Risk-Neutral and Risk-Averse Transmission Switching for Load Shed Recovery

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***Abstract*—Maintaining an uninterrupted supply of electricity is a fundamental goal of power systems operators. However, due to critical outage events, customer demand or load is at times disconnected or shed temporarily. While deterministic optimization models have been devised to help operators expedite load shed recovery by harnessing the flexibility of the grid's topology (i.e., transmission switching), an important practical issue that remains unaddressed is how to cope with the uncertainty in demand encountered during the recovery process. This paper introduces two-stage stochastic programming models to deal with uncertain load parameters with known probability distribution, and one of these also incorporates conditional value-at-risk (CVaR) to measure the risk level of unrecovered load shed. The models are implemented using a scenario-based approach where the objective is to maximize load shed recovery in the bulk transmission network by switching transmission lines and performing other corrective actions (e.g., generator re-dispatch) after the topology is modified. The benefits of the proposed stochastic programming models are quantified via comparisons with a deterministic mean-value model, using the IEEE 118-bus test case. Experiments and discussion highlight how the proposed approach can serve as an offline contingency analysis tool.**

***Index Terms*—Load shed recovery, stochastic programming, conditional value-at-risk, transmission switching.**

NOMENCLATURE

A. Sets

G	Generators.
$\dot{G}$	Generators out of service due to a contingency.
K	Transmission lines.
$\hat{K}$	Transmission lines in service.
$\bar{K}$	Transmission lines out of service.
$\dot{K}$	Transmission lines out of service due to a contingency.
$\mathcal{N}$	Buses.
Ω	Possible scenarios.

B. Decision variables

P_g^ω	Output of generator $g \in G$ in scenario $\omega \in \Omega$.
P_k^ω	Flow through line $k \in K$ in scenario $\omega \in \Omega$.
s_k	Switching for line $k \in K$ (1: switch, 0: otherwise).
θ_n^ω	Angle of bus $n \in \mathcal{N}$ in scenario $\omega \in \Omega$.
u_n^ω	Unfulfilled demand at bus $n \in \mathcal{N}$ in scenario $\omega \in \Omega$.

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C. Parameters

$P_g^{\max}, P_g^{\min}$	Max and min generation for generator $g \in G$.
B_k	Susceptance of line $k \in K$.
M_k	Large constant ("M-value") for line $k \in K$.
$P_k^{\max}, P_k^{\min}$	Max and min power flow for line $k \in K$.
$LS_{\dot{G}\cup\dot{K}}$	Load shed tied to contingency set $\dot{G} \cup \dot{K}$.
d_n^ω	Demand at bus $n \in \mathcal{N}$ in scenario $\omega \in \Omega$.
$\theta^{\max}, \theta^{\min}$	Max and min bus angle difference.
r	Line switching limit.
λ	Risk coefficient.
α	Confidence interval.

I. INTRODUCTION

MAINTAINING a reliable source of power is a fundamental goal of large electrical networks. However, disruption events continuously pose risks to the system and can ultimately trigger power outages, i.e., short-term or long-term electric power disconnections of certain areas/customers. Power outages caused more than \$1 trillion worth of damage due to weather from 1980 to 2014 [1] and annual financial losses of \$44 billion in 2015 [2]. Their incidence can drastically reduce quality of life of large populations. Indeed, each U.S. electricity user experienced an average of 1.3 electrical outage events, or equivalently 250 minutes of disconnected service, in 2016. Duration of major outage events varied from 27 minutes in Nebraska to 6 hours in West Virginia [3].

Imbalances in total energy supply and demand are among the main causes of power outages, at times forcing electric utilities to interrupt service in certain areas by automatic and/or manual disconnection or shedding of loads. The Federal Energy Regulatory Commission (FERC) requires transmission/independent system operator (TSO/ISO) to recover from an outage and replenish reserves within 90 minutes [4]; otherwise, longer waits may cause more severe power outages and damages. Hence, soon after an outage occurs, effective corrective actions must be executed—e.g., activating reserves, performing system-specific measures, etc. [5]—to mitigate power disconnections. These actions must also adapt to potential changes in the demand profile. In pursuit of these directives, this paper leverages the inherent flexibility of the bulk transmission network through the systematic switching of transmission lines in/out of service while accounting for uncertainty in demand.

In order to lower generation costs while ensuring grid stability, FERC has pursued a policy of electric transmission incentives to help encourage proper infrastructure development and congestion reduction. As recently as spring of 2019, FERC sought comments from public and stakeholders for suggested improvements to this policy, and "flexible transmission system operation" was targeted for its capability to incorporate resilience in the face of changing circumstances. Indeed, various scientific works have demonstrated that transmission line switching (TS) could engender numerous benefits as a corrective action including relieving line overloads [6, 7], mitigating voltage violations [8, 9, 10], and recovering load shed (LS) following an emergency [11, 12]. It is also recognized as a promising mechanism for improving system reliability when facing critical contingencies [13]. Readers can refer to Hedman et al. [13] for a thorough introduction of the standardized TS methodology, otherwise known as topology control. Before its standardization through mathematical models, TS had been performed by operators based on historical experience recorded in operations manuals (e.g., [14]). Nowadays, topology control is implemented in practice but only on a limited basis, partly on account of the continuing need for adequate decision-making tools.

