Optimal time-and-level-of-use price setting for an energy retailer

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Abstract

This paper presents a novel price setting optimization problem for an energy retailer in the smart grid. In this framework the retailer buys energy from multiple generators via bilateral contracts, and sells it to a population of smart homes using Time-and-Level-of-Use prices (TLOU). TLOU is an energy price structure recently introduced in the literature, where the prices vary depending on the time and the level of consumption. This problem is formulated as a bilevel optimization problem, in which the energy retailer wants to set the prices that maximize the profit, anticipating the reaction of a population that wants to minimize the total cost. We explicitly consider the users load shifting preferences, their shifting decisions, and the level of consumption in the definition of the price structure. The optimization problem is reformulated as a single-level problem to be solved by off-the-shelf solvers. Computational experiments validate the performance of TLOU and show that the retailer's economical benefit is enhanced through the implementation of this type of demand response program, while providing savings for the consumers.

Keywords: Demand-Response, Price-Setting, Bilevel Optimization, TLOU, Smart Buildings, Smart Grid.

1. Notation

Sets:

$t \in T$: Time frames

$i \in I$: Generation levels

Parameters:

π_i^G : Generation cost of the level i (¢/kWh)

π_t^R : Ramping cost in time frame t (¢/kWh)

K_i : Generation capacity of resource i (kWh)

$\bar{Z}$: Ramping allowed at no cost (kWh)

Ψ : Maximum number of changes in the energy prices

Γ_t : Shifting cost in time frame t (¢/kWh)

D_t : Energy demand in time frame t (kWh)

C : Power capacity limit (kW)

Ω_t : Max additional energy consumption in time frame t (kWh)

Variables:

π_t^H : Higher energy price in time frame t(¢/kWh)

π_t^L : Lower energy price in time frame t (¢/kWh)

x_t : Energy bought by the aggregator at time frame t (kWh)

z_t : Energy ramping penalized in time frame t (kWh)

$$\alpha_t : \begin{cases} 1 & \text{There is a change in the energy price between periods } t \text{ and } t+1 \\ 0 & \text{Otherwise} \end{cases}$$

y_t^H : Energy consumption at higher price in time frame t (kWh)

y_t^L : Energy consumption at lower price in time frame t (kWh)

w_t^+ : Over-consumption with respect to energy demand in time frame t (kWh)

w_t^- : Under-consumption with respect to energy demand in time frame t (kWh)

v_t : Energy bought to the retailer's competitor in time frame t (kWh)

Auxiliary Variables:

λ_t^a, λ^b : Dual variables associated to equality constraints

$\mu_t^{(a,\dots,g)}$: Dual variables associated to inequality constraints

$\rho_t^{(a,\dots,g)}$: Binary variables to linearize the complementary slackness condition

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2. Introduction

Energy is a vital resource for modern societies. We can profit from electricity thanks to the power systems that generate, transport and deliver electricity on real-time basis. Ensuring this real-time balance between supply and demand is one of the most important tasks of the systems operators since it affects the operational costs, the user satisfaction, the reliability of the grid. In this context, demand response (DR) is one of the most effective ways to facilitate the aforementioned balance.

According to [1], DR is defined as "changes in electric usage by demand-side resources from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized ". The potential benefits of DR increase in systems with large penetration of energy renewable sources, since the presence of renewable generation requires a more flexible demand due to its intermittent and non controllable nature. Additionally, the continuous development of smart grids (SGs) allows a multidirectional communication among the multiple players in the power grid. This exchange of information enables better decisions and the possibility to movilize DR resources in a more effective way.

Typically, the DR programs can be classified in two groups: incentive-based programs and price-based programs [2]. In the incentive-based programs, the consumers commit to provide consumption reduction over a specified period of time in exchange of a financial incentive. In the price-based programs, the energy supplier or the system operator defines a set of varying energy prices. Normally, these prices reflect the generation marginal cost and therefore the load in the system during the day. Additionally, since there are different energy prices during the day, the end-users are encouraged to shift load in order to profit from the periods of time when the prices are cheaper.

Currently there are different price-based programs available in different jurisdictions around the world [3]. One of the most common pricing scheme is Time-Of-Use (TOU). A TOU program divides a day into several time windows and fixes energy prices for each time window. In a similar way, Critical Peak Pricing (CPP) programs identify a time window during the day in which a critical situation can arise in the system, and set a significantly higher price during the aforementioned time window. The surcharge is only applied if the critical situation occurs.

We can find other DR pricing schemes in the literature. The survey presented in [4] contains a comprehensive review of DR pricing programs and how these programs are integrated with different optimization-based approaches. Additionally, this survey classifies DR programs in task-scheduling and energy-management programs, based on the granularity of the consumption of energy. In the task-scheduling programs [5], the energy consumption is planned at the level of individual loads. On the other hand, the energy-management programs allocates a budget of energy to the end users who is in charge of manage his inner loads [6]. Finally the survey highlights the importance and the potential of game theory approaches in the proper configuration of pricing programs.

Authors in [7] proposed a TOU frame work in which different prices are determined for different segments of the population with specific characteristics. The goal is to better incentivize the end-users and to obtain the expected response and avoid rebound peaks.

Recently, the authors in [8] presented a new price scheme called Time-and-Level-of-Use. TLOU offers a similar time-windows-structure to that of the traditional TOU, and includes power capacity limits that enable multiple energy prices in each time window depending on the consumption level. The consumption below a capacity limit is charged at a cheaper price and the consumption above the capacity limit is charged at a more expensive price. Although TLOU was initially developed for the residential and commercial energy sectors, authors in [9] have demonstrated its benefits for industrial customers.

Defining energy prices is a key factor for having an effective DR implementation of programs mentioned before. A proper DR price-based program should bring benefits for the supply and the consumption sides. If the prices are not attractive enough users will not respond in an effective way and the system will not profit from the DR program. On the other hand, if the prices are very appealing, the users can shift more load than required by the grid creating rebound peaks that can worsen the initial state of the system. This effect can specially occur in the context of smart buildings in which multiple end-users make optimal decisions locally.

