

Multi-period investment pathways - Modeling approaches to design distributed energy systems under uncertainty

Markus Bohlayer^{a,b}, Adrian Bürger^{a,c}, Markus Fleschutz^{a,d}, Marco Braun^a, Gregor Zöttl^{b,e,*}

^aKarlsruhe University of Applied Sciences, Institute of Refrigeration, Air-Conditioning, and Environmental Engineering, Moltkestraße 30, 76133 Karlsruhe, Germany

^bFriedrich-Alexander-Universität Erlangen-Nürnberg, Professorship of Economics (Industrial Organization and Energy Markets), Lange Gasse 20, 90403 Nürnberg, Germany

^cUniversity of Freiburg, Department of Microsystems Engineering (IMTEK), Systems Control and Optimization Laboratory, Georges-Koehler-Allee 102, 79110 Freiburg im Breisgau, Germany

^dCork Institute of Technology, Department of Process, Energy and Transport Engineering, Rossa Ave, Bishopstown, Cork, T12 P928, Ireland

^eEnergie Campus Nürnberg, Fürther Str. 250, Nürnberg 90429, Germany

Abstract

Multi-modal distributed energy system planning is applied in the context of smart grids, industrial energy supply, and in the building energy sector. In real-world applications, these systems are commonly characterized by existing system structures of different age where monitoring and investment are conducted in a closed-loop, with the iterative possibility to invest. The literature contains two main approaches to approximate this computationally intensive multi-period investment problem. The first approach simplifies the temporal decision-making process collapsing the multi-stage decision to a two-stage decision, considering uncertainty in the second stage decision variables. The second approach considers multi-period investments under the assumption of perfect foresight. In this work, we propose a multi-stage stochastic optimization problem that captures multi-period investment decisions under uncertainty and solves the problem to global optimality, serving as a first-best benchmark to the problem. To evaluate the performance of conventional approaches applied in a multi-year setup and to solve the multi-period problem at lower computational effort, we propose a rolling horizon heuristic that on the one hand reveals the performance of conventional approaches applied in a multi-period set-up and on the other hand enables planners to identify approximate solutions to the original multi-stage stochastic problem. Additionally, we consider an open-loop version of the rolling horizon algorithm to evaluate how single-period investments perform with respect to the entire scenario tree and compared to multi-period investments. We conduct a real-world case study and investigate solution quality as well as the computational performance of the proposed approaches. Our findings indicate that the approximation of multi-period investments by two-stage stochastic approaches yield the best results regarding constraint satisfaction, while deterministic multi-period approximations yield better economic and computational performance.

1. Introduction

1.1. Motivation

Multi-modal distributed energy systems serve local electricity, heat, and cold demands. These systems are exposed to changing framework conditions, such as fluctuating energy carrier prices and rapidly decreasing investment cost of renewable energies and storage technologies. Equally important are the shifting target dimensions of decision-makers aiming for an eco-efficient energy supply, which optimally balances energy supply cost and emission

of greenhouse gases¹. Energy system planners face the challenge to identify investment strategies and pathways towards a fully decarbonized energy supply. Thereby, they need to transform existing, grown infrastructures of different age step by step into sustainable solutions, considering multidimensional uncertainty.

Within this work, we present different modeling approaches to identify multi-period investments under uncertainty. On the one hand, we build on deterministic and two-stage stochastic approaches commonly used in the literature, and extend them to be applied within a rolling horizon. On the other hand, we propose a multi-stage stochastic program that can be solved to global optimality and therefore serves as a first-best benchmark. Within a quantitative comparison of the investment strategies, we

*Corresponding author. Phone: +49 9115302-767, Fax: +49 9115302-96281

Email addresses: markus.bohlayer@hs-karlsruhe.de (Markus Bohlayer), adrian.buerger@hs-karlsruhe.de (Adrian Bürger), markus.fleschutz@hs-karlsruhe.de (Markus Fleschutz), marco.braun@hs-karlsruhe.de (Marco Braun), gregor.zoettl@fau.de (Gregor Zöttl)

¹Lately, global industrial players like Tesla, Google and Bosch declared ambitious corporate emission objectives [1, 2, 3].

investigate the economic performance, the constraint violations, and the computational effort of the different approaches and compare the performance of multi-period investments to single-period investments using an adapted, "open loop" version of the rolling horizon approach.

1.2. Background and relevant literature

Distributed energy system planning is applied in the planning process of microgrids and industrial or urban energy supply systems [4]. In all three cases, Distributed Energy Resource (DER) investments and dispatch are determined to obtain a specific objective (e.g., minimization of system cost or system emissions), provided that all local energy demands are met [5]. For detailed reviews on the design of DERs and microgrids the reader is referred to the reviews of *Erdinc and Uzunoglu* [6] as well as *Gamarra and Guerrero* [7]. Prominent technologies in this context are Photovoltaik (PV) panels, Wind turbines, Gas Turbines (GT), Combined Heat and Power (CHP) plants, Absorption Heat Pumps (AHP), Heat Pumps (HP), Compression Refrigeration Systems (CRS), gas and biomass boilers in combination with Organic Rankine Cycle (ORC) systems as well as energy distribution infrastructure. Typically, the proposed models utilize one single representative year for system optimization and identify the optimal selection, sizing, and dispatch of the energy supply system within a Deterministic Synthesis Problem (DSP). Examples of such models include commercial solutions such as *HOMER Energy* [8] and *XENDEE* [9]. The increasing capabilities of commercial optimization solvers, such as *Gurobi* [10], *CPLEX* [11] or *Xpress* [12] have led to more complex models which include further aspects, such as the inclusion of passive investment options [4], detailed approximations of non-linear CHP efficiency curves [13] or the explicit consideration of temperature requirements [14].

One strand of literature investigates the impacts of uncertainties on the optimal operation and design of distributed energy systems. Within these works, a set of uncertain parameters, such as energy demands, energy carrier prices, or the availability of renewable energy, is identified and described with a finite number of scenarios [15]. Typically, these models are modeled as Two-stage Stochastic Programming (TSP) problems. In the first stage, the investment decisions are determined under uncertainty, and after the revelation of information (the realization of a specific scenario), the operational decisions are determined under perfect information [16]. In this context, *Sharafi and Elmekkawy* utilize the sampling average method and particle swarm optimization to design hybrid renewable energy systems [17]. *Fuentes-Cortés et al.* present a multi-objective optimization method for designing cogeneration systems accounting for the involved uncertainty associated with the ambient temperature, energy demands, and prices of the local energy market [18]. By using a two-stage model, *Yang et al.* [19] illustrate

that the economy of DER systems is overestimated if uncertainties are neglected. *Narayan and Ponnambalam* include the idea of mean-variance to the objective function of their two-stage stochastic optimization problem, which allows them to include risk-aversion [20]. *Mavromatidis et al.* extensively studied the uncertainties for the optimal design of DER systems. Within a first work, they identify the uncertain parameters with the highest impact on the economic performance of DER systems using the global sensitivity analysis [21]. This work was followed by the presentation of a two-stage stochastic program utilizing the beforehand identified most influential uncertain parameters [22]. And finally, they compared the adoption of different decision-making criteria as the objective function within their two-stage model [23]. *Pickering and Choudhary* utilize a 3-step methodology to generate, reduce, and optimize scenarios within a two-stage stochastic model. The focus of this work is to preserve interdependence between hourly demand profiles [24]. *Yu et al.* [25] illustrate the superiority of clustering algorithms to select demand profiles in stochastic optimization models. *Onishi et al.* focus on the generation of correlated distributions of uncertain parameters within TSP [26]. *Afzali et al.* use moment matching to discretize uncertain distributions utilized within a two-stage stochastic optimization approach [27]. While the aforementioned publications consider numerous different realizations of uncertain parameters, the temporal development of the energy infrastructure over multiple years is not considered. Investments are done prior to the realization of all scenarios and only one system configuration is determined. The stochastic approaches are typically benchmarked to a deterministic model formulation, however, an investigation of the approaches applied in a multi-period set-up has not been carried out.

Lately, *Pecenak et al.* have identified the need for multi-period investment models [28]. They argue that an economic planning model that does not capture the effects of changing framework conditions identifies system configurations that become suboptimal after short periods of time. They, therefore, propose an adaptive and forward-looking model. These models allow to consider the existing system configuration and, in case of the forward-looking model, a forecast for future price developments when identifying the system configuration. However, these approaches assume perfect foresight on future realizations of investment cost and energy carrier prices, neglecting any source of uncertainty. The multi-period model is solely benchmarked against a single-period model,

In the literature on large scale electricity generation and expansion planning, multi-stage stochastic investment planning problems can be found [29, 30]. Typically, these models neglect inter-temporal constraints where operating periods are coupled. Addressing this fact, *Liu et al.* [31] propose a multi-stage model that captures inter-temporal constraints as well as multi-scale representations of uncertainty. We build on this formulation to develop a new model that captures the multi-modal characterization of

distributed energy systems as well as specific decision and uncertainty characteristics. This problem serves as a first-best benchmark within our comparison since it can be solved to global optimality.

1.3. Contribution

Our work makes several contributions to this existing literature on energy system planning.

First, we fill a gap in the literature on multi-modal distributed energy systems by proposing a multi-stage stochastic mixed-integer program that facilitates the identification of multi-period investments, enabling the implementation of an eco-efficient decarbonization-strategy under the consideration of multiple sources of uncertainty. Then, we propose a rolling horizon approach that allows us to identify multi-period investments under uncertainty at much lower computational effort and enables the evaluation of different modeling approaches when applied in a multi-period set-up. Finally, we conduct a quantitative and qualitative analysis of the presented modeling approaches on a real-world dataset to evaluate their applicability as part of an iteratively conducted planning process and their performance regarding computational effort as well as economic and ecologic efficiency.

1.4. Paper organization

The remainder of this paper is organized as follows. First, we introduce the different modeling approaches to determine investments in distributed energy systems in Section 2. In Section 3, we present the utilized approach to decrease computational complexity by time series and scenario aggregation. Within a real-world case study in Section 4, we investigate the economic and computational performance of the presented modeling approaches. We conclude with a summary of our findings and identify future research questions in Section 5

2. Energy system synthesis problem formulations

The problem of interest in this work is a multi-period investment problem under uncertainty to determine investment pathways for multi-modal distributed energy systems. Within these systems, a set of energy carriers $k \in K := \{\text{Electricity, Hot water, Cold water, Gas, Biomass, Oil}\}$ contain the energy which is distributed in between conversion technologies $i \in I := \{\text{PV, Boiler, CHP, GT, AHP, HP, CRS, etc.}\}$, storages $n \in N := \{\text{Hot Storage, Cold Storage, Battery}\}$ and final demands. Besides the decision on the type and dimension of a specific technology, energy system planners need to decide on the optimal timing of investments in order to achieve specified emission objectives at minimal cost. The considered horizon consist of multiple investment periods $y \in Y := \{1, \dots, |Y|\}$ and each investment period is followed by an operation period, consisting of multiple time steps $t \in T := \{1, \dots, |T|\}$. Since future energy carrier prices, technology prices and

other system parameters are unknown, the decisions are made under uncertainty.

The literature provides different approaches to identify investment decisions. However, most approaches utilize a two-stage problem formulation, that identifies investment decisions in the first stage and operational decisions in the second stage. These approaches can be further subdivided into deterministic approaches, assuming all operational parameters are known with certainty, and stochastic approaches, considering uncertain operational parameters. We state the deterministic single-period problem in Section 2.1 and the stochastic single-period problem in Section 2.2. To identify multi-period investment decisions with the stochastic or deterministic single-period problem, they need to be applied in an iterative manner. Therefore, we propose a Rolling Horizon Algorithm (RHA) in Section 2.5 yielding problem formulations to identify approximate solutions to the multi-period problem. To further analyze the performance of single-period approaches often found in the literature, we utilize an adapted version of the RHA in Section 2.6, which iterates over all periods to identify the operational variables excluding the possibility for any further investments.