In last decade, researchers have been focusing on developing the mathematical optimization models to determine the best TS actions. O'Neill et al. [15] introduced a deterministic model that systematically applies TS into the transmission network and studied the economic benefits of optimal TS actions under appropriate market rules. They also provided general linear (nonlinear) formulations of an auction model. On the other hand, Hedman et al. [16] not only considered the economic benefits of TS but also its ability to maintain reliability and stability. The authors proposed a mixed integer program for N-1 DC optimal dispatch with TS, and showed that TS can reduce generation costs while ensuring N-1 reliability. Note that both O'Neill et al. [15] and Hedman et al. [16] embedded TS decisions as binary variables within the well-known linear programming formulation for DC optimal power flow problem (DCOPF) [17], thereby leading to mixed binary formulations with the objective to minimize generation costs. Recently, Escobedo et al. [12] explored the concept of *load shed recovery* (LSR) and proposed a static transmission switching model for the DC optimal load shedding recovery (DCOLSR) problem with deterministic demand. Their computational results showed that by reconfiguring the topology of the transmission lines via TS, the resulting load shed from line and/or generator contingencies could be reduced significantly.

In the aforementioned papers, it was assumed that demand is a fixed known quantity. However, out of various factors of uncertainties in power systems (see [18]), electric power demand is among the most important and difficult to forecast, especially during emergency conditions. This results in uncertainty in the requirements of the system, which could impact the validity of decisions obtained from deterministic models in practice. Therefore, to make these decision making tools more effective, it is important to consider the demand uncertainty in power system problems, as overestimation or underestimation of uncertainties may lead to excessively high costs and component damages.

Different methods, such as robust optimization and probability approaches, have been applied in different cases to handle uncertainties in power systems. Dehghanian and Kezunovic [19] studied the optimal transmission switching problem with uncertain load and renewable generation. They incorporated the two-point estimation method with deterministic DCOPF to handle the uncertainties and approximate the probability distribution function of generation costs. The optimal decision was made after considering both generation costs and system reliability. Zhang and Li [20] studied both DCOPF and ACOPF with load uncertainty and utilized chance-constrained programming to handle uncertainty, which sought a balance between reliability and total cost efficiency of the power system. Summers et al. [21] formulated a multistage stochastic chance-constrained DCOPF problem. They also provided a conservative convex approximation of the chance constraints. Rabiee et al. [22] proposed a stochastic model to minimize energy procurement costs in the DCOPF. They considered uncertainties in load, wind power generation, and energy price. They compared results of different models (e.g., two stage stochastic programming and stochastic programming with variance index and with CVaR) and concluded that the risk-averse solution is more conservative, which leads to a worse objective value. Phan and Ghosh [23] proposed a two-stage stochastic programming model to handle the generation uncertainty in ACOPF. Their model considered the power extracted from all conventional generators as the first stage, and the re-dispatch of generators following the realization of uncertainties as the second stage. Note that the majority of the aforementioned papers concentrate primarily on economic objectives for normal operation conditions.

This paper considers an emergency-related objective to maximize the amount of load shed that can be recovered following contingency-triggered imbalances between supply and demand while incorporating uncertainty in the demand profile. It is important to remark that minimizing total unmet demand could be of vital importance to maintain services of public facilities such as rescue stations, police stations, and hospitals during severe disaster events. This work provides a decision making methodology that would help TSOs in the pursuit of this directive. The proposed method could serve as an offline contingency analysis tool for aiding decision makers to balance maximal gains and risk during the recovery process. A prospective application could be used to identify crucial elements in the system and corresponding corrective actions for maintaining effective and up-to-date operator manuals.

This paper builds on the previous work of [12] by proposing stochastic load shed recovery models (S-LSR) to handle the uncertain demand and incorporate conditional value-at-risk (CVaR) for measuring the risk level of unrecovered load. No work in the existing literature has considered the uncertain demand and the risk of unrecovered LS in DCOLSR. In deterministic models, data parameters are assumed to be fixed and known, whereas in reality, the parameters are mostly uncertain. Therefore, S-LSR seeks a solution which is feasible for a representative variety of the parameter values, while optimizing the expected LSR over the sample space (the set of all possible values of the parameters) of the random variables associated with the uncertain demand. A decision maker utilizing the expectation of LSR is referred to as risk-neutral. In contrast, when facing serious disasters, it is reasonable to make more conservative decisions to avoid extreme scenarios of uncertain demand. Such decision-making models are referred to as risk-averse models. In this paper, we also utilize CVaR measure within S-LSR so that a decision maker can avoid the scenarios with large unrecovered LS in the worst-case quantile. In the rest of the paper, the risk-neutral and CVaR-based risk-averse models (presented in sections II-A and II-B) are referred to as RN-LSR and CVaR-LSR, respectively.

The main contributions of this research are the introduction of two stochastic models to handle demand uncertainties and minimize load shedding: RN-LSR and CVaR-LSR. Secondly, we demonstrate the value of these two models by comparing them to related deterministic models. A takeaway from these comparisons is that the optimal solution of RN-LSR recovers more LS in comparison to deterministic models. Furthermore, although the LS recovered using CVaR-LSR is almost as good as RN-LSR, its optimal solution avoids the worst-case quantile with large LS. Thirdly, we perform analyses that support the usage of the proposed methodology as an offline contingency tool. For instance, the models can be configured to suggest a sequence of individual switching operations.

The rest of this paper is organized as follows: In Section II, we provide two models for S-LSR: RN-LSR and CVaR-LSR. In Section III, we introduce the criterion to evaluate the proposed models and a deterministic mean-value model. In Section IV, we illustrate the computational setup and in Section V, we provide computational results for the IEEE 118-bus test case. We compare the impact of different contingencies, number of scenarios, risk coefficient, and analyze the decision making process. In Section VI, we conclude the work and discuss potential future research directions.

