A Unified Approach to Solve Convex Hull Pricing and Average Incremental Cost Pricing with Large System Study

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***Abstract*— This paper introduces a unified approach to solving convex hull pricing (CHP) and average incremental cost (AIC) pricing problems. By developing a convex hull and convex envelope formulation for individual resources, a CHP model that minimizes uplift can be solved by linear programming (LP) using relaxation of the binary terms of the security constrained unit commitment (SCUC) problem. This paper proves that by adjusting resource upper bounds based on the SCUC solution, the one-pass LP relaxation of the SCUC problem can also be used to derive AIC prices, eliminating make-whole payments. Case studies using both small systems and the MISO day ahead system are presented to compare make-whole payments, uplift and generator profit under LMP, CHP and AIC.**

Index Terms— convex hull pricing, average incremental cost pricing, mixed-integer programming, security-constrained unit commitment, uplift payment, make whole payment.

I. INTRODUCTION

In this paper, we propose a unified approach to solve convex hull pricing (CHP) [1] and average incremental cost (AIC) pricing. In [2], the CHP-Primal method was proposed to solve CHP using LP relaxation of the unit commitment economic dispatch (UCED) problem under certain formulations on individual resource constraints. In this paper, we prove that, through adding valid cuts on individual resource maximum limits, AIC problems can also be solved through LP relaxation in a manner similar to CHP. We define this method as AIC-Primal. The AIC-Primal method proposed in this paper provides a novel systematic method to determine AIC prices. Using the AIC price for settlement can result in no make-whole payments (MWP). The pros and cons of the prices are beyond the scope of the paper. A unified consistent solution method can make it easy to compare and understand the pricing outcomes.

A. Uplift, make-whole payment (MWP) and opportunity cost (OC)

The energy only unit commitment and economic dispatch (UCED) problem without transmission constraints minimizes total cost while satisfying resource constraints and power balance constraint.

For the T-period UCED problem, denote $p_g \in R_+^T$ as the vector of dispatch variables and $u_g \in \{0,1\}^T$ as the vector of binary variables for unit g. $\mathrm{X}_g \subseteq R_+^T \times \{0,1\}^T$ is the feasible commitment and dispatch region for unit g, with $d \in R_+^T$ as the demand vector.

Let $v(d) = min \quad \sum_{g\in G} C_g(p_g, u_g)$ (1)

$s.t. \quad \sum_{g\in G} p_g$=d (2)

$(p_g, u_g) \in \mathrm{X}_g \quad \forall g \in G$ (3)

Assume an optimal decision from (1)-(3) is (p^*, u^*).

Given any price vector π, the profit maximization problem for unit g is:

$\omega_g(\pi) = max_{g\in G} \, [\pi' p_g - C_g(p_g, u_g)]$ (4)

$s.t. \; (p_g, u_g) \in \mathrm{X}_g$ (5)

[π' is the transpose of vector π. Assume an optimal solution for (4)-(5) is (p", u"). The uplift under prices π is defined as:

$U_g(\pi, p^*, u^*) = \omega_g(\pi) - [\pi' p_g^* - C_g(p_g^*, u_g^*)] \geq 0$ (6)

The RTOs adopted Locational Marginal Pricing (LMP) as the as the pricing mechanism. LMP is calculated from the dual variables of the Security Constrained Economic Dispatch (SCED) problem with the commitment variables fixed at the UCED solution. It reflects the marginal cost of online generators committed to meet system wide constraints. It works well when the market formulation is convex. For markets with non-convexity, uplift is usually unavoidable under LMP.

Positive uplift indicates that under the market clearing price, a generator may not be able to maximize its profit or even recover its costs by following the commitment and dispatch instructions from the RTO. The RTO needs to introduce MWP to ensure non-confiscatory settlement and penalties to discourage uninstructed deviations.

High uplift under LMP may also be an indication of sub-optimal unit commitment solution. Under sub-optimal unit commitment, the number of committed generators deviates from what is needed under the optimal solution. It usually causes LMP to be too high or too low, which can both result in high uplift.

The uplift under price π in (6) is the difference of the maximum profit that the generator can earn in (4) and the profit it earns under the UCED solution (p_g^*, u_g^*). Note the dispatch solution from the profit maximization problem may violate system wide constraints and be infeasible. The profit the generator earns under the UCED solution is as follows:

$\varphi_g(\pi, p_g^*, u_g^*) = \pi' p_g^* - C_g(p_g^*, u_g^*)$ (7)

If $\varphi(\pi, p^*, u^*) < 0$, the generator's profit would be negative by following the RTO's instructions. To incentivize the generator to follow instructions, all RTOs compensate the generator with MWP. Some RTOs (e.g. ISO-NE) also compensate opportunity cost under their fast-start pricing design. However, it causes revenue inadequacy and RTOs must allocate this deficit to another participant, such as load serving entities. The derivation and allocation of the MWP and opportunity cost is not transparent.

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When solving a multi-period model, a generator may have multiple commitment blocks. It may have one block with positive profit and another block with negative profit. Assume there are k_g commitment blocks $B_{g,j} \subseteq T,\ j = 1, \dots, k\ with\ B_{g,j1} \cap B_{g,j2} = \emptyset$ and $for\ j1 \neq j2$. Define the commitment variable as $uc_g \in \{0,1\}^T$.

Let $uc_{gt}^* = 1\ for\ t \in B_{g,j}, \forall j = 1, \dots, k_g$

$uc_{gt}^* = 0\ for\ t \notin \{B_{g,1} \cup \dots \cup B_{g,k_g}\}$.

The profit under commitment block $B_{g,j}$ is:

$$\varphi_{g,B_{g,j}}(\pi, p^*, u^*) = \sum_{t \in B_{g,j}} [\pi_t p_{gt}^* - C_{gt}(p_{gt}^*, u_{gt}^*)]$$

The total profit is: $\varphi_g(\pi, p_g^*, u_g^*) = \sum_{j=1}^{k_g} \varphi_{g,B_{g,j}}(\pi, p_g^*, u_g^*)$

Consistent with our definition of uplift in (6), the minimum profit for a resource in any block is bounded from below by zero, as it could no worse than not participating in the market. It's unreasonable to expect a generator to operate at a loss when following the RTOs' commitment and dispatch instructions. Therefore, all RTOs compensate generators if the MWP is positive. Under profit maximization, the unprofitable block can be de-committed. Hence, the uplift for each generator is no less than its MWP.

