Optimal Residential Users Coordination Via Demand Response: An Exact Distributed Framework

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Abstract—This paper proposes a two-phase optimization framework for users that are involved in demand response programs. In a first phase, responsive users optimize their own household consumption, characterizing not only their appliances and equipment but also their comfort preferences. Subsequently, the aggregator exploits in a second phase this preliminary non-coordinated solution by implementing a coordination strategy for the aggregated loads while preserving users' privacy. The second phase relies on the solution of a bilevel program in which the aggregator's profit is maximized in the upper level while ensuring that the aggregated residential users do not incur any economic or comfort losses by participating in the demand response program. The lower level models the users' reaction to the aggregator's requests. As major complicating aspects, the resulting bilevel problem features nonlinear terms and lower-level binary variables. This challenging problem is addressed by a mixed-integer linear single-level reformulation and the application of an exact solution technique based on Dantzig-Wolfe decomposition. Simulations with up to 10,000 residential users illustrate the advantages of the proposed two-phase framework in terms of users' privacy, computational efficiency, and scalability.

Index Terms—Bilevel optimization, Dantzig-Wolfe decomposition, demand response, users coordination.

Nomenclature

The symbols used in this paper are listed below, except for those related to the Dantzig-Wolfe Decomposition (DWD), which are defined in Section III.

A. Sets

N	Set of users.
T	Set of time intervals.
$\mathcal{F}_n$	Feasible space of all variables for user n.

B. Parameters

$\hat{C}^{DA}$	Day-ahead energy cost under no coordination [\$].
$\hat{C}^{Dev}$	Load deviation cost under no coordination [\$].

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D_n^{max}	Maximum discomfort level for user n.
E_t^{des}	Target value of the aggregated power consumed at time t [W].
$\hat{E}_{n,t}^p$	Electric power consumed by user n at time t under no coordination [W].
$\hat{E}_{n,t}^s$	Electric power sold by user n at time t under no coordination [W].
$\hat{f}^U$	Users' aggregated profit under no coordination [\$].
$\hat{f}_n^U$	Profit of user n under no coordination [\$].
$\hat{p}^A$	Fixed component of the aggregator's profit [\$].
Δ_t	Duration of time interval t [h].
κ^{max}	Maximum incentive paid to users [\$].
κ_n^{min}	Minimum incentive paid to user n [\$].
λ_t^{DA}	Day-ahead market price for energy at time t [\$/Wh].
λ_t^s	Selling price of energy for users at time t [\$/Wh].
λ_t^{TOU}	Time-of-use (TOU) tariff for users at time t [\$/Wh].
μ_t^{Dev}	Price to adjust the load deviation at time t [\$/Wh].

C. Variables

$C_{n,t}^p$	Cost of user n for purchasing energy at time t [\$].
$C_{n,t}^{chp}$	Cost of user n related to the combined heat and power (CHP) operation at time t [\$].
C_t^{DA}	Day-ahead energy cost at time t under coordination [\$].
C_t^{Dev}	Load deviation cost at time t under coordination [\$].
C_n^{ev}	Daily hybrid vehicle fuel cost for user n [\$].
$E_{n,t}^p$	Electric power purchased by user n at time t [W].
$E_{n,t}^s$	Electric power sold by user n at time t [W].
e_t^+, e_t^-	Positive and negative power imbalance at time t [W].
$\boldsymbol{e}^A$	Vector of decision variables for the aggregator.
f^A	Aggregator's total cost under coordination [\$].
f_n^U	Profit of user n under coordination [\$].
$R_{n,t}^s$	Revenue of user n for selling energy at time t [\$].
$\boldsymbol{y}_n^U$	Vector of decision variables controlled by user n.
κ_n	Incentive paid to user n [\$].

D. Function

$g_n^U(\boldsymbol{y}_n^U)$	Discomfort level of user n.

I. Introduction

COORDINATING distributed energy resources is a challenge for electric utilities given the large number of resources involved [1]. If the residential sector comes into play, the challenge is even greater due to issues with householders'

privacy and constraints on the social utility [2]. A possible way to maximize social benefits is incentivizing users to change their consumption patterns to fit a load shape via Demand Response (DR) programs. A detailed literature review of DR is available in [3] and its recent application worldwide in the residential sector is reported in [4]. A survey on the application of 5G technology for a more reliable implementation of DR programs in terms of cyber security and consumer's privacy is included in [5].

To increase the contribution of residential users in the output of DR strategies, aggregators may play a crucial role [6] by managing DR programs for a group of users. Moreover, aggregators can be intermediary agents between users and the wholesale market. Many entities in the power market can act as an aggregator [1], including transmission and distribution system operators, retailers, or third parties [1]. Here, we assume that an aggregator is a demand service provider, as done in [7].

DR programs assume that users can change their electricity consumption in response to either price signals or incentive payments, giving rise to the so-called price-based or incentive-based DR programs, respectively. Assuming that the goal of any aggregator is the maximization of its profit, flattening the electricity consumption curve is a beneficial option to reduce the costs associated with peak-price hours [8]. When a price-based DR program is implemented, users optimize their consumption by responding to the price signals issued by the aggregator. However, if they all simultaneously schedule their loads to low-price periods, new rebound peaks may appear [9]. Furthermore, for the particular case of domestic electric water heaters, price-based DR programs are shown in [10] to be insufficient to change the users' behavior.

Alternatively, to increase the number of users participating in a DR scheme, incentive-based DR programs emerge to coordinate the consumption of users. In these programs, users are paid to shift and/or reduce their consumption over a given period of time. In particular, Direct Load Control (DLC) is reported as the most used program in the residential sector, which is composed of small consumers, while Curtailable Load (CL) predominantly involves medium and large consumers, such as commercial buildings and industries, respectively [4]. However, residential users participating in these programs have reported that incentives are not attractive enough and their comfort levels were negatively impacted [11]. Hence, from the users' perspective, DR requests with a lower impact on users' comfort or larger incentives are necessary. Excessive incentives, in turn, may result in unacceptable financial losses for the aggregator. In addition to the mentioned disadvantages, computational and privacy issues for DLC were also reported in [12]. Requirements, challenges, and activities regarding the introduction of incentive-based DR programs into retail electricity markets are discussed in [13].

Within the context of residential DR programs, references [14] and [15] adopt a centralized framework relying on the direct use of available commercial solvers based on mathematical programming. However, both privacy and scalability issues arise in these centralized approaches [16]. On the other hand, decentralized approaches decomposing the problem into subproblems improve the scalability but not necessarily the privacy needs if users' data are shared between the subproblems. In practice, the problem should be decomposed but also solved in a distributed mode so that each user optimizes its own consumption using its particular home energy management system (HEMS). Thus, in order to both mitigate privacy concerns and improve scalability, a distributed approach should be adopted.

