Affine Decision Rule Approximation to Immunize against Demand Response Uncertainty in Smart Grids' Capacity Planning

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Abstract

Generation expansion planning (GEP) is a classical problem that determines an optimal investment plan for existing and future electricity generation technologies. GEP is a computationally challenging problem, as it typically corresponds to a very large-scale problem that contains several sources of uncertainties. With the advent of demand response (DR) as a reserved capacity in modern smart power systems, recent versions of GEP problems model DR as an alternative for the expansion of generation, transmission and distribution networks. This adds extra uncertainties, since the availability of this resource is not known at the planning phase.In this paper, we model demand response uncertainty in a large-scale multi-commodity energy model, called ETEM, to address the GEP problem. The resulting model takes the form of an intractable multi-period adjustable robust problem which can be conservatively approximated using affine decision rules. To tackle instances of realistic size, we propose a Bender's decomposition scheme that exploits valid inequalities and favors Pareto robustly optimal solutions at each iteration. The performance of our new robust ETEM is evaluated in a realistic case study that surveys the energy system of the Swiss "Arc Lémanique" region. Our results show that an adjustable robust capacity expansion strategy can potentially reduce the expected total cost of the energy system by as much as 33% compared to a deterministic approach when accounting for electricity shortage penalties Moreover, an adjustable procurement strategy can be responsible for a 9 billion Swiss francs cost reduction compared to a naive static robust strategy. Finally, the proposed decomposition scheme improves the run time of the solution algorithm by 31% compared to the traditional Bender's decomposition.

Keywords : Multi-period adaptive robust optimization, Affine decision rule, Bottom-up energy model, Demand response

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1 Introduction

Generation expansion planning (GEP) is a classical energy problem that aims at determining the required power capacity to satisfy demand over a long-term horizon at minimum cost, while satisfying economic, environmental and technical constraints (see Koltsaklis and Dagoumas 2018, for a recent review). For this, one must consider existing and future electricity generation technologies to determine an optimal investment and retirement plan for the power sector. Computationally speaking, in a deterministic setting, the GEP problem is a large-scale optimization problem that typically accounts for more than hundreds of thousands of decision variables and constraints in order to identify realistic solutions. Motivated by advances in the structure of smart grids, recent versions of GEP problems have become even more complex as they attempt to model the notion of demand response (DR) (see e.g. Babonneau et al. 2017, Lohmann and Rebennack 2017). DR can be defined as the shifting of demand by consumers, in response to supply-side incentives offered at times of high wholesale market prices or when system reliability is jeopardized (2006). Development of sophisticated data-gathering devices such as smart meters and voltage sensors, as well as increasing penetration of flexible loads such as electric vehicles (EV) and heating pumps, provide the technical possibility of implementing DR programs through utilities. In the wholesale capacity market, the participants bid on providing DR resources as an alternative for expensive generation and transmission expansion strategies. DR resources can drastically reduce marginal production costs, as they are substitutes for technologies that help meet peak loads. However, including DR in the GEP problem is challenging. Indeed, DR is a price-sensitive resource, whose availability is not fully known, especially in the capacity expansion planning phase. As the planning horizon in a GEP problem is typically several decades, any planning or prediction of the availability of DR resources will be affected by two sources of uncertainties: i) errors in the prediction of total demand; and ii) changes in the response behavior. Failure to properly consider DR uncertainty (DRU) is likely to lead to situations in which there are insufficient capacity resources to satisfy realized demands.

Robust Optimization (RO) and Stochastic Programming (SP) are the two main methodologies to address uncertainty. The underlying assumption in SP is that the probability distribution of an uncertain parameter is known (Birge and Louveaux 2011). Conversely, RO does not assume information about the distribution of an uncertain parameter, but instead assumes that uncertain parameters lie in a user-defined uncertainty set. RO aims at finding solutions that are immunized to all perturbations of the uncertain parameters in the uncertainty set (Bertsimas et al. 2011). In multi-stage versions of SP and RO, it is assumed that decisions are taken at different points of time. The simplest form, known respectively as two-stage SP or Adjustable RO (ARO), considers *here-and-now* decisions that need to be made before having any information about the uncertain parameters and *wait-and-see* decisions that can be adjusted according to the realized uncertain parameters at a later time. It is generally known that ARO (Ben-Tal et al. 2004) provides better optimal solutions than RO, since it has more flexibility to adjust the decisions with respect to the uncertain parameters. However, the better performance comes at the price of problem intractability. To circumvent this issue, Ben-Tal et al. (2004) propose using affine decision rules for the delayed decisions, a technique also referred as the affinely adjustable robust counterpart (AARC). Following Ben-Tal et al. (2004), other types of decision rules have been proposed for ARO problems (Bertsimas and Georghiou 2018, Georghiou et al. 2020). For a review of the ARO approach, we refer the reader to Yanıkoğlu et al. (2019).

A number of different multi-stage robust or stochastic approaches have been previously used in the literature to address uncertainties in GEP problems (Dehghan et al. 2014, Mejía-Giraldo and McCalley 2014, Domínguez et al. 2016, Amjady et al. 2018, Baringo and Baringo 2018, Han et al. 2018, Zou et al. 2018). Because of the inherent computational challenges that emerge with these models, there should be a compromise between the level of details and the size of the problem. For example, Han et al. (2018) propose a two-stage stochastic program that only considers a single-period planning horizon, while the multi-stage model proposed in Domínguez et al. (2016) is only implemented in a case study involving four periods. It therefore appears that tackling uncertainty in long-term industrial-size GEP problems is out of reach for currently available methods. Improving the computational efficiency of algorithms for solving such problems is also crucial in helping to understand better the interactions between demand and supply side, as such studies can require iterating through the resolution of a number of GEP problems to identify market equilibria (see Babonneau et al. 2016, 2020, for examples of this approach).

In this paper, we propose a numerical method for addressing DRU on a realistic large-scale GEP problem. To do so, we use the Energy-Technology-Environment Model (ETEM) proposed in Babonneau et al. (2017) to model the market mechanism that matches flexible load and supply, where we introduce for the first time DRU as an implementation error of the DR decision variable. We formulate the problem as a robust multi-period linear program. In a first stage, the policy-maker decides how to invest in the capacity of each generation technology and plans for a target DR for each future time periods. Then, periodically, depending on the actual level of contribution in DR programs for this period, the planner decides on optimal energy procurement based on available resources. We evaluate the performance of the proposed *robust ETEM* model on a real-world case study based on the energy system of the "Arc Lémanique" region in Switzerland. To solve the problem, we derive a Robust Multi-Period Conservative Approximation (RMPCA) of the problem, and develop a Bender's decomposition algorithm (inspired from Ardestani-Jaafari and Delage 2018) to solve it.

Overall, the contributions of this paper can be summarized as follows:

1. We introduce DRU in a GEP problem, addressed using the ETEM energy model, and propose a robust multi-period linear program in which the procurement decisions are adjustable to the actual DR. The approach that we propose is flexible in the sense that, with little modifications, it could accommodate other sources of uncertainties in GEP problems such as uncertainty of intermittent energy resources.

2. We propose, for the first time, to seek Pareto Robustly Optimal (PRO) solutions (see Iancu and Trichakis 2014) of the master problem in a Bender's decomposition algorithm in order to accelerate convergence. When combined with the valid inequalities proposed in Ardestani-Jaafari and Delage (2017), solution time is improved by 31% on average in our randomly generated instances. Note that this idea differs from the idea of Pareto optimal cut, introduced by Magnanti and Wong (1981), in the way that the latter seeks a Pareto optimal solution among multiple optimal solutions of the sub-problems. In contrast, we seek a PRO solution in the master problem, i.e., a solution that is guaranteed not to be dominated by any other optimal solutions in terms of its constraint margin profile.

3. Compared to previous work in the literature, we solve an adjustable robust multi-commodity GEP problem whose size, in deterministic form, is two orders of magnitudes larger than the largest instance addressed in previous case studies. Moreover, our case study confirms in a simulation that adjustable robust policies can significantly improve (i.e., around 33%) the expected total cost of the energy system compared to the solution of the deterministic version when accounting for electricity shortage penalties. Furthermore, it shows that the flexibility of energy procurement decisions can be responsible for a reduction of 9 billion Swiss francs (CHF) in expected total expansion planning costs.

The remainder of the paper is organized as follows. In the next section, we present an overview of different approaches to model DRU, and summarize studies that use a multi-stage approach to model uncertainties in GEP problems. In Section 3, we introduce briefly the ETEM model. In Section 4, we present how we have modeled DRU in ETEM and derive the multi-period robust optimization problem as well as its conservative approximation. In Section 5, the Bender's decomposition is detailed. Section 6 presents the case study and provides numerical results. Finally, Section 7 provides concluding remarks.

2 Literature Review

In this section, we first review papers that use a multi-period approach to model a robust or stochastic GEP problem. We specifically summarize their solution method and the size of the instances they solve. Usually, the size of a GEP problem is dependent on the number of time periods, load duration curve (LDC) steps, technologies, and commodities. Hence, we will summarize the size of studied instances with the product of these quantities, referred as the Deterministic Size Indicator (DSI) index. In a second part, we review recently proposed strategies to model DRU in different energy related problems. Finally, we review the literature of ETEM model.

Han et al. (2018) develop a two-stage stochastic program to model the uncertainty of load demand and wind output in a GEP problem. Using affine decision rules, they reformulate the problem as a deterministic second-order cone optimization problem. They solve an instance with 1 period, 5 LDC steps, and 32 technologies to test their algorithm (DSI=160). Mejía-Giraldo and McCalley (2014) propose an adjustable robust optimization framework to address the uncertainty of fuel price, demand, and transmission capacities in a GEP problem. In this setting, investment decisions are set as affine function of fuel price, whereas voltage-angle (decision) is parameterized as an affine function of demand and transmission capacities. They reformulate the problem as a LP and test it on a simplified version of the US power system with 20 periods, 3 LDC steps, and 13 technologies (DSI=780). Domínguez et al. (2016) use linear decision rules to reformulate a multi-stage GEP problem and cast it as a tractable LP. They solve a GEP problem with 4 decision periods, 18 technologies and 100 LDC steps (DSI=7200). Zou et al. (2018) propose a partially adaptive multi-stage stochastic GEP problem. In this setting, the capacity expansion plan is adaptive to the uncertain parameter up to a certain point. Fuel price and demand are two sources of uncertainties in this study. They test their algorithm on a case study with 10 periods, 3 LDC steps, and 6 technologies (DSI=180). Dehghan et al. (2014) develop a two-stage robust optimization problem in which the load demand and investment cost are uncertain. They solve their problem, using a cutting plane method. The case study is a 10-period planning of a network with 120 technologies and 3 LDC steps (DSI=3600). None of the above studies model neither DR nor DRU. In addition, all of them are a single-commodity GEP problem, meaning that they only model the electricity network and account for an aggregated electricity demand over the planning horizon. By contrast, in a multi-commodity energy model such as ETEM, one also models the interactions of the electricity sector with other sectors of the energy system (e.g., heat, transportation sectors), as well as the demands for final energy services (such as heat and lighting). This increases the size of the deterministic problem. In this paper, we solve a case study with 10 decision-making periods, 12 LDC steps, 142 technologies, and 57 energy commodities (DSI=971,280).

DR is also modeled in other power system planning problems such as unit commitment, transmission or distribution expansion planning, and whole-sale electricity market problems. In general, there are two strategies one can follow to implement DR programs: dispatchable and non-dispatchable. In the former, the utility directly controls the timing of some loads of the customers who voluntarily participate in the program. More precisely, the utility cuts down the consumption during peak periods to ensure system reliability, and in return, remunerates participants with annual payments. In non-dispatchable strategies, the utility sends to customers a price signal with the purpose of flattening the demand curve. Automatic price-responsive devices adjust the timing of the consumption. While developing an optimal consumption schedule is the focus of some research (Liu et al. 2019), another stream studies the conditions under which the dynamic price signals improve social welfare (see, e.g., Adelman and Uçkun 2019, Venizelou et al. 2018). Elaboration on the different strategies to implement DR and an overview on the success of these programs in the US electricity market is presented in Shariatzadeh et al. (2015).

The effect of DR on the wholesale and capacity market is another stream of research. Vatani et al. (2017) introduce DR to a capacity market problem in which the cost-minimizer capacity planner trades off between new generation and transmission capacity expansion and DR expenses. Arasteh et al. (2015) model a trade-off between DR expenses and distribution expansion costs. Zhang and Zhang (2019) model DR as an alternative procurement strategy for an electricity retailer. Namely, the electricity retailer must procure energy in a day-ahead wholesale market. Given that the prices in this market are highly uncertain, DR is introduced as a leverage to manage the retailer's risk. In this approach, the retailer solves its problem to determine a procurement strategy as well as DR incentive prices. Then, consumers respond to the signaled price in order to minimize their consumption cost. Babonneau et al. (2016) model DR in a capacity expansion planning considering distribution constraints. They develop a game framework between the retailer and many small consumers. The retailer solves a minimization model to determine his desired DR and capacity expansion plan. Then, using marginal cost of production, which corresponds to dual variables, he signals consumers to adjust their consumption. They argue that, if the price function, with which the retailer signals consumers, is convex with respect to both demand and required level of reserved capacity, their approach can be cast as a linear programming problem. Finally, Lohmann and Rebennack (2017) model DR in a long-term capacity expansion planning problem.

