A decision theoretic approach for waveform design in joint radar communications applications

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***Abstract*—In this paper, we develop a decision theoretic approach for radar waveform design to maximize the joint radar communications performance in spectral coexistence. Specifically, we develop an adaptive waveform design approach by posing the design problem as a *partially observable Markov decision process* (POMDP), which leads to a hard optimization problem. We extend an approximate dynamic programming approach called *nominal belief-state optimization* to solve the waveform design problem. We perform a numerical study to compare the performance of the proposed POMDP approach with the commonly used myopic approaches.**

I. Introduction

Spectral congestion is forcing legacy radar band users to investigate methods of cooperation and co-design with a growing number of communications applications [1]. The co-design of radar and wireless communications systems faces several challenges such as interference, radar and communications decoupling, and dynamic user (radar and communications) requirements. The studies in [2], [3] provide a detailed overview of the challenges and research directions in the "spectral" coexistence of radar and communications. From the study in [4], the quality of the radar return and also the communications rate is mainly determined by the spectral shape of the waveform. Moreover, one of the key challenges for any waveform design method is to meet dynamic user needs. To address these challenges, in this paper, we develop waveform shaping methods that are adaptive, and can trade-off between competing performance objectives. A waveform design method can most effectively meet the dynamic user needs if it predicts the future user needs and allocates the resources accordingly. Previous research has considered waveform design for joint radar-communications systems, for example [5], [6]. However, existing methods often do not meet dynamic performance requirements, as they tend to be *greedy* in that they only maximize short-term performance for instantaneous benefits. For problems with dynamic performance requirements, long-term performance is critical as decisions (to choose a particular waveform) at current time epoch may lead to regret in the future. To address these challenges, we develop an adaptive waveform design method for joint radar-communications systems based on the theory of *partially observable Markov decision process* (POMDP) [7]. Specifically, we formulate the waveform design problem as a POMDP, after which the design problem becomes a matter of solving an optimization problem. In essence, the POMDP solution provides us with the optimal decisions on the waveform design parameters [8]. However, the optimization problems resulting from POMDPs are hard to solve exactly. There is a plethora of approximation methods called *approximate dynamic programming* methods or ADP methods, as surveyed in [7]. To this end, we extend one of the computationally least intensive ADP approaches called *nominal belief-state optimization* (NBO) [8].

The POMDP framework has a natural look-ahead feature, i.e., it can trade-off short-term for long-term performance. This feature lets the POMDP naturally anticipate the dynamic user needs and optimize the resources (waveforms) to actively meet the user's needs. Typically, one studies these adaptive methods under "cognitive radio (radar)," which has a rich literature. However, this project brings formalism to these methods by posing the waveform design problem as a POMDP. This particular waveform design problem has not been studied before. Recently, POMDPs were used in [9] to develop adaptive methods for "cognitive radar," but in a different context, where the focus was on optimizing radar measurement times and not on waveform shaping. The current waveform design problem is related to a class of problems called *adaptive sensing*, where POMDP was already a proven effective framework [8], [10]. We assume that the environment consists of a maneuvering radar target, obstacle blocking radar line-of-sight, a communications user, and a joint radar-communications system node. The joint radar-communications node can sense the environment to extract target parameter information or can communicate with other communications nodes, and can also act as communications relay. The joint node can simultaneously estimate the target parameters from the radar return and decode a received communications signal. We co-design the joint radar-communications system so that radar and communications systems can cooperatively share

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TABLE I: Survey of Notation

Variable	Description
B	Total System Bandwidth
P_{radar}	Radar power
T_{temp}	Effective temperature
b	Communication propagation loss
P_{com}	Communications power
a	Combined antenna gain
N	Number of targets
σ^2_{CRLB}	Cramer-Rao lower bound
σ^2_{noise}	Thermal noise
σ^2_{proc}	Process noise variance
TB	Time–bandwidth product
δ	Radar duty factor
w	Measurement noise
ζ_k	Mean vector noise
τ	Time delay to m^{th} target
α	Weighting parameter
R_{comm}	Communications rate
R_{est}	Radar estimation rate
P_k	Error covariance matrix
T_{pri}	Pulse repetition interval
H	Planning horizon length

information with each other and mutually benefit from the presence of the other. In this paper, we consider target range or time-delay to be the target parameter of interest. Table I shows the notations employed in this paper.

