Optimal Residential Users Coordination Via Demand Response: An Exact Distributed Framework

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***Abstract*—This paper proposes a two-phase optimization framework for users that are involved in demand response (DR) programs. In a first phase, responsive users optimize their own household consumption, characterizing not only their appliances and equipments but also their comfort preferences. Subsequently, the aggregator exploits in a second phase this preliminary non-coordinated solution by implementing a coordination strategy for the aggregated loads while preserving users' privacy. The second phase relies on the solution of a bilevel program in which the aggregator profit is maximized in the upper level while ensuring that the aggregated residential users do not incur any economic or comfort losses by participating in the demand response program. The lower level models the users' reaction to the aggregator requests. As major complicating aspects, the resulting bilevel problem features nonlinear terms and lower-level binary variables. This challenging problem is addressed by a mixed-integer linear single-level reformulation and the application of an exact solution technique based on Dantzig-Wolfe decomposition. Simulations with up to 10,000 residential users illustrate the advantages of the proposed two-phase framework in terms of users' privacy, computational efficiency, and scalability.**

***Index Terms*—Dantzig-Wolfe decomposition, demand response, bilevel optimization, users coordination.**

NOMENCLATURE

The symbols used in this paper are listed below, except for those related to the Dantzig-Wolfe Decomposition (DWD), which are defined in Section III.

A. Sets

N	Set of users.
T	Set of time intervals.
$\mathcal{F}_n$	Feasible space of all variables for user n.

B. Parameters

$\hat{C}^{DA}$	Day-ahead energy cost under no coordination [$].
$\hat{C}^{Dev}$	Load deviation cost under no coordination [$].
D_n^{max}	Maximum discomfort level for user n.
E_t^{des}	Target value of the aggregated energy consumed at time t [W].
$\hat{E}_{n,t}^{p}$	Electric power consumed by user n at time t under no coordination [W].
$\hat{f}^A$	Aggregator cost under no coordination [$].
$\hat{f}^U$	Users' aggregated profit under no coordination [$].
$\hat{f}_n^U$	Profit of user n under no coordination [$].
Δ_t	Duration of time interval t [h].
κ^{max}	Maximum incentive paid to users [$].
κ_n^{min}	Minimum incentive paid to user n [$].
λ_t^{DA}	Day-ahead market price for energy at time t [$/Wh].
λ_t^{s}	Selling price of energy for users at time t [$/Wh].
λ_t^{TOU}	Time-of-use (TOU) tariff for users at time t [$/Wh].
μ_t^{Dev}	Price to adjust the load deviation at time t [$/Wh].

C. Variables

$C_{n,t}^{p}$	Cost of user n for purchasing energy at time t [$].
$C_{n,t}^{chp}$	Cost of user n related to the combined heat and power (CHP) operation at time t [$].
C_t^{DA}	Day-ahead market cost at time t under coordination [$].
C_t^{Dev}	Load deviation cost at time t under coordination [$].
C_n^{ev}	Daily hybrid vehicle fuel cost for user n [$].
C_n^{U}	Total cost for user n [$].
$E_{n,t}^{p}$	Electric power purchased by user n at time t [W].
$E_{n,t}^{s}$	Electric power sold by user n at time t [W].
e_t^+, e_t^-	Positive and negative power imbalance at time t [W].
$\boldsymbol{e}^A$	Vector of decision variables for the aggregator.
f^A	Aggregator's total cost under coordination [$].
f_n^U	Profit of user n under coordination [$].
$R_{n,t}^{s}$	Revenue of user n for selling energy at time t [$].
$\boldsymbol{y}_n^U$	Vector of decision variables for user n.
$\boldsymbol{y}_n^A$	Vector of decision variables for the aggregator associated with user n.
κ_n	Incentive paid to user n [$].

D. Function

$g_n^U(\boldsymbol{y}_n^U)$	Discomfort level of user n.

I. INTRODUCTION

COORDINATING distributed energy resources is a challenge for electric utilities given the large number of resources involved [1]. If the residential sector comes into play,

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the challenge is even greater due to issues with householders' privacy and constraints on the social utility [2]. A possible way to maximize social benefits is incentivizing users to change their consumption patterns to fit a load shape via Demand Response (DR) programs. Literature reviews of DR are available in [3] and its recent application worldwide in the residential sector is reported in [4].

To increase the contribution of residential users in the output of DR strategies, aggregators may play a crucial role [5] by managing DR programs for a group of users. Many entities in the power market can act as an aggregator [1], including transmission and distribution system operators, retailers or third parties [1]. Here, we assume that an aggregator is a demand service provider as done in [6].

DR programs assume that users can change their electricity consumption in response to either price signals or incentive payments, giving rise to the so-called price-based or incentive-based DR programs, respectively. Assuming that the goal of any aggregator is the maximization of its profit, flattening the electricity consumption curve is a beneficial option for reducing the costs associated with peak-price hours [7]. When a price-based DR program is implemented, users optimize their consumption by responding to the price signals issued by the aggregator. However, if they all simultaneously schedule their loads to low-price periods, new rebound peaks may appear [8]. Furthermore, for the particular case of domestic electric water heaters, price-based DR programs are shown in [9] to be insufficient to change the user behavior.