The existing literature on distributed methods for demand response is classified in [7] according to the mathematical complexity of the models for appliances and whether their time-coupled operation is characterized. Similar to [12], references [17] and [18] present mathematical programming models neglecting the time coupling of users' appliances. This modeling aspect was considered in [19] and [20], wherein appliances were characterized by simplistic mixed-integer linear programming (MILP) formulations. Reference [21] improves upon [19] and [20] not only by integrating specific models for each appliance with their time-coupling constraints but also by using practical models. Additionally, from a user's perspective, [21] shows that more detailed models increase the users' profitability while guaranteeing their levels of comfort. Using detailed models, a distributed approach relying on a heuristic procedure rather than using commercial optimization software is proposed in [22]. Although such a heuristic showed reasonable optimality gaps for small instances that were also solvable via exact methods, optimality cannot be guaranteed in general.

Apart from the user-related modeling aspects, coordination strategies can be found in the literature focusing on the decrease in electricity peak load while also satisfying users' needs. One strategy is to formulate the DR problem as a single-level problem (SLP) in which a trade-off between users' costs or/and a utility discomfort function and a third party's performance criterion (revenues, community's living comfort, etc.) are both imposed. As described in [2], SLP-based approaches can be classified in the following three categories according to the trade-off considered: (i) a single-objective optimization that considers the perspective of either users or a third party but not both [21]; (ii) models considering both perspectives through a weighted-sum multiobjective optimization [7], [11], [12], and (iii) approaches relying on Pareto-front multiobjective optimization and simplified models to reduce the computational burden when a large number of users are accounted for [23] and [24].

Falling outside the categories in [2], reference [8] proposed a two-phase framework. In the first phase, each user individually minimizes its total electricity cost and the resulting costs and consumption profile are both the inputs for the next phase. The second phase involves the solution of a bilevel problem [25] with an upper level modeling the aggregator's DR strategy and $|N|$ lower levels representing the response of the $|N|$ residential users to this strategy. In this phase, the aggregator announces a modified aggregated load profile for each householder. Then, each user re-optimizes its consumption for each period of time considering both the new energy limits and a measure of the aggregated demand variability. Iteratively, the modified consumption patterns are sent back to

the aggregator until no further improvement of the aggregated demand profile is experienced.

Bilevel programming was applied in [26] to consider competition among rival aggregators.

Although bilevel problems are $\mathcal{NP}$-hard [25], a vast number of applications have been addressed in the literature. Among others, some approaches consider a Stackelberg game in which the upper- and lower-level problems are solved iteratively until reaching an equilibrium solution [12], [19]. Unfortunately, these works consider neither specific appliance models nor the CL option.

Motivated by the above-described state of the art, this work proposes a practical and exact distributed framework for an aggregator that coordinates a group of residential users. The proposed framework allows the joint maximization of the aggregator's and users' profits while preserving users' comfort and privacy. This off-line tool should be run the day before the decisions are implemented in practice, which is consistent with existing day-ahead electricity markets. Note that individual load profiles and the incentives given to each user are the outcomes of the proposed tool.

Representative and specific appliances for users are modeled considering as inputs the home thermal mass, the solar radiation, and the wind speed, among others. All the assumptions made for the users characterization are justified in [21]. As for the aggregator, it is assumed that electricity prices can be estimated and are thus known a day in advance. Moreover, the demand variability is optimized by the aggregator according to the so-called Peak-to-Average Ratio (PAR) [27]. This nonlinear function is typically used at a residential level to measure how flat the load profile is. Note that flattening the demand curve, as in the Brazilian case [28], is not always beneficial. In particular, high variability in demand can be advantageous for residential regions largely dependent on renewable-based generation and without enough storage capacity.

Similar to [8], the proposed framework is divided into two phases. In the first phase, profits of individual users are maximized under a price-based DR scheme. By contrast, as done in [26], in the second phase, a combination of an incentive-based DR via CL and a price-based DR program is considered. Thus, the limitations of carrying out each strategy independently are overcome. In this second phase it is assumed that all users participating in the DR program will only accept CL requests if they do not incur any financial losses or discomfort. This last phase is formulated as a bilevel programming problem, wherein the upper level characterizes the aggregator's decisions and, unlike [26], each lower-level problem models the response of a particular user. As in [8], the resulting problem structure allows deriving an alternative single-level equivalent.

As a salient methodological feature, Dantzig-Wolfe Decomposition is applied in a distributed fashion, thereby overcoming the scalability and privacy issues of [8] and [26]. Additionally, users are encouraged, via monetary incentives, to adopt a new consumption profile while maximizing the aggregator's profit. Unlike [26], we propose a deterministic bilevel model considering a single aggregator who guarantees the privacy of users' data by defining a lower-level problem for each

user. To sum up, this work substantially improves upon [8] and [26] in four aspects: (i) a monetary incentive is devised for each user, (ii) specific and practical models for home appliances from [21] are used, (iii) the aggregator's objective function consists in optimizing not only its profits and the load variability but also the incentives paid to the users, and (iv) the aggregator, rather than the users, is in charge of optimizing the load variability. Moreover, in contrast to both [8] and [22], here we propose the application of an exact distributed approach that converges to optimality in a finite number of iterations. None of the aforementioned studies in the literature considers all of these features.

The main contributions of this work are twofold:

- From a modeling perspective, a combination of incentive-based and price-based DR programs is proposed to overcome the disadvantages featured by each program. To that end, load variability, users' incentives, and electricity costs are jointly optimized by the aggregator at no extra cost for users. In addition, practical, detailed, and specific models for users are taken into account.
- From a methodological perspective, an exact, distributed, and scalable approach is applied to effectively solve the resulting problem while preserving users' privacy. Moreover, the proposed method allows mitigating the issues associated with communication interruptions.

The remainder of this paper is organized as follows. Section II presents the two-phase framework. The proposed distributed approach is presented in Section III. In Section IV, numerical results are shown and discussed. Concluding remarks are provided in Section V. Finally, an appendix is included to provide the proof for a linear approximation of the nonlinear Peak-to-Average Ratio.

II. TWO-PHASE FRAMEWORK

This section describes the optimization problems representing the building blocks of the proposed two-phase framework. In addition, the aggregator and users interaction is presented.