Section 2.3 presents a deterministic multi-period problem. Since this problem considers no uncertainty in the operational or investment parameters, we solve the problem again with the proposed RHA to evaluate the iterative revelation of new information on the investment decisions. With the adapted version of the RHA presented in Section 2.6, we investigate the performance of the investments determined in the first iteration.

Finally, we propose a stochastic multi-period problem formulation that considers the information structure in terms of a scenario tree and allows for scenario-specific investments in Section 2.4. The solution to this problem yields optimal investment pathways with respect to the considered scenario tree and therefore serves as a first-best benchmark in the upcoming sections.

2.1. Deterministic single-period problem

The most commonly used formulation is the deterministic synthesis problem, which can be found frequently in the literature and is commonly used in commercial software solutions. The formulation considers one single investment period followed by multiple operational periods and all system parameters are assumed to be known with certainty. Investment costs, as well as energy procurement costs and energy availability, are assumed to be known and invariant within the considered time horizon. Typically current market prices are used, since these are readily available. The objective of the problem is to minimize the total annual costs which consists of annualized capital expenditure and the operational expenditure within one representative year. The problem is stated in Equation (1).

In Equation (1a), the capital expenditure is calculated as the sum of variable and fixed costs multiplied with the

annualization factor a^a . The variable costs are the products of the variable cost vector $\mathbf{c}^v = [c_{\text{Hot storage}}, \dots, c_{\text{CRS}}]^\top$ and the technology vector, consisting of the conversion technology capacity vector \mathbf{p}^c and the storage technology capacity vector \mathbf{s}^c .

P1:

$$\min a^a \left((\mathbf{c}^v)^\top \begin{bmatrix} \mathbf{s}^c \\ \mathbf{p}^c \end{bmatrix} + (\mathbf{c}^f)^\top \begin{bmatrix} \mathbf{d}^s \\ \mathbf{d}^c \end{bmatrix} \right) + \sum_{t \in T} \Delta t_t ((\mathbf{c}_t^b)^\top \mathbf{p}_t^b - (\mathbf{c}_t^s)^\top \mathbf{p}_t^s) \quad (1a)$$

s. t.

$$\sum_{t \in T} \Delta t_t ((\mathbf{e}_t)^\top (\mathbf{p}_t^b - \mathbf{p}_t^s)) \leq e^{\max} \quad (1b)$$

$$\mathbf{p}_t^b - \mathbf{p}_t^s = \mathbf{p}_t^d + \sum_{i \in I} (\mathbf{p}_{i,t}^i - \mathbf{p}_{i,t}^o) + \frac{1}{\Delta t_t} \mathbf{S}(\mathbf{s}_{t+1} - (\mathbf{n}^s)^\top \mathbf{s}_t) \quad (1c)$$

$$\mathbf{N}_i \mathbf{p}_{i,t}^i = \mathbf{p}_{i,t}^o \quad (1d)$$

$$\mathbf{a}_i^\top \mathbf{p}_{i,t}^i \leq p_i^c \leq p_i^m d_i^c \quad (1e)$$

$$\mathbf{s}_t \leq \mathbf{s}^c \leq s^m \mathbf{d}^s \quad (1f)$$

$$\mathbf{p}^c \in \mathbb{R}_{\geq 0}^{|I|}, \mathbf{s}^c \in \mathbb{R}_{\geq 0}^{|N|}$$

$$\mathbf{p}_{i,t}^i, \mathbf{p}_{i,t}^o, \mathbf{p}_t^b, \mathbf{p}_t^s \in \mathbb{R}_{\geq 0}^{|K|}$$

$$\mathbf{d}^c \in \{0, 1\}^{|I|}, \mathbf{d}^s \in \{0, 1\}^{|N|}$$

The fixed costs are the product of the fixed cost vector \mathbf{c}^f and the technology existence vector, consisting of the conversion technology existence vector \mathbf{d}^c and the storage technology existence vector \mathbf{d}^s . The operational expenditure of each time step t is calculated as the sum of bought and sold power multiplied by the cost vector of energy purchases \mathbf{c}_t^b and of energy sales \mathbf{c}_t^s , respectively. Similarly, in Equation (1b) the emissions are calculated as the sum of energy entering and leaving the company on energy carrier k multiplied by the emission factor of the respective energy carrier \mathbf{e}_t . Here, emissions are limited within the constraint to a maximum emission limit e^{\max} . The term could also be included in the objective function yielding a multi-objective optimization problem. Equation (1c) states that the sum of energy purchases and energy sales need to equal the sum of energy demands represented by the demand vector \mathbf{p}_t^d , the difference between energy entering and energy leaving all conversion technologies i , represented by the vectors $\mathbf{p}_{i,t}^i$ and $\mathbf{p}_{i,t}^o$, respectively, and the change of the state of charge of the storages, described by the vector \mathbf{s}_t . The matrix \mathbf{S} maps the energy carriers on the storage technologies. Storage losses of the specific storage technologies are included in the storage efficiency vector \mathbf{n}^s . The matrix \mathbf{N}_i of each technology i includes the efficiency factors $\eta_{i,k,l}$ for the transfer of energy from energy carrier k to energy carrier l . The capacity limits of conversion and storage technologies are stated in Equation (1e) and Equation (1f), respectively.

2.2. Two-stage stochastic single-period problem

Energy systems synthesis usually considers time horizons of multiple decades, therefore operational parameters such as energy carrier prices or emission factors are likely to strongly vary. These variations are not considered within the DSP, which can lead to non-optimal design configurations. Within the TSP this downside is addressed by the inclusion of uncertain parameters. Typically the probability distribution of the stochastic variables is approximated with a finite number of scenarios $|J|$. Each scenario $j \in J := 1, \dots, |J|$ has a probability of occurrence π_s . The utilization of discrete scenarios enables the reformulation of the TSP as a deterministic-equivalent problem, which is stated as P2. For a more theoretical background on stochastic programming, the reader is referred to the work of Birge [32]. Within Equation (2a) the first term remains unchanged since investment costs are assumed to be known with certainty and investment decisions are so-called first-stage decision variables which are made before the realization of the uncertain parameters and are therefore independent of the outcome of a specific scenario. Within the second term, the expected operational costs is minimized which includes second-stage decision variables and the uncertain parameters of energy procurement and sales costs $\gamma_{j,t}^{b/s}$ which are described with a finite number of scenarios $|J|$. Additionally, some of the constraints contain stochastic parameters, namely the emission factors $\epsilon_{j,t}$ and the energy demands $\delta_{j,t}$. This formulation ensures that the decision variables are feasible for all possible realizations of the stochastic parameters.

P2:

$$\min a^a \left((\mathbf{c}^v)^\top \begin{bmatrix} \mathbf{s}^c \\ \mathbf{p}^c \end{bmatrix} + (\mathbf{c}^f)^\top \begin{bmatrix} \mathbf{d}^s \\ \mathbf{d}^c \end{bmatrix} \right) + \mathbb{E}_{j \in J} \left[\sum_{t \in T} \Delta t_t ((\gamma_{j,t}^b)^\top \mathbf{p}_{j,t}^b - (\gamma_{j,t}^s)^\top \mathbf{p}_{j,t}^s) \right] \quad (2a)$$

s. t.

$$\sum_{t \in T} \Delta t_t ((\epsilon_{j,t})^\top (\mathbf{p}_{j,t}^b - \mathbf{p}_{j,t}^s)) \leq e^{\max} \quad (2b)$$

$$\mathbf{p}_{j,t}^b - \mathbf{p}_{j,t}^s = \delta_{j,t} + \sum_{i \in I} (\mathbf{p}_{j,i,t}^i - \mathbf{p}_{j,i,t}^o) + \frac{1}{\Delta t_t} \mathbf{S}(\mathbf{s}_{j,t+1} - (\mathbf{n}^s)^\top \mathbf{s}_{j,t}) \quad (2c)$$

$$\mathbf{N}_i \mathbf{p}_{j,i,t}^i = \mathbf{p}_{j,i,t}^o \quad (2d)$$

$$\mathbf{a}_i^\top \mathbf{p}_{j,i,t}^i \leq p_i^c \leq p_i^m d_i^c \quad (2e)$$

$$\mathbf{s}_{j,t} \leq \mathbf{s}^c \leq s^m \mathbf{d}^s \quad (2f)$$

$$\mathbf{p}^c \in \mathbb{R}_{\geq 0}^{|I|}, \mathbf{s}^c \in \mathbb{R}_{\geq 0}^{|N|}$$

$$\mathbf{p}_{j,i,t}^i, \mathbf{p}_{j,i,t}^o, \mathbf{p}_{j,t}^b, \mathbf{p}_{j,t}^s \in \mathbb{R}_{\geq 0}^{|K|}$$

$$\mathbf{d}^c \in \{0, 1\}^{|I|}, \mathbf{d}^s \in \{0, 1\}^{|N|}$$

2.3. Deterministic multi-period problem

While the TSP formulation addresses the uncertain characteristics of multiple input parameters, three important aspects can not be taken into consideration. First, the approach does not take any existing supply infrastructure into account. However, most of the supply system in micro-grids and industrial companies are grown structures of different age-structure that need to be transferred into a low-carbon supply system.

Second, the TSP formulation does not consider any temporal distribution or development of uncertain parameters nor any emission reduction paths. And finally, only one investment decision is considered, which might lead to suboptimal decisions, especially with regard to investments in renewable energies, a market characterized by strong learning effects and economies of scale.

P3:

$$\begin{aligned} \min \quad & \sum_{y \in Y} a_y^d \left((\mathbf{c}_y^v)^\top \begin{bmatrix} \mathbf{s}^c \\ \mathbf{p}^c \end{bmatrix}_{y,0} + (\mathbf{c}_y^f)^\top \begin{bmatrix} \mathbf{d}^s \\ \mathbf{d}^c \end{bmatrix}_y + \right. \\ & \left. \sum_{t \in T} \Delta t_t ((\mathbf{e}_{y,t}^b)^\top \mathbf{p}_{y,t}^b - (\mathbf{c}_{y,t}^s)^\top \mathbf{p}_{y,t}^s) \right) + \\ & \sum_{z \in Z} a_{|Y|}^d (\mathbf{c}_{|Y|}^v)^\top (\mathbf{L}_z \begin{bmatrix} \mathbf{s}^c \\ \mathbf{p}^c \end{bmatrix}_{|Y|,z}) \end{aligned} \quad (3a)$$

s. t.