Alternatively, to increase the number of users participating in a DR scheme, incentive-based DR programs emerge to coordinate the consumption of users. In these programs, users are paid to shift and/or to reduce their consumption over a given period of time. In particular, Direct Load Control (DLC) is reported as the most used program in the residential sector, while Curtailable Load (CL) predominantly involves medium and large users [4]. However, users participating in these programs have reported that incentives are not attractive enough and their comfort levels were negatively impacted [10]. Hence, from the users' perspective, DR requests with a lower impact on users' comfort or larger incentives are necessary. Excessive incentives, in turn, may result in unacceptable financial losses for the aggregator. In addition to the mentioned disadvantages, computational burden and privacy issues for DLC were also reported in [11].

Within the context of residential DR programs, some references adopt a centralized framework relying on the direct use of available commercial solvers based on mathematical programming [12], [13]. However, both privacy and scalability issues arise in these centralized approaches [14]. On the other hand, decentralized approaches decomposing the problem into subproblems improve the scalability but not necessarily the privacy needs if users' data are shared between subproblems. In practice, the problem should be decomposed but also solved in a distributed mode so that each user optimizes its own consumption using its particular home energy management system (HEMS). Thus, in order to both mitigate privacy concerns and improve scalability a distributed approach should be adopted.

The existing literature on distributed methods for demand response is classified in [6] according to the mathematical complexity of the models for appliances and whether their time coupled operation is characterized. References [11], [15], and [16] present mathematical programming models neglecting the time coupling of users' appliances. This modeling aspect was considered in [17], [18], wherein appliances were characterized by simplistic mixed-integer linear programming formulations. Reference [19] improves upon [17], [18] not only by integrating specific models for each appliance with their time-coupling constraints but also by using practical models. Additionally, from a user's perspective, [19] shows that more detailed models increase the users' profitability while guaranteeing their levels of comfort. Using detailed models, a distributed approach relying on a heuristic procedure rather than using a commercial optimization software is proposed in [20]. Although such a heuristic showed reasonable optimality gaps for small instances that were also solvable via exact methods, optimality cannot be guaranteed in general.

Apart from the user-related modeling aspects, coordination strategies can be found in the literature focusing on the decrease in electricity peak load while also satisfying users' needs. One strategy is to formulate the DR problem as a single-level problem in which a trade-off between users' costs or/and a utility discomfort function and a third party's performance criterion (revenues, community's living comfort, etc.) are imposed. As described in [2] can be classified in the following three categories according to the trade-off considered: (i) approaches in which a single-objective optimization is carried out considering the perspective of either users or the third party but not both [19]; (ii) models considering both perspectives through a weighted-sum multiobjective optimization [6], [10], [11], and (iii) approaches relying on Pareto-front multiobjective optimization and simplified models to reduce the computational burden when a large number of users are considered [21], [22].

Falling outside the categories in [2], reference [7] proposed a two-phase framework. In the first phase, each user individually minimizes its electricity cost and computes the associated costs and consumption profile, which are both the inputs for the next phase. The second phase involves the solution of a bilevel problem [23] with an upper level modeling the aggregator's DR strategy, and $|N|$ lower levels representing the response of the $|N|$ residential users to this strategy. In this phase the aggregator announces a modified aggregated load profile for each householder. Then, each user re-optimizes its consumption for each period of time considering both the new energy limits and a measure of the aggregated demand variability. Iteratively, the modified consumption patterns are sent back to the aggregator until no further improvement of the aggregated demand profile is experienced.

Although bilevel problems are $\mathcal{NP}$-hard [23], a vast number of applications have been addressed in the literature. Among them, some approaches consider a Stackelberg game in which the upper and lower level problems are solved iteratively until reaching an equilibrium solution [11], [17]. Notwithstanding, these works consider neither specific appliance models nor the CL option.

Motivated by the above-described state of the art, this work contributes to the literature by proposing a practical and exact distributed framework for an aggregator that coordinates a group of residential users. The proposed tool allows the joint maximization of aggregator's and users' profits while preserving users' comfort and privacy. Representative and specific appliances for users are modeled considering as inputs the home thermal mass, the solar radiation, and the wind speed, among others. All the assumptions made for the user characterization are justified in [19]. As for the aggregator, it is assumed that electricity prices can be estimated and are thus known a day in advance. Moreover, the demand variability is optimized by the aggregator according to the so-called Peak to Average Ratio (PAR) [26]. This nonlinear function is typically used at a residential level to measure how flat the load profile is. Note that flattening the demand curve is not always beneficial, as is the Brazilian case [24]. In general, this situation may arise in residential regions largely dependent on renewable-based generation and without enough storage capacity.

Similar to [7], the proposed framework is divided into two phases. In the first phase, profits of individual users are maximized under a price-based DR scheme. By contrast, in the second phase, a combination of an incentive-based DR via CL and price-based DR is considered. Thus, the limitations of carrying out each strategy independently are overcome. In this second phase it is assumed that all users participating in the DR program will only accept CL requests if they do not incur any financial losses or discomfort. This last phase is formulated as a bilevel programming problem, wherein the upper level characterizes the aggregator decisions and each lower-level problem models the response of a particular user. As in [7], the resulting problem structure allows deriving an alternative single-level equivalent.

