Service Network Design for Same-Day Delivery with Hub Capacity Constraints

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We study a new service network design problem for an urban same-day delivery system in which the number of vehicles that can simultaneously load or unload at a hub is limited. Due to the presence of both time constraints for the commodities and capacity constraints at the hubs, it is no longer guaranteed that a feasible solution exists. The problem can be modeled on a time-expanded network and formulated as an integer program. To be able to solve real-world instances, we design and implement three heuristics: (1) an integer programming based heuristic, (2) a metaheuristic, and (3) a hybrid matheuristic. An extensive computational study using real-world instances (with different geographies, market sizes, and service offerings) from one of China's leading comprehensive express logistics service providers demonstrates the efficacy of the three heuristics.

Key words: service network design; hub capacity constraints; same-day delivery; heuristics

1. Introduction

Package delivery represents a significant part of the transportation industry. Measured by revenue, package delivery has been the fastest growing segment of the freight transport business in the United States in the 21st century, with one representative player (UPS) reporting a revenue increase from \$30 billion in 2000 to \$72 billion in 2018. A critical aspect of package delivery is the service. Driven by the growth of e-commerce which relies heavily on faster delivery times (Turban et al. 2015), package express carriers have long been aiming to provide same-day delivery service with wide coverage, especially in major metropolitan areas. Being able to do so is expected to increase demand (and profit) significantly given the compelling value proposition of same-day delivery for consumers (Savelsbergh and Van Woensel 2016).

To provide an economically viable (same-day) delivery service, carriers need to carefully allocate and utilize their resources. The challenge is to identify consolidation opportunities (so as to keep the costs down) while satisfying the service guarantees offered to customers (so as to maintain or increase market share). Service network design problems (Crainic 2000, Wieberneit 2008) have long been used to aid in this process, and are usually modeled on a time-expanded network, especially when the time aspect is critical. The goal is to minimize cost through consolidation by choosing the right shipment paths in both space and time. A consolidation transportation system typically employs a complex hub network, in which vehicles transport packages between hubs, and packages are unloaded, sorted, and loaded at hubs. Routing packages through intermediate hubs is key to consolidation, but requires additional time and additional loading and unloading.

The novel feature in the service network design problem investigated in this paper is a restriction on the number of vehicles that can simultaneously be loaded and unloaded at a hub, i.e., the presence of hub capacity constraints. These hub capacity constraints can cause waiting at hubs, which, in turn, can result in certain shipment paths no longer being time feasible, especially in urban same-day delivery environments with aggressive service offerings. In long-haul transportation systems of consolidation carriers such hub capacity constraints are usually not relevant, because the hubs are large (some very large) with many loading and unloading docks. However, in short-haul transportation systems in urban areas such hub capacity constraints are critical, because real-estate is expensive (and will continue to become more expensive) and the hubs are small (some very small) with only a few loading and unloading docks. Although service network design models and methods have been developed for various settings, to the best of our knowledge, none of these have considered accommodating these hub capacity constraints, especially not in a same-day delivery environment.

The restriction on the number of vehicles that can simultaneously be loaded and unloaded at a hub has an important consequence, which is typically not encountered in service network design problems: the existence of a feasible solution is no longer guaranteed. For example, it may not be feasible to send all packages directly from their origin to their destination as this requires a large number of vehicles and the limit on the number of vehicles that can be loaded and unloaded simultaneously at hubs may result in so much waiting that some packages will no longer be able to meet their service guarantee. Therefore, this service network design variant has two objectives: (1) maximize the number of origin-destination markets that can be served, and (2) given the origin-destination markets that are served, minimize the cost of serving these markets.

The variant of the service network design problem investigated is motivated by the settings encountered in China's major mega-cities at package express carrier SF Express (and the instances used in our computational study are representative of these settings). The same-day service products of interest include the "12-18 service", the "14-20 service", and the "12-20 service". Each service product specifies the latest time customers should present their packages to the carrier and the latest time that the carrier guarantees delivery takes place. For example, a customer choosing the "12-18 service" product has to present their packages to the carrier before noon, and the carrier guarantees that these packages will be delivered no later than 6 PM (the same day). This does not mean that all packages for service product *12-18* are available at an origin hub at noon, because in the SF Express system couriers collect packages from customers and deliver these packages to an *access point* (a parcel box), and then riders deliver packages from the access point to the origin hub. Similarly, it does not mean that all packages for service product *12-18* need to be at the destination hub by 6 PM, because riders need to deliver the packages to an access point, where they are picked up by couriers to handle the ultimate delivery. The assignment of access points to hubs, i.e., deciding which access points send packages to and receive packages from a particular hub, has an impact on the consolidation opportunities in the system (and so does, although to a lesser extent, the assignment of customer locations to access points). Ideally, these assignment decisions should be incorporated in the design of the service network connecting the hubs. In this study, however, we focus only on the design of the service network connecting the hubs, and, therefore, implicitly assume that the access point to hub assignments have already been made and are reflected in the number of packages passing through a hub (arriving at or departing from the hub). Similarly, the routing and scheduling of couriers and riders has an impact on the time when packages become available at an origin hub and the time when they have to reach a destination hub. Again, we do not explicitly model decisions involving these "pre-processing" and "post-processing" activities when designing the service network connecting the hubs, but assume that a certain amount of time (1.5 to 2 hours) is reserved for these activities. For example, if 2 hours are reserved for the pre-processing and post-processing activities for service product *12-18*, then the carrier needs to transport a package from its origin hub to its destination hub between 2 PM and 4 PM.

The service network design problem with hub capacity constraints that we consider seeks for every market, i.e., for every origin – destination pair of hubs, a path specifying how the packages in this market will be transported from their origin to their destination, and a minimum cost vehicle schedule for transporting the packages that specifies for each vehicle movement, its origin and destination, when loading at the origin starts (and ends), when it departs from the origin (and when it arrives at the destination), and when unloading at the destination starts (and ends), while ensuring that service constraints are satisfied, i.e., loading of packages at their origin starts after they become available and unloading of packages at their destination completes before they are due, and satisfies the operating restrictions at hubs, i.e., the restrictions on the number of vehicles that can simultaneously be loaded and unloaded.

We model this service network design problem using a time-expanded network in which arcs represent the loading, travel, waiting, and unloading of a vehicle and commodities represent packages in a market, and show how this model can be formulated as an integer program (IP). We develop three heuristic algorithms for its solution, because the IP cannot be solved in an acceptable amount of time for real-world instances: (1) an IP-based heuristic, (2) a metaheuristic, and (3) a hybrid matheuristic, which takes advantage of the strengths of the IP-based heuristic and the metaheuristic. Computational experiments show that the hybrid matheuristic produces high-quality solutions in a reasonable amount of time. The contributions of this research are summarized as follows:

- A new service network design variant is introduced for same-day delivery systems in urban areas and a non-trivial integer programming model for its solution is developed;
- An IP-based heuristic, a metaheuristic, and a hybrid matheuristic, which combines the strengths of the IP-based heuristic and the metaheuristic, are designed and implemented; and
- The efficacy of the heuristics is demonstrated on a number of real-world instances.

The remainder of the paper is organized as follows. In Section 2, we review relevant prior research. In Section 3, we provide a formal description of the same-day delivery system and present a model and formulation using an appropriately constructed time-expanded network. In Section 4, we introduce three heuristics: an IP-based heuristic, a metaheuristic, and a hybrid matheuristic. In Section 5, we present and interpret the results of an extensive computational study. Finally, in Section 6, we finish with conclusions and a discussion of future work.

2. Literature Review

Since we are not aware of any literature addressing the design of a service network for same-day delivery of packages with loading and unloading capacities at hubs, we briefly review literature on same-day delivery, on service network design for package express carriers, on service network design with capacity limits, on service network design modeled on time-expanded networks, and on advances in solving service network design problems.

There is a growing body of literature on same-day delivery, but it is focused almost exclusively on the delivery of packages from a fulfillment center, which gives rise to challenging dynamic routing and scheduling problems (see, e.g., (Pillac et al. 2013), (Psaraftis, Wen, and Kontovas 2016), (Sampaio et al. 2019), and (Alnaggar, Gzara, and Bookbinder 2019)), but is quite different from the same-day delivery environment studied in this paper, which is characterized by package flows between hubs. There is literature on courier operations and dial-a-ride services in urban areas, which do involve taking packages or passengers from a pickup location to a drop-off location (see, e.g., (Berbeglia, Cordeau, and Laporte 2010), (Bouros et al. 2011), and (Ho et al. 2018)), but consolidation and transfers are usually not as critical to feasibility and profitability as they are in the same-day delivery environment studied in this paper.

There is a fair amount of literature on optimizing the design and operations of package express carriers service networks, but the focus has been on inter-city package flows rather than same-day delivery of packages within a city. Examples include Kim et al. (1999), Barnhart et al. (2002), Yildiz and Savelsbergh (2019), and Lin, Zhao, and Lin (2020).

Various types of capacity limits have been incorporated in network design models. Capacity can often be "installed" in the network in discrete units, e.g., trucks transporting goods between two locations. Such discrete units of capacity can be accommodated straightforwardly in an integer programming formulation (with integer variables representing the number of units of capacity to install on an arc). In such formulations, it is easy to distinguish between existing capacity and additional capacity, and, thus, it is possible to model settings with a fixed capacity as well as settings in which capacity can be expanded at a cost; see, for example, Frangioni and Gendron (2009), Gendron, Crainic, and Frangioni (1999). Conceptually, accommodating discrete units of capacities can be viewed as allowing multiple parallel arcs. Another way to enforce capacity limits in a network design problem is to impose maximum in- and/or out-degrees on nodes. For example, Barnhart, Jin, and Vance (2000) formulate a railroad blocking problem as a network design problem, with nodes and arcs representing yards and candidate blocks, respectively, and the capacity of a classification yard is incorporated by imposing maximum in- and out-degrees at the nodes. In the context of air service network design for express shipments, the number of arriving and departing fights is limited by landing and taking-off capacities at the hubs, which can also modeled by imposing maximum in- and out-degrees at the nodes. Again, it is straightforward to extend these models to accommodate installation of additional capacity at a cost.