In all above research, DR was mostly modeled in a deterministic way. In other words, when DR was modeled using a price elastic demand curve, no uncertainty affected this curve. Alternatively, when it was modeled as a decision variable, no implementation error was affecting optimal DR decisions. However, due to a variety of reasons, including demand prediction errors and changes in response behavior, in practice DR necessarily ends up different from what the supply side expected. According to Chatterjee et al. (2018), a lack of DR forecasting and estimation tools is one of the most important barriers to wider adoption of DR. Aligned with this issue, a recent stream of research investigates the influence of DRU on its integration to the network. Li et al. (2015) model incentive-based DR as an alternative for transmission upgrade investments and consider consumer's bid for load reduction uncertainty. They develop a stochastic programming model and use Monte Carlo simulation and Bender's decomposition for resolution. He et al. (2019) develop a two-stage distributionally robust problem to model DRU in a distribution network expansion problem. Prices are modeled as a decision variable to optimally shape the desired demand response. However, the price elasticity is considered to be a random parameter. Moreover, distributed generation resources are also affected by uncertainty. They propose a column and constraint generation approach to solve the resulting problem. Zhao et al. (2013) model the uncertainty of DR and wind generation in a unit commitment problem. DR is modeled through a price-elastic demand curve with price uncertainty. They propose a three-stage robust optimization formulation. First stage decisions consider unit commitment planning, i.e., turning on and off the generators. Second stage decisions include the dispatch of electricity and are implemented after the actual wind power output is realized. Finally, in a third-stage the demand response is observed. Moreover, they propose a Bender's decomposition to solve the three-stage problem. An overview on the different techniques to model ancillary services in unit commitment problem is presented in Knueven et al. (2020). Asensio et al. (2018) model the DRU through a price-sensitive demand curve in a distribution network expansion problem and use a scenario-tree approach to solve the resulting stochastic programming model. Roveto et al. (2020) develop a data-driven distributionally robust optimization model of demand response auction, where the risk of the retailer is controlled using value-at-risk and conditional value-at-risk measures. They model DRU by considering DR bid uncertainty. In other words, the aggregator receives bids from consumers to reduce their consumption in a specific time. However, consumers might not actually reduce their consumption to this full amount. The aggregator aims to ensure a certain level of demand response procurement at minimum cost. Huang et al. (2020) develop a multi-stage stochastic programming model to optimally schedule power generation assets when the system operator has access to ancillary service market. In this paper, we model DRU in a multi-commodity energy planning model, called ETEM (Babonneau et al. 2017), to address a GEP problem. In ETEM, DR is modeled through a decision variable which is optimized by the supply-side. It is assumed that the supply-side can shape consumer's response behavior through different long-term DR programs. For the first time, we model DRU as an implementation error of the DR decision in ETEM, and develop an adaptive robust multi-period conservative approximation formulation of the problem.

ETEM first was formulated by Babonneau et al. (2012). Although the developed formulation is deterministic, they also provide a stochastic programming model with different scenarios for the electricity import cost. As an extension to the primary ETEM model, Babonneau et al. (2016) propose a linear approximation of distribution system and power flow constraints in ETEM. The new formulation, called ETEM-SG, accounts for provision of reserve capacities provided by demand response in smart grids. Babonneau et al. (2017) and Babonneau and Haurie (2019) propose robust versions of ETEM to address the uncertainty of investment costs and of availability of technologies and transmission capacity lines. In both papers, a static robust version of ETEM is developed and a relaxed version of the model is solved to reduce conservatism of the solution. Babonneau et al. (2020) use a robust version of ETEM to couple the distribution model with Wardrop and mean-field game equilibrium models that represent the charging behavior of electric vehicle owners. A static robust formulation of ETEM is considered to model the uncertainty about the demand response's feasible range, and over a single period horizon. Our paper could be seen as presenting a new and significant extension of ETEM that develops an adaptive and multi-period model that immunizes the energy system againts DR deviations.

3 The ETEM Energy Model

As proposed in Babonneau et al. (2012), ETEM is a long-term, bottom-up energy model cast as a linear programming problem. It is a member of the *MARKAL-TIMES* family of models, and represents the entire energy sector from primary resources to useful energy demands. ETEM is a demand-driven model in which the objective is to provide energy services at minimum cost. The planning horizon is typically more than 50 years and the model gives insights on both long-term strategic and short-term operational decisions. While the long-term decisions include capacity expansion and strategic targeting for a desired demand response, short-term decisions consist of energy procurement planning, which includes optimal generation, import, export and regional transmissions. ETEM can also account for technical, economic and market constraints (see Babonneau et al. 2017).

ETEM is based on the concept of a reference energy system (see Figure 1 for the depiction of the energy system of the Arc Lémanique region in Switzerland considered in our case study). The model considers different energy commodities (captured by vertical lines in the figure) namely "resources" (i.e., a mix of primary energy resources and some imported secondary energy forms), selected secondary forms (i.e., electricity and heat), and useful energy demands. More precisely, resources correspond to coal, oil products, gas, hydro, intermittent renewables (wind and solar), as well as other sources such as municipal solid waste and wood. Useful energy demands belong to four categories: industry, residential electricity, residential heat, and transportation. The boxes correspond to the different energy technologies, from generators to technologies providing energy services to end-users. In our implementation, we also add an energy resource (denoted "ExR") with a high cost that represents the cost of shortage in the system. A dummy technology (denoted "EXT") links this expensive resource to the useful demands (see Assumption 1 and following discussion for more details).

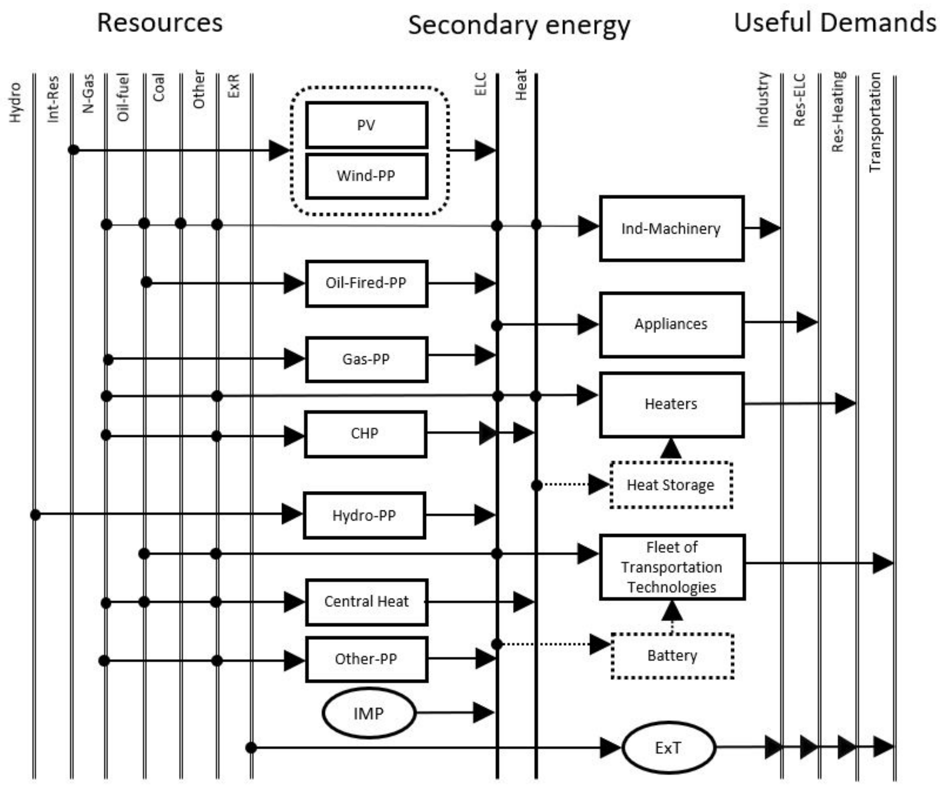


Figure 1: Arc Lémanique reference energy system, where Int-Res stands for intermittent resources (solar and wind), N-Gas for natural gas, Oil-fuel for processed oil products, Other for hydrogen and additional sources of energy (such as solid waste and wood and geothermal), ELC for electricity, RES for residential, PP for power plant, CHP for combined heat and power plant, IMP for electricity import, Ind-Machinery for industrial machinery, ExR for a dummy expensive resource added to avoid in-feasibility, ExT for a dummy expensive technology used to avoid infeasibility. Other-PP includes geothermal, fuel-cell and municipal waste power plants

The planning horizon in ETEM spans over several decades to simulate the long investment cycles typical of the energy sector. We denote by $t \in \mathbb{T} := \{1, \ldots, |\mathbb{T}|\}$ the index representing the decision-making periods. In addition, to capture short-term operational patterns of the demand load, each period is divided into smaller time-slices $s \in \mathbb{S}$, representing load duration curve steps. In ETEM, 12 time-slices are assumed ($\mathbb{S} = \{1, \cdots, 12\}$), which are partitioned among a set $\mathbb{J}$ of 3 typical seasons (winter, summer, and intermediate), and each day of these seasons is divided into 4 time slots (morning, midday, midnight, and night), as shown in Figure 2.

Before describing ETEM's mathematical formulation, it is worth introducing our nomenclature. Let $\mathbb{C}$ be the set of energy commodities and let $\mathbb{P}$ be the set of energy technologies. The set of indices $\mathbb{F}$ identifies the different input-output energy flows associated to each technology. For instance, in Figure 1, Combined Heat and Power (CHP) uses natural gas, solid waste and wood as input to

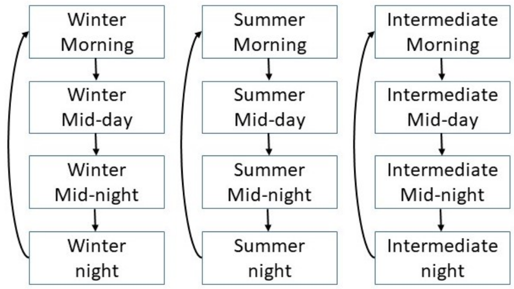


Figure 2: Sequence of time-slices

generate both electricity and heat. Finally, let $\mathbb{L}$ be a set of buses in different geographical zones. A full nomenclature is presented below.

Nomenclature

$t \in \mathbb{T}$ Index for time period

$s \in \mathbb{S}$ Index for time-slices

$p \in \mathbb{P}$ Index for technologies

$c \in \mathbb{C}$ Index for energy commodities

$c_s \in \mathbb{CS}$ Index for energy storage

$f \in \mathbb{F}$ Index for energy flows

$l \in \mathbb{L}$ Index for buses (geographical zones)

$j \in \mathbb{J}$ Index for seasons

$i \in \mathbb{I}$ Index for period-seasons (t, j)

$\mathbb{P}_c^C \subseteq \mathbb{P}$ Set of technologies consuming c

$\mathbb{P}_c^P \subseteq \mathbb{P}$ Set of technologies producing c

$\mathbb{P}^R \subseteq \mathbb{P}$ Set of intermittent technologies

$\mathbb{C}^I \subseteq \mathbb{C}$ Set of imported commodities

$\mathbb{C}^{\mathcal{D}} \subseteq \mathbb{C}$ Set of useful demands

$\mathbb{C}^{EX} \subseteq \mathbb{C}$ Set of exported commodities

$\mathbb{C}^{TR} \subseteq \mathbb{C}$ Set of transmitted commodities

$\mathbb{C}_f \subseteq \mathbb{C}$ Set of commodities linked to flow f

$\mathbb{C}^{\mathcal{G}} \subseteq \mathbb{C}$ Set of commodities with margin reserve

$\mathbb{S}^j \subseteq \mathbb{S}$ Set of time-slices s in season j

$\mathbb{S}^s \subseteq \mathbb{S}$ Set of successive time-slices of s

$\mathbb{S}^{\mathcal{G}} \subseteq \mathbb{S}$ Set of time-slices in peak period

$\mathbb{FI}_p \subseteq \mathbb{F}$ Set of inputs to technology p

$\mathbb{FO}_p \subseteq \mathbb{F}$ Set of outputs from technology p

$\alpha_{t,p}$ Investment cost

$\beta_{t,s,p}$ Capacity factor

η_c Network efficiency

$\eta^t_{f,f'}$ Technology efficiency

$\lambda''_{t,s,l,l',c}$ Transmission cost

$\lambda'_{t,s,c}$ Export cost

$\lambda_{t,s,c}$ Import cost

$\nu_{t,s,c}$ Maximum deviation from nominal demand response

$\nu_{t,p}$ Variable cost

$\Omega_{t,l,p}$ Available capacity of technology p

$\pi_{t,p}$ Fixed production cost

ρ Discount factor

θ^c_p Proportion of output c from technology p that can be used in peak period

$\Theta_{t,l,d}$ Annual final demand

$\upsilon_{t,s,c}$ Nominal demand response

$\varrho_{t,s,c}$ Required reserve for commodity $c \in \mathbb{C}^{\mathcal{G}}$

l_p Life duration of technology p

$\boldsymbol{C}_{t,l,p}$ Variable for new capacity addition

$\boldsymbol{E}_{t,s,l,c}$ Variable for export

$\boldsymbol{I}_{t,s,l,c}$ Variable for import

$\boldsymbol{P}_{t,s,l,p,c}$ Variable for activity of technology p

$\boldsymbol{T}_{t,s,l,l',c}$ Variable for regional transmission

$\boldsymbol{V}_{t,s,d}$ Variable for demand response.

