Competing Objective Optimization in Networked Swarm Systems

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***Abstract*—In this paper, we develop a decentralized collaborative sensing algorithm where the sensors are located on-board autonomous unmanned aerial vehicles. We develop this algorithm in the context of a target tracking application, where the objective is to maximize the tracking performance measured by the mean-squared error between the target state estimate and the ground truth. The tracking performance depends on the quality of the target measurements made at the sensors, which depends on the relative location of the sensors with respect to the target. Our goal is to control the motion of the swarm of vehicles with on-board sensors to maximize target tracking performance. Each sensor (on-board the vehicle) generates local noisy measurements of the target location, and the sensors maintain and update target state estimates via Bayesian data fusion rules using local measurements and the information received from neighboring sensors. The quality of the data fusion depends on the network graph over which the sensors exchange information, and this determines the overall target tracking performance. We also assume that each sensor is powered by a limited energy source; which we assume is drained by how frequently sensors exchange information. The goal is to optimize the collective motion of the vehicles/sensors (also determines the network graph connectivity) such that the mean-squared target tracking error and the network energy costs are jointly minimized. This problem belongs to a class of hard optimization problems called *conflicting objective limited resource optimization* (COLRO). We develop a fast heuristic algorithm, using dynamic programming principles, to solve this COLRO problem in real-time.**

***Index Terms*—Swarm systems, target tracking, competing objectives, sensor network**

I. INTRODUCTION

There is a growing interest in decentralized and distributed autonomous sensing methods [1], [2], where the network connecting the sensors may be time-varying. With increasing number of sensor and surveillance systems in public places, there is a need for decentralized methods to track moving targets (e.g. movement of an intruder, movement of enemy tanks in battle field) with a network of sensors. However, the decentralized collaborative sensing in a wireless multi-sensor network is a challenging problem, especially when there are network energy costs involved. Since the battery-powered sensor nodes have limited energy, there is a need for methods that can trade off between the target tracking performance and the energy costs of acquiring the measurements and sharing them (with peers) over a network. If a distributed set of autonomous vehicles are connected via a wireless network (vehicle is considered a wireless node), due to the movement of the vehicles, the links in the network graph may form and break as the relative distances between the nodes change over time, thus leading to a time-varying graph. There is a growing interest in controlling the motion of the vehicles with on-board sensors for various applications such as formation control [3], [4], target tracking [5]. With this motivation, we develop a stochastic decision optimization framework to control the motion of a swarm of autonomous vehicles (e.g., unmanned aerial systems) to track a moving object, where the swarm is connected via a wireless network.

As swarm-based systems tend to have a large number of vehicles, optimizing each motion control variable may lead to computationally expensive optimization problems; instead, we optimize the centroid location of the swarm. Once a desired centroid and network graph are obtained, the vehicles may choose one of infinitely many paths to achieve the desired centroid and the network graph.

As mentioned earlier, we also optimize the network graph of the swarm, which determines how well the sensors (on-board the vehicles) fuse their local sensor measurements with the measurements received from the neighboring sensors, as depicted in Figure 1. Clearly, the objectives of maximizing the tracking performance and minimizing the network energy costs are competing, i.e., emphasizing one objective deteriorates the other. We refer to these problems as *competing objective limited resource optimization* (COLRO) problems. In this paper, we focus on solving COLRO problems in real-time in the context of networked swarm systems.

II. Problem Specification and Approach

Let k represent the time index. A target moves on a 2-D plane according to the *constant velocity* model [6]. Let χ_k represent the target state at time k, which includes its location, velocity, and acceleration. According to the *constant velocity*

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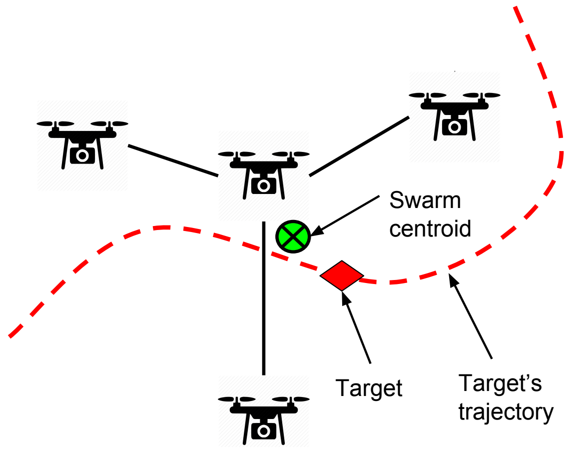