One exception is that one block "A" can be committed at a loss in order to save the startup cost of the next block "B" (e.g., from cold start to hot start). In this case, block "A" usually also has a loss in the profit maximization problem in order to maximize the total profit from both blocks. The profit from both blocks is higher than the profit from only committing block "B". We define the two blocks as "linking blocks". lt is not always straightforward to tell whether two blocks are committed together to maximize the total profit. We define the MWP as the amount required to compensate the net profit loss from all commitment blocks:

$$M_g(\pi, p_g^*, u_g^*) = \max\{0, -\sum_{j=1}^{k_g} \varphi_{g,B_{g,j}}(\pi, p_g^*, u_g^*)\} \qquad (8)$$

We define $\omega_g^+(\pi, p_{gt}^*)$ as the maximum profit the generator can make under price π without exceeding p_{gt}^*. Define $X_{p_g^*}^+ = \{ (p_g, u_g) \in X_g, p_{gt} \leq p_{gt}^*,\ \ \forall g \in G, t \in T\}$

$$\omega_g^+(\pi, p_{gt}^*) = max \quad \{\pi' p_g - C_g(p_g, u_g) \mid \{p_g, u_g\} \in X_{p_g^*}^+\}$$

Let $p"_{gt}$ be the maximum profit schedule under uniform price π. We define the opportunity cost as the part of the maximum profit that requires increasing the output from the UCED dispatch p_{gt}^*.

Hence, the opportunity cost is:

$$O_g(\pi, p^*, u^*) = \omega_g(\pi) - \omega_g^+(\pi, p_{gt}^*) \qquad (9)$$

B. Average Incremental Cost pricing to eliminate make-whole payment

Average incremental cost (AIC) pricing is proposed in non-convex market as the rough equivalent to marginal cost pricing in convex markets [4][8]. The AIC pricing mechanism produces prices that incorporate both the marginal costs and the avoidable fixed operating costs of a dispatched resource. Prior to the work presented in Section II.B of this paper, a multi-pass method was required to derive the AIC price. It modifies a resource's marginal cost offer by dividing avoidable fixed costs by the optimal dispatch (energy and reserves) and adding it to the marginal costs and relaxing the minimum operating level to zero. If a resource is dispatched down to zero in any of the intervals in one pass, the avoidable fixed costs allocated to these intervals need to be re-allocated and the problem re-solved. The ultimate solution may require several such passes to solve and this method is not straightforward to implement in a large system. The generator with the highest AIC and a non-zero AIC dispatch in the final pass of the pricing run sets the energy price. AIC pricing ensures offer cost recovery, eliminates MWP, and sends an efficient signal for entry.

AIC pricing can achieve the goal of eliminating MWP, better signaling efficient entry using the given schedule. Opportunity cost are eliminated by imposing liquidated damages and/or penalties on excursions from the dispatch signal. Self-committed units may receive different pricing under AIC than economically offered generators.

C. Convex hull pricing to minimize uplift

The convex hull prices minimize uplift. It can be determined by a Lagrangian relaxation of the UCED problem [1]. The Lagrangian dual function after dualizing power balance equation is:

$$q(\pi, d) = \sum_{g \in G} min\{C_g(p_g, u_g) - \pi' p_g \mid (p_g, u_g) \in X_g]\} + \pi' d \qquad (10)$$

The Lagrangian dual problem is:

$$Q(d) = max_\pi\ q(\pi, d) \qquad (11)$$

It's proved in [1] that the convex hull price is the dual maximizer π^*.

Under any price π, the gap between the UCED problem and its dual (10) is exactly the total uplift.

$$D(d, \pi) = v(d) - q(\pi, d) = \sum_{g \in G} U_g(\pi, p^*, u^*) \qquad (12)$$

Hence, the convex hull price vector π^* that maximizes the Lagrangian dual should minimize total uplift under any UCED solution. CHP minimizes total uplift including MWP and opportunity costs for both online and offline resources. But it may not be revenue adequate for on-line generators and may require MWP.

MISO implemented an approximate CHP in its day ahead and real time SCED software and called it Extended LMP (ELMP). After the day ahead and real time SCED dispatch run, the SCED pricing run is executed with single interval LP relaxation [10].

II. A Unified Approach to Solve CHP and AIC

In this paper, we developed a unified approach to solve CHP and AIC prices. With this new approach, both prices can be solved with a single LP using relaxation. A unified consistent solution method makes it easy to compare and understand the pricing outcomes.

A. Solving CHP with CHP-Primal method

The theoretical convex hull price is the maximizer of the Lagrangian dual problem. It's very difficult to achieve convergence using traditional Lagrangian relaxation methods. Under an alternative approach incorporating the convex hull and convex envelope of individual resources, the CHP prices can be found through solving a linear programming (LP) relaxation problem. In [2], it is proved that the convex hull pricing problem can be solved with LP relaxation (i.e. CHP-Primal) if the individual generator objective cost and resource constraints can be formulated properly as follows.

Let $conv(\cdot)$ be the convex hull of a set and $C^{**}_{g,\mathrm{X}_g}(\cdot)$ be the convex envelope of $C_g(\cdot)$ over X_g. The CHP-Primal problem is defined as:

$$min \sum_{g\in G} C^{**}_{g,\mathrm{X}_g}(p_g,u_g) \quad (13)$$

$$s.t. \quad \sum_{g\in G} p_g = \mathrm{d} \quad (14)$$

$$(p_g,u_g) \in conv(\mathrm{X}_g),\ \ u_g \in [0,1]^T \quad \forall g \in G \quad (15)$$

From [2], the optimal dual vector associated with (14) is the optimal solution to (11).

(13)-(15) is an LP problem and the expectation is that it is easier to solve than (11). It requires the development of the convex envelope formulation of the cost function and the convex hull formulation of the constraints associated with binary variables.

In [3], such a formulation is developed for MISO day-ahead SCUC with a few simplifications. The so-called extended formulation requires a large number of binary variables and solving the LP for (13)-(15) can be computationally intensive. An efficient iterative algorithm is therefore developed simplifying the problem by identifying a small sub-set of generators requiring the extended formulation. The algorithm can then solve the CHP problem using multiple LPs in an acceptable amount of time.

A theoretically complete formulation finding CHP prices may be difficult to achieve due to the myopic study window (e.g., single interval real time SCED) and complicated modeling. For example, regulation commitment and energy limit constraints make it difficult to derive the full convex hull formulation on individual resources. The day-ahead and real-time study windows may not cover the entire commitment block requiring amortization of startup cost. In this paper, we define ELMP (or extended LMP) as the approximation of CHP through solving the LP relaxation of the unit commitment and economic dispatch (UCED) problem.