A. General Model for Users

Based on [21], the following MILP is formulated for each user $n \in N$:

$$\max_{\boldsymbol{y}_n^U} \quad f_n^U = \sum_{t \in T} \left(R_{n,t}^s - C_{n,t}^p - C_{n,t}^{chp} \right) - C_n^{ev} \qquad (1)$$

subject to:

$$R_{n,t}^s = \lambda_t^s \Delta_t E_{n,t}^s, \ \forall \ t \in T \qquad (2)$$

$$C_{n,t}^p = \lambda_t^{TOU} \Delta_t E_{n,t}^p, \ \forall \ t \in T \qquad (3)$$

$$g_n^U(\boldsymbol{y}_n^U) \leq D_n^{max} \qquad (4)$$

$$\boldsymbol{y}_n^U \in \mathcal{F}_n \qquad (5)$$

The optimization goal in (1) is the maximization of the profit of user n. The revenues from selling energy $R_{n,t}^s$ are defined in (2), where λ_t^s are the energy selling prices, which are given or estimated. The electricity costs $C_{n,t}^p$ are defined in (3), where λ_t^{TOU} are the TOU tariffs, known with certainty ahead of time. $C_{n,t}^{chp}$ and C_n^{ev} represent fuel costs for the CHP and

the electric vehicle models, respectively. Constraint (4) sets the maximum discomfort D_n^{max}. This constraint can be formulated using the framework in [29]. $R_{n,t}^s$, $C_{n,t}^p$, $C_{n,t}^{chp}$, C_n^{ev}, $E_{n,t}^s$, and $E_{n,t}^p$ are elements of the vector of decision variables $\boldsymbol{y}_n^U$. Finally, expression (5) provides a compact formulation to characterize the energy flow conservation, the balance between supply/demand, the appliances operation, the pricing policies, and the energy limits. Note that f_n^U, $R_{n,t}^s$, $C_{n,t}^p$, $C_{n,t}^{chp}$, and C_n^{ev} are included here for explanation purposes.

Note that if an appliance/equipment (A/E) is not eligible for user n the corresponding decision variables are set to zero in $\boldsymbol{y}_n^U$. Furthermore, binary variables are components of this vector if user n owns a CHP system. Accordingly, each user characterizes its own set of A/E and imposes its own constraints, modeled by the feasible region $\mathcal{F}_n$. Representative and specific appliances (including solar panels, electric vehicles, batteries, heating, etc.) for users are modeled considering as inputs the home thermal mass, the solar radiation, and the wind speed, among others. Additionally, the energy consumed from (or injected into) the grid is limited in practice and network capacity constraints are implicitly formulated in (5). The full list of variables in $\boldsymbol{y}_n^U$ and constraints in $\mathcal{F}_n$ can be found in [21].

It is worth mentioning that every A/E from a specific residential user $n \in N$ can be used for DR, which is decided by its own HEMS. For more details about pricing policies, hypotheses, and justifications related to the model for users, the interested reader is referred to [21].

B. Model for the Aggregator

In this work, the aggregator's decisions are characterized as a bilevel MILP problem in which the aggregator's profit is maximized in the upper level provided that the aggregated users do not incur any economic or comfort losses by participating in the incentive-based DR program. Users' reactions are characterized in the lower level in which their consumption levels are optimized under their individual no-loss criterion.

The following assumptions are made in the context of residential energy consumption. The aggregator's revenues are proportional to the users' energy consumption and equal to the incomes received through the electricity bills paid by users. Additionally, payments for the users' self-generation are assumed by the aggregator. As explained in Section III.D, the above terms are constants in the second phase of the proposed two-phase framework. Hence, the maximization of the aggregator's profits can be achieved by minimizing the variable expenses incurred. Second, the Peak-to-Average Ratio function [27] is approximated and included in the aggregator's model. This function is defined as the ratio between the peak load and the average load, and can be formulated as $f^{PAR} = \left(\max_{t \in T} \left\{ \sum_{n \in N} E_{n,t}^p \right\} \right) \left(\sum_{t \in T} \sum_{n \in N} E_{n,t}^p / |T| \right)^{-1}$. In the context of the residential sector, system efficiency can be increased by reducing the magnitude of the peak energy load through the minimization of PAR, as done in [27]. Alternatively, the minimization of the consumption variance is considered in the MILP-based aggregator's model as a suitable measure to reduce the residential peak load. As shown in the

Appendix, both efficiency measures are related, since the minimization of the nonlinear PAR can be locally approximated by solving a linear programming problem corresponding to the minimization of the consumption variance.

The proposed bilevel MILP formulation for the aggregator is cast as follows:

$$\min_{\substack{C_t^{DA}, C_t^{Dev}, \boldsymbol{e}^A, \\ E_{n,t}^p, f^A, \kappa_n}} f^A = \sum_{t \in T} C_t^{DA} + \sum_{n \in N} \kappa_n + \sum_{t \in T} C_t^{Dev} \quad (6)$$

subject to:

$$C_t^{DA} = \lambda_t^{DA} \Delta_t \sum_{n \in N} E_{n,t}^p, \ \forall \ t \in T \quad (7)$$

$$\kappa_n^{min} \leq \kappa_n \leq \kappa_n^{max}, \ \forall \ n \in N \quad (8)$$

$$C_t^{Dev} = \mu_t^{Dev} \Delta_t (e_t^+ + e_t^-), \ \forall \ t \in T \quad (9)$$

$$\sum_{n \in N} E_{n,t}^p - \frac{\sum_{n \in N} \sum_{t' \in T} E_{n,t'}^p}{|T|} + e_t^+ - e_t^- = 0,$$
$$\forall \ t \in T \quad (10)$$

$$e_t^+, e_t^- \geq 0, \ \ \forall \ t \in T \quad (11)$$

$$f^A \leq \hat{C}^{DA} + \hat{C}^{Dev} \quad (12)$$

$$\text{where } E_{n,t}^p \in \arg\max_{\boldsymbol{y}_n^U} \left\{ f_n^U \right. \quad (13)$$

$$\text{subject to:}$$

$$\text{Constraints } (2) - (5) \quad (14)$$

$$\left. f_n^U + \kappa_n \geq \hat{f}_n^U \right\}, \forall \ n \in N \quad (15)$$

where symbol $\hat{\ }$ is used to refer to the optimal values of the variables resulting from Phase I as explained in Section II-C1.

The aggregator's total cost is minimized in (6), which comprises: (i) the costs of purchasing electricity in the day-ahead market, C_t^{DA}, (ii) the costs related to the incentives paid by the aggregator to the users for changing their load consumption patterns under a DR request, κ_n, and (iii) C_t^{Dev}, characterizing the level of variability of the aggregated energy consumed according to the aggregator's target. The upper-level decision variables are represented by $E_{n,t}^p$, κ_n, and the vector $\boldsymbol{e^A}$, which includes e_t^+ and e_t^-. Note that f^A, C_t^{DA}, and C_t^{Dev} are auxiliary variables only included for explanation purposes.

