax and MS-Relax. In terms of gain against two-stage approach, we examine higher gains in 3-branch scenario trees demonstrating the effect of higher variance on the performance of the proposed approach.

6.2.3. Discussion on Capacity Expansion Plans In this section, we examine a particular instance to analyze the generation capacity expansion plan of the adaptive two-stage model and compare it with that of the two-stage model. Figure 5 illustrates a generation expansion plan under the adaptive two-stage model over a five-year planning horizon. For each node of the tree, we show the acquired maximum effective capacity amount of each generation resource type $i \in \{1, \dots, 6\}$ in the order of nuclear, coal, NG-CC, NG-GT, wind and solar. The revision times of each resource type are determined by the model as 2, 3, 5, 5, 4, 5, respectively, and the expansion amounts of those periods are denoted in bold. We report the capacity expansion decisions of a certain resource type i in node n if period $t_n = t_i^*$ or $x_{in} > 0$. For notation simplicity, we omit the node index in the illustration. Figure 6 shows the generation expansion plan of the same instance under two-stage model. We note that the initial effective capacities of the resources are 4860, 2618, 2004, 1883, 134, 131 MWh respectively.

Figure 5 Generation expansion plan under adaptive two-stage model.

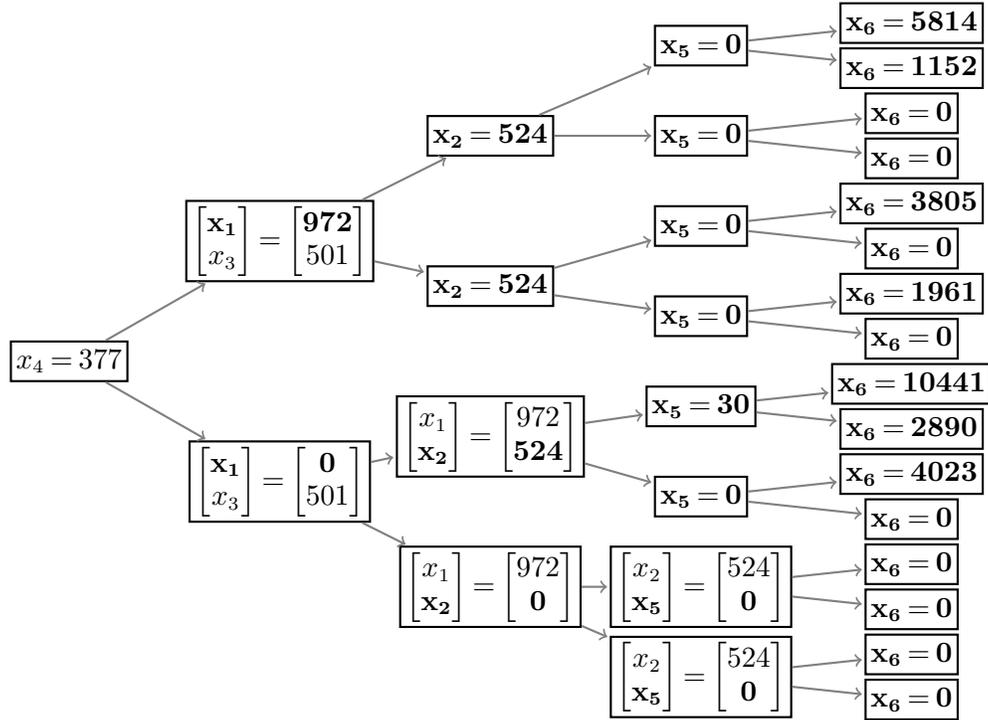
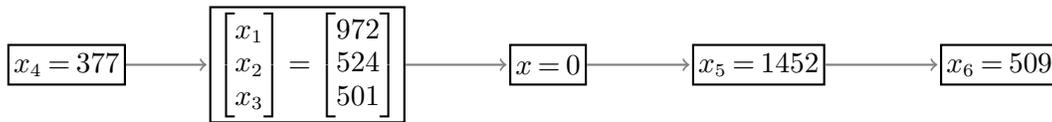


Figure 6 Generation expansion plan under two-stage model.



We observe that the adaptive two-stage approach provides more flexibility compared to the two-stage by allowing an update for each generation type’s expansion plan. Since the two-stage approach is less adaptive to the uncertainty, it brings forward the expansion times of the resources to satisfy the overall demand of each stage. We note that the expected total capacity expansion and allocation cost of the adaptive two-stage model is \$47.3 billion and the two-stage model is \$56.2 billion, resulting in 15.79% relative gain of the adaptive two-stage approach.

As the investment costs of the renewable resources decrease over time, both of the models tend to expand wind and solar generation capacities at later times in the planning. Due to this decrease in investment costs and increasing variability of demand in the scenario tree, the revision time of wind and solar capacities are in 4th and 5th periods to adjust the expansion decisions based on the underlying demand. The fuel prices and operating costs associated with traditional generation types increase over time. Thus, the capacities of these types of resources are expanded in the early periods of the planning horizon. To adapt to the demand uncertainty, we observe revisions

of expansion decisions at 2^{nd} and 3^{rd} periods for nuclear and coal generation. Since natural gas type generation resources are only expanded in first and second periods, their revision times do not necessarily have a practical implication.