Next, we present the main elements of ETEM. Interested readers can refer to Babonneau et al. (2017) for more details:

$$W^* := \min \sum_{t,l} \rho^t \cdot \left(\sum_p \alpha_{t,p} \boldsymbol{C}_{t,l,p} + \pi_{t,p} (\sum_{k=0}^{l_p-1} \boldsymbol{C}_{t-k,l,p} + \Omega_{t,l,p}) + \sum_{s,c} \nu_{t,p} \boldsymbol{P}_{t,s,l,p,c} + \right.$$
$$\left. \sum_{s,c} (\lambda_{t,s,c} \boldsymbol{I}_{t,s,l,c} - \lambda'_{t,s,c} \boldsymbol{E}_{t,s,l,c}) + \sum_{s,l',c} \lambda''_{t,s,l,l',c} \boldsymbol{T}_{t,s,l,l',c} \right), \tag{1a}$$

s.t.

$$(\sum_{p \in \mathbb{P}_c^P} \boldsymbol{P}_{t,s,l,p,c} + \boldsymbol{I}_{t,s,l,c}) \eta_c + \sum_{l' \neq l} (\eta_c \boldsymbol{T}_{t,s,l',l,c} - \boldsymbol{T}_{t,s,l,l',c}) \geq \sum_{p \in \mathbb{P}_c^C} \boldsymbol{P}_{t,s,l,p,c} + \boldsymbol{E}_{t,s,l,c}$$
$$\forall\, t,s,l,c \in \mathbb{C}/\mathbb{C}^{\mathcal{D}} \tag{1b}$$

$$\Big(\sum_{l,p \in \mathbb{P}^c} \theta_p^c \cdot \beta_{t,s,p} (\sum_{k=0}^{l_p-1} \boldsymbol{C}_{t-k,l,p} + \Omega_{t,l,p}) + \sum_{l,p \in \mathbb{P}_c^P/\mathbb{P}_c} \theta_p^c \cdot \boldsymbol{P}_{t,s,l,p,c} + \boldsymbol{I}_{t,s,l,c} \Big) \cdot \varrho_{t,s,c}$$
$$\geq \sum_{l,p \in \mathbb{P}_c^C} \boldsymbol{P}_{t,s,l,p,c} + \boldsymbol{E}_{t,s,l,c} \quad \forall t, s \in \mathbb{S}^{\mathcal{G}}, c \in \mathbb{C}^{\mathcal{G}} \tag{1c}$$

$$\eta_c \sum_{p \in \mathbb{P}^{P_c}} \boldsymbol{P}_{t,s,l,p,c} = \sum_{p \in \mathbb{P}^{C_c}, s' \in \mathbb{S}^s} \boldsymbol{P}_{t,s',l,p,c} \quad \forall t,s,l,c \in \mathbb{CS} \tag{1d}$$

$$\sum_{p \in \mathbb{P}_c^P} \boldsymbol{P}_{t,s,l,p,c} \geq \Theta_{t,l,c} \boldsymbol{V}_{t,s,c} \qquad \forall t,s,l,c \in \mathbb{C}^{\mathcal{D}} \tag{1e}$$

$$\upsilon_{t,s,c}(1 - \nu_{t,s,c}) \leq \boldsymbol{V}_{t,s,c} \leq \upsilon_{t,s,c}(1 + \nu_{t,s,c}) \quad \forall t,s,c \in \mathbb{C}^{\mathcal{D}} \tag{1f}$$

$$\sum_{s \in \mathcal{S}_j} \boldsymbol{V}_{t,s,c} = \sum_{s \in \mathcal{S}_j} \upsilon_{t,s,c} \qquad \forall t, c \in \mathbb{C}^{\mathcal{D}}, j \in \mathbb{J} \tag{1g}$$

$$\sum_{c: p \in \mathbb{P}_c^P} \boldsymbol{P}_{t,s,l,p,c} \leq \beta_{t,s,p} \left(\sum_{k=0}^{l_p-1} \boldsymbol{C}_{t-k,l,p} + \Omega_{t,l,p} \right) \qquad \forall t,s,l,p \notin \mathbb{P}^R, \tag{1h}$$

$$\sum_{c \in \mathbb{C}_m^p} \boldsymbol{P}_{t,s,l,p,c} = \beta_{t,s,p} \left(\sum_{k=0}^{l_p-1} \boldsymbol{C}_{t-k,l,p} + \Omega_{t,l,p} \right) \qquad \forall t,s,l,p \in \mathbb{P}^R \tag{1i}$$

$$\sum_{c \in \mathbb{C}_{f'}} \boldsymbol{P}_{t,s,l,p,c} = \eta_{f,f'}^t \cdot \sum_{c \in \mathbb{C}_f} \boldsymbol{P}_{t,s,l,p,c} \quad \forall f \in \mathbb{FI}_p, f' \in \mathbb{FO}_p, t,s,l,p \tag{1j}$$

$$LB_{t,j,c} \leq \sum_{s \in \mathbb{S}^j, l, p \in \mathbb{P}_c^P} \boldsymbol{P}_{t,s,l,p,c} \leq UB_{t,j,c} \quad \forall t,j,c \in \mathbb{C} \tag{1k}$$

$$LB_{t,j,c} \leq \sum_{l, s \in \mathbb{S}^j} \boldsymbol{I}_{t,s,l,c} \leq UB_{t,j,c} \quad \forall t,j,c \in \mathbb{C}^I \tag{1l}$$

$$\sum_{s \in \mathbb{S}^j, l} (\sum_{p \in \mathbb{P}_c^P} \boldsymbol{P}_{t,s,l,p,c} + \boldsymbol{I}_{t,s,l,c} - \sum_{p \in \mathbb{P}_c^C} \boldsymbol{P}_{t,s,l,p,c} - \boldsymbol{E}_{t,s,l,c}) \leq UB_{t,j,c} \quad \forall t,j,c \in \mathbb{C} \tag{1m}$$

$$(\boldsymbol{P}, \boldsymbol{I}, \boldsymbol{E}, \boldsymbol{T}) \in \mathcal{Y} \ , \ \boldsymbol{C} \in \mathcal{X} \ , \ \boldsymbol{P} \geq 0, \boldsymbol{I} \geq 0, \boldsymbol{E} \geq 0, \boldsymbol{T} \geq 0, \boldsymbol{C} \geq 0, \boldsymbol{V} \geq 0 \, . \tag{1n}$$

The objective function (1a) minimizes a discounted sum of all costs of the system over all regions ($l \in \mathbb{L}$) and time periods ($t \in \mathbb{T}$), where parameters $\alpha_{t,p}$, $\nu_{t,p}$, $\lambda_{t,s,c}$, $\lambda'_{t,s,c}$, and $\lambda''_{t,s,l,l',c}$ are unit costs of investment, fixed, variable, import, export, and regional transmission respectively, while $\pi_{t,p}$ is the fixed production cost per unit of capacity.

Constraint (1b) is a commodity balance constraint. It ensures that the regional procurement of each energy commodity c is greater or equal than its total consumption at each period t and time-slice s. Specifically, the left-hand side of this constraint consists of i) total production of commodity c in region l by all technologies producing it ($\mathbb{P}_c^P$) ii) import of commodity c, and, iii) net transmission of commodity c into region l. Parameter η_c is the network efficiency with respect to commodity c, e.g., the efficiency of electricity transmission lines. On the right-hand side, the consumption is equal to the total consumption by technologies consuming the commodity, i.e., $\mathbb{P}_c^C$, added to the amount of commodity that is exported.

Constraint (1c) introduces a safety margin in procurement of commodity $c \in \mathbb{C}^{\mathcal{G}}$, mostly electricity, during peak time-slices $s \in \mathbb{S}^{\mathcal{G}}$, to protect against random events not explicitly represented in the model. Parameter $\varrho_{t,s,c} \in [0,\ 1]$ represents the fraction of reserved capacity needed to ensure covering the peak load. The left-hand side of this constraint models the maximum amount of commodity $c \in \mathbb{C}^{\mathcal{G}}$ that can be procured in period t and time-slice s. This amount is equal to the sum of i) the maximum production capacity of commodity c by technologies that produce it as their main output ($\mathbb{P}_c$), ii) the production of commodity c by technologies that produce c as their by-product of their main activity, and iii) the import of commodity c. The left-hand side is the total consumption similar to constraint (1b). Parameter θ_p^c is the proportion of technology production that can be used during the peak period. Constraint (1d) models the balance of energy storage between consecutive time-slices. Amount of storage at time-slice c can be consumed at subsequent time-slice s'. The notion of subsequency of time-slices is presented in Figure 2. Constraints (1e) - (1g) are the key constraints that model the use of demand response. Parameter $\Theta_{t,l,c}$ is the total demand of service $c \in \mathbb{C}^{\mathcal{D}}$ in the period t and zone l. Variable $\boldsymbol{V}_{t,s,c}$ is the demand response which optimally distributes the total demand of period t into all time-slices s inside period t. Constraint (1f) limits the demand response to vary between an interval around the nominal value, i.e., $\upsilon_{t,s,c}$. In addition, the sum of the demand response must be equal to the sum of the nominal values in each season, according to constraint (1g).

Constraint (1h) limits the maximum production or consumption of each technology to the available capacity of that technology. The parameter $\beta_{t,s,p}$ is the capacity factor. The capacity factor of technology p is defined as the average power generated divided by the rated peak power. This simply represents the fraction of the total capacity which is available at each time-slice. In addition, since renewable generation is usually given priority in dispatch over conventional forms of generation, their production is imposed to take its maximum production capacity in constraint (1i). Constraint (1j) models the efficiency of technology p. In ETEM, the efficiency of a technology is modeled through the annual parameter $\eta^t_{f,f'}$ that links each output flow $f' \in \mathbb{FO}_p$ to an input flow $f \in \mathbb{FI}_p$. The constraint ensures that the sum of all output energy commodity c which is linked to output flow f', i.e., $\mathbb{C}_{f'}$, is equal to a fraction of the sum of all input energy commodities c which is linked to input flow f, i.e., $\mathbb{C}_f$.

We note that constraints (1k) and (1l) impose upper and lower bounds for seasonal production of technology p in region l, and for seasonal import of energy commodity c in region l. Moreover, in constraint (1m) the seasonal procurement of commodity c, over all regions in $\mathbb{L}$, is imposed an upper bound. This departs slightly from the original ETEM formulation, which only imposed annual bounds but is reasonable and enables the decomposition scheme presented in Section 5. Finally, space $\mathcal{X}$ and $\mathcal{Y}$ represent other operational, technical and economical constraints that defines a desirable space for capacity and procurement decisions. Moreover, $\mathcal{X}$ and $\mathcal{Y}$ contain constraints that defines the structure of the energy network. In particular, these constraints enforce that energy productions, $\boldsymbol{P}_{t,s,l,p,c}$, that do not exist in the energy network to be zero. Since these constraints do not affect our analysis, we omit to report them and refer the reader to Babonneau et al. (2017) for a complete list of constraints.

4 A Demand Response Robust Version of ETEM

The ETEM model presented in Section 3 optimizes the future evolution of the energy system by assuming full information. However, in reality, there is always uncertainty. In particular, demand response is an important source of uncertainty. ETEM assumes that demand response is completely controlled by the supply side. The latter can indeed influence the demand load using DR programs, but one must expect some realized deviations from the plans. Such deviations can either be caused by differences in the actual overall daily demand loads, as time evolves over a 50-year horizon, or by the level of participation and compliance of consumers in DR programs. In this paper, we model both types of uncertainties as implementation errors of the planned demand response $\boldsymbol{V}$ and we immunize ETEM against such error (see Ben-Tal et al. 2015, for a seminal work on robust optimization with implementation error). Specifically, given a set $\mathbb{C}^{\mathcal{U}} \subseteq \mathbb{C}^{\mathcal{D}}$ of useful demand, such as residential electricity, with error-prone demand response, the demand $\Theta_{t,l,c}\boldsymbol{V}_{t,s,c}$ for time period $t \in \mathbb{T}$, time-slice $s \in \mathbb{S}$, region $l \in \mathbb{L}$, and commodity $c \in \mathbb{C}^{\mathcal{U}}$, is replaced with $\Theta_{t,l,c}\big(\boldsymbol{V}_{t,s,c} + \boldsymbol{\delta}_{t,s,c}\big)$, where $\boldsymbol{\delta}_{t,s,c}$ captures the Relative Demand Response Deviation (RDRD) from the planned response. In a general presentation, we will consider the vector of relative deviations $\boldsymbol{\delta} \in \mathbb{R}^d$, to be the vector that is obtained from arranging the parameters $\boldsymbol{\delta}_{t,s,c}$ for all $t \in \mathbb{T}, s \in \mathbb{S}$ and $c \in \mathbb{C}^{\mathcal{U}}$ into a vector. For simplicity of exposure, we assume that the vector of each seasonal subset of RDRDs lies in a scaled budgeted uncertainty set (as introduced in Bertsimas and Sim 2004) defined as:

$$\Delta = \left\{ \boldsymbol{\delta} \in \mathbb{R}^d \middle| \exists \boldsymbol{\zeta} \in [-1,\,1]^d,\, \boldsymbol{\delta}_{t,s,c} = \beta_{t,s,c}\boldsymbol{\zeta}_{t,s,c},\ \forall t,s,c \in \mathbb{C}^U\ , \sum_{s\in\mathbb{S}^j}\sum_{c\in\mathbb{C}^U} |\boldsymbol{\zeta}_{t,s,c}| \leq \Gamma_{t,j}\ \forall t,j \in \mathbb{J} \right\}, \quad (2)$$

where $d := |\mathbb{T}| \cdot |\mathbb{S}| \cdot |\mathbb{C}^{\mathcal{U}}|$, $\beta_{t,s,c}$ captures the maximum possible RDRD , while $\Gamma_{t,j}$ is a seasonal budget that represents the fact that we expect a maximum of $\Gamma_{t,j}$ RDRDs to take on their extreme values in season j of time period t. The idea of decomposing the uncertainty information structure over each season has three important advantages. First, from a numerical perspective, it will help by enabling us to employ decomposition schemes based on Bender's decomposition. Second, it is also attractive from a statistical perspective as it allows us to calibrate the size of each seasonal uncertainty sets using seasonal data which is usually readily available, while data over joint realizations over many seasons can be scarce. Finally, imposing uncertainty budgets on subsets of perturbed parameters is known to lead to less conservative solutions in robust optimization as it enforces the worst-case scenario to distribute the damage over parameters of different subsets instead of focusing on a single one (see Bertsimas and Thiele 2006, where similar techniques are used).