Unlike [7], Dantzig-Wolfe Decomposition is applied in a distributed fashion for the sake of scalability and privacy needs. Additionally, users are encouraged, via monetary incentives, to adopt a new consumption profile while maximizing the aggregator's profit. To sum up, this work substantially improves upon [7] in four aspects: (i) a monetary incentive is devised for each user, (ii) specific and practical models for home appliances are used, (iii) the aggregator objective function consists in minimizing not only the electricity costs and the load variability but also the incentives paid to the users and, (iv) the aggregator, rather than users, is in charge of minimizing the load variability. Moreover, in contrast to both [7] and [20], here we propose the application of an exact distributed approach that converges to optimality in a finite number of iterations. None of the aforementioned studies in the literature consider all of these features.

The remainder of this paper is organized as follows. Section II presents the two-phase framework. The proposed distributed approach is presented in Section III. In Section IV, numerical results are shown and discussed. Concluding remarks are provided in Section V. Finally, an appendix is included to provide the proof for a linear approximation of the nonlinear Peak to Average Ratio.

II. Two-Phase Framework

This section describes the optimization problems representing the building blocks of the proposed two-phase framework. In addition, the aggregator and users interaction is presented.

A. General Model for Users

Based on [19], the following MILP problem is formulated for each user n:

$$\max_{\boldsymbol{y}_n^U} \quad f_n^U = \sum_{t \in T} \left(R_{n,t}^s - C_{n,t}^p - C_{n,t}^{chp} \right) - C_n^{ev} \tag{1}$$

subject to:

$$R_{n,t}^s = \lambda_t^s \Delta_t E_{n,t}^s, \ \forall t \in T \tag{2}$$

$$C_{n,t}^p = \lambda_t^{TOU} \Delta_t E_{n,t}^p, \ \forall t \in T \tag{3}$$

$$g_n^U(\boldsymbol{y}_n^U) \leq D_n^{max} \tag{4}$$

$$\boldsymbol{y}_n^U \in \mathcal{F}_n \tag{5}$$

where $R_{n,t}^s$, $C_{n,t}^p$, $C_{n,t}^{chp}$, C_n^{ev}, $E_{n,t}^s$, and $E_{n,t}^p$ are elements of the vector of decision variables $\boldsymbol{y}_n^U$. Note that if an appliance/equipment (A/E) is not eligible for user n the corresponding decision variables are set to zero in $\boldsymbol{y}_n^U$. Furthermore, binary variables are components of this vector if user n owns a CHP system. Accordingly, each user characterizes its own set of A/E and imposes its own constraints, modeled by the feasible region $\mathcal{F}_n$. The full list of variables in $\boldsymbol{y}_n^U$ and constraints in $\mathcal{F}_n$ can be found in [19]. Note that f_n^U, $R_{n,t}^s$, $C_{n,t}^p$, $C_{n,t}^{chp}$ and C_n^{ev} are included here for explanation purposes.

The optimization goal in (1) is the maximization of the profit of user n. The revenues from selling energy $R_{n,t}^s$ are defined in (2), where λ_t^s are the energy selling prices, which are given or estimated. The electricity costs $C_{n,t}^p$ are defined in (3), where λ_t^{TOU} are the TOU tariffs, known with certainty ahead of time. $C_{n,t}^{chp}$ and C_n^{ev} represent fuel costs for the CHP and the electric vehicle models, respectively. Constraint (4) sets the maximum discomfort D_n^{max}. This constraint can be formulated using the framework in [25]. Finally, expression (5) provides a compact formulation to characterize the energy flow conservation, the balance between supply/demand, the appliances operation, the pricing policies, and the energy limits.

It is worth mentioning that every A/E from a specific residential user $n \in N$ can be used for DR, which is decided by its own HEMS. For more details about pricing policies, hypotheses, and justifications related to the model for users, the interested reader is referred to [19].

B. Model for the Aggregator

In this work, the aggregator's decisions are characterized as a bilevel MILP problem in which the aggregator's profit is maximized in the upper level provided that the aggregated users do not incur any economic or comfort losses by participating in the incentive-based DR program. Users' reactions are characterized in the lower level in which their consumption levels are optimized under their individual no-loss criterion.

The following assumptions are made according to the context of residential energy consumption. First, aggregator's revenues are fixed and equal to the incomes earned via the electricity bills paid by the users. Thus, the only way to maximize the aggregator's profit is by minimizing its cost. Second, the nonlinear Peak to Average Ratio function [26], defined as $f^{PAR} = |T| \left(\max_{t\in T}\left\{\sum_{n\in N} E^p_{n,t}\right\}\right)\left(\sum_{t\in T}\sum_{n\in N} E^p_{n,t}\right)^{-1}$, is approximated by a linear formulation. Note that the optimal value of the aggregated consumption is attained when this ratio is minimized.