7. Conclusion

In this paper, we propose a stochastic programming approach that determines the best time to revise decisions for problems with partial adaptability. We first present a generic formulation for the proposed adaptive two-stage approach, and demonstrate the importance of the revision times by deriving and illustrating adaptive policies for the multi-period newsvendor problem. We then develop a mixed-integer linear programming reformulation for the generic approach through scenario trees, and show that solving the resulting adaptive two-stage model is NP-Hard. Then, we focus our analyses on a specific problem structure that includes the capacity expansion planning problem under uncertainty. We derive the value of the proposed approach in comparison to two-stage and multi-stage stochastic programming models in terms of the revision times and problem parameters. We also provide a sensitivity analysis on these relative values with respect to the cost and demand parameters of the problem structure studied. We propose solution algorithms that selects the revision times by minimizing the objective gap of the adaptive two-stage approach in comparison to the multi-stage and maximizing the corresponding gap with the two-stage models by benefiting from our analytical analyses. We present an additional solution algorithm based on the relaxation of the adaptive two-stage model, for which we develop an approximation guarantee. In order to illustrate our results, we study a generation capacity expansion planning problem over a multi-period planning horizon. Our extensive computational study highlights up to 21% gain of the adaptive two-stage approach in objective in comparison to two stage model, demonstrating the importance of optimizing revision decisions. We show that this relative gain even increases in scenario trees with higher variability, and provide a computational study illustrating the performance and convergence of the proposed solution algorithms. Finally, we present the practical implications of utilizing the adaptive two-stage approach by examining sample generation capacity expansion plans.

As a future research direction, we plan to study the extensive model associated with the adaptive two-stage stochastic program for developing effective decomposition algorithms. Additionally, our approach can be extended to various problem settings to address partially adaptive policies that need to determine the best time to revise decisions.

Acknowledgments

This research has been supported in part by the National Science Foundation grant 1633196 and the Office of Naval Research grant N00014-18-1-2075.

Appendix A: Omitted Proofs

A.1. Proof of Theorem 1

To obtain an optimal policy for the adaptive two-stage model (1), we first focus on the problem (2) where the decision maker observes the inventory at hand at the beginning of period t^* to determine the order schedule of periods t^*, \dots, T .

PROPOSITION 10. For the problem (2), we obtain an optimal policy in the following form:

$$\tilde{F}_{t^*,t}(s + X_{t^*,t}) = \frac{-c_t + c_{t+1} + b_t}{h_t + b_t}, \quad t = t^*, \dots, T. \quad (29)$$

Proof: We first observe that the objective function of the problem (2) is convex as it can be represented as sum of convex functions. Then, we rewrite the objective function in the following form:

$$H(x) = \sum_{t=t^*}^T (c_t x_t + h_t \int_0^{s + \sum_{t'=t^*}^t x_{t'}} (s + \sum_{t'=t^*}^t x_{t'} - z) \tilde{f}_{t^*,t}(z) dz - b_t \int_{s + \sum_{t'=t^*}^t x_{t'}}^{\infty} (s + \sum_{t'=t^*}^t x_{t'} - z) \tilde{f}_{t^*,t}(z) dz),$$

where $\tilde{f}_{i,j}$ is the convolution probability density function corresponding to $\sum_{t=i}^j d_t$. We then take derivative of this expression with respect to the order quantities of the periods t^*, \dots, T as follows:

$$\begin{aligned} \frac{\partial H(x)}{\partial x_{t^*}} &= c_{t^*} + h_{t^*} \tilde{F}_{t^*,t^*}(s + x_{t^*}) - b_{t^*} (1 - \tilde{F}_{t^*,t^*}(s + x_{t^*})) + \dots \\ &\quad + h_T \tilde{F}_{t^*,T}(s + \sum_{t=t^*}^T x_t) - b_T (1 - \tilde{F}_{t^*,T}(s + \sum_{t=t^*}^T x_t)) \\ &\quad \vdots \\ \frac{\partial H(x)}{\partial x_{T-1}} &= c_{T-1} + h_{T-1} \tilde{F}_{t^*,T-1}(s + \sum_{t=t^*}^{T-1} x_t) - b_{T-1} (1 - \tilde{F}_{t^*,T-1}(s + \sum_{t=t^*}^{T-1} x_t)) \\ &\quad + h_T \tilde{F}_{t^*,T}(s + \sum_{t=t^*}^T x_t) - b_T (1 - \tilde{F}_{t^*,T}(s + \sum_{t=t^*}^T x_t)) \\ \frac{\partial H(x)}{\partial x_T} &= c_T + h_T \tilde{F}_{t^*,T}(s + \sum_{t=t^*}^T x_t) - b_T (1 - \tilde{F}_{t^*,T}(s + \sum_{t=t^*}^T x_t)) \end{aligned}$$

By equating the above expressions to 0, we obtain the desired conditions, concluding the proof.

□

As a next step, we aim extending our result to the adaptive two-stage case. We observe that the objective function denoted in (1) is convex. Let the corresponding objective function be $G(x)$, and we take its derivative with respect to the order quantities of the periods $1, \dots, t^* - 1$ as follows:

$$\frac{\partial G(x)}{\partial x_1} = c_1 + h_1 \tilde{F}_{1,1}(x_1) - b_1 (1 - \tilde{F}_{1,1}(x_1)) + \dots$$

$$\begin{aligned}
 & + h_{t^*-1} \tilde{F}_{1,t^*-1} \left(\sum_{t=1}^{t^*-1} x_t \right) - b_{t^*-1} \left(1 - \tilde{F}_{1,t^*-1} \left(\sum_{t=1}^{t^*-1} x_t \right) \right) + \frac{\partial E[Q_{t^*}(\sum_{t=1}^{t^*-1} (x_t - d_t))]}{\partial x_1} \\
 & \vdots \\
 \frac{\partial G(x)}{\partial x_{t^*-1}} & = c_{t^*-1} + h_{t^*-1} \tilde{F}_{1,t^*-1} \left(\sum_{t=1}^{t^*-1} x_t \right) - b_{t^*-1} \left(1 - \tilde{F}_{1,t^*-1} \left(\sum_{t=1}^{t^*-1} x_t \right) \right) + \frac{\partial E[Q_{t^*}(\sum_{t=1}^{t^*-1} (x_t - d_t))]}{\partial x_{t^*-1}}
 \end{aligned}$$

