Quasi-Stochastic Electricity Markets

Jacob Mays[*]

School of Civil and Environmental Engineering, Cornell University, Ithaca, NY 14850

August 5, 2020

Abstract

With wind and solar becoming major contributors to electricity production in many systems, wholesale market operators have become increasingly aware of the need to address uncertainty when forming prices. While implementing theoretically ideal stochastic market clearing to address uncertainty may be impossible, the use of operating reserve demand curves allows market designers to inject an element of stochasticity into deterministic market clearing formulations. The construction of these curves, which alter the procurement of reserves and therefore the pricing of both reserves and energy, relies on contentious administrative parameters that lack strong theoretical justification. This paper proposes instead to link their construction to outcomes that would be expected in efficient stochastic markets. The analysis considers the potential of these "quasi-stochastic" market clearing approaches to improve efficiency relative to the deterministic status quo, as well as ways in which they are unable to fully replicate the stochastic ideal. Further, the paper argues that efficiently managing uncertainty entails a reexamination of the discriminatory uplift payments and enhanced pricing schemes currently employed to address non-convexity.

Keywords: Electricity market design, operating reserves, price formation, stochastic competitive equilibrium, uplift

1 Introduction

In most commodity markets, we conceive of production decisions as being driven by prices. In real-time electricity markets, the need to tightly manage frequency, voltage, and line flows precludes such a decentralized control mechanism. Prices are instead formed by the system operator shortly after the optimal production schedule has been determined. In this context, the engineering challenge is to commit units and dispatch the system in a way that maximizes gains from trade while ensuring reliability. The pricing challenge is to distribute that surplus in a manner consistent with the commitment and dispatch decisions that have been made. Ideally, the prices produced should meet the goals of efficiency, transparency, and simplicity. Unfortunately, electricity markets struggle to achieve all three [14].

[*]jacobmays@cornell.edu.

This paper connects three aspects of the price formation problem, each of which has caused significant disagreements among market designers, monitors, regulators, and participants. The first aspect is that market and reliability processes use a deterministic representation of an underlying stochastic reality. It is well understood that taking uncertainty into account can improve decisions, whether they are made algorithmically or by human operators exercising engineering judgment. It is equally well understood that if the reason for a decision is not explicitly represented in the market clearing engine, it will not be reflected in prices. Accordingly, while debates about stochastic market clearing have largely been confined to academic outlets [3, 6, 24, 25, 32, 42, 47, 48, 49], the effects of a failure to explicitly represent uncertainty in market models are already felt. In the context of this literature, the first goal of this paper is to specify two mechanisms by which inefficient and inconsistent prices may arise in real-world systems. The second aspect is the treatment of start-up and no-load costs and minimum operating levels, which lead to non-convex generator production functions. System operators have proposed a variety of schemes to incorporate these avoidable fixed costs in prices [26]. These schemes, however, can lead to a number of non-intuitive market outcomes [43]. To date, there are no universally agreed-upon metrics with which we can compare the performance of the various schemes on offer. The second goal of this paper is to illustrate how deterministic analyses of non-convex price formation may miss important aspects of the problem. The third aspect is the valuation of reserves through an operating reserve demand curve (ORDC). While system operators commonly penalize a shortage of reserves, only recently have they begun to consider the incremental value of holding reserves above a fixed requirement [19, 37, 39]. Some reject the premise altogether: in response to a recent proposal from PJM, for instance, the Independent Market Monitor claims that "ORDCs are an attempt to directly manage energy prices at levels that exceed LMP on a consistent basis," and instead recommends that PJM "focus on fundamental improvements to energy market efficiency" [30].

An underlying source of disagreement about ORDCs is their reliance on administrative parameters having unclear connection to the traditional goal of market efficiency. The third goal of this paper is to put ORDCs on firmer theoretical ground by connecting them to outcomes we would expect in efficient stochastic markets operating on the principle of maximizing expected surplus. In doing so, we illustrate why existing proposals may fail to solve the fundamental issues with price formation and sketch a new approach that may better address some of these issues. Deterministic models result in an energy price under the expected operating conditions, while stochastic models calculate energy prices under a range of potential operating conditions. ORDCs in principle allow construction of a deterministic model that yields the expected price (or revenue) from the stochastic model. Absent high-quality real-time information on uncertainty distributions, the expected price in the stochastic model is difficult to calculate. However, operator decisions to commit units in non-market reliability processes imply an estimated price in future intervals. We propose to tune ORDCs to match these implied prices, effectively tying ORDCs to operator decisions rather than relying on administrative parameters. Implementing an ORDC that underestimates this expected price will tend to result in units failing to cover their avoidable fixed costs. An ORDC that overestimates this value, on the other hand, may lead to increased self-commitment of inefficient units and price-responsive demand or to lost opportunity costs for non-committed units.

One implication of the analysis is to challenge the justification typically used for uplift payments and enhanced pricing to address non-convexity. On longer timescales, it is accepted that efficient markets include the possibility of losses. Investors in competitive markets therefore take responsibility for managing the risk associated with building long-term assets given uncertain future returns. The same logic can be applied to the short-term markets: a slow-moving generator might incur a fixed start-up cost due to an expectation of high prices later in the day, but then fail to recover that cost if demand fails to materialize. Arguments for make-whole payments and enhanced pricing typically rely on ex post analyses of generator profitability, seeking to ensure that market participants have appropriate retrospective incentives to follow the socially optimal commitment and dispatch schedule. This retrospective problem, however, does not adequately characterize the actual decisions that must be made under uncertainty. This paper argues for a shift in focus to the ex ante choices facing market participants. Pricing strategies that fail to distinguish between losses due to non-convexity and losses due to stochasticity run the risk of inefficiently socializing losses and privatizing gains. Specifically, the paper illustrates how convex hull pricing, sometimes described as an "ideal" because of its uplift-minimizing property in deterministic settings, can perform poorly in cases with uncertainty.

Section 2 provides background on recent pricing debates in centrally committed US markets. We present the ideal case of stochastic competitive equilibrium in Section 3, and introduce an example in Section 4 to help guide the remainder of the discussion. Section 5 identifies pricing issues caused by using deterministic instead of stochastic models for market clearing, and Section 6 addresses the potential of current ORDC formulations to resolve these issues. We outline our proposed approach in Section 7 and conclude in Section 8.

2 Price Formation

A curious feature of US wholesale electricity markets is that system operators, on behalf of the demand side of the market, regularly commit units that are unable to recover costs through the sale of energy and ancillary services. Market operators use discriminatory side payments to encourage generators to follow commitment and dispatch instructions despite operating at a loss. Uplift payments are widely agreed to be a necessary component of electricity markets, going by the names of bid cost recovery, revenue sufficiency guarantees, and make-whole payments in different regions [9]. While uplift payments are small relative to total energy expenditures, the larger concern is the degree to which they could indicate that prices are inefficiently low: insufficient energy market revenue leads to poor incentives for investment, contributing to the need for supplementary payments for capacity [21].

Explanations for side payments start from the observation that wholesale electricity markets lack uniform clearing prices due to the non-convex production costs characteristic of most generation technologies [17, 35]. The total revenue earned by a generator may be insufficient to cover the avoidable fixed costs associated with starting up and operating at its minimum output. Despite this condition, the decision to commit the unit can still be efficient in the sense of maximizing expected surplus. To encourage their participation in the market despite this risk of shortfall, system operators have authorized a variety of uplift payments that guarantee cost recovery [11]. Dissatisfaction with the discriminatory and non-transparent nature of these side payments, as well as concern about potential long-term consequences of price suppression, has led many to propose enhanced price formation scheme to reduce or eliminate these side payments [26]. Strategies that have been or soon will be implemented include variants of Fast-Start Pricing in ISO-NE [20], PJM [13], and NYISO [12], as well as Extended Locational Marginal Pricing in MISO [46].

Non-convexity itself, however, is only a partial explanation for uplift. It has been observed in several test systems [8, 29, 44] that the LMP can be quite sensitive to the particular commitment solution chosen by the optimization solver. Any bias toward holding additional units online can therefore result in lower prices on average. While difficult to model in a rigorous way, upward load adjustments in both unit commitment and economic dispatch models are made on a regular basis (see, e.g., Attachment E within [39] and Section 9 of [7]). Simulations on the PJM system for 2018 result in uplift of $30M under an ex post optimal unit commitment and economic dispatch solution using actual demand, versus the $200M actually incurred in 2018 [31]. A direct comparison would be unfair, since perfect knowledge of future demand is impossible. Nevertheless, the difference demonstrates the degree to which uplift is driven not purely by non-convexity but also by the exigencies of decision making under uncertainty.

Independent System Operators (ISOs) and Regional Transmission Organizations (RTOs) in the United States fulfill a dual role as reliability coordinators and market operators for the transmission grid. Along the lines of [1], this paper suggests that the uplift gap between the $30M attributable directly to non-convexity and the $200M actually observed represents a failure to fully reflect the needs of the reliability function in pricing models. Price formation is driven by a three-step process involving commitment, dispatch, and pricing [43]. In the language of [41], the models used for each of these processes are parametric cost function approximations, i.e., deterministic models that employ parametric adjustments to account for uncertainty. While these "quasi-stochastic" models can achieve high quality commitment and dispatch solutions, this paper discusses how their construction may affect prices arising in the market.

In the commitment stage, operators solve a series of mixed-integer programs (MIPs) to determine the optimal start-up and shut-down schedule of units on the system. Importantly, the day-ahead market is just one of these MIPs. It is accompanied by a series of reliability processes in which the bid-in load is replaced by an operator demand forecast. The participation of virtual bidders imparts some features of stochastic models to the day-ahead market, allowing prices to converge between day-ahead and real-time [22, 23, 28]. Accordingly, the use of a deterministic model in the day-ahead market does not necessarily pose problems for price formation. Virtual bidders do not participate in subsequent reliability processes, making it more likely for operators to commit units that will not be profitable in real time. In PJM, for example, balancing generator uplift credits are roughly three times larger than day-ahead credits, even though the day-ahead market on average clears enough energy to meet real time needs [31]. By allowing market participants to update their financial positions based on new information, introduction of intraday markets in conjunction with non-market reliability processes could help resolve some of the incentive issues discussed in this paper and improve overall efficiency [18].

Apart from handling non-convexity, a separate trend in price formation addresses the value of maintaining reserves beyond the required minimum on the system. In contrast to typical forward markets, which in the electricity setting would estimate the expected incremental cost over a range of possible operating conditions, current "real-time" auctions occur several minutes in advance of actual operations and calculate the incremental cost under the expected operating conditions, leaving the potential for those conditions to materially change between pricing and operation. The ORDC construct attempts to rectify this issue by inserting the expected cost of deploying reserves in the objective function of the pricing problem described above [19, 37, 39]. While not strictly necessary, ORDCs could also be included in commitment and dispatch models.

In theory, two key inputs govern the shape of current ORDCs. The first is the cost of recourse actions that will be taken if the system falls below a mandated level of contingency reserves. In ERCOT, which has no capacity market, this is the value of lost load, currently assessed at \$9,000/MWh since the assumed recourse action is involuntary load shedding. PJM, in which the capacity market is intended to virtually guarantee the availability of supply, proposes a lower value of \$2,000/MWh. The second is the probability that the assumed recourse action will take place given a certain level of reserves. Determining these probabilities is a challenging problem. The basic strategy in ERCOT and PJM is to use the distribution of historical forecast errors over a chosen lookahead period. The length of this lookahead period is a largely unjustified parameter in both markets. The most natural choice is a lookahead period equal to the frequency at which electricity is priced (i.e., five minutes in US markets). The argument in [39] appeals to the future value of unused reserves at the end of a pricing interval to defend the choice of a much longer lookahead period. In theory this is not required, since any this value should be incorporated in the new price of electricity at the time of repricing. However, including uncertainty over a longer horizon might be warranted if there are intertemporal constraints omitted from current single-period dispatch and pricing models. Incorporating the residual value could therefore be interpreted as a proxy for a better representation of, e.g., the opportunity cost of storage or demand-side resources. A more explicit representation of these constraints, such as that embedded in the ramping product implemented in MISO [45], would offer a stronger justification for cross-temporal pricing effects.

In practice, the difficulty of linking reserve levels in one period to the probability of recourse actions in future periods makes it a weak foundation for pricing logic. It is therefore necessary to seek other ways to validate choices made in the construction of administrative ORDCs. In ERCOT, the choice of metric is clear: since the primary goal of the ORDC is to guarantee resource adequacy, it is natural to assess it according to the reserve margin that it can support in equilibrium [34]. The resulting curve has only loose connection to its theoretical mooring in the loss of load probability (see, e.g., the administrative parameters described in [10]). In other markets resource adequacy is guaranteed by other means, necessitating a different metric. In the following sections, we propose such a metric, bid cost recovery in expectation, by connecting ORDCs with the ideal of stochastic competitive equilibrium.