Upper-level constraints comprise expressions (7)–(12). Constraints (7) define the costs of purchasing electricity in the day-ahead market, where λ_t^{DA} is an estimation of the hourly clearing price in the day-ahead electricity market. This estimation represents the interface between the aggregator and the wholesale market. Constraints (8) bound the incentives paid by the aggregator, where κ_n^{min} are parameters that can be negotiated between the aggregator and the users via a contractual agreement and κ^{max} is the aggregator's budget limit. The costs related to the load profile deviation are defined in (9). μ_t^{Dev} is a tuning parameter set by the aggregator to adjust the smoothness of the aggregated consumption profile. Note that the larger μ_t^{Dev}, the flatter the aggregated consumption. By contrast, small values of μ_t^{Dev} imply that the aggregated consumption is mainly driven by day-ahead

prices. Equations (10) characterize the consumption variance. Surplus variables e_t^+ and e_t^-, which are nonnegative (11), quantify the deviation from the expected energy consumption along the time span. Here, the variance costs rather than f^{PAR} are minimized in (6), since (9)–(11) represent a local approximation of f^{PAR} as shown in the Appendix. Finally, constraints (12) ensure that by coordinating users' consumption the aggregator does not any incur financial losses. Thus, as per (12), the aggregator's total cost, f^A, associated with the implementation of the incentive-based DR program, cannot be greater than the total cost that would be incurred without any coordination, $\hat{C}^{DA} + \hat{C}^{Dev}$. In other words, the incentives given by the aggregator cannot exceed the savings obtained by implementing the DR program.

The lower-level problem is formulated in (13)–(15). Similar to (12), constraint (15) guarantees that the profit obtained by user n by participating in the proposed DR program, $f_n^U + \kappa_n$, is greater than or equal to the profit that would be achieved under no coordination, $\hat{f}_n^U$. Note that the energy consumed by each user $E_{n,t}^p$ is individually optimized in the corresponding lower-level problem.

In the context of residential consumption, several DR targets have been defined in the literature. In [27], the PAR function is the target defined to reduce the peak load. Alternatively, in [8], the aggregator minimizes the deviation of the aggregated load from a desired profile. Note that any linear DR target can be accommodated into the aggregator's model through minor modifications. Also, minor modifications to the above formulation are required in order to accommodate the specific case in which the aggregator has certainty on the most beneficial consumption profile. In this case, μ_t^{Dev} in (9) should be replaced with a large penalty constant and constraints (10) should be replaced by the following expressions:

$$\sum_{n \in N} E_{n,t}^p + e_t^+ - e_t^- = E_t^{des}, \ \forall \ t \in T. \quad (16)$$

C. Agent Interaction

Based on [8], the proposed framework comprises two phases. The first phase only involves the users' HEMS, while the second phase involves both the users' HEMS and the aggregator's energy management system. Both phases are described next.

1) Phase I – No Coordination: In this phase, the users are not coordinated by the aggregator. Each user n solves problem (1)–(5) on its HEMS to maximize its own profit. The solution of these $|N|$ problems provides the optimal values of the energy consumed under no coordination, $\hat{E}_{n,t}^p$, and the optimal values of the uncoordinated users' profits, $\hat{f}_n^U$. Such values are used as inputs of Phase II.

2) Phase II – Coordination: In this phase, the aggregator coordinates users' targets with its DR targets by proposing specific incentives to change load consumption patterns for each user.

First, the aggregator computes the following values using the results of Phase I:

- The total cost of purchasing electricity in the day-ahead market:

$$\hat{C}^{DA} = \sum_{t \in T} \lambda_t^{DA} \Delta_t \sum_{n \in N} \hat{E}_{n,t}^p. \quad (17)$$

- The total cost associated with load deviations:

$$\hat{C}^{Dev} = \sum_{t \in T} \mu_t^{Dev} \Delta_t \left(\sum_{n \in N} \hat{E}_{n,t}^p - \frac{\sum_{n \in N} \sum_{t' \in T} \hat{E}_{n,t'}^p}{|T|} \right). \quad (18)$$

- The aggregator's profit:

$$\hat{p}^A = \sum_{n \in N} \sum_{t \in T} \lambda_t^{TOU} \Delta_t \hat{E}_{n,t}^p - \sum_{n \in N} \sum_{t \in T} \lambda_t^s \Delta_t \hat{E}_{n,t}^s. \quad (19)$$

In this paper, private data refer to users' information related to their flexibility/comfort levels and consumption habits that could be used to identify and monitor behavior patterns (e.g., type of appliances, preference between oven and cooktop for dinner, and even TV channels watched). Note that the above formulation contributes to preserve users' privacy by avoiding sharing private information with any other agent. Under the proposed solution process described in Section III, the only pieces of information sent by each user to the aggregator are the own scheduled consumption and the individual incentives, which do not reveal any private information.

Two technical aspects closely related to the proposed framework deserve special attention: baseline load and users' eligibility to participate in the DR program. Regarding the first aspect, a discussion on the importance of determining an adequate baseline load for any DR implementation is provided in [30]. The authors define the baseline load as an estimate of the electricity that would have been consumed by a customer in the absence of a demand response event. The baseline load thus corresponds to the outcome of Phase I in our proposed approach for a flat TOU tariff. As for the second aspect, users are eligible to participate in the demand response program if their overall uncoordinated consumption is greater than 0. Note, however, that as a result of Phase II, $E_{n,t}^p$ may be equal to 0 for a user n willing to consume at a particular time period t, i.e., with $\hat{E}_{n,t}^p > 0$. In other words, the optimal solution to the coordination procedure may incentivize consumers to fully reduce their consumption despite their willingness to consume at a specific time period, which is consistent with current industry practice.

D. Single-Level Transformation

Next, the bilevel problem (6)–(15) is transformed into a SLP by adopting the following practical assumption. Users and the aggregator are both willing to participate in the proposed DR program if they do not face any economic loss. In other words, the implementation of Phase II is beneficial for all of them. Under this assumption, problem (6)–(15) is a special type of bilevel problem in which the upper-level decisions only modify the value of the lower-level variables but the lower-level objective function value remains the same.

The bilevel problem (6)–(15) can be equivalently transformed into a single-level MILP problem as follows. The maximization of each user's profit f_n^U in (13) can be equivalently replaced by constraint (15).

As a result, the aggregator's problem (P) is cast as the following single-level MILP:

$$(P) \quad \min_{\substack{C_t^{DA}, C_t^{Dev}, \\ e^A, f^A, \boldsymbol{y}_n}} f^A = \sum_{t \in T} C_t^{DA} + \sum_{n \in N} \kappa_n + \sum_{t \in T} C_t^{Dev} \quad (20)$$

subject to:

$$\text{Constraints } (7) - (12), (14), \text{ and } (15)$$

where vector $\boldsymbol{y}_n$ denotes all the decision variables indexed by n, comprising κ_n, $E_{n,t}^p$, and $\boldsymbol{y}_n^U$.

This problem could be addressed by implementing different methodologies. Existing centralized approaches make use of available commercial solvers to solve it straightforwardly. As described in the next section, an exact distributed approach based on DWD [31] is alternatively proposed. The described solution methodology effectively mitigates the concerns on both scalability and users' privacy usually associated with centralized approaches.