To derive $\frac{\partial Q_{t^*}(s)}{\partial x_i}$ for $i = 1, \dots, t^* - 1$, we use the chain rule as $\frac{\partial Q_{t^*}(s)}{\partial x_i} = \frac{\partial Q_{t^*}(s)}{\partial s} \cdot \frac{\partial s}{\partial x_i}$, where $s = \sum_{t=1}^{t^*-1} (x_t - d_t)$. For that purpose, we need to identify the derivative of $Q_{t^*}(s)$ with respect to s .

LEMMA 1. $\frac{\partial Q_{t^*}(s)}{\partial s} = -c_{t^*}$.

Proof: We let the optimal solution of the problem (2) as $(x_{t^*}^*, \dots, x_T^*)$. Then, we can represent $\frac{\partial Q_{t^*}(s)}{\partial s}$ as follows:

$$\begin{aligned}
 \frac{\partial Q_{t^*}(s)}{\partial s} & = h_{t^*} \tilde{F}_{t^*,t^*}(s + x_{t^*}^*) - b_{t^*} \left(1 - \tilde{F}_{t^*,t^*}(s + x_{t^*}^*) \right) + \dots \\
 & \quad + h_T \tilde{F}_{t^*,T}(s + \sum_{t=t^*}^T x_t^*) - b_T \left(1 - \tilde{F}_{t^*,T}(s + \sum_{t=t^*}^T x_t^*) \right)
 \end{aligned}$$

Using the optimality conditions derived in the proof of Proposition 10, we can prove that $\frac{\partial Q_{t^*}(s)}{\partial s} = -c_{t^*}$. \square

Combining the above, we obtain the desired result.

A.2. Proof of Theorem 2

To prove the theorem, it suffices to show that the feasibility problem associated with the adaptive two-stage problem (9) is \mathcal{NP} -Complete. For this purpose, we define the feasible region constructed in (30), and refer to this feasibility problem as P. Next, we demonstrate that the subset sum problem, which is known to be \mathcal{NP} -Complete (Garey and Johnson 1990), can be reduced to P in polynomial time. We specify the subset sum problem as follows: Given the non-negative integers $w'_1, w'_2, \dots, w'_{N'}$, and W' , does there exist a subset $S \subseteq \{1, \dots, N'\}$ such that $\sum_{i \in S} w'_i = W'$?

$$\min_{\alpha, \beta, t^*} 0 \tag{30a}$$

$$\text{s.t. } \alpha_{i2} - \beta_{i2} = \alpha_{i1} \quad i = 1, \dots, N \tag{30b}$$

$$\alpha_{i3} - \beta_{i3} = -\alpha_{i1} \quad i = 1, \dots, N \tag{30c}$$

$$\alpha_{i4} - \beta_{i4} = \alpha_{i1} \quad i = 1, \dots, N \tag{30d}$$

$$\alpha_{i5} - \beta_{i5} = 2 - \alpha_{i1} \quad i = 1, \dots, N \tag{30e}$$

$$\alpha_{i6} - \beta_{i6} = \alpha_{i1} \quad i = 1, \dots, N \tag{30f}$$

$$\alpha_{i7} - \beta_{i7} = 2 - \alpha_{i1} \quad i = 1, \dots, N \tag{30g}$$

$$\sum_{i=1}^N w_i \alpha_{i1} = W \tag{30h}$$

$$\alpha_{im} = \alpha_{in}, \beta_{im} = \beta_{in} \quad \forall m, n \in S_t, \quad t < t_i^*, i = 1, \dots, N \tag{30i}$$

$$\alpha_{im} = \alpha_{in}, \beta_{im} = \beta_{in} \quad \forall m, n \in S_t \cap \mathcal{T}(j), \quad j \in S_{t_i^*}, \quad t \geq t_i^*, i = 1, \dots, N \tag{30j}$$

$$t_i^* \in \{1, 2, 3\} \quad i = 1, \dots, N. \tag{30k}$$

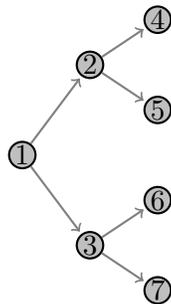
Clearly, the problem (30) is an instance of the adaptive two-stage problem. Specifically, we let $I = N$, $J = 0$, and $x_{in} = \begin{bmatrix} \alpha_{in} \\ \beta_{in} \end{bmatrix}$. We consider the scenario tree \mathcal{T} with $T = 3$ stages as depicted in Figure 7. We let $\mathcal{T} = \{1, \dots, 7\}$ be the set of nodes in ascending order, where node 1 represents the root node. We also let $N = N'$, $W = W'$, and $w_i = w'_i$ for all $i = 1, \dots, N$. We note that constraints (30b) - (30g) correspond to constraint (7b), and constraint (30h) represent the constraint (7c). Similarly, constraints (30i), (30j) and (30k) refer to the constraints (9b), (9c) and (9d), respectively.

LEMMA 2. The following holds for the problem (30):

1. If $t_i^* = 1$ for any $i = 1, \dots, N$, then the problem (30) is infeasible.
2. If $t_i^* = 2$, then $\alpha_{i1} = 1$ for all $i = 1, \dots, N$.
3. If $t_i^* = 3$, then $\alpha_{i1} = 0$ for all $i = 1, \dots, N$.