In this regard, bilevel optimization is an effective way to define energy prices [10]. Bilevel optimization is an optimization paradigm in which one problem is embedded within another. The outer optimization problem is commonly referred to as the leader's problem and the embedded optimization problem is commonly referred to as the follower's problem. The leader makes optimal decisions considering the follower's optimal reaction to the leader's decisions. This anticipation process can be specially useful to define DR pricing problems. Additionally, this type of problems are typically difficult to solve due it computationally complexity [11].

Bilevel optimization has been used in the DR energy context by several works in the literature [12]. The approach presented in [13] defines DR contracts between a wind power producer (leader) and a DR aggregator (follower). In this case the DR aggregator provides demand flexibility to the leader to handle the uncertainty generated by the wind behavior. This problem is defined as a bilevel problem and reformulated as a mixed integer linear problem (MILP).

In a similar way, [14] reformulates a bilevel problem in which the energy supplier (leader) desires to define prices that minimizes the peak of demand generated by a population (follower) who reacts to the prices. The stochastic nature of real time market prices is considered in the approached presented in [15]. In this case, the leader buys electricity in the market and define a dynamic pricing scheme to supply electricity to a population of space heaters.

The iterative method proposed in [16] allows the energy supplier (leader) to coordinate multiple residential users (followers) in order to flatten the aggregated energy consumption profile. The authors in [17] propose a bilevel problem in which a wind producer (leader) has the capability to affect the market. In this case the leader maximizes the expected profit subject to the operation of the day-ahead market clearing (follower).

A bilevel approach to define incentive-based DR programs is presented in [18]. In this work the leader sets premiums to encourage the users active participation.

The authors in [19] proposed a trilevel energy market that generates load shifting by the implementation of TOU prices. The trilevel structure involves, energy suppliers, aggregators and end-users. The end-users models are replaced by an explicit formula and the resulting bilevel problem is solved by using optimistic and semi-optimistic approaches.

Note that in the works previously mentioned the energy prices depend on the hour of the day but that neglect the user's consumption level. We propose a novel framework that explicitly considers the users' shifting capabilities and the level of consumption in the definition of DR price-based programs. The framework allows an energy supplier, or retailer, to determine the optimal price structure and the optimal energy prices in a TLOU environment. The resulting optimal prices help the retailer to maximize the profit, encouraging in a more effective way the users' load shifting, and avoiding undesired effects such as rebound peaks.

This paper is structured as follows. The proposed approach is described in Section 3, the computational experiments and results analysis are presented in Section 4, and the conclusion is given in Section 5.

3. Optimization problem

In this paper we define a framework to optimize the operations of an electricity retailer that sells energy using TLOU price structure. TLOU is a DR price-based program where energy prices vary depending on the time frame t and the amount of energy consumed. If the energy consumed in t is lower than the defined power capacity limit C_t the user pays the basic price π_t^L. If the user overshoots C_t he is charged π_t^H for the fraction of energy consumed beyond the limit. The TLOU price policy was introduced in [8] and [20] from the consumer perspective. These previous works analyze the benefits obtained by committing to consume electricity below a threshold. Additionally, [8] presents some evidence of potential benefits on the generation side such as a more homogeneous consumption, and reduction of rebound peaks.

The general operation of the energy retailer is presented in Figure 1. The retailer buys electricity from energy generators via bilateral contracts and sets the energy prices in order to supply a population of smart homes. The retailer makes decisions in a competitive environment in which the competitor offers a pre-defined energy flat rate. In other words, the population has the possibility to buy energy v_t from a retailer's competitor at a price Υ. This feature becomes specially significant due to the inelastic behavior of the electricity demand [21]. The presence of the competitor plays an important role in retailer's price setting since it represents a reference cost for the population and a boundary for the potential retailer's profit.

We assume that the retailer is able to estimate the aggregated population's demand profile D_t and the population's shifting limit profile $D_t + \Omega_t$. Gathering this information is possible nowadays thanks to the massive adoption of smart meters and the continuous development of the smart grid. This information is very relevant for the retailer since it allows a better planning of the inner operations and improve the financial results.

Under these conditions, the retailer wants to determine π_t^L, and π_t^H to maximize the profit knowing that the population will minimize its own costs and generate load shifting. This structure is suitable for the use of bilevel programming [12]. Although the capacity C_t is an important part of the definition of TLOU, we have assumed this parameter as given and constant during the day. We discuss the implications on the selection of C in Section 4.

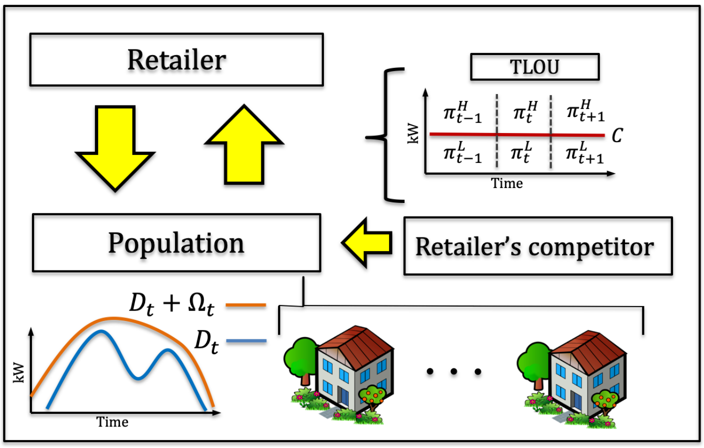

Figure 1: Retailer's operation

3.1. Shifting parameters

3.1.1. Shifting limit

The parameter Ω_t is defined as the additional energy consumption accepted by the population in time frame t. Consequently $D_t + \Omega_t$ represents the maximum consumption possible in each time frame.