III. PROPOSED DISTRIBUTED TWO-PHASE FRAMEWORK

The proposed framework differs from the above-described centralized approach in the solution of (P), thus Phase I and the updating steps (17)–(19) remain the same. For expository purposes, problem (P) is rewritten as follows:

$$\min_{\boldsymbol{y}_n, e^A} \sum_{t \in T} \lambda_t^{DA} \Delta_t \sum_{n \in N} E_{n,t}^p + \sum_{n \in N} \kappa_n \\ + \sum_{t \in T} \mu_t^{Dev} \Delta_t (e_t^+ + e_t^-) \quad (21)$$

subject to:

$$\sum_{n \in N} E_{n,t}^p - \frac{\sum_{n \in N} \sum_{t' \in T} E_{n,t'}^p}{|T|} + e_t^+ - e_t^- = 0, \ \forall \, t \in T \quad (22)$$

$$\sum_{t \in T} \lambda_t^{DA} \Delta_t \sum_{n \in N} E_{n,t}^p + \sum_{n \in N} \kappa_n \\ + \sum_{t \in T} \mu_t^{Dev} \Delta_t (e_t^+ + e_t^-) \leq \hat{C}^{DA} + \hat{C}^{Dev} \quad (23)$$

$$e_t^+, e_t^- \geq 0, \ \forall \, t \in T \quad (24)$$

$$\boldsymbol{y}_n \in \Upsilon_n, \ \forall \, n \in N \quad (25)$$

where Υ_n defines the space formed by the intersection of constraints (8), (14), and (15). Note that the set of extreme rays of the convex hull of Υ_n is empty since f_n^U is bounded. In addition, since variables in constraints (22) and (23) are bounded, these constraints are a polytope and they can be reformulated as a convex combination of its extreme points. Let Ω_n be the set of extreme points of this polytope, in which each extreme point $h \in \Omega_n$ is associated with the auxiliary variable α_n^h. Let us use the notation $\overline{i^h}$ for the fixed value of coordinate i for an extreme point h. Therefore, (P) can be

equivalently formulated as:

$$\min_{\boldsymbol{y}_n, e^A, \boldsymbol{\alpha}} \sum_{n \in N} \sum_{h \in \Omega^n} \left(\sum_{t \in T} \lambda_t^{DA} \Delta_t \overline{E_{n,t}^{p,h}} + \overline{\kappa_n^h} \right) \alpha_n^h \\ + \sum_{t \in T} \mu_t^{Dev} \Delta_t (e_t^+ + e_t^-) \quad (26)$$

subject to:

$$\sum_{n \in N} \sum_{h \in \Omega^n} \overline{E_{n,t}^{p,h}} \alpha_n^h - \frac{\sum_{n \in N} \sum_{h \in \Omega^n} \sum_{t' \in T} \overline{E_{n,t'}^{p,h}} \alpha_n^h}{|T|} \\ + e_t^+ - e_t^- = 0, \ \forall \, t \in T \quad (27)$$

$$\sum_{n \in N} \sum_{h \in \Omega^n} \left(\sum_{t \in T} \lambda_t^{DA} \Delta_t \overline{E_{n,t}^{p,h}} + \overline{\kappa_n^h} \right) \alpha_n^h \\ + \sum_{t \in T} \mu_t^{Dev} \Delta_t (e_t^+ + e_t^-) \leq \hat{C}^{DA} + \hat{C}^{Dev} \quad (28)$$

$$\sum_{h \in \Omega^n} \alpha_n^h = 1, \ \forall \, n \in N \quad (29)$$

$$\sum_{h \in \Omega^n} \overline{E_{n,t}^{p,h}} \alpha_n^h = E_{n,t}^p, \ \forall \, n \in N, \ \forall \, t \in T \quad (30)$$

$$\sum_{h \in \Omega^n} \overline{\kappa_n^h} \alpha_n^h = \kappa_n, \ \forall \, n \in N \quad (31)$$

$$e^A, \boldsymbol{\alpha} \geq 0 \quad (32)$$

$$\boldsymbol{y_n} \in \Upsilon_n, \ \forall \, n \in N \quad (33)$$

A suitable solution to address the resulting exponential number of variables is the Column-Generation (CG) algorithm. According to [32], the application of the CG algorithm to problem (26)–(33) involves the iterative solution of the restricted master problem (RMP) and one pricing subproblem per user n, referred to as SP_n. The RMP (34)–(37) is obtained by eliminating constraints (30)–(33) and using a subset of columns $\Omega_n^s \subset \Omega_n$ for each user n as follows:

$$\min_{e^A, \boldsymbol{\alpha}} \sum_{n \in N} \sum_{h \in \Omega_n^s} \left(\sum_{t \in T} \lambda_t^{DA} \Delta_t \overline{E_{n,t}^{p,h}} + \overline{\kappa_n^h} \right) \alpha_n^h \\ + \sum_{t \in T} \mu_t^{Dev} \Delta_t (e_t^+ + e_t^-) \quad (34)$$

subject to:

$$\sum_{n \in N} \sum_{h \in \Omega_n^s} \overline{E_{n,t}^{p,h}} \alpha_n^h - \frac{\sum_{n \in N} \sum_{h \in \Omega_n^s} \sum_{t' \in T} \overline{E_{n,t'}^{p,h}} \alpha_n^h}{|T|} \\ + e_t^+ - e_t^- = 0, \ \forall \, t \in T \ (v_{1,t}^d) \quad (35)$$

$$\sum_{n \in N} \sum_{h \in \Omega_n^s} \left(\sum_{t \in T} \lambda_t^{DA} \Delta_t \overline{E_{n,t}^{p,h}} + \overline{\kappa_n^h} \right) \alpha_n^h \\ + \sum_{t \in T} \mu_t^{Dev} \Delta_t (e_t^+ + e_t^-) \leq \hat{C}^{DA} + \hat{C}^{Dev} \ (v_2^d) \quad (36)$$

$$\sum_{h \in \Omega_n^s} \alpha_n^h = 1, \ \forall \, n \in N \ (v_{3,n}^d) \quad (37)$$

6

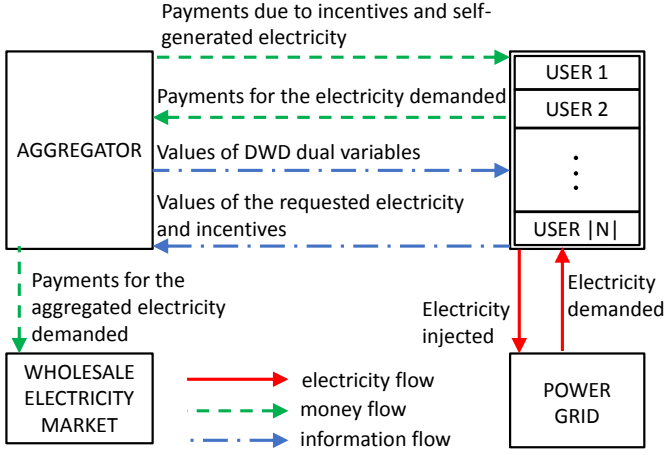Figure 1: Flows of electricity, money, and information of the proposed framework.