Another layer of complexity in service network design problems arises when there is a need to incorporate time constraints. Time-expanded networks have been used to model a variety of service network design problems (see, e.g, (Zawack and Thompson 1987), (Haghani and Oh 1996), and (Kennington and Nicholson 2010)). The most recent advances in time-expanded network applications have been focusing on partial time discretization and dynamic time discretization adjustment mechanisms. Examples include Fleischer and Skutella (2007), Boland et al. (2017), and Belieres et al. (2021).

Recent research advances in exact solution methods for solving service network design problems have come from iterative refinement techniques (Boland et al. 2017, Clautiaux et al. 2017) and branch-and-price-and-cut algorithms (Gendron and Larose 2014, Rothenbächer, Drexl, and Irnich 2016). More effective matheuristics, i.e., approaches integrating metaheuristic concepts and mathematical programming techniques, continue to appear. Gendron, Hanafi, and Todosijević (2018) present a matheuristic that iterates between linear programming and slope scaling for multicommodity capacitated fixed-charge network design. Crainic et al. (2014) combine column generation, metaheuristic, and exact optimization techniques in dealing with service network design with resource constraints. In liner shipping network design, Brouer, Desaulniers, and Pisinger (2014) use an integer program to search for improvements in their matheuristic. For less-than-truckload load plan design, Lindsey, Erera, and Savelsbergh (2016) develop an effective neighborhood search heuristic for solving a natural integer programming model where a modified and restricted version of the integer program is solved to find improving changes during each iteration of the matheuristic.

As eluded to in the introduction, to offer the urban same-day service product considered in this paper, a package express carrier needs to make several other decisions in addition to the service network design decisions investigated, e.g., the assignment of demand nodes (e.g., customer locations and access points) to hubs and the routing and scheduling of transportation resources (e.g., couriers and riders) between demand nodes and hubs. That is, we are focusing on just one layer in a hierarchical hub network. These other decisions are explored in a number of studies on hub network design in the literature. Examples include Smilowitz and Daganzo (2007), Yaman, Karasan, and Kara (2012), Dukkanci and Kara (2017), and Masaeli, Alumur, and Bookbinder (2018). Recently, Alumur et al. (2021) have observed and argued that to better align strategic and tactical decisions, there is a need to integrate hub location models with service network design research. Even though we focus solely on the service network design component, we fully agree with the need to explore integrated models. Alumur et al. (2021) also observe the importance of time, which is critical in our setting: "The time dimension has not received the attention it deserves in hub location research, in spite of its capital importance in the design and operation of hub systems."

3. Problem Description

Let $D = (N, A)$ be a flat network with node set N modeling physical locations or hubs and directed arc set A modeling travel between locations. A travel time $\tau_{ij} \in \mathbb{N}_+$ and a travel cost $c_{ij} \in \mathbb{R}_+$ are associated with each arc $a = (i, j) \in A$. Let K denote a set of commodities to be served, each of which has a single source node $o_k \in N$ (later referred to as the commodity's origin), a single sink node $d_k \in N$ (later referred to as the commodity's destination), and a quantity $q_k \in \mathbb{R}_+$ that needs to be routed from its origin to its destination along a single geographic path $P_k = (o_k, \ldots, d_k)$ using a homogeneous fleet of vehicles with capacity $Q \in \mathbb{N}_+$. We assume that $q_k \leq Q$ for all $k \in K$. Commodity $k \in K$ becomes available at its origin at time $e_k \in \mathbb{N}_+$ and is due at its destination at time $l_k \in \mathbb{N}_+$.

We assume that each commodity $k \in K$ will be loaded every time it departs from a hub $i \in N$ and will be unloaded every time it arrives at a hub $i \in N$. This assumption is in line with practices at many package express carriers. It simplifies operations at hubs; selective loading and unloading packages is prone to human error, especially under time pressure. We assume the loading of vehicle takes $\tilde{\tau}_l \in \mathbb{N}_+$ and the unloading of a vehicle takes $\tilde{\tau}_u \in \mathbb{N}_+$. For ease of presentation, we assume these times are the same for all hubs, but it is easy to accommodate hub-dependent loading and unloading times. Each hub $i \in N$ has loading capacity $L_i \in \mathbb{N}_+$ and an unloading capacity $U_i \in \mathbb{N}_+$, which represent the maximum number of vehicles that can be loaded and unloaded at the same time, respectively. At hubs, we assume that (unlimited) parking space is available for vehicles to wait when loading or unloading operations cannot start upon arrival, and that (unlimited) storage space is available to temporally hold packages. Finally, we assume, without loss of generality, that a vehicle departs as soon as it is loaded. Thus, a vehicle may only wait before it gets loaded or before it gets unloaded.

The Service Network Design with Hub Capacities (SNDHC) problem seeks to determine a path P_k for each commodity $k \in K$ and a vehicle schedule that implies loading and unloading start times at each hub in the path P_k, such that the loading at the origin hub starts at or after e_k and the unloading at the destination hub starts at or before $l_k - \tilde{\tau}_u$ and that satisfies the hub capacity constraints, i.e., for every hub $i \in N$ and at any time $t \in [\min_{k \in K} e_k, \max_{k \in K} l_k]$, there are no more than L_i vehicles loading and no more than U_i vehicles unloading. The restriction on the number of vehicles that can simultaneously be loaded and unloaded at a hub implies that the existence of a feasible solution is no longer guaranteed. Therefore, the objective of SNDHC is to minimize the cost of a vehicle schedule that maximizes the number of commodities for which a feasible path can be found.

We use a time-expanded network to model SNDHC. We derive a time-expanded network $\mathcal{D} = (\mathcal{N}, \mathcal{A} \cup \mathcal{H})$ from flat network D and a set of time points $T = \bigcup_{i \in N} T_i$ with $T_i = \{t_1^i, \ldots, t_{n_i}^i\}$. The timed node set $\mathcal{N}$ has a node (i,t) for each node $i \in N$ and $t \in T_i$. The holding arc set $\mathcal{H}$ contains arcs $((i,t_g^i),(i,t_{g+1}^i))$ for all $i \in N$ and $g = 1, \ldots, n_i - 1$. A holding arc $((i,t_g^i),(i,t_{g+1}^i))$ models the possibility of holding packages at hub i for a period of time while they wait to be loaded onto a vehicle. The movement arc set $\mathcal{A}$ contains arcs of the form $((i,t),(j,\bar{t}))$, where $(i,j) \in A$, $t \in T_i$, and $\bar{t} \in T_j$. An arc $((i,t),(j,\bar{t}))$ models the possibility of sending packages from hub i to hub j with the loading of packages starting at i at time t and the unloading of packages finishing at j at time $\bar{t}$. This implies that the vehicle departs from i at time $t + \tilde{\tau}_l$ and waits at j from $t + \tilde{\tau}_l + \tau_{ij}$ until $\bar{t} - \tilde{\tau}_u$. (Thus, we must have that $(\bar{t} - \tilde{\tau}_u) - (t + \tilde{\tau}_l) \geq \tau_{ij}$.)

Observe that arc $((i,t),(j,\bar{t}))$ and arc $((i,t),(j,\bar{t}'))$ represent a different set of activities even though the vehicle travels at the exact same time (departing at i at $t + \tilde{\tau}_l$ and arriving at j at $t + \tilde{\tau}_l + \tau_{ij}$). This differs from most service network design problems, where it suffices to model vehicle movements, i.e., have arcs of the form $((i,t),(j,t+\tau_{ij}))$. However, to be able to accurately model the hub capacity constraints, it is necessary to explicitly embed the loading and unloading of a vehicle into the arc. Different packages traveling from i to j on their journey from their origin to their destination can start loading at i at the same time t but start unloading at j at different times $\bar{t}-\tilde{\tau}_u$ at $\bar{t}'-\tilde{\tau}_u$ due to differences in waiting time at j of the vehicle that transports them.

We use a time-expanded network $\mathcal{D}^{\Delta}$ derived from $\mathcal{D}$ with a regular time discretization controlled by parameter $\Delta \in \mathbb{N}_+$. Specifically, we let $T_i = \{E\Delta, (E+1)\Delta, \cdots, L\Delta\}$ for all $i \in N$ where $E, L \in \mathbb{N}_+$ with $\min_{k\in K} e_k/\Delta - 1 < E \leq \min_{k\in K} e_k/\Delta$ and $\max_{k\in K} l_k/\Delta \leq L < \max_{k\in K} l_k/\Delta + 1$. In the time-expanded network $\mathcal{D}^{\Delta}$, for every pair of nodes (i,t) and $(j,\bar{t})$ where $(i,j) \in A$ and $(\bar{t}-\tilde{\tau}_u)-(t+\tilde{\tau}_l) \geq \tau_{ij}$, there is a movement arc $((i,t),(j,\bar{t}))$ in $\mathcal{A}$, i.e., we consider all possible loading and unloading time options for every possible vehicle travel option. To handle the discretization of time, we adopt a standard mapping that rounds up travel times, rounds up loading and unloading times, rounds up commodity availability times, and rounds down times commodity due times, so that a feasible solution to the service network design model on the time-expanded network $\mathcal{D}^{\Delta}$ can always be converted to a feasible schedule in continuous time.