The proof of the Lemma 2 follows from the construction of the program (30). Specifically, when $t_i^* = 1$ for any $i = 1, \dots, N$, then due to constraints (30i) and (30j), $\alpha_{i1} = -\alpha_{i1} = 2 - \alpha_{i1}$ resulting in infeasibility. When $t_i^* = 2$, then we have $\alpha_{i1} = 2 - \alpha_{i1}$ due to constraints (30d) and (30g), resulting in $\alpha_{i1} = 1$. Similarly, when $t_i^* = 3$, we have $\alpha_{i1} = 2 - \alpha_{i1}$ due to constraints (30b) and (30c) making $\alpha_{i0} = 0$.

Figure 7 Scenario tree for the problem (30).



We first show that given a solution S to the subset-sum problem, we can construct a feasible solution for problem P. Specifically, we set $\alpha_{i1} = 1$ if $i \in S$, and 0 otherwise for all $i = 1, \dots, N$. If $\alpha_{i1} = 1$, then $t_i = 2$ resulting in $\alpha_{i2} = 1$, $\beta_{i2} = 0$, $\alpha_{i3} = 0$, $\beta_{i3} = 1$, $\beta_{i4} = \beta_{i5} = \beta_{i6} = \beta_{i7} = 0$, $\alpha_{i4} = \alpha_{i5} = \alpha_{i6} = \alpha_{i7} = 1$ for all $i = 1, \dots, N$. If $\alpha_{i1} = 0$, then $t_i = 3$ resulting in $\alpha_{i2} = \beta_{i2} = \alpha_{i3} = \beta_{i3} = 0$,

$\alpha_{i4} = \beta_{i4} = 0$, $\alpha_{i5} = 2$, $\beta_{i5} = 0$, $\alpha_{i6} = 0$, $\beta_{i6} = 0$, $\alpha_{i7} = 2$, $\beta_{i7} = 0$. Secondly, given a feasible solution (α, β, t^*) to the problem P, we can construct a feasible solution S for the subset-sum. We note that in a feasible solution of P, we have either $t_i^* = 2$ or $t_i^* = 3$ for $i = 1, \dots, N$, as noted in Lemma 2. To construct a feasible solution, we select $S = \{i : t_i^* = 2\}$. As (α, β, t^*) is a feasible solution of P, we satisfy the condition $\sum_{i \in S} w'_i = W'$. Combining the above, we prove that P is \mathcal{NP} -Complete, completing the proof.

A.3. Proof of Proposition 1

We observe that the coefficient matrix of (17) is composed of 0 and 1 values. Additionally, number of 1's in each column j is equal to $|\mathcal{T}_i(t_i^*)(j)|$, where $\mathcal{T}_i(t_i^*)(j)$ represents the subtree rooted at node j . More specifically, an entry corresponding to i^{th} row and j^{th} column of the coefficient matrix is 1 if and only if node $i \in \mathcal{T}_i(t_i^*)(j)$. Consequently, we can rearrange the coefficient matrix to obtain an interval matrix. This demonstrates that the desired matrix is totally unimodular.

A.4. Proof of Proposition 2

The decision variables $\{x_n\}_{\{n \in \mathcal{T}\}}$ can be categorized into two groups according to the revision time t^* . First of all, for $t < t^*$, the solutions $\{x_m : m \in S_t\}$ have the same value. Thus, we denote variables until period t^* as \hat{x}_t for all $t = 1, \dots, t^* - 1$. Secondly, for $t \geq t^*$, we have the same solutions for $\{x_m : m \in S_t \cap \mathcal{T}(n)\}$ for all $n \in S_{t^*}$. We refer to these variables as \hat{x}_{nt} for any $n \in S_{t^*}$ and $t \geq t^*$.

In order to find an upper bound on $v^R(t^*)$, we construct a feasible solution where $\hat{x}_1 = \delta_1$, $\hat{x}_t = \max_{m \in \mathcal{T}(1,t)} \{\delta_m\} - \max_{m \in \mathcal{T}(1,t-1)} \{\delta_m\}$ for $2 \leq t \leq t^* - 1$, and $\hat{x}_{nt} = \max\{\delta_m : m \in \mathcal{T}(1, t^* - 1) \cup \mathcal{T}(n, t)\}$ for $t \geq t^*$. Using the relationship $v^R(t^*) \leq \sum_{n \in \mathcal{T}} p_n a_n x_n$ for any feasible $\{x_n\}_{\{n \in \mathcal{T}\}}$, we can obtain the following:

$$\begin{aligned}
 v^R(t^*) &\leq \sum_{t=1}^{t^*-1} \sum_{n \in S_t} p_n a_n \hat{x}_t + \sum_{t=t^*}^T \sum_{n \in S_{t^*}} p_n a_n \hat{x}_{nt} \\
 &\leq \bar{a}^-(t^*) \sum_{t=1}^{t^*-1} \hat{x}_t + \bar{a}^+(t^*) \sum_{n \in S_{t^*}} p_n \sum_{t=t^*}^T \hat{x}_{nt} \\
 &= \bar{a}^-(t^*) \sum_{t=1}^{t^*-1} \left(\max_{m \in \mathcal{T}(1,t)} \{\delta_m\} - \max_{m \in \mathcal{T}(1,t-1)} \{\delta_m\} \right) \\
 &\quad + \bar{a}^+(t^*) \sum_{n \in S_{t^*}} p_n \sum_{t=t^*}^T \left(\max_{m \in \mathcal{T}(1,t^*-1) \cup \mathcal{T}(n,t)} \{\delta_m\} - \max_{m \in \mathcal{T}(1,t^*-1) \cup \mathcal{T}(n,t-1)} \{\delta_m\} \right) \\
 &= \bar{a}^-(t^*) \max_{m \in \mathcal{T}(1,t^*-1)} \{\delta_m\} + \bar{a}^+(t^*) \sum_{n \in S_{t^*}} p_n \left(\max_{m \in \mathcal{T}(1,t^*-1) \cup \mathcal{T}(n)} \{\delta_m\} - \max_{m \in \mathcal{T}(1,t^*-1)} \{\delta_m\} \right) \\
 &= \bar{a}^-(t^*) \max_{m \in \mathcal{T}(1,t^*-1)} \{\delta_m\} + \bar{a}^+(t^*) \sum_{n \in S_{t^*}} p_n \max_{m \in \mathcal{T}(1,t^*-1) \cup \mathcal{T}(n)} \{\delta_m\} - \bar{a}^+(t^*) \max_{m \in \mathcal{T}(1,t^*-1)} \{\delta_m\} \\
 &= (\bar{a}^-(t^*) - \bar{a}^+(t^*)) \delta^-(t^*) + \bar{a}^+(t^*) \delta^+(t^*).
 \end{aligned}$$

Let \hat{x}_t^* for $t < t^*$ and \hat{x}_{nt}^* , for $t \geq t^*$ and $n \in S_{t^*}$ denote the optimal solution for the problem (16). Then, we can derive the following:

$$\begin{aligned}
 v^R(t^*) &= \sum_{t=1}^{t^*-1} \sum_{n \in S_t} p_n a_n \hat{x}_t^* + \sum_{t=t^*}^T \sum_{n \in S_{t^*}} p_n a_n \hat{x}_{nt}^* \\
 &\geq \underline{a}^-(t^*) \sum_{t=1}^{t^*-1} \hat{x}_t^* + \underline{a}^+(t^*) \sum_{n \in S_{t^*}} p_n \sum_{t=t^*}^T \hat{x}_{nt}^* \\
 &\geq \underline{a}^-(t^*) \sum_{t=1}^{t^*-1} \hat{x}_t^* + \underline{a}^+(t^*) \sum_{n \in S_{t^*}} p_n \left(\max_{m \in \mathcal{T}(1, t^*-1) \cup \mathcal{T}(n)} \{\delta_m\} - \sum_{t=1}^{t^*-1} \hat{x}_t^* \right) \\
 &= (\underline{a}^-(t^*) - \underline{a}^+(t^*)) \sum_{t=1}^{t^*-1} \hat{x}_t^* + \underline{a}^+(t^*) \sum_{n \in S_{t^*}} p_n \left(\max_{m \in \mathcal{T}(1, t^*-1) \cup \mathcal{T}(n)} \{\delta_m\} \right) \\
 &\geq (\underline{a}^-(t^*) - \underline{a}^+(t^*)) \delta^-(t^*) + \underline{a}^+(t^*) \delta^+(t^*).
 \end{aligned}$$

The second and fourth inequalities follow from constraint (14b). In particular, any feasible solution \hat{x}_t and \hat{x}_{nt} for any $n \in S_{t^*}$ should satisfy $\sum_{t=1}^{t^*-1} \hat{x}_t + \sum_{t=t^*}^T \hat{x}_{nt} \geq \max_{m \in \mathcal{T}(1, t^*-1) \cup \mathcal{T}(n)} \{\delta_m\}$. Additionally, $\sum_{t=1}^{t^*-1} \hat{x}_t \geq \max_{m \in \mathcal{T}(1, t^*-1)} \{\delta_m\} = \delta^-(t^*)$.

Combining above, we derive upper and lower bounds to $v^R(t^*)$.

A.5. Proof of Proposition 4

If $\hat{t}^D = 1$, then $\delta^+(t) = \delta^*$ for any $t \in \{1, \dots, T\}$. Otherwise, $\delta^+(1) = \delta^*$ as $\mathcal{T}(1)$ corresponds to the full scenario tree including the maximum demand value. For any node n in stages $t > \hat{t}^D$, we have $\max_{m \in \mathcal{T}(1, t-1) \cup \mathcal{T}(n)} \{\delta_m\} = \delta^*$. Hence, we obtain $\delta^+(t) = \delta^*$ for $t > \hat{t}^D$. For an intermediate stage t between 1 and \hat{t}^D , we have the relationship $\delta^+(t) \leq \delta^*$. Combining the above, under a general demand structure for δ_n when $\hat{t}^D > 1$, we conclude that the minimizer of $\delta^+(t)$ is in $\{2, \dots, \hat{t}^D\}$.

A.6. Proof of Proposition 5

We observe that for the cases $t < \hat{t}^D$ and $t > \hat{t}^D$, we have $\delta^+(t) = \delta^*$. For $t = \hat{t}^D$, we obtain the relationship $\max_{m \in \mathcal{T}(1, t-1) \cup \mathcal{T}(n)} \{\delta_m\} < \delta^*$ for some $n \in S_{\hat{t}^D}$. Hence, we have $\delta^+(\hat{t}^D) < \delta^*$ making \hat{t}^D the minimizer of $\delta^+(t)$.