II. Stochastic Optimal Load Shed Recovery

In this section, we present stochastic optimization models to maximize LSR (or minimize LS) using TS actions; these models are extensions of the deterministic DCOLSR model of Escobedo et al. [12]. In [12], it is assumed that the demand at each bus after an emergency situation is the same as the demand during normal conditions. However, during emergency conditions, demand is especially hard to forecast. Ignoring the uncertainty in load may lead to overloading or under-loading of lines [24]. In order to handle this uncertainty in demand, we propose two-stage stochastic programming models where uncertain demand is defined using a random variable with known probability distribution. The first one is referred to as the risk-neutral model (denoted by RN-LSR), where the risk preference of the decision makers is not considered, while the second model specifies their risk preferences by incorporating CVaR for a more conservative decision (denoted by CVaR-LSR). For more details about general risk-neutral and risk-averse stochastic models, readers can refer to [25].

A. Risk-Neutral Stochastic DC Optimal Load Shed Recovery With Transmission Switching

In RN-LSR, we make first-stage transmission-line switching decisions, called here-and-now decisions, without any knowledge of demand at each bus after the occurrence of a contingency. Thereafter, when the demand is revealed in the future, we make operational decisions, referred to as second-stage decisions, which include adjusting bus phase-angles and generator dispatch levels, with the goal to maximize LSR. In other words, we seek a solution (i.e., corrective actions) that maximizes the expected value of the objective function (i.e., LSR) for the known probability distribution followed by the uncertain demand. Formally, given a set of generators G, a set of transmission lines K, a set of buses $\mathcal{N}$, and sets of out-of-service generators and lines due to a contingency ($\dot{G} \subseteq G$ and $\dot{K} \subseteq K$, respectively), RN-LSR is formulated as follows:

$$\max \left\{ \mathbb{E}_\xi[f(s,\omega)] : \sum_{k \in K} s_k \leq r, s \in \{0,1\}^{|K|} \right\} \tag{1}$$

where binary variable s_k, $k \in K$, denotes the switching action for line k made before the uncertain demand, defined by random vector ξ with sample space Ω, is realized, and data parameter r denotes the switching limit, i.e., the maximum number of transmission line switches allowed. Problem (1) is referred to as the first-stage problem and we denote the set of first-stage feasible solutions, i.e., TS actions, by S. The objective is to maximize LSR associated with a given contingency $(\dot{G}, \dot{K})$ and therefore, we set $\mathbb{E}_\xi[f(s,\omega)] = LS_{\dot{G}\cup\dot{K}} - \mathbb{E}_\xi[Q(s,\omega)]$ where $LS_{\dot{G}\cup\dot{K}}$ (a constant for a given contingency) captures the total LS due to the contingency from the forecasted demand without any corrective (TS or re-dispatching) operations, and $\mathbb{E}_\xi[Q(s,\omega)]$ provides the expected unmet demand for a given topology configuration (controlled by TS decisions s_k, $k \in K$). The latter is computed by solving the following linear programs (also referred to as the second-stage problems) for $(s,\omega) \in (S,\Omega)$:

$$Q(s,\omega) = \min \sum_{n \in N} u_n^\omega \tag{2}$$

Subject to:

$$\theta^{min} \leq \theta_n^\omega - \theta_m^\omega \leq \theta^{max}, \qquad (m,n) \in K, m,n \in \mathcal{N} \tag{3}$$

$$\sum_{\forall k(n,\dots)} P_k^\omega - \sum_{\forall k(\dots,n)} P_k^\omega + \sum_{\forall g(n)} P_g^\omega = d_n^\omega - u_n^\omega, n \in \mathcal{N} \tag{4}$$

$$P_k^{min}(1-s_k) \leq P_k^\omega \leq P_k^{max}(1-s_k), \qquad k \in \hat{K} \tag{5}$$

$$B_k(\theta_n^\omega - \theta_m^\omega) - P_k^\omega + s_k M_k \geq 0, \qquad k \in \hat{K} \tag{6}$$

$$B_k(\theta_n^\omega - \theta_m^\omega) - P_k^\omega - s_k M_k \leq 0, \qquad k \in \hat{K} \tag{7}$$

$$P_k^{min} s_k \leq P_k^\omega \leq P_k^{max} s_k, \qquad k \in \bar{K} \tag{8}$$

$$B_k(\theta_n^\omega - \theta_m^\omega) - P_k^\omega + (1-s_k) M_k \geq 0, \qquad k \in \bar{K} \tag{9}$$

$$B_k(\theta_n^\omega - \theta_m^\omega) - P_k^\omega - (1-s_k) M_k \leq 0, \qquad k \in \bar{K} \tag{10}$$

$$P_g^{min} \leq P_g^\omega \leq P_g^{max}, \qquad g \in G \setminus \dot{G} \tag{11}$$