We denote the service network design problem with hub capacities defined on the time-expanded network $\mathcal{D}^{\Delta}$ described above by SNDHC($\mathcal{D}$). Let $y_{ij}^{t\bar{t}}$ represent the number of times arc (i,j) is used to accommodate dispatches that start loading at hub i at time t and finish unloading at hub j at time $\bar{t}$. Because multiple vehicles can perform the same movement, the $y_{ij}^{t\bar{t}}$ variables have to be integer. Let $x_{ij}^{kt\bar{t}}$ represent whether commodity $k \in \mathcal{K}$ travels on movement arc $((i,t),(j,\bar{t}))$. For convenience, let $\mathcal{A}^k$ and $\mathcal{H}^k$ denote the movement and holding arc sets containing movement and holding arcs that can possibly be used by commodity $k \in K$, respectively. Let $\mathcal{N}'$ be a timed node set that contains a node (i,t) for each node $i \in N$ and $t \in T_i'$ where $T_i' = \{\min_{k\in K} e_k, \min_{k\in K} e_k + 1, \ldots, \max_{k\in K} l_k\}$. Because each commodity must follow a single path from its origin to its destination, the $x_{ij}^{kt\bar{t}}$ variables have to be binary. Finally, let z_k be a binary variable indicating whether commodity $k \in K$ is served or not. With these variables, SNDHC($\mathcal{D}$) can be formulated as follows:

$$\max \sum_{k\in K} z_k \qquad \text{(Phase 1)}$$

$$\min \sum_{((i,t),(j,\bar{t}))\in\mathcal{A}} c_{ij} y_{ij}^{t\bar{t}} \qquad \text{(Phase 2)}$$

s.t.

$$\sum_{((i,t),(j,\bar{t}))\in\mathcal{H}^k\cup\mathcal{A}^k} x_{ij}^{kt\bar{t}} - \sum_{((j,\bar{t}),(i,t))\in\mathcal{H}^k\cup\mathcal{A}^k} x_{ji}^{k\bar{t}t} = \begin{cases} +z_k & (i,t)=(o_k,e_k), \\ -z_k & (i,t)=(d_k,l_k), \ \forall k \in K, (i,t) \in \mathcal{N}; \\ 0 & \text{otherwise}; \end{cases} \tag{1a}$$

$$\sum_{k\in K} q_k x_{ij}^{kt\bar{t}} \leq Q y_{ij}^{t\bar{t}} \qquad \forall ((i,t),(j,\bar{t})) \in \mathcal{A}; \tag{1b}$$

$$\sum_{((i,s),(j,\bar{s}))\in\mathcal{A}: t-\tilde{\tau}_l < s \leq t} y_{ij}^{s\bar{s}} \leq L_i \qquad \forall (i,t) \in \mathcal{N}'; \tag{1c}$$

$$\sum_{((i,s),(j,\bar{s}))\in\mathcal{A}:\bar{t}\leq\bar{s}<\bar{t}+\bar{\tau}_u} y_{ij}^{s\bar{s}} \leq U_j \qquad \forall (j,\bar{t})\in\mathcal{N}'; \tag{1d}$$

$$x_{ij}^{kt\bar{t}} \in \{0,1\} \qquad \forall ((i,t),(j,\bar{t}))\in\mathcal{A}^k\cup\mathcal{H}^k \quad \forall k\in K; \tag{1e}$$

$$y_{ij}^{t\bar{t}} \in \mathbb{N}_{\geq 0} \qquad \forall ((i,t),(j,\bar{t}))\in\mathcal{A}; \tag{1f}$$

$$z_k \in \{0,1\} \qquad \forall k\in K; \tag{1g}$$

That is, SNDHC($\mathcal{D}$) problem seeks to maximize the total number of commodities to be served on time in Phase 1, and to minimize the total travel cost in Phase 2 while ensuring the maximum number of served commodities. Constraints (1a) ensure that each served commodity departs from its origin after it becomes available and arrives at its destination before it is due. The presence of holding arcs allows a commodity to arrive early at its destination or depart late from its origin. Constraints (1b) ensure that a sufficient number of vehicles is available for the commodities that are sent from hub i to hub j starting loading at time t and completing unloading at time $\bar{t}$. Constraints (1c) and (1d) ensure that the number of vehicles that are loading or unloading simultaneously does not exceed the specified hub capacity limits across all locations at any time during the planning horizon. Constraints (1e), (1f) and (1g) define the variables and their domains.

Since there is a movement arc $((i,t),(j,\bar{t}))\in\mathcal{A}$ for every pair of nodes (i,t) and $(j,\bar{t})$ with $(i,j)\in A$ and $(\bar{t}-w_u)-(t+w_l)\geq\tau_{ij}$, the number of variables $x_{ij}^{kt\bar{t}}$ and $y_{ij}^{t\bar{t}}$ is huge; prohibitively large for real-life instances. Furthermore, to ensure the hub capacity restrictions across all locations and at any time of the planning horizon, the number of loading and unloading capacity constraints is huge too; prohibitively large for real-life instances. Therefore, solving SNDHC($\mathcal{D}$), or even obtaining high-quality solutions, using a standard commercial solver is practically impossible, which motivates the development of heuristics.

4. Methodology

In this section, we start by introducing two heuristic approaches for solving SNDHC($\mathcal{D}$), one IP-based heuristic and one metaheuristic. Then, we present a hybrid matheuristic that takes advantage of the strengths of both heuristics.

4.1. An IP-based heuristic

Our IP-based heuristic (IP-H) for obtaining a high-quality solution solves multiple small IPs, derived from the original IP, which greatly reduces the computational effort. IP-H solves an IP for every hub that serves as origin or destination for at least one commodity in a pre-specified sequence. Let $h\in N$ be the hub under consideration. Let $K_h^1\subseteq K$ denote the set of commodities for which h is either the origin or the destination, i.e., $K_h^1=\bigcup_{k\in K:o_k=h}k\cup\bigcup_{k\in K:d_k=h}k$. Let $K_h^2\subseteq$

$K \backslash K_h^1$ denote the set of commodities (for which h is not the origin or the destination) which have been assigned a feasible path when the previous hub in the sequence was considered (if h is the first hub in the sequence, then $K_h^2 = \emptyset$). The set of commodities considered for hub h is $K_h = K_h^1 \cup K_h^2$. We formulate an arc-based hierarchical model (the small IP) that seeks to serve as many commodities $k \in K_h$ as possible, i.e., seeks a movement path $\mathcal{P}^k = (a_1 = ((i_1, t_1), (j_1, \bar{t}_1)), a_2 = ((i_2, t_2), (j_2, \bar{t}_2)), \ldots, a_g = ((i_g, t_g), (j_g, \bar{t}_g)))$ for each commodity $k \in K_h$ to be served with timed arc $a_p \in \mathcal{A} \cup \mathcal{H}$, $(j_p, \bar{t}_p) = (i_{p+1}, t_{p+1})$ for $p = 1, \ldots, g-1$, $(i_1, t_1) = (o_k, e_k)$ and $(j_g, \bar{t}_g) = (d_k, l_k)$. The associated geographic path is $P^k = (a_1 = (i_1, j_1), a_2 = (i_2, j_2), \ldots, a_g = (i_g, j_g))$ with $a_p \in A$, $j_p = i_{p+1}$ for $p = 1, \ldots, g-1$, $i_1 = o_k$ and $j_g = d_k$. For notational convenience, we let $P^{k'}$ denote the geographic path that is assigned to commodity $k \in K_h^2$ when processing the previous hub in the sequence (if hub h is not the first hub in the sequence).

Commodities in K_h^1 are allowed to follow any feasible timed path in the network, but commodities in K_h^2 are forced to follow either the previously assigned path (but possibly a different timed copy) or the direct path from the commodity's origin to its destination. For each commodity $k \in K_h$, we denote the set of feasible timed arcs by $\mathcal{A}^{hk}$ and the set of feasible holding arcs by $\mathcal{H}^{hk}$. Let $\mathcal{A}^h = \bigcup_{k \in K_h} \mathcal{A}^{hk}$ and $\mathcal{H}^h = \bigcup_{k \in K_h} \mathcal{H}^{hk}$. Let z_k for $k \in K_h$ be a binary decision variable denoting whether commodity k is served in the optimization problem for hub h. In Phase 1, we maximize the number of commodities served (from among the commodities in K_h). In Phase 2, we minimize the total travel distance, while enforcing that the number of served commodities is the number of commodities found in Phase 1. This hierarchical optimization approach captures the objectives of SNDHC($\mathcal{D}$), but also facilitates serving a large number of commodities as minimizing the total distance in the second phase tends to free up unloading and loading capacity for commodities that have not yet been considered (as they will only be considered when we process hubs that are later in the sequence).

The formulation of the arc-based hierarchical model is as follows:

$$\max \sum_{k \in K_h} z_k \qquad \text{(hierarchical model phase 1)}$$

$$\min \sum_{((i,t),(j,\bar{t})) \in \mathcal{A}^h} c_{ij} y_{ij}^{t\bar{t}} \qquad \text{(hierarchical model phase 2)}$$

s.t.

$$\sum_{((i,t),(j,\bar{t})) \in \mathcal{H}^{hk} \cup \mathcal{A}^{hk}} x_{ij}^{kt\bar{t}} - \sum_{((j,\bar{t}),(i,t)) \in \mathcal{H}^{hk} \cup \mathcal{A}^{hk}} x_{ji}^{k\bar{t}t} = \begin{cases} +z_k & (i,t) = (o_k, e_k) \\ -z_k & (i,t) = (d_k, l_k) \;\; \forall k \in K_h, (i,t) \in \mathcal{N}; \\ 0 & \text{otherwise}; \end{cases} \quad (2a)$$

$$\sum_{k \in K_h} q_k x_{ij}^{kt\bar{t}} \le Q y_{ij}^{t\bar{t}} \qquad \forall ((i,t),(j,\bar{t})) \in \mathcal{A}^h; \quad (2b)$$

$$\sum_{((i,s),(j,\bar{s}))\in\mathcal{A}^h : t-\tilde{\tau}_l < s \le t} y_{ij}^{s\bar{s}} \le L_i \qquad \forall (i,t) \in \mathcal{N}^{'}; \tag{2c}$$

$$\sum_{((i,s),(j,\bar{s}))\in\mathcal{A}^h : \bar{t} \le \bar{s} < \bar{t}+\tilde{\tau}_u} y_{ij}^{s\bar{s}} \le U_j \qquad \forall (j,\bar{t}) \in \mathcal{N}^{'}; \tag{2d}$$

$$x_{ij}^{kt\bar{t}} \in \{0,1\} \qquad \forall ((i,t),(j,\bar{t})) \in \mathcal{H}^{hk} \cup \mathcal{A}^{hk} \quad \forall k \in K_h; \tag{2e}$$

$$y_{ij}^{t\bar{t}} \in \mathbb{N}_{\ge 0} \qquad \forall ((i,t),(j,\bar{t})) \in \mathcal{A}^h; \tag{2f}$$

$$z_k \in \{0,1\} \qquad \forall k \in K_h; \tag{2g}$$

Constraints (2a) ensure that each commodity departs from its origin after it becomes available and arrives at its destination before it is due. Constraints (2b) ensure that sufficient vehicle capacity is available for planned transportation activities on each arc. Constraints (2c) and (2d) ensure that hub capacities are respected at all locations. Constraints (2e), (2f) and (2g) define the variables and their domains.