B. Solving AIC with the AIC-Primal method

First, define the unprofitable generator and the associated unprofitable commitment block under LMP:

$$S^{MWP} = \{(g,t) \mid p^*_{gt} > 0, t \in B_{g,j}, \varphi_{g,B_{g,j}}(LMP, p^*_g, u^*_g) < 0\}$$

Then, define $X^{AIC}_{p^*_{g,\varepsilon}}$ as a sub-region of the original feasible region: $X^{AIC}_{p^*_{g,\varepsilon}} = \{(p_g,u_g) \in \mathrm{X}_g, p_{gt} \le p^{AIC-max}_{gt}),\ \forall g \in G, t \in T\}$, where $p^{AIC-max}_{gt} =$

$$\begin{cases} \min(p^*_{gt}+\varepsilon, p^{max}_{gt}) & if\ (g,t) \in S^{MWP} \\ 0 & if\ p^*_{gt} = 0 \\ p^{max}_{gt} & if\ p^*_{gt} > 0, and\ (g,t) \notin S^{MWP} \end{cases} \quad (16)$$

It essentially changes the maximum limits of unprofitable commitment blocks to $\min(p^*_{gt}+\varepsilon, p^{max}_{gt})$ and changes the maximum limits of uncommitted intervals to 0MW. We define the maximum limit constraints from (16) as AIC P-cuts. In this paper, we focus on the conditions for solving AIC with only P-cuts defined in (16), the properties of the resulting AIC and its relationship to CHP. The conditions include: i) a similar formulation as in CHP-primal and ii) an optimal UCED solution. In [9], additional cuts are introduced to achieve zero MWP when the conditions are not satisfied.

Define UCED on $X^{AIC}_{p^*_{g,\varepsilon}}$ as:

$$v^{AIC}(d, p^*_{gt}, \varepsilon) = min \sum_{g\in G} C_g(p_g,u_g) \quad (17)$$

$$s.t. \quad \sum_{g\in G} p_g = \mathrm{d} \quad (18)$$

$$\{p_g,u_g\} \in X^{AIC}_{p^*_{g,\varepsilon}} \quad \forall g \in G \quad (19)$$

Given $X^{AIC}_{p^*_{g,\varepsilon}} \subseteq \mathrm{X}_g$ and it includes the optimal solution (p^*_g,u^*_g) under X_g, (p^*_g,u^*_g) is also the optimal solution of $v^{AIC}(d, p^*_{gt}, \varepsilon)$

$$v(d) = v^{AIC}(d, p^*_{gt}, \varepsilon) \quad (20)$$

The difference between $X^{AIC}_{p^*_{g,\varepsilon}}$ and X_g is only the maximum limits of p_{gt}. If we replace p^{max}_{gt} in (13)-(15) with $p^{AIC-max}_{gt}$ the convex hull and convex envelop condition should not change. Hence, the CHP price for $v^{AIC}(d, p^*_{gt}, \varepsilon)$ can also be solved with the formulation and methods developed in [3] with adjusted p^{max}_{gt}.

The revised problem is then:

$$v\_rel^{AIC}(d, p^*_{gt}, \varepsilon) = min \quad \sum_{g\in G} C^{**}_{g,\mathrm{X}_g}(p_g,u_g) \quad (21)$$

$$s.t. \quad \sum_{g\in G} p_g = \mathrm{d} \quad (22)$$

$$(p_g,u_g) \in conv(X^{AIC}_{p^*_{g,\varepsilon}}),\ \ u_g \in [0,1]^T \quad \forall g \in G \quad (23)$$

We assume the optimal primal solution from (21)-(23) is (p^{**}_g, u^{**}_g) and the price from (21)-(23) is π^{**}.

Next, we prove the price π^{**} from LP problem (21)-(23) is the AIC price that can reduce MWP to \$0 when $\varepsilon \to 0$. We define this method of solving AIC price as AIC-Primal.

Proposition 1: There is no opportunity cost for $v^{AIC}(d, p^*_{gt}, \varepsilon)$ under LMP. The total uplift equals the total MWP.

Proof: First, assume there are no linking blocks. Based on the definition of $p^{AIC-max}_{gt}$, $if\ (g,t) \notin S^{MWP}$ and $p^*_{gt} = 0$, $p^{AIC-max}_{gt} = 0$. The profit maximization problem for $v^{AIC}(d, p^*_{gt}, \varepsilon)$ cannot increase the output and there is no opportunity cost for this set and the uplift is \$0.

If $p^*_{gt} > 0, and\ (g,t) \notin S^{MWP}$, based on the definition of LMP, the generator should make the maximum profit under LMP if its fixed cost is ignored. Given the LMP can also cover the total cost (i.e., on the commitment block not requiring MWP), the block should also be committed in the profit maximization problem. Hence, the generator should also make the maximum total profit and the uplift is \$0.

Therefore, only the subset $(g,t) \in S^{MWP}$ can have uplift. The generator commitment block can be decommitted in the profit maximization problem. It can also be dispatched up to $p^{AIC-max}_{gt} = \min(p^*_{gt}+\varepsilon, p^{max}_{gt})$. With LMP, the generator should make the maximum profit under p^*_{gt} if fixed costs are ignored. The profit from prices based on marginal costs cannot cover fixed costs. Hence, the profit maximum solution for $(g,t) \in S^{MWP}$ is to decommit the block. The uplift equals to the MWP.

The proof can be easily extended to linking blocks and is omitted here.

Proposition 2: For the commitment block requiring MWP under LMP, i.e., the subset $(g,t) \in S^{MWP}$, the solution in $v\_rel^{AIC}(d, p_{gt}^*, \varepsilon)$ cannot be $u_{g,t}^{**} = 0$.

Proof: We prove by contradiction. Based on the definition of $p_{gt}^{AIC-max}$, no new commitment can happen. If $u_{g,t}^{**} = 0$ for a commitment block in S^{MWP}, other on-line generators need to be dispatched up to meet power balance. The cost of dispatching up other on-line generators has to be lower than the cost of committing the block in $v\_rel^{AIC}(d, p_{gt}^*, \varepsilon)$ problem. Given decommitting the block creates a new binary solution, it's contradictory to the fact that (p_g^*, u_g^*) is the optimal solution of $v^{AIC}(d, p_{gt}^*, \varepsilon)$. Therefore, under the price π^{**}, this set of generators either make positive profit or break even under $(\pi^{**}, p_g^{**}, u_g^{**})$ with $p_g^{**} > 0$.