$$0 \leq u_n^\omega \leq d_n^\omega, \qquad n \in \mathcal{N} \tag{12}$$

$$P_k^\omega = P_g^\omega = 0, \qquad k \in \dot{K}, g \in \dot{G}, \tag{13}$$

where variable u_n^ω denotes the unmet demand at bus n for $n \in \mathcal{N}$. In Problem (2)-(13), the objective is to minimize the total unmet demand at all buses, u_n^ω for $n \in \mathcal{N}$, for a known realization of demand at each bus, $\{d_n^\omega\}_{n \in \mathcal{N}}$ for $\omega \in \Omega$, and TS actions $s \in S$, while ensuring that the following constraints hold. Constraints (3) set lower and upper limits on the difference between bus angles of adjacent buses for all lines $k \in K$ with incident nodes n and m. Constraints (4) are node balance constraints, which ensure that the total power flow into a node/bus, including the power generation, $n \in \mathcal{N}$ equals the total power flow out of the node, including the demand consuming at that node. Notice that the partial unfulfilment of demand at each bus is allowed by introducing a non-negative continuous variable u_n^ω. Constraints (5) and (8) set lower and upper limits on power flow through each transmission line $k \in K$. These constraints ensure that if a line k is in service before performing the TS operation, that is $k \in \hat{K}$, and is set to be switched by the first-stage TS solution, i.e., $s_k = 1$ (or if a line k is out of service before performing the TS operations, that is $k \in \bar{K}$, and is not switched during TS actions, i.e., $s_k = 0$), then this line is open and thus does not allow any power flow. Otherwise, power flow is allowed to pass through this line. Constraints (6)-(7) and (9)-(10) incorporate Kirchhoff's laws, where B_k is the susceptance of line $k \in K$ and M_k is large enough to ensure the above constraints are satisfied regardless of the corresponding bus angles when a transmission line is switched to closed and open, respectively. Constraints (11) set the lower and upper limits on power generated by generator g which is not in contingency status, i.e., $g \in G \setminus \dot{G}$ and Constraints (12) make sure that the unmet demand at each bus $n \in \mathcal{N}$ does not exceed the demand at the bus. Constraints (13) restrict power flow of lines and output of generators, respectively, impacted by the contingency.

B. Risk-Averse Stochastic DC Optimal Load Shed Recovery With Transmission Switching

In RN-LSR, the objective is to provide a topology configuration that maximizes the total expected LSR over the set of all possible demand realizations/scenarios for a given contingency. However, for a given contigency, even though the total expected LSR is large, it is possible that for some of the randomized scenarios, the unmet demand is extremely large while for the others it is extremely small, i.e., the unmet demand corresponding to each scenario is distributed with large variance. For such contingencies, power system operators (or decision makers) would likely opt for TS actions that avoid the risk of occurrence of outcomes with large unmet demand for certain scenarios. Therefore, in this paper, we also incorporate a risk measure, CVaR, into RN-LSR to specify the risk preference of the decision makers and avoid the risk of outcomes within the worst-case quantile.

1) Conditional Value-at-Risk: CVaR, also known as the mean excess loss or tail Value-at-Risk (VaR), is a popular function of risk measurement, which was first introduced by [26]. It has been widely applied in the finance industry [26] but used infrequently in the energy sector. Statistically, it represents the weighted average (i.e., expected value) of the losses for a given confidence interval of the distribution of total gain. Put otherwise, by optimizing CVaR, we minimize the risk of incurring huge losses. Formally, CVaR is defined as follows [26],

$$\text{CVaR}_\alpha(X) = \mathbb{E}(X | X \geq \text{VaR}_\alpha(X))$$

where $\text{VaR}_\alpha(X)$ represents the expected worst loss of random variable X within a given confidence interval α. In this paper, since demand is assumed to have a known distribution, the corresponding outcome, i.e., unrecovered LS or unmet demand, also follows a certain distribution. We apply CVaR to measure the risk of outcomes in α-quantile by minimizing the expectation of unrecovered LS over this quantile.

2) Risk-averse Stochastic Formulation: We present a risk-averse two-stage stochastic programming model with CVaR which is defined as follows:

$$\min_{s \in \mathcal{S}} \left\{ \mathbb{E}_\xi[f(s,\omega)] + \lambda \times \text{CVaR}_\alpha(f(s,\omega)) \right\} \tag{14}$$

where $\text{CVaR}_\alpha(f(x,\omega))$ is the cost of risk, i.e., the expectation over the α-quantile of the distribution of outcomes $f(s,\omega)$ and λ denotes the exchange rate of the cost for risk. Here, $f(s,\omega)$ is defined the same as in RN-LSR for $s \in \mathcal{S}$ and $\omega \in \Omega$. For a finite sample space, i.e., $|\Omega| < \infty$, Noyan [27] presented a reformulation for general risk-averse two-stage stochastic linear programs where both stages have only continuous variables. We utilize this result to reformulate (14) and derive its equivalent formulation as follows:

$$\begin{aligned} \max\ & (1+\lambda) LS_{\dot{G} \cup \dot{K}} - \sum_{\omega \in \Omega} p_\omega \sum_{n \in N} u_n^\omega - \lambda \eta + \frac{\lambda}{1-\alpha} \sum_{\omega \in \Omega} p_\omega z_\omega \\ \text{s.t.}\ & z_\omega \geq \sum_{n \in N} u_n^\omega - \eta, \quad \omega \in \Omega \\ & (3)-(13) \text{ hold }, s \in \mathcal{S}, \eta \in \mathbb{R}, z_\omega \in \mathbb{R},\ \omega \in \Omega. \end{aligned}$$

In other words, solving problem (14) is equivalent to solving aforementioned linear programming.

III. Evaluation of Stochastic Models for LSR

In this section, we present a criterion to evaluate RN-LSR and CVaR-LSR, in comparison to a deterministic mean-value (DMV) model for the S-LSR problem. The DMV model is a simple way to handle uncertainty in LSR problem by considering the deterministic DCOLSR model of Escobedo et al. [12] with the means of the uncertain demands used as the fixed demand parameters. It is defined as follows:

$$\min \left\{ LS_{\dot{G} \cup \dot{K}} - Q(s, \mathbb{E}_\xi(\omega)) : s \in \mathcal{S} \right\}. \tag{15}$$

where $Q(\cdot,\cdot)$ is defined by (2). Let the optimal switching solution of above mean-value problem be $\hat{s}$. Even though this decision does not consider random variations of demand, in reality, the decision makers will have to make the second-stage decisions (after the realization of uncertain demand) based on $\hat{s}$ by solving:

$$f(\hat{s},\omega) = \min \quad LS_{\dot{G} \cup \dot{K}} - Q(\hat{s},\omega)$$

for realized scenario $\omega \in \Omega$. Since we do not know which scenario will be realized, we calculate the expectation over all scenarios of the optimal value of the second stage problem, i.e., $\mathbb{E}_\xi[f(\hat{s},\omega)]$, to evaluate the performance of $\hat{s}$. In general, compared to calculating the foregoing expectation, solving RN-LSR and CVaR-LSR takes longer time. The ensuing results help determine when it is worthwhile to solve the S-LSR models proposed in this paper instead of the deterministic problem.

