Optimal Residential Coordination Via Demand Response: A Distributed Framework

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***Abstract*—This paper proposes an optimization framework for retailers that are involved in demand response (DR) programs. In a first phase responsive users optimize their own household consumption, characterizing not only their appliances and equipment but also their comfort preferences. Then, the retailer exploits in a second phase this preliminary non-coordinated solution to implement a strategy for the aggregated loads. This strategy is formulated as a bilevel problem in which the retailer costs are minimized in the upper level assuring that the aggregated residential users do not incur in any economic or comfort losses by participating in the DR program. The retailer requests are considered by each user in the lower level in order to optimize its consumption under its particular non-financial loss criterion. The resulting problem is cast as a mixed-integer non linear bilevel problem with lower-level binary variables. A mixed-integer linear single-level reformulation is presented and an exact solution technique based on Dantzig-Wolfe decomposition is proposed. Simulations with up to 10000 residential users illustrate the advantages of the proposed two-phase framework in terms of privacy, effectiveness and scalability.**

Keywords: Column Generation, Dantzig-Wolfe decomposition, demand response, bilevel optimization, user coordination.

Nomenclature

For reference, symbols used in this paper are listed below.

A. Sets

$\mathcal{N}$	Set of users.
T	Set of time intervals.
Υ_n	Set of feasible solutions for user n.
Ω_n	Set of extremes points of $conv(\Upsilon_n)$.
Γ_n	Set of extremes rays of $conv(\Upsilon_n)$.
Θ_n	Set of variables related to appliances/equipment not eligible for user n.

B. Parameters

$\overline{C}_n$	Optimal cost of user n without coordination [\$].
$\overline{C}^p$	Retailer electricity cost without coordination [\$].
D_n	Maximum discomfort level for user n.
I_n	Minimum incentive for user n for changing its load consumption pattern [\$].
$\overline{R^c}$	Retailer load volatility cost without coordination [\$].
Δ_t	Length of the time interval [h].
λ_t	Electricity retail price for users at time t [\$/Wh].
μ_t	Load volatility price for retailer at time t [\$/Wh].
ν_t	Selling electricity price for users at time t [\$/Wh].
π_t	Retailer day-ahead electricity price at time t [\$/Wh].

C. Variables

$C^b_{n,t}$	Cost of user n for purchasing electricity at time t [\$].
$C^{chp}_{n,t}$	Cost of user n related to the combined heating power operation at time t [\$].
C^{ev}_n	Daily hybrid vehicle fuel cost for user n [\$].
C^p_t	Retailer electricity cost at time t [\$].
$C^s_{n,t}$	Revenue of user n for selling electricity at time t [\$].
$E_{n,t}$	Electric power purchased by user n at time t [W].
$E^s_{n,t}$	Total electric power sold by user n at time t [W].
e^+_t	Positive power imbalance at time t [W].
e^-_t	Negative power imbalance at time t [W].
κ_n	Incentive paid to user n for changing its load consumption pattern [\$].
v^d_1, v^d_2	Dual variable vectors.
$\mathcal{X}$	Feasible space of all variables for a user model.
Ξ_n	Vector of variables for user n.

D. Functions

f_c	User total cost function [\$].
f_d	User total discomfort level.

I. Introduction

Coordinating distributed energy resources is a challenge for electric utilities given the large number of resources involved [1]. If the residential sector comes into play the challenge is even greater due to issues on the householders privacy and constraints on the social welfare. A possible way to face this challenge is incentivizing users to change their consumption patterns to fit a load shape that benefits the whole system via Demand Response (DR) programs. DR programs claim that users can change their electricity consumption in response to incentive payments or electricity tariffs warnings. Literature reviews of DR are available in [2] and its recent application worldwide in the residential sector is reported in [3].

To increase the contribution of residential users in the DR strategies output, aggregators may play a crucial role [4] by managing DR programs for a group of users. Many entities in the power market can act as an aggregator [1], including a third party retailer [3], [1] or the generator company via vertical integration between generation and retail [1].

Assuming that the target of any retailer is the maximization of its profits, flattening the electricity consumption curve is a beneficial option for reducing the costs associated to high-peak hour demands [5]. When a price-based DR is implemented, users optimize their consumption responding to the price signals issued by the retailer. If they all simultaneously schedule their loads to low-price periods, new rebound peaks may appear [6]. In particular, [7] shows that price-based DR is not enough to change the user behavior, specifically for domestic electric water heaters.

Alternatively, to avoid new peaks and to increase the number of users participating in a DR program, incentive-based DR programs emerge to coordinate users consumption. In these programs, users are paid to shift and/or to reduce their consumption for a given period of time. In particular, Direct Load Control (DLC) is reported as the most used program in the residential sector, while Curtailable Load (CL) predominantly involves medium and large users [3]. However, users participating in these programs have reported that incentives are not attractive enough and their comfort levels were negatively impacted [8]. Moreover, Fan *et al.* [9] added another drawback: DLC always incurs computation burden and privacy issues. Hence, larger incentives or DR requests with a lower impact on users comfort are necessary from the user perspective. Note, however, that excessive incentives may result in unacceptable financial losses for the retailer.