In what follows, we present first, in Section 4.1, how we modify ETEM in order to identify capacity and demand response plans that are immunized against RDRDs. The modified model is cast as a robust multi-period linear program. Unfortunately, this class of problem is known to be generally intractable. To address this issue, Section 4.2 presents a tractable conservative approximation that can be reformulated as a large-scale linear program.

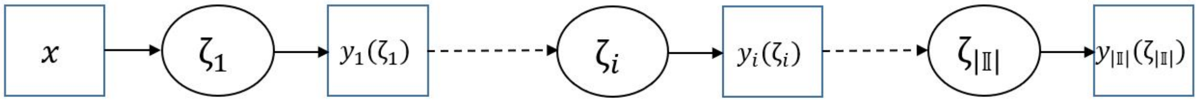


Figure 3: Sequence of decisions and uncertainty observations in our multi-period problem

4.1 A Robust Multi-Period Linear Programming Formulation

With the introduction of the RDRDs in ETEM, the problem has the potential of turning into a multi-stage decision-making problem where different decisions are allowed to depend on the observed realization of RDRDs . Indeed, we will assume that in the first stage the energy network planner decides on the capacity expansion of the system (variable $\boldsymbol{C}$) and the planned demand response (variable $\boldsymbol{V}$). Then, in later stages, on a season-by-season basis, the seasonal RDRDs are revealed to the planner who responds by deciding on the optimal energy production, energy transfers between regions, and energy imports and exports. To simplify the analysis of such a multi-stage model, we will further assume that the planner's response only depends on the current seasonal RDRDs instead of reacting to all the observations made since the first season. In fact, given the proposed formulation and choice of first stage decisions, this is without loss of generality since there always exists an optimal policy with this structure. Such an approach to multi-stage modeling was referred to, in Ardestani-Jaafari and Delage (2018), as a multi-period model; hence we adopt this naming convention throughout.

To be mathematically precise and help with presentation, decision variable $\boldsymbol{x} \in \mathbb{R}^m$, with $m = |\mathbb{T}| \cdot |\mathbb{L}| \cdot |\mathbb{P}| + |\mathbb{T}| \cdot |\mathbb{S}| \cdot |\mathbb{C}^{\mathcal{D}}|$, will capture the vector of all first stage decisions $(\boldsymbol{C}, \boldsymbol{V})$ in the demand response robust version of ETEM. Furthermore, for each time period t and season j, let $i \in \mathbb{I}$ denote the seasonal period (t, j), and consider that $\boldsymbol{y}_i \in \mathbb{R}^{k_i}$, with $k_i = |\mathbb{S}^j| \cdot |\mathbb{L}| \cdot |\mathbb{P}| \cdot |\mathbb{C}| + |\mathbb{S}^j| \cdot |\mathbb{L}| \cdot |\mathbb{C}^I| + |\mathbb{S}^j| \cdot |\mathbb{L}| \cdot |\mathbb{C}^{EX}| + |\mathbb{S}^j| \cdot |\mathbb{L}|^2 \cdot |\mathbb{C}^{TR}|$, captures all adaptive "procurement" decisions $(\boldsymbol{P}_{t,s,l,p,c}, \boldsymbol{I}_{t,s,l,c\in\mathbb{C}^I}, \boldsymbol{E}_{t,s,l,c\in\mathbb{C}^{EX}}, \boldsymbol{T}_{t,s,l,l',c\in\mathbb{C}^{TR}})_{t,s\in\mathbb{S}_j,l\in\mathbb{L},l'\in\mathbb{L}}$ implemented in season j and time period t. For simplicity, we will refer to seasonal periods as $i \in \mathbb{I}$ instead of $(t, j) \in \mathbb{T} \times \mathbb{J}$ and consider $\mathbb{A}_i := \{(t, s)\}_{s\in\mathbb{S}_j}$ as the set of all (t, s) pairs associated to seasonal period i. Finally, we refer to $\boldsymbol{\zeta}_i \in [-1, 1]^{d_i}$ as the vector of seasonal RDRDs perturbances $\{\boldsymbol{\zeta}_{t,s,c}\}_{(t,s)\in\mathbb{A}_i,c\in\mathbb{C}^{\mathcal{U}}}$.

The chronology of our multi-period decision-making model is depicted in Figure 3. In particular, this model can be shown to take the following form:

$$\min_{\boldsymbol{x},\{\boldsymbol{y}_i\}_{i=1}^{|\mathbb{I}|}} \max_{\{\boldsymbol{\zeta}_i\in\mathcal{Z}_i\}_{i=1}^{|\mathbb{I}|}} f^\top \boldsymbol{x} + \sum_{i=1}^{|\mathbb{I}|} h_i^\top \boldsymbol{y}_i(\boldsymbol{\zeta}_i) \tag{3a}$$

$$\text{s.t.} \quad A_i\boldsymbol{x} + B_i\boldsymbol{y}_i(\boldsymbol{\zeta}_i) \leq b_i + C_i\boldsymbol{\zeta}_i \quad \forall \boldsymbol{\zeta}_i \in \mathcal{Z}_i(\Gamma_i),\ \forall i \in \{1...|\mathbb{I}|\} \tag{3b}$$

$$D\boldsymbol{x} \leq e\,, \tag{3c}$$

where constraint (3c) describes the polyhedral feasible set for $\boldsymbol{C}$ and $\boldsymbol{V}$ based on $\mathcal{X}$ and constraints (1f) and (1g), while constraint (3b) describes for each i the constraints imposed on $\{\boldsymbol{P}_{t,s,l,p,c}, \boldsymbol{I}_{t,s,l,c\in\mathbb{C}^I}, \boldsymbol{E}_{t,s,l,c\in\mathbb{C}^{EX}}, \boldsymbol{T}_{t,s,l,l',c\in\mathbb{C}^{TR}}\}_{(t,s)\in\mathbb{A}_i,(l,l')\in\mathbb{L}^2}$. Namely, the latter include the constraints in $\mathcal{Y}$, which in Babonneau et al. (2017) decompose over each season, and constraints (1b)-(1e), (1h)-(1m). Finally, $\mathcal{Z}_i(\Gamma_i) := \{\boldsymbol{\zeta}_i \in \mathbb{R}^{d_i} \,|\, \|\boldsymbol{\zeta}_i\|_\infty \leq 1,\ \|\boldsymbol{\zeta}_i\|_1 \leq \Gamma_i\}$ is the traditional budgeted uncertainty set while $C_i \in \mathbb{R}^{n_i \times d_i}$ models the linear effect of the standardized perturbances on each constraint. For completeness, we note that $f \in \mathbb{R}^m$, $h_i \in \mathbb{R}^{k_i}$, $A_i \in \mathbb{R}^{n_i\times m}$, $B_i \in \mathbb{R}^{n_i\times k_i}$, and $b_i \in \mathbb{R}^{n_i}$.

4.2 Affinely Adjustable Approximation

Given the known numerical intractability of problem (3) (see Ben-Tal et al. 2004), one can instead solve a conservative approximation model obtained by enforcing that adjustable variables $\boldsymbol{y}_i(\boldsymbol{\zeta}_i)$ be affine functions of $\boldsymbol{\zeta}_i$; i.e., $\boldsymbol{y}_i(\boldsymbol{\zeta}_i) = \boldsymbol{y}_i + \boldsymbol{Y}_i\bar{\boldsymbol{\zeta}}_i$, where $\boldsymbol{y}_i \in \mathbb{R}^{k_i}$ and $\boldsymbol{Y}_i \in \mathbb{R}^{k_i \times r_i}$ become the decision variables. We follow this approach after employing a commonly used lifting of the uncertainty set:

$$\mathcal{Z}_i = P_i\bar{\mathcal{Z}}_i\,,$$

where

$$\bar{\mathcal{Z}}_i = \left\{\bar{\boldsymbol{\zeta}}_i \in \mathbb{R}^{r_i} \middle| \, W_i\bar{\boldsymbol{\zeta}}_i \leq \nu_i, \bar{\boldsymbol{\zeta}}_i \geq 0\right\}\,,$$

with

$$W_i := \begin{bmatrix} I_{d_i} & I_{d_i} \\ \mathbf{1}_{1\times d_i} & \mathbf{1}_{1\times d_i} \end{bmatrix}\,, \qquad \nu_i := \begin{bmatrix} \mathbf{1}_{d_i\times 1} \\ \Gamma_i \end{bmatrix}\,, \qquad P_i := \begin{bmatrix} I_{d_i} & -I_{d_i} \end{bmatrix}\,,$$

and I_{d_i} is the $d_i \times d_i$ identify matrix, $r_i = 2d_i$. In other words, problem (3) is conservatively approximated by the following robust linear program (referred as the Robust Multi-Period Conservative Approximation model):

$$\text{(RMPCA)} \quad \min_{\boldsymbol{x},\{\boldsymbol{Y}_i,\boldsymbol{y}_i\}_{i=1}^{|\mathbb{I}|}} \max_{\{\bar{\boldsymbol{\zeta}}_i\in\bar{\mathcal{Z}}_i\}_{i=1}^{|\mathbb{I}|}} f^\top\boldsymbol{x} + \sum_{i=1}^{|\mathbb{I}|} h_i^\top\Big(\boldsymbol{y}_i + \boldsymbol{Y}_i\bar{\boldsymbol{\zeta}}_i\Big) \tag{4a}$$

$$\text{s.t.} \quad A_i\boldsymbol{x} + B_i\Big(\boldsymbol{y}_i + \boldsymbol{Y}_i\bar{\boldsymbol{\zeta}}_i\Big) \leq b_i + C_iP_i\bar{\boldsymbol{\zeta}}_i \quad \forall\bar{\boldsymbol{\zeta}}_i \in \bar{\mathcal{Z}}_i \; \forall i \in \{1...|\mathbb{I}|\} \tag{4b}$$

$$D\boldsymbol{x} \leq e\,. \tag{4c}$$

Applying standard robust reformulation techniques, we obtain the following tractable linear program:

$$\min_{\boldsymbol{x},\{\boldsymbol{\psi}_i,\boldsymbol{\Phi}_i,\boldsymbol{Y}_i,\boldsymbol{y}_i\}_{i=1}^{|\mathbb{I}|}} f^\top\boldsymbol{x} + \sum_{i=1}^{|\mathbb{I}|} \nu_i^\top\boldsymbol{\psi}_i + h_i^\top\boldsymbol{y}_i \tag{5a}$$

$$\text{s.t.} \quad D\boldsymbol{x} \leq e \tag{5b}$$

$$W_i^\top\boldsymbol{\psi}_i \geq (h_i^\top\boldsymbol{Y}_i)^\top \quad \forall i \in \{1,..,|\mathbb{I}|\} \tag{5c}$$

$$\boldsymbol{\Phi}_i\nu_i \leq b_i - A_i\boldsymbol{x} - B_i\boldsymbol{y}_i \quad \forall i \in \{1,..,|\mathbb{I}|\} \tag{5d}$$

$$\boldsymbol{\Phi}_iW_i \geq B_i\boldsymbol{Y}_i - C_iP_i \quad \forall i \in \{1,..,|\mathbb{I}|\} \tag{5e}$$

$$\boldsymbol{\Phi}_i, \boldsymbol{\psi}_i \geq 0\,, \tag{5f}$$

where $\boldsymbol{\psi}_i \in \mathbb{R}^{(d_i+1)\times 1}$ and $\boldsymbol{\Phi}_i \in \mathbb{R}^{n_i\times(d_i+1)}$ are new variables added to the model to guarantee the feasibility of the decision variables in the worst-case scenario. It is worth mentioning that problem (5) provides an upper-bound and feasible first-stage decision for problem (3) as the space of decision rule has been reduced to affine functions.

5 Improving Numerical Efficiency using Bender's Decomposition

Because of the capacity expansion decisions with a horizon of several decades, alongside with the procurement decisions with daily resolution, ETEM is already in its deterministic form a large-scale

LP model. The size of LP that needs to be solved is further exacerbated when considering the robust reformulation presented in (5). In this regard, Section 6 will consider instances where the number of decisions and constraints is increased twenty-fold and nine-fold respectively. For this reason, we propose here a decomposition scheme to reduce the computational burden.