A. Evaluation of Risk-Neutral LSR Solution

The concept of value of stochastic solution (VSS) is used to quantify the quality of the optimal solution of RN-LSR [25]. It measures the benefits of solving RN-LSR in comparison to the DMV problem and is calculated as follows:

$$\text{VSS} = \mathbb{E}_\xi[f(s,\omega)] - \mathbb{E}_\xi[f(\hat{s},\omega)].$$

Since the optimal solution $\hat{s}$ of the DMV model is a feasible solution for RN-LSR, VSS is always non-negative. If $\text{VSS} > 0$, the optimal solution of RN-LSR is strictly better, since it provides larger expected LSR. Otherwise, the optimal solutions of RN-LSR and DMV are of the same quality.

B. Evaluation of Risk-Averse CVaR-LSR Solution

Although, theoretically, the expected LSR obtained by RN-LSR is always better than the total LSR using CVaR-LSR, we cannot conclude that the solution of CVaR-LSR is "bad" compared to the solution of RN-LSR as the worst-case scenarios in α-quantile are avoided by solving RN-LSR. Therefore, instead of comparing the optimal objective values, we compare the optimal solutions of these two stochastic models. If the optimal solutions of RN-LSR and CVaR-LSR coincide, then clearly the optimal solution of RN-LSR not only recovers the most LS, but also avoids the occurrence of outcomes with large unmet demand for certain scenarios. Otherwise, the solution of CVaR-LSR model is more conservative and it cannot recover as much LS as the solution of RN-LSR.

IV. Computational Experiments

In this section, we discuss the scenario-generation process and the setup for our computational experiments, which were performed to evaluate the efficiency and effectiveness of the proposed stochastic models, RN-LSR and CVaR-LSR.

A. Instance Generation using the IEEE 118-Bus Test System

In the computational experiments, we consider the IEEE 118-bus test case, which contains 19 generators and 186 transmission lines, with stochastic demand variations. The total base load of IEEE 118-bus test case is 4519 MW. We follow the settings in [12], where the initial states of all non-faulty lines are closed and the emergency line and generator ratings are 125% of the normal ratings. Also, we borrow the concept of the contingency list (CL) to specify the contingencies considered in the test instances. The CL is a collection of valid contingencies periodically examined by contingency analysis programs which in practice contain all N-1 contingencies and a number of N-2 contingencies. The featured experiments consider a diverse selection of the latter type, specifically double generator failures, mixed generator and non-radial line failures, and double non-radial line failures, denoted by (G-2), (G-1+L-1), and (L-2), respectively. From each foregoing contingency category, we select the five contingencies that result in the highest LS before any generation re-dispatch and TS actions are executed. For these contingencies, generator re-dispatch alone (no TS) is insufficient to recover the unserved demand in full. Refer to Table I for details, where each row lists a contingency and columns list its name, category, and components out of service. As an example, C.1 represents a contingency of type G-1+L-1, namely the outage of generator 14 and line 12.

For each instance, the demand $\{d_n^\omega\}_{n\in\mathcal{N}}$ is randomly drawn from a normal distribution with the original deterministic demand $\{d_n^0\}_{n\in\mathcal{N}}$ as the mean and 20% of the mean as the standard deviation. This is similar to the parameters chosen in [28], where the authors presented data of electric consumption during disasters for 41 countries.

TABLE I
A List of Contingencies

Name	Category	Components out of Service
C.1	G-1+L-1	Generator 14 and Line 12
C.2		Generator 13 and Line 43
C.3		Generator 13 and Line 40
C.4		Generator 13 and Line 12
C.5		Generator 13 and Line 1
C.6	G-2	Generator 13 and Generator 14
C.7		Generator 13 and Generator 11
C.8		Generator 13 and Generator 1
C.9		Generator 13 and Generator 12
C.10		Generator 13 and Generator 4
C.11	L-2	Line 12 and Line 142
C.12		Line 12 and Line 152
C.13		Line 142 and Line 154
C.14		Line 142 and Line 156
C.15		Line 154 and Line 156

B. Computational Experiment Framework

For a given stochastic IEEE 118-bus test instance, we perform five experiments: RN-LSR-DE, CVaR-LSR-DE, RN-LSR-BD, CVaR-LSR-BD, and DMV-EX, each of which is represented by the name of the model and its solution method applied. In RN-LSR-DE and CVaR-LSR-DE, we solve the deterministic equivalent of RN-LSR and CVaR-LSR, respectively, i.e., large-scale block-angular structured mixed binary programs, using CPLEX 12.8, with its default settings. While in RN-LSR-BD and CVaR-LSR-BD, we solve the RN-LSR and CVaR-LSR instances, respectively, using Benders' decomposition routine (BD) of CPLEX 12.8. In DMV-EX, we obtain the optimal solution of DMV model, i.e., $\hat{s}$, solve $f(\hat{s},\omega)$ for $\omega\in\Omega$, and calculate $\mathbb{E}_\xi[f(\hat{s},\omega)]$ as the optimal value. All experiments were performed on a 16-core Intel Xeon 3.2GHz machine with 32GB RAM running with Windows 10. The code was written in C++ using callable library of CPLEX 12.8. The time limit, which is denoted as TL, was set to three hours.