We assume that the energy demand is always satisfied at the end of the day ($\sum_{t \in T}(y_t^H + y_t^L + v_t) = \sum_{t \in T} D_t$). Therefore if the population shifts load from time frame a ($y_a^L + y_a^H + v_a < D_a$ and $w_a^- > 0$), there exists as well at least one time frame b where the load is being shifted to ($y_b^L + y_b^H + v_b > D_b$ and $w_a^+ > 0$). The parameter Ω_t thus bounds the over-consumption in certain time frames as well as indirectly the under-consumption along the day. Notice that the population does not shift any load if $y_t^H + y_t^L + v_t = D_t, \quad \forall t \in T$.

3.1.2. Shifting penalty

The parameter Γ_t represents economically the discomfort generated from shifting. We define the shifting activity as the increase of consumption with respect to the demand (variable w_t^+). The impact of this increase on the user discomfort depends on when it occurs. According to [22], and [23], the price elasticity of energy demand is lower during off-peak periods than during on-peak periods. In other words, generating a change in the demand during off-peak hours requires a larger change in price. In our case, this price change is a discount that must account for the discomfort that the users perceive as result of the increase of their consumption during off-peak periods. Therefore, we assume that the discomfort is higher in the periods of time when the known demand is lower (i.e. the users do not want to use many electric devices or appliances).

Following this idea, we define the parameter Γ_t as:

$$\Gamma_t = \frac{\beta_t}{D_t}, \tag{1}$$

where β_t is a weight that accounts for the conversion of the dissatisfaction units to monetary units, ensuring the coherence in the order of magnitude with respect to the other cost parameters in the objective function.

Note that when D_t is equal to zero no consumption is possible, and Γ_t becomes infinity (replaced by a large enough cost in the optimization steps).

3.2. Price setting problem

In this section we define the bilevel model for the TLOU price setting problem. The leader (i.e the retailer) sets the energy prices considering the decisions made by the the follower (i.e. the population).

3.2.1. Leader formulation

As mentioned previously, the retailer seeks to maximize his profit by charging the users TLOU prices. The objective function (2) is composed by the income obtained from the energy sold to the users, the generation cost and the ramping cost. The generation cost is an increasing function that accounts for the type of generation: baseline, load following, and peaking plant. The ramping cost accounts for pronounced variations in the energy generation curve beyond the technical limitations of the generation technology. For example when the demand grows rapidly, the retailer could be forced to buy electricity from a third party supplier which generates a penalty π_t^R. In a similar way if the demand decreases at a great rate the retailer will face shut-down costs.

$$\max_{(\pi,x,z)} \sum_{t\in T}(\pi_t^H y_t^H + \pi_t^L y_t^L) - \sum_{i\in I}\sum_{t\in T}\pi_i^G x_{it} - \sum_{t\in T}\pi_t^R z_t \tag{2}$$

subject to

$$y_t^H + y_t^L = \sum_{i\in I} x_{it} + z_t, \quad \forall \ t \in T \tag{3}$$

$$x_{it} \leq K_i, \quad \forall \ i \in I, \ \ t \in T \tag{4}$$

$$\mid \sum_{i\in I} x_{it} - \sum_{i\in I} x_{i,t-1} \mid \leq \bar{Z}, \quad \forall \ \ t \in T : t > 1 \tag{5}$$

$$\mid \pi_t^L - \pi_{t+1}^L \mid \leq M\alpha_t, \quad \forall \ \ t \in T : t < \mid T \mid \tag{6}$$

$$\mid \pi_t^H - \pi_{t+1}^H \mid \leq M\alpha_t, \quad \forall \ \ t \in T : t < \mid T \mid \tag{7}$$

$$\sum_{t\in T} \alpha_t \leq \Psi \tag{8}$$

$$\sum_{n=t-N+1}^{t} \alpha_n \leq 1, \forall \ \ t \in [N, \ldots, \mid T \mid] \tag{9}$$

$$\pi_t^L \leq \pi_t^H, \quad \forall \ \ t \in T \tag{10}$$

$$\pi_t^H, \pi_t^L, x_t, z_t, \geq 0, \quad \alpha_t \in \{0,1\} \tag{11}$$

Constraint (3) ensures the balance between supply and demand for each time frame including the option to buy electricity to a third party. Constraint (4) is a capacity bound for each generator level.

Constraint (5) establishes the ramping limit. For the sake of simplicity, ramp-up and ramp-down events are assumed to have the same limits and costs. Constraints (6)-(8) ensure that the TLOU structure is user-friendly, allowing up to $\Psi+1$ different prices along the day for each level of price. The price structure must be simple to facilitate the end-users understanding and the acceptance of this novel DR program. The value of M is assumed to be large enough to account for the maximum allowed difference between two consecutive prices of the same level. In a similar way, constraint (9) guarantees that the price remains constant by at least N time frames. Constraint (6) can be replace by $\pi_t^L - \pi_{t+1}^L \leq M\alpha_t$ and $\pi_{t+1}^L - \pi_t^L \leq M\alpha_t$ in order to remove the absolute value. Constraints (5) and (7) are linearized in a similar fashion.

Constraint (10) ensures that the higher price is always greater or equal to the lower price. Finally, constraint (11) are the nonnegativity constraints and binary variables definition.