II. Joint Radar-Communications Preliminaries

A. Successive Interference Cancellation Receiver Model

In this section, we present the receiver model called *Successive Interference Cancellation* (successive interference cancellation (SIC)). SIC is the same optimal multiuser detection technique used for a two user multiple-access communications channel [2], [11], except it is now reformulated for a communications and radar user instead of two communications users. We assume we have some knowledge of the radar target range (or time-delay) up to some random fluctuation (also called process noise) from prior observations. We model this process noise, $n_{\text{proc}}(t)$, as a zero-mean random variable. Using this information, we can generate a predicted radar return and subtract it from the joint radar-communications received signal. After suppressing the radar return, the receiver then decodes and removes the communications signal from the radar return suppressed received waveform to obtain a radar return signal free of communications interference. This method of interference cancellation is called SIC. It is this receiver model that causes communications performance to be closely tied to the radar waveform spectral shape. It should be noted that since the predicted target location is never always accurate, the predicted radar signal suppression leaves behind a residual contribution, $n_{\text{resi}}(t)$. Consequently, the receiver will decode the communications message from the radar-suppressed joint received signal at a lower rate. The block diagram of the joint radar-communications system considered in this scenario is shown in Figure 1. When applying SIC,

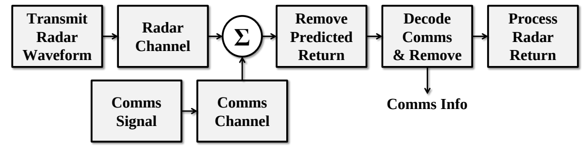


Fig. 1: Joint radar-communications system block diagram for SIC scenario. The radar and communications signals have two effective channels, but arrive converged at the joint receiver. The radar signal is predicted and removed, allowing a reduced rate communications user to operate. Assuming near perfect decoding of the communications user, the ideal signal can be reconstructed and subtracted from the original waveform, allowing for unimpeded radar access.

the interference residual plus noise signal $n_{\text{int+n}}(t)$, from the communications receiver's perspective, is given by [3], [12]

$$\begin{aligned} n_{\text{int+n}}(t) &= n(t) + n_{\text{resi}}(t) \\ &= n(t) + \sqrt{\|a\|^2 P_{\text{rad}}}\, n_{\text{proc}}(t)\, \frac{\partial x(t-\tau)}{\partial t}, \end{aligned} \quad (1)$$

and

$$\|n_{\text{int+n}}(t)\|^2 = \sigma^2_{\text{noise}} + a^2 P_{\text{rad}} \left(2\pi B_{\text{rms}}\right)^2 \sigma^2_{\text{proc}}, \quad (2)$$

where $n_{\text{proc}}(t)$ is the process noise with variance σ^2_{proc}.

B. Radar Estimation Rate

To measure spectral efficiency for radar performance, we developed a new metric recently called *radar estimation rate*, which is formally defined as the minimum average data rate required to provide time-dependent estimates of system or target parameters, for example, target range [3], [12], [13]. The radar estimation rate is expressed as follows:

$$R_{\text{est}} = I(\mathbf{x};\mathbf{y})/T_{\text{pri}}, \quad (3)$$

where $I(\mathbf{x};\mathbf{y})$ is the mutual information between random vectors $\mathbf{x}$ and $\mathbf{y}$, and $T_{\text{pri}} = T_{\text{pulse}}/\delta$ is the pulse repetition interval of the radar system, T_{pulse} is the radar pulse duration, and δ is the radar duty factor. This rate allows construction of joint radar-communications performance bounds, and allows future system designers to score and optimize systems relative to a joint information metric. For a simple range estimation problem with a Gaussian tracking prior, this takes the form [2], [3], [14]:

$$R_{\text{est}} = (1/2T)\log_2(1 + \sigma^2_{\text{proc}}/\sigma^2_{\text{CRLB}}), \quad (4)$$

where σ^2_{proc} is the range-state process noise variance and σ^2_{CRLB} is the Cramér-Rao lower bound (CRLB) for range estimation given by [3], [12], [13] $\sigma^2_{\text{CRLB}} = \sigma^2_{\text{noise}}/8\pi^2 B^2_{\text{rms}} T_p\, B\, P_{\text{rad,rx}}$, where σ^2_{noise} is the noise variance or power, T_p is the radar pulse duration, B_{rms} is the radar waveform root mean square (RMS) bandwidth, and $P_{\text{rad,rx}}$ is the radar receive power, which is inversely proportional to the distance of the target from the joint node. Immediately apparent is the similarity of above equation to Shannon's channel capacity equation [3], [12], [13], where the ratio of the source uncertainty variance to the range estimation noise variance forms a pseudo-signal-to-noise ratio (SNR) term. In Eq. 4, the estimation rate is inversely proportional to the distance of the target from the joint node. As discussed later, we design the waveform parameters over the planning horizon while accounting for the varying estimation rate due to target's motion.