V. Results and Analysis

In this section, we report and analyze the following statistics: time taken to solve RN-LSR-DE, CVaR-LSR-DE, RN-LSR-BD, and CVaR-LSR-BD, optimal LS (denoted by $\texttt{OptLS}$) by solving RN-LSR and CVaR-LSR, percentage of LSR (i.e., $(\text{LS}_{\dot{G}\cup\dot{K}}-\texttt{OptLS})/\text{LS}_{\dot{G}\cup\dot{K}}\times 100\%$), VSS, and the number of instances with positive VSS (#$\text{VSS}_{>0}$). For each $\dot{G}\cup\dot{K}$ in Table I, the value of $\text{LS}_{\dot{G}\cup\dot{K}}$ was obtained from [12].

A. Computational Results for Risk-Neutral Stochastic LSR

We evaluate the effectiveness of the RN-LSR model to handle demand uncertainty after the occurrence of contingencies considered in Table I. We report the results of our first set of experiments in Table II, where each row is the average over results of five randomly generated instances. For each instance, we generate 10 scenarios ($|\Omega|=10$) and perform experiments with different contingencies and different switching limits, i.e., $r\in\{2,3,4,5\}$. Columns labeled as T-DE and T-BD provide the time taken (in seconds) by RN-LSR-DE and RN-LSR-BD, respectively.

Table II omits five individual contingencies: C.6, C.11, C.13, C.14, and C.15. For instances associated with C.6, either the optimal value of RN-LSR and DMV models are same or RN-LSR cannot be solved within the time limit. For the other four contingencies, the solution of RN-LSR does not suggest that any line should be switched; in other words, the current/given topology is the optimal topology. However, we emphasize that these contingencies are not trivial as even though no line is suggested being switched, the unmet demand cannot be recovered entirely only by re-dispatching generators. One possible reason for such results is when two lines are out of service, they might cause network islanding with high probability. Thus, not switching lines may be optimal.

TABLE II
RESULTS OF COMPUTATIONAL EXPERIMENTS FOR RN-LSR MODEL

	Risk-Neutral LSR					
Name	r	T-DE	T-BD	%LSR	VSS	#VSS$_{>0}$
C.1	2	1802	146	84.57	0	0
	3	3501	TL	85.63	0.93	2
	4	5334	TL	86.81	1.01	2
	5	6861	TL	89.00	4.23	4
C.2	2	1650	89	97.40	1.73	2
	3	3485	TL	98.97	2.26	3
	4	4245	TL	99.26	25.90	5
	5	7259	TL	99.47	27.42	5
C.3	2	992	90	93.74	0	0
	3	3984	TL	97.41	29.09	5
	4	5588	TL	98.04	21.67	5
	5	7366	TL	98.35	31.86	5
C.4	2	3025	130	95.13	3.76	2
	3	3777	TL	97.97	13.43	5
	4	3319	TL	98.72	39.35	5
	5	7949[1]	TL[1]	99.04[1]	30.59[1]	4
C.5	2	1692	117	96.91	14.19	5
	3	4398	TL	99.17	26.63	5
	4	4182	TL	99.56	30.69	5
	5	4484	TL	99.65	32.93	5
C.7	2	945	82	96.43	4.16	4
	3	4606	TL	96.91	10.57	5
	4	1881	TL	96.95	2.92	5
	5	1457	TL	96.95	10.98	5
C.8	2	1955	132	91.74	0	0
	3	3059	TL	93.82	0	0
	4	5286	TL	94.33	3.90	4
	5	7612[1]	TL[1]	94.66[1]	8.34[1]	3
C.9	2	1456	117	95.33	0	0
	3	6366[1]	TL[1]	96.60[1]	13.52[1]	3
	4	2729	TL	96.48	10.29	5
	5	4084[1]	TL[1]	96.89[1]	16.12[1]	4
C.10	2	1806	162	92.43	4.13	1
	3	4142	TL	95.64	0	0
	4	5083	TL	96.77	0.89	1
	5	6278[1]	TL[1]	97.42[1]	17.96[1]	4
C.12	2	166	9	95.67	7.19	5
	3	82	9	95.72	25.82	5
	4	235	11	95.72	24.56	5
	5	78	10	95.72	26.52	5

The deterministic equivalent (DE) of 195 out of 200 test instances in Table II are solved to optimality in 3384 seconds on average, and 84 out of above 195 instances are solved to optimality by BD method in 1134 seconds on average. For instances with $r = 2$ (50 instances), BD is 14.4 times faster than solving DE; whereas BD could solve only 14 instances with $r \geq 3$ within the three-hour time limit. This is because the number of first stage feasible solutions increases rapidly with the increase in r, thereby increasing the number of iterations in the BD algorithm. Nonetheless, when the switching limit increases, LSR% and VSS also increase for a fixed contingency. This suggests a trade-off between LS recovery and computational time for the operators to take into account.

[1]Average over four instances because one of the five instances was not solved within the time limit (TL)

From Table II, we observe that for 139 out of 195 test instances, the optimal solution of RN-LSR model provides a better optimal value compared with the LS obtained using the DMV model. More specifically, the average VSS for these instances is 18.27 MW. Interestingly, RN-LSR and DMV models provide the same solution with the same unmet demand for 56 instances, mainly for instances with $r = 2$ (31 instances). This shows that when switching more lines is permitted, RN-LSR provides a better solution for most cases. In addition, we notice that the average %LSR is 93.59%, which implies that a large portion of the LS incurred by the contingency is recovered by performing the optimal operations suggested by RN-LSR.