3.2.2. Follower formulation

The follower's problem is embedded in the constraints of the leader problem. This type of formulation is commonly known as 'optimistic' formulation since the leader can chose the most convenient solution in the case when the follower has multiple optimal solutions. In this particular context, we can assume that the population will not act deliberately against the retailer's interests and therefore the retailer's optimal solution can be optimistic. In this case the consumers aim to minimize the total cost from the objective function (12). The total cost is composed by the sum of the energy cost from the TLOU program, the shifting cost, and the cost of buying energy from the competitor.

$$\min_{(y,w)} \sum_{t\in T}(\pi_t^H y_t^H + \pi_t^L y_t^L) + \sum_{t\in T}\Gamma_t w_t^+ + \sum_{t\in T}\Upsilon v_t \quad (12)$$

subject to

$$y_t^H + y_t^L - w_t^+ + w_t^- + v_t = D_t, \quad \forall \ t\in T : (\lambda_t^a) \quad (13)$$

$$\sum_{t\in T}(y_t^H + y_t^L) + \sum_{t\in T} v_t = \sum_{t\in T} D_t \quad : (\lambda^b) \quad (14)$$

$$y_t^L \le C, \quad \forall \ t\in T : (\mu_t^a) \quad (15)$$

$$w_t^+ \le \Omega_t, \quad \forall \ t\in T : (\mu_t^b) \quad (16)$$

$$y_t^H \ge 0, \quad \forall \ t\in T : (\mu_t^c) \quad (17)$$

$$y_t^L \ge 0, \quad \forall \ t\in T : (\mu_t^d) \quad (18)$$

$$w_t^+ \ge 0, \quad \forall \ t\in T : (\mu_t^e) \quad (19)$$

$$w_t^- \ge 0, \quad \forall \ t\in T : (\mu_t^f) \quad (20)$$

$$v_t \ge 0, \quad \forall \ t\in T : (\mu_t^g) \quad (21)$$

Constraint (13) establishes the balance between the energy bought from the retailer and the competitor, and the demand. Constraint (14) accounts for the total demand satisfaction at the end of the day. Constraint (15) limits the energy consumption at the lower level of the TLOU. In a similar way, constraint (16) enforces maximum overconsumption. Finally, constraints (17)-(21) are the nonnegativity constraints.

3.2.3. MILP Reformulation

The resulting price setting optimization model is a bilinear bilevel mixed integer linear problem. For fixed leader variables, the follower problem becomes convex and is therefore suitable for classical reformulation as a single level mixed integer linear program (MILP). The follower problem can be replaced by its Karush-Kuhn-Tucker conditions [10]. The KKT conditions include the primal and dual feasibility constraints, the complementary slackness constraints, and the stationary constraints.

The primal feasibility is given by constraints (13)-(21). In a similar way the dual feasibility is ensured by:

$$\mu_t^a, \mu_t^b, \mu_t^c, \mu_t^d, \mu_t^e, \mu_t^f, \mu_t^g \ge 0, \quad \forall \ t\in T \quad (22)$$

Constraint (22) ensures the non-negativity for dual variables of the primal inequality constraints.

The complementary slackness is given by the following non-linear equations:

$$(C - y_t^L)\mu_t^a = 0, \quad (\Omega_t - w_t^+)\mu_t^b = 0, \quad y_t^H \mu_t^c = 0,$$
$$y_t^L \mu_t^d = 0, \quad w_t^+\mu_t^e = 0, \quad w_t^-\mu_t^f = 0, \quad v_t\mu_t^g = 0, \quad \forall \ t\in T$$

These equations can be transformed into linear constraints by using big-M as follows:

$$C(1-\rho_t^a) \ge C - y_t^L, \quad \mu_t^a \le M\rho_t^a, \quad \forall \ t\in T \quad (23)$$

$$\Omega_t(1-\rho_t^b) \ge \Omega_t - w_t^+, \quad \mu_t^b \le M\rho_t^b, \quad \forall \ t\in T \quad (24)$$

$$(D_t + \Omega_t - C)(1-\rho_t^c) \ge y_t^H, \quad \mu_t^c \le M\rho_t^c, \quad \forall \ t\in T \quad (25)$$

$$C(1-\rho_t^d) \ge y_t^L, \quad \mu_t^d \le M\rho_t^d, \quad \forall \ t\in T \quad (26)$$

$$\Omega_t(1-\rho_t^e) \ge w_t^+, \quad \mu_t^e \le M\rho_t^e, \quad \forall \ t\in T \quad (27)$$

$$D_t(1-\rho_t^f) \ge w_t^-, \quad \mu_t^f \le M\rho_t^f, \quad \forall \ t\in T \quad (28)$$

$$(D_t + \Omega_t)(1-\rho_t^g) \ge v_t, \quad \mu_t^g \le M\rho_t^g, \quad \forall \ t\in T \quad (29)$$

The big-M was replaced in all the constraints that do not contain dual variables (the ones on the left side), with upper bounds obtained directly from the structure of the problem to have a tighter formulation. For example, the maximum value for the expression $C - y_t^L$ in constraint (23), equals C when $y_t^L = 0$ since we know that $0 \le y_t^L \le C$ from constraints (15) and (18). Finally the stationary conditions are:

$$\pi_t^H - \lambda_t^a - \lambda^b - \mu_t^c = 0, \quad \forall \ t\in T \quad (30)$$

$$\pi_t^L - \lambda_t^a - \lambda^b + \mu_t^a - \mu_t^d = 0, \quad \forall \ t\in T \quad (31)$$

$$\Gamma_t + \lambda_t^a + \mu_t^b - \mu_t^e = 0, \quad \forall \ t\in T \quad (32)$$

$$-\lambda_t^a - \mu_t^f = 0, \quad \forall \ t\in T \quad (33)$$

$$\Upsilon - \lambda_t^a - \lambda_t^b - \mu_t^g = 0, \quad \forall \ t\in T \quad (34)$$

The bilinear terms in the objective function (2) of the leader's problem can be linearized by using the strong duality theorem. Equation (35) is the strong duality equation for the follower.

$$\sum_{t\in T}(\pi_t^H y_t^H + \pi_t^L y_t^L) + \sum_{t\in T}\Gamma_t w_t^+ + \sum_{t\in T}\Upsilon v_t$$
$$= \sum_{t\in T} D_t\lambda_t^a + (\sum_{t\in T} D_t)\lambda^b - \sum_{t\in T} C\mu_t^a - \sum_{t\in T}\Omega_t\mu_t^b \quad (35)$$

By reorganizing constraint (35) and replacing in (2), we obtain the new leader's objective function:

$$\max \quad \Big[\sum_{t\in T} D_t\lambda_t^a + (\sum_{t\in T} D_t)\lambda^b - \sum_{t\in T} C\mu_t^a - \sum_{t\in T}\Omega_t\mu_t^b$$
$$- \sum_{t\in T}\Gamma_t w_t^+ - \sum_{t\in T}\Upsilon v_t\Big] - \sum_{i\in I}\sum_{t\in T}\pi_i^G x_{it} - \sum_{t\in T}\pi_t^R z_t \quad (36)$$

4. Computational experiments

In this section we present and discuss the numerical results based on real life data. We first define the parameters and the instances generation process in Section 4.1. Next, we discuss the results of the bilevel problem in Section 4.2.