Specifically, we start by reformulating problem (4) as follows:

$$\min_{\boldsymbol{x},\boldsymbol{\rho},\{\boldsymbol{y}_i\}_{i=1}^{|\mathbb{I}|}} \; f^\top \boldsymbol{x} + \sum_{i=1}^{|\mathbb{I}|} h_i^\top \boldsymbol{y}_i + \boldsymbol{\rho}_i \tag{6a}$$

$$\text{s.t.} \quad D\boldsymbol{x} \leq e \tag{6b}$$

$$\boldsymbol{\rho}_i \geq g_i(\boldsymbol{x}, \boldsymbol{y}_i) \quad \forall i \in \{1...|\mathbb{I}|\} \tag{6c}$$

$$A_i\boldsymbol{x} + B_i\boldsymbol{y}_i \leq b_i \quad \forall i \in \{1...|\mathbb{I}|\}\,, \tag{6d}$$

with

$$g_i(\boldsymbol{x}, \boldsymbol{y}_i) := \min_{\boldsymbol{Y}_i} \max_{\{\bar{\boldsymbol{\zeta}}_i \in \bar{\mathcal{Z}}_i\}_{i=1}^{|\mathbb{I}|}} h_i^\top \boldsymbol{Y}_i \bar{\boldsymbol{\zeta}}_i \tag{7a}$$

$$\text{s.t.} \quad (4\text{b})\,,$$

where constraint (6d) is a redundant constraint that describes the fact that (4b) must be satisfied for $\bar{\boldsymbol{\zeta}}_i = 0$ since $0 \in \bar{\mathcal{Z}}_i$. Given that $g_i(\boldsymbol{x}, \boldsymbol{y}_i)$ is jointly convex, we identify a supporting plane representation that will enable the use of a constraint generation approach, also commonly referred to as a Bender's decomposition scheme. It is worth noting that our scheme is inspired by the one used in Ardestani-Jaafari and Delage (2018) and Ardestani-Jaafari and Delage (2017), yet we depart from that work by keeping in problem (6) the $\boldsymbol{y}_i$ variables, which describe what the affine strategy prescribes for the nominal scenario $\bar{\boldsymbol{\zeta}} = 0$. Note also that constraint (6c) should be considered violated if $\boldsymbol{x}$ and $\boldsymbol{y}_i$ are such that problem (7) is infeasible.

In what follows, we start by describing the algorithm, then present two schemes that can be employed to improve its numerical efficiency by refining the quality of the upper and lower bounding process. In the latter case, the refinement will make use for the first time of the notion of Pareto robustly optimal solution (Iancu and Trichakis 2014) in decomposition schemes for robust optimization.

5.1 The Bender's Decomposition Algorithm

To simplify the presentation of this algorithm we make the following assumption.

Assumption 1. *(Uncapacitated technology) ETEM includes a technology $p_x \in \mathbb{P}_c^P/\mathbb{P}^R$ with $c \in \mathbb{C}^\mathcal{U}$ generated from a resource $c_x \in \mathbb{C}^I/\mathbb{C}^\mathcal{D}$, with infinite capacity, i.e., $\Omega_{t,l,p_x} = \infty$. Furthermore, $\mathcal{X}$ and $\mathcal{Y}$ do not impose additional constraints on $\boldsymbol{P}_{t,s,l,p_x,c}$ and $\boldsymbol{I}_{t,s,l,c_x}$.*

The role of the above assumption is to ensure that when removing constraint (6c) from problem (6), we do not end up with solutions for $(\boldsymbol{x}, \{\boldsymbol{y}_i\}_{i=1}^{|\mathbb{I}|})$ that make problem (7) infeasible. Note also that in our description of ETEM, the EXT technology and ExR energy resource respectively play the role of p_x and c_x.

Lemma 1. *Given that an uncapacitated technology exists (i.e., Assumption 1), problem* (7) *is feasible for any $\boldsymbol{x}$ and $\{\boldsymbol{y}_i\}_{i=1}^{|\mathbb{I}|}$ that satisfy* (6d)*.*

Proof. Proof: One needs to show that for any given $\boldsymbol{x}$ and $\boldsymbol{y}_i$, $i = 1, \cdots, |\mathbb{I}|$, that satisfy (6b) and (6d), there exists a $\boldsymbol{Y}_i$ so that the constraint (4b) is feasible. To do so, given that we already now that $\boldsymbol{x}$ and

$\boldsymbol{y}_i$ are such that they can cover the nominal demand response, i.e., $\boldsymbol{\zeta}^+ = \boldsymbol{\zeta}^- = 0$, we will show how to construct a $\boldsymbol{Y}_i$ that will simply cover any deviation of the demand coming from the RDRDs using the uncapacited technology. Specifically, we let all values of $\boldsymbol{Y}$ equal to zero except for the terms that model the influence of $\boldsymbol{\delta}^+$ on $\boldsymbol{P}_{t,s,l,p_x,c}$ with $c \in \mathbb{C}^{\mathcal{U}}$, $\boldsymbol{P}_{t,s,l,p_x,c_x}$, and $\boldsymbol{I}_{t,s,l,c_x}$. In particular, when setting all other terms to zero, most constraints reduce exactly to the constraints imposed in (6d) and are straightforwardly satisfied. The only constraints that need to be verified consist of a few members of (1b), (1e), (1h) and (1j), which can be described as follows:

$$(1b) \rightarrow \quad (\sum_{p\in\mathbb{P}^{P_{c_x}}} \bar{\boldsymbol{P}}_{t,s,l,p,c_x} + \bar{\boldsymbol{I}}_{t,s,l,c_x} + \sum_{c\in\mathbb{C}^{\mathcal{U}}} \boldsymbol{I}^+_{t,s,l,c_x,c}\boldsymbol{\delta}^+_{t,s,c})\eta_{c_x} + \sum_{l'\neq l}(\eta_{c_x}\bar{\boldsymbol{T}}_{t,s,l',l,c_x} - \bar{\boldsymbol{T}}_{t,s,l,l',c_x})$$
$$\geq \sum_{p\in\mathbb{P}^C_c} \bar{\boldsymbol{P}}^+_{t,s,l,p,c} + \sum_{c\in\mathbb{C}^{\mathcal{U}}} \boldsymbol{P}^+_{t,s,l,p_x,c_x,c}\boldsymbol{\delta}^+_{t,s,c} + \bar{\boldsymbol{E}}_{t,s,l,c_x} \quad \begin{array}{l}\forall(\boldsymbol{\delta}^+_i,\boldsymbol{\delta}^-_i)\in\bar{\mathcal{Z}}_i,\ \forall l\in\mathbb{L}\\ \forall(t,s)\in\mathcal{A}_i, \forall i\in\mathbb{I}\end{array} \tag{8}$$

$$(1e) \rightarrow \quad \sum_{p\in\mathbb{P}^P_c} \bar{\boldsymbol{P}}_{t,s,l,p,c} + \boldsymbol{P}^+_{t,s,l,p_x,c}\boldsymbol{\delta}^+_{t,s,c} \geq \Theta_{t,l,c}\big(\bar{\boldsymbol{V}}_{t,s,c} + \boldsymbol{\delta}^+_{t,s,c} - \boldsymbol{\delta}^-_{t,s,c}\big) \quad \begin{array}{l}\forall(\boldsymbol{\delta}^+_i,\boldsymbol{\delta}^-_i)\in\bar{\mathcal{Z}}_i, \forall c\in\mathbb{C}^{\mathcal{U}}\\ \forall(t,s)\in\mathcal{A}_i, \forall i\in\mathbb{I}\end{array} \tag{9}$$

$$(1h) \rightarrow \quad \sum_{c:p_x\in\mathbb{P}^P_c} \bar{\boldsymbol{P}}_{t,s,l,p_x,c} + \sum_{c\in\mathbb{C}^{\mathcal{U}}} \boldsymbol{P}^+_{t,s,l,p_x,c}\boldsymbol{\delta}^+_{t,s,c} \leq \infty \quad \begin{array}{l}\forall(\boldsymbol{\delta}^+_i,\boldsymbol{\delta}^-_i)\in\bar{\mathcal{Z}}_i,\ \forall l\in\mathbb{L}\\ \forall(t,s)\in\mathcal{A}_i, \forall i\in\mathbb{I}\end{array} \tag{10}$$

$$(1j) \rightarrow \quad \sum_{c\in\mathbb{C}_{f'}} \bar{\boldsymbol{P}}_{t,s,l,p_x,c} + \sum_{c\in\mathbb{C}^{\mathcal{U}}} \boldsymbol{P}^+_{t,s,l,p_x,c}\boldsymbol{\delta}^+_{t,s,c} = \eta^t_{f,f'}(\sum_{c\in\mathbb{C}_f} \bar{\boldsymbol{P}}_{t,s,l,p_x,c} + \sum_{c'\in\mathbb{C}^{\mathcal{U}}} \boldsymbol{P}^+_{t,s,l,p_x,c_x,c'}\boldsymbol{\delta}^+_{t,s,c'})$$
$$\begin{array}{l}\forall(\boldsymbol{\delta}^+_i,\boldsymbol{\delta}^-_i)\in\bar{\mathcal{Z}}_i,\ \forall l\in\mathbb{L}\\ \forall(t,s)\in\mathcal{A}_i, \forall i\in\mathbb{I}\end{array}\ , \tag{11}$$

where $\bar{\boldsymbol{P}}$, $\bar{\boldsymbol{V}}$, $\bar{\boldsymbol{C}}$, $\bar{\boldsymbol{I}}$, $\bar{\boldsymbol{T}}$, and $\bar{\boldsymbol{E}}$ refer to the assignments made through the fixed $\boldsymbol{x}$ and $\boldsymbol{y}_i$'s, while $\boldsymbol{P}^+_{t,s,l,p_x,c}$, $\boldsymbol{P}^+_{t,s,l,p_x,c_x,c}$, and $\boldsymbol{I}^+_{t,s,l,c_x,c}$ model the influence of $\boldsymbol{\delta}^+_{t,s,c}$ on $\boldsymbol{P}_{t,s,l,p_x,c}$, $\boldsymbol{P}_{t,s,l,p_x,c_x}$, and $\boldsymbol{I}_{t,s,l,c_c}$ respectively, for $c \in \mathbb{C}^{\mathcal{U}}$. In order to get a feasible assignment for $\boldsymbol{Y}$, for all t, s, $c \in \mathbb{C}^{\mathcal{U}}$, we let $\boldsymbol{P}^+_{t,s,l,p_x,c} := \Theta_{t,l,c}$, $\boldsymbol{P}^+_{t,s,l,p_x,c_x,c} := \boldsymbol{P}^+_{t,s,l,p_x,c}/\eta^t_{f,f'}$, and $\boldsymbol{I}^+_{t,s,l,c_x,c} = \boldsymbol{P}^+_{t,s,l,p_x,c_x,c}/\eta_{c_x}$. One can readily verify that constraints (8), (10), and (11) reduce to the constraint already accounted for in (6d). On the other hand, constraint (9) reduces to:

$$\sum_{p\in\mathbb{P}^P_c} \bar{\boldsymbol{P}}_{t,s,l,p,c} \geq \Theta_{t,l,c}\big(\bar{\boldsymbol{V}}_{t,s,c} - \boldsymbol{\delta}^-_{t,s,c}\big) \quad \begin{array}{l}\forall(\boldsymbol{\delta}^+_i,\boldsymbol{\delta}^-_i)\in\bar{\mathcal{Z}}_i, \forall c\in\mathbb{C}^{\mathcal{U}}\\ \forall(t,s)\in\mathcal{A}_i, \forall i\in\mathbb{I}\end{array}\ ,$$

which is necessarily satisfied since $\Theta_{t,l,c} \geq 0$ and $\boldsymbol{\delta}^-_i \geq 0$. A similar argument can be used to confirm that all non-negative constraints on $\bar{\boldsymbol{P}}_{t,s,l,p_x,c} + \boldsymbol{P}^+_{t,s,l,p_x,c}\boldsymbol{\delta}^+_{t,s,c}$, $\bar{\boldsymbol{P}}_{t,s,l,p_x,c_x} + \sum_{c\in\mathbb{C}^{\mathcal{U}}} \boldsymbol{P}^+_{t,s,l,p_x,c_x,c}\boldsymbol{\delta}^+_{t,s,c}$, and $\bar{\boldsymbol{I}}_{t,s,l,c_x} + \sum_{c\in\mathbb{C}^{\mathcal{U}}} \boldsymbol{I}^+_{t,s,l,c_x,c}\boldsymbol{\delta}^+_{t,s,c}$ are also satisfied. □

In practice, if Assumption 1 is not satisfied, a few remedies exist. First, one can create an artificial technology with either infinite or very large capacity, whose price is set to an arbitrary large amount. One can interpret the price of this artificial technology has the marginal cost of shortages in the system. If this technology is not used in the optimal solution that is identified, then shortages do not occur and the solution is optimal for the problem without this artificial technology. Alternatively, one could also modify the algorithm that is presented below to have it identify “feasibility cuts” when the current candidate solution makes (7) infeasible (see Rahmaniani et al. 2017, for details) .

We now focus on presenting a support plane representation of $g(\boldsymbol{x}, \boldsymbol{y}_i)$.