3 Stochastic Ideal

This section shows how the property of bid cost recovery in expectation is a direct consequence of a commitment to the traditional goal of market efficiency, i.e., maximizing expected surplus. An underlying assumption of the paper is that implementation of a fully stochastic market design will be impossible, at least in the foreseeable future. That said, the ideal of stochastic competitive equilibrium provides a target that we might hope to approximate with ORDCs. Here we develop a simple model of ISO/RTO operations to demonstrate this ideal. Variants of these results appear in several related works [6, 24, 47, 48, 49]; for completeness, as well as to introduce subsequent results, we present them here as well.

While the results here serve as a theoretical foundation, the remainder of the article is structured such that readers who are more interested in policy implications will be able to follow the core arguments of the paper without reference to this section.

3.1 Unit Commitment

Significant progress has been made on large-scale unit commitment models that incorporate uncertainty using stochastic [5, 15] or robust [2, 27] optimization, with performance substantially enhanced by tight formulations of the underlying technical constraints [33, 36]. If uncertainty can be appropriately characterized, these models offer significant performance improvements over current deterministic implementations. As a benchmark for the best possible decision process, we formulate a stochastic program with perfect knowledge of the distribution of the random variables. Throughout the paper, the commitment process under consideration is assumed to be a non-market reliability process in which no virtual bidders participate and from which no financial positions result.

Here we define the notation and model formulation for a two-stage stochastic unit commitment problem in which commitment variables are shared across all scenarios while the dispatch is scenario-dependent. This two-stage model simplifies real-world operations, in which such models are solved on a rolling horizon basis and only decisions from the first period are binding.

3.1.1 Notation

Sets:

- $g \in \mathcal{G}$: generators
- $s \in \mathcal{S}$: dispatch scenarios
- $l \in \mathcal{L}$: demand bids
- $t \in \mathcal{T}$: time periods

Decision Variables:

- u_{gt}: (binary) commitment status of generator g in time period t
- v_{gt}: (binary) start-up decision for generator g time period t
- q_{gt}: (binary) shut-down decision for generator g in time period t
- p_{sgt}: power output of generator g in scenario s and time period t (MW)
- r_{sgt}: reserve provided by generator g in scenario s and time period t (MW)
- z_{st}: total quantity of reserves provided by all generators in scenario s and time period t (MW)
- d_{slt}: amount of demand l cleared in scenario s and time period t (MW)

Parameters:

- C_g^{NL}: no-load cost for generator g (\$/period)
- C_g^{SU}: start-up cost for generator g (\$)
- C_g^{EN}: marginal cost for generator g (\$/MWh)
- UT_g: minimum up time for generator g

- V_l^D: value of demand l (\$/MWh)
- V^R: penalty cost for reserve shortfall (\$/MWh)
- R^+: reserve requirement (MW)

Random Variables:

- P_{sgt}^+, P_{sgt}^-: maximum and minimum availability for generator g in scenario s and time period t (MW)
- D_{slt}^+: quantity of demand bid l in scenario s and time period t (MW)

Information:

- $\mathcal{F}_t$: information available immediately before the beginning of period t
- $\phi(s|\mathcal{F}_t)$: conditional probability of scenario s given information in $\mathcal{F}_t$
- ϕ_s: unconditional probability of scenario s

3.1.2 Formulation

The stochastic unit commitment problem can be stated as

$$
\begin{aligned}
&(SUC): \\
&\underset{u,v,q,p,r,d,z}{\text{maximize}} \quad && \sum_{s\in\mathcal{S}}\sum_{l\in\mathcal{L}}\sum_{t\in\mathcal{T}}\phi_s V_l^D d_{slt} \\
& && +\sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}\phi_s V^R z_{st} \\
& && -\sum_{s\in\mathcal{S}}\sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\phi_s C_g^{EN} p_{sgt} \\
& && -\sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\left(C_g^{NL}u_{gt} + C_g^{SU}v_{gt}\right) && && \text{(1a)} \\
&\text{subject to} && \phi_s\left(\sum_{l\in\mathcal{L}} d_{slt} - \sum_{g\in\mathcal{G}} p_{sgt}\right) = 0 && \forall s\in\mathcal{S}, t\in\mathcal{T} && (\lambda_{st}) && \text{(1b)} \\
& && \phi_s\left(z_{st} - \sum_{g\in\mathcal{G}} r_{gst}\right) = 0 && \forall s\in\mathcal{S}, t\in\mathcal{T} && (\mu_{st}) && \text{(1c)} \\
& && d_{slt} \le D_{slt}^+ && \forall s\in\mathcal{S}, l\in\mathcal{L}, t\in\mathcal{T} && (\alpha_{slt}^D) && \text{(1d)} \\
& && z_{st} \le R_t^+ && \forall s\in\mathcal{S}, t\in\mathcal{T} && (\alpha_{st}^R) && \text{(1e)} \\
& && P_{sgt}^- u_{gt} \le p_{sgt} && \forall s\in\mathcal{S}, g\in\mathcal{G}, t\in\mathcal{T} && (\alpha_{sgt}^{P-}) && \text{(1f)} \\
& && p_{sgt} + r_{sgt} \le P_{sgt}^+ u_{gt} && \forall s\in\mathcal{S}, g\in\mathcal{G}, t\in\mathcal{T} && (\alpha_{sgt}^{P+}) && \text{(1g)}
\end{aligned}
$$

$$v_{gt} + u_{g,t-1} - u_{gt} - q_{gt} = 0 \qquad \forall g \in \mathcal{G}, t \in \mathcal{T} \qquad (\gamma_{gt}) \tag{1h}$$

$$\sum_{t'=\max(1,t-UT_g+1)}^{t} v_{gt'} - u_{gt} \leq 0 \qquad \forall g \in \mathcal{G}, t \in \mathcal{T} \qquad (\delta_{gt}) \tag{1i}$$

$$d_{slt} \geq 0 \qquad \forall s \in \mathcal{S}, l \in \mathcal{L}, t \in \mathcal{T} \tag{1j}$$

$$z_{st} \geq 0 \qquad \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{1k}$$

$$p_{sgt}, r_{sgt} \geq 0 \qquad \forall s \in \mathcal{S}, g \in \mathcal{G}, t \in \mathcal{T} \tag{1l}$$

$$u_{gt}, v_{gt}, q_{gt} \in \{0, 1\} \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}. \tag{1m}$$

The commitment problem is to determine the commitment status of each generator $g \in \mathcal{G}$ in each period $t \in \mathcal{T}$, given a finite number of dispatch scenarios $s \in \mathcal{S}$ characterizing uncertain demand and generator availability. The system is willing to spend V^R for reserves up to a quantity of R^+; stated differently, V^R is the cost of recourse actions that will be taken to maintain security in the event of a reserve shortfall. Provision of reserves is assumed to be costless; incorporating offer prices would require small changes. Inclusion of minimum down times and ramping constraints would similarly require only small changes to the model. We assume a single reserve product and do not include network or security constraints. As indicated by the subscript s, uncertainty arises from the minimum and maximum availability of generators, represented by the random variables P^-_{sgt} and P^+_{sgt}, as well as demand D^+_{slt}. In the case of typical wind and solar generators, $P^-_{sgt} = 0$ while P^+_{sgt} can fluctuate between 0 and the rated capacity. For thermal generators, $P^-_{sgt} = P^+_{sgt} = 0$ represents a failure.

Constraints in this MIP are used in several subsequent linear programs. For convenience, we include Greek letters to the right of each constraint denoting dual variables that will be used in these linear programs. The objective function in (1a) calculates the value of serving load and supplying reserves minus the three-part cost of meeting that load. Power balance is enforced in constraint (1b), while constraint (1c) calculates the total reserves supplied. These equations are scaled by probability ϕ_s in order to produce unscaled prices λ_{st} and μ_{st} in each scenario. Combined with the objective function, constraint (1d) models load curtailment, while constraints (1c) and (1e) model a reserve shortfall. Constraints (1f) and (1g) represent the minimum and maximum output of generators, which depends on their commitment status u_{gt}. Equation (1h) and (1i) ensure consistency between start-up, shut-down, and commitment variables given generator minimum run times. In the first time period, the variable $u_{g,t-1}$ in Eq. (1h) is replaced by the current commitment status of each generator; similar substitutions can be made in Eq. (1i) if generators have not already met their minimum run time.

3.2 Relaxation and Duality

To establish formal results, we will instead work with a convex relaxation of model (SUC). We postpone discussion of the relevance of these results for the true non-convex problem to Section 4. The relaxation of (SUC) is stated as

$$
\begin{aligned}
(RSUC): \quad & \underset{u,v,q,p,r,d,z}{\text{maximize}} & & (1a) & & & (2a)\\
& \text{subject to} & & (1b)\text{–}(1l) & & & (2b)\\
& & u_{gt} &\leq 1 & \forall g \in \mathcal{G}, t \in \mathcal{T} & \quad (\beta_{gt}^{ON}) & (2c)\\
& & v_{gt} &\leq 1 & \forall g \in \mathcal{G}, t \in \mathcal{T} & \quad (\beta_{gt}^{SU}) & (2d)\\
& & q_{gt} &\leq 1 & \forall g \in \mathcal{G}, t \in \mathcal{T} & \quad (\beta_{gt}^{SD}) & (2e)\\
& & u_{gt}, v_{gt}, q_{gt} &\geq 0 & \forall g \in \mathcal{G}, t \in \mathcal{T}. & & (2f)
\end{aligned}
$$

The dual to model $(RSUC)$ is stated as follows:

$$
\begin{aligned}
&(DRSUC):\\
&\underset{\lambda,\mu,\alpha,\beta,\gamma,\delta}{\text{minimize}} \quad \sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}}\left(\beta_{gt}^{ON}+\beta_{gt}^{SU}+\beta_{gt}^{SD}\right)\\
&\qquad + \sum_{s\in\mathcal{S}}\sum_{l\in\mathcal{L}}\sum_{t\in\mathcal{T}}\alpha_{slt}^{D}D_{slt}^{+}\\
&\qquad - \sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}}\alpha_{st}^{R}R_{t}^{+} && (3a)\\
&\text{subject to} \quad -\phi_s\lambda_{st}-\alpha_{sgt}^{P-}+\alpha_{sgt}^{P+}\geq-\phi_s C_g^{EN} \quad \forall s\in\mathcal{S}, g\in\mathcal{G}, t\in\mathcal{T} \quad (p_{sgt}) && (3b)\\
&-\phi_s\mu_{st}+\alpha_{sgt}^{P+}\geq 0 \quad \forall s\in\mathcal{S}, g\in\mathcal{G}, t\in\mathcal{T} \quad (r_{sgt}) && (3c)\\
&\phi_s\mu_{st}+\alpha_{st}^{R}\geq\phi_s V^R \quad \forall s\in\mathcal{S}, t\in\mathcal{T} \quad (z_{st}) && (3d)\\
&\phi_s\lambda_{st}+\alpha_{slt}^{D}\geq\phi_s V_l^D \quad \forall s\in\mathcal{S}, l\in\mathcal{L}, t\in\mathcal{T} \quad (d_{slt}) && (3e)\\
&\sum_{s\in\mathcal{S}}(P_{sgt}^{-}\alpha_{sgt}^{P-}-P_{sgt}^{+}\alpha_{sgt}^{P+})\\
&+\gamma_{g,t+1}-\gamma_{gt}+\beta_{gt}^{ON}-\delta_{gt}\geq-C_g^{NL} \quad \forall g\in\mathcal{G}, t\in\mathcal{T} \quad (u_{gt}) && (3f)\\
&\gamma_{gt}+\sum_{t'=t}^{\min(t^+,t+UT_g-1)}\delta_{gt}+\beta_{gt}^{SU}\geq-C_g^{SU} \quad \forall g\in\mathcal{G}, t\in\mathcal{T} \quad (v_{gt}) && (3g)\\
&-\gamma_{gt}+\beta_{gt}^{SD}\geq 0 \quad \forall g\in\mathcal{G}, t\in\mathcal{T} \quad (q_{gt}) && (3h)\\
&\alpha_{slt}^{D}\geq 0 \quad \forall s\in\mathcal{S}, l\in\mathcal{L}, t\in\mathcal{T} && (3i)\\
&\alpha_{sgt}^{P-},\alpha_{sgt}^{P+}\geq 0 \quad \forall s\in\mathcal{S}, g\in\mathcal{G}, t\in\mathcal{T} && (3j)
\end{aligned}
$$

$$\alpha^R_{st} \geq 0 \qquad \forall s \in \mathcal{S}, t \in \mathcal{T} \tag{3k}$$

$$\beta^{ON}_{gt}, \beta^{SU}_{gt}, \beta^{SD}_{gt}, \gamma_{gt} \geq 0 \qquad \forall g \in \mathcal{G}, t \in \mathcal{T}. \tag{3l}$$