A.7. Proof of Proposition 7

We first observe that $V^{TS} \geq \sum_{n \in \mathcal{T}} p_n b_n^\top y_n^{TLP} + \sum_{i=1}^I v_i^T$. Next, we note that $\{y_n^{TLP}\}_{n \in \mathcal{T}}$ is a feasible solution to the adaptive two-stage problem under t^* . Thus, $V^{ATS}(t^*) \leq \sum_{n \in \mathcal{T}} p_n b_n^\top y_n^{TLP} + \sum_{i=1}^I o_i^R(t_i^*)$, where

$$o_i^R(t_i^*) = \min \left\{ \sum_{n \in \mathcal{T}_i(t_i^*)} \hat{p}_n \hat{a}_{in} x_{in} : \sum_{m \in P(n)} x_{im} \geq \hat{\delta}_{in}, x_{in} \in \mathbb{Z}_+ \forall n \in \mathcal{T}_i(t_i^*) \right\}.$$

Additionally, we can represent $v_i^R(t_i^*)$ as follows

$$\begin{aligned} v_i^R(t_i^*) &= \min\left\{ \sum_{n \in \mathcal{T}_i(t_i^*)} \hat{p}_n \hat{a}_{in} x_{in} : \sum_{m \in P(n)} x_{im} \geq \hat{\delta}_{in}, x_{in} \in \mathbb{R}_+ \forall n \in \mathcal{T}_i(t_i^*) \right\} \\ &= \max\left\{ \sum_{n \in \mathcal{T}_i(t_i^*)} \hat{\delta}_{in} \pi_{in} : \sum_{m \in \mathcal{T}_i(t_i^*)} \pi_{im} \leq \hat{p}_n \hat{a}_{in}, \pi_{in} \in \mathbb{R}_+ \forall n \in \mathcal{T}_i(t_i^*) \right\}. \end{aligned}$$

Using Proposition 1 and linear programming duality, we can reexpress $o_i^R(t_i^*)$ as

$$\begin{aligned} o_i^R(t_i^*) &= \min\left\{ \sum_{n \in \mathcal{T}_i(t_i^*)} \hat{p}_n \hat{a}_{in} x_{in} : \sum_{m \in P(n)} x_{im} \geq \lceil \hat{\delta}_{in} \rceil, x_{in} \in \mathbb{R}_+ \forall n \in \mathcal{T}_i(t_i^*) \right\} \\ &= \max\left\{ \sum_{n \in \mathcal{T}_i(t_i^*)} (\hat{\delta}_{in} + (\lceil \hat{\delta}_{in} \rceil - \hat{\delta}_{in})) \pi_{in} : \sum_{m \in \mathcal{T}_i(t_i^*)} \pi_{im} \leq \hat{p}_n \hat{a}_{in}, \pi_{in} \in \mathbb{R}_+ \forall n \in \mathcal{T}_i(t_i^*) \right\} \\ &\leq v_i^R(t_i^*) + \max_{n \in \mathcal{T}_i(t_i^*)} \{ \lceil \hat{\delta}_{in} \rceil - \hat{\delta}_{in} \} \max\left\{ \sum_{n \in \mathcal{T}_i(t_i^*)} \pi_{in} : \sum_{m \in \mathcal{T}_i(t_i^*)} \pi_{im} \leq \hat{p}_n \hat{a}_{in}, \pi_{in} \in \mathbb{R}_+ \forall n \in \mathcal{T}_i(t_i^*) \right\} \\ &= v_i^R(t_i^*) + \max_{n \in \mathcal{T}_i(t_i^*)} \{ \lceil \hat{\delta}_{in} \rceil - \hat{\delta}_{in} \} \min\left\{ \sum_{n \in \mathcal{T}_i(t_i^*)} \hat{p}_n \hat{a}_{in} x_{in} : \sum_{m \in P(n)} x_{im} \geq 1, x_{in} \in \mathbb{R}_+ \forall n \in \mathcal{T}_i(t_i^*) \right\} \\ &= v_i^R(t_i^*) + \max_{n \in \mathcal{T}_i(t_i^*)} \{ \lceil \hat{\delta}_{in} \rceil - \hat{\delta}_{in} \} a_{i1}. \end{aligned}$$

Here, the last equality follows from the fact that $x_{i1} = 1$ and $x_{in} = 0$ for $n \in \mathcal{T}_i(t_i^*) \setminus \{1\}$ is an optimal solution of the resulting single-resource problem. Combining the above, we demonstrate the desired result.

A.8. Proof of Proposition 8

We first observe that $V^{MS} \geq \sum_{n \in \mathcal{T}} p_n b_n^\top y_n^{MLP} + \sum_{i=1}^I v_i^M$. Next, we have $V^{ATS}(t^*) \leq \sum_{n \in \mathcal{T}} p_n b_n^\top y_n^{MLP} + \sum_{i=1}^I o_i^R(t_i^*)$ as before. Using the same techniques as in the proof of Proposition 7, we obtain the desired result.

A.9. Proof of Proposition 9

We denote the objective value of the adaptive two-stage version of the stochastic capacity expansion planning problem (11) under a given (x, y, t) decision as

$$f(x, y, t) := \sum_{i=1}^I \sum_{n \in \mathcal{T}_i(t_i)} \hat{p}_n \hat{a}_{in} x_{in} + \sum_{n \in \mathcal{T}} p_n b_n^\top y_n,$$

where $\mathcal{T}_i(t_i)$ corresponds to the compressed tree under the revision decision t_i , and $\hat{p}_n = \sum_{m \in \hat{C}_n} p_m$, $\hat{a}_{in} = \sum_{m \in \hat{C}_n} p_m a_{im}$ as discussed in formulation (17).