$$\sum_{y \in Y} \sum_{t \in T} \Delta t_t ((\mathbf{e}_{y,t})^\top (\mathbf{p}_{y,t}^b - \mathbf{p}_{y,t}^s)) \leq e^{\max} \quad (3b)$$

$$\mathbf{p}_{y,t}^b - \mathbf{p}_{y,t}^s = \mathbf{p}_{y,t}^d + \sum_{i \in I} (\mathbf{p}_{i,y,t}^i - \mathbf{p}_{i,y,t}^o) \quad (3c)$$

$$+ \frac{1}{\Delta t_t} \mathbf{S}(\mathbf{s}_{y,t+1} - (\mathbf{n}^s)^\top \mathbf{s}_{y,t}) \quad (3d)$$

$$\mathbf{N}_i \mathbf{p}_{i,y,t}^i = \mathbf{p}_{i,y,t}^o \quad (3e)$$

$$\mathbf{a}_i^\top \mathbf{p}_{i,y,t}^i \leq \sum_{z \in \{0, \dots, L_i\}} p_{i,y,z}^c \quad (3f)$$

$$p_{i,y,0}^c \leq p_i^m d_{i,y}^c \quad (3g)$$

$$p_{i,y+1,z+1}^c = p_{i,y,z}^c \quad (3h)$$

$$p_{i,0,z}^c = p_{i,z}^{\text{initial}} \quad (3i)$$

$$\mathbf{s}_{y,t} \leq \sum_{z \in \{0, \dots, L_s\}} \mathbf{s}_{y,z}^c \quad (3j)$$

$$\mathbf{s}_{y,0}^c \leq \mathbf{s}_y^m \mathbf{d}_y^s \quad (3k)$$

$$\mathbf{s}_{y+1,z+1}^c = \mathbf{s}_{y,z}^c \quad (3l)$$

$$\mathbf{s}_{0,z}^c = \mathbf{s}_z^{\text{initial}} \quad (3m)$$

$$\mathbf{p}_{y,z}^c \in \mathbb{R}_{\geq 0}^{|I|}, \mathbf{s}_{y,z}^c \in \mathbb{R}_{\geq 0}^{|N|}$$

$$\mathbf{p}_{i,y,t}^i, \mathbf{p}_{i,y,t}^o, \mathbf{p}_{y,t}^b, \mathbf{p}_{y,t}^s \in \mathbb{R}_{\geq 0}^{|K|}$$

$$\mathbf{d}_y^c \in \{0, 1\}^{|I|}, \mathbf{d}_y^s \in \{0, 1\}^{|N|}$$

To address these shortcomings, we build on the approach presented by *Pecenak et al.* [28]. We adapt their multi-

year formulation to include the age structure of the energy supply system and an initial system structure. First, we adapt the approach to cover multiple investment periods $y \in Y := \{1, \dots, |Y|\}$. Further, we consider the age structure of the technology capacity. The vector $\mathbf{p}_{y,z}^c$ hereby represents the existing capacity within period y with age $z \in Z := 1, \dots, L$. Investments at each period have an age of zero and are therefore denoted by $\mathbf{p}_{y,0}^c$. The objective function is adapted to cover the investment costs and operational costs of all investment periods y . All payments are discounted with the specific discount factor $a_y^d = \frac{1}{(1+r)^y}$. The last term of the objective function includes the discounted residual value of all technologies in the last period. The Matrix \mathbf{L}_z is used to calculate the residual value based on the nominal capacity and the age z of the technologies. Furthermore, additional constraints are introduced to ensure that the power output of each technology is below the total installed capacity (compare Equations (3f) and (3j)), to track the age structure of the technology capacity (compare Equations (3h) and (3l)) and to initialize the system structure (compare Equations (3i) and (3m)). This approach enables the identification of investments for multiple periods dependent on the changing framework conditions, while the existing structure of energy supply systems can be taken directly into consideration.

2.4. Stochastic multi-period problem formulation

The downside of the formulation presented in P3 is that it is assumed to have perfect foresight on future framework conditions. To overcome the shortcoming of the formulations presented in P2 and P3, we propose a combined formulation in which we consider uncertainties in terms of stochastic processes. Our model considers an optimization horizon of multiple investment periods $y \in Y := \{1, \dots, |Y|\}$, where each investment period is followed by an operational period consisting of time intervals $t \in T(y) := \{1, \dots, |T|\}$. We introduce the multivariate stochastic process $\{\mathbf{x}_y\}_{y \in Y}$ on some filtered probability space (Ω, F, P) where Ω is the sample space, F is the filtration and P is the probability measure. The stochastic process includes uncertain emission factors ϵ_y , energy demands δ_y , energy prices $\gamma_y^{b/s}$ and investment costs $\gamma_y^{v/f}$. The filtration describes how information is revealed over the considered time horizon. The information of the first investment decision is given by F_1 , after investments are determined the market prices of the first set of operational periods are revealed and again decisions on investments can be determined under information F_2 . Since within each investment period new information is revealed the sigma algebras are $F_1 \subset F_2 \subset \dots \subset F_{|Y|}$. The decision structure and scenario tree is also illustrated in Figure 1

We define the investment costs of period y as

$$C_y^{\text{inv}}(\mathbf{s}_y^c, \mathbf{p}_y^c) = (\gamma_y^v)^\top \begin{bmatrix} \mathbf{s}^c \\ \mathbf{p}^c \end{bmatrix}_{y,0} + (\gamma_y^f)^\top \begin{bmatrix} \mathbf{d}^s \\ \mathbf{d}^c \end{bmatrix}_y, \quad (4)$$

and further introduce the operational costs of period y as

$$C_y^{\text{op}}(\mathbf{p}_{y,t}^b, \mathbf{p}_{y,t}^s) = \sum_{t \in T} \Delta t_t ((\gamma_{y,t}^b)^\top \mathbf{p}_{y,t}^b - (\gamma_{y,t}^s)^\top \mathbf{p}_{y,t}^s). \quad (5)$$

This yields the multistage stochastic optimization problem with the objective to minimize the investment and operational costs within each period under the given information.

$$\begin{aligned} \min C^{\text{total}} = & \left\{ C_1^{\text{inv}}(\mathbf{s}_1^c, \mathbf{p}_1^c) + \mathbb{E} \left[\min \left\{ C_1^{\text{op}}(\mathbf{p}_{t,1}^b, \mathbf{p}_{t,1}^s + \dots \right. \right. \right. \\ & + \mathbb{E} \left[\min \{ C_{|Y|-1}^{\text{inv}}(\mathbf{s}_{|Y|-1}^c, \mathbf{p}_{|Y|-1}^c) + \right. \\ & \left. \left. \mathbb{E}[\min \{ C_{|Y|}^{\text{op}}(\mathbf{p}_{t,|Y|}^b, \mathbf{p}_{t,|Y|}^s \mid F_{|Y|}) \} \mid F_{|Y|-1}) \} \right. \right. \\ & \left. \left. \dots \mid F_2 \right] \mid F_1 \right] \left. \right\} \quad (6) \end{aligned}$$

For computational reasons, we represent the stochastic process by a finite number of scenarios. This yields the stochastic process $\{[\epsilon, \delta, \gamma^{b/s}, \gamma^{v/f}]_{y,j}^\top\}_{y \in Y, j \in J} := \{1, \dots, |J|\}$.

Given this discrete distribution, the stochastic problem can be equivalently reformulated as the so-called deterministic equivalent, which is presented as problem P4. The objective function Equation (7a) is to minimize the total costs, which consist of the total costs of each scenario weighted by its probability of occurrence. The total costs of each scenario consist of the cumulative investment and operational costs of all years, each multiplied with the respective discount factor $a_y^d = \frac{1}{(1+r)^y}$.

The last term of the objective function represents the residual value of all investments in the last considered period. Within Equations (7b) to (7h) as well as Equations (7m) to (7o) a copy for each constraint for each scenario $j \in J$ is introduced. Since the problem is a multistage decision problem, decisions on each stage have to be made before any information on further stages is revealed. Therefore, we introduce nonanticipativity constraints in Equations (7i), (7k), (7p) and (7r).

In the first stage, decisions on the investment in period 1 are made. At this stage no further information is available and therefore investment decisions need to be equal for all scenarios, this is ensured in Equations (7i) and (7p). On all the following stages of the problem, additional information is available, and decisions are made based on this available information. However, if two paths in the scenario tree share the same history, they must share the same decision on investments for the specific investment period. This is ensured in Equations (7k) and (7r).

P4:

$$\begin{aligned} \min \sum_{j \in J} \pi_j & \left(\sum_{y \in Y} a_y^d \left((\gamma_{j,y}^v)^\top \begin{bmatrix} \mathbf{s}^c \\ \mathbf{p}^c \end{bmatrix}_{j,y,0} + (\gamma_{j,y}^f)^\top \begin{bmatrix} \mathbf{d}^s \\ \mathbf{d}^c \end{bmatrix}_{j,y} + \right. \\ & \left. \sum_{t \in T} \Delta t_t ((\gamma_{j,y,t}^b)^\top \mathbf{p}_{j,y,t}^b - (\gamma_{j,y,t}^s)^\top \mathbf{p}_{j,y,t}^s) \right) + \\ & \sum_{z \in Z} a_{|Y|} (\gamma_{j,|Y|}^v)^\top (\mathbf{L}_z \begin{bmatrix} \mathbf{s}^c \\ \mathbf{p}^c \end{bmatrix}_{j,|Y|,z}) \quad (7a) \end{aligned}$$

s. t.

$$\sum_{y \in Y} \sum_{t \in T} \Delta t_t ((\epsilon_{j,y,t})^\top (\mathbf{p}_{j,y,t}^b - \mathbf{p}_{j,y,t}^s)) \leq e^{\max} \quad (7b)$$

$$\begin{aligned} \mathbf{p}_{j,y,t}^b - \mathbf{p}_{j,y,t}^s &= \delta_{j,y,t} + \sum_{i \in I} (\mathbf{p}_{i,j,y,t}^i - \mathbf{p}_{i,j,y,t}^o) \\ &+ \frac{1}{\Delta t_t} \mathbf{S}(\mathbf{s}_{j,y,t+1} - (\mathbf{n}^s)^\top \mathbf{s}_{j,y,t}) \quad (7c) \end{aligned}$$

$$\mathbf{N}_i \mathbf{p}_{i,j,y,t}^i = \mathbf{p}_{i,j,y,t}^o \quad (7d)$$

$$\mathbf{a}_i^\top \mathbf{p}_{i,j,y,t}^i \leq \sum_{z \in \{0, \dots, L_i\}} p_{i,j,y,z}^c \quad (7e)$$

$$p_{i,j,y,0}^c \leq p_i^m d_{i,j,y}^c \quad (7f)$$

$$p_{i,j,y+1,z+1}^c = p_{i,j,y,z}^c \quad (7g)$$

$$p_{i,j,0,z}^c = p_{i,z}^{\text{initial}} \quad (7h)$$

$$p_{i,j,1,0}^c = p_{i,l,1,0}^c \quad \forall j, l \in J \quad (7i)$$

$$p_{i,j,y,0}^c = p_{i,l,y,0}^c \quad \forall j, l \in J, \quad (7j)$$

$$y \in Y \setminus \{1\} \text{ with } \gamma_{j,y-1} = \gamma_{l,y-1} \quad (7k)$$

$$\mathbf{s}_{j,y,t} \leq \sum_{z \in \{0, \dots, L_s\}} \mathbf{s}_{j,y,z}^c \quad (7l)$$

$$\mathbf{s}_{j,y,0}^c \leq s^m \mathbf{d}_{j,y}^s \quad (7m)$$

$$\mathbf{s}_{j,y+1,z+1}^c = \mathbf{s}_{j,y,z}^c \quad (7n)$$

$$\mathbf{s}_{j,0,z}^c = \mathbf{s}_z^{\text{initial}} \quad (7o)$$

$$\mathbf{s}_{j,1,0}^c = \mathbf{s}_{l,1,0}^c \quad \forall j, l \in J \quad (7p)$$

$$\mathbf{s}_{j,y,0}^c = \mathbf{s}_{l,y,0}^c \quad \forall j, l \in J, \quad (7q)$$

$$y \in Y \setminus \{1\} \text{ with } \gamma_{j,y-1} = \gamma_{l,y-1} \quad (7r)$$

$$\begin{aligned} \mathbf{p}_{j,y}^c &\in \mathbb{R}_{\geq 0}^{|I|}, \mathbf{s}_{j,y}^c \in \mathbb{R}_{\geq 0}^{|N|} \\ \mathbf{p}_{i,j,y,t}^i, \mathbf{p}_{i,j,y,t}^o, \mathbf{p}_{j,y,t}^b, \mathbf{p}_{j,y,t}^s &\in \mathbb{R}_{\geq 0}^{|K|} \\ \mathbf{d}_{j,y}^c &\in \{0, 1\}^{|I|}, \mathbf{d}_{j,y}^s \in \{0, 1\}^{|N|} \end{aligned}$$