This paper proposes a framework that jointly minimizes costs for a retailer and for users by combining price-based and incentive-based DR programs. The framework keeps the desired comfort for each user and alleviates privacy concerns. In this framework, profit maximization is same as cost minimization from the retailer perspective. Results show that a flatted demand curve may lead to financial losses for the retailer.

The organization of this paper is as follows. Section II presents the literature review and the contributions of this work. The mathematical models are introduced in Section III as well the solution methodology. Section IV provides the computational results and discussion on the case studies. Concluding remarks are provided in Section V. Finally, Dantzig-Wolfe decomposition details are outlined in the Appendix.

II. Related Works on Residential DR

Different approaches to coordinate residential users via DR programs are presented in the literature. Among them, some references address the problem in a centralized fashion by directly using available commercial solvers based on mathematical programming [10], [11]. However, both privacy and scalability issues arise in these centralized approaches [12]. On the other hand, decentralized approaches that decompose the problem into subproblems, improve the scalability but not necessarily the privacy needs if subproblems are all solved in a single machine. In practice, the problem should be decentralized but also solved in a distributed mode so that each user optimizes its own consumption with its particular home management system. Thus, in order to both mitigate privacy concerns and improve scalability the latter approach should be adopted.

The existing literature on distributed methods for demand response is classified in [13] according to the mathematical complexity of the models for appliances and whether time couplings between them are characterized. Although this classification is only for distributed methods, it can be extended to other solution methodologies. References [9], [14], [15] present mathematical programming models with variables that neglect the time couplings of users' appliances. Alternatively, mixed-integer programming models for appliances that consider their intertemporal couplings can be found in [16], [17], however the models adopted for the appliances are simplistic. Reference [18] improves former works [16], [17] not only by integrating specific models for each appliance with their coupling constraints but also by using models that are representative in practice. Additionally, from a user perspective, [18] shows that more detailed models increase the users profitability while guaranteeing their levels of comfort.

Apart from the users modeling aspects, coordination strategies can be found in the literature focusing on the decrease of electricity peak load while also satisfying users needs. One strategy is to formulate the DR problem as a single-level social welfare maximization problem (SWMP) in which a trade-off between cost or/and a utility discomfort function for end-users and a performance criterion (incomes, community's living comfort, etc.) for the retailer are imposed. Lu *et al.* in [19] classify the SWMP methods into three categories:

1) Single-objective optimization considering either users or the retailer perspective but not both. See [18];
2) Weighted sum multiobjective optimization considering both perspectives. The drawback of this category lies on the selection of appropriate weights and/or the pricing of discomfort levels. Moreover, if integer variables are considered, the Pareto front can be non-convex and some efficient points cannot be identified. See [8], [9], [13];
3) Pareto front multiobjective optimization considering simplified models to reduce the computation burden if a large number of users are considered. See [20], [21].

This work mainly differs from the aforementioned ones by proposing an alternative framework in which a bilevel problem is formulated with an upper level representing the retailer/aggregator DR strategy, and N lower levels representing the response of the N residential users to this strategy. Note that although bilevel problems are $\mathcal{NP}$-hard [22] a vast number of applications have been addressed in the literature [23].

Under certain convexity conditions on the lower-level problem, an equivalent single-level Mixed-Integer Linear Program (MILP) of a bilevel problem can be obtained by applying Karush-Kunh-Tucker (KKT) conditions [22], as done in [24], [25].However, the problem addressed here precludes the use of the KKT-based transformation since the lower level includes integer variables. Another approaches used in the literature consider a Stackelberg game in which the upper and lower level problems are solved iteratively until reaching an equilibrium solution [9], [16], [26], [27], [28], [29]. However, these works do not consider specific appliance models for users, neither the CL option, which are both characterized in the proposed framework.

Similar to this work, a two-phase framework is proposed in [5], in which a set of users are coordinated using a DR iterative strategy. In the first phase, each user individually minimizes its electricity expenses and computes the associated costs and load profile, which are both the inputs for the next phase. In the second phase, the aggregator announces a modified aggregated load profile for each householder. Then, each user re-optimizes its consumption for each period of time considering both the new energy limits and a measure of the aggregated demand variability. Iteratively, the modified consumption patterns are sent back to the aggregator until no further improvement is experienced on the aggregated demand profile. Note that this paper differs from [5] in four aspects: (i) a monetary incentive is devised for each customer, (ii) specific models for home appliances are used, (iii) the aggregator objective function consists in minimizing not only the electricity costs but also the incentives paid to the users and, (iv) the aggregator, rather than users, is in charge of minimizing the load variability.

In [30], a distributed approach was proposed to coordinate a set of users under a DR program that maximizes the social welfare via a heuristic procedure. Although this heuristic showed reasonable optimality gaps for small instances that were also solvable via exact methods, in general optimality cannot be guaranteed. In contrast, this work proposes the application of an exact distributed approach.