Figure 2: Solution framework flowchart.

$$e^{A}, \boldsymbol{\alpha} \geq 0 \quad (38)$$

where $v_{1,t}^{d}$, v_{2}^{d}, and $v_{3,n}^{d}$ are the dual variables associated with constraints (35)–(37), respectively. Once an RPM is solved, the resulting values for the dual variables are denoted by parameters $\overline{v_{1,t}^{d}}$, $\overline{v_{2}^{d}}$, and $\overline{v_{3,n}^{d}}$.

For a user n and a given h, RC_n denotes the reduced cost associated with variable α_{n}^{h}, which is calculated as:

$$RC_n = \sum_{t \in T} \lambda_{t}^{DA} \Delta_t E_{n,t}^{p} + \kappa_n - \sum_{t \in T} \overline{v_{1,t}^{d}} \left(E_{n,t}^{p} - \frac{\sum_{t' \in T} E_{n,t'}^{p}}{|T|} \right)$$

$$- \overline{v_{2}^{d}} \left(\sum_{t \in T} \lambda_{t}^{DA} \Delta_t E_{n,t}^{p} + \kappa_n \right) - \overline{v_{3,n}^{d}}. \quad (39)$$

The goal of each subproblem is to find the extreme point h with the minimum reduced cost, that is:

$$h^{*} \in \inf_{h \in \Omega^{n}} \{RC_n\}. \quad (40)$$

To that end, subproblem SP_n is conveniently formulated as:

$$\min_{\boldsymbol{y_n} \in \boldsymbol{\Upsilon_n}} RC_n. \quad (41)$$

The CG algorithm starts by solving RMP with some columns, which can be generated using the heuristic proposed in [22], which provides initial feasible columns. New columns are iteratively added to Ω_{n}^{s} in the RMP if the extreme point h obtained from solving SP_n has a negative reduced cost. The CG algorithm stops when there are no negative reduced costs.

The final solution proposed by the CG algorithm may have fractional values for binary variables, since they are relaxed in the RMP. A branch-and-price algorithm is conveniently used to obtain an integer solution by considering the RMP as the root node of the branching tree. As the CG algorithm proposes solutions with small integrality gaps for our instances, the diving heuristic procedure from [33] in coordination with the CG algorithm are considered to prove optimality.

Fig. 1 illustrates the flows of electricity, money, and information that take place under the proposed DR strategy. Apart from the coordination of users made by the aggregator,
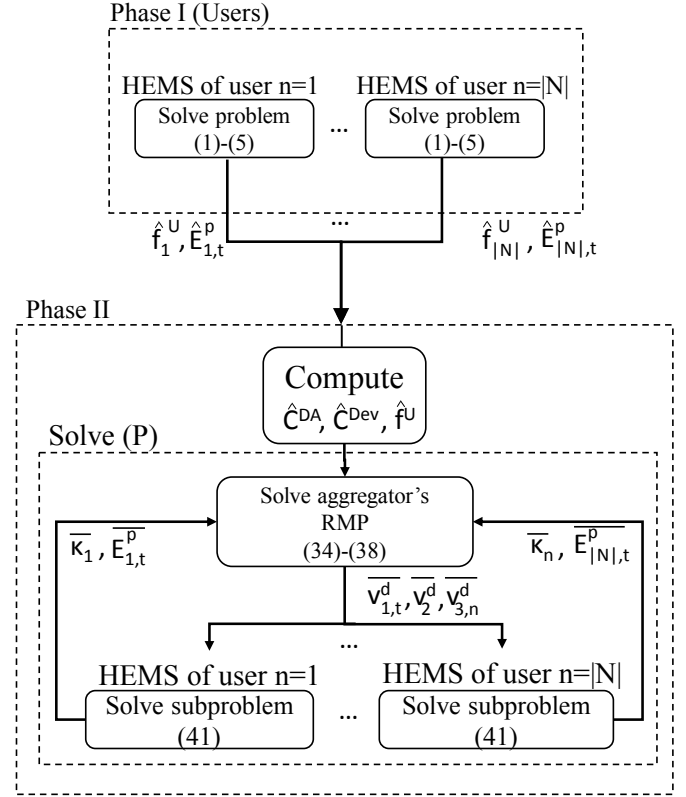
this figure also depicts the economic interface between the aggregator and the wholesale electricity market as well as the bidirectional electricity flow between users and the power grid. The information flow is more detailed in Fig. 2, which also depicts the proposed two-phase distributed framework where (P) is solved under DWD. As shown in the bottom part, the corresponding HEMS of each user n solves its own subproblem and sends information to the aggregator. As can be seen, the aggregator only sends to users information on DWD dual variables, $\overline{v_{1}^{d}}$, $\overline{v_{2,n}^{d}}$ and $\overline{v_{3,n}^{d}}$, while each user only sends to the aggregator the amount of kWh per time step to be consumed, $\overline{E_{n,t}^{p}}$, and $\overline{\kappa_n}$, thereby preserving users' privacy. Note that $\overline{\kappa_n}$ are constants that should be interpreted as the incentives requested by users at each iteration of the Column-Generation algorithm. Although these values are iteratively updated by solving SP_n, the optimal values of users' incentives are determined by the RMP, which is associated with the aggregator. Note also that variables α_{n}^{h} in the RMP allow determining the optimal combination of $\overline{\kappa_{n}^{h}}$.

It is worth emphasizing that in comparison with existing methodologies [22], which also decompose (P) into subproblems, the proposed decomposition guarantees optimality within a finite number of iterations and preserves the privacy of users' data. Moreover, the proposed approach is particularly adequate to mitigate some communication issues as a feasible and hence implementable solution will still be available in the event of a communication interruption.

IV. RESULTS

This section presents the results from the proposed framework using both the centralized and distributed approaches respectively described in Sections II and III. For assessment purposes, results from only executing Phase I are also reported. Hereinafter, "Phase I" is used to refer to the case without any users' coordination by only carrying out Phase I, and the acronyms "CE" and "DI" refer respectively to the centralized and distributed approaches. In order to gain insight into the advantages featured by the proposed users coordination over the uncoordinated situation, we have compared the results from both phases. The comparison is carried out in terms of the Peak-to-Average Ratio, the limits on users' incentives, as well as the price given to the load deviation. The time horizon comprises one day divided in intervals of 10 minutes, i.e., $|T| = 144$. This case study considers real data for users from [21] and assumes forecasted values for uncertain parameters. In contrast to [21], λ_n^s is set to 0 for all $n \in N$. We assume that this selling price has a decreasing tendency in the market. For instance, the tariff scheme (Smart Export Guarantee) adopted in April 2019 to replace the Feed-in-Tariff in the United Kingdom has reduced the selling price by 36% along one year. The reduction mainly depends on the electricity company supplier, such as EDF Energy [34]. For the sake of simplicity, the aggregator's prices remain unchanged over the time span, being $\lambda_t^{DA} = \$0.01/\text{kWh}$ and $\mu_t^{Dev} = \$0.002/\text{kWh}$, for all $t \in T$. Note that having a flat electricity price is a practical assumption in hydropower dominated countries like Brazil [28]. Besides, in order to avoid discrimination, κ_n^{min} is the same for all $n \in N$ and set to $\$0.01$.