2.5. Rolling horizon problem formulations

The problems P1 and P2 identify investment decisions for one single investment period and problems P1 and P3 identify investment decisions under the assumption of perfect foresight. To utilize these problems to identify multi-period investment decisions under uncertainty, they need

Problem formulation	Investment decisions	Uncertainty consideration	Optimization horizon
P4	multi-period	scenario tree	all periods
P1_ol	single-period	current framework conditions	one period
P2_ol	single-period	scenario tree	one period
P3_ol	multi-period	deterministic forecast	all periods
P1_rh	multi-period	current framework conditions	one period
P2_rh	multi-period	scenario tree	one period
P3_rh	multi-period	deterministic forecast	all periods

Table 1: Overview of the investigated problem formulations

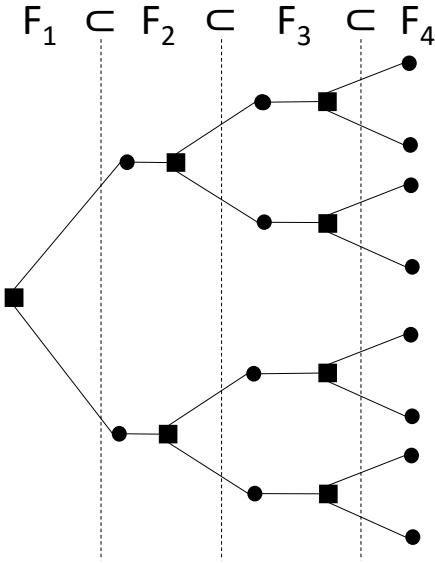


Figure 1: Decision structure of the multi-stage SP. A square indicates an investment period, a circle represents operational periods

to be applied within an iterative procedure. Therefore, we propose a RHA, which enables the identification of multi-period investment decisions under uncertainty using the conventional problems P1, P2, and P3. In the RHA, the respective problem is solved repeatedly for different years and scenario realizations. The problems are approximations of the original problem that either reduce the complexity by the assumption of perfect foresight (in case of P3), the reduction of investment and operational periods considered (in case of P2), or both (in case of P1). Within each iteration, the problem is solved for one period (in case of P1 and P2) or multiple periods (in case of P3) and optimal investments and/or operation of the energy supply system are determined. Within each iteration, the solution depends on the system configuration identified in the

temporally preceding node and on the forecast of model parameters at the specific node of the scenario tree. After each iteration of the rolling horizon algorithm, new information on the system's performance and parameter forecast is revealed. Based on this information, investment decisions for the respective period are identified to achieve the desired performance. This closed-loop feedback allows to situationally adapt the investment pathway.

The algorithm to identify investments and operation of the system is depicted in Figure 2 and the procedure is further illustrated in Figure 3. Within the first step, a deterministic or stochastic forecast is determined and the initial system configuration is defined. Based on this information, the problem P1, P2 or P3 is solved and the optimal investments (indicated by squares in Figure 3), as well as the optimal operation decisions (indicated by circles in Figure 3), are determined. Hereby, only the investment decisions of the respective period are fixed (as indicated by the red colored squares) while investment and operational decisions of subsequent periods may be identified but not fixed for further iterations. After the investment decision, uncertain information is revealed (as indicated by the branching of the trees in Figure 3) and the final operational decisions are determined solving P with fixed investments. Based on the revealed information, a new stochastic or deterministic forecast is obtained and investment decisions of the subsequent period are determined. Dependent on the problem $P \in \{P1, P2, P3\}$ the rolling horizon problem formulation is stated as

P_rh: Execute the RHA(ol=False) utilizing problem formulation P to obtain an approximate solution to problem P4.

The RHA reflects the practice of energy system management, which is often conducted in a closed Plan-Do-Check-Act cycle, in which energy system planners continuously monitor the system performance and iteratively check for new investment opportunities to improve the systems economic and ecologic efficiency². Therefore, the

²In many cases, industrial companies establish an energy management system to continuously improve the energy performance. This process is certified by the ISO 50001 standard [33].

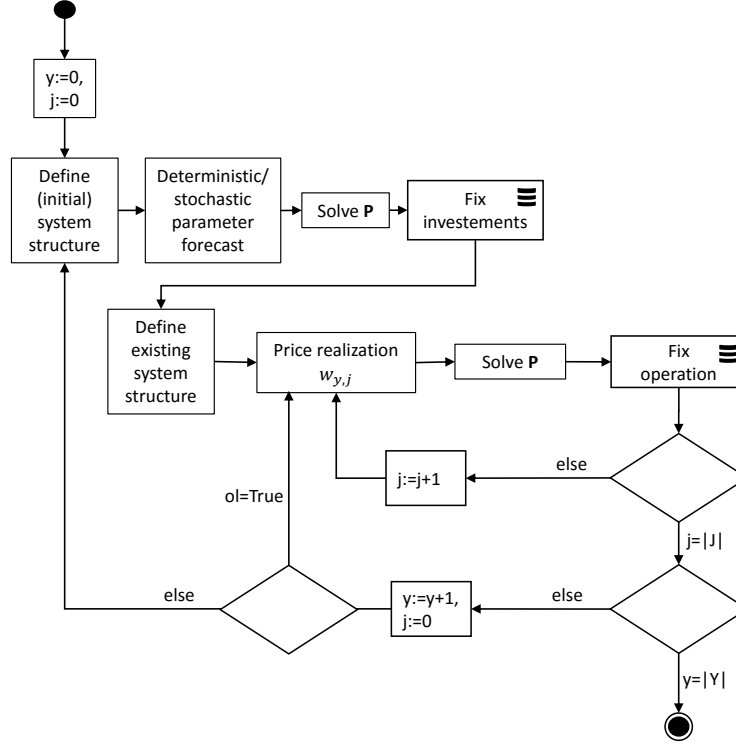


Figure 2: Flowchart of the Rolling Horizon Algorithm (RHA) to identify multi-year investment and operational decisions utilizing problem $P \in \{P1, P2, P3\}$

proposed methodology does not only allow to identify approximate solutions to the multi-period problem but also to evaluate the different modeling approaches for the application in a corporate energy system management process.

2.6. Open Loop problem formulations

To evaluate the performance of conventional planning approaches in a single-period investment setup, we include the boolean parameter ol which is set to True if investment decisions are determined in an open-loop (ol) investment identification process. This ensures, that within the RHA the investment decisions are solely determined in the first iteration under the problem respective parameter approximation. Afterward, the information within each scenario path is revealed and the respective optimal operational parameters are determined. This procedure allows to evaluate the performance of investments identified in the first iteration for the entire scenario tree. Dependent on the problem $P \in \{P1, P2, P3\}$ the open-loop problem formulation is stated as

P_{ol}: Execute the RHA($ol=True$) utilizing problem formulation **P** to evaluate the solutions obtained by **P** for the entire scenario tree.

Table 1 one gives an overview of the problem formulations investigated in the following chapters.

3. Time series and scenario aggregation

With the inclusion of a high number of variables and constraints, stochastic programming problems are computationally hard and expensive to solve within appropriate solution times. One popular approach to handle the computational complexity of large scale stochastic optimization problems is to reduce the number of variables significantly by the careful selection of both representative operational time periods and scenarios. The major advantage of this approach is that the complexity reduction not only yields decreased computational effort in case of stochastic programming problems but also when solving the scenario tree within a rolling horizon. In the following, we present the approaches for time series aggregation and scenario reduction utilized in this work.

3.1. Time series aggregation

Within energy synthesis problems, typical periods are commonly selected to represent the entire operational horizon of one year. Multiple studies showed that clustering algorithms are favorable when selecting typical periods [34]

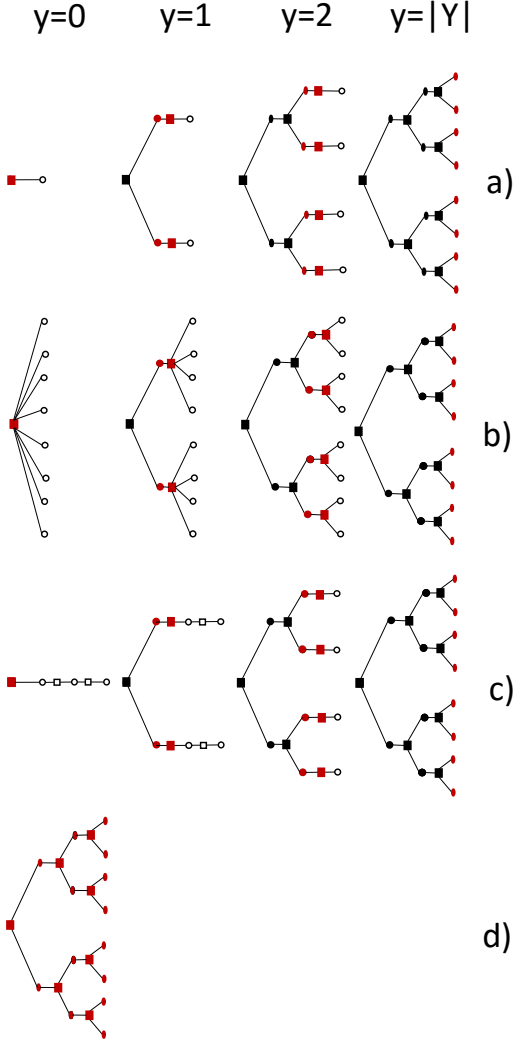


Figure 3: Representation of the information approximations over the annual iterations of the rolling horizon algorithm, with (a) showing the deterministic single year approximation (P1), (b) the two-stage stochastic approximation (P2), (c) the deterministic multi-year approximation (P3) and (d) the multi-stage stochastic approximation (P4)

and that the error of the optimization with typical periods is limited [35]. Based on these findings, we select typical periods and aggregate time steps to segments as proposed by *Bahl et al.* [36] and briefly described in the following.

We define the vector \mathbf{b}_t that includes all relevant time series, such as irradiation and energy demands on a given time horizon $t \in T := \{1, \dots, |T|\}$ in a normalized form. We assume that the time series is characterized by periodic patterns (e.g., daily or weekly). According to this periodic pattern, the time series is split up into a number N_p of periods with an equidistant periodic length. Each period $d \in D$ consists of multiple time steps $t \in T_p := \{1, \dots, \frac{|T|}{N_p}\}$. In the next step, we reduce the number of periods by clustering to a smaller set of representative periods $D' \subset D$. The

clustering problem can be stated as an integer problem:

$$\begin{aligned}
 \min_{z_{i,j}, y_j} \quad & \sum_{i \in D} \sum_{j \in D} z_{i,j} \sum_{t \in T_p} (|\mathbf{b}_{i,t} - \mathbf{b}_{j,t}|) \\
 \text{s.t.} \quad & \sum_{j \in D} z_{i,j} = 1 \quad \forall i \in D \\
 & z_{i,j} \leq y_j \\
 & \sum_{j \in D} y_j = |D'| \\
 & y_j \in \{0, 1\}, z_{i,j} \in \{0, 1\}
 \end{aligned} \tag{8}$$

The objective is to minimize the Euclidean distance between the medoids and the assigned scenarios. The assignment matrix $z_{i,j}$ assigns each scenario to one medoid, and the binary variable y_j selects the medoids from the set of periods D .