3.3 Market Participants

Model $(RSUC)$ represents the relaxation of the problem faced by the wholesale market operator. A decentralized perspective focuses on the problems faced by the market participants. Given prices λ_{st} and μ_{st} for energy and reserves, the relaxed problem faced by generator $g \in \mathcal{G}$ is stated as

$$(RGEN)_g:$$

$$\begin{aligned}
\underset{u_g,v_g,q_g,p_g,r_g}{\text{maximize}} \quad & \sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}} \phi_s p_{sgt}(\lambda_{st} - C^{EN}_g) \\
& + \sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}} \phi_s r_{sgt}\mu_{st} \\
& - \sum_{t\in\mathcal{T}} \left(C^{NL}_g u_{gt} + C^{SU}_g v_{gt}\right) && \text{(4a)} \\
\text{subject to} \quad & P^-_{sgt} u_{gt} \leq p_{sgt} \qquad \forall s \in \mathcal{S}, t \in \mathcal{T} \qquad (\alpha^{P-}_{sgt}) && \text{(4b)} \\
& p_{sgt} + r_{sgt} \leq P^+_{sgt} u_{gt} \qquad \forall s \in \mathcal{S}, t \in \mathcal{T} \qquad (\alpha^{P+}_{sgt}) && \text{(4c)} \\
& v_{gt} + u_{g,t-1} - u_{gt} - q_{gt} = 0 \qquad \forall t \in \mathcal{T} \qquad (\gamma_{gt}) && \text{(4d)} \\
& \sum_{t'=\max(1,t-UT_g+1)}^{t} v_{gt'} - u_{gt} \leq 0 \qquad \forall t \in \mathcal{T} \qquad (\delta_{gt}) && \text{(4e)} \\
& u_{gt} \leq 1 \qquad \forall t \in \mathcal{T} \qquad (\beta^{ON}_{gt}) && \text{(4f)} \\
& v_{gt} \leq 1 \qquad \forall t \in \mathcal{T} \qquad (\beta^{SU}_{gt}) && \text{(4g)} \\
& q_{gt} \leq 1 \qquad \forall t \in \mathcal{T} \qquad (\beta^{SD}_{gt}) && \text{(4h)} \\
& p_{sgt}, r_{sgt} \geq 0 \qquad \forall s \in \mathcal{S}, t \in \mathcal{T} && \text{(4i)} \\
& u_{gt}, v_{gt}, q_{gt} \geq 0 \qquad \forall t \in \mathcal{T}. && \text{(4j)}
\end{aligned}$$

Generators choose a production schedule that maximizes expected profits while following all of the generator-specific constraints described in model $(RSUC)$, but have no responsibility for system-level constraints. The simpler problem for loads is stated as follows:

$$(LOAD)_l:$$

$$\begin{aligned}
\underset{d_l}{\text{maximize}} \quad & \sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}} \phi_s d_{slt}(V^D_l - \lambda_{st}) && \text{(5a)} \\
\text{subject to} \quad & d_{slt} \leq D^+_{slt} \qquad \forall s \in \mathcal{S}, t \in \mathcal{T} \qquad (\alpha^D_{slt}) && \text{(5b)} \\
& d_{slt} \geq 0 \qquad \forall s \in \mathcal{S}, t \in \mathcal{T} && \text{(5c)}
\end{aligned}$$

3.4 Stochastic Competitive Equilibrium

Using these problem definitions, we define a stochastic competitive equilibrium for the relaxed problems characterizing the market.

Definition 1. A relaxed stochastic competitive equilibrium is a set of prices (λ_{st}, μ_{st}) $\forall s \in \mathcal{S}, t \in \mathcal{T}$, generator schedules (u_{gt}, v_{gt}, q_{gt}) $\forall g \in \mathcal{G}, t \in \mathcal{T}$ and production quantities (p_{sgt}, r_{sgt}) $\forall s \in \mathcal{S}, g \in \mathcal{G}, t \in \mathcal{T}$, load consumption quantities d_{slt} $\forall s \in \mathcal{S}, l \in \mathcal{L}, t \in \mathcal{T}$, and operator reserve procurements z_{st} $\forall s \in \mathcal{S}, t \in \mathcal{T}$ such that:

1. the generator schedules and production quantities solve $(RGEN)_g$ $\forall g \in \mathcal{G}$ and the load consumption quantities solve $(LOAD)_l$ $\forall l \in \mathcal{L}$ at the given prices, and
2. the market clears in every scenario, i.e.,

$$\sum_{l \in \mathcal{L}} d_{slt} = \sum_{g \in \mathcal{G}} p_{sgt} \quad \forall s \in \mathcal{S}, t \in \mathcal{T}$$

and

$$z_{st} = \sum_{g \in \mathcal{G}} r_{gst} \quad \forall s \in \mathcal{S}, t \in \mathcal{T}.$$

In other words, a relaxed stochastic competitive equilibrium occurs when all market participants maximize their individual expected surplus and system constraints are satisfied. We now show that this is the case for an optimal solution to the relaxed system operator's problem.

Theorem 1. Assume all market participants $g \in \mathcal{G}$ and $l \in \mathcal{L}$ agree on probabilities ϕ_s $\forall s \in \mathcal{S}$. Let $(u^*, v^*, q^*, p^*, r^*, d^*, z^*)$ be an optimal solution to $(RSUC)$ and let $(\lambda^*, \mu^*, \alpha^*, \beta^*, \gamma^*, \delta^*)$ be an optimal solution to $(DRSUC)$. Then the prices $(\lambda^*_{st}, \mu^*_{st})$ $\forall s \in \mathcal{S}, t \in \mathcal{T}$, generator schedules $(u^*_{gt}, v^*_{gt}, q^*_{gt})$ $\forall g \in \mathcal{G}, t \in \mathcal{T}$, generator production quantities (p^*_{sgt}, r^*_{sgt}) $\forall s \in \mathcal{S}, g \in \mathcal{G}, t \in \mathcal{T}$, load consumption quantities d^*_{slt} $\forall s \in \mathcal{S}, l \in \mathcal{L}, t \in \mathcal{T}$, and operator reserve procurements z^*_{st} $\forall s \in \mathcal{S}, t \in \mathcal{T}$ represent a stochastic competitive equilibrium.

Proof. The theorem follows from duality theory. For each generator $g \in \mathcal{G}$, the dual problem to $(RGEN)_g$ can be stated as follows:

$(DRGEN)_g$:

$$
\begin{aligned}
\underset{\alpha_g,\beta_g,\gamma_g,\delta_g}{\text{minimize}} \quad & \sum_{t\in\mathcal{T}} \left(\beta_{gt}^{ON} + \beta_{gt}^{SU} + \beta_{gt}^{SD}\right) && && \text{(6a)}\\
\text{subject to} \quad & -\alpha_{sgt}^{P-} + \alpha_{sgt}^{P+} \geq \phi_s(\lambda_{st} - C_g^{EN}) && \forall s\in\mathcal{S}, t\in\mathcal{T} \quad (p_{sgt}) && \text{(6b)}\\
& \alpha_{sgt}^{P+} \geq \phi_s\mu_{st} && \forall s\in\mathcal{S}, t\in\mathcal{T} \quad (r_{sgt}) && \text{(6c)}\\
& \sum_{s\in\mathcal{S}}(P_{sgt}^{-}\alpha_{sgt}^{P-} - P_{sgt}^{+}\alpha_{sgt}^{P+}) \\
& +\gamma_{g,t+1} - \gamma_{gt} + \beta_{gt}^{ON} - \delta_{gt} \geq -C_g^{NL} && \forall t\in\mathcal{T} \quad (u_{gt}) && \text{(6d)}\\
& \gamma_{gt} + \sum_{t'=t}^{\min(t^+, t+UT_g-1)} \delta_{gt} + \beta_{gt}^{SU} \geq -C_g^{SU} && \forall t\in\mathcal{T} \quad (v_{gt}) && \text{(6e)}\\
& -\gamma_{gt} + \beta_{gt}^{SD} \geq 0 && \forall t\in\mathcal{T} \quad (q_{gt}) && \text{(6f)}\\
& \alpha_{sgt}^{P-}, \alpha_{sgt}^{P+} \geq 0 && \forall s\in\mathcal{S}, t\in\mathcal{T} && \text{(6g)}\\
& \beta_{gt}^{ON}, \beta_{gt}^{SU}, \beta_{gt}^{SD}, \gamma_{gt} \geq 0 && \forall t\in\mathcal{T}. && \text{(6h)}
\end{aligned}
$$

Similarly, for each load $l\in\mathcal{L}$, the dual problem to $(LOAD)_l$ can be stated as follows:

$$
\begin{aligned}
& (DLOAD)_l : \\
& \underset{\alpha_l}{\text{minimize}} \quad \sum_{s\in\mathcal{S}}\sum_{t\in\mathcal{T}} \alpha_{slt}^{D} D_{slt}^{+} && && \text{(7a)}\\
& \text{subject to} \quad \alpha_{slt}^{D} \geq \phi_s(V_l^D - \lambda_{st}) && \forall s\in\mathcal{S}, t\in\mathcal{T} \quad (d_{slt}) && \text{(7b)}\\
& \qquad \alpha_{slt}^{D} \geq 0 && \forall s\in\mathcal{S}, t\in\mathcal{T}. && \text{(7c)}
\end{aligned}
$$

Since $(u^*, v^*, q^*, p^*, r^*, d^*, z^*)$ is an optimal solution to $(RSUC)$ and $(\lambda^*, \mu^*, \alpha^*, \beta^*, \gamma^*, \delta^*)$ is an optimal solution to $(DRSUC)$, the complementary slackness conditions hold. While we omit these conditions for brevity, the variables corresponding to each constraint in $(RSUC)$, $(DRSUC)$, $(RGEN)_g$, $(DRGEN)_g$, $(LOAD)_l$, and $(DLOAD)_l$ are shown. It can be readily seen that the complementary slackness conditions for $(RSUC)$ and $(DRSUC)$ are a superset of those for the individual market participant problems. Accordingly, the generator schedules (u_g^*, v_g^*, q_g^*) and production quantities (p_g^*, r_g^*) solve $(RGEN)_g$ $\forall g\in\mathcal{G}$, while the load consumption quantities d_l^* solve $(LOAD)_l$ $\forall l\in\mathcal{L}$ given prices (λ^*, μ^*). Furthermore, since $(u^*, v^*, q^*, p^*, r^*, d^*, z^*)$ is an optimal solution to $(RSUC)$, the market clears. Thus, the prices $(\lambda_{st}^*, \mu_{st}^*)$ $\forall s\in\mathcal{S}, t\in\mathcal{T}$, generator schedules $(u_{gt}^*, v_{gt}^*, q_{gt}^*)$ $\forall g\in\mathcal{G}, t\in\mathcal{T}$ and production quantities (p_{sgt}^*, r_{sgt}^*) $\forall s\in\mathcal{S}, g\in\mathcal{G}, t\in\mathcal{T}$, load consumption quantities d_{slt}^* $\forall s\in\mathcal{S}, l\in\mathcal{L}, t\in\mathcal{T}$, and operator reserve procurements z_{st}^* $\forall s\in\mathcal{S}, t\in\mathcal{T}$ represent a stochastic competitive equilibrium. □

A key assumption for stochastic competitive equilibrium is that market participants agree on the probability ϕ_s that each scenario $s \in \mathcal{S}$ will arise. We return to the consequences of this assumption in Section 7.

3.5 Bid Cost Recovery in Expectation

One implication of to Theorem 1 is the property of bid cost recovery in expectation for the relaxed problem, i.e., a guarantee that the expected profit of every generator is at least zero in its optimal solution. Scenario-specific profit for generator $g \in \mathcal{G}$ can be calculated as

$$\pi_{sg} = \sum_{t \in \mathcal{T}} \left(p_{sgt}(\lambda_{st} - C_g^{EN}) + r_{sgt}\mu_{st} - C_g^{NL} u_{gt} - C_g^{SU} v_{gt} \right) \tag{8}$$

Corollary 1. Let $(u^*, v^*, q^*, p^*, r^*, d^*, z^*)$ be an optimal solution to $(RSUC)$ and $(\lambda^*, \mu^*, \alpha^*, \beta^*, \gamma^*, \delta^*)$ be an optimal solution to $(DRSUC)$. Then, given probabilities ϕ_s $\forall s \in \mathcal{S}$, expected profit $\mathbb{E}[\pi_{sg}] \geq 0$ $\forall g \in \mathcal{G}$.

Proof. By Theorem 1, expected profit $\mathbb{E}[\pi_{sg}]$ can be calculated as the objective function value for $(RGEN)_g$. By strong duality, this value can also be calculated through the dual problem $(DRGEN)_g$ as

$$\mathbb{E}[\pi_{sg}] = \sum_{t \in \mathcal{T}} \left(\beta_{gt}^{ON} + \beta_{gt}^{SU} + \beta_{gt}^{SD} \right). \tag{9}$$

Since $\beta_{gt}^{ON}, \beta_{gt}^{SU}, \beta_{gt}^{SD} \geq 0$ $\forall g \in \mathcal{G}, t \in \mathcal{T}$, we can conclude that $\mathbb{E}[\pi_{sg}] \geq 0$. □

3.6 Profitability of Marginal Generators

A second implication of Theorem 1 is that expected profit is exactly zero for what we might consider marginal generators.