Fig. 1. Autonomous vehicle swarm tracking a target while jointly minimizing the tracking error and the energy consumption.

model, the target state evolves according to the following equation:

$$\chi_{k+1} = F\chi_k + v_k,\ v_k \sim \mathcal{N}(0, Q)$$

where F is the state-transition matrix, v_k is the process noise, which is drawn from a zero-mean normal distribution with the co-variance matrix Q. Let n represent the number of vehicles in the swarm. We assume that the each vehicle in the swarm has an on-board sensor that generates noisy measurements of the target's location. The vehicles in the network are connected by a time-varying graph, represented by $\mathcal{G}_k$, where

$$\mathcal{G}_k = \begin{bmatrix} 0 & a_{12} & \dots & a_{1n} \\ a_{21} & 0 & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & 0 \end{bmatrix}$$

$a_{ij,i\neq j} = 1$ represents the ability of the sensors i and j to exchange measurements for data fusion at time k, and $a_{ij,i\neq j} = 0$ otherwise. Let C_k represent the centroid of the swarm at time k. We assume that the presence of a link between two sensors at time k lets the sensors exchange local measurements (generated at time k) for data fusion purpose. The sensors on-board the vehicles generate noisy measurements of the target positions in each time step. We use the standard Kalman filter to track the target state. Since the swarm is a decentralized system, each vehicle runs a local target tracking algorithm (Kalman filter), which is updated using the measurements generated locally and received from the neighboring nodes, where the measurement at ith sensor is given by:

$$z_k^i = H_{\text{pos}}\chi_k + w_k,\ w_k \sim \mathcal{N}(0, R_k(s_k^i, \chi_k)), \tag{1}$$

where H_{pos} is a matrix that captures just the position information in the target state vector χ_k, w_k is the measurement noise, and s_k^i is the position of the ith vehicle. We assume that the angular uncertainty is better than the range uncertainty; which is captured in the definition of the covariance matrix R_k, also

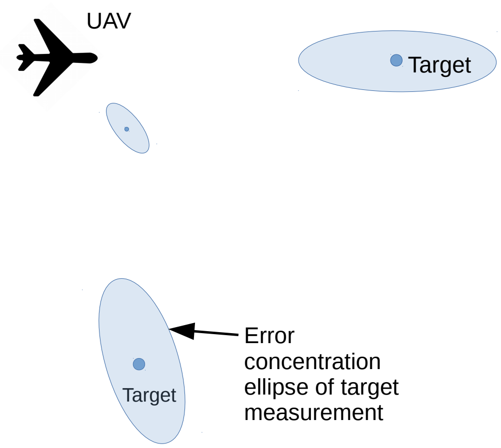

Fig. 2. Measurement error model.

captured in Figure 2. The state of the tracking algorithm is given by (ξ_k^i, P_k^i), where ξ_k^i and P_k^i represent the mean vector and the error covariance matrix corresponding to target state estimation at the ith sensor.

Let $f_{track}(\mathcal{G}_k, C_k)$ and $f_{energy}(\mathcal{G}_k, C_k)$ be functions representing target tracking error and the energy consumed respectively from sensor i's perspective, as defined below:

$$\begin{aligned} f_{track}(\mathcal{G}_k, C_k) &= \left\|\chi_k - \xi_k^i\right\|_2^2 \\ f_{energy}(\mathcal{G}_k, C_k) &= \sum_i \sum_j \mathcal{G}_k(i,j)\ \text{linkcost}(i,j) \end{aligned} \tag{2}$$

where $\text{linkcost}(i,j)$ represents the cost of using the link between sensors i and j for data fusion purpose. For simplicity, we assume the link cost is a constant and does not depend on i and j. As this is a decentralized system, each sensor in the system evaluates these functions using their own local target state estimates.

The goal of this study is to optimize the variables $\mathcal{G}_k$ and C_k such that the objectives f_{track} and f_{energy} are jointly minimized over a long time horizon H. In other words, the goal boils down to solving a COLRO problem as described below:

$$\begin{aligned} \min_{\mathcal{G}_k, C_k, k=0,..,H-1} \sum_{k=0}^{H-1} \mathrm{E}[p f_{track}(\mathcal{G}_k, C_k) + \\ (1-p) f_{energy}(\mathcal{G}_k, C_k)] \end{aligned} \tag{3}$$