A. Contribution

This work proposes a practical framework for a retailer that coordinates a group of residential users with the goal of jointly minimizing retailer and users costs while preserving users comfort. Representative and specific appliances are modeled for users considering as inputs the home thermal mass, the solar radiation, the wind speed, etc. All the assumptions made for the user characterization are justified in [18].

The proposed framework is divided into two phases. In the first phase, energy expenses of individual customers are minimized considering only price-based DR. In the second phase, incentive-based DR via CL is considered beyond price-based DR. It is assumed that all users participating in the DR program will only accept CL requests if they do not incur any financial losses or discomfort. This last phase is formulated as a bilevel programming problem, wherein the upper level characterizes the retailer decisions and each lower level problem models the response of a particular customer. Then, a single-level reformulation is proposed as done in [5]. Finally, for the sake of practicality in terms of scalability and privacy needs, Dantzig-Wolfe Decomposition (DWD) is applied in a distributed fashion. The proposed framework encourages users, via monetary incentives, to adopt a new specific load consumption while minimizing both peak loads and costs for the retailer. None of the aforementioned studies in the literature consider all of these features.

III. Framework

A. Model for Users

Models for users appliances are based on [18]. The scheduling optimization model in [18] needs 10 pages, so Figure 1 summarizes it. In the Figure, various appliances appliances/equipment (A/E) are considered and arrows between them represents relationships, which are modeled as constraints in [18]. Thermal mass of building, solar radiation, wind speed, etc., considered in [18], are also characterized in this work. Note that a householder may have any combination of A/E. For more details about pricing policies, hypotheses and justifications related to the model for users, please see [18]. Note that this reference provides a model for a single user, thus $|\mathcal{N}| = 1$. The aforementioned model is extended in this paper to consider $|\mathcal{N}| \geq 1$. Let vector Ξ_n characterize all the decision variables for user n. Note that if an appliance a is not eligible for user n, $a \in \Theta_n$, then the corresponding variables are set to zero in Ξ_n. Accordingly, each user has its own set of A/E and imposes its own constraints, characterized by the feasible region $\mathcal{F}_n \subseteq \mathcal{X}$. Thus, for each user $n \in \mathcal{N}$, the following MILP is formulated:

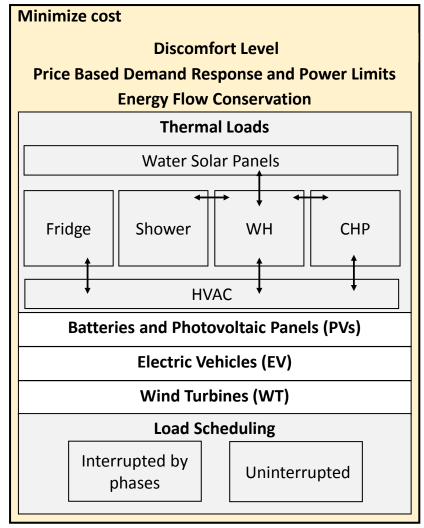

Figure 1: Scheme representation for Model from [18].

$$\min_{\Xi_n} \quad f_c(\Xi_n) = \sum_{t \in T} \left(C^b_{n,t} - C^s_{n,t} + C^{chp}_{n,t} \right) + C^{ev}_n \quad (1)$$

$$\text{s.t.} \quad f_d(\Xi_n) \leq D_n \quad (2)$$

$$\Xi_n \in \mathcal{F}_n \quad (3)$$

where $C^b_{n,t}$, $C^s_{n,t}$, $C^{chp}_{n,t}$ and C^{ev}_n, are elements of Ξ_n. The full list of variables in Ξ_n and constraints in $\mathcal{X}$ can be found in [18]. Here, electricity costs are $C^b_{n,t} = \lambda_t \Delta_t E_{n,t}$ and $C^s_{n,t} = \nu_t \Delta_t E^s_{n,t}$. Costs $C^{chp}_{n,t}$, C^{ev}_n include fuel costs, and are respectively defined for the CHP and the electric vehicle models. Note that binary variables are needed in the CHP model.

The objective function (1) minimizes the cost of user n. Constraint (2) limits the maximum discomfort D_n. This constraint can be constructed using the framework in [31]. Finally, constraints (3) represent the energy flow conservation, balance between supply/demand, the appliances operation, the pricing policies, and the energy limits. It is worth to mention that every A/E from a specific residential user $n \in \mathcal{N}$ is able to be used for DR, which is decided by its own home energy management system.

B. Phase I - no coordination

In this phase, tariffs are known a day ahead and there is no coordination between users and the retailer. Each user n solves problem (1)-(3) to optimize its cost. The optimal solution of this problem provides $\overline{E}_{n,t}\ \forall\ n \in \mathcal{N}, \forall\ t \in T$, and the optimal value $\overline{C}_n = f_c(\boldsymbol{\Xi}_n)$. These outputs are sent to the retailer. The top of Figure 2 represents this process.