Proposition 3: Under π^{**}, the commitment block under subset $(g,t) \in S^{MWP}$ has \$0 MWP when $\varepsilon \to 0$.

Proof: Given $v\_rel^{AIC}(d, p_{gt}^*, \varepsilon)$ defined in (21)-(23) has only continuous variables and linear equations, it is a LP, and thus the duality gap is \$0. With the convex hull and convex envelope formulation, the relaxation of the profit maximization problem should solve at integral values. Hence, the same set of generators should also make positive profits or break even under the profit maximization (MIP) solution. These blocks should be committed under the profit maximization problem.

Since these blocks make a negative profit under LMP, the price π^{**} must be higher than the LMP in order to generate non-negative profit. Hence, under π^{**}, the profit maximization solution $p'_{gt} \in [p_{gt}^*, p_{gt}^* + \varepsilon]$. The uplift for the commitment block under subset $(g,t) \in S^{MWP}$ in the problem $v^{AIC}(d, p_{gt}^*, \varepsilon)$ is the profit difference between p'_{gt} and p_{gt}^*. With a piece wise linear incremental energy function, uplift is $O(\varepsilon) \to 0\ when\ \varepsilon \to 0$. Hence, the price π^{**} satisfies the definition of an AIC price.

Apparently π^{**} is the CHP price for $v^{AIC}(d, p_{gt}^*, \varepsilon)$. It should minimize uplift under $v^{AIC}(d, p_{gt}^*, \varepsilon)$. Based on the problem definition $v^{AIC}(d, p_{gt}^*, \varepsilon)$, for the set of generators with $p_{gt}^* = 0$, the uplift is zero. The uplift for $(g,t) \in S^{MWP}$ is $O(\varepsilon)$ from Proposition 3. Therefore, under π^{**}, the opportunity cost for on-line profitable blocks (i.e., $p_{gt}^* > 0\ and\ (g,t) \notin S^{MWP}$) is minimized among all prices that can produce zero MWP.

Given that the opportunity costs for $\{(g,t)|p_{gt}^* = 0$ or $(g,t) \in S^{MWP}\}$ are essentially ignored under problem $v^{AIC}(d, p_{gt}^*, \varepsilon)$, for the original UCED problem, the opportunity costs under the AIC price can be much higher than those under CHP.

Transmission constraints can also be included in $v\_rel^{AIC}(d, p_{gt}^*, \varepsilon)$ (21)-(23).

$$\sum_{g \in G} s_{kg} p_g \le L_k \quad for\ k = 1, \dots, K \tag{24}$$

Here L_k is the transmission flow limit subtracting flow impacts from fixed injections. We assume in UCED solution, the flow is solved at $f_k^* = \sum_{g \in G} s_{kg} p_g^*$.

In [7], it is shown that the uplift under CHP also include an "FTR uplift" component. It is because transmission constraints not binding in the UCED solution may become binding in the CHP solution (13)-(15) when the binary variables are relaxed.

Similarly, with (21)-(23), the generator committed for this constraint and dispatched at its minimum limit in the UCED can be solved at a fractional commitment level to set price. It can result in price separation even though this constraint is not binding in the UCED and therefore can cause FTR uplift similar to CHP from the original UCED problem as shown in [7]. But it gives a good investment signal for transmission capacity expansion. This can be illustrated in example 3 in section III.

III. Case Studies

A. Multi-commitment blocks

When there are multiple commitment blocks, the profit maximizing solution may bridge two blocks together to avoid the second startup cost. As shown in Table 5, the AIC price can eliminate MWP with the underlying assumption that two startups are considered independent.

Assume the system has two generators with load profile as follows.

	Pmin	Pmax	\$/MWh	Startup	NoLoad
Gen1	0	20	10	0	0
Gen2	50	130	0	1500	0

Hour	1	2	3	4	5
LD	10	100	10	100	140

The UCED optimal solution shown in Table 1 is to commit Gen2 in hour 2 and hour 4-5. These two commitment blocks both have negative profits under LMP and requires MWP. MWP for the first block is \$1500 and for the second block is \$200. Gen2 has high minimum limit and has to shut down in hour 3 when the load is low.

The individual resource profit maximizing solution under LMP is shown in Table 2. Gen2 is committed for all hours to avoid the second startup cost. Gen2 has the opportunity to earn \$2400 under profit maximization. This part of the uplift is referred to as opportunity cost. In hours 1 and 3, the LMP is \$10/MWh even though the load is very low, 10MW. The LMP is \$0/MWh on hour 2 and 4 when load is much higher at 100MW.

Under CHP, the hour 3 price is -\$30/MWh as shown in Table 3. It's the avoided startup cost for hour 4 by increasing load by 1 MW in hour 3. It prorates the startup cost to Pmin of Gen2: $-\frac{1500}{50} = -30/MWh$. This is the sufficiently low price to avoid incentivizing Gen2 to come on-line by self-committing. It is also a correct signal to incentivize more load to come on line. At hour 3, a 50MW price sensitive demand willing to pay higher than -\$30/MWh can be cleared to avoid the second startup even when the fixed load is zero. However, it causes a negative profit for Gen1 which requires MWP. The price in hour 2 and 5 are also increased. The increase in hour 5 is enough to cover the cost in the second commitment block since $p_{25} = 130\ MW$ is at its maximum. CHP prices of \$11.538/MWh at hours 2 and 5 prorate the startup cost of \$1500 over Pmax=130MW. The price in hour 2 is not high enough to cover Gen2's cost in the first commitment block. Both generators require MWPs and the total MWP is \$830.77. The opportunity cost is \$61.54 as shown in Table 4. It results in much lower total uplift.