To further decrease the number of variables we aggregate the hourly time intervals into segments of multiple hours. The identification of segments can also be described as optimization problem:

$$\begin{aligned}
 \min_{b_j^s, y_{t,j}} \quad & \sum_{j \in S} \sum_{t \in T_p} y_{t,j} (|\mathbf{b}_j^s - \mathbf{b}_{j,t}|) \\
 \text{s.t.} \quad & y_{1,1} = 1 \\
 & \sum_{j \in S} y_{t,j} = 1 \quad \forall t \in T_p \\
 & y_{t,j} \leq y_{t-1,j} + y_{t-1,j-1} \quad \forall t \in T_p \setminus \{1\}, j \in S \setminus \{1\} \\
 & y_{j,t} \in \{0, 1\}, \mathbf{b}_j^s \in \mathbb{R}_{\geq 0}^{|M|}
 \end{aligned} \tag{9}$$

The objective is to minimize the distance between the segment center b_j^s and the initial time series for each segment and time interval. The binary variable $y_{j,t}$ assigns each period to segment $j \in S := \{1, \dots, |S|\}$. The constraints ensure that each time interval is assigned to one segment and that the segments consist of consecutive time intervals.

Using this approach, the time series is first reduced by the factor $|D'|/|D|$ by the selection of typical periods and afterward by the factor $|S|/|T_p|$ by the aggregation of time intervals to segments. The number of total time intervals of the reduced time series is then given by:

$$|T_r| = \frac{|D'|}{|D|} \frac{|S|}{|T_p|} |T| \tag{10}$$

3.2. Scenario generation and reduction

Most Stochastic Programming (SP) problems in the literature employ two-stage approaches. Uncertain parameters are considered as random variables, which are described by a given probability distribution. Typically, the Scenario Average Approximation (SAA) method is used to generate a representative sample of possible realizations of the set of random variables. In case of multi-stage stochastic programs, uncertainty is described by the stochastic process $\{X_y\}_{y \in Y}$, a collection of random variables indexed

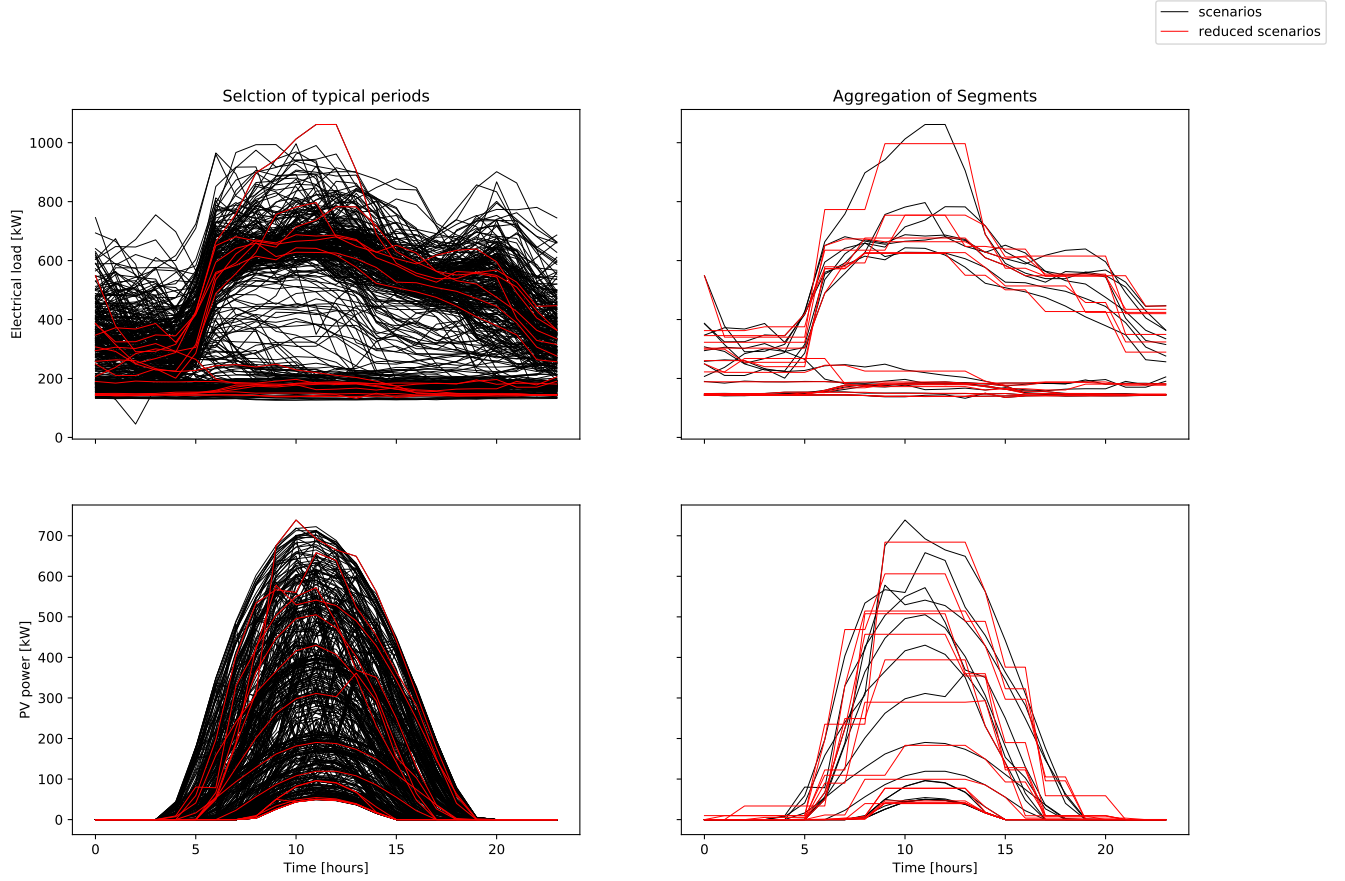


Figure 4: Demand data and solar radiation before time series and segment aggregation (gray lines) and after aggregation (red lines). The figure indicate the selection of typical periods on the left and the aggregation of the hourly intervals into segments on the right.

by a set of points in time $y \in Y$ as introduced in Section 2.4. Similar to the SAA method, in this case a model describing the underlying stochastic process is used to generate a set of sample paths $\{X_{y,j}\}_{y \in Y, j \in J^{\text{Ini}}}$. Since a large set of sample paths leads to an increasing computational complexity, clustering algorithms are applied to reduce the set of scenarios to a smaller but still representative number $|J^{\text{Red}}|$. Central references of this approach can be found in [37, 38]. To preserve autocorrelation and for computational reasons, we iteratively sample and reduce scenario tree branches for each node of the scenario tree, starting with the root node and ending with the leaf-nodes. This procedure enables the generation of a scenario tree, which reflects the filtration of the process. For more detailed description of the approach, the reader is referred to recent applications in the literature [39, 40, 41].

4. Case study

Within this case study, we investigate the energy supply system of an industrial complex located in the south-west of Germany. Distributed energy resources serve the local electricity and thermal demands. The distributed energy system is connected to the public grid to consume additional electrical energy or feed-in surplus generation of renewable resources. The selected candidate technologies within this case study include gas-fired and biomass-fired boilers, CHPs, HPs, an ORC unit, PV as well as heat and

electricity storages. Please note that the proposed method is however not limited to this set of technologies, but can be extended to include other types of conversion and storage technologies.

Within this case study, we compare different approaches to identify multi-year investment decisions under uncertainty. The objective is to minimize the total cost of energy provision while complying with the specified emission limits. Since the problem formulation of P1 and P3 are not capable to include the existing system's age structure, we compare the approaches for a greenfield design planning problem with no existing energy supply infrastructure. However, we qualitatively discuss the advantages of explicit age structure tracking in the model formulation, as proposed in P2 and P4, in Section 4.5.

The aim of the case study is to evaluate the different planning approaches with respect to the total cost of energy supply, the fulfillment of ecological objectives, constraint violations as well as selected infrastructure.

4.1. Demand data and time series aggregation

The demand data is characterized by the shift planning of the company and seasonal effects. Most production lines are run on two shifts, while some production lines are operated continuously during the week, which is reflected in the daily electricity demands as indicated in Figure 4. Thermal demands are partially caused by production and partially by heating and cooling demands of the building.

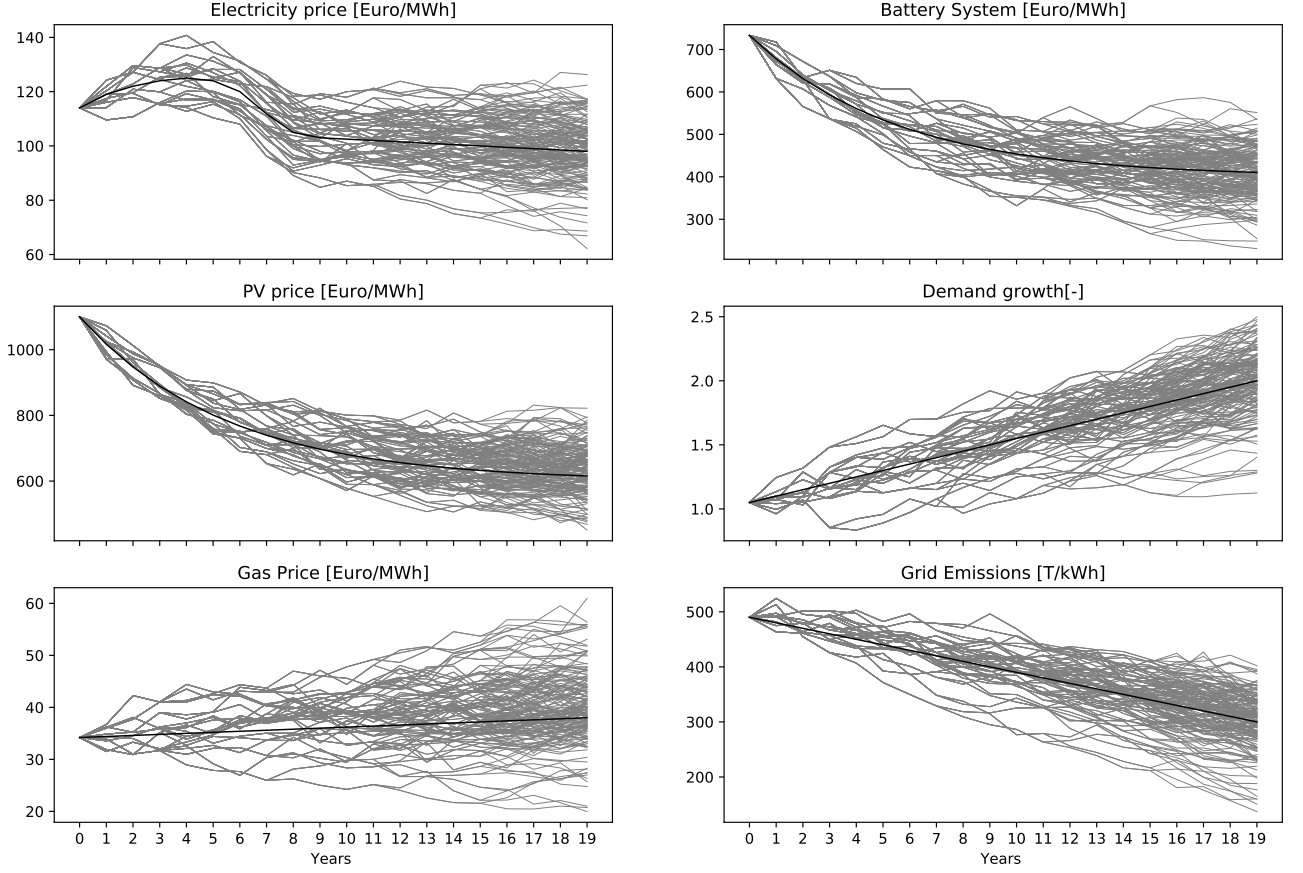


Figure 5: Deterministic projections (indicated by black lines) and scenario trees (indicated by gray lines) of energy carrier prices and investment costs

As these demands as well as the availability of solar irradiation are characterized by seasonal and daily patterns, we utilize the approach presented in Section 3.1 to identify representative periods and segments of the original time series. To solve the clustering problems in Equation (8), we utilize the Partition Around Medoids (PAM) algorithm, a greedy search that identifies local optimal solutions. To solve Equation (8) we use an adapted version of this algorithm. Figure 4 illustrates the time series aggregation procedure by means of two features of the time series. The plots on the left show the electrical demand and PV power profiles of all 365 days (indicated by black lines) as well as selected medoids (indicated by red lines). The plots on the right show the trajectories of the selected periods before and after the aggregation of hourly time intervals to multi-hour segments. The number of representative periods and segments is set to twelve and eight, respectively. The initial time series is thereby reduced from 8760 time intervals to 96 time intervals.