where $\mathrm{E}[\cdot]$ is the expectation, and p is a weighting parameter. The above optimization problem resembles a *long-horizon optimal control* problem. These problems are notorious for high computational complexities, especially due to the presence of $\mathrm{E}[\cdot]$, which is hard to evaluate explicitly. To overcome these computational issues, a class of approximation techniques called *approximate dynamic programming* (ADP) approaches are used. With this motivation, we adopt an ADP approach called *nominal belief-state optimization* (NBO) [6], which allows us to approximate the expectation making its evaluation tractable. According to the NBO approach, the expectation is approximated by assuming the "future" noise variables take *nominal* or mean values from the probability distributions they are drawn from. Since we model the noise variables as zero-mean Gaussian, the nominal values are zeros. After the approximation, the COLRO problem reduces to

$$\min_{\mathcal{G}_k, C_k, k=0,..,H-1} \sum_{k=0}^{H-1} [p\tilde{f}_{track}(\mathcal{G}_k, C_k)+ (1-p)\tilde{f}_{energy}(\mathcal{G}_k, C_k)] \tag{4}$$

where $\tilde{f}_{track}$ and $\tilde{f}_{energy}$ are deterministic approximations to f_{track} and f_{energy} obtained from the NBO method. The reduced COLRO problem in Eq. 4 is highly nonlinear and non-convex, and also a mixed integer program since $\mathcal{G}_k$ contains discrete variables. We use a numerical optimization solver called *Knitro*, which allows solving mixed integer programs such as the above reduced COLRO problem.

With the NBO approach, $\tilde{f}_{track}(\mathcal{G}_k, C_k)$ is given by the trace of the error covariance matrix corresponding to the target state, which is obtained by running the Kalman filter by assuming: 1) the future process and measurement noise variables as zero; 2) the data fusion rules are applied according to the network graph state $\mathcal{G}_k$.

A. Evaluation of Optimal UAV Kinematic Controls

The decision variables $\mathcal{G}_k$ and $\mathcal{C}_k$ depend on the positions of the UAVs over time. Of course, once the optimal values for $\mathcal{G}_k$ and $\mathcal{C}_k$ are evaluated in Eq. 4, we still need to achieve the desired graph state and the desired swarm centroid by appropriately controlling the motion of the UAVs. Since the UAV kinematic control decisions depend on the optimal values of $\mathcal{G}_k$ and $\mathcal{C}_k$, we introduce a hierarchical model with two levels, where $\mathcal{G}_k$ and $\mathcal{C}_k$ are optimized in the upper level (by solving Eq. 4) and the UAV kinematic controls are optimized in the lower level according to the following artificial potential field approach.

Let $\mathcal{G}_k^*$ and $\mathcal{C}_k^*$ be the optimized network graph and the centroid location. At time k, on each UAV we apply an attractive potential field with the center at $\mathcal{C}_k^*$, another attractive potential field between UAVs i and j ($j \neq i$) if $\mathcal{G}_k^*(i,j) = 1$ and the repulsive field otherwise. These two potential fields allow the UAVs to approach the desired centroid while forming/breaking network links to achieve $\mathcal{G}_k^*$. In addition, we also apply short-range repulsive potential fields between each pair of UAVs to avoid collisions.

III. Results and Discussion

We implement the above-discussed methods in MATLAB for a scenario with three UAVs tracking a single target. We set the time horizon $H = 6$ and apply the *receding horizon control* [6] approach for planning and implementing the optimized decisions. For bench-marking, we also implement the centralized UAV motion planning approach discussed in [6]; we call this *centralized fusion approach*.

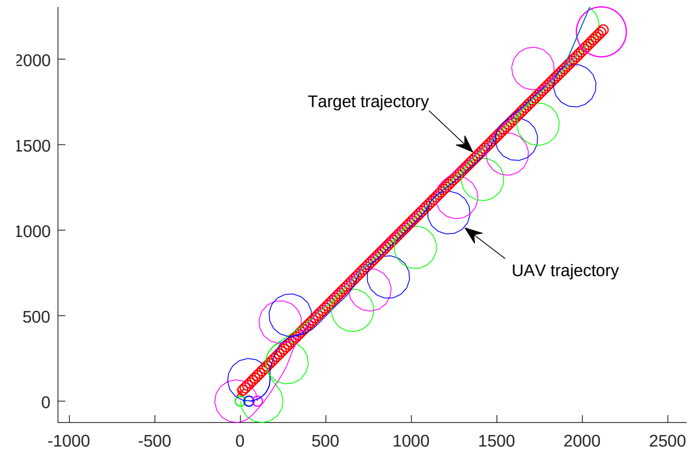

Fig. 3. Three UAVs tracking a target.