Observe that if we only allow geographic paths to use at most one intermediate hub (which we do in our computational study), then at most three different (geographic) paths are assigned to a commodity $k \in K$ when processing a hub sequence: one when processing the commodity's origin, one when processing the commodity's destination, and one when processing the intermediate hub on a non-directed path assigned earlier.

A simple, but important observation regarding IP-H is captured in the following proposition.

Proposition 1. *If for some hub sequence, it is not possible to serve all commodities $k \in K_h$ for the first hub h in the sequence, then there exists no feasible solution to SNDHC($\mathcal{D}$) that serves all commodities $k \in K$.*

Proof. Suppose there exists a feasible solution to SNDHC($\mathcal{D}$) where all commodities $k \in K$ are served, such that each commodity has at least one feasible timed path from its origin to destination, and restrictions including vehicle capacities and and hub capacities are all respected. By removing arcs that are only used by commodities $k \in K \backslash K_h$ from that feasible solution, we still keep the feasible timed paths for commodities $k \in K_h$, and all restrictions are still satisfied. Since all possible timed paths are considered for $k \in K_h$ (the network is 'empty' when we process the first hub h in a sequence), the above set of timed paths should have constituted a feasible solution when we process h. Therefore, we have reached a contradiction. Q.E.D.

The quality of solution produced by IP-H, of course, depends on the chosen hub sequence. Preliminary computational experiments have shown that processing the hubs in nonincreasing distance from the centroid or geographic center tends to produce high-quality solutions, and this is the sequence used in our computational study.

4.2. Metaheuristic Approach

We next discuss a multi-start iterated local search (MILS) metaheuristic for solving instances of SNDHC($\mathcal{D}$). Its primary benefit is that it is faster than the IP-based heuristic. The shorter solve time allows MILS to be used for sensitivity analysis, but, maybe more importantly, it can be used to guide construction of the hub sequence used in the IP-based heuristic.

MILS focuses on maximizing the number of commodities served and only uses cost information for breaking ties. MILS has two main loops: an *outer* loop and an *inner* loop; see Algorithm 1. In the outer loop, we perform It^O_{MAX} iterations where, in each iteration, we start from a greedy

Algorithm 1 Multi-Start Iterated Local Search

```
1: input: The sets of hubs and commodities and the travel time matrix;
2: output: A set of paths $\mathcal{S}^*$ serving a large number of commodities;
3: $\mathcal{S}^* \leftarrow \{\}$
4: $It_{\text{MS}} \leftarrow 0$;
5: while $It_{\text{MS}} < It^O_{\text{MAX}}$ do                                   # Outer loop
6:    $\mathcal{S} \leftarrow$ InitialSolution();
7:    $\{P, R\} \leftarrow$ ResetProbabilities();
8:    while termination criteria is not satisfied do                    # Inner loop
9:       for all pairs of hubs (i,j) in random order do
10:          $\mathcal{S} \leftarrow$ NeighborhoodSearch(i, j, $\mathcal{S}$, $P$, $R$);
11:          $\mathcal{S} \leftarrow$ RemoveInfeasiblePaths($\mathcal{S}$);
12:          $\mathcal{S} \leftarrow$ ConnectPaths($\mathcal{S}$);
13:          if $c_1(\mathcal{S}) > c_1(\mathcal{S}^*)$ then
14:             $\mathcal{S}^* \leftarrow \mathcal{S}$;
15:          end if
16:       end for
17:       $P \leftarrow$ UpdateProbabilities($P, R$);
18:    end while
19:    $It_{\text{MS}} \leftarrow It_{\text{MS}} + 1$;
20: end while
21: return $\mathcal{S}^*$;
```

initial solution $\mathcal{S}$ and perform an iterated neighborhood search to improve it, always keeping track of the best solution found during the execution of the algorithm. After an initial solution has been constructed, an attempt is made to improve that solution by iteratively examining pairs of hubs i and j that either have at least one timed path that connects them, or for which there is a commodity originating at i and destined for j that is not served in $\mathcal{S}$. The inner loop terminates after It^I_{max} consecutive iterations without an improvement.

The loading and unloading operations at a hub can be viewed as a sequence of time periods, each with a given start time. For implementation efficiency, such a sequence of time periods is stored in a binary search tree, as illustrated in Figure 1. The binary search trees allows us to quickly detect conflicts between any loading or unloading operations at a hub, which is critical to ensure that the

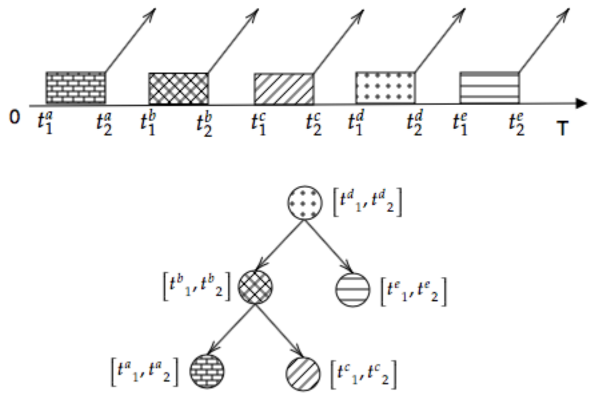


Figure 1 Binary Search Tree representing the loading operations over time at a loading dock of a hub

hub capacity constraints are respected. More specifically, determining whether a given loading or unloading operation (a time period with a given start time) can be added at a hub can be done in $\mathcal{O}(log\, w)$ time, where w is the number of loading or unloading operations in the search tree. Updating the search tree, i.e., adding or deleting a loading or unloading operation, can also be done in $\mathcal{O}(log\, w)$ time.

A solution $\mathcal{S}$ is represented as a set of timed paths and an assignment of commodities to these paths. Each timed path is composed of a sequence of sets of two actions: (1) a vehicle loading period v_i^l (the time period representing when vehicle v starts and finishes loading at location i) and (2) a vehicle unloading period v_j^u (the time period representing when the vehicle v starts and finishes unloading at location j). For example, a timed path $(i \to j \to k)$ is represented as the sequence $\{v_i^l, v_j^u, v_j^l, v_k^u\}$, where each element is stored in the binary search tree representing the activities at the loading or unloading dock used by the vehicle.

Next, we will describe how we obtain an initial solution in each iteration of the outer loop, the neighborhoods used in the local search procedure, and how the neighborhoods are selected in the local search procedure.

4.2.1. Initial solution construction. We start by listing all possible pairs of hubs (i, j) with i representing the origin of a commodity and j representing its destination, respectively, and partition them into three groups, G_1, G_2 and G_3, based on the travel time between the hubs. The travel time is short for pairs in G_1, moderate for pairs in G_2, and long for pairs in G_3. We start constructing the initial solution $\mathcal{S}$ by processing the commodities associated with pairs in G_1 in random order, one by one. Given a pair $(i, j) \in G_1$, we try to create feasible timed paths for all commodities originating at i and destined to j (there can be more than one such commodity because of the different service classes) departing as early as possible from i, always respecting hub capacities and the available time at the origin and the due time at the destination. If a feasible timed path is identified, then $(i \to j)$ is added to the list of timed paths. Before considering the next pair, we check if the path $(i \to j)$ enables the creation of additional paths, i.e., if a timed path $(j \to k)$ exists and the time path $(i \to j \to k)$ is feasible, then $(i \to j \to k)$ is created as it can potentially serve commodities originating at i and destined to k. Once all pairs in G_1 are processed, we repeat the same procedure for the pairs in G_2 and G_3.

4.2.2. Local search. MILS employs several neighborhoods to try and improve a given solution, each involving simple modifications of timed paths or of loading and unloading sequences. The exploration of the neighborhoods is exhaustive, i.e., the solution modifications or moves are performed for all hub pairs (i, j), in random order. Whenever a pair of hubs (i, j) is selected in the neighborhood search, we first check if all commodities originating from i and destined for j are served. In case there is a commodity that is not served, we first try to create a direct timed path from i to j at the earliest possible departure time from i. If no direct timed path is possible, we try to create an indirect timed path $(i \to k \to j)$, where k is chosen randomly from among the hubs that are within given distance from either i or j. Specifically, we check if there exists a timed path $(k \to j)$ that arrives at j before the due time of the commodity and if it is possible to send a vehicle from i to k such that it arrives at k in time, i.e., we only consider creating a new timed path from i to k. In case all commodities are served, or, if we are unable to create paths for all the commodities that are not yet served, we apply one of the following neighborhoods:

– MoveLoadingPeriod(i,j) and MoveUnloadingPeriod(i,j): Each neighborhood tries to delay or advance (chosen with equal probability) the loading or unloading periods of a timed path $(i \to j)$ as much as possible. New docks may be assigned at either i or j, if beneficial.

– ChangeLoadingDock(i,j) and ChangeUnloadingDock(i,j): Each neighborhood tries to change the loading or unloading dock of a timed path $(i \to j)$ in order to create space at the dock to add new loading or unloading operations (which can then be used for other timed paths). The start time of the loading or unloading operation is kept the same.