4.2. Investment and operating costs scenario generation and reduction

Mavromatidis *et. al.* [21] carried out a global sensitivity analysis to identify the most influential uncertain parameters. Their findings indicate that energy carrier prices and demand uncertainty have the major impact on the cost

and optimal design of a DES, while investment costs and emission factors have only minor influences. Within their setup they assume that only one investment is made after a short planning phase, therefore they assume that the distribution of the investment costs has only a small standard deviation of 7%. In contrast, we consider an investment horizon of 20 years, which leads to high uncertainty with respect to investment costs. Building on the findings of Mavromatidis and taking the long investment horizon into account, we consider uncertain energy carrier prices, grid emission factors, demand, and investment costs.

Since the implementation of a sophisticated forecasting model would exceed the scope of this work and elaborated price projection can be found in the literature, we utilize these projections and predict the forecasting error using an ARMA-model, similar as in [41, 42, 43]. The ARMA-series is defined as:

$$W_t^{\text{err}} = a_1 W_{t-1}^{\text{err}} + b_1 r_{t-1} + r_t \quad (11)$$

where W_t^{err} is the forecasting error of the respective uncertain parameter in period t and r_t are the innovations which are independent and identically distributed with $N(0, \sigma^2)$. This leads to a forecasting error which increases with the forecasting time. Since the identification of correlations of different uncertain parameters is out of the scope of this work, we assume that the forecasting errors are uncorre-

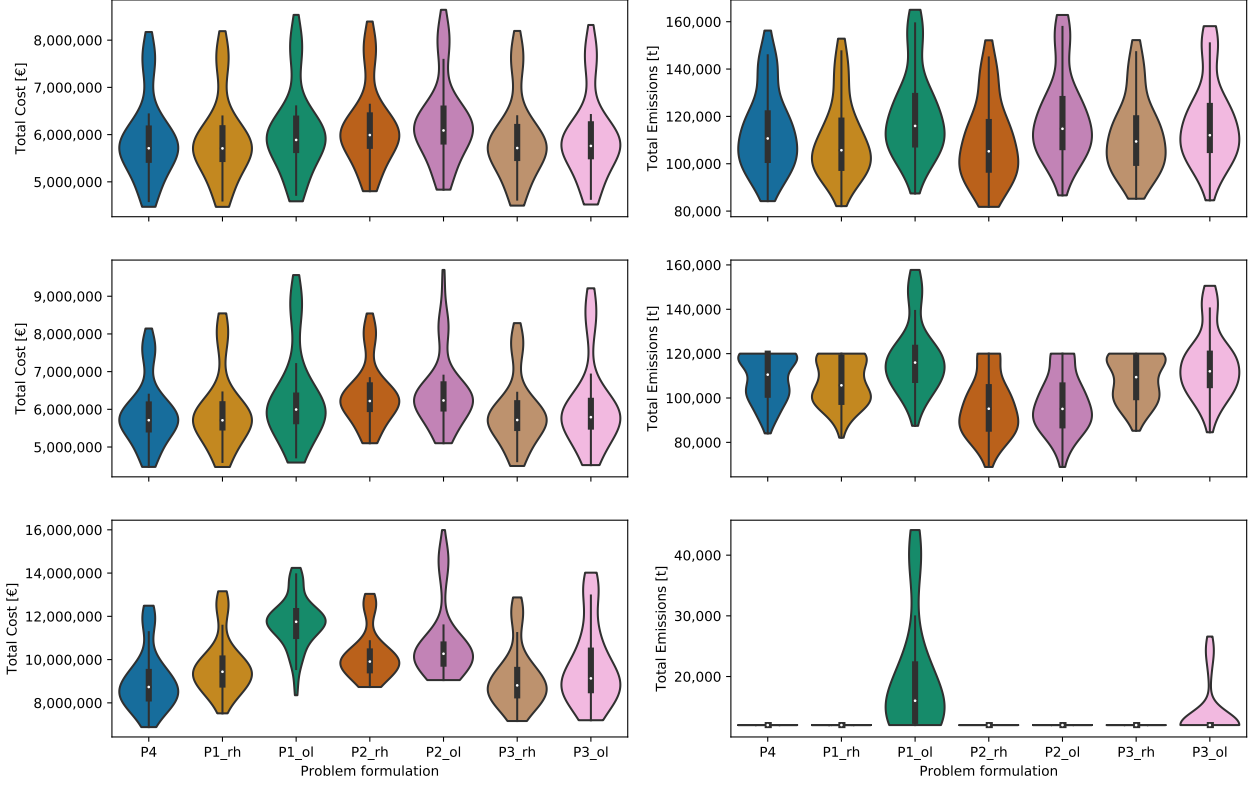


Figure 6: Cost and emission distributions for the solutions of the problem formulations. The left figures illustrate the cost distribution for formulations under increasing strict emission restrictions from top to bottom, the right-hand side figures illustrate the corresponding emission distributions. Each distribution is illustrated as violin plot which show rotated kernel density plots, together with the mean value (indicated by the white dot), the second and third quartile, (indicated by the black bar) as well as the 5th and 95th percentile (indicated by the black line)

lated. To account for the planning, approval, and construction process, we consider 5 investment periods within the time horizon of 20 years. Each investment period is followed by a 4-year operating period. We represent the multivariate stochastic input process in the form of a scenario tree, which reflects the filtration of the process, following the approach utilized, e.g., in [39, 40, 41].

We iteratively alternate between scenario generation and reduction. This procedure allows to preserve autocorrelations and, furthermore, reduces the memory usage as well as computation times of the scenario sampling and reduction procedure. For scenario sampling, we use the forecasting error model and the price projections in the literature, for scenario reduction we solve the clustering problem in Equation (8), using an own implementation of the PAM algorithm in Python. The projection of the energy carrier prices and investment costs as well as the parameters used in Equation (11) are chosen to match the projections presented in [44, 45, 46, 47].

We first sample $J_1^{\text{ini}} = 1500$ scenarios of price projections for the first 4 year period and reduce them to $J_1^{\text{Red}} = 10$ scenarios. For each subsequent operating period, we sample $J_1^{\text{Sam}} = 1500$ scenarios for each preceding

scenario path and reduce them to $J_1^{\text{Red}} = 2$. To further reduce the number of variables we aggregate operating periods to segments and select only the $J^{\text{fin}} = 120$ largest clusters with respect to their weight. The scenario tree of annual uncertain parameters is depicted in Figure 5.

4.3. Investigated problem formulations and emission mitigation strategies

We compare the seven different problem formulations proposed in Section 2 and summarized in Table 1. Since more and more system operators establish emission reduction or decarbonization strategies, we investigate the results of the proposed problem formulations under three degrees of emission limits. First, we investigate the problem solutions with no carbon emission limit. Then, we set the emission limit to a 50 % reduction compared to conventional supply system³ and finally we set the emission limit to a 100 % reduction, cumulatively yielding zero emissions. Please note, that the feed-in of renewable energy can result in negative emissions.

³Within the conventional supply system, electricity is purchased and drawn from the grid, heat is supplied by a conventional gas boiler and cold by a compression refrigeration system.

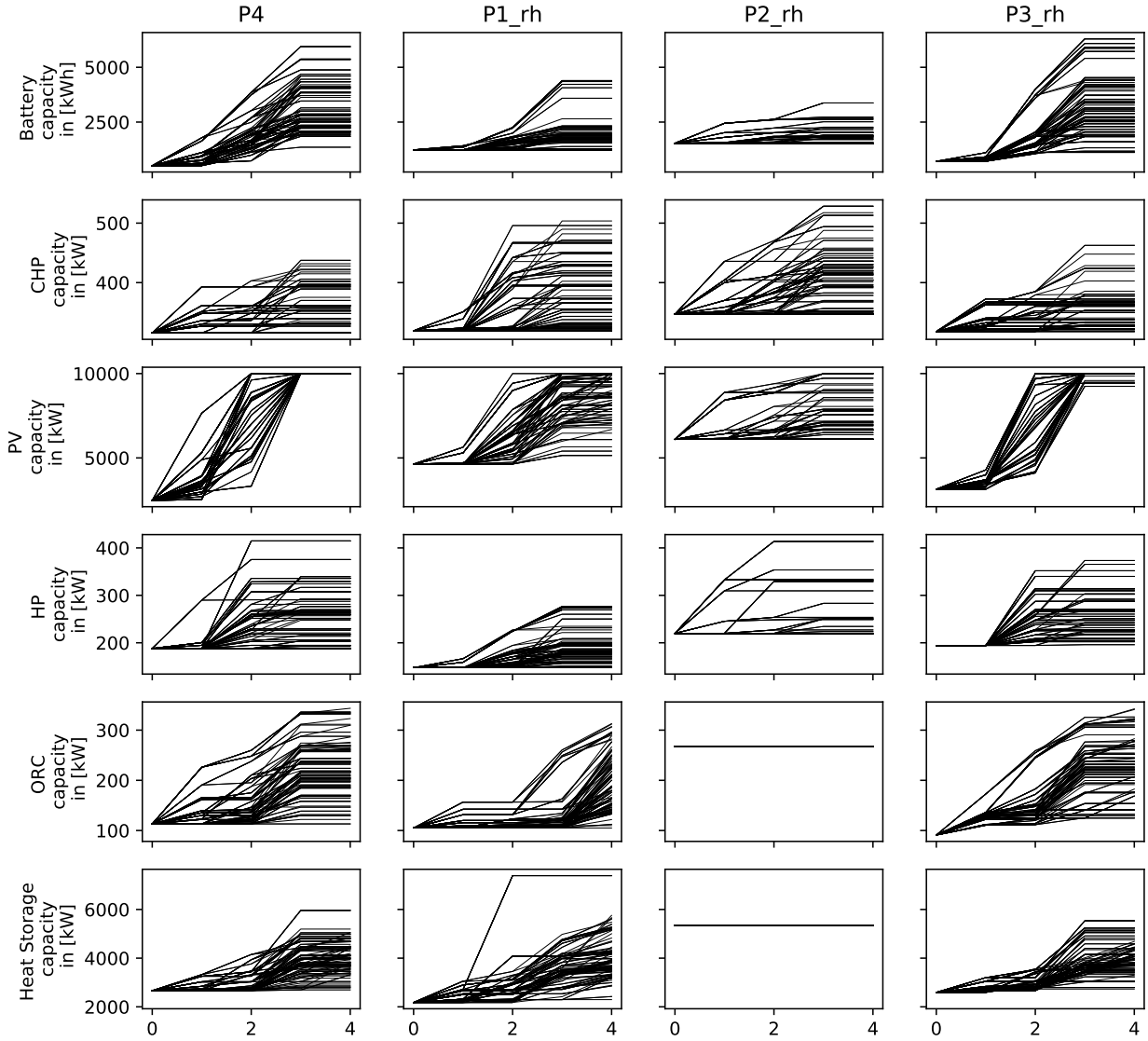


Figure 7: Investments identified using problem formulation P4, P1_rh, P2_rh, P3_rh

4.4. Results

In the following, we present the results of our case study. First, we analyze the cost and emission distributions resulting from the different problem formulations, followed by an analysis of the identified investments within each formulation.