Corollary 2. Let $(u^*, v^*, q^*, p^*, r^*, d^*, z^*)$ be an optimal solution to $(RSUC)$ and $(\lambda^*, \mu^*, \alpha^*, \beta^*, \gamma^*, \delta^*)$ be an optimal solution to $(DRSUC)$. For some generator $g \in \mathcal{G}$, let $u_{gt}^*, v_{gt}^*, q_{gt}^* < 1$ $\forall t \in \mathcal{T}$. Then expected profit $\mathbb{E}[\pi_{sg}] = 0$.

Proof. In the optimal solution, $u_{gt}^*, v_{gt}^*, q_{gt}^* < 1$ $\forall t \in \mathcal{T}$. By complementary slackness, we know that $\beta_{gt}^{ON}(1 - u_{gt}) = 0$, $\beta_{gt}^{SU}(1 - v_{gt}) = 0$, and $\beta_{gt}^{SD}(1 - q_{gt}) = 0$ $\forall t \in \mathcal{T}$. Therefore, $\beta_{gt}^{ON} = 0$, $\beta_{gt}^{SU} = 0$, and $\beta_{gt}^{SD} = 0 \; \forall t \in \mathcal{T}$. As in Corollary 1, expected profit for generator g can then be calculated as

$$\mathbb{E}\left[\pi_{sg}\right] = \sum_{t \in \mathcal{T}} \left(\beta_{gt}^{ON} + \beta_{gt}^{SU} + \beta_{gt}^{SD}\right) = 0. \tag{10}$$

□

We note that the generators considered in Corollary 2 are not necessarily marginal in the sense of providing the incremental unit of energy at a given point of time. Instead, their entrance into the optimal commitment solution is marginal in the relaxed problem. In the binary context, these generators cannot be partially on or off. With this non-convexity, expected profits will not precisely equal zero. There is no theoretical guidance predicting whether profits will increase or decrease when moving to the optimal integer solution. Absent such guidance, the net impact of these shifts is ultimately an empirical question outside the scope of this paper. Section 4 develops an example that illustrates the importance of considering uncertainty when performing such an analysis.

3.7 Uplift and Stochasticity

We close Section 3 with a definition of lost opportunity costs appropriate for the stochastic setting. Due to non-convexity, there is in general no set of uniform prices that support the efficient commitment and dispatch solution [17, 35]. Many systems have implemented enhanced pricing schemes with the nominal goal of aligning the incentives of market participants with those of the system operator. Lost opportunity costs for a given market participant can be calculated as the difference between the surplus it would receive under its individually preferred schedule and what it receives by following the socially optimal solution. Crucially, most analyses of uplift implicitly assume that generators know all future prices at the time commitment decisions must be made. This paper argues that an accurate picture of generator incentives to follow the socially optimal commitment and dispatch schedule must include uncertainty.

3.7.1 Notation

Set:

- $j \in \mathcal{J}$: pricing schemes (e.g. Locational Marginal Pricing, Convex Hull Pricing, and Fast-Start Pricing)

Variables:

- λ_{st}^j: price of energy in scenario s and time period t under pricing scheme j
- μ_{st}^j: price of reserves in scenario s and time period t under pricing scheme j

3.7.2 Ex Ante Lost Opportunity Costs

Suppose we have an optimal solution $(u^*, v^*, q^*, p^*, r^*, d^*, z^*)$ to model (SUC), i.e., the full stochastic unit commitment problem including binary constraints. Total lost opportunity costs for all markets participants under pricing scheme j can be calculated as

$$
\begin{aligned}
&(LOC)_j : \\
&\underset{u,v,q,p,r,d}{\text{maximize}} \quad \sum_{s\in\mathcal{S}}\sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}} \phi_s \left[p_{sgt}(\lambda_{st}^j - C_g^{EN}) + r_{sgt}\mu_{st}^j - C_g^{NL}u_{gt} - C_g^{SU}v_{gt}\right] \\
&\qquad - \sum_{s\in\mathcal{S}}\sum_{g\in\mathcal{G}}\sum_{t\in\mathcal{T}} \phi_s \left[p_{sgt}^*(\lambda_{st}^j - C_g^{EN}) + r_{sgt}^*\mu_{st}^j - C_g^{NL}u_{gt}^* - C_g^{SU}v_{gt}^*\right] \\
&\qquad + \sum_{s\in\mathcal{S}}\sum_{l\in\mathcal{L}}\sum_{t\in\mathcal{T}} \phi_s d_{slt}(V_l^D - \lambda_{st}^j) \\
&\qquad - \sum_{s\in\mathcal{S}}\sum_{l\in\mathcal{L}}\sum_{t\in\mathcal{T}} \phi_s d_{slt}^*(V_l^D - \lambda_{st}^j) && \text{(11a)} \\
&\text{subject to} \quad (1d), (1f)\text{–}(1j), (1l), (1m). && \text{(11b)}
\end{aligned}
$$

Model $(LOC)_j$ omits the system-wide constraints from model (SUC), leading to a separable problem in which each market participant can maximize its surplus given its technical constraints. However, as a departure from the deterministic setting, the variables u_{gt}, v_{gt}, and q_{gt} are shared across all scenarios, reflecting the necessity that generators decide on a commitment schedule in advance.

4 Example System

In this section, we develop a simple example with a single load, single time period, and single source of uncertainty to illustrate the pricing issues caused by disconnects between commitment and dispatch in the presence of uncertainty. The lone source of uncertainty is a wind generator. The availability of wind is represented by the random variable W, which is uniformly distributed between 0 and 100, i.e., $W \sim \mathcal{U}(0, 100)$. Where appropriate, we also use W to indicate a realization of this random variable. For ease of exposition, we construct a simplified version of the stochastic unit commitment problem in model (1). Along these lines, we assume wind is costless and does not provide reserves, remove it from the set of generators $\mathcal{G}$, and define a new decision variable w indicating the quantity of wind dispatched. Wind can be curtailed at no cost. We retain variable and parameter definitions but drop the unneeded subscripts s, l, and t from model (1). Demand $D^+ = 200$. The reserve requirement R^+ is set at $20 - \varepsilon$, where ε is a small positive number; this quantity is perturbed to avoid multiple optimal dual solutions, but we use $R^+ = 20$ in calculations of probabilities and revenues. The unit penalty for a reserve shortfall $V^R = \$950$, while the value of demand $V^D = \$10{,}000$. In addition to wind, the system is served by 101 thermal generators. The first has capacity $P_g^+ = 120$ and can operate flexibly, i.e., $P_g^- = 0$. Its no-load cost $C_g^{NL} = \$0$, while its energy cost $C_g^{EN} = \$50$. The remaining 100 generators are all block loaded units of size 1; i.e., $P_g^- = P_g^+ = 1$. The units are conveniently arranged by no-load costs increasing from 51 to 150 in increments of 1, while energy costs C_g^{EN} for each are zero. There are no start-up costs, i.e., $C_g^{SU} = \$0 \ \forall g \in \mathcal{G}$, and minimum up times are omitted. We refer to the largest generator as generator 0, and the remaining 100 in order according their cost (e.g., the generator with $C_g^{NL} = \$70$ is referred to as generator 20).

The stochastic unit commitment problem for this example can be stated as

$$\underset{u:\ u_g \in \{0,1\}}{\text{maximize}} \qquad -\sum_{g\in\mathcal{G}} C_g^{NL} u_g + \mathbb{E}\left[H(u;W)\right], \tag{12}$$

where $u = (u_g)_{g\in\mathcal{G}}$ and $\mathbb{E}\left[H(u;W)\right]$ represents the expected surplus from dispatching the system given uncertain wind availability. The subproblem for a specific realization of wind W is stated as follows:

$$H(u;W) = \quad \underset{p,r,d,z,w}{\text{maximize}} \qquad V^D d + V^R z - \sum_{g\in\mathcal{G}} C_g^{EN} p_g \tag{13a}$$

$$\text{subject to} \qquad d - w - \sum_{g\in\mathcal{G}} p_g = 0 \tag{13b}$$

$$z - \sum_{g\in\mathcal{G}} r_g = 0 \tag{13c}$$

$$d \le D^+ \tag{13d}$$

$$z \le R^+ \tag{13e}$$

$$w \le W \tag{13f}$$

$$P_g^- u_g - p_g \le 0 \qquad \forall g \in \mathcal{G} \tag{13g}$$

$$p_g + r_g - P_g^+ u_g \leq 0 \qquad \forall g \in \mathcal{G} \tag{13h}$$

$$d, o, w \geq 0 \tag{13i}$$

$$p_g, r_g \geq 0 \qquad \forall g \in \mathcal{G} \tag{13j}$$

4.1 Optimal Solution

The system is constructed such that it can be solved exactly with a simple rule. Since $C_0^{NL} = 0$, generator 0 will be committed. Moreover, only generator 0 will supply reserves, since all remaining thermal units will be producing energy equal to their maximum output and it will always be optimal to dispatch wind at its full availability. We commit the remaining units in order until the no-load cost associated with model (12) exceeds the expected improvement of the dispatch in model (13). Let u^n indicate the solution to model (12) in which the cheapest n block-loaded units are committed. Then, we can commit units until

$$C_{n+1}^{NL} \geq \mathbb{E}\left[H(u^{n+1}; W)\right] - \mathbb{E}\left[H(u^n; W)\right]. \tag{14}$$

The optimal solution is to commit 90 block loaded units. This leads to a ten percent chance of a reserve shortfall, which will occur if wind availability turns out to be less than 10, but no chance that load will need to be curtailed. Though non-convexity and the increasing cost of the block-loaded units complicates the analysis, the example is similar to the classic newsvendor problem. If committing generator 90 at a cost of \$140 allows us to avoid a unit of reserve shortfall, the system has a benefit of \$1000: \$50 from reducing the energy output of generator 0 plus \$950 from supplying an additional unit of reserve. If there is no shortfall, however, the system only saves \$50 in production costs. Because the overage cost of \$90 is much smaller than the underage cost of \$860, the optimal solution is to commit far more units than would be required if we saw average wind availability of 50.

4.2 Dispatch and Pricing

As the actual operating interval approaches, the system operator solves the economic dispatch problem with the commitment decisions fixed. In principle, the uncertainty in the system may evolve between the time of commitment and the time of dispatch. Since the generators in our example system have no constraints on their flexibility apart from their commitment status, the distinction does not matter for dispatch. However, the timing of uncertainty realization has important implications for pricing.

We first consider the ideal case, in which uncertainty is completely resolved before the time of dispatch. In practice, this would require clearing markets in real time at a frequency far higher than the current five minutes. In this case, the optimal dispatch is found by solving a version of model (13) with the realized value of W and the chosen commitment solution. Emphasizing that instead of a subproblem this is a separate, standalone problem with commitment variables fixed to the solution $u = \hat{u}$, we formulate the economic dispatch problem as

$$
\begin{aligned}
(ED - VERT): \quad & \underset{p,r,d,z,w}{\text{maximize}} && (13a) && (15a)\\
& \text{subject to} && d - w - \sum_{g \in \mathcal{G}} p_g = 0 && (15b)\\
& && z - \sum_{g \in \mathcal{G}} r_g = 0 && (15c)\\
& && (13d)\text{–}(13j) && (15d)\\
& && u_g = \hat{u}_g \qquad \forall g \in \mathcal{G}. && (15e)
\end{aligned}
$$

The name $(ED - VERT)$ is chosen in light of the vertical demand curve for reserves, which we will later contrast with the sloped demand curve created by an ORDC. We write the outputs of this model as functions of the random variable, further conditioned by the commitment solution $\hat{u}$ chosen prior to dispatch. We first consider the basic LMP pricing scheme, in which the energy price $\lambda^{LMP}(W; \hat{u})$ is the dual variable associated with the power balance constraint, Eq. (15b), while the price of reserves $\mu^{LMP}(W; \hat{u})$ is given by the dual variable associated with the reserve constraint, Eq. (15c). Assume the operator has chosen the optimal commitment solution, i.e., $\hat{u} = u^{90}$. As shown in Table 1, there are two possible pricing outcomes: if wind $W < 10$, the reserve shortfall leads to prices $\lambda^{LMP}(W; u^{90}) =$ \$1,000 for power and $\mu^{LMP}(W; u^{90}) =$ \$950 for reserves. If wind $W \geq 10$, generator 0 sets the price at $\lambda^{LMP}(W; u^{90}) =$ \$50.

Table 1: Dispatch and prices given optimal commitment

Range	Probability	Wind Availability	$p_0^*(W; u^{90})$	$r_0^*(W; u^{90})$	$\lambda(W; u^{90})$	$\mu(W; u^{90})$
1	0.1	$0 \leq W < 10$	$110 - W$	$10 + W$	\$1,000	\$950
2	0.9	$10 \leq W \leq 100$	$110 - W$	$20 - \varepsilon$	\$50	\$0

The profitability of a given generator is similarly a function of the random variable W. In this single-period example, we can compute profitability $\pi_g(W;\hat{u})$ as

$$\pi_g(W;\hat{u}) = p_g^*(W;\hat{u})\lambda(W;\hat{u}) + r_g^*(W;\hat{u})\mu(W;\hat{u}) - C_g^{EN} p_g^*(W;\hat{u}) - C_g^{NL}\hat{u}_g, \tag{16}$$

In the optimal solution u^{90}, consider the profitability of the most expensive committed unit, generator 90. It incurs a no-load cost of \$140. Since it is block loaded, we know $p_{90}^*(W;u^{90}) = 1$ and $r_{90}^*(W;u^{90}) = 0$ for any value of W. If it receives only the LMP, generator 90 will therefore earn a profit of \$1,000 − \$140 = \$860 ten percent of the time, while it loses \$140 − \$50 = \$90 ninety percent of the time. On average, this means that it earns a profit of $.1 \cdot \$860 - .9 \cdot \$90 = \$5$.