C. Phase II - with coordination

With the information provided by Phase I, before implementing the users coordination the retailer can compute the cost for purchasing electricity in the day-ahead (DA) market as $\overline{C}^p = \sum_{t\in T}\sum_{n\in\mathcal{N}}\pi_t\Delta_t\overline{E}_{n,t}$ and the cost associated to the demand variability as $\overline{R}^c = \sum_{t\in T}\mu_t\Delta_t\left(\sum_{n\in\mathcal{N}}\overline{E}_{n,t} - \sum_{t'\in T}\sum_{n\in\mathcal{N}}\overline{E}_{n,t'}/|T|\right)$.

In this phase, the retailer coordinates user targets with its DR targets by proposing specific incentives to change load consumption patterns for each user.

The retailer goal is to maximize its profit. Any linear objective function for the retailer is able to be used in the proposed framework. Retailer's revenues are the total amount of bills paid by the customers, expressed as $\sum_{n\in\mathcal{N}}\overline{C}_n$. This work assumes that users' electricity bills cannot be increased. The only way to increase profits for retailers is by reducing costs. In this work, retailer's expenses comprise three costs: (i) the load cost due to the variability of the energy consumed within the considered time horizon; (ii) the cost of purchasing electricity in the day-ahead market and (iii) the cost of the incentive given to customers for changing its load consumption patterns under a DR request.

Next, the retailer formulation is gradually presented by explaining first the objective function, then the constraints and finally the optimization model.

According to Burger *et al* [1], "*aggregation has system value if it increases the economic efficiency of the power system as a whole.*" Among the services creating such a value is the provision of electricity balancing. However, smart homes do not have flexibility enough for balancing services unless they are aggregated. Thus, imbalances should be computed/estimated by the aggregator, as currently done in Europe and U.S electricity markets. Within this context, the minimization of the load variability can be modeled considering the energy variance for the whole time horizon as follows:

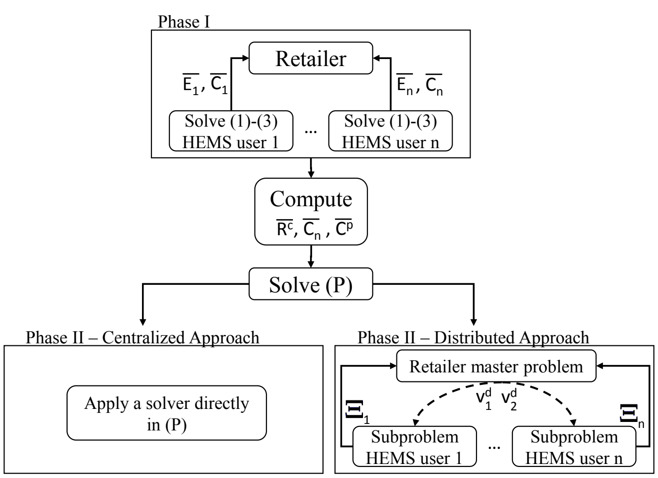

Figure 2: Framework flowchart

$$\min_{E_{n,t}} \frac{1}{(|T|-1)}\sum_{t\in T}\left(\sum_{n\in\mathcal{N}}E_{n,t} - \frac{\sum_{t'\in T}\sum_{n\in\mathcal{N}}E_{n,t'}}{|T|}\right)^2 \quad (4)$$

Adding constraints (2)-(3) to the above problem would result in a nonlinear mixed-integer problem, which is hard to solve even for a small number of users. Note that variable $E_{n,t}$ is an element of $\boldsymbol{\Xi}_n$, so a trivial solution is not possible. Considering a local approximation of (4), the problem of minimizing the variability costs can be cast as follows:

$$\min_{E_{n,t},e_t^+,e_t^-} \sum_{t\in T}\mu_t\Delta_t(e_t^+ + e_t^-) \quad (5)$$

$$\text{s.t.} \left(\sum_{n\in\mathcal{N}}E_{n,t} - \frac{\sum_{t'\in T}\sum_{n\in\mathcal{N}}E_{n,t'}}{|T|}\right) + e_t^+ - e_t^- = 0 \ \forall\ t\in T \quad (6)$$

$$e_t^+, e_t^- \geq 0 \quad \forall\ t\in T, \quad (7)$$

where μ_t is the marginal cost due to the deviation from average in either direction, which can be considered as an estimation for the marginal cost of deployed reserves or balancing prices. It allows the retailer to consider the risk for participating in the DA electricity market. Variables e_t^+ and e_t^- act as gap and surplus variables from the aggregated consumption average. In addition, constant (|T|-1) is removed from (4) since the optimal solution remains unchanged.