Table 1 Example 1 -- UCED & profit under LMP

LMP	10	0	10	0	10	
Hour	1	2	3	4	5	Total
P2 from SCUC	0.00	100.00	0.00	100.00	130.00	
Gen2 Revenue	0.00	0.00	0.00	0.00	1300.00	1300.00
Gen2 cost	0.00	1500.00	0.00	1500.00	0.00	3000.00
Gen2 Profit	0.00	-1500.00	0.00	-1500.00	1300.00	-1700.00
uc2 from SCUC	0.00	1.00	0.00	1.00	1.00	
P1 from SCUC	10.00	0.00	10.00	0.00	10.00	
Gen1 Revenue	100.00	0.00	100.00	0.00	100.00	300.00
Gen1 cost	100.00	0.00	100.00	0.00	100.00	300.00
Gen1 Profit	0.00	0.00	0.00	0.00	0.00	0.00
					Net Profit	-1700.00

Table 2 Example 1 – Individual generator profit maximization under LMP

LMP	10	0	10	0	10	
Hour	1	2	3	4	5	Total
P2 profit max	130.00	50	130	50	130	
Gen2 Revenue profit max	1300	0	1300	0	1300	3900
Gen2 cost profit max	1500.00	0	0	0	0	1500
Gen2 profit max	-200	0	1300	0	1300	2400
u2 profit max	1	1	1	1	1	
P1 profit max	0	0	0	0	0	
Gen1 Revenue profit max	0	0	0	0	0	0
Gen1 cost profit max	0	0	0	0	0	0
Gen1 profit max	0	0	0	0	0	0
					NetProfit	2400

In Table 5, the AIC price increases the hour 2 price to \$15 and hour 4 price to \$2. It's just sufficient to reduce MWP to zero. However, the opportunity cost is high at \$4710 as shown in Table 6. It sends the correct signal for the committed blocks. However, for hour 3, the price stays high at \$10/MWh.

Table 3 Example 1 –UCED Profit under CHP

CHP	0	11.53846154	-30	0	11.53846154	
Hour	1	2	3	4	5	Total
P2 from SCUC	0	100	0	100	130	
Gen2 Revenue	0	1153.846154	0	0	1500	2653.8
Gen2 cost under SCUC	0	1500	0	1500	0	3000
Gen2 Profit	0	-346.153846	0	-1500	1500	-346.15
uc2 from SCUC	0	1	0	1	1	
P1 from SCUC	10	0	10	0	10	
Gen1 Revenue	0	0	-300	0	115.3846154	-184.62
Gen1 cost	100	0	100	0	100	300
Gen1 Profit	-100	0	-400	0	15.38461538	-484.62
					NetProfit	-830.77

Table 4 Example 1 – Individual generator profit maximization under CHP

CHP	0	11.53846154	-30	0	11.53846154	
Hour	1	2	3	4	5	Total
P2 profit max	0	0	0	0	0	
Gen2 Revenue profit max	0	0	0	0	0	0
Gen2 cost profit max	0	0	0	0	0	0
Gen2 profit max	0	0	0	0	0	0
u2 profit max	0	0	0	0	0	
P1 profit max	0	20	0	0	20	
Gen1 Revenue profit max	0	230.7692308	0	0	230.7692308	461.5385
Gen1 cost profit max	0	200	0	0	200	400
Gen1 profit max	0	30.76923077	0	0	30.76923077	61.53846
					NetProfit	61.53846

Table 5 Example 1 – UCED Profit under AIC ($\varepsilon = 0.00001$)

AIC	10	14.9999985	10	1.9999998	10	
Hour	1	2	3	4	5	Total
P2 from SCUC	0	100	0	100	130	
Gen2 Revenue	0	1499.99985	0	199.99998	1300	2999.99983
Gen2 cost under SCUC	0	1500	0	1500	0	3000
Gen2 Profit	0	-0.00015	0	-1300.00002	1300	-0.00017
uc2 from SCUC	0	1	0	1	1	
P1 from SCUC	10	0	10	0	10	
Gen1 Revenue	100	0	100	0	100	300
Gen1 cost	100	0	100	0	100	300
Gen1 Profit	0	0	0	0	0	0
					NetProfit	-0.00017

Table 6 Example 1 – Individual generator profit maximization under AIC

AIC	10	14.9999985	10	1.9999998	10	
Hour	1	2	3	4	5	Total
P2 profit max	130	130	130	130	130	
Gen2 Revenue profit max	1300	1949.999805	1300	259.999974	1300	6110
Gen2 cost profit max	1500	0	0	0	0	1500
Gen2 profit max	-200	1949.999805	1300	259.999974	1300	4610
u2 profit max	1.00	1.00	1.00	1.00	1.00	
P1 profit max	0	20	0	0	0	
Gen1 Revenue profit max	0	299.99997	0	0	0	300
Gen1 cost profit max	0	200	0	0	0	200
Gen1 profit max	0	99.99997	0	0	0	100
					NetProfit	4710

B. *Ramping constraint*

In Example 2, we assume a system with two generators, where Gen2 has a restrictive 5MW/h ramp rate. We follow MISO's rules for the startup interval: the generator is at its minimum limit Pmin at the beginning of the startup interval and considered to require one half period (30 min) ramping up. It's equivalent to 22.5MW/h startup ramp rate. In the shutdown interval, we assume it can ramp down to 0MW from any dispatch MW in one interval, which is equivalent to having a shutdown ramp rate equal to the maximum limit Pmax=35MW/h. Other parameters and the load profile are as follows.

	Pmin	Pmax	\$/MWh	Startup	NoLoad	ramprate
Gen1	0	100	10	0	0	
Gen2	20	35	50	1000	30	5

Loads in each hourly period are 95MW, 100MW and 130 MW respectively. First, we use the 3-binary formulation for the UCED and the relaxation for CHP and AIC.

Assume Gen2 was initially offered with a 1-hour minimum run and minimum down time. The 3-binary formulation is as follows:

$$\min \sum_{t=1}^{3} 10 \cdot p_{1,t} + \sum_{t=1}^{3} (30 \cdot u_{2,t} + 50 \cdot p_{2,t} + 1000 \cdot v_{2,t})$$

Limit constraints:

$$0 \le p_{1,t} \le 100 \qquad for\ \ 1 \le t \le 3 \qquad \text{(a1)}$$

$$20u_{2,t} \le p_{2,t} \le 35\, u_{2,t} \qquad for\ \ 1 \le t \le 3 \qquad \text{(a2)}$$

Ramping constraints:

$$p_{2,t} - p_{2,t-1} \le 5u_{2,t-1} + 22.5v_{2,t} \qquad for\ \ 1 \le t \le 3 \qquad \text{(a3)}$$

$$p_{2,t-1} - p_{2,t} \le 5u_{2,t} + 35e_{2,t} \qquad for\ \ 2 \le t \le 3 \qquad \text{(a4)}$$

Binary constraints:

$$u_{2,t} - u_{2,t-1} = v_{2,t} - e_{2,t} \qquad for\ 1 \le t \le 3$$

$$with\ \ u_{2,0} = 0\ for\ initially\ off \qquad \text{(a5)}$$

$$v_{2,t} \le u_{2,t} \qquad for\ 1 \le t \le 3 \qquad \text{(a6)}$$

$v_{2,t} \le 1 - u_{2,t-1} \quad for\ 1 \le t \le 3$ (a7)

Power balance constraint:

$p_{1,t} + p_{2,t} = LD_t \quad for\ 1 \le t \le 3$ (a8)

$v_{2,t}, u_{2,t}, e_{2,t}\ are\ binary \quad for\ 1 \le t \le 3$ (a9)

$v_{2,t}$, $u_{2,t}$ and $e_{2,t}$ are respectively the startup, commitment and shutdown variable for Gen2. $p_{1,t}$ and $p_{2,t}$ are dispatch variables for Gen1. The 3-binary formulation is used to solve the UCED and LMP. As shown in Table 7, under LMP, Gen2 needs MWP of $1690. The maximum profit for Gen2 is $0. Uplift is also $1690.