4.3 Uplift Payments

US markets guarantee bid cost recovery, ensuring that no generator loses money over some defined interval (e.g., each block for which it is committed). In the example, generator 90 would be entitled to a make-whole payment ninety percent of the time, i.e., whenever $W \geq 10$, to cover the difference between its no-load cost and the revenue it earns from selling power. Including such payments, generator 90 would break even in the ninety percent of cases where it previously lost, leading its expected profit to increase to \$86. In the stochastic case, these payments have the effect of socializing generator 90's losses and privatizing its gains. Accordingly, if the stochastic ideal could be achieved, it would be preferable to relax the standard of bid cost recovery to the less stringent bid cost recovery in expectation. This goal can be stated

$$\mathbb{E}\left[\pi_g(W;\hat{u})\right] \geq 0 \qquad \forall g \in \mathcal{G}. \tag{17}$$

Since the potential for losses is inherent to efficient markets under uncertainty, make-whole payments cannot be justified solely by the existence of scenarios with losses. We showed in Section 3 that the property of bid cost recovery in expectation holds for an optimal solution to a convex relaxation of the stochastic unit commitment problem, while the expected profits are precisely zero for marginal generators. While this property cannot be guaranteed in general due to non-convexity, it does hold for the example system. Despite a 90 percent chance of incurring a loss, generator 90 has sufficient incentive to follow its dispatch signal if it is risk neutral, has sufficient tools available to manage its risk, and shares the operator's beliefs regarding the distribution of future prices.

4.4 Enhanced Pricing

The analysis in Section 4.2 assumes the use of the basic LMP pricing scheme, in which prices result directly from the economic dispatch problem. In the example, the LMP scheme gives rise to ex ante lost opportunity costs for generators 91, 92, 93, and 94: each would profitable in expectation given the expected price of $145. Enhanced pricing schemes calculate prices after the economic dispatch problem is solved through a separate pricing problem. Here we consider the performance of two enhanced pricing schemes if applied to the example system. While implementation details vary by market, the intent is to capture the key features of these schemes.

The convex hull pricing (CHP) problem for a realized value of W in the example system can be formulated as follows:

$$
\begin{aligned}
(CHP): \quad & \underset{u,p,r,d,z,w}{\text{maximize}} \quad && V^D d + V^R z - \sum_{g\in\mathcal{G}} C_g^{EN} p_g - \sum_{g\in\mathcal{G}} C_g^{NL} u_g && \text{(18a)}\\
& \text{subject to} && d - w - \sum_{g\in\mathcal{G}} p_g = 0 && \text{(18b)}\\
& && z - \sum_{g\in\mathcal{G}} r_g = 0 && \text{(18c)}\\
& && \text{(13d)–(13j)} && \text{(18d)}\\
& && 0 \le u_g \le 1 \qquad \forall g \in \mathcal{G}. && \text{(18e)}
\end{aligned}
$$

Binary variables are relaxed, allowing the no-load cost of the thermal generators to be incorporated in the dual variables associated with constraints (18b) and (18c). Under the ex post CHP scheme in the example system, the energy price $\lambda^{CHP}(W) = \lceil 150 - W \rceil$, i.e., the no-load cost of the most expensive block-loaded generator that would have been committed had demand been known in advance. This lower average price of $100.50 over all possible realizations of W would lead to losses in expectation for generators 51 through 90.

Fast-Start Pricing (FSP) schemes differ from CHP by specifying a subset of generators for which binary variables are eligible to be relaxed. As an example, we formulate a pricing problem in which the binary variables of all online generators are relaxed. For a realized value of W and chosen commitment solution $\hat{u}$ in the example system, this variant of FSP can be formulated as follows:

$$
(FSP): \quad \underset{u,p,r,d,z,w}{\text{maximize}} \quad V^D d + V^R z - \sum_{g\in\mathcal{G}} C_g^{EN} p_g - \sum_{g\in\mathcal{G}} C_g^{NL} u_g \qquad \text{(19a)}
$$

$$\text{subject to} \qquad d - w - \sum_{g \in \mathcal{G}} p_g = 0 \tag{19b}$$

$$z - \sum_{g \in \mathcal{G}} r_g = 0 \tag{19c}$$

$$\text{(13d)–(13j)} \tag{19d}$$

$$0 \leq u_g \leq \hat{u}_g \qquad \forall g \in \mathcal{G}. \tag{19e}$$

Assuming the optimal commitment solution u^{90}, FSP results in the same prices as CHP for $W \geq 10$, but registers a reserve shortage and sets a price of $\lambda^{FSP}(W; u^{90}) = \$1{,}000$ when $W < 10$. This yields an average energy price of $185.95 over all possible realizations of W. This higher average price leads to additional ex ante lost opportunity costs and could induce the self-commitment of inefficient generators (generator 91, for example, has a total cost of $141 to operate).

Absent uniform clearing prices, market designers may seek to identify a pricing scheme that minimizes lost opportunity costs. In particular, CHP is seen by some as an ideal solution because it minimizes lost opportunity costs in the deterministic setting [4]. It can be seen through the example, however, that ex post CHP does not in general minimize lost opportunity costs when the decision problem of market participants includes uncertainty. To emphasize the contrast, we state this result formally.

Proposition. Assume the set of eligible pricing schemes includes LMP and CHP, i.e., $\{LMP, CHP\} \subseteq \mathcal{J}$, and consider the lost opportunity cost calculations for pricing schemes $j \in \mathcal{J}$ given by models $(LOC)_j$. Computed ex post, CHP does not in general minimize ex ante lost opportunity costs, i.e., there exists at least one instance of model (SUC) in which $LOC_{CHP} > LOC_{LMP}$.

Proof. The example system developed in this section is sufficient for proof by counterexample. Total lost opportunity costs under LMP equals $10, since the average expected price of $145 is higher than the no-load cost of four uncommitted generators. Specifically, ex ante lost opportunity costs arise for generator 91 ($4), generator 92 ($3), generator 93 ($2), and generator 94 ($1). Meanwhile, under CHP, generator 90 alone incurs loss of $\$140 - \$100.50 = \$39.50$ on average. Since the socially optimal schedule is always feasible, lost opportunity costs are non-negative for every market participant. Thus $LOC_{CHP} > LOC_{LMP}$ in the example system. □

Given the centrality of the uplift minimization property to arguments made in favor of CHP, it is important to note the limitations of ex post or deterministic analyses when making such a claim. In order to preserve the uplift minimization property, we could imagine implementing "ex ante convex hull pricing," under which prices would be determined by a relaxation of the full stochastic unit commitment problem. In the example, this would lead to generator 91 being partially committed at 0.42, resulting in an energy price of \$50 whenever $W \geq 9.58$, and an energy price of \$1,000 whenever $W < 9.58$. The average price of \$141 in this case is determined by the no-load cost of generator 91 and results in no ex ante lost opportunity costs. Since commitment variables would be shared across scenarios in the ex ante version of CHP, the distribution of prices resembles LMP more than ex post CHP.

When the decision problem of market participants includes uncertainty, there is no theoretical reason to expect convex hull prices determined ex post to provide appropriate incentives ex ante. While theoretical arguments for make-whole payments and enhanced pricing appear weak based on the the example system, however, this analysis leaves open the possibility that they could outperform LMP empirically. The following section discusses two reasons the basic LMP scheme may fail in practice.

5 Two Defects of Deterministic Pricing

While significant progress is being made in stochastic commitment and dispatch, the optimization routines currently used by system operators are deterministic. This section highlights two mechanisms that may suppress prices below the level required for bid cost recovery in expectation. These deterministic routines give rise to the possibility that operators may commit units for which the property of bid cost recovery in expectation does not hold. First, in unit commitment models, operators regularly adjust the load forecast upwards to account for uncertainty that has not been explicitly represented. Second, in economic dispatch models, operators use a point forecast for grid conditions several minutes before the actual operating interval. In this section, we discuss the consequences these workarounds could have for pricing.

5.1 Unit Commitment with Biasing

For the deterministic unit commitment, we modify the stochastic model (12) in two ways. First, we substitute the random variable W with its expected value $\overline{W}$. Second, we add a load biasing term b to the demand. Reusing constraints from the subproblem in model (13), the deterministic unit commitment with load biasing can be stated as

$$\begin{aligned}
\underset{u,p,r,d,z,w}{\text{maximize}} \quad & V^D d + V^R z - \sum_{g\in\mathcal{G}} (C_g^{NL} u_g + C_g^{EN} p_g) && \text{(20a)}\\
\text{subject to} \quad & (13c)\text{–}(13e), (13g)\text{–}(13j) && \\
& d + b - w - \sum_{g\in\mathcal{G}} p_g = 0 && \text{(20b)}\\
& w \leq \overline{W} && \text{(20c)}\\
& u_g \in \{0,1\} \qquad \forall g \in \mathcal{G}. && \text{(20d)}
\end{aligned}$$

Equipped with a deterministic model, the challenge for human operators is to convert uncertainty into an appropriate bias b_t in each time period t. In the single-period example system, expected wind $\overline{W} = 50$, while the optimal solution to the true stochastic commitment problem is to commit the cheapest 90 block-loaded units. Solving a bias-free model would lead to poor results: with 50 units of wind available along with 120 units of capacity from generator 0, the remaining demand for energy and reserves can be satisfied with 50 block-loaded units. Ideally, the operator will instead choose $b = 40$, which will induce the solver to commit 90 block-loaded units.

5.2 Stochastic Pricing with Suboptimal Commitments

While the optimal bias can be determined in the example system, there is no general method to calculate appropriate biases for more complex systems. As such, the choice of bias is left to the engineering judgment of system operators. It bears emphasizing that, despite any negative connotations associated with the word "bias," some amount of biasing is necessary to achieve a good commitment solution with a deterministic model. However, conservatism on the part of operators could lead to excess biasing. This choice can have important consequences for the prices observed in the system.

We first consider the ideal pricing situation from Section 4.2, in which uncertainty resolves before the time of dispatch and prices are calculated through model $(ED - VERT)$. The energy price $\lambda^{LMP}(W; u^{90})$ takes values of \$1,000 and \$50 depending on the availability of wind. The relative probabilities of the two prices, however, depends on the commitment solution. Table 2 shows the decline in average price, $\mathbb{E}\left[\lambda^{LMP}(W; \hat{u})\right]$, as additional units are committed. At the optimal commitment solution, u^{90}, there is a ninety percent chance of avoiding a reserve shortfall. As discussed in Section 3, this leads to bid cost recovery in expectation for generator 90. In the

Table 2: Dependence of reserve shortfall and prices on commitment

Commitment Solution	Probability of Reserve Shortfall	Average Price
u^{90}	0.10	$145.00
u^{91}	0.09	$135.50
u^{92}	0.08	$126.00
u^{93}	0.07	$116.50
u^{94}	0.06	$107.00
u^{95}	0.05	$97.50
u^{96}	0.04	$88.00
u^{97}	0.03	$78.50
u^{98}	0.02	$69.00
u^{99}	0.01	$59.50
u^{100}	0.00	$50.00

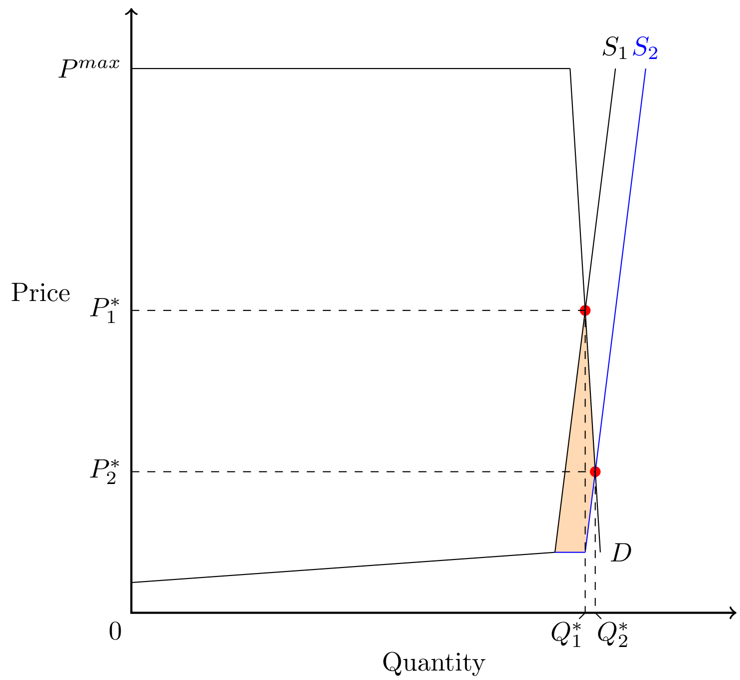
solution u^{91}, generator 91 becomes the most expensive unit. Bid cost recovery in expectation no longer holds: the average price is $135.50, lower than this unit's $141 no-load cost. The situation deteriorates with additional commitments. In the most conservative case, in which the operator chooses bias $b = 50$ and avoids any chance of a reserve shortfall, the expected price drops to $\lambda^{LMP}(W; u^{100}) = \$50$.