III. Technical Approach

We measure the performance of the system with two metrics: communications information rate bound and radar estimation rate bound (discussed in the previous section). The joint radar-communications performance bounds developed in [3], [12], [13] considered only local radar estimation error, therefore making simplified assumptions about the radar waveform. In [4], the results were generalized to include formulation of an optimal radar waveform for both global radar estimation rate performance and consideration of in-band communications users forced to mitigate radar returns. To demonstrate a point solution of joint radar-communications information inner bounds, we recently developed the notion of SIC [3], [15], [16]. The key to joint radar-communications is SIC, which is to predict and subtract the radar target return, where the prediction variance would therefore drive an additional residual noise term for the in-band communications user, which reduces the communications rate from the normal interference-free bound. The communications signal is then decoded and reconstructed (reapplication of forward error correction), and subtracted from the original return. The radar user can then operate unimpeded. As a result, the radar estimation rate is the same as given in (3). Radar users would like to increase the RMS bandwidth to the point where the range estimation error is minimized, but not at the expense of significant global error. The communications user, however, suffers from the additional residual noise source [3]:

$$\tilde{R}_{\text{com}} \leq B \log_2 \left[1 + \frac{b^2 P_{\text{com}}}{\sigma^2_{\text{noise}} + a^2 \, P_{\text{rad}} \, (2\pi \, B_{\text{rms}})^2 \, \sigma^2_{\text{proc}}} \right] . \quad (5)$$

A. POMDP Formulation of Joint Waveform Design problem

We consider a particular case study, with a radar target, communications user, and the joint node, as shown in Figure 2. The line-of-sight between the radar target and the joint node may be lost as the target moves around an obstacle (e.g., urban structure). We will develop our POMDP framework for this case study, which can be easily generalized and extended to other problem scenarios. This particular case study allows us to show qualitative and quantitative benefits of POMDP in adaptive waveform design. The key components in the waveform design algorithm based on POMDP are shown in Figure 3. The POMDP planner evaluates the belief-state (posterior distribution over the state space updated according to Bayes' rule) of the system, uses an ADP method to solve the POMDP approximately, and produces optimal or near-optimal decisions on waveform parameters; details are discussed later. Our objective is to design the shape of the waveforms over time to maximize the system performance.

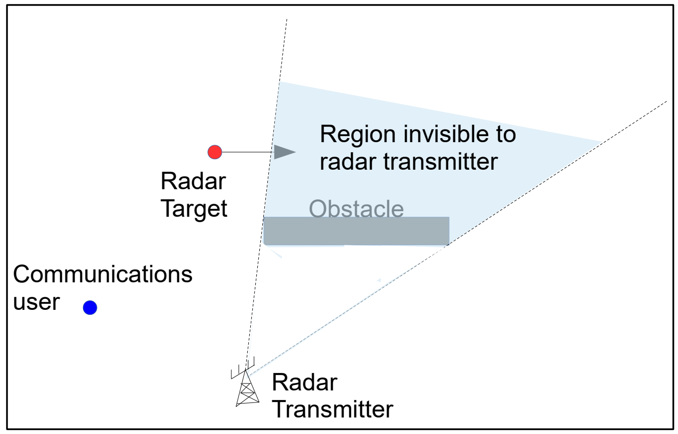


Fig. 2: Problem Scenario

Here, we choose a weighted average of the estimation rate and the communications rate as the performance metric. First, we begin with a unimodular chirp waveform $\exp[j(\pi B/T)(t^2)]$. We control the spectral shape of this chirp signal to maximize joint performance. To achieve this, we first sample the chirp signal, and collect N samples in the frequency domain. Let $X = (X(f_1), \ldots, X(f_N))^T$ be the discretized signal in the frequency domain at frequencies $f_1, \ldots, f_N$. Let $u = (u_1, \ldots, u_N)^T$ be an array of spectral weights we will optimize as discussed below, where $u_i \in [0, 1], \forall i$. We control the spectral shape of the chirp signal by multiplying (i.e., dot product) the signal with the spectral weights in the frequency domain, i.e., the resulting signal is given by $X(f_i)u_i, \forall i$. To pose any decision making problem as a POMDP, we need to define the POMDP ingredients, which are states, actions, state-transition law, observations & observation law, and reward function, in the context of the particular problem at hand. The following is a description of the POMDP ingredients as defined specific to our waveform design problem. Hereafter, we model the system dynamics as a discrete event process, where k represents the discrete time index.