In this example, the 3-binary formulation is not convex hull for Gen2 due to the ramping constraints. Using LIP to solve CHP with this formulation results in the price [$10, $10, $182.7] with an uplift of $864.7.

Using LP relaxation of (a1)-(a9) to solve AIC by setting Pmax of Gen2 at $p^*_{2,t} + \varepsilon$ with $\varepsilon = 0.001$ in (a2) based on (16) results in a very high price $1161/MWh in hour 3. Even though it can eliminate MWP, the uplift is very high as shown in Table 8. It's arguable if the generator can take action to deviate from the commitment based on the price. The high price is definitely not the correct entry signal since it only requires $146.33/MWh to eliminate MWP and a large enough flexible resource with marginal cost less than $146.33/MWh can replace Gen2.

Next, we apply the extended formulation that is convex hull for the individual generator from [3]. First, define the binary variables:

$o_{2,t}$: representing Gen2 staying off through t and starting up at the beginning of t+1, for t=0,1,2

$w_{2,t}$: representing Gen2 starting at the beginning of t+1 and staying on until the end, for t=0,1,2.

When $w_{2,t} = 1$, Gen 2 is on for s=t+1, …, 3. Define the dispatch variable as $qw^s_{2,t}, s \in [t+1,3]$

$y_{2,tk}$: representing Gen2 starting at the beginning of t+1 and shutting down at the beginning of k, for $tk \in \{02,03,13\}$. Define the dispatch variable $qy^s_{2,tk}, s \in [t+1, k-1]$

$z_{2,tk}$: representing Gen2 shut down at the beginning of t and staying off until the beginning of k+1, for $tk \in \{22,23,33\}$.

The extended formulation is:

$$\min \sum_{t=1}^{3} 10 \cdot p_{1,t} + 1000 \cdot \left(\sum_{tk\in\{02,03,13\}} y_{2,tk} + \sum_{t\in\{1,2,3\}} w_{2,t}\right) + 30 \cdot \left(\sum_{t\in\{0,1,2\}} \sum_{s\in[t+1,3]} w_{2,t} + \sum_{tk\in\{02,03,13\}} \sum_{s\in[t+1,k-1]} y_{2,tk}\right) + 50 \cdot \left(\sum_{t\in\{0,1,2\}} \sum_{s\in[t+1,3]} qw^s_{2,t} + \sum_{tk\in\{02,03,13\}} \sum_{s\in[t+1,k-1]} qy^s_{2,tk}\right)$$

Limit constraints

$0 \le p_{1,t} \le 100 \quad for\ 1 \le t \le 3$ (b1)

$20 w_{2,t} \le qw^s_{2,t} \le 35\, w_{2,t} \quad t \in [0,2], s \in [t+1,3]$ (b2)

$20 y_{2,tk} \le qy^s_{2,tk} \le 35\, y_{2,tk} \quad tk \in \{02,03,13\}, s \in [t+1, k-1]$ (b3)

Ramping constraints

$qy^{t+1}_{2,tk} \le 22.5\, y_{2,tk},\ qw^{t+1}_{2,t} \le 22.5\, w_{2,t}$ (b4)

$qy^2_{2,03} - qy^1_{2,03} \le 5\, y_{2,03},\ qy^1_{2,03} - qy^2_{2,03} \le 5\, y_{2,03},$

$qw^{s+1}_{2,t} - qw^s_{2,t} \le 5\, w_{2,t},\ t \in [0,2], s \in [t+1,3]$

$qw^s_{2,t} - qw^{s+1}_{2,t} \le 5\, w_{2,t},\ t \in [0,2], s \in [t+1,3]$

Binary constraints

$-o_{2,0} + y_{2,02} + y_{2,03} + w_{2,0} = 0, \quad -o_{2,1} + y_{2,13} + w_{2,1} = 0,$

$-o_{2,2} + w_{2,2} = 0, \quad y_{2,02} - z_{2,22} - z_{2,23} = 0,$

$y_{2,03} + y_{2,13} - z_{2,33} = 0, \quad o_{2,0} + o_{2,1} + o_{2,2} \le 1$

The final dispatch MW of Gen2:

$p_{2,1} = qy^1_{2,02} + qy^1_{2,03} + qw^1_{2,0}$

$p_{2,2} = qy^2_{2,03} + qy^2_{2,13} + qw^2_{2,0} + qw^2_{2,1}$

$p_{2,3} = qw^3_{2,0} + qw^3_{2,1} + qw^3_{2,2}$

Power balance constraint:

$p_{1,t} + p_{2,t} = LD_t \quad for\ 1 \le t \le 3$ (a8)

The SCUC problem solves with $o_{2,0} = 1$, $w_{2,0} = 1$, $qw^{1*}_{2,0}$=20, $qw^{2*}_{2,0}$=25, $qw^{3*}_{2,0}$=30. The commitment and dispatch is the same as the results from the 3-binary formulation.

Using LIP to solve CHP with this extended formulation results in the price [$10, $10, $276] with an uplift of $365.

To solve AIC from extended formulation LIP, Pmax in (b2) and (b3) is replaced based on UCED solution according to (16). Table 9 shows the result. The AIC price at hour 3 is $146.333/MWh. The resulting MWP is -$0.016. This small residual MWP is a function of ε. The price is just enough to eliminate MWP. Under this price, Gen2 is better off to only come online on hour 3 under the individual resource profit maximization solution. The uplift is $1137.48.

From this example, it's important to solve for the AIC price using the individual resource convex hull formulation. When the constraint is not the convex hull, price from LIP may be unreasonable.