Instance	Population Total cost	Population Shifting cost	Population Energy cost	Competitor Income	Retailer Income	Retailer Op. cost	Retailer Profit
S^0	100.0	0.0	100.0	100.0	0.0	0.0	0.0
$S^{1,0}$	97.2	1.9	95.3	7.7	87.6	41.6	46.0
$S^{1,150}$	97.2	1.9	95.3	7.7	87.6	41.6	46.0
$S^{1,300}$	97.6	1.9	95.8	7.1	88.6	42.5	46.1
$S^{2,0}$	99.7	0.0	99.7	16.6	83.1	39.1	44.0
$S^{2,150}$	97.6	1.1	96.5	11.9	84.6	39.3	45.3
$S^{2,300}$	97.2	3.4	93.8	0.0	93.8	45.3	48.5

Table 1: Economical performance of the implementation of TLOU.

4.1. Parameters and instances definition

We consider a population of 1000 households. Their consumption profiles are based on the daily consumption profiles obtained from the independent system operator in the province of Ontario, Canada. The resulting demand profile will be used for all the experiments and it is equivalent to a daily demand of 6.8 MWh. Additionally we define two flexibility profiles Ω_t^f where $f \in \{1, 2\}$. This information is presented in Figure 2. We see that Ω_t^1 corresponds to a population that provides low shifting flexibility, while Ω_t^2 corresponds to a population that provides high shifting flexibility.

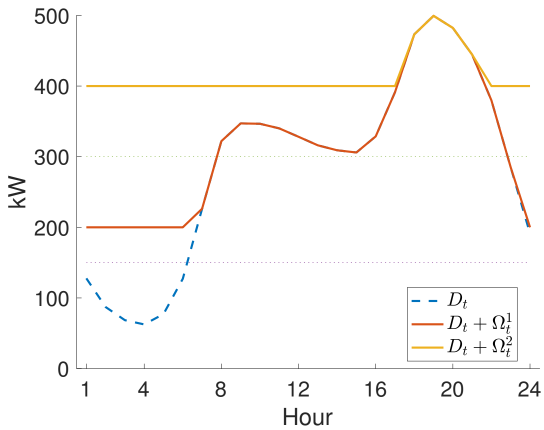

Figure 2: Demand and shifting capabilities

The generation cost $\pi_i^G : \{4, 7, 20\}$ is taken from [24], assuming that three types of generators are available: nuclear, hydro, and gas turbines. The generation capacities are equal to 150 kW for the two first resources. The last generation capacity is assumed to be infinite. Figures 2 to 7 shows these capacities (accumulated) as dotted lines. The retailer's competitor energy price is 12 ¢/kWh. Finally, we consider different TLOU capacities $C = \{0, 150, 300\}$. Note that having a TLOU with $C = 0$ is equivalent to have the traditional TOU.

4.2. Bilevel optimization results

We report in Table 1 the performance metrics (in dollars) for the 6 instances. The instances are denoted as $S^{f,C}$. The optimization models have 529 continuous variables, 192 binary variables, 1392 constraints, and are solved with Gurobi version 8.1 on a computer with 2.3 GHz Intel Core i5 CPU and 8 GB RAM. The average solution time is 0.65 seconds.

Table 1 presents the optimal values of the retailer and the population objective functions. The population's total cost (i.e. values of the follower's objective function) is the sum of the shifting cost and the energy cost. In a similar way, The energy cost is the sum of the cost of the energy bought from the competitor (competitor's income) and the cost of energy bought from the retailer (retailer's income). Finally, column "Profit" shows the retailer's profit (i.e. the value of the leader's objective function) which is the retailer's income minus the operational cost.

We use as reference the case in which the population must buy all the electricity from the competitor. In this case, the population pays the flat rate offered by the competitor, generating no shifting and the retailer has no participation. This instance is denoted as S^0 and its total cost is set to be equivalent to \$ 100. The results of the other instances were normalized accordingly.

We start the analysis with the population Ω^1 with low shifting flexibility. Increasing the capacity of TLOU does not have significant changes in the performance of the retailer and the population. More precisely, we observe the same TOU structure in instances $S^{1,0}$ and $S^{1,150}$ regardless of the increase of C. Although instance $S^{1,150}$ includes the possibility of having TLOU, the optimal decision is to offer a traditional TOU ($\pi_t^L = \pi_t^H, \quad \forall \quad t \in T$). Figure 3 shows for $S^{1,150}$, the shifting effect during the first hours of the day, where the energy bought from the retailer ($y_t^L + y_t^H$) increases up to the shifting flexibility ($D_t + \Omega_t^1$) as reaction to cheaper energy prices. These cheaper prices are offered as compensation during the time frames when the users have higher shifting cost Γ_t.

Let $\pi_{(t,t')}^L$ and $\pi_{(t,t')}^H$ be the optimal energy prices in the time window starting at time frame t until t'. The difference between $\pi_{(1,6)}^H$ and $\pi_{(7,24)}^H$ corresponds to the maximum value of Γ_t in the time frames 1 to 6. This encourages shifting towards this segment of the day since $\pi_{(1,6)}^H + \Gamma_t \leq \pi_{(7,24)}^H, \forall \quad t \in \{1, \ldots, 6\}$. This analysis can be easily extended for the other instances.