The proposed bilevel MILP formulation for the aggregator is cast as follows:

$$\min_{\boldsymbol{y}_n^A, \boldsymbol{e}^A} f^A = \sum_{t\in T} C_t^{DA} + \sum_{n\in N} \kappa_n + \sum_{t\in T} C_t^{Dev} \tag{6}$$

subject to:

$$C_t^{DA} = \lambda_t^{DA}\Delta_t \sum_{n\in N} E^p_{n,t},\ \forall t\in T \tag{7}$$

$$\kappa_n^{min} \leq \kappa_n \leq \kappa^{max},\ \forall n\in N \tag{8}$$

$$C_t^{Dev} = \mu_t^{Dev}\Delta_t(e_t^+ + e_t^-),\ \forall t\in T \tag{9}$$

$$\sum_{n\in N} E^p_{n,t} - \frac{\sum_{n\in N}\sum_{t'\in T} E^p_{n,t'}}{|T|} + e_t^+ - e_t^- = 0,\quad \forall t\in T \tag{10}$$

$$e_t^+, e_t^- \geq 0,\quad \forall t\in T \tag{11}$$

$$f^A \leq \hat{C}^{DA} + \hat{C}^{Dev} \tag{12}$$

$$\text{where } E^p_{n,t} \in \arg\max_{\boldsymbol{y}_n^U} \left\{ f_n^U \right. \tag{13}$$

subject to:

$$\text{Constraints (3)} - \text{(5)} \tag{14}$$

$$\left. R^s_{n,t} = 0,\ \forall t\in T \right\}, \forall n\in N \tag{15}$$

where symbol $\hat{}$ is used to refer to the optimal values of the variables resulting from Phase I as explained in Section II-C1.

The aggregator's total cost is minimized in (6), which comprises: (i) the costs of purchasing electricity in the day-ahead market, C_t^{DA}, (ii) the costs related to the incentives paid by the aggregator to the users for changing their load consumption patterns under a DR request, κ_n, and (iii) C_t^{Dev}, characterizing the level of variability of the aggregated energy consumed according to the aggregator's target. The upper-level decision variables are represented by the vector $\boldsymbol{y_n^A}$, which comprises variables $E^p_{n,t}$, and κ_n, and the vector $\boldsymbol{e^A}$, which includes e_t^+, e_t^- . Note that f^A, C_t^{DA} and C_t^{Dev} are auxiliary variables only included for explanation purposes.

Upper-level constraints comprise expressions (7)–(12). Constraints (7) define the costs of purchasing electricity in the day-ahead market, where λ_t^{DA} is an estimation of the hourly clearing price in the day-ahead electricity market. Constraints (8) bound the incentives paid by the aggregator, where κ_n^{min} are parameters that can be negotiated between the aggregator and the users via a contractual agreement and κ^{max} is the aggregator's budget limit. The costs related to the load profile deviation are defined in (9). μ_t^{Dev} is a tuning parameter set by the aggregator to adjust the smoothness of the aggregated consumption profile. Note that the larger is μ_t^{Dev} the flatter is the aggregated consumption. Surplus variables e_t^+ and e_t^-, which are as per (11), quantify the deviation from the expected energy consumption along the time span. As explained in the Appendix, equations (10) are the linear expression of the consumption variance. Here, the variance costs rather than f^{PAR} are minimized in (6), since (9), (10) and (11) represent a local approximation of f^{PAR} as shown in the Appendix. Finally, constraints (12) ensure that by coordinating users' consumptions the aggregator does not incur financial losses. Thus, condition (12) imposes that the aggregator's total cost, f^A, associated with the implementation of the incentive-based DR program, cannot be greater than the total costs that would be incurred without any coordination, $\hat{C}^{DA} + \hat{C}^{Dev}$. In other words, the incentives given by the aggregator cannot exceed the savings obtained by implementing the DR program.

The lower-level problem is formulated in (13)–(15). This problem is similar to (1)–(5) except for constraints (2), which are replaced with constraints (15) to prevent users from obtaining revenues from selling electricity not consumed under a DR strategy. This condition is imposed to guarantee that users engaged in the proposed DR program are committed to consuming the energy scheduled by this program.

The bilevel problem (6)–(15) can be equivalently transformed into a single-level MILP problem as follows. By setting the users' revenues to 0 in (15), the maximization of each user's profit in (13) can be equivalently reformulated as the minimization of their total cost, given by $C_n^U = \sum_{t\in T}\left(C^p_{n,t} + C^{chp}_{n,t}\right) + C_n^{ev}$. Accordingly, expressions (13) can be equivalently replaced with:

$$\kappa_n - C_n^U \geq \hat{f}_n^U,\ \forall n\in N. \tag{16}$$

Constraints (16) guarantee that the profit obtained by each user by participating in the proposed DR program, $\kappa_n - C_n^U$, is greater than or equal to the profit that would be achieved under no coordination, $\hat{f}_n^U$.

As a result, the aggregator problem (P) is cast as the following single-level MILP:

$$(P)\quad \min_{\boldsymbol{y}_n, \boldsymbol{e}^A} f^A = \sum_{t\in T} C_t^{DA} + \sum_{n\in N} \kappa_n + \sum_{t\in T} C_t^{Dev} \tag{17}$$

subject to:

Constraints (7) – (12) and (14) – (16)

where vector $\boldsymbol{y}_n$ denotes all the decision variables associated with user n, comprising vectors $\boldsymbol{y}_n^A$ and $\boldsymbol{y}_n^U$.

Minor modifications to the above formulation are required in order to accommodate the specific case in which the aggregator has certainty on the most beneficial consumption profile. In this case, μ_t^{Dev} in (9) should be replaced with a large penalty constant and constraints (10) should be replaced by the following expressions:

$$\sum_{n\in N} E^p_{n,t} + e_t^+ - e_t^- = E_t^{des},\ \forall t\in T. \tag{18}$$

C. Agent Interaction

Based on reference [7], the proposed framework comprises two phases. The first phase only involves the users' HEMS, while the second phase involves both users' HEMS and the aggregator energy management system. Both phases are described next.