States: State at time k is defined as $x_k = (\chi_k, \xi_k, P_k)$, where χ_k represents the target state, which includes the location, velocity, and the acceleration of the target; and (ξ_k, P_k) represents the state of the tracking algorithm, e.g., Kalman filter, where ξ_k is the mean vector, and P_k is the covariance matrix.

Actions: Actions are the waveform spectral weights vector u_k as defined above.

State-Transition Law: χ_k evolves according to a motion model called *near-constant velocity model* captured by $\chi_{k+1} = F\chi_k + n_k$, where F is a transition matrix, and $n_k = n_{\text{proc}}(t = k)$ is the process noise described in Section II-A, which is modeled as a Gaussian process. ξ_k and P_k evolve according to Kalman filter equations.

Observation Law: $z_k^{\text{Targ}} = G\chi_k + w_k$ (if not occluded) and $z_k^{\text{Targ}} = w_k$ (if occluded), where G is a transition matrix, and w_k is the measurement noise, modeled as a Gaussian process. Specifically, $w_k \sim \mathcal{N}(0, R_k)$, where R_k is the noise covariance matrix, where the entries in the matrix scale

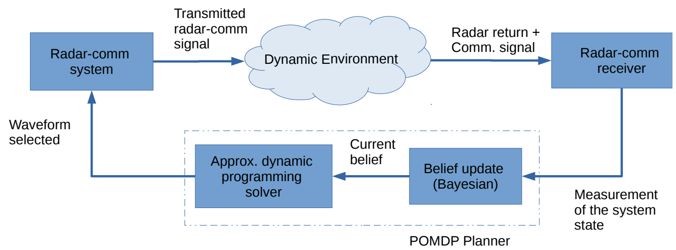


Fig. 3: Adaptive waveform optimization in a dynamic environment.

(increase) with the distance between the joint node (or sensor node) and the target. We assume the other state variables to be fully known.

Reward Function: The reward function rewards the decision u_k taken at time k given the state of the system is x_k as defined below:

$$R(x_k, u_k) = \alpha R_{\text{est}}(x_k, u_k) + (1 - \alpha) R_{\text{comm}}(x_k, u_k),$$

where R_{est} is the radar estimation rate [4], R_{comm} is the communications data rate, and $\alpha \in [0, 1]$ is a weighting parameter.

Belief State: We maintain and update the posterior distribution over the state space (as the actual state is not fully observable), also known as the "belief state" given by $b_k = (b_k^{\chi}, b_k^{\xi}, b_k^{P})$, where $b_k^{\xi}(x) = \delta(x - \xi_k)$, $b_k^{P}(x) = \delta(x - P_k)$, and $b_k^{\chi} = \mathcal{N}(\xi_k, P_k)$. Here, we know the state of the tracking algorithm, so belief states corresponding to these states are just delta functions, whereas the target state is modeled as a Gaussian distribution with ξ_k and P_k as the mean vector and the error covariance matrix respectively.

B. POMDP Solution

Our goal is to optimize the actions over a long time-horizon (of length H) to maximize the expected cumulative reward. The objective function (to be maximized) is given by $J_H = \mathrm{E}\left[\sum_{k=0}^{H-1} R(x_k, u_k)\right]$. But, we can also write J_H in terms of the belief states as

$$J_H = \mathrm{E}\left[\sum_{k=0}^{H-1} r(b_k, u_k) \,\middle|\, b_0\right],$$

where, $r(b_k, u_k) = \int R(x, u_k) b_k(x)\, \mathrm{d}x$ and b_0 is the initial belief state. Let $J_H^*(b)$ represent the optimal objective function value, given the initial belief-state b. Therefore, the optimal action policy at time k is given by $\pi^*(b_k) = \arg\max_u Q(b_k, u)$, where $Q(b_k, u) = r(b_k, u) + \mathrm{E}\left[J_H^*(b_{k+1}) \,|\, b_k, u\right]$ which is also called the Q-value. [7], [8] give a detailed description of POMDP and its solution. POMDP formulations are known for their high computational complexity, particularly because it is near impossible to obtain the above-discussed Q-value in real-time [8]. There exist a plethora of approximation methods called *approximate dynamic programming* (ADP) methods that approximate the Q-value [7]. We adopt a fast ADP approach called *nominal belief-state optimization* (NBO), which we previously developed in the context of another adaptive sensing problem [8]. With NBO approximation, the POMDP formulation leads to the following optimization problem:

TABLE II: Parameters for Waveform Design Methods

Parameter	Value
Bandwidth (B)	5 MHz
Center frequency	3 GHz
Effective temperature (T_{temp})	1000 K
Communications range	10 km
Communications power (P_{com})	1 W
Communications antenna Gain	20 dBi
Communications receiver Side-lobe Gain	10 dBi
Radar antenna gain	30 dBi
Target cross section	10 m^2
Target process standard deviation (σ_{proc})	100 m
Time–bandwidth product (TB)	128
Radar duty factor (δ)	0.01

$$\min_{u_k, k=0,\ldots,H-1} \sum_{k=0}^{H-1} r(\tilde{b}_k, u_k), \tag{6}$$

where $\tilde{b}_k, k = 0, \ldots, H-1$ is a sequence of readily available "nominal" belief states, as opposed to b_ks which are random variables, obtained from the NBO approach.

IV. Simulation Results and Discussion

We implement the POMDP and the NBO approaches in MATLAB to solve the waveform design problem in the above described scenario. We study the methods in a scenario with two obstacles blocking the line-of-sight (LOS) between the joint node and the target as the target moves from the left to the right as shown in Figure 2. We use MATLAB's *fmincon* [17] to solve the optimization problem in Eq. 6. Additionally, we implement the receding horizon control approach while optimizing the decision variables over the moving planning horizon. The parameters used in this study are shown in Table II

The following are the main objectives of this numerical study.

- Study the impact of the planning horizon H on the joint performance with respect to the estimation and the communications rates.
- Quantitative comparison of the myopic approach ($H = 1$) and the non-myopic approach ($H > 1$).

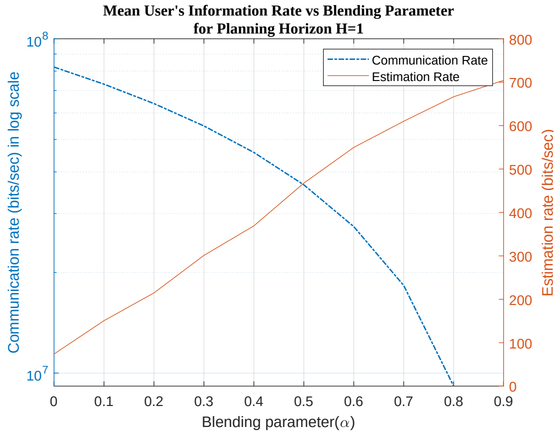


Fig. 4: Average rate vs. α

TABLE III: Planing horizon length (H) Versus average combined rate for $\alpha = 0.5$

Planing horizon length (H)	Average combined rate (x10^6 bits/sec)
1	22.8
2	45.72
3	68.58
4	91.43
5	114.29

A. Impact of Blending Parameter on the Rates

We plot the estimation rate and communications rate of the optimized waveform against $\alpha \in [0, 1]$ as shown in Figure 4. As expected, α allows us to trade-off between the two rates. This trade-off property of the system is the reason we need to optimize the waveform parameters over a planning horizon as opposed to one-step optimization. We show a rate-rate curve showing the communications and estimation rate for different values of α where R_{comm} is the SIC communications data rate defined in Section III-A and α is a blending parameter that is varied from 0 to 1. When $\alpha = 0$, in Eq. 4 only communications rate is considered, and when $\alpha = 1$, only the radar estimation rate is considered. In between, the product is jointly maximized.

B. Impact of Planning Horizon on the Rates

Figure 5 shows the impact of planning horizon H on the joint (weighted average) radar communications rate. The joint rate significantly increases as we increase the length of the planning horizon. In Figure 6, we plot the estimation rate for $H = 1$ and $H = 4$. At around time index 25, the line of sight is lost, which leads to reduction in the estimation rate. As the line-of-sight gets established at the time index 50, the rates go up in both cases, but the rise is significantly higher for $H = 4$, which shows that our non-myopic approach plans the waveform parameters more effectively than the myopic approach ($H = 1$). Table III summarizes the average combined rates for different planning horizon lengths as discussed above. As we increase $H = 1$ to $H = 5$ the combined rate is increased by more than five times, but at the same time the computational complexity in solving Eq. 6 with $H = 5$ is significantly higher than with $H = 1$. In fact, this computational complexity grows exponentially with H. Thus, one may need to assess if it is worth trading off computational complexity for better performance, and then determine the planning horizon length H accordingly. Figure 7 shows the quantitative comparison of radar estimation rate and communication rate for five different planning horizon. Figure 8 shows the qualitative comparison of planning horizon $H = 1$ vs. $H = 5$. In both cases the size