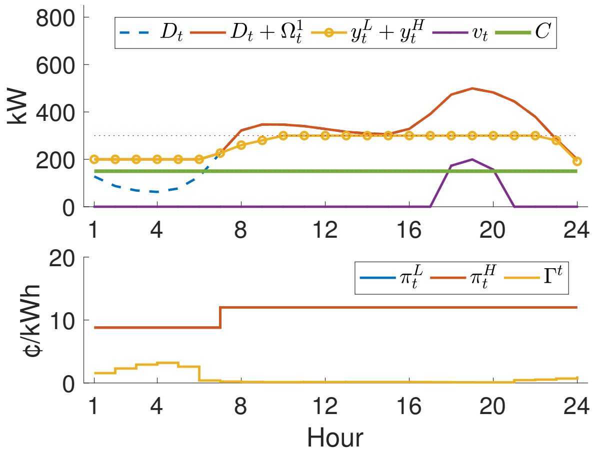

Figure 3: Results for instance $S^{1,150}$.

The retailer's competitor provides energy during the evening peak. This occurs because the retailer can not match the competitor's price when the third generation level is required (dotted line, $> 300kW$). Note that the highest optimal price obtained equals the price offered by the competitor (12 ¢/kWh).

For instance $S^{1,300}$, TLOU has a marginal effect on the results (Figure 4).

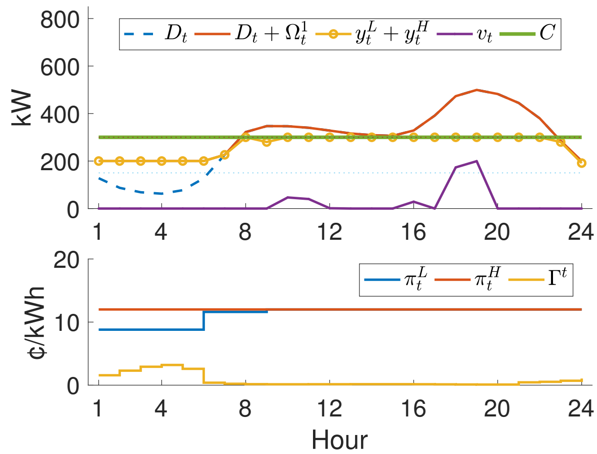

Figure 4: Results for instance $S^{1,300}$.

The retailer price $\pi^L_{(6,9)}$ is slightly cheaper than $\pi^H_{(6,9)}$. In this case $\pi^L_{(6,9)} + \Gamma_t \leq \pi^H_{(6,9)} \quad \forall \quad t \in \{6, \ldots, 9\}$.

We observe in Figure 4 a slightly increase of consumption in $t = 8$ with respect to previous instances. This additional shifting allows the retailer to serve a higher proportion of demand. The amount of energy provided by the competitor decreases for this instance. The price $\pi^H_{(1,5)}$ has no effect in the results since $D_t + \Omega^1_t < C, \quad \forall \quad t \in \{1, \ldots, 5\}$.

The lack of shifting flexibility reduces the potential impact of TLOU for the population Ω^1. The shifting effect on the generation side is bounded by the proper population shifting which allows the retailer to obtain the best performance with traditional TOU in two our of three cases. Nevertheless, our approach finds the most appropriate price structure allowing the retailer to capture most of the demand and the population to reduce the energy bill in about 4%.

Let us now comment the results for population Ω^2 with high shifting flexibility (Figures 5 to 7).

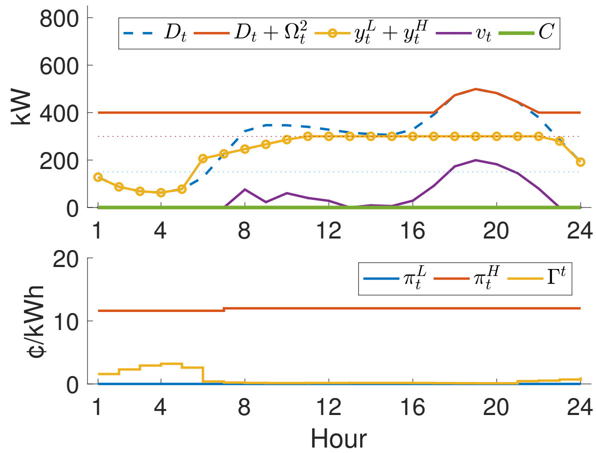

Figure 5: Results for instance $S^{2,0}$.

For instance $S^{2,0}$ (Figure 5) a significant fraction of the demand is satisfied by the retailer's competitor. In this case, the high shifting flexibility combined with $C = 0$ results into a very conservative decision by the retailer who offers a very small price incentive in the first time frames. There is consumption increase as result of shifting only in $t = 6$. Any larger incentive will result in a rebound peak reaching the population shifting limit and worsening the economical performance of the retailer.

The equality $\pi^H_{(1,6)} + \Gamma_6 = \pi^H_{(7,24)}$ encourages shifting towards $t = 6$ since Γ_6 is the lowest value in the time frames 1 to 6. Therefore, shifting makes no sense for the periods 1 to 5. This is a strict equality due to the optimistic feature previously explain in Section 3.2.2 . If both options (shifting or not) are equally good for the population, the model will always favor the leader (i.e. the retailer).

Table 1 shows that $S^{2,0}$ reports the lowest profit for the retailer and the highest total cost for the population. In fact, the total user cost is almost equal to the bound defined in the reference case.