Table 7 Example 2 – UCED solution and profit under LMP

LMP	10	10	90		
P2 from SCUC	20	25	30		
Gen2 Revenue under LMP	200	250	2700		3150
Gen2 cost under SCUC	2030	1280	1530		4840
Gen2 Profit under LMP	-1830	-1030	1170	MWP	-1690
P1 from SCUC	75	75	100		
Gen1 Revenue under LMP	750	750	9000		10500
Gen1 cost under SCUC	750	750	1000		2500
Gen1 Profit under LMP	0	0	8000		8000
				profit	6310
				max profit	8000
				uplift	1690

Table 8 Example 2 – AIC solution and profit using 3-binary formulation

AIC	10	10	1161	Total
P2 from SCUC	20	25	30	
Gen2 Revenue	200	250	34835	35285
Gen2 cost under SCUC	2030	1280	1530	4840
Gen2 Profit	-1830	-1030	33305	30445
P1	75	75	100	
Gen1 Revenue	750	750	116118	117618
Gen1 cost	750	750	1000	2500
Gen1 Profit	0	0	115118	115118
			profit	145563
			max profit	148141
			uplift	2578

Table 9 Example 2 – AIC solution and profit using extended formulation

AIC	10	10	146.33	
P2 from SCUC	20.00	25.00	30.00	
Gen2 Revenue under AIC	200	250	4389.9	4839.9
Gen2 cost under SCUC	2030.00	1280.00	1530.00	4840
Gen2 Profit under AIC	-1830	-1030	2859.9	-0.1
P1 from SCUC	75	75	100	
Gen1 Revenue under AIC	750	750	14633	16133
Gen1 cost under SCUC	750	750	1000	2500
Gen1 Profit under AIC	0	0	13633	13633
			profit	13633
			max profit	14770
			uplift	1137.5

C. Transmission Constraint

Fig. 2 is a 2-bus system connected by a single transmission line. Due to the transmission limit of 150MW, more expensive generator Gen2 needs to be committed to serve L2. The UCED results and individual generator profit maximization results under LMP are shown in Table 10. The LMP is \$40/MWh on both buses. Gen2 requires a \$5500 MWP. The flowgate marginal price (FMP) is \$0, correctly indicating there is no economic marginal expansion. There is no FTR uplift. In convex markets, the LMPs take care of all pricing issues, but in non-convex markets, the pricing issues take place on multiple levels and the LMP is not a complete price signal.

As shown in Table 11, under CHP, the bus 2 price is \$90/MWh. Gen2 MWP is reduced to \$500. Price separation across the constraint provides a signal for transmission capacity expansion. However, it causes FTR uplift of \$2500.

In Table 12, For AIC, the LIP prices for bus 1 and bus 2 are \$40 and \$95. The FIP is \$55. There is no physical congestion, but in the pricing run, the price separation and FIP indicate there may be value to an incremental expansion. An incremental transmission capacity expansion of 50 MW or more will eliminate the need for Gen2. An expansion of 50 MW or more with cost less than \$5500 increases the market surplus.

	Bus1	Flowgate	Bus2
LD	30		200
Gen	G1		G2
Startup	0		1500
Pmin	0		100
Pmax	250	150	150
\$/MWh	40		80

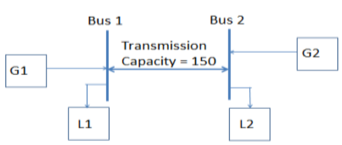

Fig. 2 Two bus system and parameter

D. MISO case study

In this section, the extended formulation and the iterative solution method from [3] are implemented and studied on seven MISO day ahead cases. The method is extended to add AIC P-cuts from (16) based on SCUC solution. For the MISO model, the generator constraints include minimum and maximum run time, minimum down time, maximum and minimum limits and ramping constraints. Economic values include three startup costs (hot, intermediate and cold), fixed cost per period (i.e., no load cost), and the piece-wise linear incremental energy cost. At the system level, power balance constraints and large number of the original transmission constraints are also included.

First, the SCUC is solved to 0.1% MIP gap with 3-binary formulation. Then the AIC solution is found from solving the LP relaxation of the extended formulation [3]. AIC2 is found by solving the LP relaxation of the 3-binary formulation. The P-cuts are applied on units with unprofitable blocks under the LMP prices determined from the SCUC solution. Table 13 shows the MWP, uplift and FTR uplift from 7 sample days under LMP, CHP, AIC and AIC2.

The MWP under CHP are between 1.64% to 26.55% compared to those derived from the standard LMP methodology as shown in Table 13. AIC and AIC2 can both result in close to zero MWP.

Table 10 Example 3 – LMP

UCED & profit				Individual generator profit maximation			
LMP	40	40		LMP	40	40	
Bus	1	2	Total	Bus	1	2	Total
P2 from SCUC		100.00		P2		0.00	
Gen2 Revenue		4000.00	4000.00	Gen2 Revenue		0.00	0.00
Gen2 cost from SCUC		9500.00	9500.00	Gen2 cost		0.00	0.00
Gen2 Profit		-5500.00	-5500.00	Gen2 Max Profit		0.00	0.00
uc2 from SCUC		1.00		uc2 from SCUC		0.00	
P1 from SCUC	130.00			P1	0.00		
Gen1 Revenue	5200.00		5200.00	Gen1 Reve	0.00		0.00
Gen1 cost from SCUC	5200.00		5200.00	Gen1 cost	0.00		0.00
Gen1 Profit	0.00		0.00	Gen1 Max	0.00		0.00
		Net profit	-5500.00			Net Max profit	0.00
						Gen uplift	5500.00
FTR sold 1->2 MW:	150	FTR payment	0				
SCUC flow MW:	100.00	**Congestion revenue**	0				
		Net FTR uplift	0				

Table 11 Example 3 -- CHP

UCED & profit				Individual generator profit maximation			
CHP	40	90		CHP	40	90	
Bus	1	2	Total	Bus	1	2	Total
P2 from SCUC		100.00		P2		0.00	
Gen2 Revenue		9000.00	9000.00	Gen2 Revenue		0.00	0.00
Gen2 cost from SCUC		9500.00	9500.00	Gen2 cost		0.00	0.00
Gen2 Profit		-500.00	-500.00	Gen2 Max Profit		0.00	0.00
uc2 from SCUC		1.00		uc2 from SCUC		0.00	
P1 from SCUC	130.00			P1	0.00		
Gen1 Revenue	5200.00		5200.00	Gen1 Reve	0.00		0.00
Gen1 cost from SCUC	5200.00		5200.00	Gen1 cost	0.00		0.00
Gen1 Profit	0.00		0.00	Gen1 Max	0.00		0.00
		Net profit	-500.00			Net Max profit	0.00
						Gen uplift	500.00
FTR sold 1->2 MW:	150	FTR payment	7500				
SCUC flow MW:	100.00	**Congestion revenue**	5000				
		Net FTR uplift	-2500				