Simulations were run to optimality using CPLEX 12.8 under AMPL 20180618 on an Intel® Xeon® X5675 at 3.07 GHz and 96 GB of RAM.

Fig. 3 depicts the system load profiles under the three approaches considered as well as the time-of-use tariff for $|N| = 500$. As can be seen, both CE and DI efficiently flatten the aggregated load profile. It is worth mentioning that the aggregated consumption profiles are not identical for CE and DI, but the same optimal value of (P) is attained by both methods, confirming the existence of multiple solutions. If only Phase I is performed, there is a large consumption peak near time step 96 anticipating the increase in λ_t^{TOU} between this time step and time step 125. Although from the users' perspective it is more economical to store energy to avoid consumption at high prices, this is not convenient for the aggregator.

Running times for CE and DI are listed in Table I for different sets of users of growing size. As can be seen, DI allows dealing with instances of up to 10,000 users whereas CE is unable to solve instances with more than 500 users (indicated with symbol -) within one month, which corroborates the scalability issues of this method. Moreover, unlike CE, DI is suitable for parallel computing, which would greatly reduce the computational burden. An approximation of the computing times required by the parallel implementation of DI can be obtained dividing the results in Table I by $|N|$. These estimated
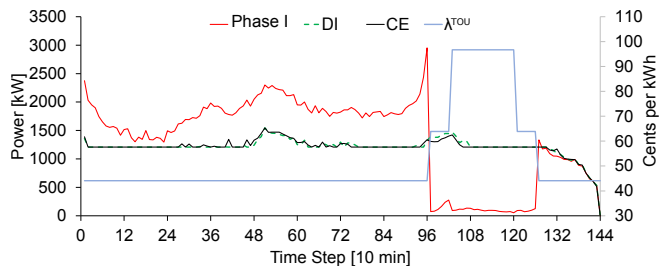

Figure 3: Scheduled load profiles for $|N| = 500$ and time-of-use tariff.

times are shown in Table II. As can be seen, the computing times are drastically reduced to a range between 17.14 s and 130.54 s, thereby backing the practical applicability of DI.

Table I: CPU times on a single machine

$|N|$	Time [s]		$|N|$	Time [s]	
	CE	DI		CE	DI
10	10	507	50	239	856
100	1000	3032	300	8244	25314
500	12522	34515	1000	-	89631
5000	-	588870	10000	-	1305448

Table II: Estimated CPU times per user with parallel computing

$|N|$	10	50	100	300
Time [s]	51.00	17.14	30.32	84.38
$|N|$	500	1000	5000	10000
Time [s]	69.00	89.63	117.77	130.54

Table III: Number of iterations for DWD

$|N|$	10	50	100	300
Number of iterations	10	50	83	56
$|N|$	500	1000	5000	10000
Number of iterations	36	35	31	31

For the cases shown in Table I, the Peak-to-Average Ratio is calculated and the largest improvement is attained for $|N| = 10,000$. In this case, f^{PAR} is equal to 2.06 if only Phase I is carried out while a value of 1.14 is achieved under DI, thereby yielding a substantial 44.7% reduction. This result is consistent with the intuition that by increasing the number of users more room to flatten their aggregated consumption is available for the aggregator.

The number of iterations required by the DWD procedure to reach convergence is shown in Table III. Note that this number does not depend on the case size.

Fig. 4 illustrates the evolution along DWD iterations of the optimality gap corresponding to the case with $|N| = 500$ users. According to this figure, the impact on solution quality of a given interruption steadily declines along the iterations of the DWD algorithm. Note that the optimality gap is crucial
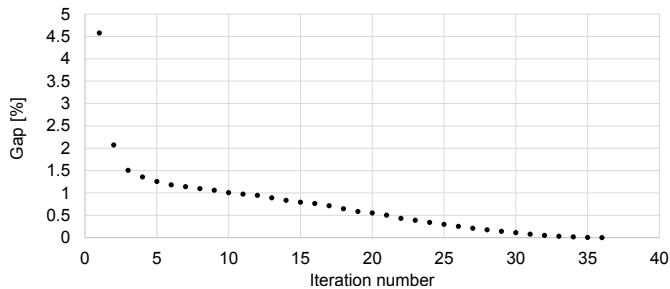
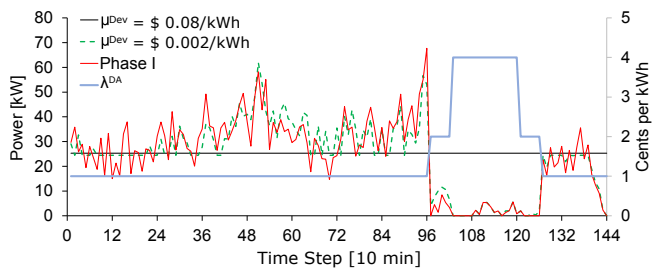Figure 4: Optimality gap along DWD iterations.

Figure 5: Impact of μ_t^{Dev} on the scheduled load profiles for $|N| = 10$.

Figure 6: Impact of κ_n^{min} on the scheduled load profiles for $|N| = 10$.

to assess the quality of the last solution stored before a communication interruption takes place.

To study the trade-off modeled in (20) between the smoothness of the aggregated consumption profile and the electricity acquisition cost, different values are assigned to μ_t^{Dev} assuming that the day-ahead electricity price follows a TOU tariff as depicted in Fig. 5. For $|N|$=10 and μ_t^{Dev} equal to $0.002 /kWh and $0.08 /kWh, $\forall \, t \in T$, the aggregator's cost respectively amounts to $40.76 and $53.11 under DI, and to $41.49 and $86.21 without any coordination. These results confirm that coordination is beneficial for the aggregator. Fig. 5 additionally shows the consumption profiles for the considered values of μ_t^{Dev}. It can be seen that, for $\mu_t^{Dev} = $0.08 /kWh, a flat consumption profile is attained while the consumption profile for $\mu_t^{Dev} = $0.02 /kWh is very similar to the non-coordinated case.