1) Phase I – No Coordination: In this phase, no coordination between users and the aggregator is implemented. Each user n solves problem (1)–(5) in its HEMS to maximize its own profit. The solution of these $|N|$ problems provides the optimal values of the energy consumed under no coordination, $\hat{E}^{p}_{n,t}$, and the optimal values of the uncoordinated users' profits, $\hat{f}^{U}_{n}$. Such values are used as inputs of Phase II.

Note that the main difference of this phase from that in [7] is that a more general model for users is considered.

2) Phase II – Coordination: In this phase, the aggregator coordinates users' targets with its DR targets by proposing specific incentives to change load consumption patterns for each user.

First, the aggregator computes the following values using the results of Phase I:

- The total cost of purchasing electricity in the day-ahead market:
$$\hat{C}^{DA} = \sum_{t\in T} \lambda^{DA}_{t} \Delta_t \sum_{n\in N} \hat{E}^{p}_{n,t}. \tag{19}$$
- The total cost associated with load deviations:
$$\hat{C}^{Dev} = \sum_{t\in T} \mu^{Dev}_{t} \Delta_t \left(\sum_{n\in N} \hat{E}^{p}_{n,t} - \frac{\sum_{n\in N}\sum_{t'\in T} \hat{E}^{p}_{n,t'}}{|T|} \right). \tag{20}$$
- The aggregated users' profit:
$$\hat{f}^{U} = \sum_{n\in N} \hat{f}^{U}_{n}. \tag{21}$$

Subsequently, (P) is solved. This problem could be addressed by different methodologies by directly using available commercial solvers. For the sake of both scalability and users' privacy, an exact distributed approach based on DWD [27] is alternatively proposed to solve (P) as described in the next section.

III. Proposed Distributed Two-Phase Framework

The proposed framework differs from the above-described centralized approach in the solution of (P), thus Phase I and the updating steps (19)–(21) remain the same. For expository purposes, problem (P) is cast in a compact way as follows:

$$\min_{\boldsymbol{e}^{A}, \boldsymbol{y}_n} \quad \sum_{n\in N} (\boldsymbol{c}_n)^T y_n + \boldsymbol{\mu} \Delta \mathbf{1}^T \boldsymbol{e}^{A} \tag{22}$$
subject to:
$$\sum_{n\in N} \boldsymbol{A}_n y_n + \boldsymbol{Q}\boldsymbol{e}^{A} \;=\; \boldsymbol{b} \tag{23}$$
$$\boldsymbol{y}^{U}_{n} \;\in\; \mathcal{F}_n, \;\forall n \in N \tag{24}$$

where $\boldsymbol{c}_n$ denotes the vector of objective function coefficients for each user n; $\boldsymbol{\mu}$ and $\boldsymbol{\Delta}$ are the vectors associated with μ^{Dev}_{t} and Δ_t, respectively, and $\mathbf{1}_{1\times|2T|}$ is a vector of ones. Note that the objective function (22) corresponds to (17). $\boldsymbol{A}_n$ and $\boldsymbol{b}$ in (23) correspond to the matrix of constraint coefficients and the vector of right-hand-side coefficients in (P). Finally, $\boldsymbol{Q}$ denotes the matrix of constraint coefficients associated with the vector $\boldsymbol{e}^{\boldsymbol{A}}$ in (P), and constraints (24) are identical to (5).

Additional notation is introduced next to obtain a reformulation of (P) suitable for a branch-and-price algorithm [27]. The polyhedron that defines the set of constraints of user n is defined as follows:
$$\Upsilon_n = \left\{ y_n \in \{\mathcal{F}_n \cap \mathcal{G}_n\} \middle|\; g^{U}_{n}(y^{U}_{n}) \leq D^{max}_{n},\; \kappa_n \geq\; \kappa^{min}_{n} \right\},$$
where $\mathcal{G}_n = \left\{ y_n \in \mathcal{F}_n \middle|\; \kappa_n \geq -\hat{f}^{U}_{n} - C^{U}_{n} \right\}$.

Note that the set of extreme rays of the convex hull of Υ_n is empty since f^{U} is bounded.

The application of DWD to (P) relies on the following formulation:

$$\min_{\boldsymbol{e}^{\boldsymbol{A}}, \boldsymbol{y}_{\boldsymbol{n}}, \alpha} \quad \sum_{n\in N}\sum_{h\in\Omega^{n}} (\mathbf{c}_n)^T h \alpha^{h}_{n} + \boldsymbol{\mu}\boldsymbol{\Delta}\mathbf{1}^T \boldsymbol{e}^{\boldsymbol{A}} \tag{25}$$
subject to:
$$\sum_{n\in N}\sum_{h\in\Omega^{n}} \boldsymbol{A}_n h \alpha^{h}_{n} + \boldsymbol{Q}\boldsymbol{e}^{\boldsymbol{A}} \;=\; \mathbf{b} \tag{26}$$
$$\sum_{h\in\Omega^{n}} \alpha^{h}_{n} \;=\; 1,\; \forall n \in N \tag{27}$$
$$\boldsymbol{\alpha} \;\geq\; 0 \tag{28}$$
$$\sum_{h\in\Omega^{n}} h\alpha^{h}_{n} \;=\; \boldsymbol{y}_{\boldsymbol{n}},\; \forall n \in N \tag{29}$$
$$\boldsymbol{y}_{\boldsymbol{n}} \;\in\; \Upsilon_n,\; \forall n \in N. \tag{30}$$