Table 12 Example 3 – AIC

UCED & profit				Individual generator profit maximation			
AIC	40.00	95.00		AIC	40.00	95.00	
Bus	1.00	2.00	Total	Bus	1.00	2.00	Total
P2 from SCUC		100.00		P2		150.00	
Gen2 Revenue		9499.85	9499.85	Gen2 Revenue		14249.78	14249.78
Gen2 cost from SCUC		9500.00	9500.00	Gen2 cost		13500.00	13500.00
Gen2 Profit		-0.15	-0.15	Gen2 Max Profit		749.78	749.78
uc2 from SCUC		1.00		uc2 from SCUC		1.00	
P1 from SCUC	130.00			P1	0.00		
Gen1 Revenue	5200.00		5200.00	Gen1 Reve	0.00		0.00
Gen1 cost from SCUC	5200.00		5200.00	Gen1 cost	0.00		0.00
Gen1 Profit	0.00		0.00	Gen1 Max	0.00		0.00
		Net profit	-0.15			Net Max profit	749.78
						Gen uplift	749.93
FTR sold 1->2 MW:	150.00	FTR payment	8249.78				
SCUC flow MW:	100.00	**Congestion revenue**	5499.85				
		Net FTR uplift	-2749.93				

Here due to the non-zero MIP gap, Proposition 2 may not be satisfied. Some units may be backed down to zero in the AIC run and it may result in small residual MWP. But the MWP are negligible when the MIP gap is small.

Since CHP has the property of minimizing uplift, generator uplift under CHP is very small, only less than 16% of the uplift under LMP. The generator uplift under AIC and AIC2 can be higher or lower than the uplift under LMP. With the extended formulation, the uplift under AIC is lower than the uplift under AIC2. Under AIC, market procedures can be implemented to disincentivize resources from deviating from the dispatch.

The FTR uplift is zero under LMP. It's about 2~6% under CHP and less than 2.5% under AIC and AIC2. The market needs to have procedures to allocate this part of the imbalance.

Generator profit mostly increases under CHP. But it may decrease under the scenario when offline units are partially committed to reduce their opportunity cost. The total generator profit of all cases increased between 5~14% under AIC and AIC2 compared to the profit under LMP.

Table 14 shows the solving time on a Linux server using an Intel Haswell processor @ 2.5 GHz, 512GB RAM, 32 sockets per CPU, 1 core per socket, 1 thread per core. The time to solve CHP can be very long even if the extended formulation is applied to a subset of resources to solve LP relaxation solution under the iterative approach in [3]. However, the CHP solution does not depend on the SCUC solution. It can be solved in parallel with the SCUC solution.

In AIC, the uncommitted intervals with $p_{gt}^{*} = 0$ are applied with P-cut to set $p_{gt}^{AIC-max} = 0$. The LP relaxation with extended formulation can solve much faster, less than 581s for all cases. AIC2 solves LP relaxation on the 3-binary formulation with added P-Cut. It can solve in less than 184s.

The 3-binary formulation can be further improved with added binary cuts based on the SCUC binary solutions and applied p-cuts on all resources with $p_{gt}^{*} > 0$. A flow cut also needs to be added [9]. It may have higher uplift compared to AIC solved with the extended formulation. But it has the advantage of fast solving time and may totally remove MWP even under sub-optimal SCUC solution. The results from this approach are shown on table 13 and 14 under AIC34.

IV. Conclusion

In this paper, a unified approach to solving convex hull pricing (CHP) and average incremental cost (AIC) pricing problems is introduced. Using similar CHP-Primal formulation with valid cuts on individual resource maximum limits, AIC price can also be solved with AIC-Primal approach through LP relaxation of the adjusted UCED problem. The pros and cons of the prices are beyond the discussion of the paper. A unified consistent solution method can make it easy to compare and understand the pricing outcomes. Cases studies on small systems and MISO cases with prevailing unit commitment constraints are presented to show the effectiveness of the method.

Table 13 MWP, Uplift and FTR Uplift on MISO cases (ε=0.0001)

Case#	1	2	3	4	5	6	7
MWP (percentage relative to LMP MWP)							
LMP	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
CHP	18.23%	11.36%	15.76%	26.55%	15.57%	1.64%	6.34%
AIC	0.00%	0.02%	0.05%	0.16%	0.02%	0.22%	0.00%
AIC2	0.00%	0.01%	0.05%	0.16%	0.02%	0.20%	0.00%
AIC34	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Gen Uplift (percentage relative to LMP Gen profit)							
LMP	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
CHP	9.00%	15.28%	9.72%	5.67%	15.90%	6.20%	5.65%
AIC	106.85%	102.80%	131.72%	79.16%	111.12%	72.32%	90.75%
AIC2	108.29%	112.72%	138.28%	83.33%	113.45%	74.83%	91.08%
AIC34	104.17%	101.58%	139.98%	92.88%	111.50%	94.62%	102.51%
FTR Uplift (percentage relative to LMP Gen profit)							
LMP	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
CHP	1.97%	4.01%	5.76%	1.19%	4.59%	1.92%	3.56%
AIC	0.20%	1.02%	3.34%	0.85%	1.18%	1.20%	2.60%
AIC2	0.20%	1.06%	3.31%	0.85%	1.22%	1.19%	2.59%
AIC34	0.03%	0.19%	3.92%	0.41%	0.03%	0.32%	1.09%
Gen Profit (percentage relative to LMP Gen profit)							
LMP	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
CHP	89.51%	104.13%	100.11%	102.37%	104.42%	104.57%	101.33%
AIC	105.44%	110.18%	108.24%	108.56%	112.71%	107.52%	111.57%
AIC2	105.48%	111.26%	109.50%	108.46%	113.61%	107.73%	113.21%
AIC34	104.61%	107.02%	111.37%	107.21%	111.64%	106.63%	110.57%

Table 14 Solving time

Solving Time (s)							
SCUC time	305	2277	3809	1533	2566	368	6777
LMP	negligible						
CHP	1208	6671	7552	5329	10500	4832	5675
AIC	344	581	337	405	456	387	344
AIC2	153	149	184	91	124	90	98
AIC34	98	103	118	101	129	103	102

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