To focus on the benefits of having a low variability of the aggregated consumption, only the associated cost, C_t^{Dev}, is minimized in (20) under DI and the terms related to the aggregator's electricity cost and the incentives are dropped in (20). Then, the minimum amount of incentives, κ_n^{min}, given by the aggregator to each user is modified in order to quantify its influence on the aggregator's total cost, computed considering all the terms in (20). In particular, for $|N|$=10 and values of κ_n^{min} equal to $0, $0.01, and $0.05, the aggregators' cost amounts, respectively, to $35.1, $36.0, and $36.9, which corroborates the fact that larger costs are associated with larger incentives. Note that by setting $\kappa_n^{min} = $0, as done in [22], or without explicitly considering κ_n^{min}, as in [8], yields a price-based DR program with the minimum value of the aggregator's cost. However, without any economic incentive, the risk of users leaving such a DR program is increased. Yet, increasing the minimum amount of incentives effectively reduces the load variability as shown in Fig. 6 for the considered values of κ_n^{min}. Note that constraint (12) prevents from setting the minimum amount of incentives too high. For this particular case, infeasible solutions are obtained when $\kappa_n^{min} \geq $0.06. Analyzing the incentives received by each user, they are all equal to the minimum incentive in any case, $\kappa_n = \kappa_n^{min}, \, \forall \, n \in N$. In particular, for $\kappa_n^{min} = $0.05 per day, the monthly electricity bill reduction is around 1% for each user.

In some power markets, like those in Europe, the load variability is optimized according to the hourly day-ahead price λ_t^{DA}. The proposed approach is able to deal with such a case by setting μ_t^{Dev} to 0. To that end, a case
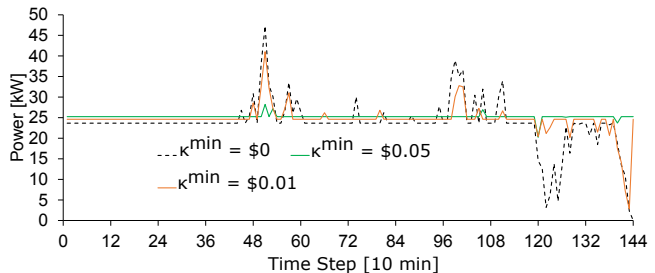
is solved considering the day-ahead prices of Spain on 22 March 2020 [35]. In addition, based on [36], a TOU tariff is considered with the following values: $\lambda_t^{TOU} = 0.04$ $/kWh for all $t \leq 73$ and $\lambda_t^{TOU} = 0.1$ $/kWh for all $t \geq 74$. The selected prices and the results obtained for both phases considering 10 householders are depicted in Fig. 7. Note that in Phase I the peak starts near time period 72 due to the transition in λ^{TOU}, while the peaks for the DI start near time periods 50 and 100, corresponding to low values of the day-ahead prices. From the aggregator's perspective, the coordinated approach using a distributed implementation leads to a saving of $5.4 as compared to Phase I. Note that aggregating only 10 householders may save $162 per month.

V. CONCLUSION

The main motivation of this work is to provide a practical tool for aggregators involved in the implementation of demand response programs. This tool coordinates the consumption of householders so that their comfort levels and costs remain unchanged after coordination. A two-phase framework is proposed in which the users' profits are first individually maximized without any coordination. Subsequently, in a second phase, the aggregator minimizes its cost while keeping the desired comfort level and profit for every user. The problem solved by the aggregator is posed as a bilevel problem that can be cast as a single-level equivalent. Privacy and scalability needs are accommodated by applying Dantzig-Wolfe decomposition. Simulation results show that the proposed framework is a useful tool for aggregators who seek to decrease the aggregated electricity demand curve peaks while maintaining the level of users' comfort at no financial loss. In addition, scalability and computational effectiveness of the
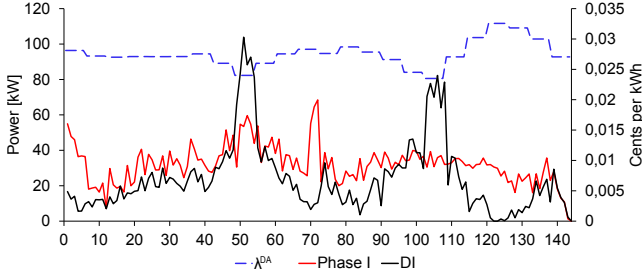
Figure 7: Impact of λ_t^{DA} on the scheduled profiles for $|N| = 10$.

proposed approach are validated through the simulation of cases of growing size. Finally, the practical applicability of the proposed approach is illustrated with a real-life instance comprising 10,000 users.

To extend this work, we propose two avenues of research. Firstly, stochastic programming or robust optimization models will be devised to handle the uncertainty in both aggregator- and users-related parameters. In particular, the interface between the aggregator and the wholesale electricity market will be captured by characterizing the uncertainty in day-ahead electricity prices. Secondly, fairness will be incorporated by considering incentives according to the participation of each user. This aspect is of utmost interest for practical implementation purposes.

APPENDIX

In this appendix we show that the minimization of the non-linear function f^{PAR} can be locally approximated via a linear programming problem corresponding to the minimization of the variance of the energy consumed along the time horizon.

Let $B_t = \sum_{n \in N} E_{n,t}^p$ and $\beta = max_{t \in T} B_t$ and let t^* denote any index in T with $B_t = \beta$. Then, the PAR function f^{PAR} can be defined by the following expression:

$$f^{PAR} = \frac{\beta |T|}{\beta + \sum_{t \in T/t^*} B_t}. \tag{42}$$

For $\beta = 0$, $B_t = 0$, $\forall\ t \in T$, and f^{PAR} is an undefined function, meaning in practice no consumption. Conversely, for $\beta > 0$, $B_t \leq \beta$, $\forall\ t \in T$, and f^{PAR} is greater than 1. Moreover, the minimum value of f^{PAR} is attained for $B_t = \beta$, $\forall\ t \in T$ and is equal to 1, which corresponds to a flat consumption profile.

Besides, the variance σ^2 of the energy consumed along the time horizon can be defined as follows:

$$\sigma^2 = \frac{1}{|T| - 1} \sum_{t \in T} \left(\sum_{n \in N} E_{n,t}^p - \frac{\sum_{n \in N} \sum_{t' \in T} E_{n,t'}^p}{|T|} \right)^2. \tag{43}$$

Theoretically, the minimum value for the variance is 0 and is attained when $\sum_{n \in N} E_{n,t}^p$ is identical $\forall\ t \in T$. With no loss of generality, minimizing this nonlinear function is locally approximated by solving the following linear programming problem:

$$\min_{E_{n,t}^p, e_t^+, e_t^-} \sum_{t \in T} e_t^+ + e_t^- \tag{44}$$

subject to:

$$\sum_{n \in N} E_{n,t}^p - \frac{\sum_{n \in N} \sum_{t' \in T} E_{n,t'}^p}{|T|} + e_t^+ - e_t^- = 0, \ \forall\ t \in T \tag{45}$$

$$e_t^+, e_t^- \geq 0, \quad \forall\ t \in T \tag{46}$$

where e_t^+ and e_t^- are nonnegative surplus variables.

For the particular case of $B_t = \beta$, $\forall\ t \in T$, the variance also attains its minimum value of 0 since $\sum_{n \in N} E_{n,t}^p = \beta$, $\forall\ t \in T$. Thus, the solution of problem (44)–(46) is equivalent to the minimization of f^{PAR}.

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