We represent the solution corresponding to *ATS-Relax* algorithm as $(x^{ATS-Relax}, y^{ATS-Relax}, t^{ATS-Relax})$, and the solution of the true adaptive two-stage program as $(x^{ATS}, y^{ATS}, t^{ATS})$. Consequently, we define the bound between the two approaches as

$$V^{ATS-Relax} - V^{ATS} = f(x^{ATS-Relax}, y^{ATS-Relax}, t^{ATS-Relax}) - f(x^{ATS}, y^{ATS}, t^{ATS})$$

$$\begin{aligned}
 &\leq f(x^{ATS-Relax}, y^{ATS-Relax}, t^{ATS-Relax}) - f(x^{LP}, y^{LP}, t^{LP}) \\
 &= f(x^{ATS-Relax}, y^{ATS-Relax}, t^{ATS-Relax}) - f(x^{ATS-Relax}, y^{LP}, t^{LP}) \\
 &\quad + f(x^{ATS-Relax}, y^{LP}, t^{LP}) - f(x^{LP}, y^{LP}, t^{LP}) \\
 &\leq f(x^{ATS-Relax}, y^{LP}, t^{LP}) - f(x^{LP}, y^{LP}, t^{LP}),
 \end{aligned}$$

where (x^{LP}, y^{LP}, t^{LP}) represents the solution of the relaxation of the true adaptive two-stage program when x decisions are relaxed to be continuous. We note that $t^{LP} = t^{ATS-Relax}$ by the definition of Algorithm 3. Next, we can state the following

$$f(x^{ATS-Relax}, y^{LP}, t^{LP}) - f(x^{LP}, y^{LP}, t^{LP}) = \sum_{i=1}^I \sum_{n \in \mathcal{T}_i(t_i^{LP})} \hat{p}_n \hat{a}_{in} (x_{in}^{ATS-Relax} - x_{in}^{LP}),$$

Here, \hat{p}_n and \hat{a}_n are computed specifically for the compressed tree under the revision decision t^{LP} . Using the analysis in the proof of Theorem 6 in Huang and Ahmed (2009), the above expression reduces to

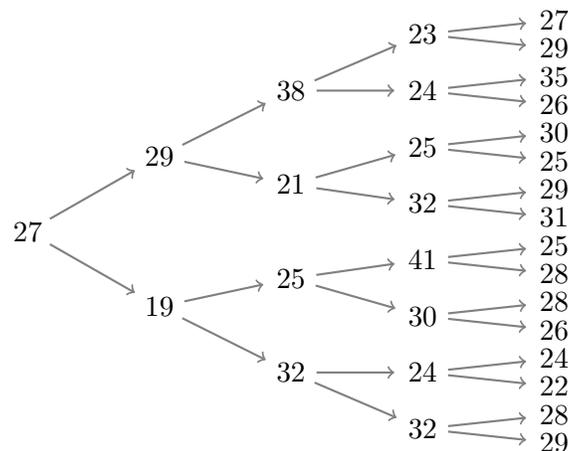
$$f(x^{ATS-Relax}, y^{LP}, t^{LP}) - f(x^{LP}, y^{LP}, t^{LP}) = \sum_{i=1}^I \left(\max_{n \in \hat{\mathcal{T}}_i(t_i^{ATS-Relax})} \{(\lceil \delta_{in} \rceil - \delta_{in})\} a_{i1} \right).$$

Combining the above, we obtain the desired result.

Appendix B: Illustrative Instance

In this section, we provide the details of the instance studied in Section 4.2.2. The cost parameter $a_n = 1$ for all $n \in \mathcal{T}$. The demand parameter $\{\delta_n\}_{n \in \mathcal{T}}$ is randomly generated from the distribution $N(\mu, \sigma^2)$, where $\mu = 30$ and $\sigma = 5$. We consider a scenario tree with 5 stages, and the resulting values for $\{\delta_n\}_{n \in \mathcal{T}}$ are presented in Figure 8.

Figure 8 Demand values for the illustrative instance.



Appendix C: Scenario Tree Generation Algorithm

Algorithm 4 Scenario Tree Generation with M branches

- 1: Obtain the demand values at the root node as $\{d_{k0}\}_{k \in \mathcal{K}}$.
 - 2: **for all** $t = 2, \dots, T$ **do**
 - 3: Let $\underline{\alpha}_t \leq \bar{\alpha}_t$ be the demand increase multiplier bounds.
 - 4: Find the equisized multiplier interval values as $\{\alpha_t^j\}_{j=0}^M$ where $\alpha_t^j = \underline{\alpha}_t + j(\bar{\alpha}_t - \underline{\alpha}_t)/M$.
 - 5: **for all** $k \in \mathcal{K}$ **do**
 - 6: **for all** node $n \in S_t$ **do**
 - 7: Find the order of the node in the stage as $j = n \bmod M$.
 - 8: Generate the demand increase multiplier $\beta_t^j \sim U(\alpha_t^j, \alpha_t^{j+1})$.
 - 9: Define $d_{kn} := \beta_t^j d_{ka(n)}$.
-