4.4.1. Cost and emission distributions

The results regarding the distribution of total cost and total emissions are depicted in Figure 6 for all problem formulations. The left hand side figures illustrate the cost distribution for formulations under increasing strict emission limits from top to bottom, the right hand side figures illustrate the corresponding emission distributions. Each distribution is illustrated as violin plot which shows rotated kernel density plots, together with the mean value

(indicated by the white dot), the second and third quartile, (indicated by the black bar) as well as the 5th and 95th percentile (indicated by the black line).

The first observation is that the cost level rises for all problem formulations with increasingly stringent emission limits. Furthermore, it can be seen, that the emission limit is violated when the deterministic approaches are applied in an open loop. All planning approaches applied in closed-loop with the iterative possibility to invest yield good results regarding emission limit satisfaction.

For all emission limits, the solution of P4 results in global optimal solutions with respect to the given scenario tree and objective function and, hence in the lowest mean cost. For all emission limits, the solution of P3_rh yields the second-best results with a mean cost of only 1-2%

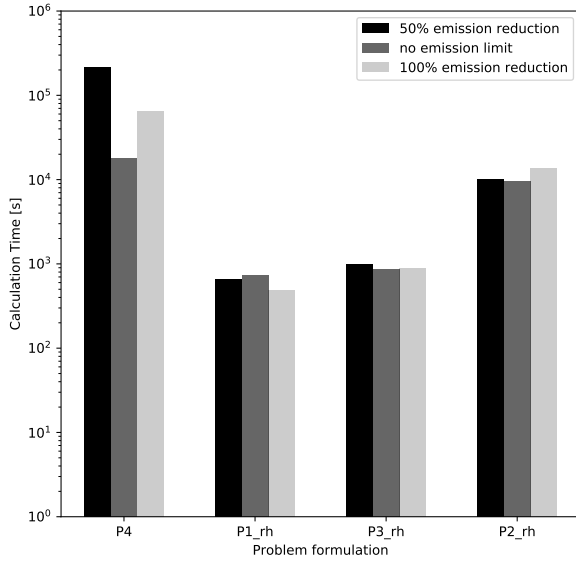


Figure 8: Computational effort to solve the problem using the different problem formulations

higher than the cost of the solution of P4. The solution of P3_ol yields minor deterioration with a mean cost of 2-8% higher than the solution of P4. The solution of P2_rh yields solutions with a mean total cost of 5-16% higher compared to the solution of P4. The solution of P2_ol yields only a slightly higher mean cost of 6-18% compared to the solution of P4. The solution of P1_ol yields a higher mean cost of 3-30% compared to the solution of P4.

In the case of no emission limit, the emission distributions do not show significant differences. In the case of moderate emission limits, the distributions of the solution of P1_rh and P3_rh are comparable to the distribution of the solution of P4, while the solution of P2_rh yields significantly lower mean emissions.

If exposed to the aggressive emission limits, all problem formulations that allow for repeated investments within a rolling horizon are able to meet the emission limits. The deterministic formulations P1_ol and P3_ol are not able to meet the emission objectives within all scenarios, while the problem formulation P2_ol yields solutions in compliance with the emission limits.

4.4.2. Investments

In the following, we analyze the investments identified by the problem formulations in detail to explain the observed effects regarding cost and emission distributions. Figure 7 illustrates the capacity developments obtained from the solution of the problem formulations. As the differences between the cost distributions are largest for the aggressive emission limit, we select this case for the

analysis. For the investment identified by the problem formulation P4, it can be observed, that the technology capacities increase over time, additional investments are selected within each scenario path. If specific scenarios share the same history of uncertain parameter realizations, they share the same investment decision for a specific investment period. On the other hand, if the scenarios have different realizations of uncertainty, investment decisions are made independently. That facilitates to invest situationally dependent on realized investment prices and the development of demands and emission factors within the specific scenario path. In comparison, in the first iteration of the P2_rh formulation, no subsequent investments are considered and therefore the investments in the first period are chosen to meet emission objectives for all possible scenario outcomes. This yields high investments in early periods for technologies which are still subject to strong ongoing learning effects. This is reflected by the high investments in electrical storages, PV, CHPs, and ORCs. Additionally, one single investment portfolio meets the emission limits in all scenarios, whereas the problem formulation P4 allows situational investments dependent on the changing framework conditions. The problem formulation P3_rh yields comparable investment strategies as the problem formulation P4 since investments over the entire horizon are anticipated in each iteration. Even though each iteration of the rolling horizon only considers one deterministic price projection, investment decisions at each stage can be adapted based on the information available up to the specific stage.

4.4.3. Computational effort

In the following, we discuss the computational effort which is required to solve the problems. All computations are conducted on a Lenovo ThinkPad with an Intel Core i5-6300U 2.5 GHz CPU and 12 GB RAM running Windows 10, using Python 3.6 and Gurobi 9.0.

In contrast to operational optimization models that are required to solve decision problems in real-time, investment decision models are less time-critical. However, low solution times are a decisive element for the user's acceptance of software. Additionally, the solution times indicate how much additional information can be included in the models such as piece-wise linearized part-load performance or investment cost modeling.

The presented modeling formulations use different approximation techniques and, hence, yield problems of different structures. This directly impacts the number of variables and constraints and consequently the solution time of the problem. The fact that stochastic optimization problems have significantly more variables is a well-known downside of stochastic programming. Even though we put effort into a careful selection of scenarios, the problem formulation P4 and P2 (within the first iteration) have 531.684 and 164.521 variables respectively. The deterministic problem formulations P1 and P3 lead to much lower numbers with 4.462 and 892 variables, respectively.

The solution time of individual instances of P1, P2, P3, and P4 are 1.6s, 14s, 227s, and 64.000s respectively for the first iteration without an emission limit. The solution of the entire decision tree requires the sequential solution of individual problem instances in the case of P1_cl, P2_cl, and P3_cl. The total solution time is represented in Figure 8. Solution times of problem P4 and P2_cl range between 4 to 18 hours and between 2 and 4 hours, respectively. In comparison, the solution of problem P1_cl and P3_cl range between 8 and 16 minutes. The solution times of the approaches solved within a rolling horizon could be further reduced by parallelization, however, within this work, no further effort has been made in this direction.

4.5. Discussion

4.5.1. Quality of approximation approaches

As indicated in the result section, the formulation P3_rh yields the lower mean cost than the formulation P2_rh. With respect to the constraint satisfaction, the P3_rh strategy results in no emission limit violations and only minimal heat supply deficits, when applied in a rolling horizon planning process.

Both formulations use approximations and simplifications of the original model, while the formulation P2_rh collapses the multi-stage decision structure to a two-stage decision with only one investment period, the formulation P3_rh neglects uncertainty and utilizes deterministic parameter forecasts. Since the formulation P2_rh anticipates the entire solution space, investments are chosen to meet emission objectives in any case. Furthermore, investments are only considered for the first period, and given the structure of the decision tree, these investments are done identically for all considered scenarios. This results in increased investment costs compared to the formulation P3_rh since investment costs in earlier periods is on a higher level. Furthermore, the average technology capacity is higher since scenario path specific investments are not considered in the formulation P2_rh. The higher investments are reflected within the emission distribution of the solution of the formulation P2_rh, yielding lower total mean emissions.

4.5.2. Discussion of the value of multi-period approaches

Besides the quantitative differences, the application of the multi-period approaches presented in this work offers multiple qualitative advantages over the single-period investment approaches. First, the formulation P3 and P4 do not only allow for a consideration of the capacity of existing aggregates but also to track their age structure. This fact enables decision-makers to actively take future replacement investments into consideration. Furthermore, the multi-period approaches provide information on the development of annual cost distributions and allow to identify potentially high-cost outliers and therefore to actively manage risk. This information allows the management

to plan expenditures within the budget planning. On the other hand, these approaches facilitate the inclusion of annual budget constraints, which enable the planner to shift investments to more eligible periods, while still complying with the emission objectives. With respect to the adoption of an emission reduction strategy, the multi-period approaches enable the planner to set emission pathways and annual reduction objectives and therefore allow the identification of a sustainable transformation strategy.

5. Conclusion

This work investigates different modeling approaches to analyze multi-period investment problems arising in the context of distributed energy system planning under uncertainty. The proposed multi-stage stochastic optimization problem is solved to global optimality and therefore serves as a first-best benchmark. To solve the problem at lower computational effort and to evaluate stochastic two-stage as well as multi-period deterministic approximations, we propose a rolling horizon approach that solves the problems sequentially to obtain multi-period investments for the entire scenario tree. An adapted, "open-loop" version of the rolling horizon approach enables us to evaluate single-period investments for the entire scenario tree and to compare the performance of single-period investments to multi-period investments. To reduce the computational complexity we utilize clustering approaches for time series and scenario aggregation. Our results indicate that the consideration of future investment periods has a stronger impact on the economic performance of the energy system than the detailed approximation of the probability distribution of the uncertain parameters. With respect to the imposed emission limits, the two-stage stochastic approximation has the tendency to overachieve emission objectives since the first-period investments are designed to meet emission objectives within all scenarios, while the deterministic multi-period approximation allows for future situational investments. Regarding the computational effort, the deterministic approximation is solved approximately ten times faster than the stochastic two-stage approximation. One major advantage of stochastic approaches is the active consideration of the probability distribution of uncertain parameters, which allows for inclusion of risk measures and the active management of risk. Future work could investigate how these measures could mitigate the risk of high-cost outliers.

Acknowledgments

This research was performed in relation to the project "Win4Climate" as part of the National Climate Initiative financed by the Federal Ministry for the Environment, Nature Conservation, Building and Nuclear Safety (BMUB) according to a decision of the German Federal Parliament (No. 03KF0094A) and as part of the Energie Campus

Nürnberg. The authors thank the Deutsche Forschungsgemeinschaft for their support within Projekt B09 in the Sonderforschungsbereich / Transregio 154 Mathematical Modelling, Simulation and Optimization using the Example of Gas Networks.

Conflicts of Interest

The authors declare no conflict of interest.