where Ω_n is the set of extreme points of the convex hull of Υ_n. Each extreme point $h \in \Omega_n$ in problem (25)-(30) is associated with the variable α^{h}_{n}, and a column $A_n h$, which corresponds to the load schedule for user n. A suitable solution to address the resulting exponential number of variables is the Column-Generation (CG) algorithm. According to [28], the application of the CG algorithm to problem (25)-(30) involves the iterative solution of the restricted master problem (RMP) and one pricing subproblem per user $n \in N$, referred to SP_n. The RMP (31)-(34) is obtained by eliminating constraints (29)-(30) and using a subset of columns $\overline{\Omega}_n \subset \Omega_n$ for each user $n \in N$:

$$\min_{\boldsymbol{e}^{\boldsymbol{A}}, \alpha} \quad \sum_{n\in N}\sum_{h\in\overline{\Omega}_n} (\mathbf{c}_n)^T h \alpha^{h}_{n} + \boldsymbol{\mu}\boldsymbol{\Delta}\mathbf{1}^T \boldsymbol{e}^{\boldsymbol{A}} \tag{31}$$
subject to:
$$\sum_{n\in N}\sum_{h\in\overline{\Omega}_n} \boldsymbol{A}_n h \alpha^{h}_{n} + \boldsymbol{Q}\boldsymbol{e}^{\boldsymbol{A}} \;=\; \mathbf{b} \;(v^{d}_{1}) \tag{32}$$
$$\sum_{h\in\overline{\Omega}_n} \alpha^{h}_{n} \;=\; 1,\; \forall n \in N \;(v^{d}_{2,n}) \tag{33}$$
$$\boldsymbol{\alpha} \;\geq\; 0 \tag{34}$$

where v^{d}_{1} and $v^{d}_{2,n}$ are the dual variable vectors associated with constraints (32) and (33), respectively.

For a user n and a given h, the reduced cost of a variable α^{h}_{n} is calculated as $(\mathbf{c}_n)^T h - (v^{d}_{1})^T A_n h - v^{d}_{2,n}$. The goal of

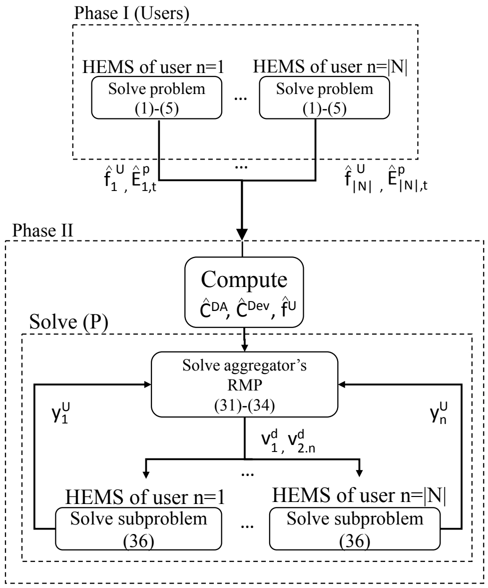

Figure 1: Solution framework flowchart

SP$_n$ is to find the extreme point with the minimal reduced cost:

$$h^* \in \inf_{h \in \Omega^n} \{(\mathbf{c}_n)^T h - (v_1^d)^T \boldsymbol{A}_n h - v_{2,n}^d\} \tag{35}$$

To that end, subproblem SP$_n$ is formulated as:

$$\min_{\boldsymbol{y_n} \in \boldsymbol{\Upsilon_n}} \quad (\boldsymbol{c}_n)^T \boldsymbol{y_n} - (v_1^d)^T \boldsymbol{A}_n \boldsymbol{y_n} - v_{2,n}^d \tag{36}$$

The CG starts by solving RMP with some columns, which can be generated using the heuristic from [20], which provides initial feasible columns. New columns are iteratively added to $\overline{\Omega}_n$ in the RMP if the extreme point h^* obtained from solving SP$_n$ has a negative reduced cost. The CG algorithm stops when there is no negative reduced costs.

The final solution proposed by the CG may have fractional values for binary variables, since they are relaxed in the RMP. A branch-and-price algorithm is used to obtain an optimal integer solution considering the RMP as the root node of the branching tree. As the CG algorithm proposes solutions with small integrality gap for our instances, the diving heuristic procedure from [29] is considered in coordination with the CG algorithm to prove optimality.

Fig. 1 depicts the proposed two-phase distributed framework. As shown in the bottom part, the corresponding HEMS of each user n solves its own subproblem and sends the solution $\boldsymbol{y}_n$ to the aggregator. The aggregator only sends to users information on DWD dual variables, v_1^d and $v_{2,n}^d$, thereby preserving users' privacy. It is worth emphasizing that this is an important distinctive aspect as compared with existing methodologies [20], which also decompose (P) into subproblems but either do not guarantee optimality within a finite number of iterations or breach the privacy of users' data.