– SwitchLoadingPeriod(i,j): The neighborhood tries to switch the loading operation of a direct timed path $(i \to j)$ with the loading operation of another, randomly selected, direct timed path $(i \to k)$ at the loading dock. New unloading times and docks may be assigned to the vehicles at both j and k, if necessary. The switch is only performed if it does not affect the feasibility of the commodities assigned to the timed paths.

– CrossoverPaths(i,j): Let $(i \to k \to j)$ and $(l \to m \to j)$ be two distinct timed paths in $\mathcal{S}$. The neighborhood generates new paths by crossing over $(i \to k \to j)$ and $(l \to m \to j)$ into two new paths: $(i \to m \to j)$ and $(l \to k \to j)$. The crossover operation is only applied if the new paths are feasible and do not violate the loading/unloading capacity constraints at the related hubs.

– SwitchDirectPath(i,j): The neighborhood tries to reassign the commodities on the direct timed path $(i \to j)$ to an indirect timed path $(i \to k \to j)$, where k is randomly selected. If all commodities can (feasibly) be reassigned, then $(i \to j)$ is removed from $\mathcal{S}$, hence opening space for a new loading operation at hub i and a new unloading at hub j.

– Re-ArrangeLoadingPeriods(i): Consider the hubs i, j and k and the movement arcs u, v, and w shown in Figure 2a. Each movement arc reflects the time that the loading starts and the time that the unloading finishes. The accompanying rectangles represent the feasible times at which the loading can start, which are determined by the latest time that one of the commodities transported becomes available at i and the departure time at i that ensures an arrival at the time unloading starts. Next, assume that there is a commodity that originates at k and is destined for j. In this example, it is possible to create an indirect timed path $(k \to i \to j)$ by starting the loading of v earlier and starting the loading of w later (but the start times remain within their respective windows, as shown in Figure 2b.

This neighborhood attempts to re-organize the loading operations at the loading dock of hub i by solving a small integer program, which seeks to maximize the number of potential new indirect timed paths for commodities not yet served while ensuring that no existing timed paths become infeasible. For a given movement arc w originating at hub i, let d_w denote the destination, let e_w denote the earliest time that loading of w can start, let l_w denote the latest time that loading of w can start, and let s_w be a variable that represents the time that the loading of w starts. Finally, let there be a commodity that originates at k and is destined for d_w that is not yet served and a vehicle that originates at k and finishes unloading at i at time a_{k,d_w}. We solve the following IP:

$$\max \sum_{k,d_w} \lambda_{k,d_w}$$

s.t

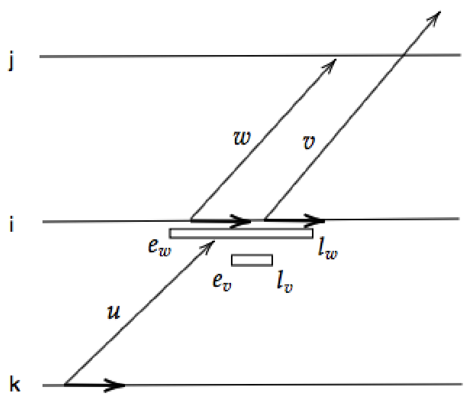
$$s_w + \bar{\tau}_l \leq s_y + M \cdot (1 - \delta_{wy}) \qquad \forall w, y \in \mathcal{W}, w \neq y \qquad (3a)$$

$$\delta_{wy} + \delta_{yw} = 1 \qquad \forall w, y \in \mathcal{W} \qquad (3b)$$

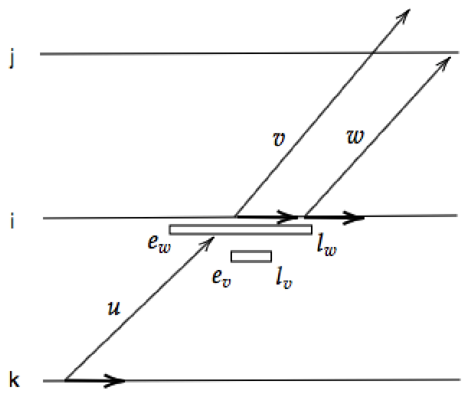
$$e_w \leq s_w \leq l_w \qquad \forall w \in \mathcal{W} \qquad (3c)$$

$$s_w \geq a_{k,d_w} \lambda_{k,d_w} \qquad \forall w \in \mathcal{W}, \forall (k, d_w) \qquad (3d)$$

$$\lambda_k \in \{0,1\} \qquad \forall k \qquad (3e)$$



(a) Initial state of the loading operations (large rectangles) w,v and u in hubs i and k, respectively, and the associated time windows $[e,l]$ where w and v can start loading. In this example, there is an unserved timed commodity path from k to j.



(b) Final state of the loading operations in hub i after the neighborhood move is applied. It is possible to create the indirect timed path $(k \to i \to j)$ by starting the loading of v earlier and starting the loading of w later, while still respecting the associated time windows.

Figure 2 Initial and final states of the loading operations at a hub after the neighborhood move ReArrangeLoadingPeriods is applied.

$$\delta_{wy} \in \{0,1\} \qquad \forall w,y \in \mathcal{W} \tag{3f}$$

$$s_w \in \mathbb{R}_+ \qquad \forall w \in \mathcal{W} \tag{3g}$$

where $\mathcal{W}$ is the set of movement arcs originating at i, λ_{k,d_w} is a binary variable indicating whether or not a new indirect timed path can be created to served a commodity originating at k and destined for d_w, δ_{wy} is a binary variable indicating whether or not the start of the loading of w precedes the start of the loading of y, and M is a large constant. Constraints (3a) and (3b) ensure that there are no overlaps between loading periods, while constraint (3c) ensure that no existing paths become infeasible, and constraint (3d) links the decision variables λ_{k,d_w} and s_w. Finally, constraints (3e),(3f) and (3g) define the decision variables and their domains.

- Re-ArrangeUnloadingPeriods(i): This neighborhood is similar to the previous one, but now focused on re-arranging the unloading operations at an unloading dock of hub i.
- OpenNewLoadingPeriod(i): The neighborhood tries to re-arrange the loading operations at a loading dock of hub i, again using a small integer program, now with the goal of opening space for a new loading operation at the dock. The integer program is similar to the one presented in Re-ArrangeLoadingPeriods. It has an additional decision variable representing the start time of the new loading operation and additional constraints that link the variable to the latest possible time of the start of a loading operation for a commodity that is not yet served. If new indirect timed paths are possible for more than one such commodity, a single new direct timed path is chosen randomly with equal probability.
- OpenNewUnloadingPeriod(i): This neighborhood is similar to the previous one, but now focused on opening space for a new unloading operation at an unloading dock of hub i.

As the neighborhood moves, which are designed to find timed paths for commodities that currently do not yet have a timed path, may also result in destruction of existing timed paths before carrying out a move, we examine the difference between the number of commodities with a timed path before and after the the move is performed. If the number of commodities with a timed path remains the same or increases, the move is performed, but if the number of commodities with a timed path decreases, but not by more than a threshold T_k, it is carried out with probability p. Finally, after performing a move, we first remove the indirect timed paths that have become infeasible, using RemoveInfeasiblePaths($\mathcal{S}$), and then try to create new indirect timed paths, using ConnectPaths($\mathcal{S}$).

4.2.3. Neighborhood selection. The probability that a neighborhood is selected depends on how many timed paths are added/removed by the neighborhood throughout the execution of the metaheuristic. The more successful a particular neighborhood is in creating new timed paths, the higher the chance this neighborhood will be chosen in subsequent iterations. Let P_n be the current probability of selecting neighborhood n and let R_n be the current reward associated with n, i.e, the difference between the total number of timed paths added and removed when using neighborhood n in past iterations. When UpdateProbabilities(P,R) is called, the selection probabilities associated with the neighborhoods are updated, as shown in Algorithm 2, where N is the number of neighborhoods and η is a parameter determining how aggressively we increase or decrease the probabilities based on the rewards vector. Before entering the inner loop, ResetProbabilities()

resets all probabilities and rewards by setting $\{P_n = \frac{1}{N}, R_n = 0\}$ $\forall n \in N$. In order to maintain an effective level of diversification, we reset all probabilities to the discrete uniform distribution $\mathcal{U}(1, N)$ if no improvements were made to the current solution for It^I_R consecutive iterations.

Algorithm 2 Update neighborhood selection probabilities

```
1: input: The probability and the rewards vectors $P$ and $R$
2: output: The updated probability vector $P$.
3: $\bar{P} \leftarrow P$
4: $W \leftarrow 0$
5: for $n = 1, \cdots, N$ do
6:     $\bar{P}_n = P_n \cdot e^{\eta R_n}$;
7:     $W \leftarrow W + \bar{P}_n$;
8: end for
9: for $n = 1, \cdots, N$ do
10:     $\bar{P}_n = \frac{\bar{P}_n}{W}$;
11: end for
12: return $\bar{P}$;
```

4.3. Hybrid Matheuristic

Given that the IP-heuristic presented above processes hubs in a given sequence, it is obvious that this sequence has an impact on the quality of the resulting solution. Therefore, in this section, we propose a hybrid matheuristic (H-MAT), in which solutions produced by the metaheuristic are used to guide the IP-heuristic. More specifically, the solutions produced by the metaheuristic are used to (1) adjust the sequence in which the hubs are processed, and (2) adjust the (geographic) paths options considered for commodities $k \in K^2_h$ when a hub is processed.