References

- [1] Emily Chasan. Teslas first impact report puts hard number on CO2 emissions. *Bloomberg*, 2019. URL: <https://www.bloomberg.com/news/articles/2019-04-17/tesla-s-first-impact-report-puts-hard-number-on-co2-emissions>.
- [2] Google. Environmental report. *Google*, 2019. URL: https://services.google.com/fh/files/misc/google_2019-environmental-report.pdf.
- [3] Bosch. Carbon neutrality by 2020. *Bosch*, 2020. URL: <https://www.bosch.com/company/sustainability/environment/>.
- [4] M. Stadler, M. Groissbock, G. Cardoso, and C. Marnay. Optimizing distributed energy resources and building retrofits with the strategic der-camod. *Applied Energy*, 132:557 – 567, 2014. URL: <https://doi.org/10.1016/j.apenergy.2014.07.041>.
- [5] Dragan Markovic, Dragan Cvetkovic, and Branislav Masic. Survey of software tools for energy efficiency in a community. *Renewable and Sustainable Energy Reviews*, 15(9):4897 – 4903, 2011. URL: <https://doi.org/10.1016/j.rser.2011.06.014>.
- [6] O. Erdinc and M. Uzunoglu. Optimum design of hybrid renewable energy systems: Overview of different approaches. *Renewable and Sustainable Energy Reviews*, 16(3):1412 – 1425, 2012. URL: <https://doi.org/10.1016/j.rser.2011.11.011>.
- [7] Carlos Gamarra and Josep M. Guerrero. Computational optimization techniques applied to microgrids planning: A review. *Renewable and Sustainable Energy Reviews*, 48:413 – 424, 2015. URL: <https://doi.org/10.1016/j.rser.2015.04.025>.
- [8] HOMER Energy LLC. The homer pro microgrid software by homer energy. 2020. URL: <https://www.homerenergy.com/products/pro/index.html>.
- [9] XENDEE. Optimal microgrid screening, configuration, technical design, and implementation. 2020. URL: <https://xendee.com/economic-optimization/>.
- [10] Gurobi Optimization, LLC. Gurobi optimizer reference manual, 2019. URL: <http://www.gurobi.com>.
- [11] CLPEX. Cplex, 2020. URL: <https://www.ibm.com/analytics/cplex-optimizer>.
- [12] FICO. FICO Xpress, 2020. URL: <https://www.fico.com/fico-xpress-optimization/docs/latest/overview.html>.
- [13] Christian Milan, Michael Stadler, Goncalo Cardoso, and Salman Mashayekh. Modeling of non-linear CHP efficiency curves in distributed energy systems. *Applied Energy*, 148:334 – 347, 2015. URL: <https://doi.org/10.1016/j.apenergy.2015.03.053>.
- [14] Markus Bohlayer and Gregor Zöttl. Low-grade waste heat integration in distributed energy generation systems - an economic optimization approach. *Energy*, 159:327 – 343, 2018. URL: <https://doi.org/10.1016/j.energy.2018.06.095>.
- [15] Xiufeng Yue, Steve Pye, Joseph DeCarolis, Francis G.N. Li, Fionn Rogan, and Brian Ó. Gallachóir. A review of approaches to uncertainty assessment in energy system optimization models. *Energy Strategy Reviews*, 21:204 – 217, 2018. URL: <https://doi.org/10.1016/j.esr.2018.06.003>.
- [16] Georgios Mavromatidis, Kristina Orehounig, and Jan Carmeliet. A review of uncertainty characterisation approaches for the optimal design of distributed energy systems. *Renewable and Sustainable Energy Reviews*, 88:258 – 277, 2018. URL: <https://doi.org/10.1016/j.rser.2018.02.021>.
- [17] Masoud Sharafi and Tarek Y. ElMekkawy. Stochastic optimization of hybrid renewable energy systems using sampling average method. *Renewable and Sustainable Energy Reviews*, 52:1668 – 1679, 2015. URL: <https://doi.org/10.1016/j.rser.2015.08.010>.
- [18] Luis Fabián Fuentes-Cortés, José Ezequiel Santibañez-Aguilar, and José María Ponce-Ortega. Optimal design of residential co-generation systems under uncertainty. *Computers and Chemical Engineering*, 88:86 – 102, 2016. URL: <https://doi.org/10.1016/j.compchemeng.2016.02.008>.
- [19] Yun Yang, Shijie Zhang, and Yunhan Xiao. Optimal design of distributed energy resource systems based on two-stage stochastic programming. *Applied Thermal Engineering*, 110:1358 – 1370, 2017. URL: <https://doi.org/10.1016/j.applthermaleng.2016.09.049>.
- [20] Apurva Narayan and Kumaraswamy Ponnambalam. Risk-averse stochastic programming approach for microgrid planning under uncertainty. *Renewable Energy*, 101:399 – 408, 2017. URL: <https://doi.org/10.1016/j.renene.2016.08.064>.
- [21] Georgios Mavromatidis, Kristina Orehounig, and Jan Carmeliet. Uncertainty and global sensitivity analysis for the optimal design of distributed energy systems. *Applied Energy*, 214:219 – 238, 2018. URL: <https://doi.org/10.1016/j.apenergy.2018.01.062>.
- [22] Georgios Mavromatidis, Kristina Orehounig, and Jan Carmeliet. Design of distributed energy systems under uncertainty: A two-stage stochastic programming approach. *Applied Energy*, 222:932 – 950, 2018. URL: <https://doi.org/10.1016/j.apenergy.2018.04.019>.
- [23] Georgios Mavromatidis, Kristina Orehounig, and Jan Carmeliet. Comparison of alternative decision-making criteria in a two-stage stochastic program for the design of distributed energy systems under uncertainty. *Energy*, 156:709 – 724, 2018. URL: <https://doi.org/10.1016/j.energy.2018.05.081>.
- [24] B. Pickering and R. Choudhary. District energy system optimisation under uncertain demand: Handling data-driven stochastic profiles. *Applied Energy*, 236:1138 – 1157, 2019. URL: <https://doi.org/10.1016/j.apenergy.2018.12.037>.
- [25] Jiah Yu, Jun-Hyung Ryu, and In beum Lee. A stochastic optimization approach to the design and operation planning of a hybrid renewable energy system. *Applied Energy*, 247:212 – 220, 2019. URL: <https://doi.org/10.1016/j.apenergy.2019.03.207>.
- [26] Viviani C. Onishi, Carlos H. Antunes, Eric S. Fraga, and Heriberto Cabezas. Stochastic optimization of trigeneration systems for decision-making under long-term uncertainty in energy demands and prices. *Energy*, 175:781 – 797, 2019. URL: <https://doi.org/10.1016/j.energy.2019.03.095>.
- [27] Sayyed Faridoddin Afzali, James S. Cotton, and Vladimir Mahalec. Urban community energy systems design under uncertainty for specified levels of carbon dioxide emissions. *Applied Energy*, 259:114084, 2020. URL: <https://doi.org/10.1016/j.apenergy.2019.114084>.
- [28] Zachary K. Pecanak, Michael Stadler, and Kelsey Fahy. Efficient multi-year economic energy planning in microgrids. *Applied Energy*, 255:113771, 2019. URL: <https://doi.org/10.1016/j.apenergy.2019.113771>.
- [29] L. Tang and M. C. Ferris. A hierarchical framework for long-term power planning models. *IEEE Transactions on Power Systems*, 30(1):46–56, 2015.
- [30] I. Konstantelos and G. Strbac. Valuation of flexible transmission investment options under uncertainty. *IEEE Transactions on Power Systems*, 30(2):1047–1055, 2015.
- [31] Y. Liu, R. Sioshansi, and A. J. Conejo. Multistage stochastic investment planning with multiscale representation of uncertainties and decisions. *IEEE Transactions on Power Systems*, 33(1):781–791, 2018.
- [32] John R. Birge and Francois Louveaux. *Introduction to Stochastic Programming*. Springer Publishing Company, Incorporated, 2nd edition, 2011.
- [33] ISO50001. *ISO 50001:2018 Energy management systems - Requirements with guidance for use*. August

2018. Available in electronic form for online purchase at <https://www.iso.org/standard/69426.html>.
- [34] Thomas Schütz, Markus Hans Schraven, Marcus Fuchs, Peter Remmen, and Dirk Müller. Comparison of clustering algorithms for the selection of typical demand days for energy system synthesis. *Renewable Energy*, 129:570 – 582, 2018. URL: <https://doi.org/10.1016/j.renene.2018.06.028>.
 - [35] Björn Bahl, Alexander Kümpel, Hagen Seele, Matthias Lampe, and André Bardow. Time-series aggregation for synthesis problems by bounding error in the objective function. *Energy*, 135:900 – 912, 2017. URL: <https://doi.org/10.1016/j.energy.2017.06.082>.
 - [36] Björn Bahl, Theo Söhler, Maike Hennen, and André Bardow. Typical periods for two-stage synthesis by time-series aggregation with bounded error in objective function. *Frontiers in Energy Research*, 5:35, 2018. URL: <https://doi.org/10.3389/fenrg.2017.00035>.
 - [37] Holger Heitsch and Werner Römisch. Scenario reduction algorithms in stochastic programming. *Computational Optimization and Applications*, 24(2):187–206, Feb 2003. URL: <https://doi.org/10.1023/A:1021805924152>.
 - [38] Holger Heitsch and Werner Römisch. Scenario tree reduction for multistage stochastic programs. *Computational Management Science*, 6(2):117–133, May 2009. URL: <https://doi.org/10.1007/s10287-008-0087-y>.
 - [39] Trine Krogh Boomsma, Nina Juul, and Stein-Erik Fleten. Bidding in sequential electricity markets: The nordic case. *European Journal of Operational Research*, 238(3):797 – 809, 2014. URL: <https://doi.org/10.1016/j.ejor.2014.04.027>.
 - [40] Nadine Kumbartzky, Matthias Schacht, Katrin Schulz, and Brigitte Werners. Optimal operation of a CHP plant participating in the german electricity balancing and day-ahead spot market. *European Journal of Operational Research*, 261(1):390 – 404, 2017. URL: <https://doi.org/10.1016/j.ejor.2017.02.006>.
 - [41] Markus Bohlayer, Markus Fleschutz, Marco Braun, and Gregor Zöttl. Energy-intense production-inventory planning with participation in sequential energy markets. *Applied Energy*, 258:113954, 2020. URL: <https://doi.org/10.1016/j.apenergy.2019.113954>.
 - [42] R Barth, L Soder, C Weber, H Brand, and DJ er. Wilmar deliverable 6.2 methodology of the scenario tree tool, documentation. tech. rep. *Institute of Energy Economics and the Rational Use of Energy, University of Stuttgart*, 2015. URL: <http://www.wilmar.risoe.dk/Results.htm>.
 - [43] Jan Abrell and Friedrich Kunz. Integrating intermittent renewable wind generation - a stochastic multi-market electricity model for the european electricity market. *Networks and Spatial Economics*, 15(1):117–147, Mar 2015. URL: <https://doi.org/10.1007/s11067-014-9272-4>.
 - [44] Christoph Kost, Shivens Shammugam, Verena Jülich, Huyen-Tran Nguyen, and Thomas Schlegl. Stromgestehungskosten erneuerbare energien. *Fraunhofer-Institut für Solare Energiesysteme ISE*, 2018. URL: https://www.ise.fraunhofer.de/content/dam/ise/de/documents/publications/studies/DE2018_ISE_Studie_Stromgestehungskosten_Erneuerbare_Energien.pdf.
 - [45] Wesley J Cole and Allister Frazier. Cost projections for utility-scale battery storage. 2019. URL: <https://doi.org/10.2172/1529218>.
 - [46] Jan Figgner, Peter Stenzel, Kai-Philipp Kairies, Jochen Linsen, David Haberschusz, Oliver Wessels, Georg Angenendt, Martin Robinius, Detlef Stolten, and Dirk Uwe Sauer. The development of stationary battery storage systems in germany - a market review. *Journal of Energy Storage*, 29:101153, 2020. URL: <https://doi.org/10.1016/j.est.2019.101153>.
 - [47] Harald Hecking, Martin Hintermayer, Dominic Lencz, and Johannes Wagner. Energiemarkt 2030 und 2050 - Der Beitrag von Gas- und Wärminfrastruktur zu einer effizienten CO₂-Minderung. *Energy Research & Scenarios gGmbH*, 2017. URL: https://www.ewi.research-scenarios.de/cms/wp-content/uploads/2017/11/ewi_ERS_Energiemarkt_2030_2050.pdf.