References

- Ahmed S (2016) Multistage stochastic optimization. <https://www.ima.umn.edu/materials/2015-2016/ND8.1-12.16/25386/mssp.pdf>.
- Ahmed S, King AJ, Parija G (2003) A multi-stage stochastic integer programming approach for capacity expansion under uncertainty. *Journal of Global Optimization* 26(1):3–24.
- Ahmed S, Sahinidis NV (2003) An approximation scheme for stochastic integer programs arising in capacity expansion. *Operations Research* 51(3):461–471.
- Balasubramanian J, Grossmann IE (2004) Approximation to multistage stochastic optimization in multi-period batch plant scheduling under demand uncertainty. *Industrial & Engineering Chemistry Research* 43(14):3695–3713.
- Bertocchi M, Vittorio M, Dupačová J (2006) Horizon and stages in applications of stochastic programming in finance. *Annals of Operations Research* 142(1):63–78.
- Black V (2012) Cost and performance data for power generation technologies prepared for the national renewable energy laboratory. Technical report, U.S. Department of Energy, National Renewable Energy Laboratory, URL <https://www.bv.com/docs/reports-studies/nrel-cost-report.pdf>.
- Bodur M, Luedtke JR (2018) Two-stage linear decision rules for multi-stage stochastic programming. <https://arxiv.org/pdf/1701.04102.pdf>.
- Bookbinder JH, Tan JY (1988) Strategies for the probabilistic lot-sizing problem with service-level constraints. *Management Science* 34(9):1096–1108.
- Chen L, de Mello TH (2010) Re-solving stochastic programming models for airline revenue management. *Annals of Operations Research* 177(1):91–114.

- Dempster MAH, Pedrón NH, Medova A, Scottnand JE, Semboš A (2000) Planning logistics operations in the oil industry. *Journal of the Operational Research Society* 51(11):1271–1288.
- Dempster MAH, Thompson G (2002) *Dynamic portfolio replication using stochastic programming* (Risk Management: Value at Risk and Beyond. Cambridge Univ. Press).
- Dupačová J (2006) Contamination for multistage stochastic programs. <http://edoc.hu-berlin.de/18452/9003>.
- Dupačová J, Bertocchi M, Moriggia V (2009) Testing the structure of multistage stochastic programs. *Computational Management Science* 6(2):161–185.
- Garey M, Johnson D (1990) *Computers and Intractability; A Guide to the Theory of NP-Completeness* (New York, NY, USA: W. H. Freeman & Co.).
- Huang K, Ahmed S (2009) The value of multistage stochastic programming in capacity planning under uncertainty. *Operations Research* 57(4):893–904.
- Jin S, Ryan SM, Watson JP, Woodruff DL (2011) Modeling and solving a large-scale generation expansion planning problem under uncertainty. *Energy Systems* 2(3):209–242.
- Koca E, Yaman H, Akturk M (2018) Stochastic lot sizing problem with nervousness considerations. *Computers & Operations Research* 94:23 – 37.
- Kuhn D (2005) *Generalized Bounds for Convex Multistage Stochastic Programs*, volume 548 (Springer).
- Lin JT, Chen TL, Chu HC (2014) A stochastic dynamic programming approach for multi-site capacity planning in tft-lcd manufacturing under demand uncertainty. *International Journal of Production Economics* 148:21 – 36.
- Liu Y, Sioshansi R, Conejo AJ (2018) Multistage stochastic investment planning with multiscale representation of uncertainties and decisions. *IEEE Transactions on Power Systems* 33(1).
- Min D, hyun Ryu J, Choi DG (2018) A long-term capacity expansion planning model for an electric power system integrating large-size renewable energy technologies. *Computers & Operations Research* 96:244 – 255.
- Ozen U, Dogru MK, Tarim SA (2012) Static-dynamic uncertainty strategy for a single-item stochastic inventory control problem. *Omega* 40(3):348 – 357.
- Pflug GC (2001) Scenario tree generation for multiperiod financial optimization by optimal discretization. *Mathematical Programming* 89(2):251–271.
- Rajagopalan S, Singh MR, Morton TE (1998) Capacity expansion and replacement in growing markets with uncertain technological breakthroughs. *Management Science* 44(1):12–30.
- Riis M, Andersen KA (2004) Multiperiod capacity expansion of a telecommunications connection with uncertain demand. *Computers & Operations Research* 31(9):1427 – 1436.
- Ruszczynski AP, Shapiro A (2003) *Stochastic programming*, volume 10 (Elsevier Amsterdam).

- Shapiro A (2003) Inference of statistical bounds for multistage stochastic programming problems. *Mathematical Methods of Operations Research* 58(1):57–68.
- Silver EA, Peterson R, Pyke DF (1988) *Inventory Management and Production Scheduling* (John Wiley & Sons).
- Singh KJ, Philpott AB, Wood RK (2009) Dantzig-wolfe decomposition for solving multistage stochastic capacity-planning problems. *Operations Research* 57(5):1271–1286.
- Swaminathan JM (2000) Tool capacity planning for semiconductor fabrication facilities under demand uncertainty. *European Journal of Operational Research* 120(3):545 – 558.
- Tarim S, Kingsman BG (2004) The stochastic dynamic production/inventory lot-sizing problem with service-level constraints. *International Journal of Production Economics* 88(1):105 – 119.
- Tunc H, Kilic OA, Tarim SA, Rossi R (2018) An extended mixed-integer programming formulation and dynamic cut generation approach for the stochastic lot-sizing problem. *INFORMS Journal on Computing* 30(3):492–506.
- Van Mieghem JA (2003) Commissioned paper: Capacity management, investment, and hedging: Review and recent developments. *Manufacturing & Service Operations Management* 5(4):269–302.
- Wang KJ, Nguyen PH (2017) Capacity planning with technology replacement by stochastic dynamic programming. *European Journal of Operational Research* 260(2):739 – 750.
- Zhang M, Kucukyavuz S, Goel S (2014) A branch-and-cut method for dynamic decision making under joint chance constraints. *Management Science* 60(5):1317–1333.
- Zipkin P (2000) *Foundations of Inventory Management* (McGraw-Hill).
- Zou J, Ahmed S, Sun XA (2018) Partially adaptive stochastic optimization for electric power generation expansion planning. *INFORMS Journal on Computing* 30(2):388–401.