Figure 3 shows the trajectories of three UAVs tracking a target. The target and the UAVs begin their motion in the bottom-left region, and move toward the top-right region. Figures 4 and 5 show the network link status (three links for three UAVs) as a function of time for the weighting parameter in Eq. 4 set to $p = 0.2$ and $p = 0.01$ respectively. Clearly, in Figure 5, the UAVs exchange information less often compared to the scenario in Figure 4. These figures clearly demonstrate our ability to smoothly trade off between the two competing performance indices. We evaluate the normed error between the actual target location (ground truth) and the target location estimate at each sensor over time for the scenario in Figure 3. In Figure 6, we compare the performance of the above-discussed approach against the *centralized fusion approach*, which clearly shows that the tracking performance of the centralized approach is just marginally better than our COLRO-based methods discussed here. Of course, in the centralized approach, the performance with respect to the network energy costs is ignored. In other words, our approach, while slightly trading off the tracking performance, gains significantly in the performance with respect to the network energy consumption.

IV. Conclusions

In this paper, we presented a real-time heuristic approach to solve a *competing objective limited resource optimization* (COLRO) problem in the context of a networked UAV/sensors system. The objective is to optimize the motion of a swarm of UAVs (equipped with sensors) to track a moving target, while jointly minimizing the tracking error and the network energy cost. This optimization problem lead to *long horizon optimal control* problem, which is known to be computationally hard. So, we extended our previously developed approximate dynamic programming approach called *nominal belief state optimization* to solve the above COLRO problem. We tested

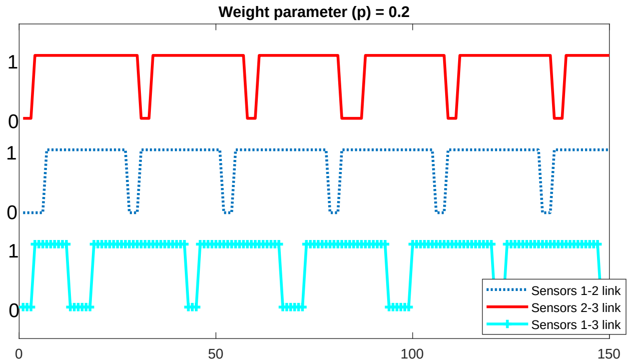

Fig. 4. Status of UAV network links over time with $p = 0.2$ (1 means active and 0 otherwise)

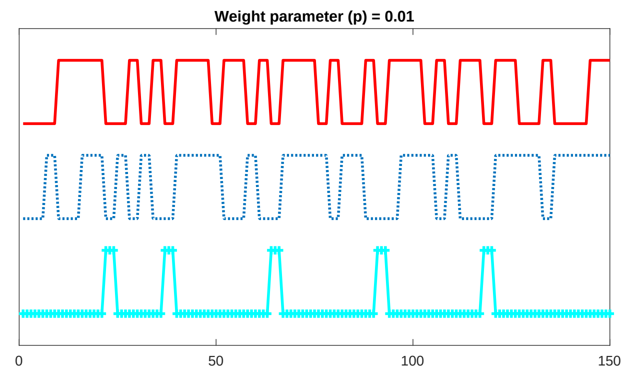

Fig. 5. Status of UAV network links over time with $p = 0.01$ (1 means active and 0 otherwise)

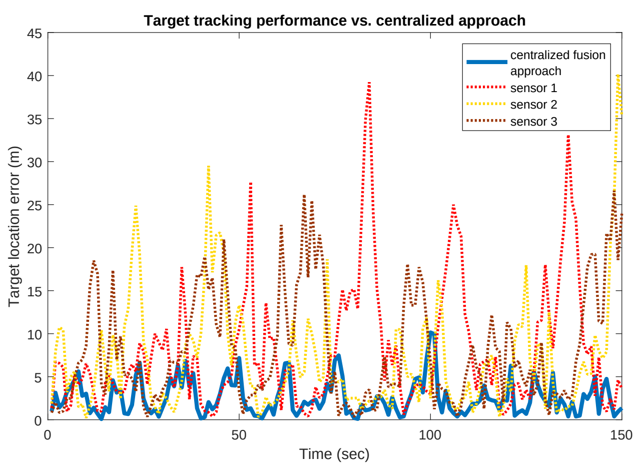

Fig. 6. Normed target location error: COLRO-based approach vs. centralized approach.

the performance of the approach in a simulated environment (implemented in MATLAB), and compared the performance of our approach with a *centralized fusion approach* (benchmark). We found our method to lose on the tracking performance only minimally compared to the centralized fusion approach, while significantly saving the network energy costs.

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