IV. Case Study

This section presents the results from the proposed framework using both the centralized and distributed approaches respectively described in Sections II and III. From assessment purposes, results from only executing Phase I are also reported. Hereinafter, "Phase I" is used to refer to the case without any users' coordination by only carrying out Phase I, and the acronyms "CE" and "DI" refer respectively to the centralized and distributed approaches. The comparison is carried out in terms of the Peak-to-Average Ratio, the limits on users' incentive, as well as the price given to the load deviation. The time horizon comprises one day divided in intervals of 10 minutes, i.e., $|T| = 144$. This case study considers real data for users from [19] and assumes forecasted values for uncertain parameters. For the sake of simplicity, aggregator prices remain unchanged over the time span, being $\lambda_t^{DA} =$ \$0.01/kWh and $\mu_t^{Dev} =$ \$0.002/kWh, $\forall t \in T$. Note that having a flat electricity price is a practical assumption in hydropower dominated countries like Brazil [24]. Besides, in order to avoid discrimination, κ_n^{min} is the same for all $n \in N$ and set to \$0.01.

Simulations were run to optimality using CPLEX 12.8 under AMPL 20180618 on an Intel® Xeon® X5675 at 3.07 GHz and 96 GB of RAM.

Fig. 2 depicts the system load profiles under the three approaches considered as well as the time-of-use tariff for $|N| = 500$. As can be seen, both CE and DI efficiently flatten the aggregated load profile. It is worth mentioning that the aggregated consumption profiles are not identical for CE and DI, but the same optimal value of (P) is attained by both methods, confirming the existence of multiple solutions. If only Phase I is performed, there is a large consumption peak near time step 96 anticipating the increase in λ_t^{TOU} between this time step and time step 125. Although from the users' perspective it is more economical to store energy to avoid consumption at high prices, this is not convenient for the aggregator.

Running times for CE and DI are listed in Table I for different sets of users of growing size. As can be seen, DI allows dealing with instances of up to 10,000 users whereas CE is unable to solve instances with more than 500 users (indicated with symbol -), which corroborates the scalability issues of this method. Moreover, unlike CE, DI is suitable for parallel computing, which would greatly reduce the computational burden. An approximation of the computing times required by the parallel implementation of DI can be obtained dividing the results in Table I by $|N|$. Note that such approximate computing times range between 51 s and 130.54 s, thereby backing the practical applicability of DI.

Table I: Computing times

$\lvert N \rvert$	Time [s]		$\lvert N \rvert$	Time [s]	
	CE	DI		CE	DI
10	10	507	50	239	856
100	1000	3032	300	8244	25314
500	12522	34515	1000	-	89631
5000	-	588870	10000	-	1305448

For the cases shown in Table I, the Peak-to-Average Ratio is calculated and the largest improvement is attained for $|N|$ =10,000. In this case, f^{PAR} is equal to 2.06 if only Phase I is carried out while a value of 1.14 is achieved under

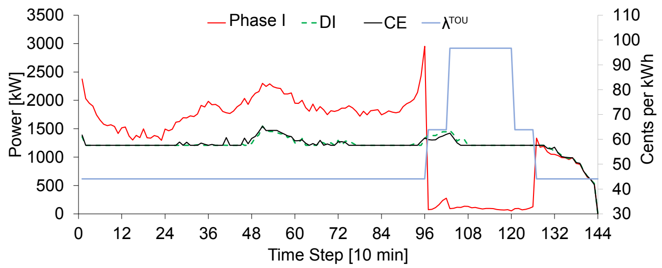

Figure 2: The scheduled load profiles for $|N| = 500$ and the time-of-use tariff.

DI, thereby yielding a substantial 44.7% reduction. This result is consistent with the intuition that by increasing the number of users more room to flatten their aggregated consumption is available for the aggregator.

To study the trade-off modeled in (17) between the smoothness of the aggregated consumption profile and the electricity acquisition cost, different values are assigned to μ_t^{Dev} assuming that the day-ahead electricity price follows a TOU tariff as depicted in Fig. 3. For $|N|$=10 and μ_t^{Dev} equal to \$0.002 /kWh and \$0.08 /kWh, $\forall t \in T$, the aggregator's cost respectively amounts to \$40.76 and \$53.11 under DI, and to \$41.49 and \$86.21 without any coordination. These results confirm that coordination is beneficial for the aggregator. Fig. 3 additionally shows the consumption profiles for the considered values of μ_t^{Dev}. It can be seen that for μ_t^{Dev} = \$0.08 /kWh, a flat consumption profile is attained while the consumption profile for μ_t^{Dev} = \$0.02 /kWh is very similar to the non-coordinated case.

To focus on the benefits of having a low variability of the aggregated consumption, only the associated cost, C_t^{Dev}, is minimized in (17) under DI and the terms related to the aggregator's electricity cost and the incentives are dropped from (17). Then, the minimum amount of incentives, κ_n^{min}, given by the aggregator to each user is modified in order to quantify its influence on the aggregator's total cost, computed considering all the terms in (17). In particular, for $|N|$=10 and values of κ_n^{min} equal to \$0, \$0.01, and \$0.05, the aggregators' cost amounts to \$35.1, \$36.0, and \$36.9, which corroborates the fact that larger costs are associated with larger incentives. Note that by setting κ_n^{min} = \$0, as in [20], or without explicitly considering κ_n^{min}, as in [7], yields a price-based DR program with the minimum value of the aggregator's cost. However, without any economic incentive the risk of users leaving such a DR program is increased. Yet, increasing the minimum amount of incentives effectively reduces the load variability as shown in Fig. 4 for the considered values of κ_n^{min}. Note that constraint (12) prevents from setting the minimum amount of incentives too high. For this particular case, infeasible solutions are obtained when $\kappa_n^{min} \geq$ \$0.06. Analyzing the incentives received by each user, they are all equal to the minimum incentive in any case, $\kappa_n = \kappa_n^{min}$, $\forall n \in N$. In particular, for κ_n^{min} = \$0.05 per day, the monthly electricity bill reduction is around 1% for each user.