Theorem 2. *Given that Assumption 1 is satisfied,*

$$g_i(\boldsymbol{x}, \boldsymbol{y}_i) = \max_{\boldsymbol{\theta}_i \geq 0, \bar{\boldsymbol{\zeta}}_i \geq 0, \boldsymbol{\lambda}_i \geq 0} (-b_i + A_i\boldsymbol{x} + B_i\boldsymbol{y}_i)^\top \boldsymbol{\theta}_i - Tr(C_iP_i\boldsymbol{\lambda}_i) \tag{12a}$$

$$s.t. \quad \boldsymbol{\theta}_i\nu_i^\top - \boldsymbol{\lambda}_i W_i^\top \geq 0 \tag{12b}$$

$$B_i^\top \boldsymbol{\lambda}_i = -h_i \bar{\boldsymbol{\zeta}}_i^\top \tag{12c}$$

$$W_i\bar{\boldsymbol{\zeta}}_i \leq \nu_i\,, \tag{12d}$$

where $\boldsymbol{\theta}_i \in \mathbb{R}^{n_i \times 1}, \boldsymbol{\lambda}_i \in \mathbb{R}^{n_i \times r_i}$ *and* $\bar{\boldsymbol{\zeta}}_i \in \mathbb{R}^{r_i \times 1}$.

Proof. Proof: In the first step, we identify the robust counterpart of constraint (4b). For each $i \in \mathbb{I}$, given that $\bar{\mathcal{Z}}_i$ is non-empty, LP duality applies on each robust constraint thus introducing auxiliary variables that we denote in matrix form in $\boldsymbol{\Phi}_i$. Namely, constraint (4b) can be said equivalent to

$$\exists \boldsymbol{\Phi}_i \in \mathbb{R}^{n_i \times (d_i+1)}, \quad \forall i \in \{1 \cdots |\mathbb{I}|\}$$

$$\boldsymbol{\Phi}_i \nu_i \leq b_i - A_i\boldsymbol{x} - B_i\boldsymbol{y}_i \qquad \forall i \in \{1 \cdots |\mathbb{I}|\} \tag{13}$$

$$\boldsymbol{\Phi}_i W_i \geq B_i \boldsymbol{Y}_i - C_iP_i \quad \forall i \in \{1 \cdots |\mathbb{I}|\}\,. \tag{14}$$

Therefore, problem (7) can be reformulated as:

$$\begin{aligned} \min_{\boldsymbol{Y}_i, \boldsymbol{\Phi}_i} \quad & \max_{\bar{\boldsymbol{\zeta}}_i \in \bar{\mathcal{Z}}_i} \left(h_i^\top \boldsymbol{Y}_i \bar{\boldsymbol{\zeta}}_i \right) \\ \text{s.t.} \quad & (13), (14)\,, \end{aligned}$$

which is a feasible minimization problem according to Lemma 1 since we assumed that Assumption 1 holds. In the next step, since $\bar{\mathcal{Z}}_i$ is a compact and convex set, Sion's minmax theorem (Sion 1958) holds and one can reverse the order of minimization over $\{\boldsymbol{Y}_i, \boldsymbol{\Phi}_i\}$ and maximization over $\bar{\boldsymbol{\zeta}}_i$'s. Finally, since the set defined by (13) and (14) is feasible, one can employ LP duality to replace the inner minimization problem with its dual maximization problem to obtain:

$$\begin{aligned} g(\boldsymbol{x}, \boldsymbol{y}_i) = \max_{\bar{\boldsymbol{\zeta}}_i \in \bar{\mathcal{Z}}_i, \boldsymbol{\theta}_i, \boldsymbol{\lambda}_i} \quad & (-b_i + A_i\boldsymbol{x} + B_i\boldsymbol{y}_i)^\top \boldsymbol{\theta}_i - Tr(C_iP_i\boldsymbol{\lambda}_i) \\ \text{s.t.} \quad & \boldsymbol{\theta}_i\nu_i^\top - \boldsymbol{\lambda}_i W^\top \geq 0 \\ & B_i^\top \boldsymbol{\lambda}_i = -h_i\bar{\boldsymbol{\zeta}}_i^\top \\ & \boldsymbol{\theta}_i \geq 0,\ \boldsymbol{\lambda}_i \geq 0\,, \end{aligned}$$

where $\boldsymbol{\lambda}_i \in \mathbb{R}^{n_i \times r_i}$, $\boldsymbol{\theta}_i \in \mathbb{R}^{n_i \times 1}$ are dual variables associated with constraints (13) and (14) respectively. □

Equipped with Theorem 2, we can define the following Bender's decomposition algorithm. Intuitively, the procedure consists in solving a so-called master problem in which constraint (6c) is removed thus producing a sequence of lower bounds for problem (4). Given a current optimal solution of the master problem, the latter is refined by progressively reintroducing the constraints that are the most violated from the set:

$$\boldsymbol{\rho}_i \geq (-b_i + A_i\boldsymbol{x} + B_i\boldsymbol{y}_i)^\top \boldsymbol{\theta}_i - Tr(C_iP_i\boldsymbol{\lambda}_i),\ \forall (\boldsymbol{\lambda}_i, \boldsymbol{\theta}_i) \in \{(\boldsymbol{\lambda}_i, \boldsymbol{\theta}_i) \in \mathbb{R}_+^{n_1 \times r_i} \times \mathbb{R}_+^{n_i} \,|\, \exists \bar{\boldsymbol{\zeta}}_i \in \bar{\mathcal{Z}}_i,\ (12b),\ (12c)\},\ \forall i \in \mathbb{I}\,.$$

For each i, the most violated constraint can be found by solving the LP presented in (12), which optimal value can be used to update an upper bound problem (4). We refer the reader to the pseudo-code presented in Algorithm 1.

Algorithm 1: Benders Decomposition (BD) algorithm

Set $UB = \infty, LB = -\infty, v = 0$;

Let i be the index for each subproblem;

Let $v = v + 1$ and solve the deterministic problem, i.e., problem (4) when $\bar{\mathcal{Z}}_i = \{0\}$, and store the value of $\boldsymbol{x}$ as $\overline{\boldsymbol{x}}^{(v)}$ and $\boldsymbol{y}_i$ as $\overline{\boldsymbol{y}}_i^{(v)}$;

while $(UB - LB)/LB \geq \varepsilon$ **do**

  $\forall i = \{1 \cdots |\mathbb{I}|\}$ solve problem (12) with $(\overline{\boldsymbol{x}}^{(v)}, \overline{\boldsymbol{y}}_i^{(v)})$ and store the optimal value in $\boldsymbol{\rho}_i^*$ and decision variables in $\overline{\boldsymbol{\theta}}_i^v, \overline{\boldsymbol{\Lambda}}_i^v, \overline{\boldsymbol{\zeta}}_i^v$;

  Update upper bound $UB = \min(UB, f^\top \overline{\boldsymbol{x}}^v + \sum_{i=1}^{|\mathbb{I}|} \boldsymbol{\rho}_i^* + h_i^\top \overline{\boldsymbol{y}}_i^v)$;

  Solve the updated master problem :

$$\min_{\boldsymbol{x}, \{\boldsymbol{\rho}_i, \boldsymbol{y}_i\}_{i=1}^{|\mathbb{I}|}} \quad f^\top \boldsymbol{x} + \sum_{i=1}^{|\mathbb{I}|} h_i^\top \boldsymbol{y}_i + \boldsymbol{\rho}_i \tag{15a}$$

$$\text{s.t.} \quad (6b), (6d),$$

$$\boldsymbol{\rho}_i \geq (-b_i + A_i \boldsymbol{x} + B_i \boldsymbol{y}_i)^\top \overline{\boldsymbol{\theta}}_i^{v'} - Tr(C_i P \overline{\boldsymbol{\Lambda}}_i^{v'}), \quad \forall v' \leq v, \forall i \in \{1 \cdots |\mathbb{I}|\}. \tag{15b}$$

  Let $v = v + 1$ and store the optimal value of the master problem variables in $\overline{\boldsymbol{x}}^v, \overline{\boldsymbol{y}}_i^v$ and $\overline{\boldsymbol{\rho}}^v$;

  Update the lower bound: $LB = f^\top \overline{\boldsymbol{x}}^v + \sum_i h_i^\top \overline{\boldsymbol{y}}_i^v + \overline{\boldsymbol{\rho}}^v$.

end

5.2 Improving the Algorithm's Convergence

In order to improve the algorithm convergence, we apply two strategies. First, as proposed in Ardestani-Jaafari and Delage (2018) and Ardestani-Jaafari and Delage (2017), we add a set of valid inequalities to the master problem (15) based on violated scenarios from previous iterations in order to tighten the lower bounding problem. Specifically, for any $i \in \mathbb{I}$ and finite set of scenarios $\{\bar{\boldsymbol{\zeta}}_i^l\}_{l \in \Omega} \subset \bar{\mathcal{Z}}_i$, the following constraint necessarily holds in problem (6):

$$\forall l \in \Omega, \exists \boldsymbol{y}_i^l \in \mathbb{R}^{d_i}, \; \boldsymbol{\rho}_i \geq h_i^\top (\boldsymbol{y}_i^l - \boldsymbol{y}_i) \; \& \; A_i \boldsymbol{x} + B_i \boldsymbol{y}_i^l \leq b_i + C_i P_i \bar{\boldsymbol{\zeta}}_i^{\,l},$$

given that for all $l \in \Omega$:

$$\begin{aligned} \boldsymbol{\rho}_i \geq g(\boldsymbol{x}, \boldsymbol{y}_i) = \min_{\mathbf{Y}_i : (4b)} \max_{\bar{\boldsymbol{\zeta}}_i \in \bar{\mathcal{Z}}_i} h_i^\top \mathbf{Y}_i \bar{\boldsymbol{\zeta}}_i &\geq \min_{\mathbf{Y}_i : A_i \boldsymbol{x} + B_i (\boldsymbol{y}_i + \mathbf{Y}_i \bar{\boldsymbol{\zeta}}_i^l) \leq b_i + C_i P_i \bar{\boldsymbol{\zeta}}_i^l} h_i^\top \mathbf{Y}_i \bar{\boldsymbol{\zeta}}_i^l \\ &= \min_{\boldsymbol{y}_i^l, \mathbf{Y}_i : \boldsymbol{y}_i^l = \boldsymbol{y}_i + \mathbf{Y}_i \bar{\boldsymbol{\zeta}}_i^l, \, A_i \boldsymbol{x} + B_i \boldsymbol{y}_i^l \leq b_i + C_i P_i \bar{\boldsymbol{\zeta}}_i^l} h_i^\top (\boldsymbol{y}_i^l - \boldsymbol{y}_i) \geq \min_{\boldsymbol{y}_i^l : \, A_i \boldsymbol{x} + B_i \boldsymbol{y}_i^l \leq b_i + C_i P_i \bar{\boldsymbol{\zeta}}_i^l} h_i^\top (\boldsymbol{y}_i^l - \boldsymbol{y}_i). \end{aligned}$$

In our implementation, we choose to set $\{\bar{\boldsymbol{\zeta}}_i^l\}_{l \in \Omega}$ to be the violated scenarios in the last v iterations of the algorithm. For completeness, we include the new master problem below:

$$\min_{\boldsymbol{x}, \{\boldsymbol{\rho}_i, \boldsymbol{y}_i, \boldsymbol{y}_i^l\}_{i=1}^{|\mathbb{I}|}} \quad f^\top \boldsymbol{x} + \sum_{i=1}^{|\mathbb{I}|} h_i^\top \boldsymbol{y}_i + \boldsymbol{\rho}_i \tag{16a}$$

$$\text{s.t.} \quad (6b), (6d), (15b),$$

$$\boldsymbol{\rho}_i \geq h_i^\top (\boldsymbol{y}_i^l - \boldsymbol{y}_i) \quad \forall l \in \Omega, \forall i \in \{1 \cdots |\mathbb{I}|\} \tag{16b}$$

$$A_i\boldsymbol{x} + B_i\boldsymbol{y}_i^l \leq b_i + C_iP_i\bar{\boldsymbol{\zeta}}_i^{\,l} \quad \forall l \in \Omega, \forall i \in \{1\cdots|\mathbb{I}|\}\,. \tag{16c}$$

The second proposed improvement has to do with how the upper bound is obtained. In particular, this bound comes from evaluating the true worst-case cost of the current best candidate based on the master problem (15), or its tighter version (16). Yet, since the master problem is itself a robust optimization problem, we can expect based on the findings of Iancu and Trichakis (2014) that at each iterations there exists a large set of optimal solutions for (15). In a traditional Bender's decomposition approach (such as in Ardestani-Jaafari and Delage 2018), the candidate that is used to calculate the upper bound and generate an optimality cut is arbitrarily chosen by the LP solver used to solve (15). While such a solution is optimal for (15), it could have a much worst performance in problem (12) compared to other optimal solutions. We therefore recommend, after solving (15), to identify a Pareto robustly optimal solution of (15) by solving the following LP:

$$\min_{\boldsymbol{x},\{\boldsymbol{\rho}_i',\boldsymbol{\rho}_i,\boldsymbol{y}_i\}_{i=1}^{|\mathbb{I}|}} \; f^\top\boldsymbol{x} + \sum_{i=1}^{|\mathbb{I}|} h_i^\top\boldsymbol{y}_i + \boldsymbol{\rho}_i' \tag{17a}$$

$$\text{s.t.} \quad (6b), (6d), (15b), (16b), (16c)\,,$$

$$f^\top\boldsymbol{x} + \sum_{i=1}^{|\mathbb{I}|} h_i^\top\boldsymbol{y}_i + \boldsymbol{\rho}_i \leq (1+\epsilon)\mathscr{M}_v^* \tag{17b}$$

$$\boldsymbol{\rho}_i' \geq \sum_{v'\leq v} \frac{1}{v}((-b_i + A_i\boldsymbol{x} + B_i\boldsymbol{y}_i)^\top\overline{\boldsymbol{\theta}}_i^{v'} - Tr(C_iP\overline{\boldsymbol{\lambda}}_i^{v'})) \quad \forall i \in \{1\cdots|\mathbb{I}|\}\,, \tag{17c}$$

where $\mathscr{M}_v^*$ is the optimal solution of the master problem (15), and $\epsilon > 0$ allows for some small ϵ sub-optimality in order to help numerically. The new candidate $(\overline{\boldsymbol{x}}^v, \overline{\boldsymbol{y}}_i^v)$ remain approximately optimal for (15) yet is guaranteed to not be Pareto dominated by other solutions of (15). We refer the reader to Iancu and Trichakis (2014) for more details about how to identify Pareto robustly optimal solutions.