Figure 1 demonstrates this mechanism graphically. The effect of additional commitments is to shift the supply curve in the economic dispatch toward the right (i.e., from S_1 to S_2). If the start-up and no-load costs associated with the commitment are comparable to the additional surplus in dispatch indicated by the shaded area, the total surplus under the two solutions may be comparable. Due to the steepness of both the supply and demand curves, however, a small change in the supply stack can result in a large change in prices. Near-vertical demand curves and "hockey-stick" supply curves are characteristic to electricity markets (in PJM, e.g., see Figure 7 in [31]).

5.3 Ex Ante Deterministic Pricing

A separate pricing issue can arise when uncertainty is not completely resolved before the pricing model is run, as is the case in real-world systems that require a lead time between execution of the dispatch model and actual system operation. Even if this lead time were zero, some uncertainty would necessarily remain, since pricing intervals are at least five minutes long in real-world systems. Interpreting our example system differently, we can imagine that wind output will fluctuate between 0 and 100 over the course of the interval. Accounting for every fluctuation would require continuous-time pricing of electricity [38]. Instead, prices are set by running a single instance of model $(ED-$

Figure 1: Effect of additional committed units on price



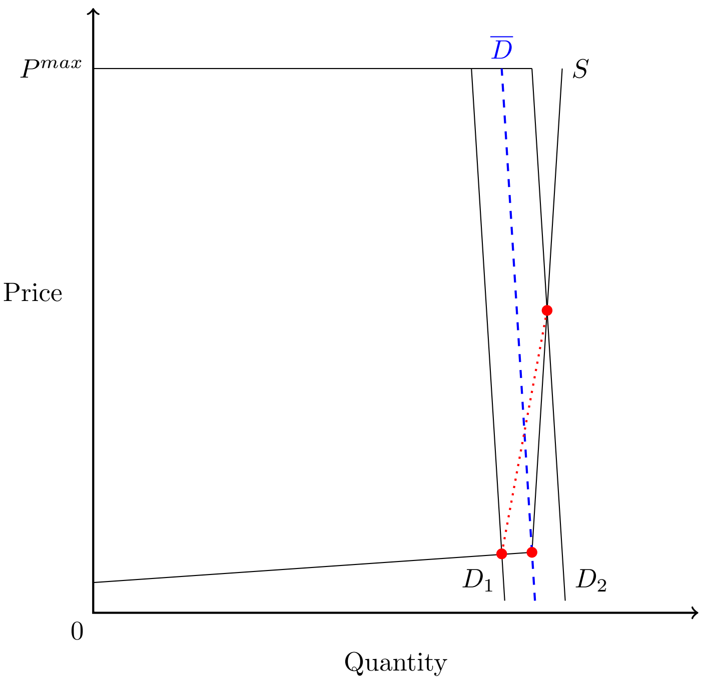
$VERT$) using the expected wind output $\overline{W} = 50$. With this output reserves are plentiful, and the price $\lambda^{LMP}(\overline{W}; u^{90}) = \$50$. In other words, the price reflects marginal cost under the expected operating conditions, rather than the expected price under a range of operating conditions. Even under the optimal commitment u^{90}, this price fails to satisfy the property of bid cost recovery in expectation.

Figure 2 offers a graphical depiction of this second mechanism, using uncertainty in demand rather than wind. As in Figure 1, the magnitude of the effect depends on the shape of the supply stack S, with the hockey-stick shape shown creating the potential for the price using the average demand curve $\overline{D}$ to be lower than the average of the prices using demand curves D_1 and D_2. Under normal operating conditions, during which the supply curve is likely to be flatter, the effect on pricing may be negligible.

6 Quasi-Stochastic Improvements

The pricing problems identified above lead to a difference between the expected price in the stochastic ideal with the optimal commitment, $\mathbb{E}\left[\lambda^{LMP}(W; u^*)\right]$, and the single price seen by market participants with the actual commitment, $\lambda^{LMP}(\overline{W}; \hat{u})$. In the example this could lead

Figure 2: Effect of uncertainty in economic dispatch on price



to significant ex ante lost opportunity costs. Without bid cost recovery in expectation, even risk-neutral generators would be reluctant to submit truthful offers and follow commitment and dispatch instructions. We propose that the goal of ORDCs should be to bridge the gap between these two prices, restoring the average price that would arise in an idealized stochastic market. The expected price under CHP calculated ex ante, mentioned in Section 4.4, could be substituted for $\mathbb{E}\left[\lambda^{LMP}(W;u^*)\right]$ without changing the remainder of the analysis. With either target, this conception generalizes the ORDC beyond its original focus on times of scarcity [19], during which the difference $\mathbb{E}\left[\lambda^{LMP}(W;u^*)\right] - \lambda^{LMP}(\overline{W};\hat{u})$ is likely greatest. This generalization helps explain why an ORDC might be active even at times when no chance of scarcity exists. In this section we present current ORDC formulations, demonstrating that they achieve this goal under ideal circumstances, and then discuss three issues with price formation they may fail to address.

6.1 Probability-Driven ORDC

The example system presents the best possible circumstances for construction of an ORDC along the lines of [19]: we have complete knowledge of the probability distribution for wind availability, are able to identify the optimal commitment solution u^{90}, can easily determine that the marginal unit has a variable cost of \$50, and have a constant recourse cost of \$950. Ninety percent of the time, holding additional reserves would have no value. The remaining ten percent of the time, additional reserves would mitigate the system's reserve shortfall, providing value of \$950. Before uncertainty is resolved and assuming risk neutrality, therefore, holding additional reserves has marginal value $\mathbb{E}\left[\mu^{LMP}(W; u^{90})\right] = \$95$. Measured against the expected wind $\overline{W}$, this commitment solution holds 60 units of reserves, much more than the required 20. The uniform distribution of W simplifies the calculation of expected value at different levels of reserve: with only 20 units of reserve online, there is a 50 percent chance of shortage, so the expected value of additional reserves is \$475. With 70 units of reserve, there is no chance of a shortfall, so the expected value is \$0. Between these reserve levels, expected value falls linearly.

The economic dispatch problem in model $(ED - VERT)$ utilizes a vertical demand curve for reserves, assigning a value of 0 to any reserves above the level R^+. The sloped ORDC formulation replaces this with a function reflecting the declining expected value of reserves. In keeping with the formulation in [19], implementation in ERCOT, and proposal for PJM, we define the ORDC for our example with respect to the expected wind $\overline{W}$ and ignore the negative net load changes that would come with wind above this level. As in model $(ED-VERT)$, we formulate the dispatch and pricing problem as a single-period problem. Instead of the linear coefficient V^R up to the quantity R^+, we express the total value of these reserves is expressed through the concave, non-decreasing function $V^O(z)$. Given the marginal value of reserves discussed above, the function $V^O(z)$ in the example is linear up to the quantity $z = 20$, quadratic in the range $20 \leq z < 70$, and constant for $z \geq 70$. The economic dispatch with sloped ORDC can be stated as

$$
\begin{aligned}
(ED - ORDC): \quad & \underset{p,r,d,z,w}{\text{maximize}} && V^D d + V^O(z) - \sum_{g \in \mathcal{G}} C_g^{EN} p && \text{(21a)} \\
& \text{subject to} && (13d), (13g)\text{–}(13j) && \\
& && d - w - \sum_{g \in \mathcal{G}} p_g = 0 && \text{(21b)} \\
& && z - \sum_{g \in \mathcal{G}} r_g = 0 && \text{(21c)} \\
& && w \leq \overline{W}. && \text{(21d)}
\end{aligned}
$$

In the deterministic pricing setting discussed in Section 5.3, prices were set using the vertical demand curve of model $(ED - VERT)$ and the expected wind output $\overline{W}$. Model $(ED - ORDC)$

similarly uses $\overline{W}$, but incorporates the uncertain W in the function $V^O(z)$. Under this model, the LMP $\lambda^{ORDC}(\hat{u})$ and reserve price $\mu^{ORDC}(\hat{u})$, calculated as the dual variables to Eqs. (21b) and (21c), are still functions of the commitment solution, but no longer functions of the random variable. Assuming the optimal commitment solution, u^{90}, solving model (21) leads to wind clearing at 50 units, each of the 90 block-loaded units providing 1 unit of energy, and generator 0 meeting the remaining 60 units of demand. However, instead of being limited to 20 units, all 60 units of generator 0's remaining capacity now clear as reserves. Since the marginal value of reserves is \$95 at this level, the reserve price $\mu^{ORDC}(u^{90}) = \$95$, while $\lambda^{ORDC}(u^{90}) = \$145$. In other words, the ORDC recovers the expected price under the stochastic ideal, $\mathbb{E}\left[\lambda^{LMP}(W; u^*)\right]$, despite being a deterministic model. Section 7.1 formalizes a methodology that achieves this result in a more general setting.

6.2 Stochasticity without Shortage

In the example system, limiting uncertainty to a one-dimensional random variable enables straightforward calculation of the probability of reserve shortage. In real-world systems, a wide array of transmission contingencies, load fluctuations, and generator failures can impact the cost of operating the system and thus the prices that would arise in the stochastic ideal. Combinations of these events could lead to a wide range of prices in a stochastic ideal setting, in which markets could be cleared in real time as conditions changed. This stochasticity in system cost is present at all times, even when the probability of lost load is negligible.

With their focus on reserve shortages, current formulations in ERCOT and PJM do not capture this more general form of stochasticity, instead assuming a constant recourse cost. By itself, this could mean that both underestimate the value of reserves. However, both markets potentially overestimate the value of reserves by incorporating forecast errors over much longer timescales than the five minutes for which prices are valid. Without an external metric to validate ORDC parameters, it is difficult to assess whether the net effect is to under- or overvalue reserves.

6.3 Robustness to Commitment Solution

With perfect knowledge, the ORDC in model (21) helps resolve the pricing issue identified in Section 5.3, namely, the need for the system operator to set prices before uncertainty is realized. It does not, however, address the issue of suboptimal commitments raised in Section 5.2. If the operator in the example commits all 100 block-loaded units, the system registers 70 units of reserve and the LMP drops to $\lambda^{ORDC}(u^{100}) = \$50$. In other words, the clearing price of reserves is not determined solely by the cost of the recourse action and the probability it will take place; an additional factor is the bias chosen by the operators. In recognition of this dependence, ERCOT utilizes a Real-Time On-Line Reliability Deployment Price Adder, which attempts to counteract the pricing impacts of operator actions [40]. The example system illustrates a potential difficulty of calculating this adder. As discussed in Section 5.1, reliability deployments at a level of 40 units restore the stochastic optimal commitment solution. It is only bias above this level that creates a problem for pricing in need of correction. Differentiating between commitments that do and do not warrant correction is not straightforward.

6.4 Flexibility Compensation

While an ideal ORDC may be able to bridge the gap between the expected price under uncertain conditions and the deterministic price in the expected conditions, it does not recover all the properties of the stochastic ideal. In the example, consider the revenue earned by the wind generator. In the stochastic setting, prices and wind output are negatively correlated: since reserve shortages only occur when $0 \leq W < 10$, average wind generation in these high price times is only 5. The average wind output when prices are lower is 55. Expected revenue for the wind generator is thus $5 \cdot \$1{,}000 \cdot .1 + 55 \cdot \$50 \cdot .9 = \$2{,}975$. In the ORDC setting, by contrast, wind earns \$145 per unit regardless of its metered output, giving an expected revenue of $50 \cdot \$145 = \$7{,}250$.

7 ORDC Construction

Given the challenges faced by current formulations, this section considers how we might construct ORDCs to bridge the gap between the expected price that would arise in a stochastic model, $\mathbb{E}\left[\lambda^{LMP}(W; u^*)\right]$, and the price in the single scenario chosen by system operators for the deterministic dispatch, $\lambda^{LMP}(\overline{W}; \hat{u})$. In doing so, we hope to approximate the outcomes that might be achieved in a stochastic competitive equilibrium, in which system operators maximize expected surplus and market participants maximize expected profits. We begin with the case where system operators are able to implement stochastic models for background reliability processes (e.g., the PJM IT-SCED and ISO-NE Real-Time Unit Commitment), but are unable to implement a fully stochastic market design. We then consider the problem faced by current system operators using deterministic commitment models. Lastly, we return to the subject of bid cost recovery in expectation, highlighting three implications of the analysis for future market design.