The results for instance $S^{2,150}$ (Figure 6) show the effect of TLOU. The retailer offers TLOU prices in the time frames 1 to 6. The price $\pi^L_{(1,6)}$ encourages population to consume electricity until reaching C. Then $\pi^H_{(1,6)}$ performs as a penalty for consumption higher than C. The population only pays π^H_t in $t = 6$.

We can see that $\pi^L_{(1,6)} + \Gamma_t <= \pi^H_{(7,24)} \forall \quad t \in \{1, \ldots, 6\}$. In a similar way, $\pi^H_{(1,6)} + \Gamma_6 = \pi^H_{(7,24)}$. The later one encourages consumption beyond C only in $t = 6$ during the time window (1,6). This occurs since Γ_6 is the lowest shifting cost in the time window.

As shown in Table 1, for instance $S^{2,150}$ the prices defined for the TLOU by our approach report benefits for

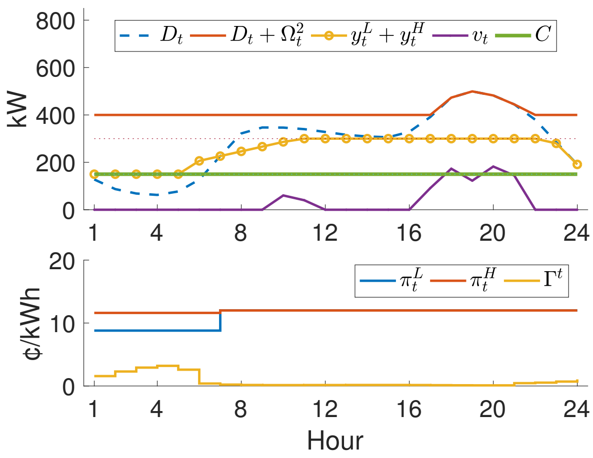

Figure 6: Results for instance $S^{2,150}$.

both parties. First, the retailer captures a higher fraction of the demand (competitor only makes $11.9) which results in a higher profit. Second, the population is rewarded in a more effective way (total cost of $97.6) which leads to a higher shifting ($1.1).

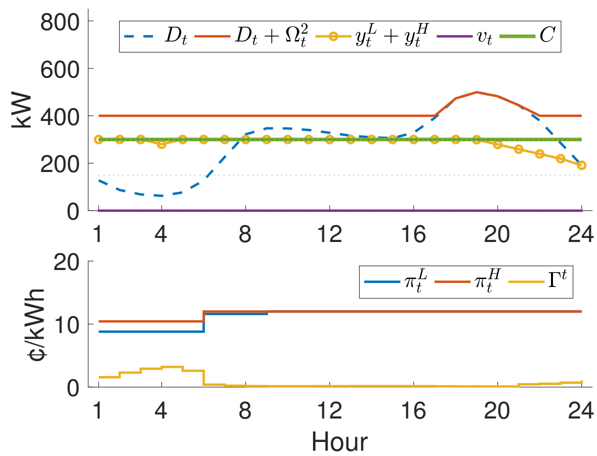

Figure 7: Results for instance $S^{2,300}$.

Finally Figure 7 puts into highlight the effect of TLOU for instance $S^{2,300}$. In this case $C = 300$ allows the retailer to satisfies completely the population demand. The retailer offers two different lower prices $\pi^L_{(1,5)}$ and $\pi^L_{(6,9)}$. Once again $\pi^L_{(1,5)} + \Gamma_t \leq \pi^H_{(6,24)} \quad \forall \quad t \in \{1, \ldots, 5\}$ and $\pi^L_{(6,9)} + \Gamma_t \leq \pi^H_{(6,24)} \quad \forall \quad t \in \{6, \ldots, 9\}$, which encourage shifting up to C in the first part of the day. On the other hand, $\pi^H_{(1,5)} + \Gamma_t \geq \pi^L_{(6,9)}$ and $\pi^H_{(1,5)} + \Gamma_t \geq \pi^H_{(6,24)}$ $\forall \quad t \in \{1, \ldots, 5\}$ since no consumption in the higher level is desired by the retailer in the time frames 1 to 5.

For instance $S^{2,300}$ the optimal TLOU prices achieve the highest performance for both retailer and population. The retailer obtains profit of $48.5 which represents an increase of about 10% with respect to instance $S^{2,0}$. On the other hand, the population obtains an energy cost of $93.8 which represents a reduction of 6.2% with respect to S^0.

In the previous experiments the TLOU were defined to mach directly the generation capacities of the retailer. Figure 8 presents the profits for a population $S^{2,C}$ where C changes from 0 to 500 with a step $\Delta C = 25$.

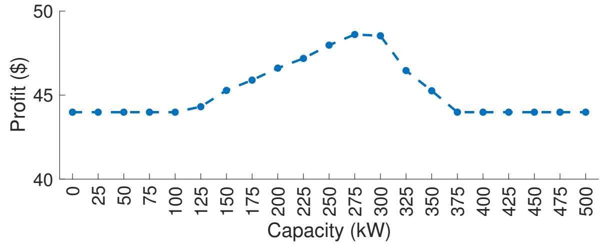

Figure 8: Results for $S^{2,C}$ with different TLOU capacities.

The maximum profit is reached for $C = 275$. We see the same cost in lower and higher capacities. Having insufficient capacity is equivalent to having a capacity greater than the highest possible shifting. In both cases the optimal decision is to implement a TOU or a flat rate. The ideal capacity limit should encourage enough shifting while keeping track of the total generation costs. This is not a trivial task since many factors must be considered such as the demand curve, shifting limits, and generation capacities. Although in this article C is considered as a parameter, we consider that in future work it must be included as an optimization variable. This will rise some non-linearities in the optimization problem that will have to be properly handled.

5. Conclusions

We have presented a new framework that allows the energy retailer to profit from a TLOU price structure in a smart grid context while generating savings for the consumers.

We formulated a bilevel optimization problem in which the retailer seeks to set TLOU energy prices to maximize the profit considering the shifting activities of a population that minimizes the total cost. The model considered the users level of consumption and their shifting preferences in the definition of the price structure. This optimization problem is reformulated as a single-level problem and solved by off-the-shelf solvers.