To control the solution time, we divide the processing of hubs in stages. There will be G stages and each stage involves the processing of a certain number of hubs. The “staging” can be represented by a vector $(m_1, m_2, \ldots, m_G)$ with $m_g > 0$ and $\sum_{g=1}^{G} m_g = |N|$, and m_g representing the number of hubs processed in Stage g. In Stage g, we start by running MILS, where we enforce that commodities that are served in the latest solution (obtained when processing the last hub in Stage $g-1$) have to remain served, and recording the commodities that are served in its solution. Then, the hubs are sorted in nondecreasing order of the number of commodities served that originate from or are destined to the hub, and the first m_g as-yet unprocessed hubs are selected to be processed (one by one). When processing hub h, each commodity $k \in K^1_h$ is allowed to follow any feasible timed path, and each commodity $k \in K^2_h$ is forced to follow either (1) the direct path from the commodity's origin o_k to its destination d_k, or (2) the previously assigned geographic path $P^{k'}$, or (3) the geographic path in the MILS solution. The arc sets $\mathcal{A}^{hk}$ and $\mathcal{H}^{hk}$ for $k \in \mathcal{K}_h$ and $\mathcal{A}^h$ and $\mathcal{H}^h$ are defined as before. Algorithm 3 shows the pseudo-code for H-MAT.

Algorithm 3 Hybrid Matheuristic

```
1: input: The sets of hubs, commodities, the travel time/distances between hubs, the number of
   stages $|G|$ and parameter vector $M$;
2: output: The set of timed paths $\mathcal{S}^*$;
3: while not all hubs $i \in N$ have been processed do
4:     Run Algorithm 1 while making sure that previously served commodities remain served, and
       record served commodities and their assigned geographic paths in the updated best MILS
       result;
5:     Sort hubs based on the number of served commodities that origin from or destine to the hub
       in the updated best MILS result (ascending);
6:     Choose the first $m_g \in M$ unprocessed hubs in the sort as the next $m_g \in M$ hubs in the hub
       sequence $\tilde{N}$ to process;
7:     for hub $h$ in the new chosen $m_g$ hubs do
8:         identify commodity set $K_h$;
9:         identify integrated arc set $\mathcal{A}^{hk}$ and holding arc set $\mathcal{H}^{hk}$ for every $k \in K_h$;
10:        solve the arc-based hierarchical model;
11:    end for
12:    $g = g + 1$
13: end while
14: return $\mathcal{S}^*$;
```

The advantage of H-MAT over IP-H is that the sequence in which the hubs are processed is determined dynamically and informed by the solution produced by MILS and that for commodities $k \in K_h^2$ another geographic path is considered (the geographic path it uses in the MILS solution).

5. Computational Results

5.1. Instances

The proposed algorithms were used to solve real-world instances of SF Express, each representing same-day delivery service offerings in one of China's mega-cities. A representative day was used to define the demand to be served by a fleet of homogeneous vehicles, each with a capacity of 400 packages. The demand consists of a set of commodities, each specifying an origin hub, a destination hub, a number of packages, the time the packages are available at the origin hub, and the time the packages are due at the destination hub. Packages that have the same origin and destination hub, but that become available at different times or that have different due times are considered to be different commodities; this happens when more than one service product is offered in a market (e.g., "12-18 service" and "14-20 service"). For each hub a limit on the maximum number of vehicles that can be simultaneously loaded and that can be simultaneously unloaded are given. A commodity will be loaded every time it departs from a hub and unloaded every time it arrives at a hub; each loading or unloading operation takes 10 minutes. Table 1 summarizes the characteristics of the four instances. For each instance, the demand information includes the number of commodities to be served ($|K|$), the length (hours) of the planning period ($|T|$), i.e., $|\max_{k \in K} l_k - \min_{k \in K} e_k|$, the average and standard deviation of the number of packages per commodity ($\bar{q}_k$, $\tilde{q}_k$), the average and standard deviation of the time that commodities become available ($\bar{e}_k$, $\tilde{e}_k$), the average and standard deviation of the time commodities are due ($\bar{l}_k$, $\tilde{l}_k$), the average and standard deviation of the direct travel time (minutes) for the commodities ($\bar{\tau}_k$, $\tilde{\tau}_k$), and an upper bound on the number of commodities that can be served ($|K'|$), obtained using Proposition 1. For each instance, the network information includes the number of hubs ($|N|$), the average and standard deviation of the travel time (minutes) and the distance (kilometers) between hubs ($\bar{\tau}_{ij}$, $\tilde{\tau}_{ij}$, $\bar{c}_{ij}$, $\tilde{c}_{ij}$), and the average and standard deviation of the unloading and loading capacity of the hubs ($\bar{U}_i$, $\tilde{U}_i$, $\bar{L}_i$, $\tilde{L}_i$). Travel times and distance information are empirical estimates provided by the carrier. Instances C and D correspond to representative days of different seasons in the same city.

Table 1 Characteristics of the test instances

Instance	Demand											Network								
	$\|K\|$	$\|T\|$	$\bar{q}_k$	$\tilde{q}_k$	$\bar{e}_k$	$\tilde{e}_k$	$\bar{l}_k$	$\tilde{l}_k$	$\bar{\tau}_k$	$\tilde{\tau}_k$	$\|K'\|$	$\|N\|$	$\bar{\tau}_{ij}$	$\tilde{\tau}_{ij}$	$\bar{c}_{ij}$	$\tilde{c}_{ij}$	$\bar{U}_i$	$\tilde{U}_i$	$\bar{L}_i$	$\tilde{L}_i$
A	270	2.5	32	28	13:09	15	15:02	8	43	13	264	17	43	14	25	13	4	0	3	0
B	986	1.75	34	32	13:15	0	15:12	15	44	13	980	32	44	13	26	12	5	2	4	3
C	1291	5.6	29	20	13:38	55	16:58	58	60	22	1286	31	58	23	29	16	3	0	3	0
D	1779	5.6	29	19	13:38	55	16:56	58	56	21	1774									

5.2. Analysis

We compare the performance of the proposed algorithms, i.e., the IP-based heuristic (IP-H), the metaheuristic (MILS) and the hybrid matheuristic (H-MAT), on the set of instances described in Section 5.1. IP-H was coded in Python and uses Gurobi 8.1.1 to solve integer programs. MILS was coded in C++ and uses IBM CPLEX Optimizer 12.6 to solve integer programs. All experiments were performed in a single thread of a dedicated Intel Xeon ES-2630 2.3GHz processor with 50GB RAM running Red Hat Enterprise Linux Server 7.4.

We conducted preliminary experiments to calibrate the parameters of the metaheuristic seeking to balance the time allocated to the different search components and returning results in less than three hours for the larger instances. We observed that the values $It^O_{max} = 10, It^I_{max} = 500, T_k = -1, p = 0.05, It^I_R = 100$ and $\eta = 0.005$ resulted in a good trade-off between run time and solution quality. This parameter setting has been used in the experiments reported in this section. The maximum time allowed for solving the integer programs was set to five seconds and the incumbent solution is discarded when the time limit is exceeded and when the integrality gap is more than 10%. The maximum distance for choosing an intermediate hubs was set to $\frac{3}{4}$ of the average distance between hubs. The same set of parameter values was used when running MILS within H-MAT.

Geographic paths for commodities are allowed to use at most one intermediate hub. The integer programs solved by IP-H and H-MAT are based on a time-expanded network with a homogeneous time discretization of 2 minutes. To control the number of variables in the integer programs solved by IP-H and H-MAT, the maximum waiting time embedded in a movement arc is 15 minutes for instances A and B and 10 minutes for instances C and D. Moreover, a maximum solution time of 4 hours is imposed for each phase of the hierarchical optimization and in the second phase we seek a solution that is within 5% of optimality. MILS uses a time discretization of 1 minute.

The stages in H-MAT are defined by vector $(4, 4, 3, 3, 3)$ for instance A, $(7, 7, 6, 6, 6)$ for instance B, and $(7, 6, 6, 6, 6)$ for instances C and D. The reason for using five stages is to control the overall solution time (by invoking MILS only five times). We process fewer hubs in the later stages because finding a timed path for a commodity is more difficult towards the end and we benefit from more frequent adjustment of the geographic paths considered for commodities.

In order to assess the performance of the proposed algorithms, we report the following statistics for the solutions produced by each algorithm: the number of commodities served ($|K^S|$), the total travel cost (C), the number of commodities served using direct and indirect paths ($|K_d^S|$, $|K_i^S|$), the average number of segments per path for commodities served ($|G^S|$), the average travel time (minutes) of the indirect paths for commodities served (τ_i^S), the average direct travel time for commodities served (τ_d^S), the average direct travel time for commodities not served (τ_d^U), and the solve time (hours) (TT).

Table 2 Performance of the Proposed Algorithms

Instance	Algorithm	$\|K^S\|$	C	$\|K_d^S\|$	$\|K_i^S\|$	$\|G^S\|$	τ_i^S	τ_d^S	τ_d^U	TT
A	MILS	264	3909.11	143	121	1.46	65.21	45.03	58.33	0.21
	IP-H	264	3900.73	149	115	1.44	62.03	42.89	60.67	0.38
	H-MAT	264	3726.94	144	120	1.45	63.58	42.96	60.17	1.56
B	MILS	965	11553.48	395	570	1.59	62.52	43.10	55.52	1.43
	IP-H	971	13339.52	542	429	1.45	63.76	41.88	65.53	15.78
	H-MAT	975	12181.09	498	477	1.49	63.18	41.35	64.36	41.50
C	MILS	1233	17408.90	606	627	1.51	79.80	58.15	67.98	2.64
	IP-H	1275	20720.57	744	531	1.42	78.22	58.10	75.38	38.08
	H-MAT	1282	18120.13	668	614	1.48	79.58	56.90	79.11	71.56
D	MILS	1626	21477.84	890	736	1.45	76.20	52.05	55.96	3.00
	IP-H	1674	20826.99	884	790	1.47	75.14	50.90	70.54	118.93
	H-MAT	1689	20059.50	851	838	1.50	74.20	51.29	69.92	174.91

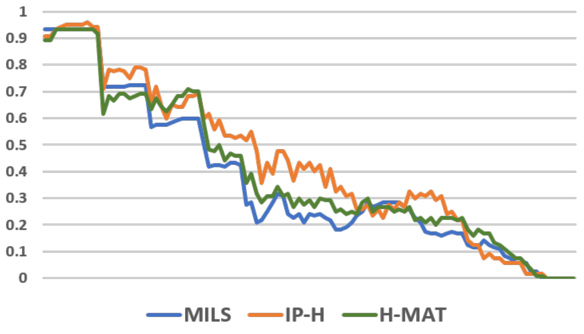
The results can be found in Table 2. We observe that MILS is significantly faster than IP-H and H-MAT, but the efficiency comes at the price of serving fewer commodities. H-MAT produces the best solutions, but the quality comes at the price of taking a long time (175 hours for Instance D), which is acceptable, however, as this is a planning problem. Interestingly, H-MAT produces solutions in which the largest number of commodities is served, but also with a low cost; the cost is always less than the cost of the solutions produced by IP-H and also less than the cost of the solutions produced by MILS for Instance A and Instance D.