7.1 Stochastic Reliability Models

If the system operator runs a stochastic model along the lines of model (12) for commitment, it can reasonably be assumed that the ex ante optimal commitment solution u^* will be chosen. The expected price $\mathbb{E}\left[\lambda^{LMP}(W;u^*)\right]$ in any period can then be calculated using the dual variables in the dispatch subproblem, model (13). After the system operator chooses a particular scenario for use in the deterministic pricing model, it is straightforward to find the marginal cost of energy and the quantity of reserves available given that choice of scenario. In the example system, $\mathbb{E}\left[\lambda^{LMP}(W;u^*)\right] = \$145$ and we chose the scenario $W = 50$. This enables us to calculate a marginal energy cost of \$50 under the vertical reserves demand curve in model $(ED-VERT)$ and to determine that a total of 60 units of reserves are online. When constructing the ORDC, this means we should choose a curve such that $V^O(60) = \$95$. Inserting this valuation in model (21) yields a reserve clearing price of $\mu^{ORDC}(u^*) = \$95$ and an energy price of $\lambda^{ORDC}(u^*) = \$145$.

Here we formalize this methodology, which extends naturally to the dynamic, multi-period setting, by returning to the more extensive model developed in Section 3. Prior to the beginning of each pricing interval, we assume the operator has access to information $\mathcal{F}_t$, from which they can calculate the probabilities $\phi(s|\mathcal{F}_t)$. The operator chooses random variable realizations from a single scenario $s \in \mathcal{S}$ in order to calculate prices with a deterministic, single-period model. Our proposed method recovers the expected energy price $\sum_{s\in\mathcal{S}} \phi(s|\mathcal{F}_t)\lambda_{st}^{LMP}$ through construction of an ORDC specific to the operator scenario choice, time period, and available information. The ORDC as constructed takes the form of a piecewise constant value function broken into segments $c \in \mathcal{C}$. Reserves up to a quantity $R_{ct}^{+}(s,\mathcal{F}_t) \geq 0$ in segment $c \in \mathcal{C}$ are valued at $V_{ct}^{O}(s,\mathcal{F}_t) \geq 0$, while the total quantity supplied in that segment is expressed as z_{sct}. While these parameters will depend on information $\mathcal{F}_t$ and choice of scenario s, we omit this notation from the formulation. We assume generator commitments are fixed at u_{gt}^*, as determined by a stochastic unit commitment performed prior to the dispatch interval. The economic dispatch problem with operating reserve demand curve for time period t given scenario choice s is stated as

$(ED - ORDC)_{st}$:

$$\underset{p,r,d,z}{\text{maximize}} \quad \sum_{l \in \mathcal{L}} V_l^D d_{slt} + \sum_{c \in \mathcal{C}} V_{ct}^O z_{sct} - \sum_{g \in \mathcal{G}} C_g^{EN} p_{sgt} \tag{22a}$$

$$\text{subject to} \quad \sum_{l \in \mathcal{L}} d_{slt} - \sum_{g \in \mathcal{G}} p_{sgt} = 0 \qquad (\lambda_{st}^O) \tag{22b}$$

$$\sum_{c \in \mathcal{C}} z_{sct} - \sum_{g \in \mathcal{G}} r_{gst} = 0 \qquad (\mu_{st}^O) \tag{22c}$$

$$d_{slt} \leq D_{slt}^+ \qquad \forall l \in \mathcal{L} \qquad (\alpha_{slt}^D) \tag{22d}$$

$$z_{sct} \leq R_{ct}^+ \qquad \forall c \in \mathcal{C} \qquad (\alpha_{sct}^R) \tag{22e}$$

$$P_{sgt}^- u_{gt}^* \leq p_{sgt} \qquad g \in \mathcal{G} \qquad (\alpha_{sgt}^{P-}) \tag{22f}$$

$$p_{sgt} + r_{sgt} \leq P_{sgt}^+ u_{gt}^* \qquad g \in \mathcal{G} \qquad (\alpha_{sgt}^{P+}) \tag{22g}$$

$$d_{slt} \geq 0 \qquad \forall l \in \mathcal{L} \tag{22h}$$

$$z_{sct} \geq 0 \tag{22i}$$

$$p_{sgt}, r_{sgt} \geq 0 \qquad g \in \mathcal{G}. \tag{22j}$$

With this problem defined, an ORDC can be constructed on the basis of Theorem 2.

Theorem 2. Suppose that for a chosen scenario $s' \in \mathcal{S}$, we are able to identify generator $g' \in \mathcal{G}$ as a marginal supplier of energy, i.e., $P_{s'g't}^- u_{g't}^* < p_{s'g't}^* < P_{s'g't}^+ u_{g't}^*$. Further suppose that we are able to determine a segment $c' \in \mathcal{C}$ for which $0 < z_{s'c't}^* < R_{c't}^+$, and let the value of reserves $V_{c't}^O = \sum_{s \in \mathcal{S}} \phi(s|\mathcal{F}_t) \lambda_{st}^{LMP} - C_{g'}^{EN}$ in this segment. Then the clearing price of energy $\lambda_{s't}^{ORDC} = \sum_{s \in \mathcal{S}} \phi(s|\mathcal{F}_t) \lambda_{st}^{LMP}$.

Proof. In the dual to $(ED - ORDC)_{s't}$, the constraint related to the primal variable $z_{s'ct}$ is

$$\mu_{s't}^{ORDC} + \alpha_{s'c't}^R \geq V_{c't}^O. \tag{23}$$

Since in the optimal solution $z_{s'c't}^* > 0$, by complementary slackness this inequality is binding. Further, since $z_{s'c't}^* < R_{c't}^+$, we know that $\alpha_{s'c't}^R = 0$. Therefore,

$$\mu_{s't}^{ORDC} = V_{c't}^O = \sum_{s \in \mathcal{S}} \phi(s|\mathcal{F}_t) \lambda_{st}^{LMP} - C_{g'}^{EN}. \tag{24}$$

Now we consider the dual constraint related to the primal variable $r_{s'g't}$, i.e.,

$$-\mu_{s't}^{ORDC} + \alpha_{s'g't}^{P+} \geq 0. \tag{25}$$

Since reserves are costless to provide and $V^O_{ct} \geq 0$, we can always choose an optimal solution in which generator g' supplies reserve up to its capacity, i.e., $p_{s'g't} + r_{s'g't} = P^+_{s'gt} u^*_{gt}$. Since $p^*_{s'g't} < P^+_{s'g't} u^*_{g't}$, this implies that $r_{s'g't} > 0$, and thus $\mu^O_{s't} = \alpha^{P+}_{s'g't}$. Lastly, the dual constraint related to the primal variable $p_{s'g't}$ gives

$$-\lambda^{ORDC}_{s't} - \alpha^{P-}_{s'g't} + \alpha^{P+}_{s'g't} \geq -C^{EN}_{g'}. \tag{26}$$

Since $P^-_{s'g't} u^*_{g't} < p^*_{s'g't}$, by complementary slackness this inequality is binding and $\alpha^{P-}_{s'g't} = 0$. Substituting $\sum_{s \in \mathcal{S}} \phi(s|\mathcal{F}_t) \lambda^{LMP}_{st} - C^{EN}_{g'} = \mu^{ORDC}_{s't} = \alpha^{P+}_{s'g't}$ gives

$$\lambda^{ORDC}_{s't} = \sum_{s \in \mathcal{S}} \phi(s|\mathcal{F}_t) \lambda^{LMP}_{st}. \tag{27}$$

□

In this construction, the marginal value of reserves $V^O_{c't}$ reflects a weighted average of the cost of all recourse actions included in the stochastic model, including the reserve penalty factor V^R. In principle, constructing an ORDC in this way allows reconstruction of the expected price in a deterministic model. The assumption that we can identify the marginal supplier of energy may seem somewhat limiting, particularly in the presence of ramp constraints. However, we note that the same assumption is made in the ORDC construction in [19]. In practice, a reasonable proxy for the marginal cost of energy is likely to be available based on the results of recent dispatch periods. The assumption that we can identify which segment of the ORDC will be active is much weaker, since the total quantity of reserves available is tracked in real time by system operators.

A complication in practice, but not in theory, is that commitment decisions do not occur at the same time as dispatch and pricing decisions, and accordingly are made with different information available. Suppose that at time t_1, the operator makes a decision to commit a generator for a future time t_2. We note that by the tower property,

$$\mathbb{E}\left[\mathbb{E}\left[\lambda_{st}|\mathcal{F}_{t_2}\right]|\mathcal{F}_{t_1}\right] = \mathbb{E}\left[\ \lambda_{st}|\mathcal{F}_{t_1}\right]. \tag{28}$$

This states that the expected price in interval t_2, given the information the operator learns between t_1 and t_2, is equal to the expected price when it was committed.

7.2 Deterministic Reliability Models

A potentially greater challenge is to calculate a marginal value of reserves consistent with operator commitments even if those commitments are made with a deterministic model. If the operator instead uses a deterministic model with biasing along the lines of model (20), we do not have access to the optimal commitment solution or expected price. We propose to address the first issue by assuming that the choice of bias made by operators is in fact optimal. To ensure consistency between operator actions and prices, the simplest choice is to trust the engineering judgment of the operators. As operators gain access to increasingly sophisticated algorithmic decision support tools, this assumption is likely to become increasingly valid over time. The second issue can be addressed by invoking the result shown in Section 3.6 in the convex case: at the time of commitment, the expected profit for a marginal generator in the commitment solution should be precisely zero. Suppose in the example that the operator chooses a bias of $b = 42$. The most expensive generator committed in the optimal solution, generator 92, has a no-load cost of \$142 and will earn all its revenues from the sale of energy, which it produces at a cost of \$0. The fact that operators committed the unit through their choice of bias reveals their preference for holding an additional unit of capacity online at a cost of \$142. When constructing an ORDC, we should therefore choose a curve such that $V^O(62) = \$92$, leading to reserve price $\mu^{ORDC}(u^{92}) = \$92$ and energy price $\lambda^{ORDC}(u^{92}) = \$142$. More generally, the goal of bid cost recovery in expectation for the most expensive committed unit offers a lower bound on expected prices. Accordingly, such a strategy could be successful as long as at least one unit in the system remains uncommitted. Translating this deterministic methodology to the dynamic, multi-period setting, however, is less straightforward than in the stochastic case.

7.3 Implications for Future Market Design

The proposed method recovers some version of the expected price, whether that is calculated explicitly in a stochastic model or implicitly in a deterministic one. In the example system, this leads to bid cost recovery: the price produced is sufficiently high that all committed units cover their avoidable fixed costs. However, three complicating factors prevent more general guarantees on bid cost recovery in expectation. Each of these factors has implications for future market design improvements beyond the ORDCs discussed in this paper.

The first is the flexibility effect discussed in Section 6.4. A generator whose production is positively correlated with prices may achieve bid cost recovery in expectation under the stochastic ideal, but not when the uncertainty in prices is removed. Moreover, as described the methodology targets the expected price, rather than the total expected cost to operate the system. Since the generation side of the market has historically been significantly more flexible than the demand side, it may be preferable to target a price that is weighted by the cleared load in the scenario. In other words, instead of the average price $\mathbb{E}\left[\lambda^{LMP}(W;u^*)\right]$, the market operator could target the average generator revenue $\mathbb{E}\left[d^*(W;u^*)\lambda^{LMP}(W;u^*)\right]$. This revenue could then be shifted to more flexible generators through a system of penalties and rewards for adherence to dispatch signals.

The second is non-convexity. Recognition of non-convexity in the deterministic context has led to the introduction of make-whole payments and enhanced pricing schemes, justified on the grounds that market participants following operator dispatch signals should not lose money. As illustrated by the example, however, the stochastic analysis offers a new interpretation of the challenge posed by non-convexity. In a stochastic setting, commitments may be profitable in expectation but result in losses under some realizations. In such cases it is difficult to justify make-whole payments or enhanced pricing, which amount to socializing losses and privatizing gains. If ORDCs effectively raise real-time prices to the level expected at the time of commitment, these efforts to address non-convexity may no longer be warranted.

The third is risk aversion and asymmetric beliefs. The model of stochastic competitive equilibrium in Theorem 1 relies on the assumption that all market participants are risk neutral and share a common assessment of the probability that each scenario materializes. If those assumptions do not hold, the market operator may believe that a unit will be profitable in expectation, but the market participant may not be willing to follow commitment and dispatch instructions. Reconciling the two beliefs may be possible with the introduction of intraday markets [18] or a broader set of market instruments [16]. For example, if in the example the system operator expected a price of \$145 while generator 90 expected a price of \$130, the system operator could buy a future at a price between \$140 and \$145 at the time of commitment. This approach introduces other challenges, since the operator would then be at risk of revenue shortfall. However, socializing net losses on these contracts may be preferable to maintaining current uplift payments.