Finally, the retailer objective function considering all the costs is given as follows.

$$\min_{\substack{E_{n,t},e_t^+,e_t^-,\\ C_t^p,\kappa_n}} \sum_{t\in T}\mu_t\Delta_t\left(e_t^+ + e_t^-\right) + \sum_{t\in T}C_t^p + \sum_{n\in\mathcal{N}}\kappa_n \quad (8)$$

$$\text{s.t.} \qquad C_t^p = \pi_t\Delta_t\sum_{n\in\mathcal{N}}E_{n,t}\ \forall\ t\in T \quad (9)$$

As mentioned in the introduction, financial losses may result from exaggerating the incentives given by the retailer. Since user electricity bills cannot be increased, the only way to increase the retailer profits is to decrease its costs. Thus, condition (10) imposes that the retailer opportunity costs, defined as the savings due to its CL program, should be lower than the amount of incentives spent.

$$\sum_{n\in\mathcal{N}}\overline{C}_n - \sum_{t\in T}\mu_t\left(e_t^+ + e_t^-\right) - \sum_{t\in T}C_t^p - \sum_{n\in\mathcal{N}}\kappa_n \geq \sum_{n\in\mathcal{N}}\overline{C}_n - \overline{R}^c - \overline{C}^p \quad (10)$$

Constraint (10) is reformulated as (11) eliminating $\overline{C}_n$ and rearranging terms as follows:

$$\overline{R}^c - \sum_{t\in T}\mu_t\Delta_t\left(e_t^+ + e_t^-\right) + \overline{C}^p - \sum_{t\in T}C_t^p \geq \sum_{n\in\mathcal{N}}\kappa_n \quad (11)$$

In this work, the retailer decision framework is characterized as a bilevel programming problem in which the retailer costs are minimized in the upper level assuring that the aggregated users do not incur in any economic or comfort losses by participating in the incentive-based DR program. Users responses are characterized in the lower level by optimize its consumption under its particular non-financial loss criterion:

$$\min_{\substack{E_{n,t},e_t^+,e_t^-,\\ C_t^p,\kappa_n}} \sum_{t\in T}\mu_t\Delta_t\left(e_t^+ + e_t^-\right) + \sum_{t\in T} C_t^p + \sum_{n\in\mathcal{N}}\kappa_n \tag{12}$$

$$\begin{aligned} \text{s.t.}\quad & (6),(7),(9),(11) \\ \forall\, n\in\mathcal{N}\quad & \min_{\Xi_n} f_c(\Xi_n) \qquad\qquad (13) \\ & \text{s.t.}\quad (2),(3) \end{aligned}$$

Here the retailer sets incentives according to its goal and users respond based on their targets. Hence, if the retailer guarantees that the users costs are not increased or apply a discount on their bills, users can change their consumption profiles to favor the system efficiency. As a consequence, after a DR request, if the cost for client n, $\overline{C}_n$, is increased, the retailer pays the extra cost via the incentive κ_n. Therefore, constraints (14)-(15) protect users against costs increases.

$$\kappa_n \geq \sum_{t\in T}\left(C_{n,t}^b - C_{n,t}^s + C_{n,t}^{chp}\right) + C_n^{ev} - \overline{C}_n \quad \forall\, n\in\mathcal{N} \tag{14}$$

$$\kappa_n \geq I_n \;\forall\; n\in\mathcal{N} \tag{15}$$

In this phase users extra costs are transferred to the retailer since retailer revenues are fixed once Phase I is completed. Accordingly, C_t^b is replaced by C_t^p in $f_c(\Xi_n)$ and (14). Additionally, objective functions (13) are replaced by constraints (14), reducing the bilevel formulation (12)-(13) into a high point problem [32, Proposition 1]. Note that this transformation is possible since minimizing $f_c(\Xi_n)$ also minimizes κ_n. Moreover, assuming that $\pi_t < \nu_t < \lambda_t,\ \forall\, t\in T$, the user revenues for selling electricity are set to 0 in Phase II, $C_t^s = 0$ $\forall\, t\in T$, since it would be unreasonable that retailers pay users for the electricity injected into the grid.

Thus, the retailer problem (P) can be cast as the following single-level MILP:

$$\text{(P)}\quad \min_{\substack{E_{n,t},e_t^+,e_t^-,\\ C_t^p,\kappa_n,\Xi_n}} \sum_{t\in T}\mu_t\Delta_t\left(e_t^+ + e_t^-\right) + \sum_{t\in T} C_t^p + \sum_{n\in\mathcal{N}}\kappa_n$$

$$\text{s.t.}\quad (6),(7),(9),(11),(14),(15),(2),(3)$$

D. Assumptions at the retailer level

This paper considers that the generation system is mainly based on hydro and thermal plants and that exists a vertical integration between retail and generation. The generator company has the monopoly in a certain region, such as many regions in Brazil. The electricity variability in those regions are costs assumed by the generator. In liberalized markets, high variability means high risk of imbalance, so high risk of costs. Incentives, I_n, may be negotiate between the retailer and the user $n\in\mathcal{N}$ via lease agreement.

E. Solution Methodology

Different methodologies can be applied to solve (P). The centralized approach would solve it directly by using a available commercial solvers.

An alternative methodology is to decompose (P) into subproblems but solve them in a single machine. Alternatively, in this work Dantzig-Wolfe decomposition is applied to decompose (P) into $|\mathcal{N}|$ subproblems and distribute their solution into each user home management system.