The use of this pricing scheme allows the retailer to improve the profit, encouraging the consumers to shift in a smarter way. This effect is more significant when the population is willing to shift more load, since TLOU controls rebound peaks and generates more homogeneous consumption profiles.

We have provided several instances to validate and analyze the performance of the implementation of this type of DR program. In instances of a population with lower shifting flexibility, TLOU does not report significant impact on the retailer's and the population performance. In fact, in some cases the optimal decision is to implement a

TOU structure. On the other hand, TLOU shows a significant improvement for both retailer and population when the population is willing to provide more shifting flexibility. In this case, the retailer profits from a more efficient operation of the system and is able to better incentivize the population, who in return shifts load in a smarter way.

A sensitive component of TLOU is the power capacity limit that draws the difference between the lower and the higher prices. Future work will include the parameter C as a variable, as well as multiple end-users that behave in a different way (multiple followers) and the incertitude in the shifting reaction to prices.

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References

[1] D. Kathan, R. Aldina, M. P. Lee, L. Medearis, P. Sporborg, M. Tita, D. Wight, and Wilkerson A. Assessment of Demand Response and Advanced Metering. *Department of Energy, Washington, DC*, 2012.

[2] Electric Power Annual 2014. Technical report, U.S. Energy Information Administration, Washington, DC, 2016.

[3] Pierluigi Siano. Demand response and smart grids: A survey. *Renewable and Sustainable Energy Reviews*, 30:461–478, 2014.

[4] J. S. Vardakas, N. Zorba, and C. V. Verikoukis. A survey on demand response programs in smart grids: Pricing methods and optimization algorithms. *IEEE Communications Surveys Tutorials*, 17(1):152–178, Firstquarter 2015.

[5] B. Celik, R. Roche, D. Bouquain, and A. Miraoui. Decentralized neighborhood energy management with coordinated smart home energy sharing. *IEEE Transactions on Smart Grid*, 9(6):6387–6397, Nov 2018.

[6] A. Khan, N. Javaid, M. N. Iqbal, N. Anwar, I. Haq, and F. Ahmad. Time and device based priority induced demand side load management in smart home with consumer budget limit. In *2018 IEEE 32nd International Conference on Advanced Information Networking and Applications (AINA)*, pages 874–881, May 2018.

[7] M. Muratori and G. Rizzoni. Residential demand response: Dynamic energy management and time-varying electricity pricing. *IEEE Transactions on Power Systems*, 31(2):1108–1117, 2016.

[8] Juan A Gomez-Herrera and Miguel F Anjos. Optimal collaborative demand-response planner for smart residential buildings. *Energy*, 161:370–380, 2018.

[9] José Luis Ruiz Duarte, Neng Fan, and Tongdan Jin. Multi-process production scheduling with variable renewable integration and demand response. *European Journal of Operational Research*, 2019.

[10] Benoít Colson, Patrice Marcotte, and Gilles Savard. Bilevel programming: A survey. *4OR*, 3(2):87–107, 2005.

[11] A. Kovács. On the computational complexity of tariff optimization for demand response management. *IEEE Transactions on Power Systems*, 33(3):3204–3206, 2018.

[12] Stephan Dempe, Vyacheslav Kalashnikov, Gerardo A Pérez-Valdés, and Nataliya Kalashnykova. Bilevel programming problems. *Energy Systems. Springer, Berlin*, 2015.

[13] Nadali Mahmoudi, Tapan K Saha, and Mehdi Eghbal. Modelling demand response aggregator behavior in wind power offering strategies. *Applied energy*, 133:347–355, 2014.

[14] Sezin Afsar, Luce Brotcorne, Patrice Marcotte, and Gilles Savard. Achieving an optimal trade-off between revenue and energy peak within a smart grid environment. *Renewable Energy*, 91:293–301, 2016.

[15] Marco Zugno, Juan Miguel Morales, Pierre Pinson, and Henrik Madsen. A bilevel model for electricity retailers' participation in a demand response market environment. *Energy Economics*, 36:182–197, 2013.

[16] A. Safdarian, M. Fotuhi-Firuzabad, and M. Lehtonen. A distributed algorithm for managing residential demand response in smart grids. *IEEE Transactions on Industrial Informatics*, 10(4):2385–2393, Nov 2014.

[17] L. Baringo and A. J. Conejo. Strategic offering for a wind power producer. *IEEE Transactions on Power Systems*, 28(4):4645–4654, Nov 2013.

[18] F. Salah, R. Henríquez, G. Wenzel, D. Olivares, M. Negrete-Pincetic, and C. Weinhardt. Portfolio design of a demand response aggregator with satisficing consumers. *IEEE Transactions on Smart Grid*, pages 1–1, 2018.

[19] Didier Aussel, Luce Brotcorne, Sébastien Lepaul, and Léonard von Niederhäusern. A trilevel model for best response in energy demand-side management. *European Journal of Operational Research*, 2019.

[20] Juan A Gomez-Herrera and Miguel F Anjos. Optimization-based estimation of power capacity profiles for activity-based residential loads. *International Journal of Electrical Power & Energy Systems*, 104:664–672, 2019.

[21] Nina Boogen, Souvik Datta, and Massimo Filippini. Dynamic models of residential electricity demand: Evidence from switzerland. *Energy Strategy Reviews*, 18:85–92, 2017.

[22] Andreas Knaut and Simon Paulus. Hourly price elasticity pattern of electricity demand in the german day-ahead market. EWI Working Paper 16/07, Köln, 2016.

[23] Talat S Genc. Measuring demand responses to wholesale electricity prices using market power indices. *Energy Economics*, 56:247–260, 2016.

[24] Electric Power Annual 2017. Technical report, U.S. Energy Information Administration, Washington, DC.