6 Computational Results

In this section, we evaluate the performance of the proposed robust ETEM approximation model (i.e., RMPCA) on a case study based on the energy system of the Arc Lémanique region in Switzerland. To do so, we compare the performance of the RMPCA formulation with a static robust model (SRM), which considers all decisions as *here-and-now* decisions, and a deterministic model (DET), which disregards demand response uncertainty. The purpose of these computational experiments is to empirically show that: i) considering DRU in a capacity expansion planning problem and solving it using robust optimization can decrease both worst-case and expected total costs of the system; and ii) assuming flexibility of the procurement decisions with respect to observed actual DR decreases the level of conservatism of the model and consequently the expected total cost of the system. Recall that in RMPCA the planner first decides on the capacities and planned DR, then in seasonal periods, after observing the actual DR, he decides on the optimal procurement of energy. Theoretically, it is known that the policies proposed by SRM are more conservative compared to policies proposed by RMPCA. In addition, we present a detailed comparison of the structure of the RMPCA, SRM, and DET policies. The purpose is to understand how considering the uncertainty of the demand response affects the long-term capacity expansion strategies. Finally, we provide a computational analysis to compare the performance of the different decomposition algorithms presented in Section 5. All numerical studies are performed on a 64-bit computer with 128 GB of RAM and Intel(R) Xeon(R) 3.1 GHz (40 CPUs). The deterministic version of ETEM is originally written in AMPL. However, we extract the standard form matrices from AMPL and carry out robust optimization using YALMIP toolbox and GUROBI solver in a MATLAB R2019a environment.

6.1 A Swiss Case Study

The ETEM model describing the Arc Lémanique region (Cantons of Geneva and Vaud, in Switzerland) encompasses 142 different technologies, including centralized electricity and heat production plants, decentralized electricity production, conventional and flexible loads, and end-use transportation technologies. In addition, 57 energy commodities, including 21 types of useful demand, are modeled. We choose to immunize ETEM against residential electricity demand response deviations (i.e., Res-ELC in Figure 1), which implies that the robust model is subject to 120 independent sources of perturbations. This choice is motivated by the fact that the response behavior of residential electricity is more prone to uncertainty in the long-run compared to industrial and large-scale electricity consumers, as the former corresponds to a larger and more diverse group of consumers. Furthermore, as we wish to avoid as much as possible energy shortages in the system, we set the cost of the additional energy source (ExR) high enough so that there is no capacity shortage in the optimal solution of RMPCA model when the largest amount of deviations are assumed. Finally, in order to better investigate the influence of robust optimization on the solutions, no upper bound limits the installation of renewable technologies (wind and photovoltaic power plants) in the model. For interested readers, we note that our deterministic version of ETEM presents 171,620 variables and 431,652 constraints while the LP reformulation of RMPCA, i.e., problem (5), presents 3,540,720 variables and 3,708,658 constraints.

While we refer the reader to Babonneau et al. (2017) for a complete description of all input parameters and assumptions, it is worth mentioning that we consider here a case where CO_2 emissions are curbed to 2.5 Mt in 2050, a 45% reduction compared to 2010 levels (5.48 Mt).

6.2 Performance Analysis

In this section, we study the performance of strategies proposed by RMPCA, SRM, and DET in the average and worst-case scenarios. For a specific budget size Γ and box size β, $\boldsymbol{x}^{\Gamma,\beta}_{RMPCA}$ represents the RMPCA strategy obtained by solving problem (5). Similarly, $\boldsymbol{x}^{\Gamma,\beta}_{SRM}$ is the static robust strategy obtained by solving problem (5) when forcing $\boldsymbol{Y}_i = 0 \; \forall i \in \{1 \cdots |\mathbb{I}|\}$. It should be noted that, because problem (5) is computationally demanding and cannot be solved in a 72-hour time limit, we have used the PRO-BD-VI algorithm proposed in Section 5. Finally, the DET strategy $\boldsymbol{x}_{DET}$ is obtained by solving the deterministic model , i.e., problem (4) when $\bar{\mathcal{Z}}_i = \{0\} \; \forall i \in \{1 \cdots |\mathbb{I}|\}$. After obtaining all the strategies from the different models, in a next step, we generate a set of 1000 DR random scenarios $\boldsymbol{\zeta}_i^j$ $(\forall i \in \{1, \cdots, |\mathbb{I}|\}, j \in [1, \cdots, 1000])$ using a uniform distribution on the $[-\beta, \beta]$ interval. For each scenario, we then re-optimize problem (4) with the value of $\boldsymbol{x}$ and $\boldsymbol{\zeta}_i$ fixed to the corresponding strategy and random scenario. In the first period (i.e., $t = 1$), we have no uncertainty given that a full information about the level of DR is available. If, for a specific policy and scenario, problem (4) becomes infeasible, this implies a capacity shortage. Table 1 reports the proportion of scenarios with capacity shortage for the different strategies and different budget sizes (expressed in percentage of $|d_i|$ and denoted as $\Gamma\%$). When the budget size increases, the chances of capacity shortage of RMPCA and SRM reduces. This is because a larger budget size increases the conservatism of the model, and consequently, the model constructs more capacity to avoid shortage, which obviously is more expensive. Nevertheless, for budget sizes in the range of [50%, 60%], the capacity shortage of RMPCA can be as low as 2%. To have a fair comparison between the expected total cost of different strategies, we report the average total cost both over all scenarios and over the scenarios without capacity shortage.

Table 1: Proportion of random scenarios with capacity shortage

$\Gamma\%$[1]	RMPCA	SRM	DET
10%	100%	100%	100%
20%	98%	96%	100%
30%	29%	0%	100%
40%	9%	0%	100%
50%	4%	0%	100%
60%	2%	0%	100%
70%	10%	0%	100%
80%	0%	0%	100%
90%	0%	0%	100%
100%	0%	0%	100%

[1] $\Gamma\%$ denotes that the units are in percent of $|d_i|$.

Figure 4a gives the average total cost for the RMPCA and SRM strategies, over all scenarios, for different budget sizes and $\beta = 0.6$. Figure 4b gives similar results when the average costs are taken over only scenarios without capacity shortage. We can first note that the average cost for the SRM strategy remains fixed for budget sizes greater than 25%. This is because when $\Gamma\% = 25\%$, $|d_i| = 1$, the SRM already considers each constraint to be maximally perturbed due to the fact that each one only involves one demand deviations. Second, we note that Figures 4 (a) and (b) show that the average RMPCA cost is considerably lower than the SRM cost for all budget sizes. This already suggests that the flexibility of energy procurement decisions can significantly reduce the expected total cost. More precisely, the maximum difference occurs when $\Gamma\% = 30\%$ yet this difference comes at the price of 29% chances of capacity shortage (see again Table 1). A better trade-off is achieved when $\Gamma\% = 50\%$, where the average total cost of RMPCA is around 4% less than for SRM (i.e., a 9 billion CHF reduction) while the chances of capacity shortage are estimated at only 4% . In comparison, the average total cost (including shortage penalties) from implementing the DET strategy is around 256 billion CHF, which is about 33% more than the total cost of RMPCA. Overall, this evidence supports the importance of: i) accounting of uncertainty of the demand response in ETEM; and ii) the importance of considering that the procurement is adjustable with respect to the actual DR.

Figure 5 is similar to Figure 4, but we have fixed the budget size to 50% and let the box size β vary. For example, $\beta = 0.2$ means that the actual demand response at each time-slice can deviate from the planned demand response by up to 20% of the total demand. As expected, increasing the support of $\boldsymbol{\zeta}$ leads to an increase of the average total cost of the system. Interestingly, the rate of increase of the DET cost is larger than for RMPCA and SRM. Indeed, as SRM and RMPCA are sensitive to worst-case values of DR, more capacity is built under these strategies. This gives more production flexibility with cheaper prices for the system when the actual DR is lower than planned DR.

Figure 6 corresponds to the Cumulative Distribution Function (CDF) diagram of the simulated total cost of RMPCA, SRM and DET strategies when $\Gamma\% = 50\%$ and $\beta = 0.6$. We can note that RMPCA stochastically dominates SRM, while SRM stochastically dominates DET. In fact, the maximum cost of the RMPCA strategy over all random scenarios is less than the minimum cost of the SRM strategy, and similarly for the DET strategy. Again, this figure shows the importance of considering uncertainty of the DR in capacity expansion models.

Next, Figure 7 shows the worst-case total cost of the system when implementing the RMPCA and

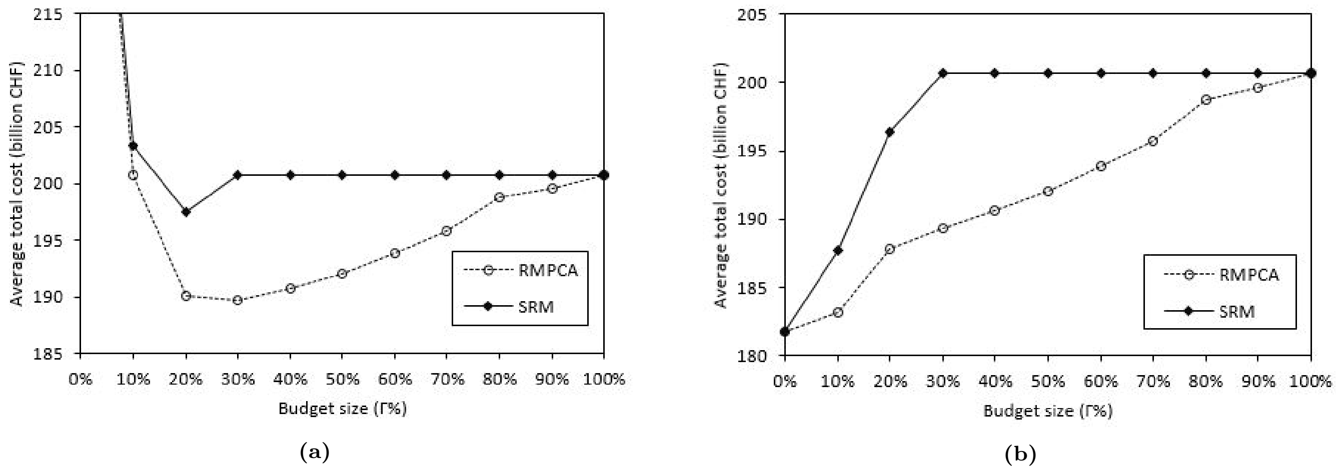


Figure 4: Average total cost of RMPCA and SRM strategies for different budget sizes and $\beta = 0.6$. (a) presents the average total cost over all 1000 random scenarios, whereas (b) presents the average total cost over the scenarios without capacity shortage.

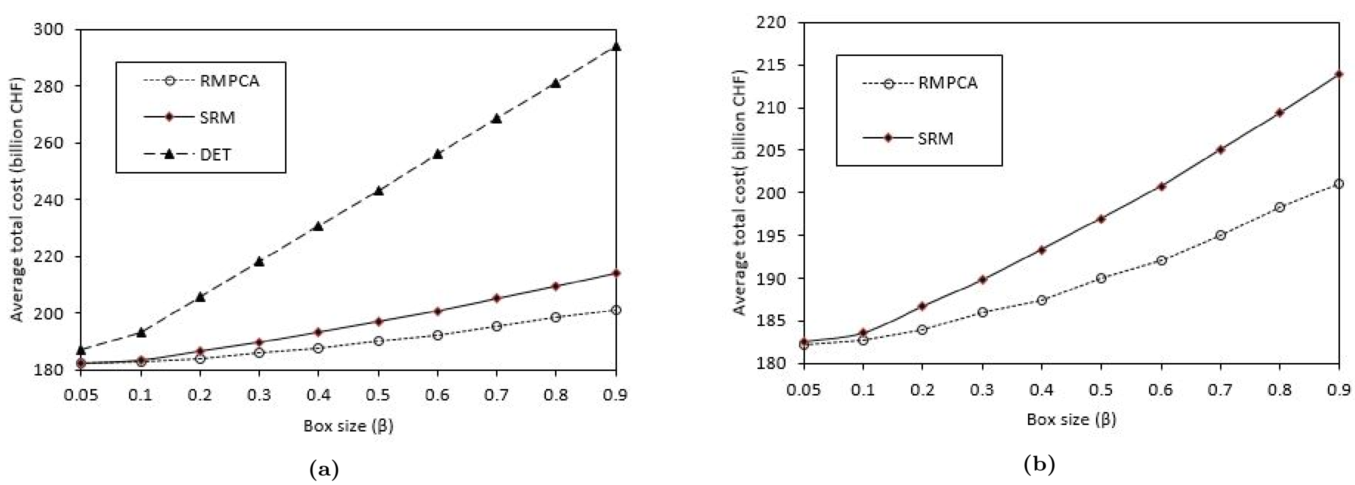
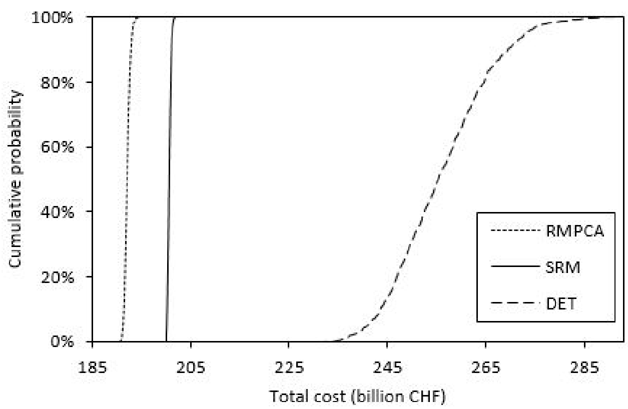
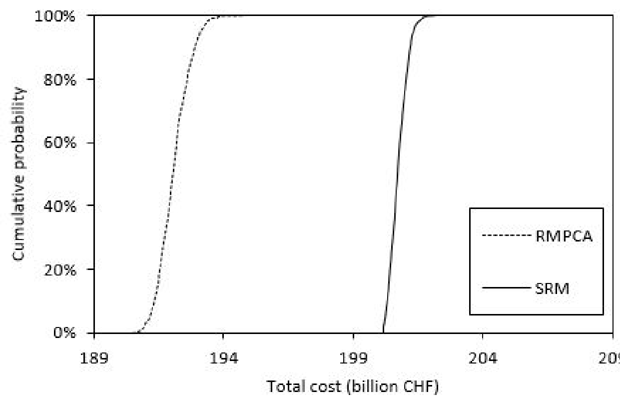