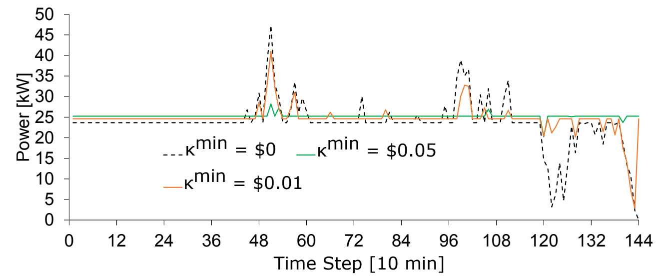

Figure 3: Impact of κ_n^{min} on the scheduled load profiles for $|N| = 10$.

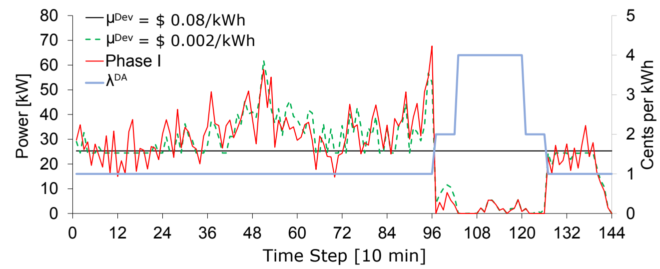

Figure 4: Impact of μ_t^{Dev} on the scheduled load profiles for $|N| = 10$.

V. Conclusion

The main motivation of this work is to provide a practical tool for aggregators involved in the implementation of DR programs. This tool coordinates the consumption of householders so that their comfort levels and costs remain unchanged after coordination. A two-phase framework is proposed in which the users' profits are first individually maximized without any coordination. Subsequently, in a second phase, the aggregator minimizes its cost while keeping the desired comfort level and profit for every user. The problem solved by the aggregator is posed as a bilevel problem that can be cast as a single-level equivalent. Privacy and scalability needs are accommodated by applying Dantzig-Wolfe decomposition. Simulation results show that the proposed framework is a useful tool for aggregators who seek to decrease the aggregated electricity demand curve peaks while maintaining the level of users' comfort at no financial loss.

To extend this work, we propose two avenues of research. Firstly, uncertainties in both aggregator and user parameters will be characterized using stochastic or robust optimization. This extension would allow implementing measures to control the risk taken by the aggregator for participating in the day-ahead electricity market. Secondly, fairness will be incorporated by considering incentives according to the participation of each user. This aspect is of utmost interest for practical implementation purposes. However, we recognize that the resulting challenging problem requires further research.

Appendix

In this appendix we show that the minimization of the nonlinear function f^{PAR} can be locally approximated via a linear programming problem corresponding to the minimization of the variance of the energy consumed along the time horizon.

Let $B_t = \sum_{n\in N} E^p_{n,t}$ and $\beta = max_{t\in T} B_t$ and let t^* denote any index in T with $B_t = \beta$. Then, the PAR function f^{PAR} can be defined by the following expression:

$$f^{PAR} = \frac{\beta|T|}{\beta + \sum_{t\in T/t^*} B_t}. \tag{37}$$

For $\beta = 0$, $B_t = 0$, $\forall t \in T$, and f^{PAR} is an undefined function, meaning in practice no consumption. Conversely, for $\beta > 0$, $B_t \leq \beta$, $\forall t \in T$, and f^{PAR} is greater than 1. Moreover, the minimum value of f^{PAR} is attained for $B_t = \beta$, $\forall t \in T$ and is equal to 1, which corresponds to a flat consumption profile.

Besides, the variance σ^2 of the energy consumed along the time horizon can be defined as follows:

$$\sigma^2 = \frac{1}{|T|-1} \sum_{t\in T} \left(\sum_{n\in N} E^p_{n,t} - \frac{\sum_{n\in N}\sum_{t'\in T} E^p_{n,t'}}{|T|} \right)^2. \tag{38}$$

Theoretically, the minimum value for the variance is 0 and is attained when $\sum_{n\in N} E^p_{n,t}$ is identical $\forall t \in T$. With no loss of generality, minimizing this nonlinear function is locally approximated by solving the following linear programming problem:

$$\min_{E^p_{n,t}, e^+_t, e^-_t} \sum_{t\in T} e^+_t + e^-_t \tag{39}$$

subject to:

$$\sum_{n\in N} E^p_{n,t} - \frac{\sum_{n\in N}\sum_{t'\in T} E^p_{n,t'}}{|T|} + e^+_t - e^-_t = 0, \ \forall t \in T \tag{40}$$

$$e^+_t, e^-_t \geq 0, \quad \forall t \in T \tag{41}$$

where e^+_t and e^-_t are nonnegative surplus variables.

For the particular case of $B_t = \beta$, $\forall t \in T$, the variance also attains its minimum value of 0 since $\sum_{n\in N} E^p_{n,t} = \beta$, $\forall t \in T$. Thus, the solution of problem (39)–(41) is equivalent to the minimization of f^{PAR}.

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