We also see that in the best solutions most commodities that are served are served using a direct path. The tight service constraints, i.e., the difference between due time and available time, implies that relatively few indirect paths, which involve one additional unloading operation and one additional loading operation, and, potentially, additional waiting time at the intermediate hub, are feasible. As expected, the average direct travel time of the commodities that are not served is high compared to the average direct travel time of the commodities that are served and the average travel time between hubs. Here too, this is due to the tight service constraints, as this implies that (too) few options are available for these commodities.

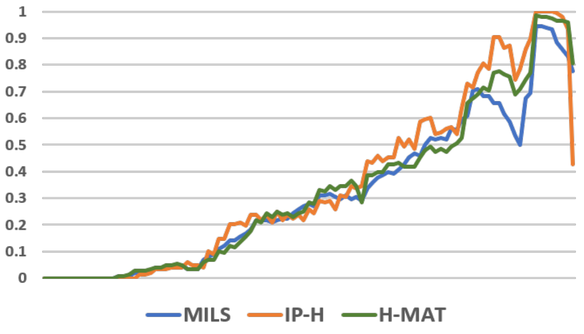
Next, we examine the differences in the solutions produced by the three algorithms from a capacity utilization perspective. Figure 3 shows the average utilization of loading (or unloading) capacity across the hubs at different times during the planning period, where the capacity utilization at a point in time is computed as the number of docks occupied at that time divided by the number of docks available at that time.

The most striking feature of the graphs shown is the difference between instances A and B and instances C and D. For Instance C and D, with a relatively long planning period, we see two loading and two unloading peaks, clearly reflecting the use of indirect timed paths, i.e., the use of hubs to transfer packages. For Instance A and B, with a relatively short planning period, we see that it is critical to get packages moving as soon as possible and the loading capacity in the early part of the planning period is almost fully utilized, and that we need all the available time as the unloading capacity in the late part of the planning period is almost fully utilized. We also observe that the solution produced by H-MAT utilizes less loading and unloading capacities than the solution produced by IP-H, while serving more commodities, which indicates that hub capacities are used more effectively. When interpreting the graphs in Figure 3, it is important to realize that two factors impact the average utilization. First, there is variation in the time that commodities become available or are due. These variations are not negligible when compared to the length of the planning horizon, especially for Instance C and D. Second, commodities originating at the more centrally located hubs, which are relatively close to the other hubs, do not have to depart as early as possible in order to be feasibly served (instead, it may be preferable to let these commodities "wait" so that they can be consolidated with other commodities that are transferred at the hub), which may mean that the utilization of loading capacity at earlier times at these hubs can be low. Similarly, utilization of unloading capacity at later times at these hubs can be low. As a result, the average utilization across all hubs at a particular time does not provide a full picture of the impact of hub capacity limits.

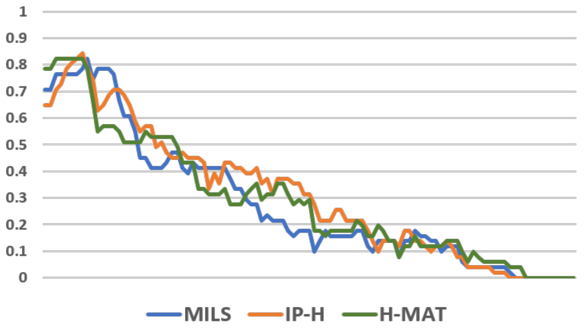
To further assess the effectiveness of the proposed heuristics, we compared the quality of the solutions produced by the heuristics to the quality of the solutions produced when solving SNDHC($\mathcal{D}$), presented in Section 3, using a commercial solver for a limited amount of time on a set of small instances. We observed that the heuristics produce high-quality solutions (within a few percent



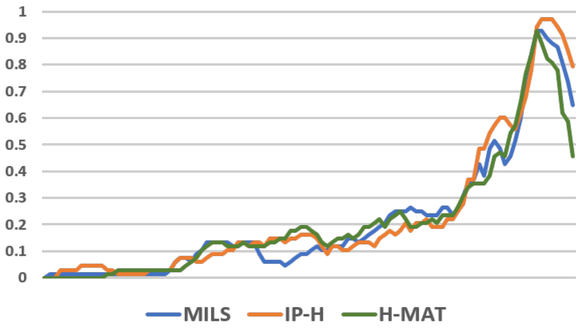
(a) Loading capacity utilization for Instance A



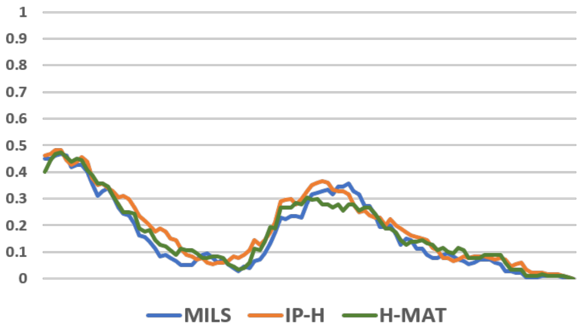
(b) Unloading capacity utilization for Instance A



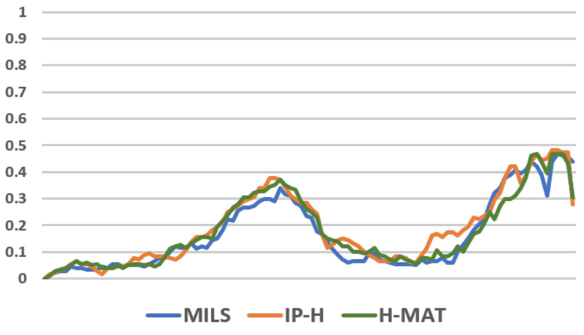
(c) Loading capacity utilization for Instance B



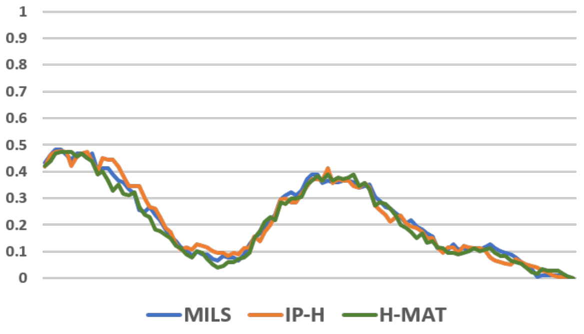
(d) Unloading capacity utilization for Instance B



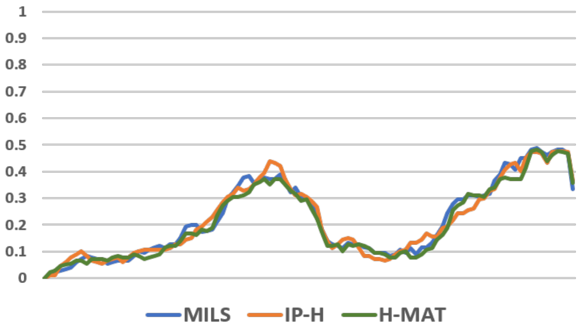
(e) Loading capacity utilization for Instance C



(f) Unloading capacity utilization for Instance C



(g) Loading capacity utilization for Instance D



(h) Unloading capacity utilization for Instance D

Figure 3 **Average hub capacity utilization.**

of optimality) in a fraction of the time, especially when the number of commodities gets larger. Details of these experiments can be found in Appendix A.

Many interacting factors affect our ability to serve the commodities and understanding which of these factors more strongly impact our ability to serve commodities is informative and valuable. MILS is most suitable for carrying out such an investigation as it is more efficient. More specifically, we explore three scenarios, in which we relax one factor: (i) we reduce the loading and unloading time of a vehicle from 10 to 8 minutes (RedLoadUnloadTime), (ii) we increase the due time of each commodity by 10 minutes (IncrDueTime), and (iii) we modify the loading and unloading capacity at each hub during the first and last ten minutes of the planning horizon by making all docks available for loading in the first ten minutes and making all docks available for unloading in the last ten minutes (AdjLoadUnloadCap).

Table 3 Sensitivity Analysis for Operational Constraints

Instance	Setting	$\lvert K^S \rvert$	C	$\lvert K^S_d \rvert$	$\lvert K^S_i \rvert$	$\lvert G^S \rvert$	τ^S_i	τ^S_d	τ^U_d
B	Default	965	11553.48	395	570	1.59	62.52	43.10	55.52
	RedLoadUnloadTime	986	10023.49	396	590	1.60	66.69	42.78	-
	IncrDueTime	984	9419.64	391	593	1.60	68.08	41.51	64.00
	AdjLoadUnloadCap	986	9998.82	400	586	1.59	66.19	42.13	-
C	Default	1233	17408.90	606	627	1.51	79.80	58.15	67.98
	RedLoadUnloadTime	1289	18163.38	654	635	1.49	85.33	59.02	73.50
	IncrDueTime	1289	17999.60	649	640	1.50	84.74	58.62	61.50
	AdjLoadUnloadCap	1291	19677.67	705	586	1.45	84.42	59.29	-

The results can be found in Table 3. As expected, reducing the loading and unloading times of vehicles, adjusting the loading and unloading capacities at hubs, and extending the due time of commodities all enable more commodities to be served. For Instance B, we are not only able to serve more commodities, but we are also able to do it at less cost. For Instance C, we see that minimally adjusting loading and unloading capacities at the hubs (only in the first and last ten minutes) allows the maximum number of commodities to be served, mostly because it allows many more commodities to be served using a direct path. Recall to that MILS always tries to create a direct path between pairs of hubs first, and only if a direct path is not possible due to the hub capacity constraints, it tries to create an indirect path. For this instance, we also see that serving more commodities comes at a significant cost; an increase of less than 5% in the number of commodities served comes at an increase in cost of more than 13%. Overall, these results demonstrate that serving all commodities is very challenging when hub capacity constraints are tight.