Next, problem (P) is conveniently reformulated to be decomposed by DWD. Let Υ_n be the polyhedron that defines the set of constraints of each client n. Let also Ω_n and Γ_n denote, respectively, the set of extremes points and extremes rays of $conv(\Upsilon_n)$. Note that adding bounds to users costs leads to $\Gamma_n = \emptyset$. For each user n, Ξ_n is the vector of decision variables, $\mathbf{c}_n$ denotes the vector of cost coefficients and A_n is the matrix with the coefficients of constraints. Let define the vectors $\mathbf{e^i} = [e_1^i, e_2^i, \ldots, e_{|T|}^i]\ \forall\, i\in\{+,-\}$ and the transpose of the variable vector $\mathbf{y}$ as $\mathbf{y}_t = [\mathbf{e}^+\ \mathbf{e}^-]$, $\mathbf{1}_{1\times|2T|}$ as a vector of ones and Q as a matrix with constraints coefficients for $\mathbf{e}^+$ and $\mathbf{e}^-$. Let also concatenate the constraints' right hand side in the vector $\mathbf{b}$. Let define $\boldsymbol{\mu}$ and $\boldsymbol{\Delta}$ as vector for variables μ_t and parameters Δ_t, respectively. Let define $\mathcal{G}_n$ and Υ_n as:

$$\mathcal{G}_n = \left\{\Xi_n\in\mathcal{X}|\ \kappa_n \geq \sum_{t\in T}\left(C_t^p - C_{n,t}^s + C_{n,t}^{chp}\right) + C_n^{ev} - \overline{C}_n\right\}.$$

$$\Upsilon_n = \{\Xi_n\in\mathcal{X}|\ f_d(\Xi_n)\leq D_n,\ \kappa_n \geq\ I_n, \Xi_n\in\mathcal{F}_n\cap\mathcal{G}_n\}.$$

As shown at the bottom of Figure 2, each residential user solves its own problem and sends the solution to the retailer. The retailer only sends to the users information on DWD dual variables thus preserving other users privacy. The details of how DWD [33] is particularized for problem (P) can be found in the Appendix.

IV. Results and Discussion

This section presents the results of applying the proposed DWD approach and compare them with those obtained with a centralized approach (CA). A time horizon of one day divided in intervals of 10 minutes, $|T| = 144$, is considered. This work considers real data for users from [18] and assumes forecasted values for uncertain parameters. For user's models, this paper considered data from Belo Horizonte, a Brazilian city. Users are served by Itutinga Hydroelectric Power Plant. For the sake of simplicity, retailer prices remain unchanged over the time span, considering $\pi_t = 0.01$ \$/kWh and $\mu_t = 0.002$ \$/kWh $\forall\, t\in T$. Due equity government law, I_n is constant for all $n\in\mathcal{N}$, initially set to 0.01\$.

Simulations were run using CPLEX 12.8 under AMPL on a Intel(R) Xeon(R) X5675-3.07GHz and 96 GB of RAM.

As can be seen in Figures 3 and 4 the framework efficiently flattens the system load profile. Without any coordination, near time step 96, there is a large consumption peak anticipating the increase in the Time of Use (TOU) tariff ($\boldsymbol{\pi}$) between time step 96-125. Although from the users perspective it is more economical to storage energy to avoid consumption at high prices, this is not convenient for the retailer.

In order to measure the system efficiency, the Peak to Average ratio is typically used. This ratio is defined as $\text{PAR} = |T|\left(\max_t\left\{\sum_{n\in N}E_{n,t}\right\}\right)\left(\sum_{t\in T}\sum_{n\in N}E_{n,t}\right)^{-1}$. Particularly, the results for $|\mathcal{N}|$ =10000 provide the largest improvement (44.7%) in PAR, being equal to 2.06 without coordination, and to 1.14 under the DR strategy.