8 Conclusion

This article explains the inconsistency between commitment, dispatch, and pricing observed in current markets as arising due to the use of deterministic models to represent a stochastic reality. A full stochastic model gives information on the expected cost to operate the system given a range of potential conditions, whereas a deterministic model estimates the cost given one set of assumed conditions. This article proposes that the goal of ORDCs should be to recover this expected cost, thus improving the quality of the price signal produced in real-time markets. This interpretation generalizes ORDCs, helping explain their relevance in markets where they are not required for resource adequacy. Restoring the expected price can be achieved with only modest changes to current market clearing mechanisms, making ORDCs promising for near-term implementation. Future empirical work, however, is needed to validate the performance of the approach against the status quo as well as other proposals.

Whereas previous ORDC implementations rely primarily on administrative parameters, this paper proposes to construct ORDCs by tying them to the commitment decisions made by system operators. Since current proposals rely on administrative choices loosely connected to the probability of involuntary load shedding, they are unlikely to fully align with operator decisions. If these administrative parameters underestimate the value placed on reserves by operators, ORDCs will fail to achieve price consistency between commitment and dispatch. If they overestimate the value of reserves, they may instead induce inefficient self-commitment of units not needed for reliability.

The ORDC construction envisioned in Section 7.1 relies on information available in stochastic unit commitment models. Implementation of these models for non-market reliability processes may be realistic in the near future. At present, however, operators in centrally committed markets rely on deterministic models with biasing. We sketch an approach in Section 7.2 to construct ORDCs on the basis of the more limited information available in these models. Starting from either a stochastic or deterministic model, translating our approach to dynamic, multi-period settings with network constraints and multiple reserve products presents several challenges. Instead of constructing ORDCs, a more immediate application of the analysis in this paper may be to assess the quality of curves generated by administrative choices. In other words, administrative ORDCs can be judged on the basis of achieving bid cost recovery in expectation while avoiding uneconomic self-commitments.

The analysis identifies three additional shortcomings of current markets connected to stochasticity. First, deterministic market models have the effect of undercompensating flexible resources. While a shift to more frequent settlement intervals helps alleviate the problem, this undercompensation could become more significant with increased contributions from variable renewable resources. Second, ORDCs weaken the justification for uplift payments. If ORDCs are successfully implemented, stakeholders may wish to reexamine current practices regarding these payments as well as efforts to implement enhanced pricing schemes. Third, the presence of uncertainty leads to the potential for risk aversion and asymmetric beliefs to influence the behavior of market participants. These asymmetries are particularly important for energy-limited resources, the offers of which are contingent on self-assessed opportunity costs. Asymmetry could lead to suboptimal commitment and dispatch and may necessitate a broader set of risk-trading instruments to facilitate market clearing. Accordingly, while ORDCs offer the potential to resolve one issue with deterministic market designs, further improvements are needed to fully reflect the stochastic reality.

Acknowledgment

The author thanks Joe Byers, Yonghong Chen, David Morton, Richard O'Neill, Feng Pan, Ivilina Popova, and staff in the Business Architecture and Technology department at ISO New England. The views expressed in this paper are those of the author and do not represent the views of ISO New England.

References

[1] Y.M. Al-Abdullah, M. Abdi-Khorsand, and K.W. Hedman. The role of out-of-market corrections in day-ahead scheduling. *IEEE Transactions on Power Systems*, 30(4):1937–1946, 2015.

[2] D. Bertsimas, E. Litvinov, X.A. Sun, J. Zhao, and T. Zheng. Adaptive robust optimization for the security constrained unit commitment problem. *IEEE Transactions on Power Systems*, 28(1):52–63, 2013.

[3] E. Bjørndal, M. Bjørndal, K. Midthun, and A. Tomasgard. Stochastic electricity dispatch: A challenge for market design. *Energy*, 150:992–1005, 2018.

[4] H.-P. Chao. Incentives for efficient pricing mechanism in markets with non-convexities. *Journal of Regulatory Economics*, 56(1):33–58, 2019.

[5] K. Cheung, D. Gade, C. Silva-Monroy, S.M. Ryan, J.P. Watson, R.J.B. Wets, and D.L. Woodruff. Toward scalable stochastic unit commitment. *Energy Systems*, 6(3):417–438, 2015.

[6] R. Cory-Wright, A. Philpott, and G. Zakeri. Payment mechanisms for electricity markets with uncertain supply. *Operations Research Letters*, 46(1):116–121, 2018.

[7] Department of Market Monitoring–California ISO. Annual report on market issues & performance. Available at `http://www.caiso.com/Documents/2019AnnualReportonMarketIssuesandPerformance.pdf`, 2020.

[8] B. Eldridge, R. O'Neill, and B.F. Hobbs. Near-optimal scheduling in day-ahead markets: Pricing models and payment redistribution bounds. *IEEE Transactions on Power Systems*, 35(3):1684–1694, 2020.

[9] Electric Power Research Institute. Independent system operator and regional transmission organization price formation working group white paper: Current practice and research gaps in alternative (fast-start) price formation modeling. Available at `https://www.epri.com/#/pages/product/3002013724/?lang=en-US`, December 2019.

[10] Electric Reliability Council of Texas. ORDC OBD revisions for PUCT project 48551. Available at `http://www.ercot.com/mktrules/issues/OBDRR011`, February 2019.

[11] Federal Energy Regulatory Commission. Staff analysis of uplift in RTO and ISO markets. Available at https://www.ferc.gov/legal/staff-reports/2014/08-13-14-uplift.pdf, August 2014.

[12] Federal Energy Regulatory Commission. Docket no. EL18-33-000. Available at https://www.ferc.gov/whats-new/comm-meet/2019/041819/E-2.pdf, April 2019.

[13] Federal Energy Regulatory Commission. Docket no. EL18-34-000. Available at https://www.ferc.gov/whats-new/comm-meet/2019/041819/E-3.pdf, April 2019.

[14] Federal Energy Regulatory Commission. Energy price formation. Available at https://www.ferc.gov/industries/electric/indus-act/rto/energy-price-formation.aspx, September 2019.

[15] Y. Feng, I. Rios, S.M. Ryan, K. Spürkel, J.P. Watson, R.J.B. Wets, and D.L. Woodruff. Toward scalable stochastic unit commitment. part 1: load scenario generation. *Energy Systems*, 6(3):309–329, 2015.

[16] M. Ferris and A. Philpott. Dynamic risked equilibria. Available at http://www.optimization-online.org/DB_HTML/2018/04/6577.html, April 2018.

[17] P. Gribik, W.W. Hogan, and S.L. Pope. Market-clearing electricity prices and energy uplift. December 2007.

[18] I. Herrero, P. Rodilla, and C. Batlle. Enhancing intraday price signals in U.S. ISO markets for a better integration of variable energy resources. *The Energy Journal*, 39(3):141–165, 2018.

[19] W.W. Hogan. Electricity scarcity pricing through operating reserves. *Economics of Energy & Environmental Policy*, 2(2), 2013.

[20] ISO New England. Real-time fast-start pricing project. Available at https://www.iso-ne.com/participate/support/customer-readiness-outlook/real-time-fast-start-pricing-project, March 2017.

[21] P. Joskow. Competitive electricity markets and investment in new generating capacity. In Dieter Helm, editor, *The New Energy Paradigm*. Oxford University Press, 2007.

[22] J. Kazempour and B.F. Hobbs. Value of flexible resources, virtual bidding, and self-scheduling

in two-settlement electricity markets with wind generation—part i: Principles and competitive model. *IEEE Transactions on Power Systems*, 33(1):749–759, 2018.

[23] J. Kazempour and B.F. Hobbs. Value of flexible resources, virtual bidding, and self-scheduling in two-settlement electricity markets with wind generation—part ii: Iso models and application. *IEEE Transactions on Power Systems*, 33(1):760–770, 2018.

[24] J. Kazempour, P. Pinson, and B.F. Hobbs. A stochastic market design with revenue adequacy and cost recovery by scenario: Benefits and costs. *IEEE Transactions on Power Systems*, 33(4):3531–3545, 2018.

[25] X. Kuang, Y. Dvorkin, A.J. Lamadrid, M.A. Ortega-Vazquez, and L.F. Zuluaga. Pricing chance constraints in electricity markets. *IEEE Transactions on Power Systems*, 33(4):4634–4636, 2018.

[26] G. Liberopoulos and P. Andrianesis. Critical review of pricing schemes in markets with non-convex costs. *Operations Research*, 64(1):17–31, 2016.

[27] Á. Lorca, X.A. Sun, E. Litvinov, and T. Zheng. Multistage adaptive robust optimization for the unit commitment problem. *Operations Research*, 64(1):32–51, 2016.

[28] J. Mather, E. Bitar, and K. Poolla. Virtual bidding: Equilibrium, learning, and the wisdom of crowds. *IFAC-PapersOnLine*, 50(1):225–232, 2017. 20th IFAC World Congress.

[29] J. Mays, D.P. Morton, and R.P. O'Neill. Investment effects of pricing schemes for non-convex markets. *European Journal of Operational Research*, 2020.

[30] Monitoring Analytics, LLC. 2019 quarterly state of the market report for PJM: January through March. Available at `http://www.monitoringanalytics.com/reports/PJM_State_of_the_Market/2019.shtml`, May 2019.

[31] Monitoring Analytics, LLC. State of the market report for PJM. Available at `http://www.monitoringanalytics.com/reports/PJM_State_of_the_Market/2018.shtml`, March 2019.

[32] J.M. Morales, A.J. Conejo, K. Liu, and J. Zhong. Pricing electricity in pools with wind producers. *IEEE Transactions on Power Systems*, 27(3):1366–1376, 2012.

[33] G. Morales-España, J.M. Latorre, and A. Ramos. Tight and compact MILP formulation for the

thermal unit commitment problem. *IEEE Transactions on Power Systems*, 28(4):4897–4908, 2013.

[34] S. Newell, R. Carroll, A. Kaluzhny, K. Spees, K. Carden, N. Wintermantel, and A. Krasny. Estimation of the market equilibrium and economically optimal reserve margins for the ercot region. Available at `http://www.ercot.com/content/wcm/lists/143980/10.12.2018_ERCOT_MERM_Report_Final_Draft.pdf`, October 2018.

[35] R.P. O'Neill, P.M. Sotkiewicz, B.F. Hobbs, M.H. Rothkopf, and W.R. Stewart. Efficient market-clearing prices in markets with nonconvexities. *European Journal of Operational Research*, 164(1):269–285, 2005.

[36] J. Ostrowski, M. F. Anjos, and A. Vannelli. Tight mixed integer linear programming formulations for the unit commitment problem. *IEEE Transactions on Power Systems*, 27(1):39–46, 2012.

[37] A. Papavasiliou and Y. Smeers. Remuneration of flexibility using operating reserve demand curves: A case study of belgium. *The Energy Journal*, 38(6):105–135, 2017.

[38] M. Parvania and R. Khatami. Continuous-time marginal pricing of electricity. *IEEE Transactions on Power Systems*, 32(3):1960–1969, 2017.

[39] PJM. Enhanced price formation in reserve markets of PJM Interconnection, L.L.C. Available at `http://www.pjm.com/directory/etariff/FercDockets/4036/20190329-el19-58-000.pdf`, 2019.

[40] Potomac Economics. 2018 state of the market report for the ERCOT electricity markets. Available at `https://www.potomaceconomics.com/wp-content/uploads/2019/06/2018-State-of-the-Market-Report.pdf`, June 2019.

[41] W.B. Powell. A unified framework for stochastic optimization. *European Journal of Operational Research*, 275(3):795–821, 2019.

[42] G. Pritchard, G. Zakeri, and A. Philpott. A single-settlement, energy-only electric power market for unpredictable and intermittent participants. *Operations Research*, 58(4-part-2):1210–1219, 2010.

[43] D.A. Schiro, T. Zheng, F. Zhao, and E. Litvinov. Convex hull pricing in electricity markets: Formulation, analysis, and implementation challenges. *IEEE Transactions on Power Systems*, 31(5):4068–4075, 2016.

[44] R. Sioshansi, R. O'Neill, and S.S. Oren. Economic consequences of alternative solution methods for centralized unit commitment in day-ahead electricity markets. *IEEE Transactions on Power Systems*, 23(2):344–352, 2008.

[45] B. Wang and B.F. Hobbs. Real-time markets for flexiramp: A stochastic unit commitment-based analysis. *IEEE Transactions on Power Systems*, 31(2):846–860, 2016.

[46] C. Wang, P.B. Luh, P. Gribik, T. Peng, and L. Zhang. Commitment cost allocation of fast-start units for approximate extended locational marginal prices. *IEEE Transactions on Power Systems*, 31(6):4176–4184, 2016.

[47] S. Wong and J.D. Fuller. Pricing energy and reserves using stochastic optimization in an alternative electricity market. *IEEE Transactions on Power Systems*, 22(2):631–638, 2007.

[48] G. Zakeri, G. Pritchard, M. Bjorndal, and E. Bjorndal. Pricing wind: A revenue adequate, cost recovering uniform price auction for electricity markets with intermittent generation. *INFORMS Journal on Optimization*, 1(1):35–48, 2019.

[49] V.M. Zavala, K. Kim, M. Anitescu, and J. Birge. A stochastic electricity market clearing formulation with consistent pricing properties. *Operations Research*, 65(3):557–576, 2017.