6. Final Remarks

Same-day and instant delivery services are the fastest growing business segments of package express carriers. These services are typically offered in urban areas and require effective consolidation to be profitable. Consolidation can only be achieved by operating hub networks and because real-estate prices in urban areas are high (and are expected to go up even more), it is only possible to operate relatively small hubs and these hubs are likely to have limited loading and unloading capacity (and limited space for vehicles waiting to be loaded or unloaded). We have introduced a variant of the traditional service network design problem that incorporates loading and unloading capacity limits at hubs to study these new delivery environments and have proposed and analyzed different heuristic solution approaches. Much more needs to be done to be able to provide truly effective decision support for companies operating in this space. Most importantly, recognizing that many of the model parameters are uncertain, e.g., vehicle loading and unloading time and travel times between hubs, and developing solution approaches that go beyond using "guessing" parameter values or using point estimates as parameter values.

Although some of the ideas explored in this paper have influenced the current planning process at SF Express, SF Express currently uses a path-based formulation as some practical considerations can be more easily accommodated in a path-based model (as they can be handled during the path generation process). Furthermore, hub capacity is treated as a soft constraint, with a penalty for exceeding capacity in the objective function, rather than a hard constraint. This is done for commercial reasons, as SF Express wants/needs to offer same-day service between all locations in this highly competitive space. The same-day service network has now been deployed in one of the first-tier cities in China for more than two years. Interestingly, by exploiting the fact that in the service network design produced by the optimization model many of the markets (origin-destination pairs) are served using relatively short direct paths and that these direct paths can depart later in the day, the service cut-off time for many markets could be set later, sometimes up to 2 hours, which resulted in an increase of same-day package demand of more than 30%.

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Appendix A Experiments on Small Instances

We compare the quality of the solutions produced by our heuristics, i.e., the IP-based heuristic (IP-H), the metaheuristic (MILS) and the hybrid matheuristic (H-MAT), with the solutions produced when solving SNDHC($\mathcal{D}$), presented in Section 3, using Gurobi 8.1.1 for a limited amount of time for a set of small instances. We let Ω denote the instance set. It contains twelve representative small instances based on the demand information from Instance A presented in Section 5.1. For each instance $\omega \in \Omega$, we extracted a random subset of hubs in the network and the set of commodities to be served between these hubs. The differences between commodity available and due times were shorten to reduce the size of the constructed time-expanded network and to create more challenging instances. The travel time and distances between hubs as well as the unloading and loading capacity of the hubs were adjusted such that the presence of hub capacity constraints makes it challenging to serve all commodities. Table 4 summarizes the characteristics of the instances, where we report: the number of hubs ($|N|$), the unloading and loading capacity of the hubs (U_i, L_i), and the number of commodities to be served ($|K|$).

Table 4 Small instances

Instance Ω	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7	ω_8	ω_9	ω_{10}	ω_{11}	ω_{12}
$\|N\|$	8	8	8	10	10	10	10	10	10	12	12	12
U_i	2	2	2	2	2	2	2	2	2	2	2	2
L_i	1	1	1	1	1	1	2	2	2	2	2	2
$\|K\|$	55	54	56	89	89	90	89	90	90	132	132	130

In IP-H, we process the hubs in nonincreasing distance from the centroid. In MILS, we use the same set of parameters as described in Section 5.2. The stages in H-MAT are defined by vectors (3, 3, 2), (3, 3, 2, 2), and (3, 3, 2, 2, 2) for the instances with 8 hubs, 10 hubs, and 12 hubs, respectively. In our heuristics, we allow geographic paths for commodities to use at most one intermediate hub. In SNDHC($\mathcal{D}$), more than one intermediate hub is allowed. When using the commercial solver, the heuristic mode is turned on, the 'MIPFocus' parameter is set to 1 to find feasible solutions quickly, and the maximum solution time for the first and second phase of the hierarchical optimization are set to 12 hours and 24 hours, respectively.

The results are summarized in Table 5. We report: the number of commodities served ($|K^S|$), the total travel cost (C), the average number of segments for commodities served ($|G^S|$), the solve time (in seconds) for the heuristics (TT), the time (in seconds) for the solver to obtain a feasible solution that is at least as good as the one given by H-MAT in terms of $|K^S|$ (TT^1), the time (in seconds) for the solver to find a best solution in terms of $|K^S|$ (TT^2), the time (in seconds) for the solver to finish the first phase of the hierarchical optimization (TT^3), the optimality gap in the first phase of the hierarchical optimization (gap^1), the time (in seconds) for the solver to obtain a feasible solution that is at least as good as the one given by H-MAT in terms of C if they have the same value of $|K^S|$ (TT^4), the time (in seconds) for the solver to find a best solution in terms of C (TT^5), the time (in seconds) for the solver to finish the second phase of the hierarchical optimization (TT^6), and the optimality gap in the second phase of the hierarchical optimization (gap^2).

Table 5 Experiments on Small Instances

Instance $\boldsymbol{\Omega}$		$\boldsymbol{\omega_1}$	$\boldsymbol{\omega_2}$	$\boldsymbol{\omega_3}$	$\boldsymbol{\omega_4}$	$\boldsymbol{\omega_5}$	$\boldsymbol{\omega_6}$	$\boldsymbol{\omega_7}$	$\boldsymbol{\omega_8}$	$\boldsymbol{\omega_9}$	$\boldsymbol{\omega_{10}}$	$\boldsymbol{\omega_{11}}$	$\boldsymbol{\omega_{12}}$
MILS	$\boldsymbol{\vert K^S\vert}$	46	45	51	64	59	72	85	82	86	117	112	110
	$\boldsymbol{C}$	1724	1708	1566	2070	2077	2072	3058	2892	3000	3641	3692	3897
	$\boldsymbol{\vert G^S\vert}$	1.09	1.07	1.16	1.14	1.08	1.21	1.14	1.16	1.16	1.23	1.20	1.15
	$\boldsymbol{TT}$	40	38	50	102	43	84	56	52	58	95	99	93
IP-H	$\boldsymbol{\vert K^S\vert}$	48	46	53	67	61	73	88	87	88	123	118	112
	$\boldsymbol{C}$	1769	1698	1620	2098	2122	1909	3115	3489	2968	3947	4013	4070
	$\boldsymbol{\vert G^S\vert}$	1.13	1.11	1.19	1.18	1.15	1.29	1.18	1.08	1.22	1.20	1.18	1.13
	$\boldsymbol{TT}$	18	14	25	54	34	111	63	59	47	208	199	129
H-MAT	$\boldsymbol{\vert K^S\vert}$	48	46	53	67	61	74	88	87	88	123	118	113
	$\boldsymbol{C}$	1703	1694	1515	2073	2060	2099	3111	3351	2941	3881	3979	4043
	$\boldsymbol{\vert G^S\vert}$	1.12	1.11	1.23	1.18	1.15	1.23	1.18	1.11	1.23	1.22	1.19	1.14
	$\boldsymbol{TT}$	109	117	103	220	217	255	233	227	215	381	392	417
Solver	$\boldsymbol{\vert K^S\vert}$	49*	47*	53*	67*	62*	76*	89*	87*	88*	124*	119	115
	$\boldsymbol{C}$	1735*	1834*	1506*	1981*	2096*	2128	3199	3245	2859*	3623	3751	4037
	$\boldsymbol{\vert G^S\vert}$	1.14	1.06	1.25	1.22	1.11	1.34	1.19	1.16	1.26	1.29	1.26	1.17
	$\boldsymbol{TT^1}$	10	19	36	42	65	492	62	59	29	1476	1011	629
	$\boldsymbol{TT^2}$	12	24	36	42	65	2806	119	59	29	1933	1011	3435
	$\boldsymbol{TT^3}$	52	24	38	182	50	40536	119	98	30	1933	+	+
	$\boldsymbol{gap^1}$	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	1.68%	0.87%
	$\boldsymbol{TT^4}$	−	−	302	193	−	−	−	304	154	−	−	−
	$\boldsymbol{TT^5}$	63	30	306	294	180	40536	1198	1392	289	60673	61287	44622
	$\boldsymbol{TT^6}$	159	50	405	942	243	+	+	+	293	+	+	+
	$\boldsymbol{gap^2}$	0.00%	0.00%	0.00%	0.00%	0.00%	9.59%	1.13%	1.29%	0.00%	5.35%	5.47%	5.20%

$*$: proven to be the optimal objective value
$-$: not applicable
$+$: the formulation is not solved to optimality within the specified time limit

We observe that H-MAT produces the best solutions among the proposed heuristics, reaching optimality in four out of the twelve instances in terms of the number of commodities served, and missing at most 2 commodities in the other eight instances (compared to the solutions obtained by the IP solver). In terms of costs, compared to the other heuristics, H-MAT also found better solutions when the same number of commodities is served (which shows the benefit of dynamically adjusting the sequence of hubs to be processed and utilizing the alternative commodity paths given by MILS). On average, the gap between the costs of the solutions found by H-MAT and the proven optimal solutions serving the same number of commodities is only 2.8%.

As expected, the solve time for the exact model increases dramatically as the number of hubs and commodities to be served increase. The optimal number of commodities served for instances ω_{11} and ω_{12} could not be found within 12 hours, which suggests that the time to find good feasible solutions is likely unacceptably long for real-world instances. In the second phase of the hierarchical optimization, the commercial solver was only able to find an optimal solution for five out of the twelve instances within 24 hours. This demonstrates that heuristics are needed to handle large-scale instances.