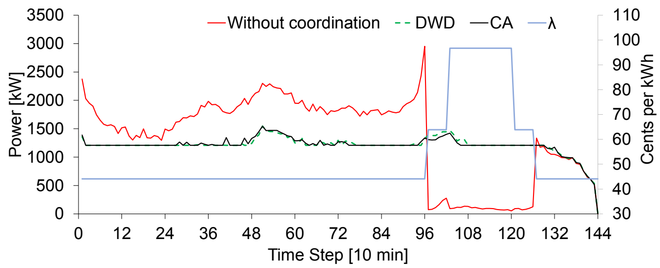

Figure 3: System load profile for $|\mathcal{N}|$ =500

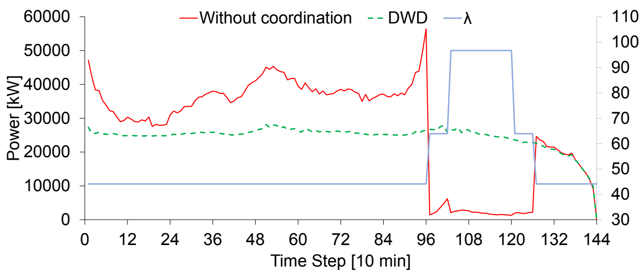

Figure 4: System load profile for $|\mathcal{N}|$ =10000

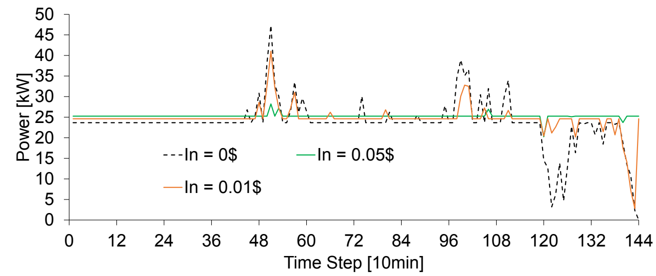

Figure 5: Impact of I_n on system load profile - $|\mathcal{N}| = 10$.

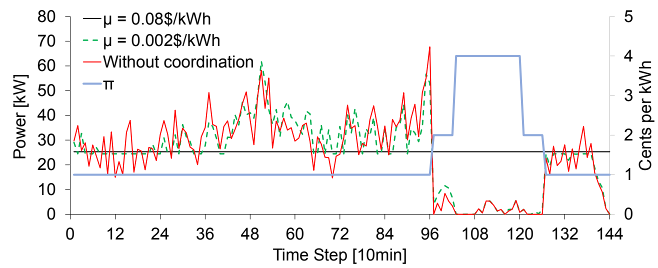

Figure 6: Impact of μ_t on system load profile - $|\mathcal{N}| = 10$.

Regarding the retailer profits, if only the aggregated demand variability is minimized lower savings are attained. As shown in Figure 5, for larger incentives, $I_n = 0.05\,\$$, the retailer succeeds in reducing the load variability at the expense of lower savings. Retailer total costs amount to 35.1 \$, 36 \$ and 36.9 \$ for $I_n = 0\,\$$, $I_n = 0.01\,\$$ and $I_n = 0.05\,\$$, respectively. For $I_n \geq 0.06\,\$$, constraint (11) cannot be satisfied, resulting in an infeasible solution. Without explicitly considering I_n, as in [5], or with $I_n = 0$, as in [30], the retailer achieves the best savings, but it would be difficult to maintain users in the DR program without incentive. Regarding the user profits, $\kappa_n = I_n\ \forall\ n \in \mathcal{N}$. When $I_n = 0.05$ per day, the monthly electricity bill reduction is around 1%.

From the retailer perspective, there is a trade-off between the electricity acquisition cost ($\boldsymbol{\pi}$) and the balancing price ($\boldsymbol{\mu}$). Assuming that $\boldsymbol{\pi}$ follows a TOU tariff as stated in Figure 6, the following two cases can be analyzed. For $\mu_t = 0.002$ \$/kWh $\forall\ t \in T$, the retailer costs sum up to 40.76\$, being the load variability minimization a second priority, since the aggregated load is not flatten as shown previously. However, for $\mu_t = 0.08$ \$/kWh $\forall\ t \in T$, the retailer costs are increased up to 53.11\$ but the load variability is reduced to the minimum. Without any coordination, the retailer total cost computed as $\overline{C}^p + \overline{R}^c$ amount to 41.49\$ and 86.21\$, respectively. Hence, coordination is beneficial for the retailer in any case.

Using one computer, computation times for CA and DWD are summarized in Table I. As can be seen, the centralized approach is faster than the DWD approach if the number of users is less than or equal to 500. However, CA is unable to solve instances of (P) with more than 500 users in less than 2 weeks, which confirms the scalability issues of this method. Furthermore, in practice each user and the retailer should solve its own problem for privacy reasons. This is like solve the user problems in parallel computing. Thus, an approximation of practical cpu times for DWD can be obtained dividing the results in Table I by $|\mathcal{N}|$. The estimated time for 10,000 users is 130.5 seconds.

It is worth to mention that the master problem and subproblems were not solved in parallel in this work. Particularly, overhead can be due to the complete reconstruction of the optimization model for each subproblem. However, in a distributed implementation ($\mathcal{N}+1$ computers), we believe that the estimated computation times can be reduced, since users data are stored in the RAM memory of their respective computers, thus avoiding reconstructing the models.

Table I: Summary Results - one computer

$\vert\mathcal{N}\vert$	Time [s]		$\vert\mathcal{N}\vert$	Time [s]	
	CA	DWD		CA	DWD
10	10,18	507	50	238,7	856
100	1000	3032	300	8244	25314
500	12522	34515	1000	-	89631
5000	-	588870	10000	-	1305448

V. Conclusions

The main motivation of this work is to provide retailers involved in the implementation of DR programs with a practical tool. This tool coordinates the consumption of householders so that their comfort and costs remain unchanged after coordination. A two-phase framework is proposed in which first the users expenses are individually minimized without any coordination. Then, in a second phase, the retailer minimizes systems costs by shaving aggregated peak loads while keeping the desired comfort level and costs for every user. The resulting problem is first posed as a bilevel problem and then a single-level reformulation is presented. Privacy and scalability needs are preserved by applying Dantzig-Wolfe decomposition to solve the problem in a distributed fashion. The simulation results show that the proposed framework is a useful tool for retailers who seek to decrease the aggregate electricity demand curve peaks while maintaining the level of users comfort at no financial loss.