Figure 5: Average total cost of RMPCA, SRM and DET strategies for different box sizes and $\Gamma\% = 50\%$. (a) presents the average total cost over all 1000 random scenarios, whereas (b) presents the average total cost over only the scenarios without capacity shortage.



(a)



(b)

Figure 6: Comparison of the cumulative distribution functions of simulated total cost associated with RMPCA, SRM and DET policies when $\Gamma\% = 50\%$ and $\beta = 0.6$. (a) presents the distribution for all 1000 random scenarios, while (b) presents the distribution for the scenarios without capacity shortage.

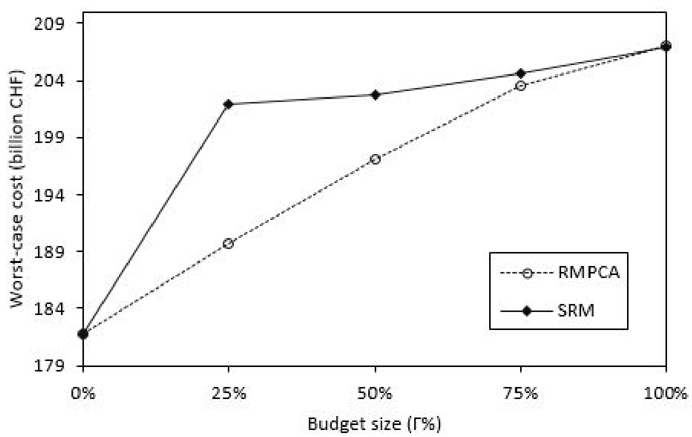


Figure 7: Worst-case performance of RMPCA and SRM as a function of budget size (for $\beta = 0.6$)

SRM strategies for different budget sizes. As this figure shows, RMPCA performs up to 6% better than SRM which is equivalent to approximately 12 billion CHF over the entire planning horizon.

Figure 8 provides finally an analysis on the trade-off between rate of capacity shortage and the average total cost for RMPCA and SRM strategies when the budget size varies from 10% to 100% and the box size $\beta = 0.6$. As it is also shown in Table 1, for budget sizes greater than 30%, the rate of capacity shortage for SRM is 0%. However, this zero-shortage comes at the cost of building too much capacity (see Figure 9, below). As Figure 8 suggests, allowing a certain degree of shortage can reduce the total cost drastically. For example, for $\Gamma\% = 60\%$ allowing a 2% chance of capacity shortage reduces the average total cost by around 7 billion CHF (equivalent to 4% of the total system cost).

6.3 Electricity Generation Structure

In this section, we discuss how the different modeling schemes impact the evolution over time of electricity generation. Figure 9 presents first the total installed capacity for the different strategies. SRM is the most conservative approach, as it yields more capacity than RMPCA and DET (around

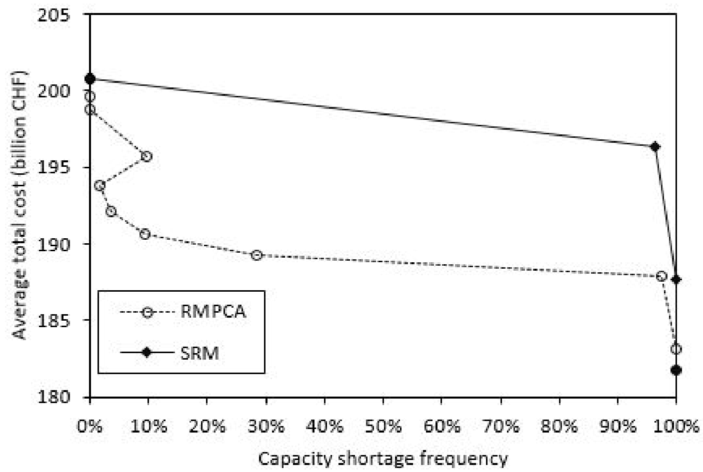


Figure 8: Trade-off between capacity shortage frequency and average total cost of RMPCA and SRM for different budget sizes and $\beta = 0.6$.

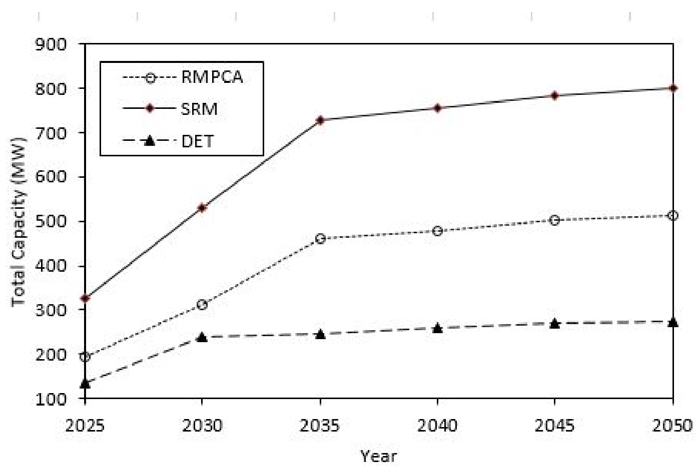


Figure 9: Total installed capacity for RMPCA, SRM and DET.

1.5 times more than RMPCA, and 3 times more than DET). It is indeed reasonable that RMPCA is less conservative than SRM, as it enjoys the flexibility of the energy procurement decisions to adjust for the actual (observed) demand response.

Table 2 presents next the installed capacity by type of technology for the RMPCA, SRM and DET strategies. It highlights the importance of wind power to decarbonize the electricity generation sector. In addition, Table 3 reports on the average annual electricity generation by different technology types for RMPCA, SRM, and DET policies.

6.4 Computational Analysis

In this section, we numerically evaluate the performance of the decomposition algorithms presented in Section 5. Specifically, we investigate how considering PRO and valid inequalities can improve the solution time and the number of iterations of Algorithm 1. In order for our results to be more general, we have designed smaller versions of *ETEM* with 16 technologies and 9 energy commodities, in which parameters are randomly assigned. Moreover, the size of the model changes with different

Table 2: Total installed capacity (in MW) by type of technology for RMPCA, SRM and DET models.

	2025			2030			2035			2040			2045			2050		
	RMPCA	SRM	DET	RMPCA	SRM	DET	RMPCA	SRM	DET	RMPCA	SRM	DET	RMPCA	SRM	DET	RMPCA	SRM	DET
Hydro-PP	21.3	21.3	18.9	22.4	22.4	22.4	22.4	22.4	22.4	22.6	22.6	22.6	22.8	22.8	22.8	23	23	23
PV-PP	0.2	0.2	0.2	0.2	0.2	0.2	0.8	0.8	0.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8	1.8
Wind-PP	168.4	297.6	110.8	282.5	504.1	211.3	431.1	697.3	216.5	446.2	721.9	230.0	471.2	750.1	239.1	480.9	767.2	240.7
CHP	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7	4.7
Other[1]	0.5	0.5	0.5	1.2	0.5	0.5	1.2	3.1	0.5	1.2	3.1	0.5	1.2	3.1	0.5	3.1	3.1	3.1

[1] Other technologies include geothermal,fuel-cell and municipal-waste power plants.

Table 3: Average annual electricity generation (in GWh) by type of technology for RMPCA, SRM and DET models over 1000 random scenarios.

	2025-2029			2030-2034			2035-2039			2040-2044			2045-2049			2050-2055		
	RMPCA	SRM	DET	RMPCA	SRM	DET	RMPCA	SRM	DET	RMPCA	SRM	DET	RMPCA	SRM	DET	RMPCA	SRM	DET
Hydro-PP	2010	1943.6	1642.8	2210.6	2086.8	2306	2153.4	2088	2146.1	2173.3	2108.7	2202.4	2188.5	2129.7	2244.5	2214.1	2152	2210.8
PV-PP	4.9	4.9	4.9	3.7	3.7	3.7	16.7	16.7	16.7	38.3	38.3	38.3	38.3	38.3	38.3	38.3	38.3	38.3
Wind-PP	9238	16323	6076	9506	21654	5744	14495	22233	6033	15233	23583	6537	16602	24892	7038	17137	25749	7124
CHP	12.5	0.5	40.3	10.2	0.0	39.4	1.6	0	15.8	1.6	0	15.8	1.5	0	16.6	1.5	0	16.6
Other[1]	16.2	0.7	32.5	13.8	0.0	29	2.4	0	0	2.3	0	0	2.2	0	6.8	2.3	0	8.9

[1] Other technologies include geothermal,fuel-cell and municipal-waste power plants.

problem horizons $\mathbb{T} \in \{3, 5, 7, 9\}$. Then, for each problem size, we generate 20 random instances and solve these random instances for different budget sizes $\Gamma\% \in \{20\%, 40\%, 60\%, 80\%\}$. Table 4 reports the average solution time (ST) and number of iterations (It) for each value of $\mathbb{T}$. In this table, BD refers to the traditional Benders decomposition, i.e., Algorithm 1. PRO-BD refers to the algorithm with improved upper bound based on PRO solutions while PRO-BD-VI additionally exploits valid inequalities. It is worth mentioning that based on our numerical experiments, restricting Ω to the last 4 worst-case scenarios led to the best solution time. Results show that PRO-BD improves the solution time of the BD by up to 20%, whereas PRO-BD-VI is able to improve it by up to 45%. On average, PRO-BD improves the solution time by 16% and PRO-BD-VI improves by 31%. Finally, a considerable improvement in terms of the number of iterations is observed when comparing PRO-BD-VI and BD.

7 Concluding Remarks

In this paper, we model demand response uncertainty in an adjustable multi-period robust generation capacity expansion problem. In a first stage, the planner decides on capacity expansion, as well as, planned demand response. Then, in sequence of seasonal stages, the actual DR is revealed, and accordingly, the optimal procurement decisions are taken. As the resulting adjustable model is

Table 4: Solution time (ST), in seconds, number of iterations (It), percentage of improvement (% impr.) for different versions of the decomposition algorithms. Four problem sizes are considered, for which the number of variables (VAR) and constraints (CON) are reported.

$\mathbb{T}$	DET Problem size		RMPCA Problem size		BD		PRO-BD			PRO-BD-VI		
	VAR	CON	VAR	CON	ST	It	ST	It	% impr.	ST	It	% impr.
3	4,644	11,830	97,112	103,095	47.0	15	37.5	10	20%	26.0	4	45%
5	7,740	19,718	161,860	171,837	92.8	15	78.1	11	16%	68.9	5	26%
7	10,836	27,601	226,583	240,534	78.0	10	68.0	7	13%	48.0	3	39%
9	13,932	35,486	291,316	309,249	97.0	9	82.1	6	15%	81.3	3	16%
Average					78.7	12	66.4	8	16%	56.0	4	31%

intractable, we use a conservative approximation scheme based on affine decision rules to solve the problem. We present a general formulation of the problem, where with slight modifications of the formulation one could accommodate other sources of uncertainty such as the level of production one can achieve from intermittent resources. We test our algorithm on a real-world case study based on the energy system of the Arc Lémanique region (Cantons of Geneva and Vaud, in Switzerland). Numerical results reinforce the importance of adjustable robust formulation. Namely, RMPCA can reduce the average total cost of the system by 33% compared to the solution of the deterministic problem, while keeping the chances of shortage near 4%. RMPCA also performs 4% better than a more naive static robust counterpart (SRM). Finally, we develop a Bender's type of decomposition to solve our large-scale RMPCA problem. In this algorithm, the speed of convergence is improved through i) adding valid inequalities to the master problem, and ii) identifying PRO solutions of the master problem. One possible direction for future work is linking the robust ETEM with a collective behavior demand model in order to more precisely adjust the size of the specific uncertainty sets of each period.

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