The main take-away from this experiment is that RN-LSR is better than DMV, since it provides a higher LSR than DMV for most instances (i.e., #VSS$_{>0}$). Most of the LS is recovered by performing the optimal operations provided by RN-LSR model. The LSR% is at least 84% and up to 99%.

B. Computational Results of Risk-Averse CVaR-Based LSR

To compare RN-LSR and CVaR-LSR models, we perform the second set of experiments on instances with a fixed number of line switches, $r = 3$, C.12 contingency from the (L-2) category, i.e., failure of line 12 and line 152, 200 scenarios of the uncertain demand, i.e., $|\Omega| = 200$, and different values of confidence interval α and risk-coefficient λ. In Table III, we provide results of our computational experiments where each row in this table corresponds to a randomly generated test instance. Columns labeled as RN-Sol and CVaR-Sol provide the optimal TS operations provided by RN-LSR and CVaR-LSR, respectively. The total LS after performing the optimal TS and re-dispatching operations of RN-LSR is reported in column labelled as `OptLS`. We observe that the total LS for RN-LSR and CVaR-LSR are the same but with different optimal solution for all the instances, except for one—in particular, for the instance with $\alpha = 0.85$ and $\lambda = 0.2$, the optimal value of CVaR-LSR is 3.52 MW greater than that of RN-LSR. This demonstrates that CVaR-LSR optimal solutions not only avoid the outcomes with large LS, but also provide the least expected LS over all possible scenarios. Even the average %LSR for CVaR-LSR is 96.31%, which implies that on average 96.31% of the LS is recovered by performing risk-averse TS and re-dispatching operations provided by CVaR-LSR, after the failure of line 12 and line 152.

Columns labeled as T-BD provide the time taken to solve RN-LSR and CVaR-LSR instances using BD algorithm. RN-LSR-DE and CVaR-LSR-DE could not be solved within the three-hour time limit and, thus, their times are not reported; three instances of CVaR-LSR were similarly not solved via BD. RN-LSR instances and CVaR-LSR instances solved by the

TABLE III
RESULTS FOR COMPUTATIONAL EXPERIMENTS WITH DIFFERENT RISK COEFFICIENT AND CONFIDENCE INTERVAL

α	λ	Risk-Neutral LSR			Risk-Averse CVaR-Based LSR		
		RN-Sol	OptLS	T-BD	CVaR-Sol	T-BD	%LSR
0.85	0.2	154 157 160	8.1	283	154 157 165	6301	96.7
	0.5	154 157 160	11.6	599	154 163 165	9722	95.3
	0.8	154 160 161	5.7	605	154 157 163	8732	97.7
	3	154 157 169	8.4	10^3	154 160 161	8949	96.6
	7	154 163 165	10.9	659	154 159 161	4959	95.6
	10	154 161 169	10.3	650	154 159 161	10^4	95.8
0.9	0.2	154 163 165	11.2	617	154 157 163	6615	95.5
	0.5	154 163 165	7.8	445	154 161 169	5983	96.8
	0.8	154 160 161	10.6	549	154 161 173	8509	95.7
	3	154 162 163	8.7	687	-	TL	-
	7	154 157 165	11.7	450	-	TL	-
	10	154 157 163	11	721	154 162 163	7497	95.6
0.95	0.2	154 161 162	8.2	483	139 154 161	5285	96.7
	0.5	154 160 161	7.7	545	154 159 161	9173	96.9
	0.8	154 161 173	7.3	778	154 157 169	7711	97.1
	3	154 161 169	10.6	364	154 157 169	9131	95.7
	7	132 154 157	6.8	582	73 154 161	5742	97.3
	10	154 161 162	7.6	531	-	TL	-

TABLE IV
COMPUTATIONAL RESULTS FOR RN-LSR AND CVAR-LSR MODELS

	Risk-Neutral LSR					Risk-Averse LSR		
Instance	Sol	OptLS	BD	VSS	%LSR	Sol	OptLS	BD
C.7.1.200	51	117.3	559	0	91	64	119.1	3662
	64	102.7	551	0.2	92.1	64	102.7	3423
C.7.1.300	51	91.6	665	0	93	64	92.5	8686
	51	100.8	901	0	92.3	64	100.8	8219
C.7.2.200	64 112	42.3	6123	5.6	96.8	115 141*	47.8*	TL
	64 112	38.3	5443	3.8	97	115 141*	42.1*	TL
C.7.2.250	64 112	40.9	7628	4.6	96.9	64 115*	71.8*	TL
	64 112	42.9	5911	5.6	96.7	51 115*	78.4*	TL
C.7.2.300	64 112	44.4	5515	3.8	96.6	51 115*	80.8*	TL
	64 112	48.6	5070	6.5	96.3	115 116*	55.1*	TL
C.7.2.400	-	-	TL	-	-	-	-	TL
	64 112	42.4	9626	5.9	96.7	51 115*	79.6*	TL
C.12.1.200	154	19.2	256	0	92.2	154	19.2	2023
	154	17.9	166	0	92.8	154	17.9	1906
C.12.1.400	154	15.5	519	0	93.7	154	15.5	6850
	154	16.4	456	0	93.4	154	16.4	7125
C.12.2.200	154 161	9.6	535	6.2	96.1	154 161	9.6	4587
	154 161	7.85	314	6.2	96.8	154 161	7.85	6184
C.12.2.250	154 161	10.4	580	6.5	95.8	154 161	10.4	8347
	154 161	10.57	568	7.1	95.7	154 161	10.57	10^4
C.12.2.300	154 161	12.6	314	7.5	94.9	154 161*	12.6*	TL
	154 161	9.4	380	6.7	96.2	154 161*	9.4*	TL
C.12.2.400	154 161	9.9	370	6.9	96	134 154*	41.9*	TL
	154 161	7	715	6	97.2	154 161*	7*	TL

BD approach took on average 586 seconds and 7643 seconds, respectively. Put otherwise, solving RN-LSR using BD was 12 times faster than solving CVaR-LSR using BD.