To extend this work, we propose two avenues of research. Firstly, uncertainties in both retailer and user parameters will be characterized using stochastic or robust optimization. This extension would allow to implement measures to control the risk taken by the retailer for participating in the day-ahead electricity market. Secondly, fairness (incentives accordingly to the participation of each user) will be used. This makes the model more difficult to solve, but makes DR more acceptable in practice.

APPENDIX

Applying DWD approach [33] in (P), we find the following Problem (16)-(21):

$$\min_{\mathbf{y}, \Xi_n, \sigma} \quad \boldsymbol{\mu}\boldsymbol{\Delta}\mathbf{1}^T\mathbf{y} + \sum_{n\in N}\sum_{h\in\Omega^n} (\mathbf{c}_n)^T h\sigma_n^h \tag{16}$$

$$Q\mathbf{y} + \sum_{n\in N}\sum_{h\in\Omega^n} A_n h\sigma_n^h = \mathbf{b} \tag{17}$$

$$\sum_{h\in\Omega^n} \sigma_n^h = 1 \ \forall\ n\in\mathcal{N} \tag{18}$$

$$\sigma \geq 0 \tag{19}$$

$$\sum_{h\in\Omega^n} h\sigma_n^h = \Xi_n \ \forall\ n\in\mathcal{N} \tag{20}$$

$$\Xi_n \in \Upsilon_n \ \forall\ n\in\mathcal{N} \tag{21}$$

In the Formulation (16)-(21), each extreme point $h\in\Omega^n$ is associated to a variable σ_n^h, and to a column $A_n h$ that corresponds to a load schedule for user $n\in\mathcal{N}$. That formulation has a exponential amount of variables; therefore, one algorithm to solve it is the Column-Generation (CG), whose details can be found in [34]. In summary, CG has two problems: the Restricted Master Problem (RMP) and at least one pricing problem denoted as sub-problem (SP). In our case, each client $n\in\mathcal{N}$ will have its own sub-problem SP^n. The RMP (22)-(25) is obtained by relaxing Constraints (20)-(21) and using a subset of columns for each client $n\in\mathcal{N}$: $\overline{\Omega}^n \subset \Omega^n$:

$$\min_{\mathbf{y}, \sigma} \quad \boldsymbol{\mu}\boldsymbol{\Delta}\mathbf{1}^T\mathbf{y} + \sum_{n\in N}\sum_{h\in\overline{\Omega}^n} (\mathbf{c}_n)^T h\sigma_n^h \tag{22}$$

$$Q\mathbf{y} + \sum_{n\in N}\sum_{h\in\overline{\Omega}^n} A_n h\sigma_n^h = \mathbf{b}\ (v_1^d) \tag{23}$$

$$\sum_{h\in\overline{\Omega}^n} \sigma_n^h = 1 \ \forall\ n\in\mathcal{N}\ (v_2^d) \tag{24}$$

$$\sigma \geq 0 \tag{25}$$

In the RMP, v_1^d and v_2^d are dual variable vectors associated with constraints (23) and (24), respectively. For a user $n\in\mathcal{N}$ and for a given $h\in\Omega$, the reduced cost of a variable σ_n^h is calculated as $(\mathbf{c}_n)^T h - (v_1^d)^T A_n h - v_2^d$. The target of SP^n is to find the extreme point that have the minimal reduced cost:

$$h^* \in \inf_{h\in\Omega^n} \{(\mathbf{c}_n)^T h - (v_1^d)^T A_n h - v_2^d\} \tag{26}$$

The point h^* is found by the problem SP^n (27)-(28).

$$\min_{\Xi_n} \quad (\boldsymbol{c}_n)^T\Xi_n - (v_1^d)^T A_n \Xi_n - v_2^d \tag{27}$$

$$\Xi_n \in \Upsilon_n \tag{28}$$

The CG starts by solving RMP with some columns, which can be generated using the heuristic from [30] that provides feasible columns. Iteratively new columns will be added in $\overline{\Omega}^n$ accordingly the result of the SP^n. Once RMP is solved, each SP^n is solved. If its reduced cost is negative, the extreme point $h^*\in\Omega^n$ will be added in the RMP as a new column. The CG stops when there is no negative reduced costs.

Since the RMP is a relaxation, the final solution proposed by the CG may have fractional values for binary variables, which violate binary constrains. Branch-and-price algorithm is used to obtain an optimal integer solution considering the RMP as the node root of the branching tree. As the CG proposes solutions with small integrity gap for our instances (the biggest integrity gap found is 0.001), the *diving heuristic procedure* from in [35] is considered in coordination with CG to prove optimality.

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