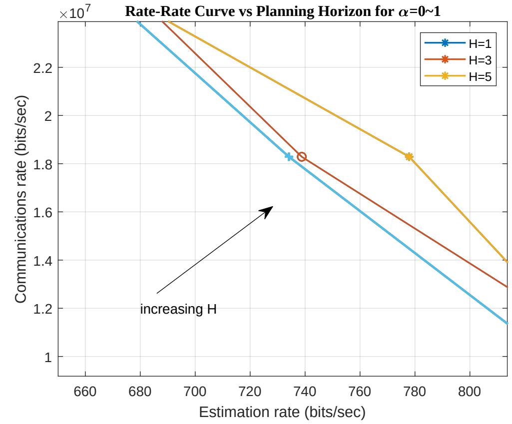


Fig. 5: Combined rate vs. planning horizon H

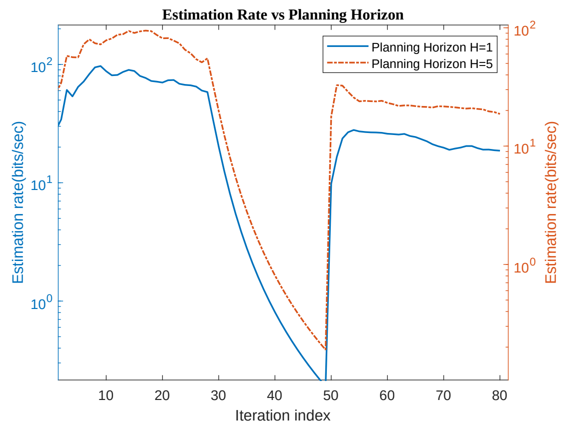


Fig. 6: Estimation rate vs. planning horizon

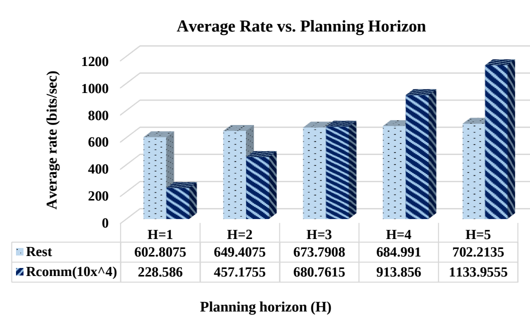
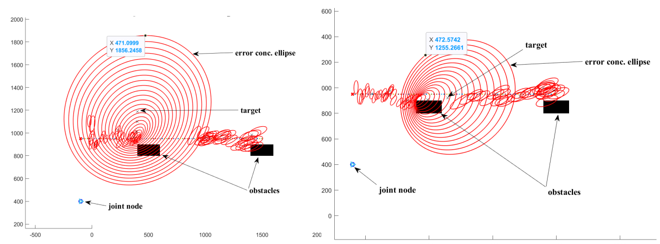


Fig. 7: Average rate vs. planning horizon



(a) Planning Horizon H=1 **(b)** Planning Horizon H=5

Fig. 8: Myopic vs. non-myopic approach

TABLE IV: Planing horizon length (H) Versus Average target location error

Planing horizon length (H)	Average target location error (m)
1	107.4344
2	102.7342
3	94.9062
4	73.7049

of the error confidence ellipse of the target increases when the target is occluded by the obstacles. But the size of the ellipse visibly reduces as we set $H = 5$. This reduction in the ellipse size is captured quantitatively in Figure 6. Table IV shows the impact of plan horizon length on the average target location error.

V. Conclusions

We developed a decision theoretic framework for adaptive waveform design in joint radar-communications systems. Specifically, we posed the waveform design problem as a *partially observable Markov decision process* (POMDP) and extended an *approximated dynamic programming approach* to solve the problem in near real-time. Particularly, we adapted an ADP approach called *nominal belief-state optimization* or NBO. The goal is to optimize the spectral shape of the radar waveform over time to maximize the joint performance of radar and communications in spectral coexistence. We presented the quantitative benefits, in terms of communications and radar estimation rates, of our POMDP-based non-myopic approach in waveform design against myopic or greedy approaches. In our future studies, we will address challenges including time-varying communication demand and target detection probability.

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