The most important result from the above experiment is that the optimal TS operation of CVaR-LSR not only avoids the outcomes with large LS, the optimal LS provided by CVaR-LSR is almost the same as RN-LSR for most instances. However, the time taken to solve CVaR-LSR is longer than RN-LSR. Hence, decision makers should consider the benefits between the time taken to solve the problem and their risk preference when executing these TS operations.

C. Computational Results using Larger Number of Scenarios

We perform a third set of experiments to evaluate the impact of increasing the number of scenarios on RN-LSR and CVaR-LSR and report results in Table IV. We denote an instance category by C.$\mu.r.\tau$ where $C.\mu$ is the contingency listed in Table I, r is the switching limit, and τ is the number of scenarios ($|\Omega| \in \{200, 250, 300, 400\}$). For CVaR-LSR model, we set the risk-coefficient $\lambda = 7$, which represents the decision maker is risk-averse, and confidence interval $\alpha = 0.95$. Each row in Tables IV corresponds to a randomly generated instance. Table IV reports the optimal TS solution and OptLS provided by RN-LSR and CVaR-LSR. For instances which could not be solved within the time limit, the table reports the best known solution and upper bound, denoted by "*"; in case these are not known, "-" is used.

Using BD, 23 of 24 RN-LSR instances were solved in 2091 seconds, and 12 of 24 CVaR-LSR instances were solved in 5926 seconds, on average. The average %LSR for solvable RN-LSR instances is 95%, and the RN-LSR and CVaR-LSR provide the same optimal TS solution for 9 of 12 instances. For the remaining three instances, we observe that although CVaR-LSR solution increases LS (0.9 MW on average), it avoids the worst quantile of outcomes with high LS.

The most salient conclusion is that as the number of scenarios increases, the optimal solutions by solving both RN-LSR and CVaR-LSR converge. This indicates when number of scenarios is large enough, i.e., $|\Omega| = 200$, the optimal solution remains stable.

D. Practical Implications

In practice, operators may prefer to switch one line at a time, with sufficient time between switches to avoid engineering issues including transient stability (e.g., see [29]). Assuming each switching operation takes approximately 10-15 minutes [12], as many as five line switches could be putatively carried out within 90s minutes, i.e., the duration required by FERC to return the system to normal state and replenish reserves following a contingency [4]. This explains the choice of $r \leq 5$ for the experiments summarized in Table II. A more stringent requirement by some TSOs (e.g., PJM [14]) states that the system must return to normal state within roughly 30 minutes [14]. Thus, we restrict $r \leq 3$ in the experiments associated with Tables III and IV. From results in Table IV, we observe that the solution does not change with different random seeds when number of scenarios is large enough, LS decreases significantly from $r = 1$ to $r = 2$, and VSS > 0 for almost all instances with $r \geq 2$. This suggests that TSOs/ISOs should consider switching at least two lines, based on the optimal solution provided by experiments with $|\Omega| \geq 200$. If the stochastic risk-averse problem, CVaR-LSR, and the stochastic risk-neutral problem, RN-LSR, provide the same optimal solution, then this solution not only provides the most load shed recovery, but also avoids the worst case scenarios, and hence is a stable optimal solution. Moreover, our results may also provide the optimal sequence of switching, which minimizes the LS during switching operations. From our results in Section IV, we observe that for some instances, as r increases, the set of optimal solutions with switching limit $r - 1$, i.e., S_{r-1}, is a subset of the optimal solution with switching limit r, i.e., $S_{r-1} \subset S_r$. Thus, the optimal r-th switch is given by $S_r \setminus S_{r-1}$. For the optimal solutions with some contingencies which do not have above properties, this methodology suggests operators to set $r = 1$ and solve RN-LSR or CVaR-LSR as desired up to a predetermined $r_{\max}$-times. The optimal TS solution provided by solving RN-LSR with $r = 1$ $r_{\max}$-times is also a feasible solution to RN-LSR with $r = r_{\max}$, and it will also provide a switching sequence which minimizes the LS during switching operations.

The proposed methodology can be utilized as an offline contingency analysis tool to provide guidance for TSO/ISO to minimize load shedding during an emergency. Its benefits were illustrated via a limited selection of nontrivial contingencies involving the outage of two components. A wider selection could be tested offline to provide optimal TS operations that could supplement TSO operator manuals. Compared to previous deterministic LSR models, this method considers demand uncertainty and avoids the worst-case scenarios of large outages. Thus, it provides more comprehensive and representative decision support for emergency situations.

VI. Conclusion

We introduced stochastic risk-neutral and risk-averse transmission switching models for LSR under uncertain demand. Compared to previously studied deterministic models with the mean of uncertain demand as their demand inputs, the stochastic models not only reduced expected LS, but also avoided the possible scenarios with extremely large LS. Most of the stochastic variants of IEEE 118-bus test cases were solved within a reasonable time, and the percentage of LS that was recovered by TS operations and re-dispatching (suggested by the stochastic models) ranges from 84.5% to 99%. As the accompanying experiments and discussion evince, this method could serve as an offline contingency analysis tool. Its outputs could help maximize the recovery of LS incurred by failures of generators and/or transmission lines effectively.

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